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This paper formally models the Public Private Investment Partnership (PPIP), a plan for U.S. government sponsored purchases of distressed assets. This paper solves both the problem of the asset manager buying toxic assets and the banks selling toxic assets. It solves for the fair market value of toxic assets implied by subsidized toxic asset sales, and it estimates the size of the government's subsidy. Moreover, this paper finds the circumstances under which banks and asset managers will meet at mutually acceptable prices. In general, healthier banks will be more willing sellers of toxic assets than zombies.

Journal of Economic Literature Codes: G12, G13, G18, G21, G28, G38

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1. Introduction

The purchases of so called toxic assets have figured prominently in the U.S. Treasury's attempts to "cleanse" the banking system of bad loans. Both Secretary Henry "Hank" Paulson, Jr., under President George W. Bush, and Secretary Timothy Geithner, under President Barak Obama, have attempted to buy bad assets from banks. Mr. Paulson's attempts culminated in the funding of the $700 billion Troubled Asset Relief Program. He abandoned the troubled assets part of the moniker when he initiated the Capital Purchase Program (CPP) in mid-October 2008 a few weeks after the legislation was signed by President Bush. The CPP bought preferred stock and warrants in "healthy banks."

U.S. Secretary of the Treasury Timothy Geithner subsequently took up this idea of buying bad assets. He released the details of this plan to buy troubled assets in March 23, 2009. Unlike the Resolution Trust Corporation, used to clean up the Savings and Loan Crisis of the 1980s and 1990s, the Public Private Investment Partnership (PPIP) announced by Mr. Geithner planned to buy troubled assets from banks that had not yet failed. This plan proposed to partner with the Federal Deposit Insurance Corporation (FDIC), the Federal Reserve Bank of New York, and the U.S. Treasury to provide inexpensive financing to private investors to buy up to $500 billion to $1 trillion of toxic assets.[1] While it seems clear that the U.S. Treasury will not participate in such a large scale of toxic asset purchases, it has already spent tens of billions of dollars to buy toxic assets on behalf of taxpayers.

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[1] Timothy Geithner, March 23, 2009, "My Plan for Bad Bank Assets," *Wall Street Journal*, accessed online on January 24, 2010, at http://online.wsj.com/article/SB123776536222709061.html.

This paper attempts to formally model the incentives of both asset managers buying toxic assets and banks selling toxic assets under the framework of the Public Private Investment Partnership (PPIP), which is currently being used to buy high yield real estate backed securities and loans with U.S. federal government funds. This is the only paper to formally model the relationship between the interest rates offered on government subsidized debt and the overbidding incentives for toxic assets. It provides closed form solutions to the prices that would prevail in the toxic asset sales, the fair market value of the assets based on prevailing prices, and the levels of expected subsidies involved in toxic asset sales.

This paper uses a binomial option pricing structure to model the joint buying and selling decisions of asset managers and banks, respectively. It proceeds as follows. In section 2, the details of the U.S. government's toxic asset plans pursued in 2008, 2009, and 2010 are discussed. In the section 3, we discuss the relevant literature. In section 4, we introduce the model and discuss the asset manager's problem. In section 5, we generate closed-form solutions for the government's subsidy. Next, in section 6, we discuss the incentives of the bank disposing of the toxic assets and pursue a numerical example. In section 7, we discuss an extension of the model in continuous time, and numeric solutions are generated from the Black and Scholes (1973) approach in that section. Finally, in section 8, the paper concludes.

2. Toxic Asset Purchase Programs

There are three toxic asset programs which have been undertaken by the U.S. government since 2009. The Term Asset-Backed Securities Loan (TALF) was sponsored by the Federal Reserve Bank of New York and the U.S. Treasury. It bought over $11 billion of commercial mortgage backed securities (CMBS). The Legacy Loans Program (LLP) of the Federal Deposit Insurance Corporation (FDIC) is ongoing and has exclusively sold troubled bank loans from the FDIC's receivership estates. The U.S. Treasury's Legacy Securities Program (LSP) is scheduled to buy $29.4 billion worth of legacy CMBS and residential mortgage backed securities RMBS. Only the first and third programs, the TALF and the LSP, are supported by taxpayer bailout funds, the TARP. More importantly for this study only the TALF and the LSP will buy toxic assets from open banks. Since this paper models the decision of an open bank to sell its toxic assets it is most relevant to studying the TALF and LSP. Nevertheless, many of the results concerning the overbidding incentives of the asset manager are also relevant to the FDIC's LLP. All three programs subsidize the purchase of toxic assets through low interest rate, non-recourse loans.[2]

The New York Federal Reserve sponsored subsidized purchases of Commercial Mortgage Backed Securities (CMBS) from June 2009 to June 2010. The Fed offered below-market, non-recourse loans to private market participants to buy CMBS. (Non-recourse loans allow the borrower to walk away from the loan without declaring bankruptcy. The lender, the Federal Reserve and the U.S. Treasury, are stuck holding the collateral but cannot go after any other assets of the borrower.) The Term Asset-Backed Securities Loan (TALF) planned to sponsor up to $200 billion in purchases of asset

[2] Through July 2010, these LSP, the LLP, and the TALF, programs have sponsored the purchase of $16.2 billion, $7.3 billion, and $11.5 billion in toxic assets, respectively. Both the LSP and the LLP are continuing to fund the purchase of distressed real estate securities and loans.

backed securities (ABS), securitizing CMBS, auto loans, credit cards, small business loans, and student loans for private sector participants. According to U.S. Treasury (2010), this program closed after only sponsoring $43 billion in deals between its commercial mortgage and consumer loan programs. The U.S. Treasury through TARP committed to contributed $20 billion to the TALF lending in the form of subordinated debt, which would absorb the first losses on any TALF loans that go bad. Yet, only $4.3 billion of subordinated debt was issued to support these loans through TARP because of the reduced size of the overall program.

Questions about the ability of the government to be a passive investor have scared away many potential participants. Moreover, many banks are reluctant to take write-downs on loans that are not always required to be held on balance sheets at fair market value. (The process of re-pricing assets is often referred to "marking-to-market." This process is required of legacy securities, but loans that are held to maturity do not have to be marked down, according to current accounting rules enacted on April 2, 2009.[3]) In addition, the $104.3 billion of common equity and $103.4 billion of preferred and trust preferred stock raised by banks and savings associations in 2009 have caused many to question whether a toxic asset plan is still necessary.[4] On June 5, 2009, Chairwoman of the FDIC Sheila Bair announced that her agency would be scaling back its efforts in the legacy loans portion of the toxic asset purchases to a pilot program.[5] Since then the LLP

[3] Yalman Onaran, June 5, 2009, "Bank Profits from Accounting Rules Masking Looming Loan Losses," *Bloomberg* accessed online on January 25, 2010, at http://www.bloomberg.com/apps/news?pid=newsarchive&sid=alC3LxSjomZ8.
[4] Ravi Panchal and Carmen Hoskin, January 12, 2010, "Capital Raises Among Banks and Thrifts in 2009," *SNL Financial*.
[5] David Enrich, Liz Rappaport, and Jenny Strasburg, June 29, 2009, "Wary Banks Hobble Toxic-Asset Plan," *Wall Street Journal*, accessed online on July 1, 2009, at http://online.wsj.com/article/SB124622976702566007.html. The pilot sale was of residential mortgage loans of a failed bank, Franklin Bank, which were owned by the FDIC in receivership was conducted in

has only been used to finance the sale of receivership assets. It has not "cleansed" the balance sheets of open banks.

The FDIC is unlikely to buy whole loans from open banks. The FDIC has a liquidity problem because the premiums that have been paid into the deposit insurance fund are insufficient to cover the losses that it has incurred from bank failures since 2008. While the FDIC does have a $500 billion line of credit with the U.S. Treasury, the Chairwoman of the FDIC Sheila Bair has expressed reluctance to use that line of credit for fear of the political ramifications.[6] Currently, the FDIC is having member banks pre-pay three years or about $45 billion of deposit insurance premiums to solve the liquidity problem of the fund.[7] The U.S. Treasury has been the subject of withering attacks for its management of the Troubled Asset Relief Program (TARP). If the FDIC taps the line of credit, the fear is that the FDIC will be perceived as the recipient of a taxpayer bailout. These funding problems make the FDIC's sponsorship of Legacy Loans Program sales of toxic assets of banks outside of receivership less likely.

September 2009 for $856.2 million. See Wilson (2010a) for a fuller description of this sale. Another sale of commercial loans of another failed bank, Corus Bank, using the legacy loans structure for $2.772 billion was conducted in October 2009. See Monica Hatcher, October 8, 2009, "There's a new player in South Florida's condo market," *Miami Herald*, accessed online on January 24, 2010, at http://www.miamiherald.com/business/story/1271817.html and FDIC, October 6, 2009, "PRESS RELEASE: Corus Bank Assets – Winning Bidder Announced, Transaction Details" accessed online on January 25, 2010, at http://www.fdic.gov/news/news/press/2009/pr183details.pdf and http://www.fdic.gov/news/news/press/2009/pr09183.html. Through October 2010 all legacy loans sales have been of FDIC receivership assets. Thus, the seller was never an open bank in the Legacy Loans Program. Instead, the seller was the FDIC. Because this paper is about the incentives of both the buyer and an open bank seller, it is not as relevant to analyzing legacy loans sales in receivership. Wilson (2010a) analyzes legacy loans transactions.

[6] Jessica Holzer, September 18, 2009, "FDIC Considers Borrowing from Treasury to Shore Up Deposit Insurance," *Wall Street Journal*, accessed online on January 24, 2010, at http://online.wsj.com/article/SB125328162000123101.html. The speech and response to the question on borrowing from the U.S. Treasury is made at the "Future of Global Finance Conference" accessible online on September 18, 2009, at http://webcast.georgetown.edu/79009.html.

[7] Stephen Labaton, September 30, 2009, "Banks to Prepay Assessments to Rescue F.D.I.C.," *New York Times*, A1, accessed online on January 25, 2010, at http://www.nytimes.com/2009/09/30/business/economy/30regulate.html.

Yet, both the Federal Reserve Bank of New York[8] and the U.S. Treasury moved forward with scaled back plans to sponsor purchases of Commercial Mortgage Backed Securities (CMBS) and Residential Mortgage Backed Securities (RMBS) through the TALF and the Public Private Investment Partnership's Legacy Securities Program (PPIP-LSP). From June 16, 2009, to June 18, 2010, the Term Asset-Backed Securities Loan (TALF) has sponsored approximately $11.4 billion in legacy CMBS deals and has sponsored $72 million in newly issued CMBS with low interest rate three-to-five-year loans.[9] Leverage ratios for TALF CMBS purchases can be as much as 85 percent of the purchase prices of the toxic assets. That is, maximum leverage ratios are 5.67-to-1.[10] That program is closed for new investment at the time of writing.

The U.S. Treasury hired nine asset managers for its Legacy Securities Program (LSP),[11] which are eligible to receive, up to $22.1 billion from taxpayer funds to purchase up to $29.4 billion of residential mortgage backed securities (RMBS) and commercial mortgage backed securities (CMBS) from open financial institutions and other holders of these assets. These nine asset managers have access to low interest rate loans of maturities of eight-to-ten years to purchase RMBS and CMBS securities.[12] Maximum leverage ratios in this program are 1-to-1.

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[8] Before becoming U.S. Treasury Secretary, Timothy Geithner was President of the Federal Reserve Bank of New York.

[9] These figures were compiled from the Federal Reserve Bank of New York's website at http://www.newyorkfed.org/markets/cmbs_operations.html on October 19, 2010.

[10] July 21, 2010, "Term Asset-Backed Securities Loan Facility: Frequently Asked Questions," Federal Reserve Bank of New York, accessed online on November 29, 2010, at http://www.ny.frb.org/markets/talf_faq.html.

[11] U.S. Treasury, December 18, 2009, "PRESS RELEASE: Treasury Department Announces Additional Initial Closing of Legacy Securities Public-Private Investment Fund," accessed online on January 24, 2010, at http://www.treas.gov/press/releases/tg457.htm.

[12] Taxpayers were to contribute a maximum of $20 billion in non-recourse debt and $10 billion in equity in the PPIP-LSP. The asset managers would have contributed a maximum of $10 billion for a total purchasing power of $40 billion in the Legacy Securities Program. Yet, the asset managers only raised $7.4 billion from private investors before the program closed to new private investors.

The full extent of the gains or losses from the TALF and LSP may not be realized for three-to-ten years when these low interest rate loans mature. The early returns for the PPIP-LSP were good.[13] From October 2009, when the first asset manager bought toxic assets, to June 30, 2010, the average return from the eight funds still in operation was 15.5 percent on the equity portions and about 1.3 percent on the taxpayers' debt investments. Thus, the blended asset returns on the first $16.2 billion of funds invested was approximately 8.4 percent. Moreover, since taxpayers own one half of the equity and all the debt in these deals, taxpayers' overall returns were closer to 6 percent.[14] A positive return is good, but these investments appear to have underperformed similar distressed mortgage funds from September 30, 2009, to June 30, 2010. Hedgefund.net reports that mortgage funds returned 21.5 percent over that period.[15] Thus, taxpayers likely have negative risk-adjusted returns over this period.

It seems that many asset managers believed they would be the beneficiaries of subsidies in the PPIP-LSP. The eagerness of asset managers to participate in the PPIP-LSP despite the fears of political scrutiny and the uncertainty surrounding the program probably reflects their belief that the profits generated from subsidized government finance outweighs the risk. According to the U.S. Treasury press release, over 100 asset

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[13] This start date is estimated from page 18 of U.S. Treasury, December 18, 2009, "Dividends and Interest Report as of November 30, 2009," accessed online on January 26, 2010, at http://financialstability.gov/docs/dividends-interest-reports/November%202009%20Dividends%20and%20Interest%20Report.pdf. It seems likely that Invesco bought the first toxic assets in October 2010 since it was the first asset manager to pay monthly interest in November 2009.

[14] Eric Dash, August 26, 2010, "A Big Surprise: Troubled Assets Garner Rewards," *New York Times*, B1, accessed online on October 19, 2010, at http://www.nytimes.com/2010/08/27/business/27toxic.html.

[15] See July 22, 2010, "HFN Industry Research June Performance Report," *Hedge Fund Net*, accessed online on October 19, 2010, at http://www.hedgefund.net/dailyemailreports/HFN_Industry_Report_June2010.pdf.

managers applied for PPIP-LSP.[16] Nine asset managers were selected. The principal managers selected were AllianceBernstein, Angelo, Gordon & Co. and GE Capital Real Estate, BlackRock. Invesco, Marathon Asset Management, Oaktree Capital Management, RLJ Western Asset Management, TCW Group, and Wellington Management Company.[17] Subsequently, the TCW Group has withdrawn as an asset manager.[18]

3. Relevant Literature

There are several papers discussing various aspects of government plans to buy toxic assets from banks. Nevertheless, unlike this paper, no other discusses the relationship between the Public Private Investment Partnership (PPIP) structure and the interest rate charged on the government subsidized loans. Further, most papers do not model the joint decision of the buyers and sellers of toxic assets. The present paper focuses on the joint incentives of a shareholder controlled bank and an asset manager to participate in the PPIP.

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[16] U.S. Treasury Department Office of Public Affairs, April 29, 2009, "Treasury Announces Receipt of Applications to Become Fund Managers under Public Private Investment Program" accessed online on May 10, 2009, at http://financialstability.gov/latest/tg109.html. The minimum requirements for an asset manager were the ability to raise $500 million dollars. Although some selected managers came just short of that goal.

[17] U.S. Treasury, July 8, 2009, "PRESS RELEASE: Joint Statement Secretary Of The Treasury Timothy F. Geithner, Chairman Of The Board Of Governors Of The Federal Reserve System Ben S. Bernanke, And Chairman Of The Federal Deposit Insurance Corporation Sheila Bair on the Legacy Asset Program," Accessed online on January 24, 2010, at http://financialstability.gov/latest/tg_07082009.html.

[18] Because of the firing of the Senior Mortgage Securities Fund's chief investment officer Jeffrey Gundlach, the US Treasury and TCW Group closed and planned to liquidate the TCW PPIP-LSP fund. See Eleanor Laise, January 4, 2010, "TCW Pulling Out of PPIP," *Wall Street Journal*, accessed online on January 24, 2010, at http://online.wsj.com/article/SB10001424052748704160504574640862583910586.html. The U.S. Treasury made a $356.25 million investment prior to liquidating the TCW fund according to U.S. Treasury, "Office of Financial Stability, Troubled Asset Relief Program Transaction Report for Period Ending January 15, 2010," accessed online on January 24, 2010, at http://financialstability.gov/docs/transaction-reports/1-20-10%20Transactions%20Report%20as%20of%201-15-10.pdf.

Wilson (2010b) less formally discusses the incentives of the asset manager and the payoffs to the FDIC from toxic asset sales from assets held in receivership estates. In contrast, this paper models toxic asset sales by banks which have not yet failed. Wilson (2010b) does not model the incentives of a solvent bank to sell toxic assets, since the bank is assumed to be controlled by the FDIC. Nor does Wilson (2010b) formally model the relationship between the interest rate offered on the government-sponsored debt and the asset manager's maximum willingness to pay.

Wilson (2009) and Wilson and Wu (2010) argue that if banks sell their toxic assets this may improve these banks' lending and investment incentives *ex post*. Yet, the government's willingness to do this creates moral hazard problems *ex ante*. Moreover, buying toxic assets will only be effective if these large and complex institutions are prevented from purchasing similarly risky assets. For example it was prudent that Goldman Sachs, Bank of America, Citigroup, J.P. Morgan, Morgan Stanley, Wells Fargo, and any other banks subject to the 2009 stress tests, were not allowed to be buyers of toxic assets under the Legacy Securities Program. Nevertheless, GE Capital, one of the first nine asset managers, was not subjected to the stress test, but its large balance sheet of over $600 billion in assets, nearly the size of Lehman Brother's balance sheet when it failed in September 2008, could conceivably pose systemic risks.[19] Moreover, there is some evidence that large banks have increased their holdings of distressed real estate

[19] The stress test moniker derives from a phrase in Secretary Geithner's, February 10, 2009, speech outlining his plans to rehabilitate the financial industry. The less popular, official title of this program administered by the Federal Reserve is the Supervisory Capital Assessment Program (SCAP). All the nineteen institutions that participated in SCAP had over $100 billion in assets. See Board of Governors of the Federal Reserve, "Supervisory Capital Assessment Program: Overview of Results," Federal Reserve, accessed online on January 25, 2010, at http://www.federalreserve.gov/newsevents/press/bcreg/bcreg20090507a1.pdf. Peter Eavis, June 18, 2009, "GE Capital's Political Minefield," *Wall Street Journal*, C10 has a good discussion of the regulatory loopholes GE capital has slipped under.

securities. These purchases ostensibly defeat the purpose of the government "cleansing" these banks of their toxic assets with subsidized purchases.[20]

Wilson (2009) and Wilson and Wu (2010) find that, in terms of minimizing the subsidies necessary for inducing efficient lending and voluntary participation, toxic asset sales and common stock sales are equally effective. Yet, if toxic asset sales are voluntary and common stock issues are compulsory for large banks, as in the Federal Reserve's stress tests or when the large banks repaid their TARP preferred stock, then compulsory common stock sales will strictly dominate voluntary, and thus subsidized, toxic asset sales.

Another related study is Wilson (2010a). That study does explain why troubled banks will not sell toxic assets outside of bankruptcy even if selling those assets for fair market value is at or above accounting book value. Wilson (2010a) argues that the volatility provided by troubled assets is valuable to the zombie bank, but it is of little no value to the buyer. This explains the purported lack of trades in the market for toxic assets. Yet, Wilson (2010a) does not directly model the PPIP and the bidding incentives of the asset manager.

Bebchuck (2009) provides much common sense guidance about not allowing big banks to be buyers of toxic assets. That study also suggests that there should be several asset managers competing to participate in the program. Yet, unlike this paper, it lacks any formal modeling of how asset prices are determined. Unlike this study, it has nothing to say about what kinds of banks will voluntarily participate in the program.

[20] Christopher Condon and Jody Shenn, January 4, 2010, "No Good Deed Goes Unpunished as Banks Seek Profits (Update1)," *Bloomberg* accessed online on January 25, 2010, at http://www.bloomberg.com/apps/news?pid=newsarchive&sid=aFyVxmuF6DTQ report that some large banks increased their holdings of toxic assets as the Legacy Securities Program began its purchases.

The potential for subsidies in the PPIP comes from the non-recourse loans provided by the U.S. Treasury in the Legacy Securities Program, the FDIC-backed, non-recourse loans in the Legacy Loans Program, and the Fed's Term Asset-Backed Securities Loan (TALF) for commercial mortgage backed securities (CMBS). Two recent Nobel Prize winners, among others, have pointed out that the non-recourse loans in the PPIP encourage private investors to overbid. Nevertheless, the respective editorial and blog post assume the guarantee fee and interest charged is zero.[21] This paper argues that the situation is more nuanced if the interest rate on the debt is positive.

Non-recourse loans allow a borrower to walk away from the loan if he or she cedes the collateral tied to the loan. The borrower's non-collateral assets are not at risk. In housing markets, lenders can only seize the home in foreclosure, but they cannot seize other assets of the borrower. Pavlov and Watcher (2002; 2009a; 2009b) demonstrate that non-recourse loans in housing markets can lead to inflated prices when lenders undervalue the put option embedded in mortgage loans.

Bhansali and Wise (2009) model the embedded put option in the non-recourse TALF loans. The loans and the collateral can be given back to the New York Federal Reserve Bank if the assets in the portfolio underperform. In particular, Bhansali and Wise (2009) model the TALF puts as being written on a portfolio of partially correlated securities. They find that the value of these puts can be in excess of 7 percent if the assets have an annual volatility of 20 percent and a correlation of 50 percent. The value of the puts rises to up to 23.6 percent of the asset price if the assets have an annual

[21] Joseph Stiglitz, April 1, 2009, "Obama's Ersatz Capitalism," *New York Times*, accessed online on April 18, 2009 at http://www.nytimes.com/2009/04/01/opinion/01stiglitz.html.; Paul Krugman, "Geithner Plan Arithmetic," Conscience of a Liberal Blog, *New York Times*, accessed online on May 9, 2009 at http://krugman.blogs.nytimes.com/2009/03/23/geithner-plan-arithmetic/.

volatility of 50 percent and a correlation of 50 percent. Since these loans charge interest rates often little more than 100 basis points than similar maturity U.S. Treasuries, five year puts that are worth more than 5 percent (5 times 100 basis points) of the loan amount entail subsidies for investors' borrowing in the TALF.

Zheng (2009), attempts to formally model PPIP bidding. Yet, ironically, Zheng (2009) does not understand that PPIP loans are non-recourse. Thus, the whole premise of Zheng (2009)—that the asset manager's non-collateral assets are at risk—is flawed, and few conclusions can be drawn from that paper. Moreover, Wilson (2011) points out that Zheng (2009) incorrectly assumes that the pricing mechanism for toxic asset sales for the Legacy Securities Program (LSP) will be a second price auction among all the government sponsored bidders. Auctions for toxic assets are not held in either the LSP or the TALF according to Wilson (2011). Asset managers in these programs are not necessarily in direct competition for the same toxic assets.

In contrast, the present paper shows that setting the guarantee fee or the interest rate premium above the risk free rate at an appropriate level is tricky business. The underlying assets of most toxic securities in the PPIP, mortgage loans and securities, are notoriously hard to value. If the guarantee fee is too high or the leverage offered is too low, then the government capital will not be attractive unless there are severe liquidity problems in credit markets. If the guarantee fee is too low or the leverage is too high, taxpayers are exposed to massive losses. Bhansali and Wise (2009) do formally model how the haircuts, the amount equity supplied by private investors, on TALF deals affects the value of the non-recourse financing. Yet, they don't model the joint decisions of buyers and sellers to engage in trade.

This study finds the conditions under which buyers and sellers will meet at a mutually acceptable price for toxic assets. Moreover, it allows the reader to estimate the costs to taxpayers. With observable parameters and the volatility of the toxic asset, it is possible to derive the fair market value of the asset. The market value of the assets is NOT the price paid in the toxic asset sale. The guarantee fee as a function of leverage can also be solved for when the volatility of the toxic asset is known. When the volatility of the bank's assets after selling the toxic securities is also known, this paper provides guidance for the circumstances under which will trades occur. This is the first study of this kind to the author's knowledge. There is no theoretical study that examines the circumstances under which trades will occur in Geithner's toxic asset plan using parameters such as the interest rate charged and the volatility of the asset purchased.

4. The Private Investor's Problem

In period 0, the private investor buys the toxic asset with the government's help. In period 1, the asset's returns are realized and the private investor either pays back the government's loan in full or she cedes the returns from the toxic asset to the government. Suppose that there is a bank that has toxic mortgages that are currently worth M. M is the discounted present value of all cash flows associated with this asset at time 0. They are worth $uM > M$ if things go well in one period in the future. They are worth $dM < M$ if things go poorly in the next period. Let us define

$$u \equiv e^{\sigma\sqrt{t}} \equiv 1/d, \tag{1}$$

where σ is the annual standard deviation of the asset and t is the time interval in years between period 0 and period 1. More concretely, in the presentation that follows, t is the time to maturity of the non-recourse loan. The risk free rate of return is r_f. This is the periodic rate. Unless otherwise mentioned, all interest rates, and guarantee fees are quoted as periodic (not annualized) rates in this paper. To prevent arbitrage opportunities where an investor can buy the assets and always earn returns equal to or better than the risk free rate or buy the risk free asset and earn returns that are always better than buying the toxic asset, $u > 1 + r_f > d$. (We exclude the possibility that there is either a zero or one probability that the up or down state will occur.) The private investor is buying a call option when participating in the PPIP. Let $0 \leq \lambda \leq 1$, be the fraction of the purchase price that is in the form of non-recourse loans. For the Legacy Loan Program, the maximum leverage, λ, is 6/7; for the Legacy Securities Program the maximum leverage is ½ of total assets; and, for the TALF's CMBS purchases, the maximum leverage is 85 percent of the purchase price. Let us denote M^* as the purchase price implied by adding up the face value of the government loan plus the call options owned by the U.S. government and the private investor. This is what the U.S. government says is the value of the asset. This is not the actual market value of the asset because the government loan is worth much less than its face value. (The value of the asset can be calculated as the fair market value of the sum of its debt and equity claims.) Bhansali and Wise (2009) and Wilson (2010b) make it clear that the value of the government loan or debt is the present value of its promised interest and principal less a put option, which reflects the chance that the loan or debt will be defaulted upon. Using puts to value risky debt is standard practice in fixed income analysis since Merton (1974).

Let us assume that the private investor pays at least the toxic assets minimum value. That is, $M^* \geq dM$. The loan from the government pays back principal and interest. It pays the risk-free rate, r_f, plus a spread over the risk-free rate, g, charged by the government for the loan, $R \equiv r_f + g$. If the toxic asset purchase is through the Legacy Securities Program R, is the interest rate that the U.S. Treasury charges the asset manager on the non-recourse loan.

The FDIC charges borrowers no interest on its loans. Thus, $g = -r_f$ in the LLP. g is the premium or discount above the risk-free rate on a U.S. Treasury issued loan. The Legacy Securities Program (LSP) charges an interest rate of the 30-day LIBOR plus 100 basis points for the maximum leverage ratio of one-to-one.[22] The TALF loans pay similar interest rates. We will use the terminology guarantee fee for the spread over the risk-free rate on the government loans.

We will assume that the private investor will not bid so high that she will always default on her non-recourse loan. Thus strike price of this call option, X, is

$$dM \leq X \equiv \lambda M^*(1 + r_f + g) \leq uM \tag{2}$$

[22] See U.S. Treasury, "Legacy Securities Public-Private Investment Partnership Summary of Indicative Terms and Conditions of UST Debt," accessed online on January 24, 2010, at http://www.financialstability.gov/docs/S-PPIP_LOI_Term-Sheets.pdf. This term sheet says that Legacy Securities Program loans have a maturity of 10 years. Exhibit A of that document says that the loans pay the one month LIBOR plus 100 basis points if the maximum leverage of one-to-one is selected. If one-half-to-one leverage is selected the loans pay the 30-day LIBOR plus 200 basis points.

Using the replicating portfolio approach an option is worth some combination of the risk-free asset, *B*, plus a fraction, Δ, of the underlying toxic asset. If In period 1, the call option is worth either

$$\begin{aligned} \Delta uM + B(1+r_f) &= uM - X \\ \Delta dM + B(1+r_f) &= 0 \end{aligned} \tag{3}$$

With two equations and two unknowns, a little algebra will verify that

$$\Delta = \frac{uM - X}{(u-d)M} > 0 \quad \& \quad B = -\frac{d(uM - X)}{(1+r_f)(u-d)} < 0 \tag{4}$$

The value of the call option today is worth

$$C_0 = \Delta M + B. \tag{5}$$

We can solve for the value of the call option today by inserting Δ and *B* in equation (4) into equation (5). Thus, the value of this call option today, C_0, is the following:

$$C_0 = \frac{(1+r_f - d)(uM - X)}{(1+r_f)(u-d)} \geq 0 \tag{6}$$

Let us define the variable *q* below:

$$0 \leq q \equiv \frac{1+r_f-d}{u-d} \leq 1 \tag{7}$$

This is often referred to as the risk-neutral probability of the option. As Copeland and Antikarov (2003, p. 98) point out, the risk-neutral probability is not the objective probability of the up state occurring. Instead, it is a mathematical convenience that allows us to discount the up and down state value of this option by the risk-free rate. Substituting in equation (7) and (6), the value of the call option simplifies to the following:

$$C_0 = \frac{q(uM-X)}{1+r_f} \geq 0 \tag{8}$$

Yet, the private investor only gets a fraction ϕ of this call option, where $0 \leq \phi \leq .1$. Under the current terms of the LSP, the private investor gets $\phi = ½$ of this call option in exchange for putting up an investment of

$$\phi(1-\lambda)M^* \tag{9}$$

For the PPIP's Legacy Loans Program, the minimum amount put down is $(1/14)M^*$. For the LSP the private investors put down a minimum of $M^*(1/4)$ or one fourth of the purchase price. [23] Thus, the private investors profit is defined as π_I below:

[23] Sources: FDIC, "Legacy Loans Program: Fact Sheet," accessed online on April 18, 2009 http://www.fdic.gov/llp/LLPFactSheet.pdf and U.S. Treasury, "Legacy Securities Public-Private Investment Partnership Summary of Indicative Terms And Conditions Of UST Debt," accessed online on January 24, 2010, at http://www.financialstability.gov/docs/S-PPIP_LOI_Term-Sheets.pdf.

$$\pi_I = \phi\left[\frac{q}{1+r_f}\left(uM - \lambda M^*(1+r_f+g)\right) - (1-\lambda)M^*\right] \geq 0 \qquad (10)$$

The private investors' will only bid a price less than or equal to M_H^* because any price paid higher than M_H^* would cause them to lose money on the transaction:

$$M^* \leq \frac{Mqu}{(1+r_f)(1-\lambda) + q\lambda(1+r_f+g)} \equiv M_H^* \qquad (11)$$

This confirms Krugman's[24] observation in his less formal numerical example, that fraction of "equity" contribution by the private investor, ϕ, is irrelevant to that investor's maximum willingness to pay.

Proposition 1
The private investor's maximum willingness to pay, M_H^*, is increasing in the amount of leverage, λ, offered as long as the guarantee fee, g, on the loan is not too large.

If we differentiate the maximum willingness to pay of the investor, M_H^*, with respect to the percent of leverage offered λ it is clear that the maximum willingness to pay is increasing in the amount of leverage offered as long as the guarantee fee, g, is not too large. That is,

[24] Paul Krugman, "Geithner Plan Arithmetic," Conscience of a Liberal Blog, *New York Times*, accessed online on May 9, 2009 at http://krugman.blogs.nytimes.com/2009/03/23/geithner-plan-arithmetic/.

$$\frac{\partial M_H^*}{\partial \lambda} = \frac{quM[(1+r)(1-q)-qg]}{\{(1+r)(1-\lambda)+q\lambda(1+g+r)\}^2} > 0,$$
$$\text{when } g < (1+r)\frac{1-q}{q} \equiv \bar{g} \tag{12}$$

$\bar{g}$ is the maximum value of the guarantee fee plus liquidity fee, such that the investor will accept any leverage. *Q.E.D.*

If $g < \bar{g}$, some levels of leverage could be profitable. The true value of the toxic mortgages is *M*. Thus the overpayment is

$$M_H^* - M = M\left(\frac{qu-[(1+r_f)(1-\lambda)+q\lambda(1+r_f+g)]}{(1+r_f)(1-\lambda)+q\lambda(1+r_f+g)}\right) \tag{13}$$

Whether or not the private investor is willing to pay above or below fair market value for the toxic assets depends on the magnitude of the leverage offered in relation to the other parameters.

Lemma 1
There is a strictly positive minimum level of leverage necessary for the investor to overbid for the toxic assets.

Namely, $M_H^* - M \geq 0$ if the numerator of (13) is positive. If the amount of leverage, λ, satisfies the following ratio, then the private investor will be willing to bid at or above the fair market value for the toxic asset if she partakes in the non-recourse financing:

$$\lambda \geq \frac{1 + r_f - qu}{(1 + r_f)(1 - q) - qg} \equiv \underline{\lambda} > 0, \qquad (14)$$

$$\text{when } g < \overline{g}$$

It can be shown that the numerator in (14) is equal to

$$\frac{1 - d(1 + r_f)}{u - d} > 0 \qquad (15)$$

Equation (15) is positive because the no arbitrage condition requires that $d < 1/(1 + r_f)$. The denominator in (14) must be positive if the guarantee fee, g, is not so large that the investor finds it unprofitable to accept any leverage.

Thus for leverage $\lambda < \underline{\lambda}$ there will be no sales through the PPIP program because the private investor is unwilling to pay at or above fair market value for the toxic assets. At levels of leverage above $\underline{\lambda}$, the private investor will be willing to pay above fair market value. If the bank is willing to sell at such prices, then trade can occur through the PPIP. When the guarantee fee is zero, equation (14) simplifies to the following relationship:

$$(1 + r_f)\lambda \geq d, \text{ when } g = 0. \qquad (16)$$

That is, the investor will only bid at or above the fair market value, with this financing, if the non-recourse loan payment $(1 + r_f)\lambda M$, is greater than or equal to the asset value in the worst case scenario, dM. If that is true, then the investor can take advantage of the fact that she can walk away if the mortgage returns turn bad. Otherwise, if $(1 + r_f)\lambda M$ <

dM, the put option embedded in the non-recourse debt is never exercised and is worthless.

The prices that prevail in the PPIP sales are the prices of call options on the asset—not the price of the asset itself—as Stiglitz[25] correctly points out. Using this analysis we can approximate the true, market value of the asset. Some caution should be given; over long time intervals, the binomial model's approximation with just one up or down movement will deviate significantly from the more accurate Black and Scholes (1973) and Merton (1973) value of an option. To address this concern in section 7 the model has been extended to the Black and Scholes (1973)'s continuous time framework. The continuous time model of section 7 can generate numerical approximations for estimation purposes, but it cannot generate the closed-form solutions which are obtained in the binomial approach used in the other parts of this paper.

Using option pricing theory, we can approximate the winning investor's belief about the true price of the toxic asset. Suppose that we believe that the pricing mechanism will have the buyer of the asset pay $\mu \in [0,1]$ of their maximum valuation of the winning asset. If the pricing is optimal from the seller's point of view, then the private investor will pay 100 percent of her maximum willingness to pay $\mu = 1$. Let,

$$M^* \equiv \mu M_H^* . \tag{17}$$

Proposition 2
The fair market value of the asset, M, diverges from the price paid in the Public-Private Investment Partnership sales price, M^*.

[25] Joseph Stiglitz, April 1, 2009, "Obama's Ersatz Capitalism," *New York Times*, accessed online on April 18, 2009 at http://www.nytimes.com/2009/04/01/opinion/01stiglitz.html.

The true price of the asset is

$$M = \frac{M^*}{\mu q u}\left\{(1+r_f)(1-\lambda) + q\lambda(1+r_f+g)\right\}, \tag{18}$$

This is obtained by substituting (17) into the expression for M_H^* in equation (11) and inverting the relationship. *Q.E.D.*

To estimate the market value of the toxic asset, *M*, the observer must estimate several parameters beyond observing M^*, the price paid for the asset. For example, if someone wants to use the legacy loans auctions to mark similar loans to market prices, that person should estimate the market value from equation (18), not from the sales price of the toxic asset, M^*. If the PPIP sale observer finds that the price is M^*, she must estimate the following and plug those into equation (18) above:

1. The fraction, μ, of the buyer's maximum willingness to pay that was actually paid. μ will likely be close to one for a well-designed and competitive auction from the perspective of the seller of the toxic asset. Yet, only the Legacy Loans Program (LLP) holds an auction among government subsidized investors. Most of the toxic assets purchased have been bought through the Legacy Securities Program (LSP) and the TALF. In those latter programs the managers select securities to buy with government loans. Thus, they may be paying far less than their maximum willingness to pay.
2. The risk free rate over the holding period. This is relatively easily found by finding a Treasury yield of similar maturity.

3. The leverage ratio. This should be disclosed by the FDIC and the U.S. Treasury. For example, in the Legacy Loans Program (LLP) auction the maximum leverage ratio, debt-to-value ratio, is 6/7. For the Legacy Securities Program (LSP), which does not use auctions, the maximum debt to value ratio is ½. For the TALF's CMBS program, the maximum debt to value ratio is 85 percent.

4. $g \equiv R - r_f$. It can be obtained by comparing the T-note rate, r_f, to the interest rate, R, in the loan agreement. For variable rate agreements, the premium will be the relevant LIBOR swap rate plus the spread in the agreement (100 or 200 basis points) less the relevant T-note rate.

5. The volatility of the toxic asset sold, σ. This must be estimated. The volatility of the toxic asset sold will determine the values of u, d, and q. u and d are defined in equation (1), and q is defined in equation (6).

Since market prices must be estimated from PPIP prices, it may be fair to say that the PPIP may obscure rather than illuminate true market prices to the casual observer.

5. The Government's Subsidy

To calculate the subsidy in PPIP, we must consider the value of the government's entire position and the price it pays for that position. We have solved for the net gain from the private investor's call option position in equation (10). The net value of government's call option position is $\frac{1-\phi}{\phi}\pi_I$. As announced in the fact sheet, the U.S.

Treasury would put in half the equity $\phi = ½$, and the private investor would put in half the equity. The second part of the government's position is a loan guarantee. The guarantee is a put option written by the government. The premium or fee for this put option is γM^*. This fee, γM^*, is only paid in full when the loan is paid back. Otherwise, the government gets the toxic asset, which is worth less than X. The put option can be derived using put-call parity:

$$P_0 = C_0 + \frac{X}{1+r_f} - M \tag{19}$$

The parameterized version of equation (19) can be obtained by combining equation (8) and (2).

The guarantee fee γM^* only is paid when returns are good. When returns are poor, $dM \leq M^*\lambda(1 + r_f + \theta)$ no guarantee fee is paid. When $M^*\lambda(1 + r_f + \theta) < dM \leq X$, then part (or all) the guarantee fee is paid in the low demand state. Let us define a variable, χ, that captures the fact that in some instances (rarely) only part of the guarantee fee will be paid in the low demand state:

$$0 \leq \chi \equiv \frac{\max\{[dM - M^*\lambda(1+r_f + \theta)], 0\}}{\gamma M^*} \leq 1 \tag{20}$$

The value of the guarantee fee can be estimated using the replicating portfolio method, where the subscript, γ, on δ and B denotes that this is the replicating portfolio for the loan guarantee fee:

$$\begin{aligned} \Delta_\gamma uM + (1+r_f)B_\gamma &= \gamma M^* \\ \Delta_\gamma dM + (1+r_f)B_\gamma &= \chi\gamma M^* \end{aligned} \tag{21}$$

If we solve for Δ_γ and B_γ above and insert those solutions into the equation (22) below,

$$V_0^\gamma = \Delta_\gamma M + B, \tag{22}$$

then his leaves us with the following expression

$$V_0^\gamma = \frac{M^*\gamma[(1+r_f-d)-\chi(u+r_f-1)]}{(1+r_f)(u-d)}. \tag{23}$$

The government's profit, π_G, is the following:

$$\pi_G = V_0^\gamma - P_0 + \frac{1-\phi}{\phi}\pi_I \tag{24}$$

A closed form solution for present value of the government's profit, which will be a loss if there is any trading with PPIP funds, can be obtained by inserting in equations (23), (19), and (10) into equation (24).   Thus, Secretary Geithner's toxic asset subsidy is approximately negative one multiplied by equation (24).

6. The Troubled Bank's Problem

Suppose that a bank has total liabilities of D_i coming due at time t, where i denotes the type of zombie bank. $i = a$, b, or c. By definition, $D_a > D_b > D_c$. For simplicity, D_i is the future value of the both interest and principal. We will assume that there are no intermediate payments on the liabilities. All the liabilities mature at time t. To make this bank a zombie or at least a potential zombie, let us assume that $dM \leq D_i$. Yet it has some hope because $uM > D_i$. Merton (1974) argues that a firm's share price is a call option written on the firm's assets, M, with a strike price that is the liabilities of the firm, D_i here. Using the replicating portfolio approach to find the binomial value of the zombie bank's share price today, the troubled bank's share price can be replicated by holding Δ_Z shares and a quantity of the risk-free asset B_Z.

$$\begin{aligned} \Delta_Z uM + B_Z(1+r_f) &= uM - D_i \\ \Delta_Z dM + B_Z(1+r_f) &= 0 \end{aligned} \tag{25}$$

We can solve for these two equations and two unknowns.

$$\Delta_Z = \frac{uM - D_i}{(u-d)M} > 0 \quad \& \quad B_Z = -\frac{d(uM - D_i)}{(1+r_f)(u-d)} < 0 \tag{26}$$

Today, the value of the call option on the bank's assets is worth

$$S_0^Z = \Delta_Z M + B_Z \tag{27}$$

Combining equations (26) and (27), the value of the troubled bank's stock is the following:

$$S_0^Z = \frac{q(uM - D_i)}{(1+r_f)} > 0 \tag{28}$$

The risk-neutral probability, q, is defined in equation (7).

Suppose the troubled bank sold its toxic mortgages for a price $M^* > M$. It would give up its risky assets and would hopefully be constrained by regulators from acquiring similarly risky assets. The volatility of its new assets would be $\underline{\sigma} \leq \sigma$, and the spread of its asset values would be $\underline{u} = e^{\underline{\sigma}\sqrt{t}}$ and $\underline{d} = 1/\underline{u}$, where $\underline{u} \leq u$ and $\underline{d} \geq d$. Using the replicating portfolio approach once again, there are three scenarios:

A. $\underline{d}M^* < \underline{u}M^* \leq D_a$, and the bank certainly fails if it sells the toxic assets. Shareholders get nothing in this scenario if they sell the toxic assets. Thus, they won't sell toxic assets. This is the well-known write-down effect. Troubled banks will not sell their bad loans because it will leave the stock worthless.

B. $\underline{d}M^* \leq D_b < \underline{u}M^*$, and the bank fails if asset returns are bad. This is similar to the situation when the bank still held the toxic assets. The potential losses to creditors are lower if the bank sells its toxic asset.

C. $D_c < \underline{d}M^* < \underline{u}M^*$, and the bank is always solvent. The troubled bank is restored to health by the toxic asset sale.[26]

a. Scenario B: the Troubled Bank is Still Potentially Insolvent

Scenario A is a non-starter. The troubled bank isn't going to sell its toxic assets and push its stock price from a positive number in (28) to zero just to sell its toxic assets. Therefore, we will turn our attention to scenario B, where $\underline{u}M^* > D_b \geq \underline{d}M$. The replicating portfolio is the following:

$$\begin{aligned} \Delta_b \underline{u}M + B_b(1+r_f) &= \underline{u}M - D_b \\ \Delta_b \underline{d}M + B_b(1+r_f) &= 0 \end{aligned} \tag{29}$$

The delta and position in the risk-free asset in the replicating portfolio are the following:

$$\Delta_b = \frac{\underline{u}M^* - D_b}{(\underline{u}-\underline{d})M^*} > 0 \quad \& \quad B_b = -\frac{\underline{d}(\underline{u}M^* - D_b)}{(1+r_f)(\underline{u}-\underline{d})} < 0 \tag{30}$$

[26] Wilson (2009) and Wilson and Wu (2010) argue that it is only in this scenario that lending decisions will dramatically improve. Yet, those papers would caution that in many instances the cost of the subsidy may outweigh the benefits even if moral hazard problems are ignored.

The risk-neutral probability is the following in both scenario B and C:

$$0 \le \underline{q} \equiv \frac{1 + r_f - \underline{d}}{\underline{u} - \underline{d}} \le 1 \tag{31}$$

The value of the stock after the toxic asset sale in scenario B is the following:

$$S_0^b = \frac{\underline{q}(\underline{u}M^* - D_b)}{1 + r_f} \ge 0 \tag{32}$$

The value of the stock is strictly increasing in the volatility of the firm's assets.

$$\frac{\partial S_0^Z}{\partial \sigma} = \frac{\sqrt{t}}{(1 + r_f)(u - d)^2}\left\{[(1 - d(1 + r_f))(uM - D_i)] + [(1 - (1 + r_f)u)(dM - D_i)]\right\} > 0 \tag{33}$$

An equivalent argument to equation (33) can be made for $\frac{\partial S_0^b}{\partial \underline{\sigma}} > 0$.

The value of the stock is strictly increasing in the volatility of the toxic assets. We know this because $1 - d(1 + r_f) > 0$ because $d < 1/(1 + r_f)$. We know this by combining our no arbitrage condition, $u > 1 + r_f$, and the relationship $u = 1/d$. $uM > D_i$. Thus, the first term in square brackets is the multiple of two positive sums and therefore is positive. In addition, $dM - D_i < 0$. The no arbitrage condition indicates that $1 - (1 + r_f)u < 0$ because $u > 1 + r_f$. Thus, the term in the second square brackets is the multiple of two negative terms and is the positive. Therefore, the sign of this derivative is unambiguously positive.

Equation (33) indicates that the troubled bank's stock price will fall when it sells its toxic assets if $M^* = M$ and $\sigma > \underline{\sigma}$.

There is a countervailing influence that makes it attractive for the troubled bank to sell. If $M^* > M$, then it is possible that the higher price outweighs the lost volatility.

$$\frac{\partial S_0}{\partial M} = \frac{q}{1 + r_f} > 0 \tag{34}$$

The troubled bank will only sell if $S_0^b - S_0^Z \geq 0$. Yet, the sign of the voluntary participation constraint for B banks, VP_b, is ambiguous. We cannot sign the constraint without further specifying the parameter values.

$$VP_b = S_0^b - S_0^Z = \frac{1}{(1 + r_f)}[\underline{q}\underline{u}M^* - quM] \gtreqless 0, \text{ when } M^* > M \tag{35}$$

The reasons for the ambiguity of the sign of equation (35) are discussed in the appendix.

b. Scenario C: The Troubled Bank is Cured by Selling its Toxic Assets

In scenario C, the overpayment for the toxic assets is sufficiently high that the bank is restored to solvency in all possible (two) states of the world. In this case the replicating portfolio is the following:

$$\begin{aligned}\Delta_c \underline{u}M^* + B_c(1+r_f) &= \underline{u}M^* - D_c \\ \Delta_c \underline{d}M^* + B_c(1+r_f) &= \underline{d}M^* - D_c\end{aligned} \tag{36}$$

In this case the replicating portfolio is created by

$$\Delta_c = 1 \quad \& \quad B_c = -\frac{D_c}{1+r_f}. \tag{37}$$

Thus the stock is worth the following:

$$S_0^c = M^* - \frac{D_c}{1+r_f} > 0 \tag{38}$$

Clearly, the stock price in scenario C is not affected by the volatility of the stock after the toxic asset sale. That is,

$$\frac{\partial S_0^c}{\partial \underline{\sigma}} = 0 \tag{39}$$

This is because the potential for default no longer exists. Therefore, the bank's shareholders can no longer benefit from dumping the bad assets, $\underline{d}M^*$, onto creditors. It certainly is good news for taxpayers if a bank that is "too-big-to-fail" has no chance of failing. Nevertheless, the subsidies necessary, $M^* - M$, to approximate that condition may be prohibitively large!

The sign of the voluntary participation constraint for a type C bank is generally ambiguous:

$$VP_c = S_0^c - S_0^Z = M^* - \frac{1}{1+r_f}(quM + (1-q)D_c) \underset{<}{\overset{\geq}{-}} 0, \text{when } M^* > M \tag{40}$$

The volatility of the assets after the PPIP sale, $\underline{\sigma}$, does not affect the maximum price that investors are willing to pay for the toxic assets, M_H^*, and the opportunity cost, S_0^Z, of selling them.

c. The Healthy Bank's Problem

Suppose that there is a healthy bank that will be selling toxic assets. This means that $uM > dM \geq D_H$. Thus, the healthy bank has fewer liabilities than the troubled banks $D_H < D$. In this case, the share price of this bank is merely the following:

$$S_0^H = M - \frac{D_H}{1+r_f} \tag{41}$$

The argument for why this is the case is analogous to the replicating portfolio problem in equations (36), (37), and (38) in the previous section. Let us assume that after the toxic asset sale the healthy bank's assets have no greater volatility than before the sale. That means that the share price before and after does not depend on the volatility of the toxic assets. The derivation of the stock value after the toxic asset sale for the healthy bank in equation (42) below is identical to the derivation of equation (38) by way of equations (36) and (37) for the scenario C bank in the previous subsection. Thus, after the toxic asset sale, the share price is

$$S_0^{H*} = M^* - \frac{D_H}{1+r_f} \tag{42}$$

The voluntary participation constraint is obtained by subtracting equation (41) from equation (42). It is unambiguously positive when $M^* > M$. The healthy bank's individual rationality constraint is the following:

$$VP_H = S_0^{H*} - S_0^H = M^* - M > 0, \text{ when } M^* > M \tag{43}$$

Proposition 3
Healthy banks will be willing to sell into the PPIP if the private investor bids at or above the asset's fair market value, $M^* \geq M$.

This follows from the voluntary participation constraint in equation (43). *Q.E.D.*

Proposition 4
Extremely unhealthy banks, scenario A banks, will never sell their toxic assets. Healthy banks will always sell their toxic assets.

The most extreme zombie bank is a scenario A bank. A scenario A bank, by definition, loses so much volatility by selling its toxic assets for greater than fair market value that it is insolvent in all states of the world. Selling its more volatile toxic assets would push its stock value from a positive number in equation (28) to zero, because the value of the assets would never exceed its liabilities after the toxic asset sale. The voluntary participation constraint of a scenario A zombie bank is never satisfied.

$$VP_A = S_0^a - S_0^Z = 0 - \frac{q(uM - D_a)}{(1 + r_f)} < 0 \tag{44}$$

Thus, the scenario A bank would never sell its toxic assets for a price $M^* > M$. From equation, (43) and proposition 3, we know healthy banks will sell their toxic assets if the price $M^* > M$. *Q.E.D.*

Corollary 1
Healthy banks have a greater incentive to sell their toxic assets than scenario C, zombie banks.

Let us prove this by first recalling the definition of the risk neutral valuation of the toxic asset. The toxic asset is equal to the following:

$$M \equiv \frac{quM + (1-q)dM}{1 + r_f} \tag{45}$$

Since $D > dM$ and $M^* > M$ by definition, then the following must be true:

$$M^* > \frac{1}{1+r_f}(quM + (1-q)D) > M \equiv \frac{1}{1+r_f}(quM + (1-q)dM) \qquad (46)$$

Equation (46) implies that the voluntary participation constraint of the healthy bank in equation (43) exceeds the voluntary participation constraint in equation (40). That is,

$$M^* - M > M^* - \frac{1}{1+r_f}(quM + (1-q)D) \qquad (47)$$

Therefore, we can conclude that healthy banks have a greater incentive to sell their toxic assets than scenario C zombie banks, which are restored to health after the toxic asset sale. *Q.E.D.*

Corollary 2
Zombie banks that become healthy after the toxic asset sale, type C banks, have a greater incentive to sell their toxic assets than scenario B zombie banks, which still are at risk of failure after the toxic asset sale.

The proof of Corollary 2 is in the appendix.

Corollary 3
The scenario B zombie bank has a greater incentive to participate than a scenario A zombie bank.

If we subtract the gain from participation in equation (44) for the scenario A zombie bank from the gain from participation in equation (35), then we are left with the following relationship:

$$VP_B - VP_A = \frac{q(uM^* + D)}{1 + r_f} > 0 \tag{48}$$

Equation (48) is unambiguously positive. Therefore, type B zombies have a greater incentive to sell their toxic assets than type A zombies. *Q.E.D.*

In terms of the banks with the greatest incentive to participate in the LSP and TALF, we can order them from the most solvent to the least solvent:

1. Healthy banks have the greatest incentive to participate.
2. Banks that are always solvent after they unload their toxic assets have the second greatest incentive to participate. These scenario C banks have a positive market value of equity even if the new assets have zero volatility.
3. Scenario B banks have less incentive to participate than healthy or scenario C banks, but scenario B banks have a greater incentive to participate than scenario A banks. Scenario B banks are only solvent if returns are high after the toxic assets are sold. Thus, they could have (but do not necessarily have) negative book value of equity if their assets were marked at fair market value. These banks will participate only if both the volatility of their assets does not decline too much after the toxic asset sale and the overpayment for assets, $M^* – M$, is large.

4. Scenario A banks definitely have a negative book value of equity if their assets were marked at fair market values. These banks will have zero market value of equity after the toxic asset sale reduces the volatility of their assets. They will not voluntarily participate in the toxic asset plan.

d. Numerical Example of the Legacy Securities Program

Here we use current security prices and the structure of the Term Asset Lending Fund's (TALF)'s commercial mortgage backed securities program to estimate if troubled banks are likely to move from risky toxic assets to safer assets. The results below are discouraging. The annual volatility of the toxic assets must exceed 22 percent for zombie banks to part with their troubled assets. The U.S. Treasury's LSP with lower levels of leverage than the TALF, 1-to1 versus 5.67-to-1, is even less likely to convince troubled banks to sell volatile assets. Either sales in the PPIP will result (a) from sales of receivership assets; (b) from sales by healthy banks, which will sell for any price at or above fair market value according to proposition 3; or (c) from sales by troubled banks unloading toxic assets temporarily. In the latter case, troubled banks will sell the assets at inflated price into the PPIP and buy essentially identical toxic securities on the open market.[27]

The New York Federal Reserve bank charged asset managers in the TALF's commercial mortgage backed securities (CMBS) program 3.6305 percent per annum on

[27] It seems regulators are doing little to curb large banks acquisition of toxic assets. See Christopher Condon and Jody Shenn, January 4, 2010, "No Good Deed Goes Unpunished as Banks Seek Profits (Update1)," *Bloomberg* accessed online on January 25, 2010, at http://www.bloomberg.com/apps/news?pid=newsarchive&sid=aFyVxmuF6DTQ.

five-year TALF loans for deals announced on March 19, 2010.[28] The five-year U.S. Treasury note rate on March 19, 2010, was 2.48 percent. Thus, the interest rate offered was 115 basis points higher than the 5-year U.S. Treasury note.

In this paper, we have used periodic interest rates.[29] Thus, if one wants to know the relationship between the period risk-free rate, r_f, and the annualized risk-free rate, r_f^A, she should convert between the two using the following formula:

$$r_f \equiv (1 + r_f^A)^t - 1 \quad (49)$$

In this case, the total periodic risk-free rate is 13.03 percent. We can calculate the periodic premium over the risk-free rate as the following, where R^A was the annual interest rate charged on the government loan to buy toxic assets:

$$g \equiv (1 + R^A - r_f^A)^t - 1 \quad (50)$$

Using equations (49) and (50), the premium above the risk-free rate is 5.89 percent in this example.

[*Insert figure 1 about here.***]**

[28] Source: March 19, 2009, "Press Release: New York Fed announces $1.3 billion in Legacy CMBS TALF loans requested at March 19 facility," New York Federal Reserve Bank, accessed online on November 29, 2010, at http://www.ny.frb.org/markets/cmbs_operations_100407a.html.

[29] Interest rates and swap rates on March 19, 2010, were obtained from the Federal Reserve at http://www.federalreserve.gov/Releases/H15/data.htm.

In figure 1, the author plots the amount the private investor is willing to pay above the fair market value of the asset and the bank's net gain from selling the toxic asset as a function of the toxic asset's volatility. The bank will only participate when its gain from participating curve in figure 1 is above the horizontal axis. In figure 1, the bank must move to much less risky assets. Thus, the overpayment must be large for the banks to participate. Indeed, no trades will occur unless the toxic asset has volatility in excess of 22 percent per annum. The overpayment for toxic assets under those circumstances is in excess of 21 percent of fair market value, leaving the troubled bank healthy after the sale.

7. Geithner's Toxic Asset Plan in Continuous Time

As we have seen, many insights about the U.S. government's toxic asset plans begun in 2009 can be had by taking a financial contracting approach, using a one-period binomial option pricing framework. One-step binomial models are attractive because their linearity and simplicity allows the researcher to used closed form solutions to analyze these toxic asset plans. Nevertheless, the approximations from such a simplification are inferior to those that can be generated from a continuous time Black and Scholes (1973) model. The trade-off is that we must resort to numerical solutions with a continuous time modeling of Geithner's toxic asset plans. This is in contrast to the one-period binomial framework which allows algebraic solutions.

Let $C_0(M,\sigma, X, t)$ denote a European call option written on an asset M with a volatility, σ, a strike price, X, and an expiration date of t. This simple call option expires at time t and must pay back the government loan plus promised interest given in equation

(2) at the maturity of the loan. The value of the call option represents both the U.S. government's and the private investor's equity stakes, or haircut in the case of the TALF, in the toxic asset purchased. (The private investors' percent ownership of this call option, ϕ, cancels out because it would be multiplied by both the left-hand and right-hand sides of the equation below.) For the private investor to make a profit or break even, the following must be true:

$$C_0(M, \sigma, X, t) = (1-\lambda)M_H^*, \quad \text{where } X \equiv (1 + r_f + g)\lambda\mu M_H^* \tag{51}$$

That is, the call option on the toxic assets held by the PPIP equity investors must be at least as high as the price paid for the equity stake. This can be numerically solved for M_H^*.

r_f and g are periodic, not annualized interest charges. Yet, the periodic rates should be only used for the strike price calculation in (51). The Black-Scholes equation uses annualized risk-free rates and volatilities.

Let us model the equity of the bank selling its toxic assets as a European call option $S_0(M, s, D, t)$. This call option has an underlying asset, M; a volatility, σ; a strike price, D, which represents all the liabilities and interest which all come due at time t; and a maturity, t. After selling the toxic assets into the PPIP, the bank's stock is a European call option, $S_1(\mu M_H^*, \underline{\sigma}, D, t)$. This call option has an underlying asset worth μM_H^*; a volatility, $\underline{\sigma}$, where $\underline{\sigma} > \sigma$; a strike price, D; and a maturity, t. For the bank selling its toxic assets the voluntary participation constraint below must be non-negative:

$$VP = S_1(\mu M^*_H, \underline{\sigma}, D, t) - S_0(M, \sigma, D, t) \quad (52)$$

Let us do a numerical example using equations (51) and (52). The Black-Scholes equations for C_0, S_0, and S_1 are written out in the appendix. As with the numerical example in the previous section, let us assume that that M = \$1000, D = \$1108, t = 5, r_f = (1 + .0248)^5 – 1 = .1303, g = (1 + .036305 – 0.0248)^5 – 1 = .0589, λ = 0.85, ϕ = 1, and $\underline{\sigma}$ = 0.02. Here we assume that the private investor pays his or her maximum willingness to pay. That is, μ = 1.

[*Insert figure 2 about here.***]**

The continuous time estimates predict less overbidding than the estimates from the binomial model at the point where the bank chooses to sell toxic assets. At just above 22 percent annual toxic asset volatility, both the binomial and Black-Scholes models predict that the bank will begin to find selling the toxic assets profitable to the TALF investors. Nevertheless, the binomial model predicts that overbidding will be about 21.8 percent compared to 18.0 percent using the Black-Scholes approximation. With a five year time horizon, the Black-Scholes estimates are more accurate. Nevertheless, the closed form solutions of the binomial model are directionally correct.

Figure 1 uses the binomial estimates, and figure 2 uses the continuous time estimates of TALF overbidding. The participation constraint in figure 1 of the zombie bank is kinked at the places where the zombie bank transitions from a type A, B, and C zombie bank. In contrast, since the range of outcomes is smooth in the Black-Scholes estimates in figure 2, the gains from participation smoothly adjust with the increasing volatility. Yet, both participation constraints have similar shapes. For low levels of volatility, increases in volatility mean that the bank loses by moving to less volatile assets because the ability to shift risks onto the bank's creditors is a significant portion of the bank stock's value at first. (In the binomial model, this region in figure 1 is where the bank is a type A zombie.) As volatility increases, the overpayment effect dominates the risk-shifting gains from retaining the toxic assets. Yet, there comes a point where the most important effect of increased volatility is to boost private investors' willingness to pay for toxic assets. At that point, the bank will be eager to sell its toxic CMBS portfolio to the TALF investors.

8. Conclusion

This paper has shown that the price that prevails from a Public Private Investment Partnership (PPIP) sale will not be the market price. It provides a formula for estimating the fair market value of toxic assets sold to investors with government backing. Further, it has shown that the extent of over or under bidding for assets depends on both the leverage and the premium paid over the risk-free rate.

In the Legacy Securities Program (LSP) and the Term Asset-Backed Securities Loan (TALF), the premium over the risk-free rate determines whether or not and to what extent asset managers are willing to overbid. This paper shows that, if this premium is under priced, private investors will have an incentive to overbid for toxic assets. Thus, the structure of the PPIP and the TALF encourages excessive risk taking by private investors purchasing assets with government-subsidized, non-recourse debt.

Healthy banks are the most likely banks to benefit from selling into the PPIP. Yet, from society's point of view, Wilson (2009) and Wilson and Wu (2010) only make the case for subsidized asset sales by troubled banks that are "to big to fail." Subsidizing a healthy bank or a systematically unimportant bank to sell its assets into the PPIP would reduce social welfare because it leads to deadweight losses from taxation. A small bank that is poorly capitalized can be more efficiently restructured in FDIC receivership. Wilson (2010b) argues in receivership, the toxic assets can be sold without subsidies.

Moreover, it only makes sense to have troubled banks sell volatile assets and retain less volatile assets. Thus, if regulators cannot prevent banks from repurchasing risky assets after toxic assets are sold in the PPIP, then it makes little sense for the government to subsidize toxic asset sales. Yet, this paper shows that it is tricky to encourage troubled banks to reduce the volatility of their assets by toxic asset sales without encouraging massive overbidding. It is the overbidding that will lead to taxpayer losses in expectation. Thus, there is a good chance that the costs of Geithner's toxic asset plan exceed even its *ex post* benefits before moral hazard issues are taken into account. Because PPIP toxic asset sales involve subsidies, Wilson (2009) and Wilson and Wu (2010) would argue that they encourage more moral hazard in the future by large banks that expect to be bailed out in a crisis.

This paper has taken the view developed by Wilson (2010a) that banks do not sell toxic assets because their shareholders are made richer by the volatility it provides. Thus, the author does not accept the premise that there are few sales due to illiquidity. There are few toxic asset sales because of the limited liability inherent in share contracts. A troubled bank's shareholders benefit from the upside of the toxic assets, but the taxpayers and the bank's other creditors absorb the losses on the downside. An asset manager, who is not subsidized, only buys the toxic asset but places little or no value on its volatility, which the zombie bank holds dear.

The author believes that the architects of the Public Private Investment Partnership (PPIP) in the Obama administration must agree with the conclusion of Wilson (2010a) that there is a substantial wedge in the value of toxic assets between buyers and sellers which is not caused by illiquidity. Otherwise, they would not have designed such a subsidy-filled process to engineer toxic asset sales. If the markets were merely illiquid, the U.S. Treasury could provide recourse loans to toxic asset buyers. Asset managers would bid for and win toxic assets based on those assets' fair market value with recourse loans. Yet, the administration chose to use non-recourse loans to finance toxic asset sales, which can contribute to asset price bubbles if the put option premium (guarantee fee or the premium over the risk-free rate) is too low, according to Pavlov and Wachter (2002; 2009a; 200b). The author assumes that it often will be because volatility, among other things, is so hard to estimate for untraded securities. Thus, overbidding will sometimes be extraordinary for very volatile assets, and taxpayer losses will be massive in those cases.

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9. Appendix

a. Deriving $\frac{\partial M_H^*}{\partial \sigma} > 0$

When differentiating with respect to σ, there are only q, u, d depend on volatility. $u = e^{\sigma\sqrt{t}}$, $d = e^{-\sigma\sqrt{t}}$, and q given in equation (7) is a function of both u and d. It can be simpler to differentiate the function by first eliminating the q's and writing M_H^* as a function of just u and d.

$$M_H^* = \frac{Mu(1+r_f)-1}{(u-d)(1+r_f)(1-\lambda)+(1+r_f-d)\lambda(1+r_f+g)} \tag{53}$$

We will not write out the exponentials because the differentiating u and d with respect to volatility yields:

$$\begin{aligned} \frac{\partial u}{\partial \sigma} &= \sqrt{t}\left(e^{\sigma\sqrt{t}}\right) = u\sqrt{t} \\ \frac{\partial d}{\partial \sigma} &= -\sqrt{t}\left(e^{-\sigma\sqrt{t}}\right) = -d\sqrt{t} \end{aligned} \tag{54}$$

b. $\underline{\lambda} > 0$ when $g < \underline{g}$

If we use the definition of q in equation (7) numerator in equation (14) is equal to the following:

$$1+r_f - qu = \frac{(u-d)(1+r_f) - u(1+r_f - d)}{u-d} \tag{55}$$

This can be simplified to the following, which is greater than zero because of the no arbitrage condition said $d < \frac{1}{1+r_f}$.

$$1-(1+r_f)d > 0 \tag{56}$$

Thus the numerator is strictly positive.

Let us turn to the denominator in equation (14) which is rewritten below on the left hand side of equation (57). On the right hand side we have substituted in $\bar{g}$. Since $\bar{g} > g$, if leverage is going to have a positive effect on bidding, the left hand side must exceed the right if leverage has any effect at all.

$$\begin{aligned} &(1+r_f)(1-q) - qg > (1+r_f)(1-q) - q\bar{g} = 0, \\ &\text{when } g < \bar{g} = \frac{1-q}{r}(1+r_f). \end{aligned} \tag{57}$$

Thus equations (56) and (57) show that both the denominator and the numerator are positive when $\bar{g} > g$, and the no arbitrage condition is satisfied, $d < \frac{1}{1+r_f}$. *Q.E.D.*

c. Ambiguity of the sign of VP_b

The sign of equation (35) depends on the sign of [$\underline{qu}M^* - quM$]. We know that $M^* > M$. Yet, if $\underline{qu} < qu$, then the sign is ambiguous. Otherwise, the sign of (35) is clearly positive. We know that the standard deviation of the assets after the toxic asset sale, $\underline{\sigma}$, is less than before those assets are sold, σ. Thus, differentiating the *qu* with respect to the asset volatility will help settle the issue. Using the definitions in equations (1) and (7), this derivative simplifies to the following:

$$\frac{d}{d\sigma}(uq) = \frac{d}{d\sigma}\left(\frac{u(1+r_f)-1}{u-d}\right) = \frac{\sqrt{t}(u+d)}{(u-d)^2} > 0, \forall\sigma \tag{58}$$

Equation (58) implies that $uq > \underline{uq}$, because $\sigma > \underline{\sigma}$. Since $M^* > M$ the sign in of the VP_b constraint in (35) is ambiguous. *Q.E.D.*

If the overpayment is sufficiently large or the volatility change is sufficiently small, then the scenario B bank would sell its toxic assets. Yet, in other circumstances, a modest overpayment for assets and large drop in volatility would make selling securities in the LSP or TALF unprofitable.

d. Ambiguity of the sign of VP_c

If the bank receives M^* and immediately buys just as toxic assets as it sold to the government, then the risk neutral valuation of those new assets would be the following:

$$M^* \equiv \frac{quM^* + (1-q)dM^*}{1+r_f} \tag{59}$$

Inserting equation (59) into equation (40) the voluntary participation constraint becomes the following after some algebra:

$$(VP_c) \qquad S_0^c - S_0^Z = \frac{qu(M^* - M)}{1+r_f} + \frac{(1-q)(dM^* - D_c)}{1+r_f} \tag{60}$$

The first term is unambiguously positive because $M^* > M$. Yet the second term has an ambiguous sign because we cannot sign ($dM^* - D_c$). Therefore, the sign of the voluntary participation constraint for a scenario C bank is ambiguous. *Q.E.D.*

e. Proof of Corollary 2

Subtracting type C bank's participation constraint in equation (35) from bank B's participation constraint in equation (40), we have the following:

$$VP_c - VP_b = M^*\left[1 - \frac{\underline{u}q}{1+r_f}\right] - \frac{(1-q)D_c}{1+r_f} \tag{61}$$

The term in square brackets (61) can be shown to be equal the following using the definition of the risk-neutral probability in equation (7) and the relationship that $\underline{u} = 1/\underline{d}$:

$$\left[1 - \frac{\underline{u}q}{1+r_f}\right] = \frac{1-\underline{d}(1+r_f)}{(1+r_f)(\underline{u}-\underline{d})} = \frac{\underline{d}(1-\underline{q})}{1+r_f} \tag{62}$$

Inserting the left-hand side of equation (62) into equation (61), we get the following relationship:

$$VP_c - VP_b = \frac{(1-\underline{q})\underline{d}M^* - (1-q)D_c}{1+r_f} \tag{63}$$

We know that for a type C bank $\underline{d}M^* > D_c$. Yet, we don't know the relationship between $(1 - \underline{q})$ and $(1 - q)$. We do know that the volatility of the new assets, $\underline{\sigma}$, is less than the volatility of the old assets, σ. If we can show that lower volatility assets have a higher risk neutral probability of failure, $(1 - \underline{q}) > (1 - q)$, then that is sufficient to show equation (63) is positive.

Let us differentiate the risk-neutral probability of failure with respect to the standard deviation of the assets. Using the definitions in equations (1) and (7), this derivative simplifies to the following after some algebra:

$$\frac{d}{d\sigma}(1-q) = -\sqrt{t}\left(\frac{2+(1+r_f)(u-d)}{(u-d)^2}\right) < 0, \forall\sigma \tag{64}$$

Equation (64), implies that $(1 - \underline{q}) > (1 - q)$. Therefore, coupled with $\underline{d}M^* > D_c$ it must be the case that equation (63) is unambiguously positive, or

$$VP_c - VP_b = \frac{(1-\underline{q})\underline{d}M^* - (1-q)D_c}{1+r_f} > 0. \tag{65}$$

This is what we wanted to show. *Q.E.D.*

f. Black and Scholes (1973) Formula Applied to Geithner's Toxic Asset Plan

The Black-Scholes equation can be used to approximate willingness to pay of the private investor receiving government subsidized non-recourse loans. In this instance, the equation is the following:

$$\begin{aligned}
&C_0 = MN(d_1) - (\mu M_H^* \lambda(1+g+r_f))\exp(-r_f t)N(d_2) = (1-\lambda)M_H^* \\
&d_1 = \frac{\ln\{M/[\mu M_H^* \lambda(1+g+r_f)]\} + (r_f + \sigma^2/2)t}{\sigma\sqrt{t}} \\
&d_2 = d_1 - \sigma\sqrt{t}
\end{aligned} \tag{66}$$

$N(d_j)$ stands for the cumulative normal density function, where j = 1 or 2. exp( ) denotes the exponential number. This equation can be numerically solved for the instance where C_0 equals the right-hand side of the top equation, $(1-\lambda)M_H^*$. This numerical solution represents the private investor's maximum willingness to pay for 100 percent of the equity in the toxic asset investment fund.

The zombie bank's initial equity, S_0, can also be valued with this approach. No numerical solutions are needed for the equation below:

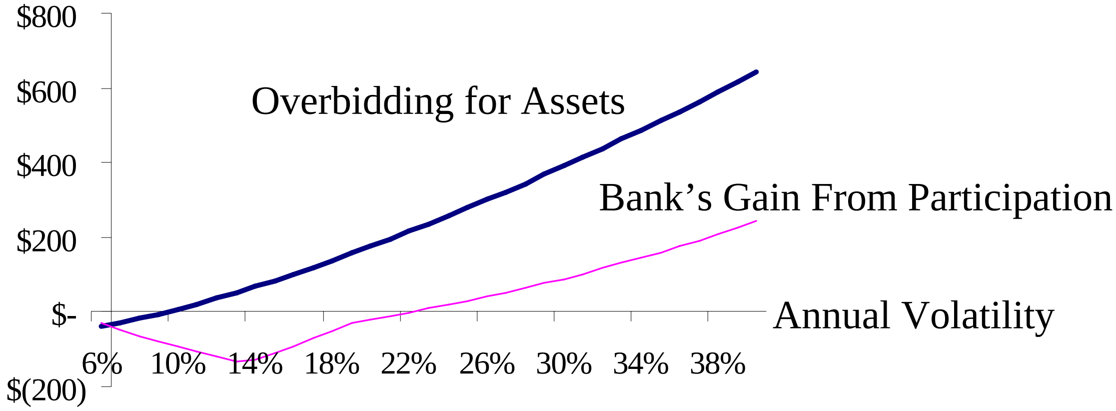
$$
\begin{aligned}
S_0 &= MN(d_1) - D\exp(-r_f t)N(d_2) \\
d_1^0 &= \frac{\ln\{M / D\} + (r_f + \sigma^2 / 2)t}{\sigma\sqrt{t}} \\
d_2^0 &= d_1 - \sigma\sqrt{t}
\end{aligned} \tag{67}
$$

The numerical solution for M_H^* can be obtained from equation (66), and S_1 below can then be solved directly. After the toxic asset sale the zombie bank's equity is worth the following:

$$
\begin{aligned}
S_1 &= \mu M_H^* N(d_1^1) - D\exp(-r_f t)N(d_2^1) \\
d_1^1 &= \frac{\ln\{\mu M_H^* / D\} + (r_f + \underline{\sigma}^2 / 2)t}{\underline{\sigma}\sqrt{t}} \\
d_1^1 &= d_1 - \underline{\sigma}\sqrt{t}
\end{aligned} \tag{68}
$$

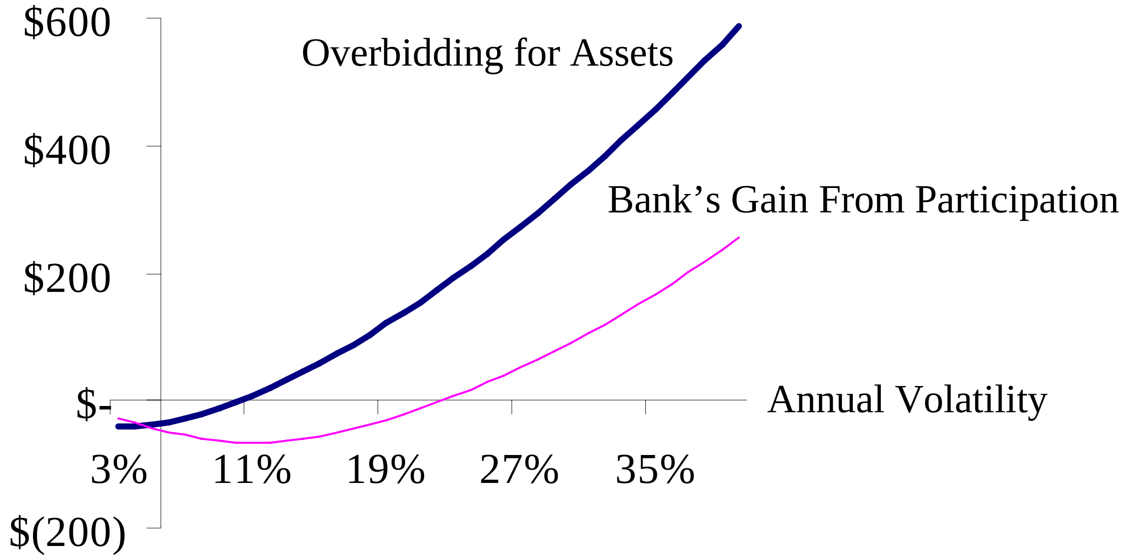
Figure 1:

Overpayment for assets and the troubled bank's gain from participation in the TALF's CMBS program as a function of the toxic asset's volatility when new assets have a low volatility using the binomial model



The parameter values in this example are modeled after those that would have prevailed for Term Asset Lending Fund's (TALF)'s commercial mortgage backed securities CMBS sales based on the program's last subscription date for legacy securities on March 19, 2010. The numerical solutions are base on the estimates from a one-step binomial model. Here we assume that M = \$1000, D = \$1108, t = 5, r_f = (1 + .0248)^5 – 1 = .1303, g = (1 + .036305 – 0.0248)^5 – 1 = .0589, λ = 0.85, ϕ = 1, μ = 1, and $\underline{\sigma}$ = 0.02. M = fair market value of the asset; D = the future value of the bank's promised debt principle and interest; t = time in years until asset must be sold and the non-recourse loan must be repaid; r_f = the periodic risk-free rate; g = periodic interest rate premium above the risk-free rate; λ = maximum leverage; ϕ = fraction of the purchase price contributed by the private investor; μ = fraction of private investors' maximum willingness to pay paid for the asset; and $\underline{\sigma}$ = annual volatility of the assets that replace the toxic assets. As long as the toxic asset's value is sufficiently volatile, 9.48 percent per annum, the private investors are willing to overbid for toxic assets. If the toxic assets volatility exceeds 13.49 percent, then overpayment for assets is high enough that the zombie bank moves from a scenario A zombie, which always fails if it sells its toxic assets, to a scenario B zombie bank, which only fails after selling troubled assets when returns are bad. If the toxic asset's volatility exceeds 19.10 percent, private investors are willing to overpay enough, 15.85 percent over fair market value, so that the zombie bank would be solvent in all possible states of the world if it sold its bad assets. Nevertheless, the private investor will only overbid enough, 21.78 percent over fair market value, for the zombie bank to voluntarily sell assets if the toxic asset's volatility exceeds 22.11 percent. In this case, the troubled bank will become healthy if it voluntarily sells its toxic assets. Thus, in this case, sales only occur in scenario C when the zombie bank becomes healthy after the toxic asset sale.

Figure 2:
Overpayment for assets and the troubled bank's gain from participation in the TALF's CMBS program as a function of the toxic asset's volatility when new assets have a low volatility using the continuous time model



The parameter values in this example are modeled after those that would have prevailed for Term Asset Lending Fund's (TALF)'s commercial mortgage backed securities CMBS sales based on the program's last subscription date for legacy securities on March 19, 2010. The solutions are based on a numerically solved Black and Scholes (1973) model. Here we assume that M = \$1000, D = \$1108, t = 5, r_f = (1 + .0248)^5 – 1 = .1303, g = (1 + .036305 – 0.0248)^5 – 1 = .0589, λ = 0.85, ϕ = 1, μ = 1, and $\underline{\sigma}$ = 0.02. M = fair market value of the asset; D = the future value of the bank's promised debt principle and interest; t = time in years until asset must be sold and the non-recourse loan must be repaid; r_f = the periodic risk-free rate; g = periodic interest rate premium above the risk-free rate; λ = maximum leverage; ϕ = fraction of the purchase price contributed by the private investor; μ = fraction of private investors' maximum willingness to pay paid for the asset; and $\underline{\sigma}$ = annual volatility of the assets that replace the toxic assets. As long as the toxic asset's value is sufficiently volatile, 10.23 percent per annum, the private investors are willing to overbid for toxic assets. If the toxic asset's volatility exceeds 22.36 percent, private investors are willing to overpay enough, 17.99 percent over the fundamental value of M = \$1000, so that the zombie bank to voluntarily sell its toxic assets.