



Yale SCHOOL OF MANAGEMENT  
*Program on Financial Stability*

## EliScholar – A Digital Platform for Scholarly Publishing at Yale

---

YPFS Resource Library

---

5-8-2013

### Subsidizing Price Discovery

Braz Camargo

Kyungmin Kim

Benjamin R. Lester

<https://elischolar.library.yale.edu/ypfs-documents/8060>

---

This resource is brought to you for free and open access by the Yale Program on Financial Stability and [EliScholar](#), a digital platform for scholarly publishing provided by Yale University Library. For more information, please contact [ypfs@yale.edu](mailto:ypfs@yale.edu).



# WORKING PAPERS

RESEARCH DEPARTMENT

**WORKING PAPER NO. 13-20  
SUBSIDIZING PRICE DISCOVERY**

Braz Camargo  
Sao Paulo School of Economics – FGV

Kyungmin (Teddy) Kim  
University of Iowa

Benjamin Lester  
Federal Reserve Bank of Philadelphia

May 8, 2013

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA

Ten Independence Mall, Philadelphia, PA 19106-1574 • [www.philadelphiafed.org/research-and-data/](http://www.philadelphiafed.org/research-and-data/)

# Subsidizing Price Discovery\*

Braz Camargo  
Sao Paulo School of Economics - FGV

Kyungmin (Teddy) Kim  
University of Iowa

Benjamin Lester<sup>†</sup>  
Federal Reserve Bank of Philadelphia

May 8, 2013

## Abstract

When markets freeze, not only are gains from trade left unrealized, but the process of information production through prices, or *price discovery*, is disrupted as well. Though this latter effect has received much less attention than the former, it constitutes an important source of inefficiency during times of crisis. We provide a formal model of price discovery and use it to study a government program designed explicitly to restore the process of information production in frozen markets. This program, which provided buyers with partial insurance against acquiring low-quality assets, reveals a fundamental trade-off for policymakers: while some insurance encourages buyers to bid for assets when they otherwise would not, thus promoting price discovery, too much insurance erodes the informational content of these bids, which hurts price discovery.

---

\*We thank Philip Bond for an excellent discussion of this paper, along with Viral Acharya, Roc Armenter, Hal Cole, Hanming Fang, Itay Goldstein, Andrew Postlewaite, and Xianwen Shi for helpful comments. All errors are our own.

<sup>†</sup>The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

*“The complete evaporation of liquidity in certain market segments of the US securitisation market has made it impossible to value certain assets fairly.”* — Official Statement of BNP Paribas, after stopping withdrawals from three investment funds on August 9, 2007.

*“For some of the securities there are just no prices [...] As there are no prices, we can’t calculate the value of the funds.”* — Alain Papiasse, Head of Asset Management, BNP Paribas.

## 1 Introduction

It is well known that informational asymmetries between buyers and sellers can disrupt the process of trade. This disruption leads to two types of welfare losses. First, gains from trade may not be realized; this is a standard result from the literature on adverse selection. Second, and equally important, the information contained in prices can be valuable to other market participants.<sup>1</sup> Hence, when markets “freeze,” an important by-product is that the process of information generation through prices, or *price discovery*, is disrupted.

A natural question is whether government intervention in markets suffering from adverse selection can help prevent the losses associated with market freezes and, if so, what form this intervention should take. Answering this question has become particularly important in light of the recent financial crisis, where informational asymmetries were believed to play a central role in the collapse of trade observed in several key financial markets. However, most of the literature that has emerged to study policy interventions in markets with adverse selection has focused exclusively on the ability of various government programs to prevent the first type of loss discussed above—unrealized gains from trade—while ignoring the effect of these programs on the amount of information being produced.

---

<sup>1</sup>There are many reasons why information contained in asset prices can be valuable to other agents. To name a few, information about a particular type of asset can (i) reduce informational asymmetries in markets for similar (or even identical) assets, thereby helping other agents realize gains from trade; (ii) reduce uncertainty surrounding the balance sheets of distressed financial institutions that own similar assets, thereby allowing existing depositors, new investors, or even government regulators to make better decisions regarding whether to withdraw funds from, lend to, or “bail out” these institutions, respectively; or (iii) better inform agents about the fundamentals of the economy, thereby providing better guidance for real investment decisions. We discuss these explanations, and others, in Section 4.

The goal of this paper is to explore the extent to which government intervention can promote (or hinder) the process of price discovery in frozen markets. To achieve this goal, we first construct a model in which adverse selection is an impediment to trade, and hence diminishes the amount of information produced. We then use this model to study the efficacy of a government program that was introduced during the recent financial crisis with an explicit objective of restoring the process of price discovery in certain financial markets. We find that a fundamental tension emerges for policymakers: while government support can encourage buyers to bid for assets, thereby “unfreezing” the market and ensuring that some transactions occur, too much government support can cause the informational content of these transaction prices to deteriorate. More generally, this tension implies that, at some point, a policy that increases gains from trade will also reduce price discovery. Hence, a policy that maximizes the gains from trade in a particular frozen market will typically not maximize total welfare when information is valuable to other market participants.

The specific program we study is the so-called Public-Private Investment Program for Legacy Assets, or PPIP, which was introduced in March 2009 with the primary goal of restoring “private sector price discovery” in the market for real estate loans and assets backed by these loans.<sup>2</sup> We focus on PPIP for two reasons. First, this is the only program (of many) implemented during the recent financial crisis with an explicitly stated intention of promoting price discovery. Therefore, from a positive point of view, PPIP is the most natural program to evaluate along this dimension.<sup>3</sup> Second, the crucial trade-off we identify by studying PPIP, described above, is likely to feature prominently in other government programs designed to deal with adverse selection. Hence, the insights that emerge from our framework should be applicable more generally.

The basic idea behind PPIP was for the government to partner with, and *subsidize*, private investors in order to buy legacy assets from distressed financial institutions. More specifically,

---

<sup>2</sup>The official statement of the program outlined three goals or “basic principles”: in addition to private sector price discovery, the program also hoped to “maximize the impact of each taxpayer dollar” and “share risk and profit with private sector participants.”

<sup>3</sup>An alternative approach would be to use mechanism design to identify properties of the optimal form of intervention. While this is an interesting exercise as well, it is accompanied by the usual question of whether the optimal policy is implementable given the various real-world constraints (political, institutional, and so on). Our approach, on the other hand, is attractive because it is both simple and relevant: as we describe below, our analysis offers guidance for policymakers regarding the choice of simple parameters within a program that has already been approved and implemented.

the government organized auctions for specific pools of assets being held on the balance sheets of distressed financial institutions, and invited a few private investors to participate in each auction. Crucially, the winner of an auction was required to finance only a fraction of the purchase price with his own equity, receiving a *nonrecourse* loan from the FDIC for the remaining amount.<sup>4</sup> Given the nature of nonrecourse loans, the government was essentially providing partial insurance to the investors against losses, which it hoped would encourage them to bid for the assets being auctioned off, thereby using “private-sector competition to determine market prices for currently illiquid assets.”<sup>5</sup>

Though this program was approved to help purchase up to five hundred billion dollars of assets, with the potential to expand to one trillion dollars over time, several fundamental questions remain unanswered. First, in a market frozen because of adverse selection, how does reducing an asset’s downside risk affect the willingness of buyers to participate, their bidding behavior, and subsequent transaction prices? Moreover, what is the information content of these prices? How does the information content of prices depend on the specifics of the policy, i.e., the fraction of equity that buyers are required to put up themselves and the number of buyers that are chosen to participate in the auction?

We construct a theoretical model that can be used to address all of these questions. Section 2 lays out the basic environment. There is a seller who possesses a single asset, and has private information about the quality of this asset. There are  $N \geq 2$  buyers, each of whom can acquire a noisy signal about the quality of the asset at some cost. Buyers decide whether or not to acquire this informative signal, update their beliefs about the quality of the asset if they acquire the signal, and then bid on the asset. The seller can then either accept the highest bid, or reject all bids. To this basic environment, we introduce a stylized version of PPIP that works as follows. The buyer who

---

<sup>4</sup>The specifics of the actual policy are best illustrated through an example taken from a fact sheet prepared by the Treasury to illustrate how PPIP worked. Suppose several private investors place a bid for an asset, and the winning bid is \$84. Under this program, the investor would be required to put up \$6 of his own equity, the Treasury would match his equity investment with an additional \$6 in exchange for a 50% stake in the asset’s returns, and the FDIC would provide a nonrecourse loan for the remaining \$72. That is, the program allows the investor to leverage up to a ratio of 6-to-1. If the asset appreciates, it is clearly optimal for the investor to repay the loan. However, should the asset fall in value, the investor can default on the loan and forfeit the asset; given the nature of nonrecourse loans, his losses are bounded above by his initial equity investment, \$6.

<sup>5</sup>Timothy Geithner, *Wall Street Journal*, March 23, 2009, “My Plan for Bad Bank Assets.”

wins the asset receives a nonrecourse loan for a fraction  $1 - \gamma$  of the purchase price. After acquiring the asset, but before deciding whether to repay the loan, the buyer learns the quality of the asset. Given our assumptions on parameters, the buyer repays the loan if he acquired a high-quality asset and defaults if he acquired a low-quality asset, or “lemon.” Therefore, the program is tantamount to a transfer that occurs to “unlucky” buyers who acquire a lemon; that is, the program is a simple form of insurance.

In Section 3, we characterize equilibria and study the relationship between the policy parameter  $\gamma$ , the buyers’ incentive to acquire information, and their subsequent bidding behavior. We show that there is a trade-off between mitigating the adverse selection problem and encouraging buyers’ information acquisition. Intuitively, when the amount of insurance against losses provided by the policy is too low, the problem of adverse selection remains severe, the expected gains from trade are small, and thus buyers are hesitant to acquire information and bid for the asset. However, when the program provides too much insurance against losses, a moral hazard problem emerges: buyers become more willing to “gamble” and bid aggressively without first acquiring the costly signal.

In Section 4, we use our equilibrium characterization to study the informational content of the winning bid.<sup>6</sup> In order to quantify the amount of information contained in the winning bid, we use the notion of *entropy informativeness*: we derive the beliefs of an agent who does not participate in the auction, but who can observe the winning bid, and we calculate the expected reduction in entropy of this agent’s beliefs. A key result is that the quantity of information contained in the winning bid is maximized at an *interior* value of the policy parameter  $\gamma$ . This value of  $\gamma$  provides enough insurance to ensure that buyers have an incentive to acquire information and place legitimate bids for the asset, but not so much insurance that they place these bids without first acquiring information. Since the policy parameter that maximizes the probability that gains from trade are realized is  $\gamma = 0$ , this result highlights the fact that ignoring the effect of intervention on price discovery can lead to substantially different conclusions for the optimal policy.

In Section 5, we discuss some of our assumptions and possible extensions to our framework. First, since the government can control the number  $N$  of buyers participating in each auction, a

---

<sup>6</sup>As we discuss in the text, our focus on the informational content of the winning bid—as opposed to, say, the vector of bids placed by buyers—is motivated by realism: PPIP reported the identity of the winning bidder and how much was paid, but not the losing bids.

natural question is how the choice of  $N$  affects the informational content of prices. We show that, much like choosing  $\gamma$ , there is a trade-off in choosing  $N$ , too. Hence, for each  $\gamma$ , there is an optimal number of participants for maximizing information production. Second, we discuss the type of information that policymakers chose to disclose about each PPIP auction, possible reasons for these choices, and how they ultimately influence the process of price discovery. Third, we discuss the robustness of our results to alternative signal structures. Finally, we illustrate how our framework can accommodate an alternative theory of market crashes. In particular, we explain how our insights can generalize to environments in which trade is frozen because of buyers' binding budget constraints, or so-called "cash-in-the-market pricing." Section 6 concludes.

**Related Literature.** The literature on optimal interventions in frozen markets is growing fast; a non-exhaustive list includes Tirole [2012], Philippon and Skreta [2012], Chari et al. [2010], Lester and Camargo [2011], Guerrieri and Shimer [2011], Chiu and Koepl [2011], Philippon and Schnabl [2011], House and Masatlioglu [2010], Diamond and Rajan [2012], and Farhi and Tirole [2012]. As we noted above, the majority of this literature focuses on how government interventions can improve allocations, while ignoring the effects of these interventions on the process of information production. In these papers, government interventions are costly for one of two reasons: either there is a direct cost of raising the funds required for the intervention or there is an implicit cost associated with introducing a moral hazard problem. Hence, one contribution of our paper is to identify a third potential cost of government intervention: disrupting the process of price discovery.

To the best of our knowledge, the only paper that explicitly studies the effects of government interventions on information production is Bond and Goldstein [2012]. The focus of their analysis is very different from ours; most notably, they are not interested in inefficiencies due to adverse selection, and thus the interventions in their model play an entirely different role than in our environment. However, they highlight an interesting feedback effect that is absent from our analysis: in their dynamic framework, the government decides how much to use market prices in formulating a policy, which affects the incentives of speculators to trade and hence changes the informational



content of these prices.<sup>7</sup>

Finally, from a technical point of view, our paper is related to two strands of the auction literature. The first strand examines the extent to which the winning bid(s) of an auction reflects the underlying value of the good(s) for sale; see, for example, Wilson [1977], Milgrom [1979], Milgrom [1981], Pesendorfer and Swinkels [1997], Pesendorfer and Swinkels [2000], Kremer [2002], and Lauermann and Wolinsky [2013]. In contrast to our paper, this literature typically treats the information set of each bidder as exogenous and focuses on a (necessary and) sufficient condition for the winning price to completely reveal the underlying value of the object.<sup>8</sup> The second strand studies the incentives of bidders to acquire information under various auction formats, and whether these incentives align with the socially optimal level; see, for example, Matthews [1984], Persico [2000], Bergemann and Valimaki [2002], and Bergemann et al. [2009]. The focus of these papers, however, is very different from ours; they are interested in neither the informational content of the winning bid, nor the effects of any form of intervention on information acquisition or the ensuing outcomes.

## 2 The Model

**Environment.** There is a single seller who possesses one indivisible asset and  $N$  ex ante homogeneous buyers who are interested in purchasing the asset. The asset is either of high ( $H$ ) or low ( $L$ ) quality. If the asset is of high quality, then the seller receives payoff  $c$  from retaining the asset, while a buyer receives payoff  $v$  from acquiring it, where  $v > c > 0$ .<sup>9</sup> If the asset is of low quality, then it is of no value to either the seller or the buyers; that is, it yields all of them zero payoff.

The quality of the asset is the seller's private information. The buyers have a common prior

---

<sup>7</sup>Their modeling approach builds on the literature that studies informational feedback from asset prices to real decisions; see Bond et al. [2012] for a nice review of this literature.

<sup>8</sup>A notable exception is Jackson [2003], who allows for endogenous information acquisition.

<sup>9</sup>Note that, e.g., as in Duffie et al. [2005], buyers and sellers receive different levels of utility from holding a high-quality asset. One could imagine that this type of asset yields a (possibly stochastic) stream of dividends, and that buyers and sellers value this stream of dividends differently. This can arise for a multitude of reasons: for example, buyers and sellers may have different levels of risk aversion, financing costs, regulatory requirements, or hedging needs. The current formulation is a reduced-form representation of such differences. For more discussion and examples in which these differences arise endogenously, see, e.g., Duffie et al. [2007], Vayanos and Weill [2008], and Gârleanu [2009].

belief that the asset is of high quality with probability  $\pi \in (0, 1)$ . Should a buyer acquire an asset of quality  $j \in \{L, H\}$  at some price  $b$ , this buyer receives payoff  $v - b$  if  $j = H$  and  $-b$  if  $j = L$ , the seller receives payoff  $b$ , and all other buyers receive payoff zero. Should no trade occur, the seller receives payoff  $c$  if  $j = H$  and zero if  $j = L$ , and all buyers receive payoff zero.

**Trading.** The game proceeds as follows. First, each buyer  $i \in \{1, \dots, N\}$  has the opportunity to inspect the asset at a cost  $k_i$ , where  $k_i$  is independently and identically drawn from the interval  $[0, \infty)$  according to a continuous and strictly increasing distribution function  $G$ . If buyer  $i$  incurs the cost  $k_i$ , then he receives a private and independently drawn signal  $s_i \in \{\ell, h\}$  about the quality of the asset. In order to deliver our results most clearly, we focus on a simple signal generating process summarized by the matrix

$$\begin{array}{|c|c|c|} \hline & H & L \\ \hline h & 1 & 1 - \rho \\ \hline \ell & 0 & \rho \\ \hline \end{array},$$

where  $\rho \in (0, 1)$ . In words, a buyer who inspects the asset always receives the “good” signal  $h$  if the asset is of quality  $H$ . However, if the asset is of quality  $L$ , then the buyer receives the “bad” signal  $\ell$  only with probability  $\rho$ .<sup>10</sup> We refer to a buyer who has decided to receive a costly signal as “informed” and a buyer who has chosen not to receive a signal as “uninformed.” For ease of exposition, we say that an uninformed buyer observes the signal  $s_i = u$ . A buyer cannot observe the other buyers’ costs, nor can he observe whether the other buyers are informed or uninformed.

After buyers have the opportunity to receive a signal, they each simultaneously submit a non-negative (sealed) bid for the asset; we denote buyer  $i$ ’s bid by  $b_i$ . The seller then decides whether to accept the highest bid or to reject all bids and retain the asset. If the highest bid is offered by two or more buyers (and the seller accepts), the asset is awarded to each of those buyers with equal probability.

---

<sup>10</sup>Although the informational structure is rather stylized, it has a natural interpretation. One can imagine that there are certain “red flags” associated with low quality assets, corresponding to signal  $\ell$  in our environment. A buyer who studies a seller’s asset will never uncover such a red flag if the asset is of high quality, while he may (with probability  $\rho$ ) find one if the asset is of low quality. Importantly, many of our results are robust to other specifications, including the case in which the bad signal occurs with positive probability when the asset is of high quality; see Section 5 for a discussion.

**Assumptions.** A buyer who receives the bad signal knows with certainty that the asset is of low quality. Alternatively, a buyer who receives the good signal is still uncertain about the quality of the asset, but updates his belief that the asset is of high quality to

$$\tilde{\pi} = \frac{\pi}{\pi + (1 - \pi)(1 - \rho)} > \pi.$$

In order to focus on the most relevant case, we make the following two assumptions:

ASSUMPTION 1. (*Severe adverse selection*)

$$\pi(v - c) - (1 - \pi)c < 0 \Leftrightarrow \pi < \frac{c}{v};$$

ASSUMPTION 2. (*Positive value of inspection*)

$$\tilde{\pi}(v - c) - (1 - \tilde{\pi})c > 0 \Leftrightarrow \tilde{\pi} > \frac{c}{v} \Leftrightarrow \rho > \frac{c - \pi v}{(1 - \pi)c}.$$

Assumption 1 implies that the adverse selection problem is severe enough that buyers are not willing to place a “serious” bid  $b \geq c$  without inspecting the asset. Assumption 2 implies that inspection is sufficiently informative about the quality of the asset to generate the potential for trade between a buyer who receives the signal  $h$  and a seller with a high-quality asset; that is, a buyer who receives the good signal is willing to bid  $b \geq c$ .

**Policy.** The government policy is captured as follows. A buyer who purchases the asset at price  $b$  must put up an amount  $\gamma b$  of his own equity, and is issued a nonrecourse loan from the government for the remaining portion of his bid,  $(1 - \gamma)b$ . Should the buyer choose not to repay the loan, the government can seize the asset, but the buyer is not liable for any additional payments. Therefore, the buyer’s loss is limited to  $\gamma b$  under this program.

We assume that a buyer who purchases the asset observes its quality before deciding whether to repay the loan. Hence, a buyer who acquires the asset at price  $b$  repays the loan if the asset is of quality  $H$ , earning a payoff of  $v - b$ , and defaults otherwise, suffering a loss of  $\gamma b$ . Thus, the government policy is tantamount to insurance: it provides a rebate of size  $(1 - \gamma)b$  to an “unlucky” buyer who pays price  $b > 0$  and receives a low-quality asset.

**Strategies and Equilibrium.** The seller’s behavior is straightforward in our model: a seller with a low-quality asset accepts any positive bid, while a seller with a high-quality asset accepts the highest bid  $b$  if  $b \geq c$  and rejects all bids otherwise. It simplifies our analysis to assume that a bid of zero is rejected by both types of sellers, even though sellers with low-quality assets are indifferent between accepting and rejecting such a bid; one could imagine, for example, an arbitrarily small transaction cost associated with trading an asset of low quality. In what follows, we take the behavior of the sellers as given and focus on the behavior of the buyers.

A strategy for buyer  $i \in \{1, \dots, N\}$  has two components. First, he must decide whether or not to inspect the asset as a function of the cost  $k_i$ . The optimal inspection strategy for a buyer is obviously a cutoff rule: inspect the asset if, and only if, the cost  $k_i$  is not greater than the value of doing so. Therefore, we represent a buyer’s inspection strategy by his cutoff cost  $k$ . Second, a buyer must formulate an optimal bidding strategy as a function of his private information: namely, the signal  $s_i \in \{u, \ell, h\}$  he receives about the quality of the asset.<sup>11</sup> We let a cumulative distribution function  $F_s$  represent the mixed bidding strategy of a buyer with signal  $s$ ;  $F_s(b)$  is the probability that a buyer with signal  $s$  bids  $b$  or less.

A symmetric equilibrium is a strategy profile  $(k, F_u, F_\ell, F_h)$  in which: (i) a buyer inspects the asset if, and only if, the cost of doing so is not greater than the benefit; and (ii) each buyer’s bid is optimal given his private information and the strategies of the other buyers.

### 3 Equilibrium Characterization

In this section, we characterize the symmetric equilibria of the trading game described in the previous section. The main challenge is to jointly characterize buyers’ equilibrium inspection and bidding strategies. Indeed, the value of inspection for buyers depends on their bidding behavior, which in turn depends on their inspection decisions.

We overcome the challenge described above by proceeding in two steps. First, we consider the model in which the probability  $\lambda$  that each buyer is informed is exogenously given and characterize

---

<sup>11</sup>In principle, a buyer can condition his bidding strategy on his cost of inspection  $k_i$ . However, at the time of bidding, the costs of inspection are already sunk and, therefore, intrinsically irrelevant to buyers’ bidding problems. Hence, we assume that a buyer’s bidding strategy can condition only on his signal. In addition, it is possible to show that allowing buyers to condition their bids on their inspection costs does not affect equilibrium payoffs and outcomes.

its equilibria for all possible values of  $\lambda$  and  $\gamma$ . Second, we use the equilibrium characterization of the model with exogenous inspection probabilities to determine the value of inspection for a buyer, conditional on the other buyers inspecting the asset with probability  $\lambda$ . This allows us to characterize the equilibria of the original game, where the probability that each buyer is informed is determined endogenously, for each choice of  $\gamma$ . In particular, we show that, for each  $\gamma \in [0, 1]$ , there exists a unique cutoff  $k^*$  such that the value of being informed is exactly  $k^*$  when the probability that each buyer is informed is  $\lambda^* = G(k^*)$ .

We restrict the analysis in the text to the case of  $\gamma > 0$ , as this is the most relevant case. For completeness, however, in the Appendix we show that if  $\gamma = 0$ , then the equilibrium cutoff cost for inspecting the asset is zero (Proposition 5). Intuitively, if  $\gamma = 0$ , then buyers do not face any risk when purchasing the asset and, therefore, do not benefit from first inspecting it.

### 3.1 Equilibrium with Exogenous Information Acquisition

Suppose each buyer inspects the asset with probability  $\lambda \in (0, 1)$ .<sup>12</sup> The behavior of a buyer who observes the signal  $\ell$  is trivial: since this signal reveals that the asset is of low quality, a weakly dominant strategy for the buyer is to bid  $b = 0$ , yielding payoff  $V_\ell = 0$ . Therefore, in what follows, we take as given the behavior of buyers who receive the signal  $\ell$ , and we concentrate on the behavior of uninformed buyers and informed buyers who receive the signal  $h$ . In a slight abuse of notation, we refer to the former as “type  $u$ ” buyers and the latter as “type  $h$ ” buyers. Note that bidding  $b \in (0, c)$  is suboptimal for both types of buyers, because such an offer is accepted only when the asset is of low quality, in which case the buyer surely suffers a payoff loss of  $\gamma b$ .

For each  $s \in \{u, h\}$ , denote the minimum and maximum of the support of  $F_s$  by  $\underline{b}_s$  and  $\bar{b}_s$ , respectively. In addition, denote by  $V_s(b)$  the expected payoff to a type  $s$  buyer who bids  $b$ , and let  $V_s$  be the equilibrium payoff of a type  $s$  buyer. We first establish several basic properties of the equilibrium bidding strategies.

**Lemma 1.** *The following holds in equilibrium for all  $\gamma > 0$ : (i)  $\bar{b}_u \leq \underline{b}_h$ ; (ii)  $\bar{b}_h > c$ ; (iii)  $F_s(b)$  is continuous and strictly increasing in  $b$  when  $b \in [\max\{c, \underline{b}_s\}, \bar{b}_s]$  for each  $s \in \{u, h\}$ ; and (iv)  $\underline{b}_u = 0$ .*

<sup>12</sup>Below, we give conditions under which  $\lambda \in (0, 1)$  for all  $\gamma > 0$  when information acquisition is endogenous.

We relegate the proof of Lemma 1 to the Appendix and sketch the intuition here. The first property is a typical single crossing property: a buyer who is more optimistic about the quality of the asset has a higher willingness to pay and therefore bids a higher price. As a result, the supports of  $F_h$  and  $F_u$  overlap at most at a single point. The second property states that type  $h$  buyers place serious bids with positive probability. Otherwise, if type  $h$  buyers always bid zero, then a profitable deviation for such a buyer would be to bid  $b = c$  and obtain a payoff  $\tilde{\pi}(v - c) - (1 - \tilde{\pi})\gamma c$ , which is positive by Assumption 2. The third property states that the mixed bidding strategies  $F_u$  and  $F_h$  have neither atoms nor gaps on  $[c, \bar{b}_h]$ . If there were an atom, then a buyer who bids slightly above the atom obtains a strictly higher payoff than a buyer who bids at the atom. Similarly, if there were a gap in the support of the distribution of bids, then a bid at the lower end of the gap would be strictly preferable to a bid at the upper end of the gap, as both bids have the same probability of winning. The last property states that  $\underline{b}_u = 0$ , which implies that  $V_u = 0$ . Indeed, if  $\underline{b}_u \geq c$ , then a type  $u$  bidder who bids  $b = \underline{b}_u$  wins only when the asset is of low quality, in which case his expected payoff is strictly negative.

Taken together, the properties in Lemma 1 imply that any equilibrium takes one of the following three forms. First, it could be that  $c < \bar{b}_u \leq \underline{b}_h$ , so that even uninformed buyers make serious bids for high-quality assets. Second, it could be that  $\bar{b}_u = 0$  and  $\underline{b}_h \geq c$ , so that uninformed bidders never make serious bids, while informed bidders who receive the signal  $h$  always place a serious bid. Finally, it could be that  $\bar{b}_u = \underline{b}_h = 0$ , so that even type  $h$  buyers make an offer of zero with positive probability. The equilibrium set can be characterized by analyzing each case separately. Since the analysis is similar for all three cases, we focus on the second case in detail here and relegate the other cases to the Appendix.

In the second case, in which uninformed buyers always bid zero, the expected payoff to a type  $s \in \{u, h\}$  buyer who bids  $b \geq c$  is

$$V_s(b) = \pi_s [1 - \lambda + \lambda F_h(b)]^{N-1} (v - b) - (1 - \pi_s) \{1 - \lambda + \lambda [\rho + (1 - \rho) F_h(b)]\}^{N-1} \gamma b, \quad (1)$$

where  $\pi_u = \pi$  and  $\pi_h = \tilde{\pi}$ . Indeed, each of the other buyers is either uninformed, and so bids less than  $b$  for sure, or is informed, and so bids less than  $b$  with probability  $F_h(b)$  if the asset is of

high quality and probability  $\rho + (1 - \rho)F_h(b)$  if the asset is of low quality. Hence, if the asset is of high quality, then the buyer wins the asset with probability  $[1 - \lambda + \lambda F_h(b)]^{N-1}$  and obtains a payoff of  $v - b$ . On the other hand, if the asset is of low quality, then the buyer wins the asset with probability  $\{1 - \lambda + \lambda[\rho + (1 - \rho)F_h(b)]\}^{N-1}$  and suffers a loss of  $\gamma b$ .

Given (1), an equilibrium in which  $\bar{b}_u = 0$  and  $\underline{b}_h \geq c$  can be constructed as follows. First, it must be that  $\underline{b}_h = c$ ; if  $\underline{b}_h > c$ , then a type  $h$  buyer strictly prefers bidding  $c$  to  $\underline{b}_h$ , as this decreases his payment without changing his probability of winning. Second, the expected payoff of a type  $h$  buyer can be found by considering a type  $h$  buyer who bids  $\underline{b}_h = c$ . From (1), it follows that

$$V_h = \tilde{\pi}(1 - \lambda)^{N-1}(v - c) - (1 - \tilde{\pi})(1 - \lambda + \lambda\rho)^{N-1}\gamma c.$$

Finally, for each  $b \in [c, \bar{b}_h]$ ,  $F_h(b)$  can be derived from (1) and the fact that the buyer must be indifferent between all bids in the support of  $F_h$ , and thus  $V_h(b) = V_h$ .

The equilibrium under consideration exists if, and only if, a type  $h$  buyer has no incentive to bid 0 and a type  $u$  buyer has no incentive to bid more than  $c$ . The first condition is that  $V_h \geq 0$ , which is equivalent to

$$\left(1 + \frac{\lambda\rho}{1 - \lambda}\right)^{N-1} \leq \frac{\tilde{\pi}(v - c)}{(1 - \tilde{\pi})\gamma c}. \quad (2)$$

The second condition is that  $V_u(b) \leq 0$  for all  $b \geq c$ . Since  $\pi < \tilde{\pi}$  implies that a type  $u$  bidder strictly prefers  $b$  to  $b' > b$  whenever a type  $h$  buyer is indifferent between  $b$  and  $b'$ , a necessary and sufficient condition for  $V_u(b) \leq 0$  for all  $b \geq c$  is that  $V_u(c) \leq 0$ , which is equivalent to

$$\left(1 + \frac{\lambda\rho}{1 - \lambda}\right)^{N-1} \geq \frac{\pi(v - c)}{(1 - \pi)\gamma c}. \quad (3)$$

Combining (2) and (3), an equilibrium in which  $\bar{b}_u = 0$  and  $\underline{b}_h \geq c$  exists if, and only if,

$$\frac{\pi(v - c)}{(1 - \pi)\gamma c} \leq \left(1 + \frac{\lambda\rho}{1 - \lambda}\right)^{N-1} \leq \frac{\tilde{\pi}(v - c)}{(1 - \tilde{\pi})\gamma c}.$$

It is clear from the reasoning above that the equilibrium is unique for each pair  $(\lambda, \gamma) \in (0, 1) \times (0, 1]$  satisfying (2) and (3).

Proposition 1 summarizes the characterization of the equilibria with  $\bar{b}_u = 0$  and  $\underline{b}_h = c$  described above and provides a characterization of the other two types of equilibria, namely, the

equilibria with  $c < \bar{b}_u \leq \underline{b}_h$  and the equilibria with  $\bar{b}_u = \underline{b}_h = 0$ . Loosely speaking, for an equilibrium with  $c < \bar{b}_u \leq \underline{b}_h$  to exist, it must be that a type  $u$  buyer obtains a non-negative payoff from bidding  $c$  when all type  $u$  buyers bid zero; a necessary and sufficient condition for this is found by simply reversing the sign in the inequality in (3). Similarly, for an equilibrium with  $\bar{b}_u = \underline{b}_h = 0$  to exist, it must be that a type  $h$  buyer obtains a negative payoff from bidding  $c$  when all other type  $h$  buyers bid  $b \geq c$ ; a necessary and sufficient condition for this is found by simply reversing the sign in (2).

**Proposition 1.** *For each  $(\lambda, \gamma) \in (0, 1) \times (0, 1]$ , there exists a unique symmetric equilibrium. Let  $\underline{\lambda}(\gamma) \geq 0$  be the smallest value of  $\lambda$  that satisfies*

$$\left[1 + \frac{\lambda\rho}{1-\lambda}\right]^{N-1} \geq \frac{\pi(v-c)}{(1-\pi)\gamma c}$$

and  $\bar{\lambda}(\gamma) \in (\underline{\lambda}(\gamma), 1)$  be the only value of  $\lambda$  that satisfies

$$\left[1 + \frac{\lambda\rho}{1-\lambda}\right]^{N-1} = \frac{\tilde{\pi}(v-c)}{(1-\tilde{\pi})\gamma c}.$$

(1) *If  $\lambda \in (0, \underline{\lambda}(\gamma))$ , then  $\underline{b}_h = \bar{b}_u > c$ . The expected payoff of a type  $h$  buyer is*

$$V_h = \tilde{\pi}\rho v \left\{ \frac{1}{(1-\pi)(1-\lambda+\lambda\rho)^{N-1}\gamma} + \frac{1}{(1-\lambda)^{N-1}} \right\}^{-1}. \quad (4)$$

(2) *If  $\lambda \in [\underline{\lambda}(\gamma), \bar{\lambda}(\gamma)]$ , then  $\bar{b}_u = 0$  and  $\underline{b}_h = c$ . The expected payoff of a type  $h$  buyer is*

$$V_h = \tilde{\pi}(1-\lambda)^{N-1}(v-c) - (1-\tilde{\pi})(1-\lambda+\lambda\rho)^{N-1}\gamma c. \quad (5)$$

(3) *If  $\lambda > \bar{\lambda}(\gamma)$ , then  $\underline{b}_h = \bar{b}_u = 0$ . In this case,  $V_h = 0$ .*

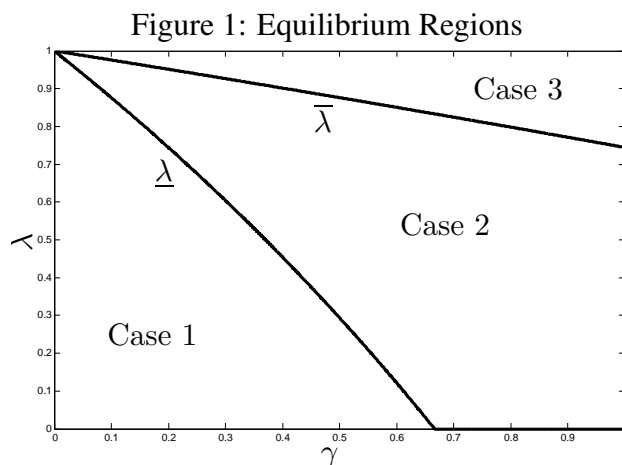
Figure 1 plots the values of  $\lambda$  and  $\gamma$  where each of the three types of equilibria exist.<sup>13</sup> To understand each type of equilibrium, it is helpful to note that, holding  $\gamma$  constant, an increase in  $\lambda$  worsens the winner's curse for both types of buyers and thus weakens their incentives to place serious bids. However, this effect is stronger for uninformed buyers, as type  $h$  buyers are better

<sup>13</sup>One can easily show that  $\bar{\lambda}(\gamma)$  is strictly decreasing in  $\gamma$  and that  $\underline{\lambda}(\gamma)$  is strictly decreasing in  $\gamma$  as long as  $\pi(v-c) > (1-\pi)\gamma c$ , with  $\underline{\lambda}(\gamma) = 0$  otherwise.



informed about the quality of the asset. Similarly, holding  $\lambda$  constant, an increase in  $\gamma$  implies that buyers are less insured against acquiring a lemon, which also leads to less aggressive bidding.

Therefore, when both  $\lambda$  and  $\gamma$  are small, both types of buyers bid aggressively; in the first case of Proposition 1, even uninformed buyers place serious bids. As  $\lambda$  and  $\gamma$  increase, the winner's curse for uninformed buyers becomes sufficiently strong that they stop placing serious bids; in the second case, only informed buyers place bids greater than  $c$ . Finally, when  $\lambda$  and  $\gamma$  are sufficiently close to one, the winner's curse becomes sufficiently strong for informed buyers, too—so much so, in fact, that they bid  $b = 0$  with positive probability in the last case.



### 3.2 Equilibrium with Endogenous Information Acquisition

We now complete the description of equilibria by endogenizing the information acquisition decision of buyers. In equilibrium, the cutoff inspection cost coincides with the value of conducting an inspection. We first derive the ex ante value of inspection as a function of the policy  $\gamma$  and the probability  $\lambda$  that other buyers inspect the asset, which we denote by  $V_I = V_I(\lambda, \gamma)$ . Then, for each  $\gamma > 0$ , we identify the cutoff inspection cost  $k^*$  that equates the value of inspection to the cost itself; that is, since  $G(k)$  is the probability that each buyer inspects the asset given a cutoff strategy  $k$ , we find the value of  $k^*$  such that  $k^* = V_I(G(k^*), \gamma)$ .

As noted above, the expected payoffs of type  $\ell$  and  $u$  buyers are always zero. Therefore,

$$V_I(\lambda, \gamma) = [\pi + (1 - \pi)(1 - \rho)] V_h(\lambda, \gamma),$$

where  $V_h(\lambda, \gamma)$  denotes the expected payoff of a type  $h$  buyer given  $\lambda$  and  $\gamma$ . The following result is then immediate from Proposition 1.

**Lemma 2.** *For each  $\gamma > 0$ ,  $V_I(\lambda, \gamma)$  is continuous in  $\lambda$ , strictly decreasing in  $\lambda$  if  $\lambda < \bar{\lambda}(\gamma)$ , and equal to zero if  $\lambda \geq \bar{\lambda}(\gamma)$ .*

Given these properties of  $V_I$ , it is straightforward to characterize the equilibrium of the original game, where information acquisition is endogenous. For simplicity, we assume that  $G(0) > 0$ . Together with the fact that  $G(v - c) < 1$ , as  $G$  is strictly increasing on  $[0, \infty)$ , the assumption that  $G(0) > 0$  ensures that in equilibrium the probability that each buyer inspects the asset lies in the open interval  $(0, 1)$ .<sup>14</sup>

To summarize the analysis so far, for each  $\gamma > 0$ , a symmetric equilibrium is a strategy profile  $(k^*, F_\ell^*, F_u^*, F_h^*)$  such that: (i) a type  $\ell$  buyer bids zero; (ii)  $F_u^*$  and  $F_h^*$  are the unique bidding strategies for type  $u$  and type  $h$  buyers, respectively, of the game with exogenous information acquisition when  $\lambda = G(k^*)$ ; and (iii)  $k^* = V_I(G(k^*), \gamma)$ . The existence of an equilibrium cutoff cost follows from the fact that  $V_I(\lambda, \gamma)$  is continuous in  $\lambda$  for all  $\gamma > 0$ . The cutoff cost is unique given that  $V_I(\lambda, \gamma)$  is nonincreasing in  $\lambda$ . Note that  $G(k^*) < \bar{\lambda}(\gamma)$  for all  $\gamma > 0$  since  $V_I(\lambda, \gamma) = 0$  if  $\lambda \geq \bar{\lambda}(\gamma)$ . Thus, the payoff to type  $h$  buyers is positive in equilibrium when  $\gamma > 0$ .

**Proposition 2.** *For each  $\gamma > 0$ , there exists a unique symmetric equilibrium. The cutoff inspection cost  $k^*$  is positive if, and only if,  $G(0) < \bar{\lambda}(\gamma)$ .*

In what follows, we assume that  $G(0) < \bar{\lambda}(1)$ , which is a sufficient condition for  $k^* > 0$  for all  $\gamma > 0$ ; this assumption simply rules out the possibility that the mass of agents who become informed at no cost is so large that the value of becoming informed is zero.<sup>15</sup>

<sup>14</sup>While the assumption that  $G(0) > 0$  helps ensure an interior probability of inspection, this assumption could easily be relaxed without changing the substance of our results.

<sup>15</sup>For a given  $\gamma > 0$ , if  $G(0) > \bar{\lambda}(\gamma)$ , then  $k^* = 0$  and the equilibrium is simply described by Proposition 1 with  $\lambda = G(0)$ . In this case, a marginal change in the policy  $\gamma$  obviously has no effect on  $k^*$ .

### 3.3 Policy and Inspection

We now examine how policy affects buyers' incentives to acquire information. In what follows, we make explicit the dependence of  $k^*$  on  $\gamma$  and write  $k^*(\gamma)$  to denote the equilibrium cutoff cost when the government policy is  $\gamma$ .

Suppose first that  $\lambda^*(\gamma) \equiv G(k^*(\gamma)) < \underline{\lambda}(\gamma)$ , so that uninformed buyers make serious bids in equilibrium. From (4) in Proposition 1, one can see immediately that  $V_I(\lambda, \gamma)$  is increasing in  $\gamma$ ; that is, a policy that provides the buyers with *less* insurance leads to an *increase* in the value of inspection. To understand the intuition behind this result, recall that the expected payoff of a type  $h$  buyer who bids  $\bar{b}_u = \underline{b}_h$  is

$$V_h(\underline{b}_h) = \tilde{\pi} (1 - \lambda)^{N-1} (v - \underline{b}_h) - (1 - \tilde{\pi}) (1 - \lambda + \lambda\rho)^{N-1} \gamma \underline{b}_h.$$

An increase in  $\gamma$  has two opposing effects on  $V_h(\underline{b}_h)$  and thus on  $V_I$ . First, less insurance against acquiring a lemon directly decreases the expected payoff of any buyer who places a serious bid. Second, less insurance also causes buyers to bid less aggressively. In particular, less insurance decreases the bids placed by type  $u$  buyers (in the sense of first-order stochastic dominance), which makes it cheaper for type  $h$  buyers to outbid uninformed buyers and increases the value of inspection.

To compare the two effects of an increase in  $\gamma$ , recall that the expected payoff of a type  $u$  bidder is always zero, so that

$$0 = \pi (1 - \lambda)^{N-1} (v - \underline{b}_h) - (1 - \pi) (1 - \lambda + \lambda\rho)^{N-1} \gamma \underline{b}_h.$$

Therefore, the two effects discussed above cancel out each other for type  $u$  buyers. In addition, the first, direct effect is relevant only when the asset is of low quality, while the second, indirect effect is relevant whether the asset is of high or low quality. It follows that the indirect effect dominates the direct effect for a type  $h$  buyer, because he assigns a higher probability to the asset being of high quality than a type  $u$  buyer.

Now suppose that  $\lambda^*(\gamma) > \underline{\lambda}(\gamma)$ . From (5) in Proposition 1, it is easy to see that an increase in  $\gamma$  causes a decrease in  $V_I$ . Indeed, since now uninformed buyers never make serious bids, the

direct effect discussed above is still present, but the indirect effect is absent. Therefore, an increase in  $\gamma$  always decreases  $V_I$ .

Figures 2 and 3 plot the effect of an increase in  $\gamma$  on  $V_I$  and  $k^*$  when  $G(k^*)$  is less than or greater than  $\underline{\lambda}(\gamma)$ , respectively. Given the response of  $V_I$  to an increase in  $\gamma$ , it follows immediately that  $k^*$  is increasing in  $\gamma$  if  $\lambda^*(\gamma) < \underline{\lambda}(\gamma)$  and decreasing in  $\gamma$  otherwise. To formalize this result, for each  $\gamma > 0$ , let  $\underline{k}(\gamma)$  denote the smallest value of  $k$  such that  $G(k) \geq \underline{\lambda}(\gamma)$ .<sup>16</sup> In Proposition 3, we establish that there exists a unique and interior value of  $\gamma$  such that  $k^*(\gamma) = \underline{k}(\gamma)$ ; see the Appendix for the proof. For the purpose of comparative statics, this allows us to characterize the regions of the parameter space in which an increase in  $\gamma$  has a positive or negative effect on information acquisition.

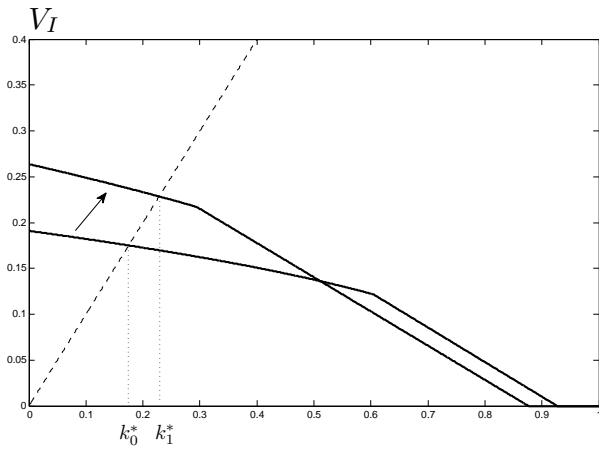


Figure 2: An Increase in  $\gamma$  When  $\lambda^* < \underline{\lambda}(\gamma)$

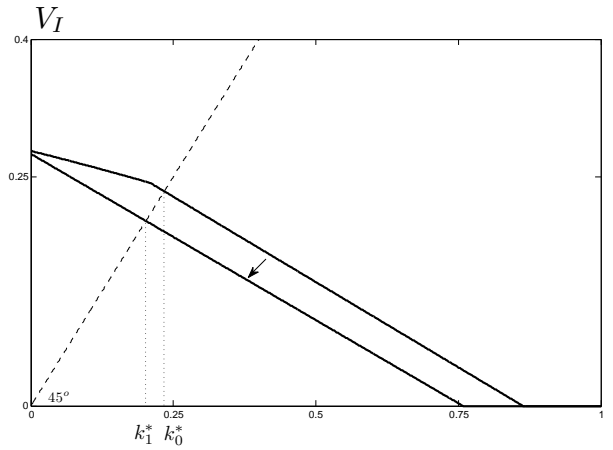


Figure 3: An Increase in  $\gamma$  When  $\lambda^* > \underline{\lambda}(\gamma)$

**Proposition 3.** *There exists a unique and interior  $\tilde{\gamma} \in (0, 1)$  such that  $k^*(\tilde{\gamma}) = \underline{k}(\tilde{\gamma})$ . The cutoff cost  $k^*(\gamma)$  is strictly increasing in  $\gamma$  when  $\gamma \in (0, \tilde{\gamma})$  and strictly decreasing in  $\gamma$  when  $\gamma \in (\tilde{\gamma}, 1]$ . Finally,  $k^*(\gamma)$  converges to zero as  $\gamma$  decreases to zero.*

**Corollary 1.**  *$k^*(\gamma)$  is maximized when  $\gamma = \tilde{\gamma}$ .*

Intuitively, when  $\gamma$  is close to one, the policy provides little insurance and hence the losses associated with acquiring a low-quality asset are large. This risk depresses the expected value of acquiring the asset—even after observing the signal  $h$ —and hence the value of becoming informed

<sup>16</sup>Note that  $\underline{k}(\gamma) = 0$  if  $\gamma \geq \pi(v - c)/(1 - \pi)c$  since  $G(0) > 0$ .

is small. On the other hand, when  $\gamma$  is close to zero, the policy insures nearly all of a buyer's downside risk and, as a result, a moral hazard problem emerges: even uninformed buyers bid aggressively. This price competition drives down the expected return from acquiring the asset and hence the value of acquiring information.

Interestingly, the policy that maximizes both prices and the probability of trade,  $\gamma = 0$ , is the policy that minimizes information acquisition.<sup>17</sup> Therefore, our results suggest that there is an important tension for policymakers even if the shadow price of government funds is zero and the risk of moral hazard is absent: namely, that there is an inherent trade-off between ensuring that gains from trade are realized and promoting price discovery. We formalize this trade-off in the next section.

## 4 Policy, Prices, and Information Production

A primary goal of PPIP was to promote price discovery by encouraging private investors to produce information about the quality of certain assets (and thus, presumably, about the quality of similar or related assets as well). In the previous section, we developed a stylized model of how an asset would be sold under PPIP. In this section, we use the equilibrium characterization of the previous section to study how the *quantity* of information produced by PPIP depends on the policy parameter  $\gamma$ .

Consistent with the actual implementation of PPIP, we assume that both the policy rule and the winning bid of the auction are observed by the public, and study the extent to which these observations reduce uncertainty about the quality of the asset.<sup>18</sup> Formally, this is accomplished by studying the expected reduction in *entropy* that results from an agent with prior  $\pi$  observing the winning bid of an auction with policy  $\gamma$ . Using the reduction in entropy as a metric for the quantity

---

<sup>17</sup>A straightforward consequence of the proof of Proposition 1 is that  $\bar{b}_u$  increases to  $v$  and  $F_u(0)$  decreases to zero as  $\gamma$ , and thus  $\lambda^*(\gamma)$ , decreases to zero.

<sup>18</sup>For example, on September 19, 2009 the FDIC issued a press release providing details of an auction that occurred on August 31, 2009. The information in the press release included the assets for sale (a pool of residential mortgage loans with an unpaid principal balance of approximately \$1.3 billion), the name of the winning bidder (Residential Credit Solutions), the number of total bidders who participated in the auction (twelve), the winning bid (approximately \$885 million), and the leverage ratio used to finance the purchase (6-to-1). More details of this auction, or others like it, are available at <http://www.fdic.gov>.

of information has become standard—and provides a natural benchmark here—because this metric is largely model-free: it does not depend on the endowments or preferences of the agents in the model, nor on the decision-making process to which the information is applied.<sup>19</sup>

Using the results from the previous section, we establish that the quantity of information is maximized at an interior value of  $\gamma$ , which provides enough insurance to encourage buyers to participate in the auction by placing serious bids, but not so much insurance that they place these bids without first acquiring information. We highlight that the value of  $\gamma$  that maximizes information production stands in stark contrast to the policy that maximizes gains from trade in the auction,  $\gamma = 0$ . This difference suggests that ignoring the effect of an intervention on price discovery can lead to substantially different prescriptions for policymakers. Finally, we also point out that the choice of  $\gamma$  that maximizes the quantity of information is not necessarily the choice of  $\gamma$  that maximizes the *value* of this information.

#### 4.1 Entropy and the *Quantity* of Information Produced

Consider an agent who does not participate in the auction, but who is able to observe both the policy choice,  $\gamma$ , and the winning bid,  $p$ ; we adopt the convention that  $p = 0$  when no trade takes place. Given the prior belief  $\pi$  that the asset is of high quality, and knowing both the signal structure and equilibrium strategies, the agent can use  $p$  to update his belief about the quality of the asset. Denote the agent’s posterior belief by  $\pi^+$ . Since  $p$  is a random variable whose distribution depends on  $\gamma$ , each choice of  $\gamma$  induces a distribution  $\Omega^*(\cdot; \gamma)$  of posterior beliefs;  $\Omega^*(\pi^+; \gamma)$  is the (unconditional) probability that the agent’s posterior belief is  $\pi^+$  or less when the policy is  $\gamma$ .

Following Sims [2003], we measure the quantity of information produced by a PPIP auction with policy choice  $\gamma$  as the expected reduction in uncertainty that results from observing  $p$ , where uncertainty is measured by the entropy of the agent’s beliefs. Recall that the entropy of a probability distribution  $q$  on a finite set  $\mathcal{J}$  of events is  $H(q) = -\sum_{j \in \mathcal{J}} q_j \log(q_j)$ , where  $q_j$  is the probability of  $j \in \mathcal{J}$ . Thus, the quantity of information or *entropy informativeness* of the auction

---

<sup>19</sup>The notion of entropy was first introduced in this context by Shannon [1948]. For early discussions of this measure’s use in economics, see Marschak [1959] and Arrow [1972]. For a more recent treatment, see Sims [2003], Veldkamp [2011], and Cabrales et al. [2013].

is

$$I(\gamma) = H(\pi) - \mathbb{E} [H(\pi^+)],$$

where  $H(\phi) = -\phi \log(\phi) - (1 - \phi) \log(1 - \phi)$  is the entropy of a belief  $\phi$  that the asset is of quality  $H$  and the expectation is taken with respect to  $\Omega^*(\cdot; \gamma)$ .

In order to evaluate how the choice of  $\gamma$  affects information production, we first deduce the distribution  $\Omega^*(\cdot; \gamma)$ . Afterwards, we study how  $I(\gamma)$  depends on  $\gamma$ .

**The Distribution of Posterior Beliefs** For each  $\gamma \in (0, 1]$  and  $\lambda \in (0, \bar{\lambda}(\gamma))$ , let  $\phi(p; \lambda, \gamma)$  denote the agent's posterior belief that the asset is of high quality after observing a winning bid  $p$  when the probability that buyers become informed is  $\lambda$  and the choice of policy is  $\gamma$ .<sup>20</sup> The following result reports basic properties of  $\phi(p; \lambda, \gamma)$ ; see the Appendix for a derivation of  $\phi(p; \lambda, \gamma)$  and a proof of Lemma 3.

**Lemma 3.** *The posterior belief  $\phi(p; \lambda, \gamma)$  satisfies the following properties: (i)  $\phi(0; \lambda, \gamma) < \phi(c; \lambda, \gamma)$ ; (ii)  $\phi(p; \lambda, \gamma)$  is strictly increasing in  $p$  when  $p \in [c, \bar{b}_h]$ ; and (iii)  $\phi(\bar{b}_h; \lambda, \gamma) = \tilde{\pi}$ .*

The first two facts in Lemma 3 are intuitive. Indeed, since type  $\ell$  buyers only bid zero, while type  $u$  and type  $h$  buyers can bid seriously, observing trade at some price  $p \geq c$  is more indicative that the asset is of high quality than observing no trade. Moreover, as  $p$  increases, so too does the conditional probability that the other buyers received signal  $h$  or  $u$  (as opposed to  $\ell$ ) but bid  $b \leq p$ . However, for any  $p < \bar{b}_h$ , bids less than  $p$  are more likely when the asset is of low quality. Hence,  $\phi(p; \lambda, \gamma) < \tilde{\pi}$  for all  $p \in [c, \bar{b}_h)$  because of what an observer infers about the losing bids. It is only when  $p = \bar{b}_h$  that observing the winning bid is equivalent to observing the high signal, for in this case bids less than  $p$  have the same probability regardless of the asset's type.

Given  $\phi(p; \lambda, \gamma)$ , along with the equilibrium characterization of Section 3, we can now construct  $\Omega^*(\cdot, \gamma)$  for each  $\gamma \in (0, 1]$ . Let  $\Omega_j^*(\pi^+; \gamma)$  be the probability that the agent's posterior belief is  $\pi^+$  or less when the policy is  $\gamma$  and the asset quality is  $j \in \{L, H\}$ . Moreover, in a slight abuse of notation, let  $\phi(p) = \phi(p; \lambda^*(\gamma), \gamma)$  and  $\phi^{-1}(\pi^+) = \phi^{-1}(\pi^+; \lambda^*(\gamma), \gamma)$  be the inverse of  $\phi(p)$ ,

<sup>20</sup>We restrict attention to  $\lambda < \bar{\lambda}(\gamma)$  since we know that in equilibrium the probability of information acquisition is smaller than  $\bar{\lambda}(\gamma)$  for all  $\gamma > 0$ .

which is well defined by Lemma 3. Then,

$$\Omega_H^*(\pi^+; \gamma) = \begin{cases} 0 & \text{if } \pi^+ \in [0, \phi(0)) \\ [(1 - \lambda^*(\gamma))F_u^*(0)]^N & \text{if } \pi^+ \in [\phi(0), \phi(c)) \\ [(1 - \lambda^*(\gamma))F_u^*(\phi^{-1}(\pi^+))]^N & \text{if } \pi^+ \in [\phi(c), \phi(\bar{b}_u)) \\ [(1 - \lambda^*(\gamma))]^N & \text{if } \pi^+ \in [\phi(\bar{b}_u), \phi(\underline{b}_h)) \\ [1 - \lambda^*(\gamma) + \lambda^*(\gamma)F_h^*(\phi^{-1}(\pi^+))]^N & \text{if } \pi^+ \in [\phi(\underline{b}_h), \phi(\bar{b}_h)] \end{cases} \quad (6)$$

if  $\gamma < \tilde{\gamma}$ , and

$$\Omega_H^*(\pi^+; \gamma) = \begin{cases} 0 & \text{if } \pi^+ \in [0, \phi(0)) \\ [(1 - \lambda^*(\gamma))]^N & \text{if } \pi^+ \in [\phi(0), \phi(c)) \\ [1 - \lambda^*(\gamma) + \lambda^*(\gamma)F_h^*(\phi^{-1}(\pi^+))]^N & \text{if } \pi^+ \in [\phi(c), \phi(\bar{b}_h)] \end{cases} \quad (7)$$

if  $\gamma \geq \tilde{\gamma}$ . Recall from the previous section that  $\tilde{\gamma}$  is the value of  $\gamma$  that maximizes buyers' incentive to acquire information, and is also the value of  $\gamma$  above which only type  $h$  buyers are willing to bid seriously. Similar calculations, relegated to the Appendix for the sake of brevity, can be used to derive  $\Omega_L^*(\cdot; \gamma)$ . Given  $\Omega_H^*$  and  $\Omega_L^*$ , the unconditional distribution of posterior beliefs is then

$$\Omega^*(\pi^+; \gamma) = \pi \Omega_H^*(\pi^+; \gamma) + (1 - \pi) \Omega_L^*(\pi^+; \gamma).$$

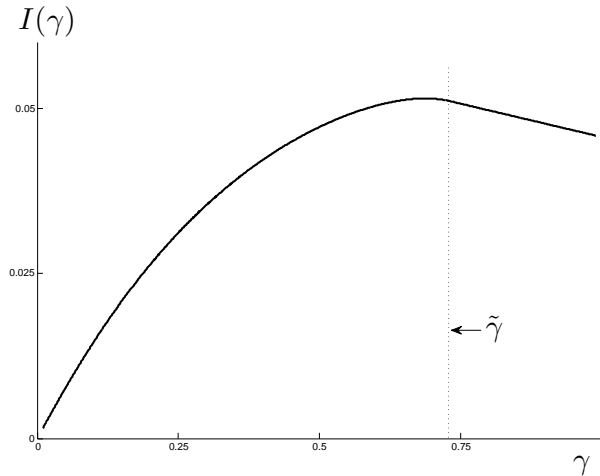
**Maximizing Information** We now establish that the quantity of information is maximized at a value of  $\gamma$  that lies strictly between 0 and  $\tilde{\gamma}$ . At this value of  $\gamma$ , the probability that each buyer is informed is not maximized, but the probability that uninformed buyers make serious bids (and distinguish themselves from type  $\ell$  buyers) is positive. We explain this intuition in greater detail below. We also reiterate the fact that the policy that maximizes gains from trade,  $\gamma = 0$ , minimizes price discovery.

**Proposition 4.**  *$I(\gamma)$  is maximized at a point that is strictly positive, but strictly smaller than  $\tilde{\gamma}$ . Moreover,  $I(\gamma)$  decreases to zero as  $\gamma$  decreases to zero.*

The intuition for why  $I(\gamma)$  is maximized at an interior value of  $\gamma$  is simple. We know from Proposition 3 that a decrease in  $\gamma$  initially increases the incentive of buyers to inspect the asset, which promotes price discovery by encouraging buyers to actively participate in the auction by



Figure 4: The Quantity of Information and Policy



placing serious bids. However, we also know from Proposition 3 that too much insurance deters information acquisition: if buyers face little risk when acquiring the asset, they have no incentive to pay the cost of inspection before placing a serious bid. As a result, if  $\gamma$  becomes too small, the probability that buyers place serious bids continues to increase, but the probability that they are informed begins to fall, so the informational content of the winning bid decreases. In fact, when  $\gamma = 0$ , the winning bid is completely uninformative, so the distribution of posterior beliefs is concentrated at the prior belief. The value of  $\gamma$  that maximizes the quantity of information produced strikes a balance between the two effects just described and ultimately encourages both participation and information acquisition.

To understand why  $I(\gamma)$  is maximized when  $\gamma$  is smaller than  $\tilde{\gamma}$ , it is helpful to note that a change in  $\gamma$  affects the distribution of posterior beliefs through two margins. First, a change in  $\gamma$  affects the probability  $\lambda^*$  that each buyer acquires information; we refer to this as the “extensive” margin. Second, a change in  $\gamma$  affects the informational content of winning bids by its effect on the equilibrium bidding strategies; we refer to this as the “intensive” margin.

Suppose first that  $\gamma > \tilde{\gamma}$ , and consider the effect of a marginal decrease in  $\gamma$ . In this case, according to Proposition 3, the additional insurance afforded by a smaller value of  $\gamma$  causes  $\lambda^*$  to rise. Ceteris paribus, this change in the extensive margin increases the informational content of the winning bid. Moreover, a marginal change in  $\gamma$  has no effect on the intensive margin when

$\gamma > \tilde{\gamma}$ . Intuitively, holding  $\lambda$  constant, a marginal decrease in  $\gamma$  will cause an upward shift in the distribution of bids placed by type  $h$  buyers, but type  $u$  buyers will continue to pool completely with type  $\ell$  buyers by always bidding zero. As a result, though the winning bid increases (in expectation), the informational content of the winning bid remains unchanged. Hence, when  $\gamma > \tilde{\gamma}$ , a marginal decrease in  $\gamma$  leads to informational gains along the extensive margin and has no effect along the intensive margin, so the quantity of information is strictly decreasing in  $\gamma$  in the range  $(\tilde{\gamma}, 1]$ .

Now, suppose  $\gamma < \tilde{\gamma}$ , and consider the effect of a marginal decrease in  $\gamma$ . According to Proposition 3, increasing the amount of insurance provided by the government policy now decreases  $k^*$ , and thus  $\lambda^*$ . Ceteris paribus, this change in the extensive margin decreases the informational content of the winning bid. However, unlike in the previous case, a marginal change in  $\gamma$  will also affect the intensive margin when  $\gamma < \tilde{\gamma}$ . In fact, as  $\gamma$  decreases from  $\tilde{\gamma}$ , the probability that an uninformed buyer bids zero decreases. As a result, the observation that  $p = 0$  becomes more informative, as it becomes easier to determine whether this outcome was caused by informed buyers seeing the signal  $\ell$  or uninformed buyers bidding  $b = 0$ . Since  $\lambda^*$  is maximized at  $\tilde{\gamma}$ , the change in the extensive margin is of second order in a neighborhood of  $\tilde{\gamma}$ , so the effect on the intensive margin dominates. Hence,  $I(\gamma)$  is decreasing in  $\gamma$  in a neighborhood of  $\tilde{\gamma}$ . However, as  $\gamma$  decreases even further, the negative effect on the extensive margin effect eventually dominates the positive effect on the intensive margin. Hence, the value of  $\gamma$  that maximizes  $I(\gamma)$  lies in the interior of  $(0, \tilde{\gamma})$ .

## 4.2 Optimality and the *Value* of Information

We conclude this section with a word of caution: the policy that maximizes the quantity of information need not coincide with the policy that maximizes the value of information. Intuitively, the value of the information generated by PPIP depends on how and in what circumstances this information is used.

The following simple example helps illustrate our point. Suppose the goal of PPIP was to produce information that would unfreeze markets for assets that were similar (or even identical) to the asset being sold in a PPIP auction. More formally, consider the problem of a buyer and a seller

that meet after the outcome of the auction is reported. Suppose the seller has a single asset of the same quality as the asset sold in the PPIP auction, but the quality is the seller’s private information. For simplicity, suppose preferences and beliefs are unchanged: (i) the buyer has a prior belief that the asset is of quality  $H$  with probability  $\pi < c/v$  and of quality  $L$  with probability  $1 - \pi$ ; and (ii) the asset yields the buyer and seller utility  $v$  and  $c$ , respectively, if it is of quality  $H$  and yields both parties zero payoff otherwise. The buyer observes the winning bid in the auction, updates his posterior to  $\pi^+$  (as described above), and makes a take-it-or-leave-it offer.<sup>21</sup>

In the example under consideration, clearly the buyer will offer  $c$  if  $\pi^+ \geq c/v > \pi$  and 0 otherwise, so the net social surplus generated in this match is simply

$$\pi [1 - \Omega_H^*(c/v; \gamma)] (v - c). \quad (8)$$

From (8), it is clear that the policy that maximizes social surplus is the policy that maximizes the probability that  $\pi^+ \geq c/v$  when the asset is of quality  $H$ . In words, given the payoff structure in the example, a “positive” signal—that is, a winning bid that increases  $\pi^+$ —is unambiguously good: there is a social benefit from the signal being accurate when the asset is of high quality, but there is no penalty from the signal being inaccurate when the asset is of low quality. As a result, in this example the policy that maximizes the surplus between the buyer and the seller, and thus the value of information, is  $\tilde{\gamma}$ . Indeed, a consequence of the proof of Proposition 4 is that  $\Omega_H^*(c/v; \gamma)$  is maximized when  $\gamma = \tilde{\gamma}$ ; the improved precision at the lower tail of the distribution of posterior beliefs that follows from decreasing  $\gamma$  from  $\tilde{\gamma}$  is inconsequential for the problem at hand. Note, incidentally, that since  $\Omega_H^*(c/v; \gamma)$  converges to zero as  $\gamma$  decreases to zero, the policy that maximizes gains from trade in the auction,  $\gamma = 0$ , minimizes the value of information.

In general, there are many reasons why information produced by a program such as PPIP might be valuable (or detrimental) to other agents in the economy. For example, it could be that the information produced about the value of certain legacy assets provides a more accurate assessment

---

<sup>21</sup>It is trivial to relax a number of assumptions that we make in this example without altering any of the insights that follow. For one, the quality of the asset for sale need not be perfectly correlated with the quality of the asset sold in the PPIP auction. In addition, it is not necessary that there is a single buyer and seller, or that the terms of trade are determined by a take-it-or-leave-it offer; the results would be unchanged if there were a large number of buyers and sellers, and the good were sold in either a competitive market or in an auction setting. For a more detailed model of “liquidity spillovers”, see Cespa and Foucault [2012].

of the balance sheets of banks that own similar assets. This information could be valuable to depositors who have to decide whether or not to withdraw their funds from such a bank, or to invest additional funds.<sup>22</sup> The same information could also be valuable to regulators who have to decide whether or not to “bail out” those banks that face financial distress.<sup>23</sup> In both cases, more accurate asset prices could reduce the incidence of liquidating banks that would ultimately be solvent or providing additional capital to banks that would ultimately find themselves insolvent. An alternative theory of information spillovers is that the winning bids from PPIP auctions could allow for better investment decisions in similar or related products. For example, if the price of a pool of mortgage-backed securities contains information about the prospect of future real estate prices or foreclosure rates, this could improve the decision-making of agents like mortgage lenders and home builders.<sup>24</sup>

For each of the examples cited above, the fine details of the environment (that is, the prior beliefs, preferences, and payoff structure) will imply a different mapping from posterior beliefs into social welfare, and thus different choices of  $\gamma$  that maximize the value of information. Our intention here is not to argue that any of these examples are the “right” model of information spillovers. Rather, our analysis highlights the crucial trade-off that lies at the core of a program such as PPIP: the government can certainly encourage participation (and increase asset prices) by providing insurance to potential buyers, but at some point this insurance will start to erode the informational content of prices, reducing the efficacy of the government intervention. A consequence of this trade-off is a potential divergence between the policy that maximizes the gains from trade that are realized in a particular market ( $\gamma = 0$  in our setting) and the policy that maximizes price discovery. Hence, a lesson from our analysis is that ignoring the trade-off between participation and information generation—and the importance of price discovery, more generally—can potentially lead to substantively different prescriptions for the optimal policy.

---

<sup>22</sup>See Goldstein and Pauzner [2005], who provide a model that describes how information about fundamentals can change the probability of a bank run.

<sup>23</sup>Furlong and Williams [2006] provide an informative summary of the literature that studies how regulators respond to information produced in financial markets; see Bond and Goldstein [2012] for a recent theoretical contribution.

<sup>24</sup>Dow and Gorton [1997] and Foucault and Gehrig [2008], among many others, study the relationship between the information contained in asset prices and real economic efficiency.

## 5 Assumptions and Extensions

**The Number of Buyers.** The analysis above treats the number of buyers who participate in the auction,  $N$ , as exogenous. In reality, however, investors were required to apply in order to participate in PPIP, so one might consider  $N$  to be a second potential instrument available to policymakers.<sup>25</sup> Since the number of buyers participating in the auction clearly affects the equilibrium outcome, a natural question is how the informational content of the winning price is affected by a change in  $N$ .

The answer is that there are two, opposing effects. On the one hand, an increase in the number of participants decreases the buyers' expected payoffs, thereby depressing their incentives to acquire information; this result can be easily derived given the expressions for  $V_h$  in Proposition 1. As  $N$  increases,  $k^*$  decreases, and thus the probability that each buyer is informed,  $\lambda^* = G(k^*)$ , also falls. As  $N$  tends to infinity, for any  $\gamma$ ,  $V_h$  approaches zero and the buyers' incentive to acquire information evaporates. On the other hand, *ceteris paribus*, having more buyers implies more effective information aggregation. In particular, holding  $\lambda$  constant, increasing  $N$  reduces the risk that no buyer acquires information and, consequently, decreases the probability that socially valuable information is not generated.

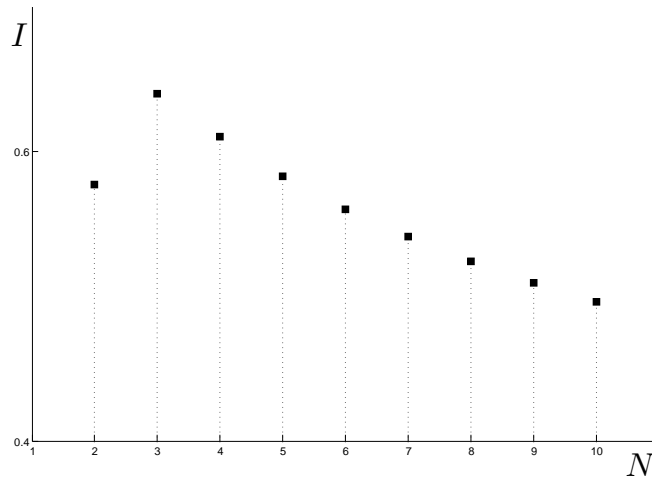
Figure 5 plots the typical shape of  $I(\gamma)$  across different values of  $N$ , where we set  $\gamma$  equal to the policy that maximizes  $I(\gamma)$  for each  $N$ . In this example,  $N = 3$  maximizes the quantity of information produced. Unfortunately, a precise characterization of the optimal  $N$  depends on, e.g., the exact shape of the distribution function  $G$ , and hence is not analytically tractable. However, as this example illustrates, the number of buyers is a second, important consideration for policymakers when designing a program of this type.

**Information Disclosure and Price Discovery.** The analysis above also assumes that the winning bid is observable, but that losing bids are not. This assumption was motivated by realism: PPIP reported the identity of the winning bidder and the price paid, but did not publish any information regarding the losing bids. From a theoretical point of view, this policy choice may seem at odds

---

<sup>25</sup>Of course, it is possible that a concern for fairness may constrain policymakers from excluding interested investors.

Figure 5: The Quantity of Information and  $N$



with the objective of maximizing price discovery; after all, given the analysis above, it should be fairly obvious that publishing the full vector of bids would be more informative about the quality of the asset.

However, the decision not to disclose losing bids may have been important for encouraging buyers to participate in the first place. More specifically, since investors in PPIP auctions could potentially compete in future auctions for similar assets, they may have found it valuable to keep their bids (and thus some of the private information they acquired) private for use at a later time. It is worth noting, however, that the decision to publish the number of bidders in each auction was a good one; using the winning bid to back out the underlying signals of the bidders is known to be considerably more complicated when the number of bidders is unknown.<sup>26</sup>

**Information Structure** We employed an extremely simple signal-generating process in our analysis: there were only two signals, and one signal was assumed to completely reveal the quality of the asset. Though this information structure greatly simplified our analysis, none of our central insights depend on it. In other words, the main lessons that arise from the analysis of our stylized environment carry over to a more general environment.

To see this, suppose an agent who chooses to incur the cost  $k_i$  can receive one of  $S$  signals.

<sup>26</sup>See, e.g., Athey and Haile [2007].

In this case, there are potentially  $S + 1$  types of bidders (including uninformed bidders) at the bidding stage. Then, as long as these signals satisfy the monotone likelihood ratio property, the single crossing property that applies to buyers' interim beliefs and their subsequent bids continues to hold: buyers who receive "better" signals place higher bids than those who receive "worse" signals. Moreover, it is straightforward to show that the bids of these agents—perhaps with the exception of agents who received the lowest signal—converge to  $v$  as  $\gamma$  converges to zero. As a result, as in Proposition 3, the incentive for buyers to acquire information vanishes as  $\gamma$  converges to zero. Similarly, for  $\gamma$  close to one, informed bidders would place serious bids only if their signal was sufficiently high. This also reduces buyers' ex ante incentives to acquire information about the quality of the asset, which hinders price discovery.

Therefore, as in our simple environment, the value of  $\gamma$  that maximizes price discovery in an environment with a more general signal structure will be interior: it will provide some insurance for buyers to participate in the auction by making serious bids, but not so much insurance that they make those bids without first inspecting the asset. Notice that this reasoning is independent of the likelihood of the lowest signal, so that our insights do not depend on the assumption that a "bad" signal perfectly reveals the low quality of the asset.

**Budget-Constrained Buyers.** Our analysis focuses on the insurance role that nonrecourse lending plays in alleviating the problem of adverse selection. There is, however, an alternative theory for why markets crash, often called "cash-in-the-market pricing" (see, e.g., Allen and Gale [1994]). According to this theory, markets can experience a sudden decrease in prices and trading volume because the buyers in the market are budget constrained: though they would like to purchase assets at the current market prices, they cannot acquire the liquid assets required to do so. Interestingly, by allowing private investors to leverage and thus relaxing their budget constraints, the loans offered by PPIP could also help address this second source of market freezes.

Suppose, for example, that each buyer  $i = 1, \dots, N$  has liquid wealth  $w_i$ , which is a random draw from a distribution with support  $[0, \bar{w}]$ . Moreover, suppose that bids are constrained by the inequality

$$\gamma b_i \leq w_i,$$

so that each buyer is required to finance a fraction  $\gamma$  of the purchase price with his liquid wealth. Clearly, if  $\bar{w}$  is sufficiently small, then buyers will not be able to bid  $b_i \geq c$ , and thus no information will be produced. Therefore,  $\gamma < 1$  certainly has the ability to promote trade and price discovery. However, as in our benchmark model, decreasing  $\gamma$  too much can potentially be counterproductive.

The reason is that *budget constraints relax the winner's curse*. In particular, in our benchmark model, the expected quality of the asset conditional on winning the auction is revised down because there is a probability that other buyers received the signal  $\ell$ . However, in a model with budget constraints, this effect is diminished; when a buyer wins the auction, it could be because other buyers received a signal  $\ell$ , but it could also be because they received a signal  $h$  but their budget constraint was binding. Therefore, since decreasing  $\gamma$  relaxes budget constraints, it also attenuates the winner's curse and thus decreases  $V_h$ . As a result, as in our benchmark model,  $k^*(\gamma)$  can also be non-monotonic in an environment where the market is frozen due to cash-in-the-market pricing. Since the analysis of this issue is fairly complex, we explore it in greater detail in a separate paper.

## 6 Conclusion

In the aftermath of the financial crisis that began in 2007, a literature has emerged to study the potential for government intervention to curb the losses associated with frozen markets. This paper contributes to this literature along two dimensions. In a narrow sense, we provide the first theoretical analysis of the Public-Private Investment Program for Legacy Assets. We believe that this, in and of itself, is important: a program that was approved to use up to one trillion dollars of taxpayer money warrants, at the very least, a formal theoretical model that allows policymakers to understand the relevant trade-offs and choose parameters to best navigate these trade-offs. More generally, our analysis sheds light on an important—yet largely overlooked—consideration for *any* market intervention: the effect on price discovery. We show that, in an environment in which information is valuable to various agents in the economy, a policy that simply maximizes gains from trade in a particular market will typically not be the policy that maximizes overall welfare.



# Appendix

## Proof of Lemma 1

Let  $V_s(b)$  be the expected payoff of a type  $s \in \{u, h\}$  buyer who bids  $b$ . In addition, denote by  $F_s(b_-)$  the probability that a type  $s$  buyer bids a price strictly less than  $b$ . For simplicity, let  $\xi_s(b) = [F_s(b_-) + F_s(b)]/2$ ; note that  $\xi_s(b)$  is nondecreasing in  $b$ . Then, for any  $b \geq c$ ,

$$\begin{aligned} V_s(b) &= \pi_s [(1 - \lambda)\xi_u(b) + \lambda\xi_h(b)]^{N-1} (v - b) \\ &\quad - (1 - \pi_s) [(1 - \lambda)\xi_u(b) + \lambda(\rho + (1 - \rho)\xi_h(b))]^{N-1} \gamma b, \end{aligned}$$

where  $\pi_u = \pi$  and  $\pi_h = \tilde{\pi}$ .

(i) The result is obvious if  $\bar{b}_u = 0$ . Suppose that  $\bar{b}_u \geq c$ . First note that if  $b \geq c$ , then

$$\begin{aligned} V_h(b) &= \frac{1}{\pi + (1 - \pi)(1 - \rho)} V_u(b) \\ &\quad + \frac{(1 - \pi)\rho}{\pi + (1 - \pi)(1 - \rho)} [(1 - \lambda)\xi_u(b) + \lambda(\rho + (1 - \rho)\xi_h(b))]^{N-1} \gamma b; \end{aligned} \quad (9)$$

recall that  $\tilde{\pi} = \pi/(\pi + (1 - \rho)(1 - \pi))$ . The second term in the right-hand side of (9) is strictly increasing in  $b$ . In addition, by the optimality of  $\bar{b}_u$  for a type  $u$  buyer, we have that  $V_u(\bar{b}_u) \geq V_u(b)$  for all  $b \in [c, \bar{b}_u]$ . It then follows that  $V_h(\bar{b}_u) > V_h(b)$  for all  $b \in [c, \bar{b}_u]$ , which implies that  $\underline{b}_h \geq \bar{b}_u$ .

(ii) Suppose  $F_h(0) = 1$ . By (i), this implies that  $F_u(0) = 1$  as well. Hence, the payoff to a type  $h$  buyer who bids  $b = c$  is equal to  $\tilde{\pi}(v - c) - (1 - \tilde{\pi})\gamma c$ , which is greater than zero by Assumption 2. Thus, bidding  $b = 0$  is suboptimal for a type  $h$  buyer, a contradiction.

(iii) First, we establish that there are no atoms on the relevant region of the support. Let

$$\alpha(\xi_u, \xi_h) = \left[ \frac{(1 - \lambda)\xi_u + \lambda(\rho + (1 - \rho)\xi_h)}{(1 - \lambda)\xi_u + \lambda\xi_h} \right]^{N-1}$$

and note that

$$V_h(b) = \tilde{\pi} [(1 - \lambda)\xi_u(b) + \lambda\xi_h(b)]^{N-1} (v - b) \times \left\{ 1 - \frac{(1 - \tilde{\pi})\gamma b}{\tilde{\pi}(v - b)} \alpha(\xi_u(b), \xi_h(b)) \right\}$$

for all  $b \geq \underline{b}_u$ . Since  $\alpha(\xi_u, \xi_h)$  is strictly decreasing in both  $\xi_u$  and  $\xi_h$ , it immediately follows that  $F_h$

cannot have an atom in  $[\underline{b}_h, \bar{b}_h]$ , for otherwise a type  $h$  bidder can obtain a strictly higher expected payoff by bidding slightly above the mass point. The same argument shows that  $F_u$  cannot have a mass point in  $[\max\{c, \underline{b}_u\}, \bar{b}_u]$ .

Now we establish that there are no gaps. Suppose  $F_u$  is constant in some interval  $[b_1, b_2] \subseteq (\max\{c, \underline{b}_u\}, \bar{b}_u]$ ; if  $b_1 = \max\{c, \underline{b}_u\}$ , then  $b_1$  is a mass point of  $F_u$ . Then, a type  $u$  bidder strictly prefers bidding  $b_1$  to  $b_2$ , for both bids imply the same and positive probability of winning, while the first bid implies a smaller payment. Thus,  $F_u(b)$  is strictly increasing in  $b$  when  $b \in [\max\{c, \underline{b}_u\}, \bar{b}_u]$ . A similar argument applies to  $F_h$ .

(iv) Suppose  $\underline{b}_u > 0$  and consider a type  $u$  buyer who bids  $\underline{b}_u$ . By (i) and (iii), the buyer wins if, and only if, all other buyers are of type  $\ell$ , which is only possible if the asset is of low quality. So, the expected payoff to the buyer is strictly negative, which cannot be the case.

### Proof of Proposition 1

We know from Lemma 1 that if  $b \geq c$ , then

$$V_s(b) = \pi_s [1 - \lambda + \lambda F_s(b)]^{N-1} (v - b) - (1 - \pi_s) [1 - \lambda + \lambda(\rho + (1 - \rho)F_s(b))]^{N-1} \gamma b,$$

where  $\pi_u = \pi$  and  $\pi_h = \tilde{\pi}$ . We also know from Lemma 1 that the following three mutually exclusive cases are also exhaustive:  $\bar{b}_u > c$ ,  $\bar{b}_u = 0$  and  $\underline{b}_h > c$ , and  $\underline{b}_h = 0$ .

**Case 1:**  $\bar{b}_u > c$ .

For each  $b \in [c, \bar{b}_u]$ ,  $F_u(b)$  is derived from the fact that  $V_u(b) = 0$ . In addition, combining  $F_u(\bar{b}_u) = 1$  with  $V_u(\bar{b}_u) = 0$ , we obtain

$$\bar{b}_u = \frac{\pi(1 - \lambda)^{N-1}}{\pi(1 - \lambda)^{N-1} + (1 - \pi)(1 - \lambda + \lambda\rho)^{N-1}\gamma} v.$$

We see immediately that  $\underline{b}_h = \bar{b}_u$  when  $\underline{b}_u > c$ . Hence,  $V_h$  is determined by considering a type  $h$  buyer who bids  $\bar{b}_u$ . From (9) in the proof of Lemma 1 and  $F_h(\bar{b}_u) = 0$ , we find that

$$V_h = \frac{(1 - \pi)\rho\gamma\bar{b}_u}{\pi + (1 - \rho)(1 - \pi)} (1 - \lambda + \lambda\rho)^{N-1}.$$

Substituting  $\bar{b}_u$  in the above expression for  $V_h$  and arranging the terms,  $V_h$  is obtained as in (4). For

each  $b \in [\underline{b}_h, \bar{b}_h]$ ,  $F_h(b)$  is derived from the fact that  $V_h(b) = V_h$ .

A necessary and sufficient condition for the equilibrium described in the above paragraph to exist is that  $F_u(c) \in (0, 1)$ . From  $V_u(c) = 0$ , we obtain

$$\left[1 + \frac{\lambda\rho}{(1-\lambda)F_u(c)}\right]^{N-1} = \frac{\pi(v-c)}{(1-\pi)\gamma c}.$$

Hence,  $F_u(c) > 0$  if, and only if,  $\gamma < \hat{\gamma} = \pi(v-c)/(1-\pi)c$ , and  $F_u(c) < 1$  if, and only if,  $\lambda < \underline{\lambda}(\gamma)$ ; note that  $\underline{\lambda}(\gamma) > 0$  if, and only if,  $\gamma < \hat{\gamma}$ .

Suppose now that  $\lambda < \underline{\lambda}(\gamma)$ , so that  $\gamma < \hat{\gamma}$  a fortiori. Then  $\bar{b}_u = 0$  implies that the payoff to a type  $u$  buyer from bidding  $b = c$  is at least

$$\pi(1-\lambda)^{N-1}(v-c) - (1-\pi)(1-\lambda+\lambda\rho)^{N-1}\gamma c,$$

which is positive given that  $\lambda < \underline{\lambda}(\gamma)$ . Thus,  $\bar{b}_u > c$  when  $\lambda < \underline{\lambda}(\gamma)$ .

**Case 3:**  $\underline{b}_h = 0$ .

Note that if type  $h$  buyers are indifferent between bidding  $b = 0$  and bidding  $b \in [c, \bar{b}_h]$ , then  $F_h(b)$  must be such that

$$V_h(b) = \tilde{\pi} [1 - \lambda + \lambda F_h(b)]^{N-1} (v - b) - \tilde{\pi} [1 - \lambda + \lambda(\rho + (1 - \rho)F_h(b))]^{N-1} \gamma b = 0.$$

A necessary and sufficient condition for this equilibrium to exist is that  $F_h(c) > 0$ . Straightforward algebra shows that  $F_h(c) > 0$  is equivalent to  $\lambda > \bar{\lambda}(\gamma)$ .

Suppose now that  $\lambda > \bar{\lambda}(\gamma)$ . Then  $\underline{b}_h > 0$  implies that

$$V_h(c) \leq \tilde{\pi}(1-\lambda)^{N-1}(v-c) - (1-\tilde{\pi})(1-\lambda+\lambda\rho)^{N-1}\gamma c < 0,$$

a contradiction. Thus,  $\underline{b}_h = 0$  when  $\lambda > \bar{\lambda}(\gamma)$ .

**Case 2:**  $\bar{b}_u = 0$  and  $\underline{b}_h > c$ .

We know from above that  $\bar{b}_u = 0$  and  $\underline{b}_h > c$  if, and only if,  $\lambda \in [\underline{\lambda}(\gamma), \bar{\lambda}(\gamma)]$ . Moreover, the analysis in the main text shows that there exists a unique equilibrium when  $\lambda \in [\underline{\lambda}(\gamma), \bar{\lambda}(\gamma)]$  and that  $V_h$  is given by (5) in this equilibrium.

### Proof of Proposition 3

Note that  $V_I(G(k), \gamma) \leq V_I(G(0), \gamma)$  for all  $k \geq 0$  by Lemma 2. Also note that  $V_I(G(0), \gamma)$  converges to zero as  $\gamma$  decreases to zero by Proposition 1 and the fact that  $G(0) < \underline{\lambda}(\gamma)$  if  $\gamma$  is small enough. Thus,  $k^*(\gamma)$  converges to zero as  $\gamma$  decreases to zero. A straightforward argument shows that  $k^*(\gamma)$  is also continuous in  $\gamma$ .

Now note that  $\underline{k}(\gamma)$  is continuous and nonincreasing in  $\gamma$ , with  $\underline{k}(0) = 1$  and  $\underline{k}(\gamma) = 0$  if  $\gamma \geq \pi(v - c)/(1 - \pi)c$ . Since  $k^*(\gamma) > 0$  for  $\gamma > 0$  and  $k^*(\gamma) < 1$  if  $\gamma$  is small enough, there exists  $\gamma \in (0, 1)$  such that  $k^*(\gamma) = \underline{k}(\gamma)$ . Let  $\tilde{\gamma} \in (0, 1)$  be the greatest value of  $\gamma$  such that  $k^*(\gamma) = \underline{k}(\gamma)$ . Then  $G(k^*(\tilde{\gamma})) = \underline{\lambda}(\tilde{\gamma})$  as  $\underline{k}(\tilde{\gamma}) = k^*(\tilde{\gamma}) > 0$ . Hence, by the reasoning in the main text,  $k^*(\gamma) < k^*(\tilde{\gamma})$  if  $\gamma < \tilde{\gamma}$ . Given that  $\underline{k}(\gamma) \geq \underline{k}(\tilde{\gamma})$ , there cannot exist any other  $\gamma$  such that  $k^*(\gamma) = \underline{k}(\gamma)$ .

### Proposition 5 and Proof

**Proposition 5.** *The equilibrium cutoff cost for inspecting the asset is  $k^* = 0$  when  $\gamma = 0$ .*

We prove that if  $\gamma = 0$ , then the expected payoff to a type  $h$  buyer is zero regardless of the probability  $\lambda$  that the other buyers become informed. Since this implies that  $V_I(\lambda, 0) = 0$ , it immediately follows that  $k^* = 0$  when  $\gamma = 0$ .

Suppose, by contradiction, that  $V_h > 0$ , so that  $\underline{b}_h \geq c$ . Given that

$$V_s(b) = \pi_s((1 - \lambda)\xi_u(b) + \lambda\xi_h(b))^{N-1}(v - b)$$

for all  $b \geq c$ , we then have  $V_u \geq V_u(\underline{b}_h) > 0$ , so that  $\underline{b}_u \geq c$  as well. A straightforward modification of the proof of (iii) of Lemma 1 shows that both  $F_u$  and  $F_h$  have no mass points on  $[c, v)$ . Since  $\underline{b}_s = v$  implies that  $V_s = 0$ , we then have that  $\underline{b} = \min\{\underline{b}_u, \underline{b}_h\} \in [c, v)$ . This, however, implies that  $V_u(\underline{b}) = V_h(\underline{b}_h) = 0$ , so that either  $V_u = 0$  or  $V_h = 0$ , a contradiction.

### Proof of Lemma 3

Consider first the case where  $\lambda \in (0, \underline{\lambda}(\gamma))$ , so that  $\bar{b}_u > c$ . Suppose  $p = 0$ . In this case, all buyers must be either of type  $\ell$  or of type  $u$ . Therefore,

$$\phi(0; \lambda, \gamma) = \frac{\pi}{\pi + (1 - \pi) \left[ 1 + \frac{\lambda\rho}{(1 - \lambda)F_u(0)} \right]^N}.$$

Now suppose  $p \in [c, \bar{b}_u]$ . In this case, the winner must be uninformed, while all other buyers are either of type  $\ell$  or of type  $u$  bidding below  $p$ , so that

$$\phi(p; \lambda, \gamma) = \frac{\pi}{\pi + (1 - \pi) \left[ 1 + \frac{\lambda\rho}{(1 - \lambda)F_u(p)} \right]^{N-1}}.$$

Since  $F_u(c) = F_u(0) > 0$  and  $F_u(p)$  is strictly increasing in  $p$  when  $p \in [c, \bar{b}_u]$ , it is easy to see that  $\phi(0; \lambda, \gamma) < \phi(c; \lambda, \gamma)$  and that  $\phi(p; \lambda, \gamma)$  is strictly increasing in  $p$  when  $p \in [c, \bar{b}_u]$ .

Finally, if  $p \in [\underline{b}_h, \bar{b}_h]$ , then the winner must be of type  $h$ , while any other buyer can be of type  $\ell$ , of type  $u$ , or of type  $h$  bidding less than  $p$ . Therefore,

$$\phi(p; \lambda, \gamma) = \frac{\pi}{\pi + (1 - \pi)(1 - \rho) \left\{ 1 + \frac{\lambda\rho[1 - F_h(p)]}{1 - \lambda + \lambda F_h(p)} \right\}^{N-1}}.$$

It is easy to see that  $\phi(\underline{b}_h; \lambda, \gamma) > \phi(\bar{b}_u; \lambda, \gamma)$ . Moreover, since  $F_h(p)$  is strictly increasing in  $p$  when  $p \in [\underline{b}_h, \bar{b}_h]$ , we have that  $\phi(p; \lambda, \gamma)$  is strictly increasing in  $p$  when  $p \in [\underline{b}_h, \bar{b}_h]$ . To finish, note that  $F_h(\bar{b}_h) = 1$  implies that  $\phi(\bar{b}_h; \lambda, \gamma) = \tilde{\pi}$ .

Consider now the case where  $\lambda \in [\underline{\lambda}(\gamma), \bar{\lambda}(\gamma)]$ . Then, since now  $F_u(0) = 1$ , we have that

$$\phi(0; \lambda, \gamma) = \frac{\pi}{\pi + (1 - \pi) \left[ 1 + \frac{\lambda\rho}{1 - \lambda} \right]^N}$$

and

$$\phi(p; \lambda, \gamma) = \frac{\pi}{\pi + (1 - \pi)(1 - \rho) \left\{ 1 + \frac{\lambda\rho[1 - F_h(p)]}{1 - \lambda + \lambda F_h(p)} \right\}^{N-1}}$$

for all  $p \in [c, \bar{b}_h]$ . We see immediately from the analysis above that  $\phi(0; \lambda, \gamma) < \phi(c; \lambda, \gamma)$ ,  $\phi(p; \lambda, \gamma)$  is strictly increasing in  $p$  when  $p \in [c, \bar{b}_h]$ , and  $\phi(\bar{b}_h; \lambda, \gamma) = \tilde{\pi}$ .

### Derivation of $\Omega_L^*(\pi^+, \gamma)$

The only change from  $\Omega_H^*(\cdot; \gamma)$  is that now each informed buyer is of type  $h$  with probability  $\lambda(1 - \rho)$  and of type  $l$  with probability  $\lambda\rho$ . Therefore, if  $\gamma < \tilde{\gamma}$ , then

$$\Omega_L^*(\pi^+; \gamma) = \begin{cases} 0 & \text{if } \pi^+ \in [0, \phi(0)) \\ [(1 - \lambda^*(\gamma))F_u^*(0) + \lambda^*(\gamma)\rho]^N & \text{if } \pi^+ \in [\phi(0), \phi(c)) \\ [(1 - \lambda^*(\gamma))F_u^*(\phi^{-1}(\pi^+)) + \lambda^*(\gamma)\rho]^N & \text{if } \pi^+ \in [\phi(c), \phi(\bar{b}_u)) \text{ ,} \\ (1 - \lambda^*(\gamma) + \lambda^*(\gamma)\rho)^N & \text{if } \pi^+ \in [\phi(\bar{b}_u), \phi(\bar{b}_h)) \\ \{1 - \lambda^*(\gamma) + \lambda^*(\gamma)[\rho + (1 - \rho)F_h^*(\phi^{-1}(\pi^+))]\}^N & \text{if } \pi^+ \in [\phi(\bar{b}_h), \phi(\bar{b}_h)] \end{cases} \quad (10)$$

and if  $\gamma \geq \tilde{\gamma}$ , then

$$\Omega_L^*(\pi^+; \gamma) = \begin{cases} 0 & \text{if } \pi^+ \in [0, \phi(0)) \\ (1 - \lambda^*(\gamma) + \lambda^*(\gamma)\rho)^N & \text{if } \pi^+ \in [\phi(0), \phi(c)) \text{ .} \\ \{1 - \lambda^*(\gamma) + \lambda^*(\gamma)[\rho + (1 - \rho)F_h^*(\phi^{-1}(\pi^+))]\}^N & \text{if } \pi^+ \in [\phi(c), \phi(\bar{b}_h)] \end{cases} \quad (11)$$

### Proof of Proposition 4

The proof consists of three steps. We first show that  $I(\gamma)$  is strictly decreasing in  $\gamma$  if  $\gamma > \tilde{\gamma}$ . We then show that  $I(\tilde{\gamma} - \varepsilon) > I(\tilde{\gamma})$  for  $\varepsilon$  positive but sufficiently small. We finally show that  $I(\tilde{\gamma}) > \lim_{\gamma \rightarrow 0} I(\gamma) = 0$ .

**Step 1.**  $I'(\gamma) < 0$  if  $\gamma > \tilde{\gamma}$ .

Suppose  $\gamma > \tilde{\gamma}$ . We begin by establishing some properties of  $\Omega^*(\cdot; \gamma)$  that are useful in the argument that follows. A straightforward consequence of Lemma 3 is that if  $\pi^+ \in [\phi(c), \tilde{\pi}]$ , then  $\phi(p) = \phi(p; \lambda^*(\gamma), \gamma) \leq \pi^+$  if, and only if,

$$a = \left[ \frac{(1 - \pi^+)\pi}{\pi^+(1 - \pi)(1 - \rho)} \right]^{\frac{1}{N-1}} \leq 1 + \frac{\lambda^*(\gamma)\rho [1 - F_h^*(p)]}{1 - \lambda^*(\gamma) + \lambda^*(\gamma)F_h^*(p)};$$

note that  $a \geq 1$  since  $\pi^+ \leq \tilde{\pi}$ . Hence,

$$F_h^*(\phi^{-1}(\pi^+)) = 1 - \frac{1}{\lambda^*(\gamma)} \frac{a-1}{a-1+\rho},$$

and so  $\lambda^*(\gamma)[1 - F_h^*(\phi^{-1}(\pi^+))]$  is independent of  $\gamma$  for all  $\pi^+ \in [\phi(c), \tilde{\pi}]$ . Therefore, (7) and (11) imply that  $\Omega^*(\pi^+; \gamma)$  is independent of  $\gamma$  when  $\pi^+ \in [\phi(c), \tilde{\pi}]$ . Another consequence of (7) and (11) is that

$$\Omega^*(\pi^+; \gamma) = \pi(1 - \lambda^*(\gamma))^N + (1 - \pi)(1 - \lambda^*(\gamma) + \lambda^*(\gamma)\rho)^N$$

for all  $\pi^+ \in [\phi(0), \phi(c)]$ . Thus, from Proposition 3, we have that  $\Omega^*(\pi^+; \gamma)$  is strictly increasing in  $\gamma$  when  $\pi^+ \in [\phi(0), \phi(c)]$ .

We now compute the derivative of  $\mathbb{E}[H(\pi^+)]$  with respect to  $\gamma$ . First note that

$$\begin{aligned} \mathbb{E}[H(\pi^+)] &= \Omega^*(\phi(0); \gamma)H(\phi(0)) + \int_{\phi(c)}^{\tilde{\pi}} H(\pi^+)d\Omega^*(\pi^+; \gamma) \\ &= H(\tilde{\pi}) + \Omega^*(\phi(0); \gamma) [H(\phi(0)) - H(\phi(c))] - \int_{\phi(c)}^{\tilde{\pi}} H'(\pi^+)\Omega^*(\pi^+; \gamma)d\pi^+, \end{aligned}$$

where the second equality follows from integration by parts and the fact that  $\Omega^*(\phi(c); \gamma) = \Omega^*(\phi(0); \gamma)$ . Given that

$$\frac{d}{d\gamma} \int_{\phi(c)}^{\tilde{\pi}} H'(\pi^+)\Omega^*(\pi^+; \gamma)d\pi^+ = -\Omega^*(\phi(c); \gamma)H'(\phi(c))\frac{d\phi(c)}{d\gamma}$$

by the Fundamental Theorem of Calculus and the fact that  $\Omega^*(\pi^+; \gamma)$  is independent of  $\gamma$  when  $\pi^+ \in [\phi(c), \tilde{\pi}]$ , we then have that

$$\frac{d\mathbb{E}[H(\pi^+)]}{d\gamma} = \frac{d\Omega^*(\phi(0); \gamma)}{d\gamma} [H(\phi(0)) - H(\phi(c))] + \Omega^*(\phi(0); \gamma)H'(\phi(0))\frac{d\phi(0)}{d\gamma}.$$

Since  $H(\phi(c)) < H(\phi(0)) + H'(\phi(0))(\phi(c) - \phi(0))$  by the strictly concavity of  $H(\phi)$  and  $d\Omega^*(\phi(0), \gamma)/d\gamma > 0$ , the equation for  $d\mathbb{E}[H(\pi^+)]/d\gamma$  derived above implies that

$$\frac{d\mathbb{E}[H(\pi^+)]}{d\gamma} > H'(\phi(0)) \left\{ -\frac{d\Omega^*(\phi(0); \gamma)}{d\gamma} \phi(c) + \frac{d}{d\gamma} [\Omega^*(\phi(0); \gamma)\phi(0)] \right\}.$$

We claim that the right-hand side of the above equation is zero. Indeed,

$$\frac{d\Omega^*(\phi(0); \gamma)}{d\gamma} = -N [\pi(1 - \lambda^*(\gamma))^{N-1} + (1 - \pi)(1 - \rho)(1 - \lambda^*(\gamma) + \lambda^*(\gamma)\rho)^{N-1}] \frac{d\lambda^*(\gamma)}{d\gamma},$$

and so Lemma 3 implies that

$$-\frac{d\Omega^*(\phi(0); \gamma)}{d\gamma} \phi(c) = N\pi(1 - \lambda^*(\gamma))^{N-1} \frac{d\lambda^*(\gamma)}{d\gamma}.$$

The desired result follows from the fact that Lemma 3 also implies that  $\Omega^*(\phi(0); \gamma)\phi(0) = \pi(1 - \lambda^*(\gamma))^N$ . We can then conclude that  $\mathbb{E}[H(\pi^+)]$  is strictly increasing in  $\gamma$  when  $\gamma \in (\tilde{\gamma}, 1]$ . This implies that  $I'(\gamma) < 0$  if  $\gamma > \tilde{\gamma}$ .

**Step 2.**  $I(\tilde{\gamma} - \varepsilon) > I(\tilde{\gamma})$  for  $\varepsilon$  positive but sufficiently small.

Suppose that  $\gamma < \tilde{\gamma}$ . By the same argument as in Step 1,  $\Omega^*(\pi^+; \gamma)$  is independent of  $\gamma$  when  $\pi^+ \in [\underline{b}_h, \tilde{\pi}]$ . In addition, as in Step 1, if  $\pi^+ \in [\phi(c), \phi(\bar{b}_h)]$ , then

$$F_u^*(\phi^{-1}(\pi^+)) = \frac{\lambda^*(\gamma)}{(1 - \lambda^*(\gamma))(\hat{a} - 1)},$$

where  $\hat{a} > 1$  is the only variable that depends on  $\pi^+$ . Hence, when  $\pi^+ \in [\phi(c), \phi(\bar{b}_u)]$ , (6) and (10) imply that  $\Omega^*(\pi^+; \gamma) = \Psi(\pi^+, \lambda^*(\gamma))$ , where  $\Psi(\pi^+, \lambda)$  is strictly increasing in  $\lambda$ . In particular,  $\Omega^*(\pi^+; \gamma)$  is strictly increasing in  $\gamma$  when  $\pi^+ \in [\phi(c), \phi(\bar{b}_u)]$  according to Proposition 3. Now observe from the proof of Proposition 1 that

$$(1 - \lambda^*(\gamma))F_u^*(0) = \frac{\lambda^*(\gamma)\rho}{(\hat{\gamma}/\gamma)^{1/(N-1)} - 1},$$

where  $\hat{\gamma} = \pi(v - c)/(1 - \pi)c$ . Hence, (6) and (10) together with Proposition 3 also imply that  $\Omega^*(\phi(0), \gamma)$  is strictly increasing in  $\gamma$ .

We now compute  $d\mathbb{E}[H(\pi^+)]/d\gamma$ . Integration by parts implies that

$$\begin{aligned} \mathbb{E}[H(\pi^+)] &= H(\tilde{\pi}) + \Omega^*(\phi(0); \gamma) [H(\phi(0)) - H(\phi(c))] + \Omega^*(\phi(\bar{b}_u); \gamma) [H(\phi(\bar{b}_u)) - H(\phi(\underline{b}_h))] \\ &\quad - \int_{\phi(c)}^{\phi(\bar{b}_u)} H'(\pi^+) \Omega^*(\pi^+; \gamma) d\pi^+ - \int_{\phi(\underline{b}_h)}^{\tilde{\pi}} H'(\pi^+) \Omega^*(\pi^+; \gamma) d\pi^+, \end{aligned}$$

where we used the fact that  $\Omega^*(\phi(c); \gamma) = \Omega^*(\phi(0); \gamma)$  and  $\Omega^*(\phi(\underline{b}_h); \gamma) = \Omega^*(\phi(\bar{b}_u); \gamma)$ . By the



Fundamental Theorem of Calculus and the fact that  $\Omega^*(\pi^+; \gamma)$  is independent of  $\gamma$  when  $\pi^+ \in [\phi(c), \tilde{\pi}]$ , we then have

$$\begin{aligned} \frac{d\mathbb{E}[H(\pi^+)]}{d\gamma} &= \frac{d\Omega^*(\phi(0); \gamma)}{d\gamma} [H(\phi(0)) - H(\phi(c))] + \Omega^*(\phi(0); \gamma) H'(\phi(0)) \frac{d\phi(0)}{d\gamma} \\ &\quad + \frac{d\Omega^*(\phi(\bar{b}_u); \gamma)}{d\gamma} [H(\phi(\bar{b}_u)) - H(\phi(\underline{b}_h))] - \int_{\phi(c)}^{\phi(\bar{b}_u)} H'(\pi^+) \frac{d\Omega^*(\pi^+; \gamma)}{d\gamma} d\pi^+. \end{aligned}$$

Since  $\Omega^*(\phi(0); \gamma)$  is strictly increasing in  $\gamma$  and  $H(\phi)$  is strictly concave in  $\phi$ ,

$$\begin{aligned} \frac{d\mathbb{E}[H(\pi^+)]}{d\gamma} &> H'(\phi(0)) \left\{ -\frac{d\Omega^*(\phi(0); \gamma)}{d\gamma} \phi(c) + \frac{d}{d\gamma} [\Omega^*(\phi(0); \gamma) \phi(0)] \right\} \\ &\quad + \frac{d\lambda^*(\gamma)}{d\gamma} \left\{ \frac{\partial \Psi(\pi^+; \lambda^*(\gamma))}{\partial \lambda} [H(\phi(\bar{b}_u)) - H(\phi(\underline{b}_h))] - \int_{\phi(c)}^{\phi(\bar{b}_u)} H'(\pi^+) \frac{\partial \Psi(\pi^+; \lambda^*(\gamma))}{\partial \lambda} d\gamma d\pi^+ \right\}. \end{aligned}$$

Now observe that  $\Omega^*(\phi(0); \gamma) \phi(0) = \pi[(1 - \lambda^*(\gamma))F_u^*(0)]^N$ . Moreover, straightforward algebra shows that

$$\frac{d\Omega^*(\phi(0); \gamma)}{d\gamma} \phi(c) = \frac{d}{d\gamma} [\Omega^*(\phi(0); \gamma) \phi(0)] + \phi(c)(1 - \pi)N[(1 - \lambda^*(\gamma))F_u^*(0) + \lambda^*(\gamma)\rho]^{N-1} \rho \frac{d\lambda^*(\gamma)}{d\gamma}.$$

Given that  $d\lambda^*(\tilde{\gamma})/d\gamma = 0$ , we can then conclude that

$$\left. \frac{d\mathbb{E}[H(\pi^+)]}{d\gamma} \right|_{\gamma=\tilde{\gamma}} > 0.$$

Hence, there exists  $\varepsilon > 0$  such that  $I(\gamma)$  is strictly decreasing in  $\gamma$  when  $\gamma \in (\tilde{\gamma} - \varepsilon, \tilde{\gamma})$ .

**Step 3.**  $I(\tilde{\gamma}) > \lim_{\gamma \rightarrow 0} I(\gamma) = 0$ .

Since  $I(\tilde{\gamma}) > 0$ , it suffices to show that  $I(\gamma)$  converges to zero as  $\gamma$  tends to zero. The result is a straightforward consequence of Proposition 3:  $\lim_{\gamma \rightarrow 0} k^*(\gamma) = 0$ , and so  $\Omega^*(\cdot; \gamma)$  converges to the degenerate distribution that assigns probability one to  $\pi$  as  $\gamma$  tends to zero.

## References

- Franklin Allen and Douglas Gale. Limited market participation and volatility of asset prices. *The American Economic Review*, pages 933–955, 1994.
- Kenneth J. Arrow. The value of and demand for information. *Decision and Organization*, 2: 131–139, 1972.
- Susan Athey and Philip A. Haile. Nonparametric approaches to auctions. *Handbook of Econometrics*, 6:3847–3965, 2007.
- Dirk Bergemann and Juuso Valimäki. Information acquisition and efficient mechanism design. *Econometrica*, 70(3):1007–1033, 2002.
- Dirk Bergemann, Xianwen Shi, and Juuso Välimäki. Information acquisition in interdependent value auctions. *Journal of the European Economic Association*, 7(1):61–89, 2009.
- Philip Bond and Itay Goldstein. Government intervention and information aggregation by prices. *Working Paper, University of Pennsylvania and University of Minnesota*, 2012.
- Philip Bond, Alex Edmans, and Itay Goldstein. The real effects of financial markets. *Annual Review of Financial Economics*, 4(1), 2012.
- Antonio Cabrales, Olivier Gossner, and Roberto Serrano. Entropy and the value of information for investors. *American Economic Review*, 103(1):360–77, 2013.
- Giovanni Cespa and Thierry Foucault. Illiquidity contagion and liquidity crashes. *Manuscript, HEC Paris*, 2012.
- V.V. Chari, Ali Shourideh, and Ariel Zetlin-Jones. Adverse selection, reputation and sudden collapses in secondary loan markets. *NBER Working Paper*, 2010.
- Jonathan Chiu and Thorsten V. Koepl. Market making of last resort: Optimal intervention when financial markets are frozen. *Working Paper, Bank of Canada and Queen’s University*, 2011.
- Douglas W Diamond and Raghuram G Rajan. Illiquid banks, financial stability, and interest rate policy. *Journal of Political Economy*, 120(3):552–591, 2012.
- James Dow and Gary Gorton. Stock market efficiency and economic efficiency: Is there a connection? *The Journal of Finance*, 52(3), 1997.
- Darrell Duffie, Nicolae Gârleanu, and Lasse Heje Pedersen. Over-the-counter markets. *Econometrica*, 73(6):1815–1847, 2005.
- Darrell Duffie, Nicolae Gârleanu, and Lasse Heje Pedersen. Valuation in over-the-counter markets. *Review of Financial Studies*, 20(6):1865, 2007.
- Emmanuel Farhi and Jean Tirole. Collective moral hazard, maturity mismatch and systemic bailouts. *The American Economic Review*, 102:60–93, 2012.

- Thierry Foucault and Thomas Gehrig. Stock price informativeness, cross-listings, and investment decisions. *Journal of Financial Economics*, 88(1):146–168, 2008.
- Frederick T. Furlong and Robard Williams. Financial market signals and banking supervision: Are current practices consistent with research findings? *Federal Reserve Bank of San Francisco Economic Review*, pages 17–29, 2006.
- Nicolae Gârleanu. Portfolio choice and pricing in illiquid markets. *Journal of Economic Theory*, 144(2):532–564, 2009.
- Itay Goldstein and Ady Pauzner. Demand–deposit contracts and the probability of bank runs. *the Journal of Finance*, 60(3):1293–1327, 2005.
- Veronica Guerrieri and Robert Shimer. Competitive equilibrium in asset markets with adverse selection. *University of Chicago, Working Paper*, 2011.
- Christopher House and Yusufcan Masatlioglu. Managing markets for toxic assets. *NBER Working Paper*, 2010.
- Matthew O. Jackson. Efficiency and information aggregation in auctions with costly information. *Review of Economic Design*, 8(2):121, 2003.
- Ilan Kremer. Information aggregation in common value auctions. *Econometrica*, 70(4):1675–1682, 2002.
- Stephan Laueremann and Asher Wolinsky. Search and adverse selection. *mimeo*, 2013.
- Benjamin Lester and Braz Camargo. Trading dynamics in decentralized markets with adverse selection. *Federal Reserve Bank of Philadelphia, Working Paper No. 11-36*, 2011.
- Jacob Marschak. *Remarks on the Economics of Information*. Defense Technical Information Center, 1959.
- Steven Matthews. Information acquisition in discriminatory auctions. *Bayesian models in economic theory*, ed. by M. Boyer, and R. Kihlstrom, 49:1477–1500, 1984.
- Paul R Milgrom. A convergence theorem for competitive bidding with differential information. *Econometrica: Journal of the Econometric Society*, pages 679–688, 1979.
- Paul R Milgrom. Rational expectations, information acquisition, and competitive bidding. *Econometrica: Journal of the Econometric Society*, pages 921–943, 1981.
- Nicola Persico. Information acquisition in auctions. *Econometrica*, 68(1):135–148, 2000.
- Wolfgang Pesendorfer and Jeroen M Swinkels. The loser’s curse and information aggregation in common value auctions. *Econometrica: Journal of the Econometric Society*, pages 1247–1281, 1997.
- Wolfgang Pesendorfer and Jeroen M Swinkels. Efficiency and information aggregation in auctions. *American Economic Review*, pages 499–525, 2000.

- Thomas Philippon and Philipp Schnabl. Efficient recapitalization. *Journal of Finance*, forthcoming., 2011.
- Thomas Philippon and Vasiliki Skreta. Optimal interventions in markets with adverse selection. *The American Economic Review*, 102(1):1–28, 2012.
- Claude E. Shannon. A mathematical theory of communication. *Bell Systems Technical Journal*, 27:379–423, 1948.
- Christopher A. Sims. Implications of rational inattention. *Journal of Monetary Economics*, 50(3): 665–690, 2003.
- Jean Tirole. Overcoming adverse selection: How public intervention can restore market functioning. *The American Economic Review*, 102(1):29–59, 2012.
- Dimitri Vayanos and Pierre-Olivier Weill. A search-based theory of the on-the-run phenomenon. *Journal of Finance*, 63(3):1361–1398, 2008.
- Laura L. Veldkamp. *Information Choice in Macroeconomics and Finance*. Princeton University Press, 2011.
- Robert Wilson. A bidding model of perfect competition. *The Review of Economic Studies*, pages 511–518, 1977.