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A Positive Theory of Fixed-Rate Funds-Supplying Operations in an Accommodative Financial Environment

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A Positive Theory of Fixed-Rate Funds-Supplying Operations in an Accommodative Financial Environment

Junnosuke Shino *

Abstract

This paper studies bidding behaviors in fixed-rate funds-supplying auctions using a simple game-theoretic model. While the existing literature argues that such auction schemes are vulnerable to the overbidding problem, the bid-to-cover ratio for the Bank of Japan's current fixed-rate operations has remained stable. We modify the stylized *repo game* by incorporating the current framework of fixed-rate funds-supplying auctions operated by the Bank of Japan and the accommodative financial environment recently experienced in Japan. It is shown that any stable bid-to-cover ratios other than either undersubscription or overbidding can be supported by an equilibrium in the modified game.

JEL classifications: C72, E58

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1 Introduction

Fixed-Rate Funds-Supplying Operations by the Bank of Japan [1]

Following the turmoil in international financial markets triggered by the "Dubai shock " in November 2009, US and European stock prices plummeted. In response to this, the yen exchange rate appreciated against the currencies of the major economies. Given that these international financial developments might adversely affect economic activity, the Bank of Japan (BOJ) called an unscheduled Monetary Policy Meeting (MPM) on December 1, 2009. In addition to funds-supplying operations against pooled collateral [2] conducted through conventional variable-rate auctions, the BOJ introduced, at the meeting, a *fixed-rate funds-supplying operation against pooled collateral* (hereafter the fixed-rate operation) to further ease monetary conditions [3].

The fixed-rate operations take the form of loans, and the loan rates are presumably fixed at the Bank's target for the uncollateralized overnight call rate stipulated in the guidelines for money market operations. Given the loan rate, the total amount of funds supplied is announced at each auction by the Bank in advance. Therefore, the rate and the amount applied to each auction are "common knowledge " to bidders. Provided with this information, every participant simply bids the amount of money they wish to obtain at that rate. If the sum of all bids is smaller than the total allotment preannounced by the Bank (the case of "undersubscription"), then each bidder gets the amount of money it bids. Otherwise, the allotment is proportionally allocated depending on their bids.

Starting from December 10, 2009, the BOJ offered 800 billion yen per operation with

[1] This subsection is based on Bank of Japan [3].

[2] Funds-supplying operations against pooled collateral are operations in which the BOJ extends loans to its counterparties. These loans are backed by pooled collateral that counterparties have submitted to the Bank. These operations are highly convenient for counterparties because a wide range of assets, including government bonds, other public liabilities, and corporate debt such as corporate bonds and CP are eligible and counterparties can easily make substitutions between collateral. Pooled collateral refers to collateral that counterparties submit to the Bank based on agreements pertaining to transactions with the Bank, such as funds-supplying operations against pooled collateral, complementary lending facilities, and intraday overdrafts, and other contracts.

[3] Funds-supplying operations against pooled collateral are of two types: operations conducted at all offices of the Bank, and operations conducted at the Head Office. Whereas conventional variable-rate operations have been conducted at all offices of the Bank and at the Head Office, fixed-rate funds-supplying operations against pooled collateral have been conducted at all offices of the Bank in which a wider range of counterparties can participate.

a term of three months, and conducted the auction about once a week until the middle of March 2010. The amount outstanding in these operations reached around 10 trillion yen at the end of February 2010. At the MPM held on March 16 and 17, 2010, given that the amount of outstanding funds provided by special funds-supplying operations to facilitate corporate financing [4] would gradually decline from April 2010 onward, the BOJ decided to expand the measure to encourage a decline in longer-term interest rates by substantially increasing the amount of funds to be provided through the fixed-rate operations. Following the decision, from March 23, 2010 onward, the BOJ increased the frequency of the fixed-rate operations to twice a week. The amount outstanding of these operations reached around 20 trillion yen by June 2010. Furthermore, at an unscheduled MPM on August 30, 2010, the BOJ introduced a six-month term in the fixed-rate operations to encourage a decline in market interest rates and further ease monetary policy. The Bank started providing additional funding of approximately 10 trillion yen with a six-month term, while maintaining the outstanding amount of funds provided by the existing three-month term operations at 20 trillion yen.

Past Experiences and Existing Studies on Fixed-Rate Funds-Supplying Operations

As noted above, fixed-rate operations are one of the BOJ's main funds-supplying measures. It is not, however, the first time that this type of operation was adopted as a central bank's funds-supplying tool: until June 21, 2000, the European Central Bank (ECB)'s main refinancing operations had been conducted as fixed-rate tenders, where the lending rate was preannounced and banks simply indicated how much refinancing they would like to receive at that rate. Typically, total bids exceeded the allotment of the operations, and banks were rationed in proportion to their bids. In these operations, it is remarkable that severe *overbidding*, a phenomenon in which total bids drastically exceed the total allotment offered by a central bank, had been observed. As a result, the *allotment ratio*,

[4] Special funds-supplying operations to facilitate corporate financing were adopted as a temporary measure at the MPM held in December 2008. These are operations in which unlimited amounts of funds are supplied against the value of corporate debt pledged as pooled eligible collateral at an interest rate equivalent to the target for the uncollateralized overnight call rate.

the ratio of the allotment of funds provided to the total amount of bids (the reciprocal of the *bid-to-cover ratio*), had approached to zero (Fig.1) [5].

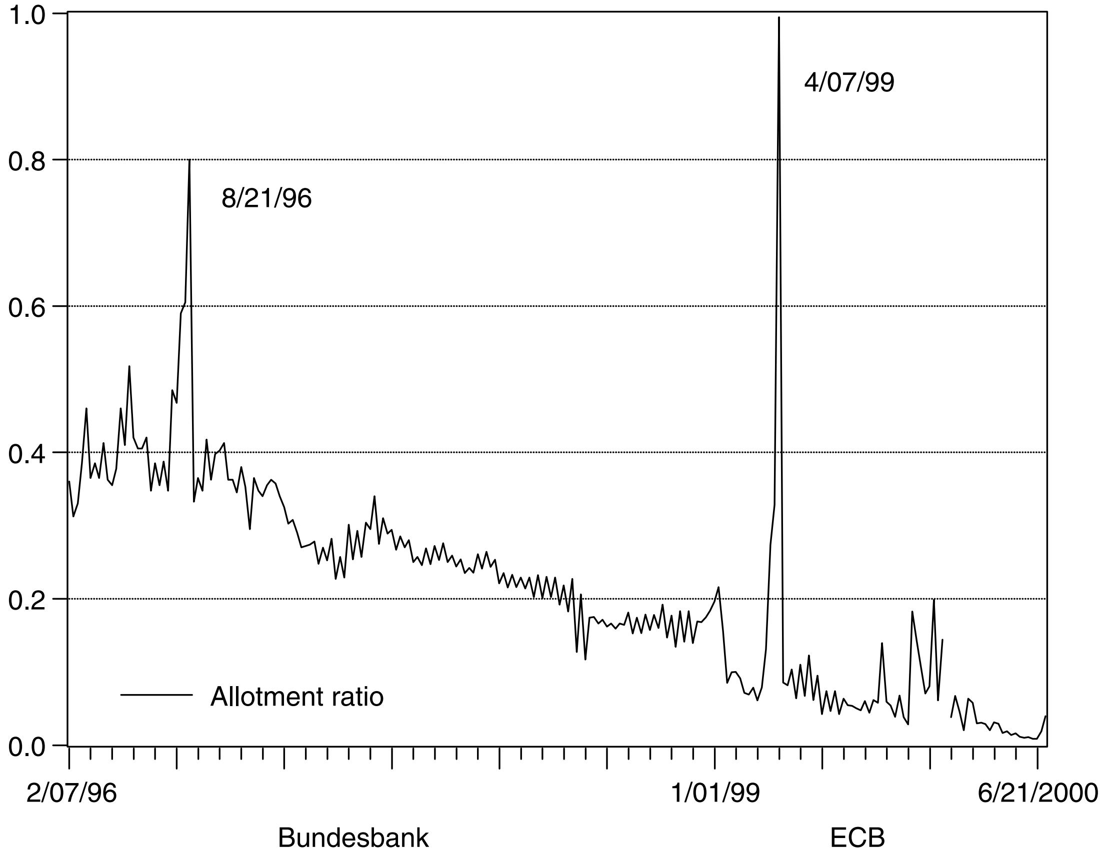

Figure 1: Allotment Ratios in ECB's Fixed-Rate Operations (Nautz and Oechssler [9])

A possible explanation for this phenomenon is that heightened expectations for an interest rate hike had enhanced the attractiveness of funding through fixed-rate operations, leading to an increase in the demand for the operations, as advanced by the ECB (see European Central Bank [6]). However, according to Nautz and Oechssler [9], such expectations did not prevail for the entire period during which serious overbidding was observed. Therefore, interest rate expectations are only part of the story. In particular, they cannot explain why the bid-to-cover ratio increases over time.

Based on this observation, Nautz and Oechssler [9] introduces a stylized game among bidders, called the *repo game*, and explains the overbidding phenomenon in terms of bidders' *myopic best-response process*. Their analysis invoked subsequent research

[5] More specifically, the bid-to-cover ratio had reached a maximum of more than 100.

about fixed-rate tender operations, as seen in Ayuso and Repullo [1] and Nautz and Oechssler [10].

We will review and modify their repo game later. Here it suffices to point out that in the fixed-rate funds auction the problem of overbidding looks inevitable. Nevertheless, returning to current BOJ's operations, an intriguing observation is that the bid-to-cover ratio for the BOJ's fixed-rate operations has remained stable in general, albeit with some fluctuations due to changes in expectations about monetary policy, and has not shown rapid changes such as surges observed in the ECB's operations or plunges leading to undersubscription. The fact that only overbidding or undersubscription can be described in Nautz and Oechssler's [10] framework implies that such a stable evolution of the ratio cannot be addressed by the exiting literature. This might be because the assumptions or construction of the game do not fit the BOJ's current framework of fixed-rate operations or the financial environment experienced in Japan.

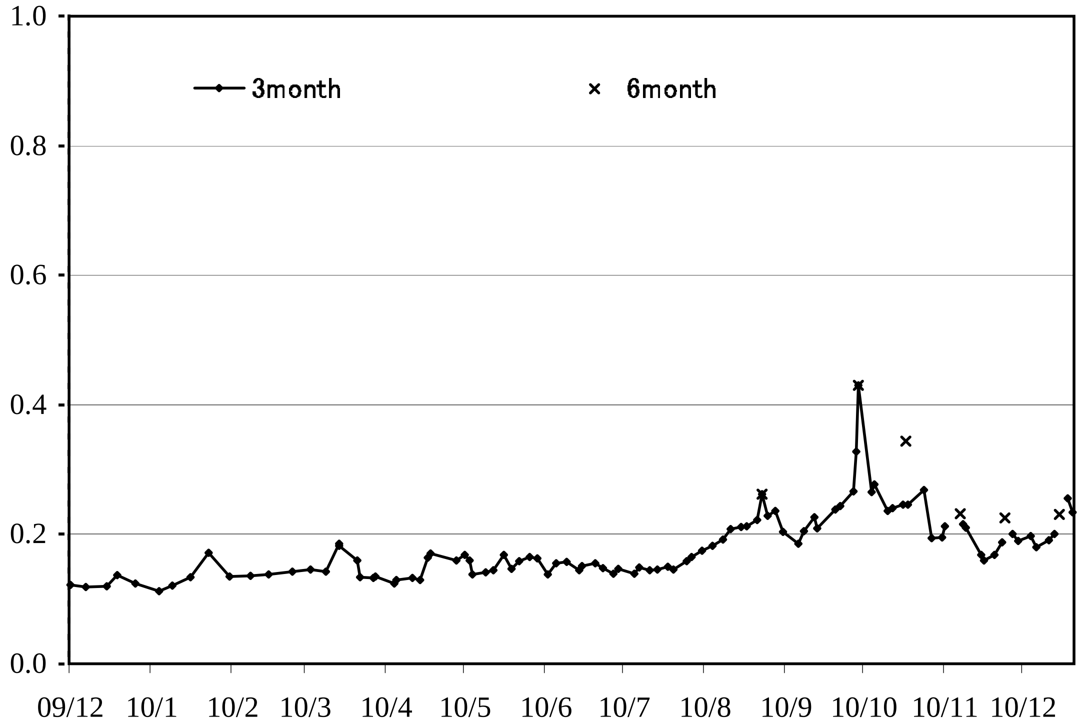

Figure 2: Allotment Ratios in Bank of Japan's Fixed-Rate Operations

In the next section, we briefly review the repo game and the mechanism of overbidding under the fixed-rate operations. In Section 3, we then modify the repo game by incorporating the current framework of fixed-rate funds-supplying auctions operated by the Bank of Japan and the accommodative financial environment recently experienced in Japan, and show that any stable bid-to-cover ratios other than either undersubscription or overbidding can be supported by an equilibrium. Some discussions and concluding remarks are made in Section 4.

2 Vulnerability of Fixed-Rate Operations to the Overbidding Problem: An Analysis using the Repo Game

In this section we review the repo game of Nautz and Oechssler [9] and the overbidding mechanism under fixed-rate operations [6].

Suppose that there are n financial institutions (typically banks). Let $\{1, 2, ..., n\} \equiv N$ be the set of the institutions and each $i \in N$ has exogenous liquidity needs l_i. An institution or bidder in fixed-rate operations may participate in the central bank's auction by demanding an amount d_i at a fixed interest rate, or it can borrow (or lend) an amount m_i on the interbank cash market (lending is denoted by a negative m_i) such that $l_i = m_i + d_i$. The decision as to where to obtain funds may depend on a number of factors, e.g., on the expected spread between the operation rate and the interbank rate and on preferences or the size of the bank, etc., and is beyond the scope of this analysis. It should only be noted that market clearing condition requires $\sum_{i \in N} m_i = 0$ thus the sum of banks' demands, $d \equiv \sum_{i \in N} d_i = \sum_{i \in N} l_i$, is an exogenous variable.

Let bank i's bid in the auctions be denoted by b_i. The game proceeds as follows:

1. Given a total allotment of a and a fixed lending rate r which are preannounced by the central bank, each bidder i chooses b_i [7].
2. The *allotment ratio* is determined. Letting q be the ratio, $q \equiv Min\left\{\frac{a}{\sum_{j \in N} b_j}, 1\right\}$.

   We also let $\beta = \dfrac{\sum_{j \in N} b_j}{a}$ be a *bid-to-cover ratio.*

[6] This section follows Nautz and Oechssler [9].

[7] Under the BOJ's fixed-rate operations, the rates are presumably fixed at the BOJ's target for the uncollateralized overnight call rate stipulated in the guidelines for money market operations on the day of disbursement of the loan.

3. The actual allotment to bidder i is determined by:

$$a_i = qb_i. \tag{1}$$

That is, if the sum of all bids b exceeds the total allotment a, the actual allotment is determined through the rationing scheme.

We denote bidder i's strategy set by B_i and define $B \equiv \times_{j\in N} B_j$ and $B_{-i} \equiv \times_{j\in N\setminus i} B_j$. As bidders choose their actions simultaneously, there is a strategic uncertainty and we assume that bidders are risk averse. Specifically, we follow Nautz and Oechssler [9] and assume the following quadratic single-peaked loss function which bidder i minimizes:

$$\pi_i \equiv (a_i - d_i)^2 = \left(Min \left\{ \frac{a}{\sum_{j\in N} b_j}, 1 \right\} b_i - d_i \right)^2. \tag{2}$$

In particular, if $a \leq \sum_{j\in N} b_j$,

$$\pi_i \equiv (a_i - d_i)^2 = \left(\frac{a}{\sum_{j\in N} b_j} b_i - d_i \right)^2, \tag{3}$$

thus the loss attains its minimum value of zero when $b_i = \frac{d_i}{a-d_i} \sum_{j\in N\setminus i} b_j$.

Without loss of generality, we hereafter assume that the game is symmetric, that is, $d_j = d$ for every $j \in N$. Note that this assumption does not affect the main results or implications for the following analysis. The notations d and d_i are both used in the following analysis in accordance with the context.

With this setup, Nautz and Oechssler [9] shows the following.

Remark 2.1 (Nautz and Oechssler [9])

1. *If* $a \geq \sum_{j\in N} d_j$ $(\equiv nd)$, *then* $[b_i = d_i \ \forall i \in N]$ *is the Nash equilibrium.*

2. *If* $a < \sum_{j\in N} d_j$ $(\equiv nd)$, *no Nash equilibrium exists.*

The first case corresponds to undersubscription, while the second case cannot explain any actual bidding behavior in an equilibrium analysis. In order to describe the overbidding phenomenon actually observed under ECB operations, Nautz and Oechssler [9] introduces a simple *myopic best-reply process* (each bidder has adaptive expectations).

To see this, consider the (symmetric) two-bidder case ($N = \{1, 2\}$) and plot each bidder's best-response function on the (b_1, b_2) plane:

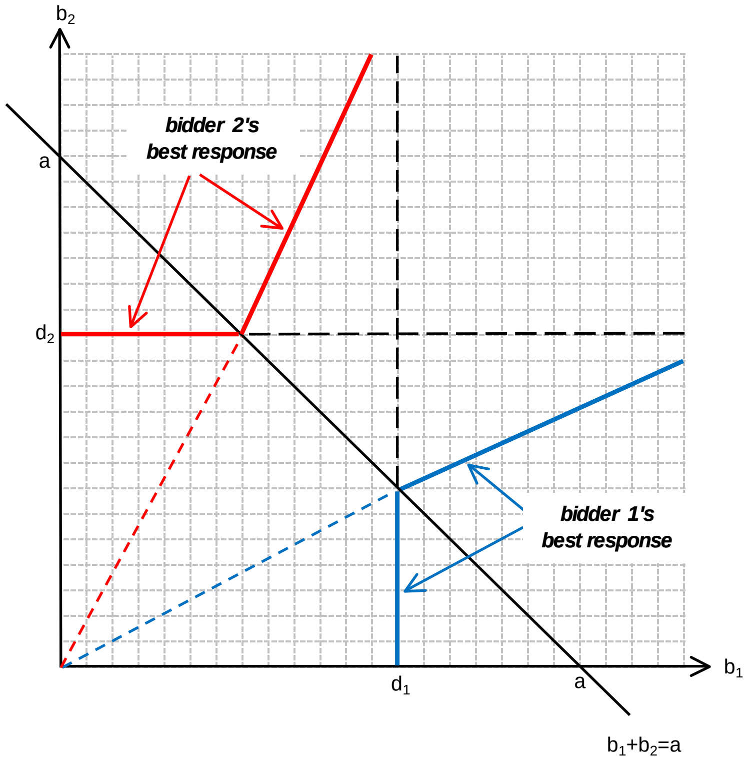

Figure 3: Best-Response Functions in the Repo Game ($d_1 + d_2 > a$)

The blue and red bold lines in Fig.3 are bidder 1 and 2's best-response functions, respectively. At any points on i's best-response function, i's loss attains its minimum value of zero, while any other points entail a strictly positive loss. As long as $b_1 + b_2 \leq a$, the allotment ratio is 1 thus $b_i = d_i$ gives the minimum loss to i. When $b_1 + b_2 > a$, on the other hand, i has to bid strictly more than d_i to attain the minimum loss ($a_i = d_i$). The fact that there is no intersection of the two lines means no Nash equilibrium exists [8].

Now suppose that the bidding profile at period t is $x = (x_1, x_2)$ (see Fig.4) and we assume a myopic best-reply process [9]. This process assumes that at $t + 1$ bidder i chooses the best response to bidder j's previous bid x_j. Thus the bidding profile at $t + 1$ would be y in Fig 4. Note that no matter how small the bidding profile x we select in period

[8]If, on the other hand, $d_1 + d_2 \leq a$, the red and blue lines intersect at $(b_1, b_2) = (d_1, d_2)$ located at the lower left of the $b_1 + b_2 = a$ line. This corresponds to the first part of Remark 2.1.

[9]Specifically, here we pick x such that bidder i's best response to x_j remains d_i in order to emphasize that even such a small bidding profile eventually results in overbidding.

t, The bidding profile $y = (y_1, y_2)$ in the next period always satisfies $y_1 + y_2 > a$ because $d_1 + d_2 > a$. Furthermore, once the sum of the bidding amount exceeds a, the process then explodes for sure. This is the overbidding mechanism that Nautz and Oechssler [9] clarifies.

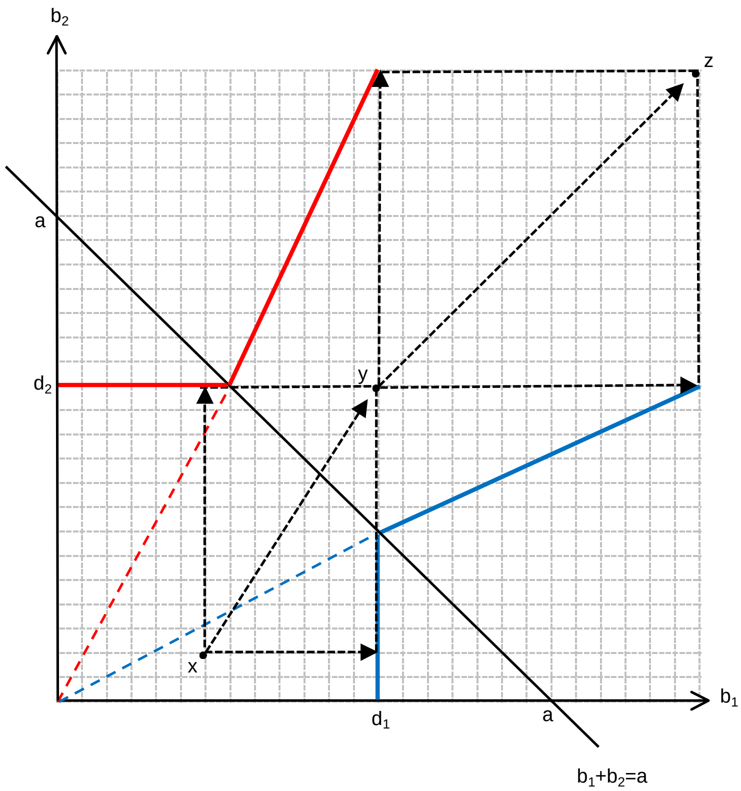

Figure 4: Myopic Best-Reply Process in the Repo Game

3 Modified Repo Game and Stable Bidding Rates

We reviewed in the previous section how the simple repo game proposed by Nautz and Oechssler [9] can describe overbidding (if $a < d_1 + d_2$), which was observed in the ECB fixed-rate auction, or undersubscription (if $a \geq d_1 + d_2$). Fig.2 shows, however, that the bid-to-cover ratio under the BOJ's fixed-rate operations has remained stable in general. This might be because the assumption or construction of the repo game does not fit the BOJ's operation framework or the current financial environment in Japan. In the following analysis, we modify the repo game so that these factors are taken into consideration. The first point that is not considered in the repo game is that the *maximum* *bidding limit* exists in the BOJ's fixed-rate operations. Since the introduction of the fixed rate funds-supplying operations in December 2009, the limit has been set at 200 billion yen per operation, independent of the size of the bidders and of the term of the operation. Letting the (common) limit be $\bar{b}$, we now redefine bidder i's strategy set as $B_i = [0, \bar{b}]$. Since the introduction of the fixed-rate operations, it has been pointed out by market participants that financial institutions bid relatively "large" amounts compared with their liquidity needs. This implies that bidder i can choose b_i satisfying $b_i > d_i$, and note that such behavior is possible only when $d_i < \bar{b}$. We impose this condition on $\bar{b}$.

With this setup, we first establish the following remark:

Remark 3.1

Suppose the common maximum bidding limit $\bar{b}$ exists and $B_i = [0, \bar{b}]$ for all $i \in N$. Then,

1. *If $a \geq \sum_{j \in N} d_j \ (\equiv nd)$, then $b_i = d_i \ \forall i \in N$ is the Nash equilibrium.*
2. *If $a < \sum_{j \in N} d_j \ (\equiv nd)$, $b_i = \bar{b} \ \forall i \in N$ is the Nash equilibrium.*

Proof. The first part is obvious. Suppose $a < \sum_{j \in N} d_j$ and consider i's best response to $b^*_{-i} = (\bar{b}, \ldots, \bar{b}) \in B_{-i}$. First, consider b_i with $b_i + (n-1)\bar{b} > a$. Then $\pi_i = \left(\frac{b_i}{(n-1)\bar{b} + b_i} a - d_i\right)^2$ holds, thus

$$\frac{\partial \pi_i}{\partial b_i} = 2\left(\frac{ab_i}{b_i + (n-1)\bar{b}} - d_i\right)\left(\frac{a\bar{b}(n-1)}{\{b_i + (n-1)\bar{b}\}^2}\right). \tag{4}$$

As the last term in (4) is strictly positive, π_i is strictly decreasing if and only if

$$\left(\frac{ab_i}{b_i + (n-1)\bar{b}} - d_i\right) < 0 \longleftrightarrow b_i < \frac{(n-1)d_i}{a - d_i}\bar{b}.$$

From the assumption $nd_i > a$, $\frac{(n-1)d_i}{a-d_i} > 1$, thus the loss function π_i is strictly decreasing on $[0, \bar{b}]$. As a result, for b^*_{-i}, $b_i = \bar{b}$ attains its minimum loss of $\left(\frac{a}{n} - d\right)^2$ when $b_i + (n-1)\bar{b} > a$ holds. To finish the argument, suppose that b_i with $b_i + (n-1)\bar{b} \leq a$. In this case, note that i's loss is $(b_i - d)^2$. We need to show $\left(\frac{a}{n} - d\right)^2 \leq (b_i - d)^2$ but note that $\frac{a}{n} < d$ and $b_i + (n-1)\bar{b} < d$ [10]. Therefore, it suffices to show that $b_i < \frac{a}{n}$, and this is true from the

[10]The latter holds from the following: $d - \left[b_i + (n-1)\bar{b}\right] = d - a + (n-1)\bar{b} \geq d - a + (n-1)d = nd - a > 0$. The first inequality comes from $d < \bar{b}$.

following:

$$
\begin{aligned}
\frac{a}{n} - b_i &\geq \frac{a}{n} - a + (n-1)\bar{b} \\
&= (n-1)\left(\bar{b} - \frac{a}{n}\right) \\
&> (n-1)\left(\bar{b} - d\right) \quad \left(\because a < nd\right) \\
&> 0.
\end{aligned}
$$

Q.E.D.

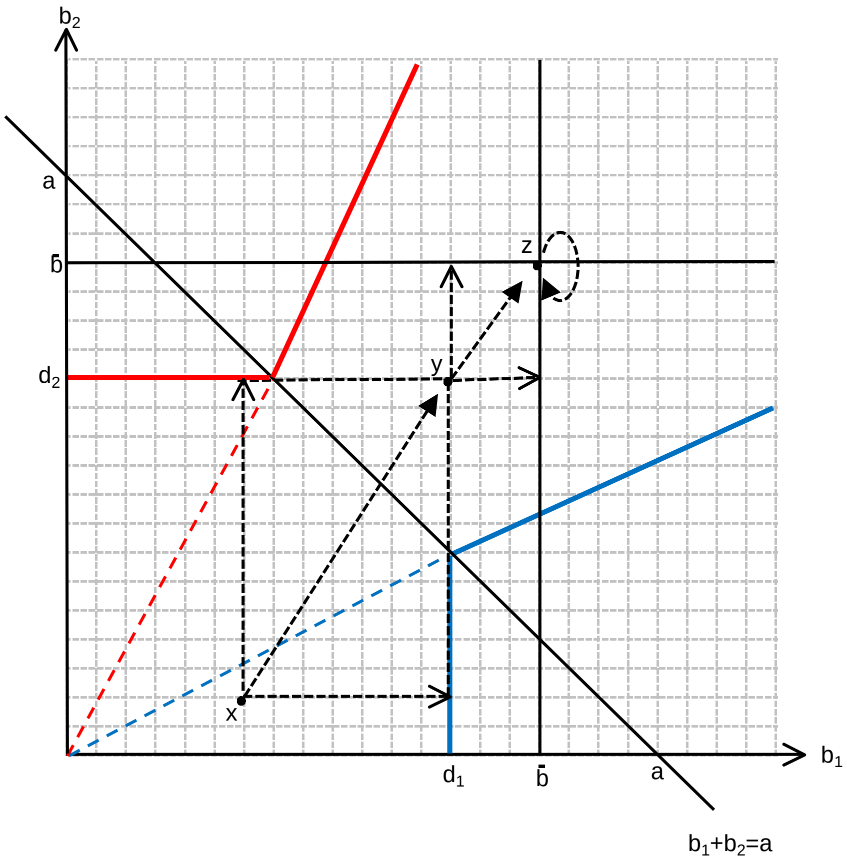

Figure 5: Myopic Best-Reply Process with Maximum Bidding Limit

The first part of Remark 3.1 corresponds to undersubscription and the second implies that all bidders choose "full bid." However, it is known that the actual bid-to-cover ratio under the BOJ's auctions is significantly lower than the case when all participants in the operation chose the maximum bidding limit. Put differently, consider the myopic best-reply process introduced in the previous section. Then the dynamics obviously suggest that the bid-to-cover ratio rapidly converges to $\frac{n\bar{b}}{a}$ (see Fig.5), which seems inconsistent with the observed ratio. These observations suggest that only introducing the maximum bidding limit into the existing repo game is not enough to describe the actual bid-to-cover ratio under the BOJ's fixed-rate operations.

As factors behind such developments, the *level of interest rates applied to the complementary deposit facility* and the *accommodative financial environment in Japan* can be identified.

At the MPM held on October 31, 2008, the BOJ decided to establish the complementary deposit facility with the aim of ensuring stability in financial markets through further facilitation of money market operations (see Bank of Japan [4]). The complementary deposit facility is a measure under which the Bank pays interest on excess reserve balances (balances held at the account with the Bank in excess of required reserves under the reserve deposit requirement system). The purpose of the facility is to increase the flexibility of the Bank's money market operations by enabling sufficient provision of liquidity. Furthermore, as the target rate was set equal to the rate applied to the complementary deposit facility, it also has the effect of reducing the opportunity cost of holding excess reserve balances to zero when the actual allotment is larger than the initial demand d_i . Indeed, an observation consistent with this view is that excess reserves have substantially increased and remained at high levels after the introduction of the facility (Fig.6).

The second factor behind the stable bid-to-cover ratio is the recent accommodative financial environment in Japan [11]. The three-month rates, whose duration is the same as for fixed-rate operations, have been pushed down with a backdrop of the persistent expectation of continued large-scale monetary easing. In particular, Fig.7 shows that these interest rates, as well as overnight rates, are approximately 0.1%, which is identical to the rate applied to the fixed-rate operations. Under such a situation, the external funding cost that is incurred when the actual allotment is smaller than the initial demand is sufficiently small.

[11] As for recent developments of Japanese financial markets, see, for example, Bank of Japan [2].

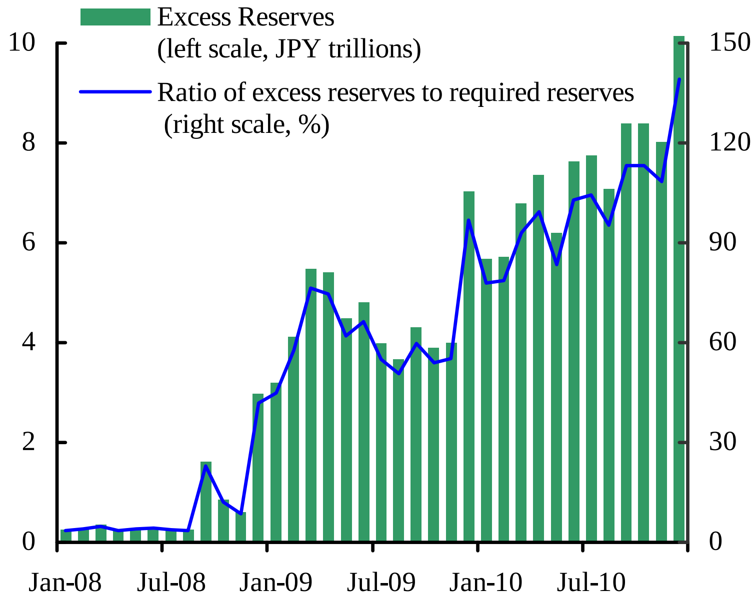

Figure 6: Excess Reserves of Institutions Subject to the Reserve Requirement System

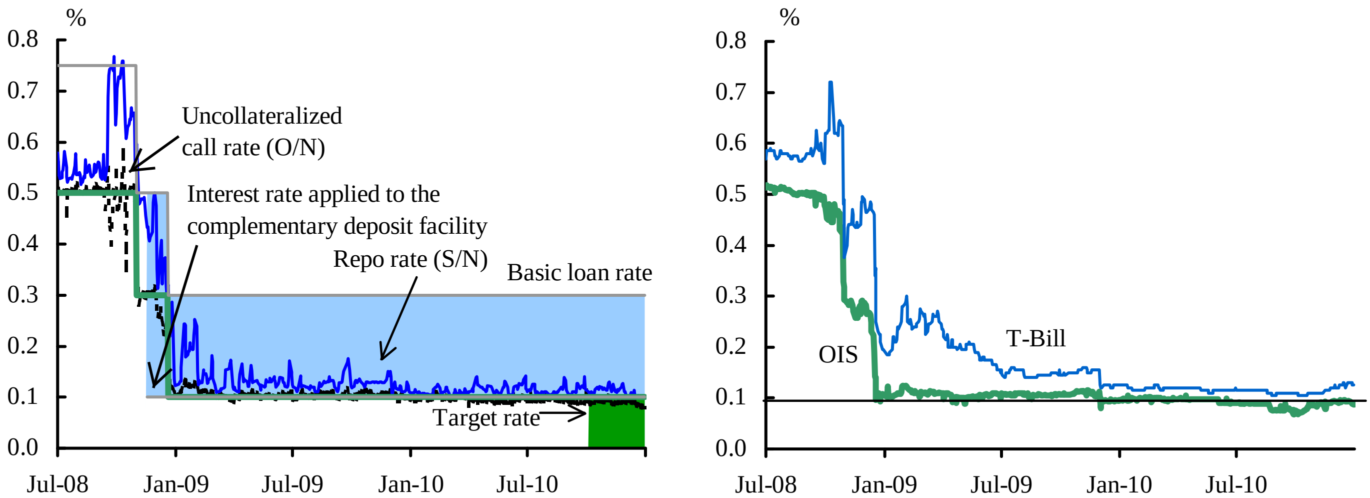

Figure 7: Overnight Rates (Left) and Three-Month Rates (Right)

On the other hand, if the difference between the actual allotment and the initial demand is substantially large, then some positive costs are incurred. For one thing, when the actual allotment is much smaller than the initial demand, the bidder needs to finance the difference at an interbank market, and such large funding may entail a premium. For another, under the BOJ's current fixed rate funds-supplying operations, a bidder's actual allotment (not necessarily the amount of the bid) has to be within the limit of the value of pooled collateral it holds. As the funds are offered on a T+2 basis, a bidder whose actual allotment exceeds its value of collateral can still cope with the constraint by purchasing financial assets eligible to be used as collateral, for example, treasury bill purchases after the auction, but before the actual funds provision two business days later. However, such purchasing behavior may also entail some costs.

In sum, these observations imply that as long as the difference between the actual allotment and the initial demand remains within a certain range, the cost is virtually zero. In particular, we assume that bidder i's loss function $\tilde{\pi}_i$ is *locally satiated* [12] around d_i and described below [13] (see also Fig.8):

$$\tilde{\pi}_i = \begin{cases} 0 & \text{if} \quad d_i - k \leq a_i \leq d_i + k \\ (a_i - d_i)^2 & \text{if} \quad a_i < d_i - k \text{ and } d_i + k < a_i \end{cases} \tag{6}$$

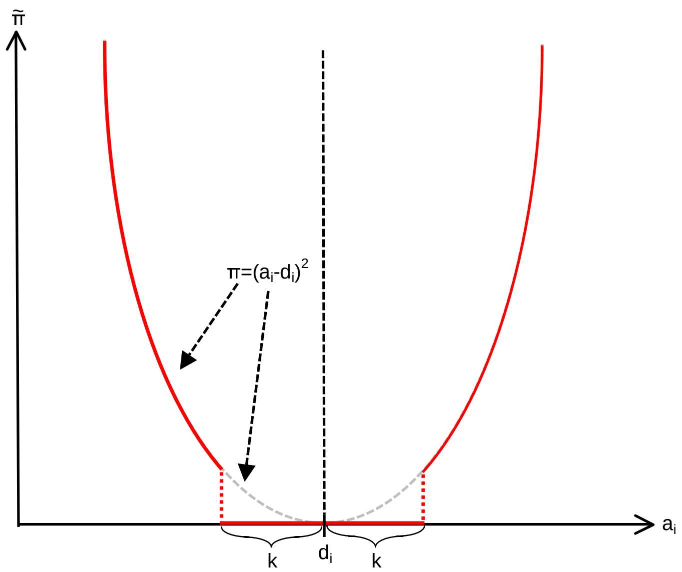

Figure 8: A Bidder's Locally Satiated Loss Function $\tilde{\pi}_i$

With the introduction of locally satiated loss functions and the upper bidding limit, we obtain the following result:

[12] As for a formal description of satiated preference relations, see, for example, Mas-Collel *et. al* [8].

[13] Here $\tilde{\pi}_i$ is assumed to be discontinuous. This construction is made only to make the analysis mathematically easy. If, for example, we alternatively define the following continuous satiated loss function $\hat{\pi}_i$, the results do not change:

$$\hat{\pi}_i = \begin{cases} 0 & \text{if} \quad d_i - k \leq a_i \leq d_i + k \\ \{a_i - (d_i - k)\}\{a_i - (d_i + k)\} & \text{if} \quad a_i < d_i - k \text{ and } d_i + k < a_i. \end{cases} \tag{5}$$

Theorem 3.1

Suppose $a < \sum_{j \in N} d_j$. Then there exists $\tilde{k}$ such that if $k \geq \tilde{k}$, then any bid-to-cover ratio that is neither overbidding nor undersubscription can be supported by a Nash equilibrium.

Proof. Suppose $d - \frac{a}{n} \leq \tilde{k}$ (note: we can actually pick such positive $\tilde{k}$) and pick a strategy profile in which all bidders choose b^* satisfying $b^* \geq \frac{a}{n}$. Note that $a_i = \frac{a}{n}$ and the following is true for k such that $k \geq \tilde{k}$:

$$d - k \leq \frac{a}{n} < d + k. \tag{7}$$

The former inequality comes from $d - \frac{a}{n} \leq \tilde{k}$ and the latter comes from $a < nd \Leftrightarrow \frac{a}{n} < d < d + k$. From (6), (7) implies that under the satiated loss function $\tilde{\pi}_i$, any symmetric bidding profile consisting of the b^* above is a Nash equilibrium. Taking the upper bidding limit $\bar{b}$ into account, we conclude that any bid-to-cover ratio x satisfying $1 \leq x \leq \frac{n\bar{b}}{a}$ can be supported by a symmetric equilibrium . **Q.E.D.**

Theorem 3.1 means that in the situation where a bidder's loss function is satiated, then the bid-to-cover ratio currently observed under BOJ's fixed-rate auction can be described as a Nash equilibrium. While this result suffices as a descriptive analysis of the auction, some additional remarks should be made. First, while we focused on a symmetric Nash equilibrium in the theorem, this constraint is not necessarily required in order to derive a stable bid-to-cover ratio. Next, in our setup, some of (but not all) bid-to-cover ratios associated with undersubscription can be supported by an equilibrium. To see this, consider a two bidders example that satisfies the following conditions: (as a more specific example, you may take $a = 18, d_1 = d_2 = 12, k = 4$ and $\bar{b} = 15$):

$$d_i - k > 0 \tag{8}$$

$$a > d_i + k \tag{9}$$

$$k < 2d_i - a \tag{10}$$

$$\bar{b} < d + k. \tag{11}$$

The third condition, meaning that k is smaller than a certain value, is imposed just to make the example graphically simple. Now consider the (b_1, b_2) plane and define lines

(a) to (d) as follows (see Fig.9) :

$$\begin{aligned} b_2 &= \frac{a-(d_1-k)}{(d_1-k)} b_1 \quad \text{[line (a)]} \\ b_2 &= \frac{a-(d_1+k)}{(d_1+k)} b_1 \quad \text{[line (b)]} \\ b_2 &= \frac{d_2-k}{a-(d_2-k)} b_1 \quad \text{[line (c)]} \\ b_2 &= \frac{d_2+k}{a-(d_2+k)} b_1 \quad \text{[line (d)]} \end{aligned}$$

Then it is easily checked that for bidder 2's strategy $\tilde{b}_2$, bidder 1's strategy $\tilde{b}_1$ satisfying (12) gives the minimum loss 0 to bidder 1, that is, best responses to $\tilde{b}_2$ (see Fig.10).

$$\max\left\{d_1-k, \frac{d_1-k}{a-(d_1-k)}\tilde{b}_2\right\} \le \tilde{b}_1 \le \bar{b}. \tag{12}$$

(12) simply implies that $(\tilde{b}_1, \tilde{b}_2)$ is located between lines (a) and (b) and $\tilde{b}_1$ is lower than $\bar{b}$.

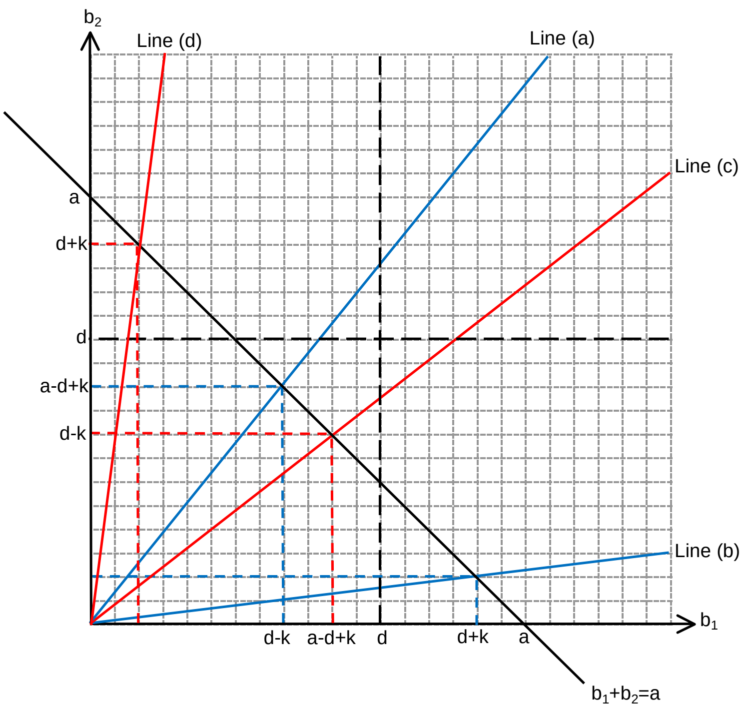

Figure 9: Line (a) to (d) on (b_1, b_2) plane

Noting that the exact same argument holds for bidder 2, the set of Nash equilibria can be described as the shaded area in Fig.11, or more formally:

$$\left\{(b_1, b_2) \in B \,\middle|\, \frac{d_2-k}{a-(d_2-k)} b_1 \le b_2 \le \frac{a-(d_1-k)}{(d_1-k)} b_1, d_1-k \le b_1 \le \bar{b}, d_2-k \le b_2 \le \bar{b}\right\}. \tag{13}$$

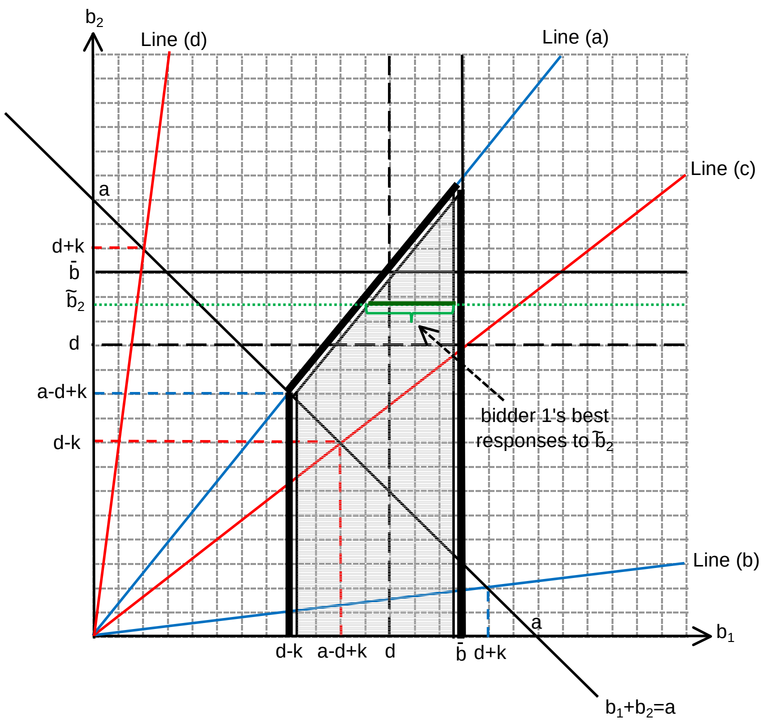

Figure 10: Bidder 1's best responses under $\tilde{\pi}_1$

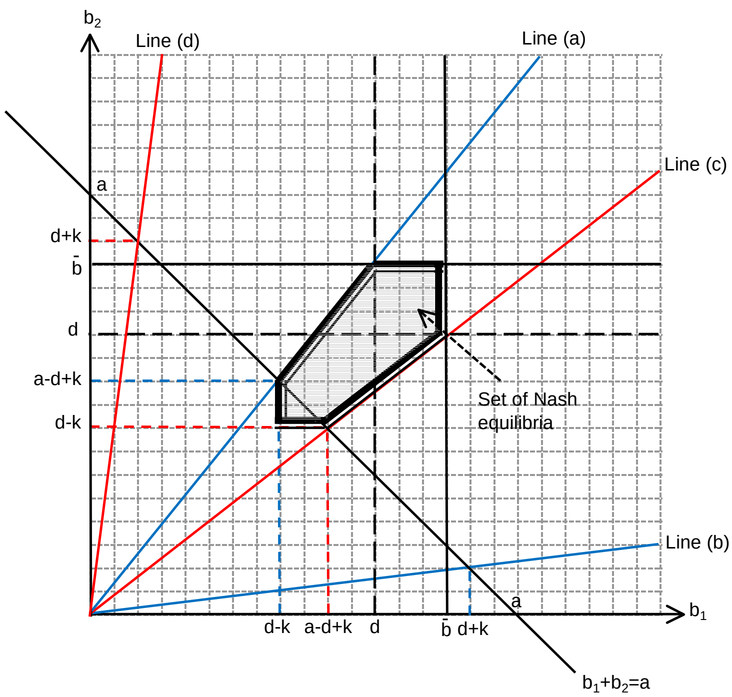

Figure 11: The Set of Nash Equilibria Under $\tilde{\pi}$

This example shows the following properties. First, any bid-to-cover ratios satisfying $1 \leq x < \frac{2\bar{b}}{a}$, which are neither undersubscription nor overbidding, are supported by a

Nash equilibrium, *including asymmetric ones*. Next, some undersubscriptions, satisfying $\frac{2(d-k)}{a} \leq x < 1$, are also supported by some equilibrium. However, as the demand for an auction d increases relative to a, such an underscription becomes unlikely to occur.

4 Discussion and Conclusion

Under normal circumstances, fixed-rate funds-supplying operations are vulnerable to the overbidding problem compared with, for example, variable-rate auctions, as pointed out by Nautz and Oechssler [9]. On the other hand, it has also been pointed out that this type of funds-supplying auction has also several advantages as a monetary policy tool. For example, the purpose of the ECB's introduction of fixed tender MRO is to convey effective policy signaling into the market (See Hartmann *et al* [7]). The BOJ introduced the scheme with the purpose of enhancing easy monetary conditions further. As we studied, under the situation where the financial environment is accommodative and the level of interest rates applied to the complementary deposit facility are appropriately set, fixed-rate operations are less subjective to the overbidding problem. Thus it can be concluded that the BOJ's current fixed-rate operations are more likely to enjoy these policy merits, rather than suffering from a severe overbidding problem.

However, it should be noted that such stable evolution of the bid-to-cover ratio requires some prerequisite conditions as we examined and fixed-rate operations cannot fundamentally avoid the overbidding problem by itself. Specifically, the modified repo game constructed in our study suggests that a change in views regarding future interest rates could affect the bid-to-cover ratio through *two different channels*. The first channel could work without making any changes in a bidder's loss function, as discussed in ECB [6]. For example, heightened expectations of an interest rate hike could enhance the attractiveness of funding through fixed-rate operations, leading to an increase in the demand for the operation. In addition to this mechanism, in this paper we identified another channel: as expectations of an interest rate hike heighten, the satiated range in the loss function (k in Fig.8) could narrow. In this case, at the point when the condition on Theorem 3.1 would not be satisfied, the ratio would suddenly explode.

We conclude the analysis by identifying several questions for further research. First, our results support not only the actual bid-to-cover ratio currently observed in Japan, which is neither undersubscription nor overbidding, but also certain underscriptions. While the result may be sufficient in terms of positive theory to explain the actual ratio in an equilibrium, introducing some dynamics regarding the bidder's behavior might exclude an equilibrium consistent with undersubscription. Second, we can drop the assumption of symmetry in our model. Such cases where the desired allotment d_i varies by bidders or some of bidders' loss functions are strictly concave could be intriguing extensions. Third, in this study we assume that the game structure is common knowledge among bidders but any type of uncertainty can be introduced. Finally, possible generalizations of our analysis should integrate this type of allocation mechanism with the standard mechanism design theory [14].

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