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By

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COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
YALE UNIVERSITY
Box 208281
New Haven, Connecticut 06520-8281

http://cowles.yale.edu/

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Jidong Zhou

Yale University

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Abstract

How will an improved information environment affects competition and market performance when consumers face search frictions? This paper provides a unified way to model information improvement that makes the search pool more "selective" (e.g., due to personalized recommendations), or more "informative" (e.g., due to the availability of more detailed product information). Information improvement tends to induce consumers to search less, intensify price competition and benefit consumers, if the search friction is small, or if information improvement truncates the match utility distribution from below. More generally, however, it is also possible for information improvement to raise the market price and harm consumers.

Keywords: consumer search, personalized recommendations, information improvement, price competition

JEL classification: D43, D83, L13

1 Introduction

Over the past two decades consumers have experienced a significantly improved information environment in their shopping process. For example, they often use online platforms to gather product information such as search engines (e.g., Google), product comparison websites (e.g., Expedia), and e-commerce marketplaces (e.g., Amazon). These platforms

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not only help consumers save on the cost of finding sellers, but also often guide consumers towards better and more relevant products. For instance, personalized recommendations, filtering, or targeted advertising enables consumers to encounter and consider more relevant products first; offering customer reviews, or using better online display technologies (e.g., virtual clothing try-on or furniture preview) makes the inspection and comparison of products more informative.

Do consumers search more or less in a more selective or informative search pool? For example, with a more relevant search pool, consumers become choosier and aim to find a better matched product before stopping their search, but it also becomes more likely for them to encounter a well-matched product at each step. Do consumers always benefit from improved information in a search environment? This should be the case if product prices remain unchanged. However, sellers usually have incentives to adjust their prices since consumer search behavior is influenced by an improved information environment. How might sellers change their price? Is it possible that improved information inflates the market price so that consumers end up worse off?

These questions are also relevant for platform design. For example, if a platform such as a search engine makes profit from per-click fees, it will have an incentive to make consumers search longer. If a platform is a product comparison website and its profit is from percentage commission fees, its interest will be more aligned with the sellers'. If a platform faces strong competition from other platforms, it will put more weight on consumer surplus.

In this paper we provide a unified way to model information improvement in a search market, and study how improved information affects consumer search behavior and firm competition. We adopt the search framework developed in Wolinsky (1986) and Anderson and Renault (1999), and consider a large number of sellers, each supplying a horizontally differentiated product. When a consumer visits a seller, she discovers both its product price and how well matched its product is. The match utility is a random draw from some distribution. As we will discuss more later, this framework has become a standard setup for studying many consumer search related economic issues. We model information improvement by assuming that consumers face a new match utility distribution which is greater than the original one in the sense of "increasing convex order." Two leading cases are when the match utility distribution becomes higher in the sense of first-order stochastic dominance (FOSD) (e.g., when consumers face more relevant products), or more dispersed in the sense of mean-preserving spread (MPS) (e.g., when the inspection of each product becomes more informative).

First, we show that consumers search longer (shoter) if the change of information environment leads to a more (less) dispersed match utility distribution in terms of "dispersive order" or "excess wealth order" (which are both stronger requirements than MPS if the mean remains unchanged). Second, we argue that information improvement usually has two opposite effects on pricing. If the reservation match utility in the consumer search rule were fixed, firms would like to raise their prices, for example, when the match utility distribution becomes higher in terms of hazard rate. However, consumers actually set a higher reservation match utility when the search pool becomes better, which is a force for firms to lower their prices. This often renders the impact of information improvement on market price ambiguous. Our analysis explains why a clear-cut price comparison result is available in some related existing works which we will discuss in detail later. (They model information improvement in a particular way so that the hazard rate of the match utility distribution remains unchanged.) Third, consumers benefit from information improvement in our model if it lowers the market price. However, it is possible that information improvement inflates the market price so much that consumers suffer.

Simple conditions for unambiguous impacts of information improvement are available in some cases. For instance, when the search friction is sufficiently small, we show that if information improvement does not change the maximum possible match utility, then consumers search less and at the same time market price drops so that consumers become better off. Also, when information improvement leads to a new match utility distribution which is a truncation of the original one from below, it benefits consumers regardless of its impact on market price (provided that the search market remains active).

Our model is built on the search framework developed in Wolinsky (1986) and Anderson and Renault (1999).[1] This framework has now become the workhorse model for many recent works on consumer search. They include, for example, prominence and ordered search (e.g., Armstrong, Vickers, and Zhou (2009)), attention-grabbing advertising (e.g., Haan and Moraga-Gonzalez (2011)), product design and the long-tail phenomenon (e.g., Bar-Isaac, Caruana, and Cuñat (2012)), multiproduct search and retail market structure (e.g., Zhou (2014), and Rhodes and Zhou (2019)), price directed search (e.g., Choi, Dai, and Kim (2018)), and paid recommendations by intermediaries (e.g., Teh and Wright

[1]The other well-known branch of the consumer search literature considers homogeneous products and focuses on price search. The classic works include Diamond (1971), Varian (1980), Burdett and Judd (1983), and Stahl (1989). However, the frameworks there are not suitable for studying product information improvement which motivates this paper.

(2022)). Our paper offers a comparative static analysis of this classic setup with respect to the match utility distribution, a question which is important in many applications but which has not been studied systematically in the literature.[2]

Given the relevance and importance of improved information in many markets (especially those intermediated by platforms), it is not surprising that some research has been done on this topic. Eliaz and Spiegler (2011) study how a profit-maximizing search engine controls the quality of the search pool. It is shown that the search engine has an incentive to encourage the entry of low-relevance firms by using a low per-click fee. This degrades the search pool and leads to longer consumer search and higher market prices. de Corniere (2016) investigates how sellers or a search engine choose the degree of targeting in the context of search advertising. He shows that targeting or an improved search pool makes consumers search less but intensifies price competition among sellers. The final market price, though, may become higher because the per-click fee charged by the search engine inflates sellers' cost. Zhong (2021) studies how an online retail platform chooses the match precision in personalized recommendations in a search environment. He shows a similar result that a higher match precision leads to more intense price competition among sellers (whenever consumer search remains active).[3] All these works are built on the Wolinsky framework or a close variant, but as mentioned before they focus on a particular type of information quality change which essentially truncates the match utility distribution from below.[4] By considering a more general way of modeling information improvement, our paper helps understand why similar clear-cut comparison results are derived in those papers (before taking into account the platform fees) and also sheds light on to what extent their results are robust to alternative ways of modelling information improvement.[5]

[2]Section 4 in Anderson and Renault (1999) discusses how the degree of product differentiation affects search and price competition. They consider the case with a finite number of firms and capture the degree of product differentiation by a multiplicative parameter in front of the match utility random variable. Given their full-market coverage assumption, the change of product differentiation is a special case of the MPS relationship.

[3]If the precision is too high so that consumers never search beyond the first encountered seller, then monopoly pricing will arise, similar to the Diamond (1971) paradox.

[4]Moraga-González and Sun (2021) study firms' quality investment incentives in the Wolinsky search framework, and they model product quality improvement as an increase of the match utility distribution in the sense of FOSD. This is the same as when the search pool becomes more selective in our model. In particular, they also derive conditions for how a (small) quality improvement across all firms affects consumer search duration. Their conditions are related to the excess-wealth-order result in our paper.

[5]In this paper we do not model strategic platforms explicitly. Instead we focus on an exogenous change of the information environment and aim to understand its market implications more deeply. Considering

There is also growing empirical research on how more targeted recommendations in e-commerce platforms affect consumer behavior and market performance. For example, Donnelly et al. (2022) document evidence that introducing personalized rankings on Wayfair.com induces more consumers to search and improves purchase diversity (i.e., shifting demand from bestsellers to niche items), and it also benefits both the retailer and consumers. However, their assessment does not take into account the potential effect of personalized recommendations on product pricing. Zhou et al. (2021) highlight an interesting side effect of having a more precise search algorithm which is not examined in this paper. When TaoBao.com returns more targeted search results to consumers, as expected they search less and buy the product under study more likely, but meanwhile they also spend less time in exploring other products, reducing unplanned purchases.

2 The model

We first introduce the Wolinsky-Anderson-Renault search framework, and then describe how to model information improvement in this framework. There is a continuum of firms, each supplying a differentiated product at a constant marginal cost which is normalized to zero. There is a continuum of consumers, each having at most a unit demand for one of the products. We normalize the measure of consumers per firm to one. Both firms and consumers are risk neutral, and each consumer has a zero outside option. A product's match utility for a consumer, denoted by X, is a random draw from a distribution with CDF $H(x)$ and support $[\underline{x}, \overline{x}]$. The realization of X is assumed to be i.i.d. across consumers and products. This implies that firms are *ex ante* symmetric and consumers have idiosyncratic preferences.

Consumers initially have imperfect information about each product's match utility and price. They can, however, search sequentially to gather information: by incurring a cost $s > 0$ a consumer can visit a firm and discover both its match utility and price. During the search process, consumers know the common match utility distribution across products and hold a rational belief of firms' pricing strategy. Since there are no common shocks across firms, we assume that upon observing an off-equilibrium price from a firm, consumers believe that the other firms still adopt their equilibrium pricing strategy. We look for a perfect Bayesian equilibrium where firms set their prices simultaneously to

strategic platforms is certainly important, but the additional insight from doing so often depends on the modelling details such as the platform market structure, the fee structure, and whether sellers and consumers multihome or not.

maximize their own profit given their rational expectation of consumer search behavior, and consumers search sequentially and optimally given the match utility distribution and their rational expectation of firms' pricing strategy. We focus on a symmetric equilibrium where all firms charge the same price and consumers search actively and randomly. Note that in this setup with a continuum of symmetric firms, consumers will never return to retrieve a product inspected before, and so it does not matter whether consumers have free recall or not.

We now describe two types of possible information improvement in this setup:

More selective search pool. Consider first the case when consumers encounter more relevant products first or face a more "selective" search pool. One simple way to capture this type of information improvement is to assume that a product will be displayed or recommended to a consumer if and only if its match utility for this consumer exceeds a threshold, say, $\hat{x}$. Suppose there is no further ranking among the qualified products, and firms do not observe which consumers their products are displayed to and so they cannot price discriminate accordingly. Since there is still a continuum of qualified products, the consumer will never search beyond this pool of products. Therefore, the situation in this case is as if consumers face a search market where each product has a match utility distribution

$$\frac{H(x) - H(\hat{x})}{1 - H(\hat{x})} ,$$

which is a truncation of the underlying distribution H from below. This distribution increases in the sense of FOSD as $\hat{x}$ increases.[6] As we will discuss in more detail later, this is essentially the setup used in de Corniere (2016) to study targeted search advertising and in Zhong (2021) to study personalized recommendations.

This FOSD relationship holds more generally if better matched products are more likely to be displayed to consumers. Consider two regimes where in regime $i \in \{1, 2\}$ a product with match utility x is displayed with probability $\rho_i(x)$, and suppose $\frac{\rho_2(x)}{\rho_1(x)}$ increases in x. Then it can be shown that consumers perceive a higher match utility distribution in regime 2 in the sense of FOSD.[7]

More informative search pool. Consider now the case when product inspection becomes more informative (e.g., when a platform starts offering customer reviews or introducing

[6] A random variable X_2 with CDF $H_2(x)$ is greater than another random variable X_1 with CDF $H_1(x)$ in the sense of FOSD, denoted by $X_2 \geq_{\text{FOSD}} X_1$, if $H_2(x) \leq H_1(x)$ for all x.

[7] The perceived match utility density function in regime i is $\frac{\rho_i(x)h(x)}{\int_{\underline{x}}^{\overline{x}} \rho_i(t)h(t)dt}$. When $\frac{\rho_2(x)}{\rho_1(x)}$ increases in x, the density in regime 2 crosses that in regime 1 once and from below. This implies that the distribution in regime 2 is an FOSD of that in regime 1.

VR shopping). In this case, we assume that when a consumer samples a product, she only observes a signal of its true match utility. Information improves when the signal becomes more precise. One convenient example is the often-used "truth-or-noise" signal structure. Suppose the signal perfectly reveals the match utility with probability θ and is otherwise a pure noise (e.g., a random draw from distribution H independent of the true match utility). Then conditional on a signal realization $\tilde{s}$, the consumer's estimate of the match utility is $\theta\tilde{s} + (1-\theta)\mu$, where μ is the mean of the true match utility. Therefore, the situation is as if consumers face a match utility distribution

$$H\left(\frac{x-(1-\theta)\mu}{\theta}\right) .$$

When the precision θ increases, this distribution becomes more dispersed in the sense of MPS.[8] This MPS relationship remains true more generally whenever the signal becomes more informative in the Blackwell sense.

More generally, we model information improvement in our setup by "increasing" the match utility distribution from $F(x)$ to $G(x)$ as described in the following assumption, where X_F is the random variable associated with $F(x)$ and X_G is the random variable associated with $G(x)$.

Assumption 1. *X_G is greater than X_F in the "increasing convex order," i.e.,*

$$\mathbb{E}[\psi(X_G)] \geq \mathbb{E}[\psi(X_F)]$$

for any increasing and convex function ψ whenever the expectations exist.

See, for example, section 4.A in Shaked and Shanthikumar (2007) for a comprehensive discussion of increasing convex order.[9] It includes FOSD and MPS as two special cases, and it also captures the case when the search pool becomes *both* more selective and informative. This is easy to see from an alternative definition of increasing convex order: there exists a random variable Y such that $X_G \geq_{\text{FOSD}} Y \geq_{\text{MPS}} X_F$ or $X_G \geq_{\text{MPS}} Y \geq_{\text{FOSD}} X_F$. Notice, however, that there are cases of increasing convex order which are neither

[8] A random variable X_2 with CDF $H_2(x)$ is greater than another random variable X_1 with CDF $H_1(x)$ in the sense of MPS, denoted by $X_2 \geq_{\text{MPS}} X_1$, if $\int_{-\infty}^{x} H_2(\tilde{x})d\tilde{x} \geq \int_{-\infty}^{x} H_1(\tilde{x})d\tilde{x}$ for all x and the equality holds at $x = \max\{\overline{x}_1, \overline{x}_2\}$.

[9] It implies that a risk-seeking decision maker prefers X_G over X_F. If the ψ function in Assumption 1 becomes increasing and concave, we then have the more familiar concept of Second Order Stochastic Dominance.

FOSD nor MPS.[10] Suppose both F and G are differentiable, and their associated densities are f and g, respectively. Let $[\underline{x}_i, \overline{x}_i]$, $i \in \{F, G\}$, be the support of X_i, and the support is allowed to be unbounded.

Our research question then boils down to how an increase of the match utility distribution in the sense of increasing convex order affects consumer search behavior, price competition, and consumer welfare.

For convenience, for a random variable X with CDF $H(x)$, we write

$$\mathbb{E}[(X-u)_+] \equiv \int_{\underline{x}}^{\overline{x}} \max\{0, x-u\} dH(x) = \int_u^{\overline{x}} [1-H(x)]dx \ , \tag{1}$$

where the second equality is from integration by parts. This expression captures the expected benefit from an additional search when the match utility distribution is H and the best match utility so far is u. We call the price p_H^M which maximizes $p[1-H(p)]$ the standard monopoly price associated with the match utility distribution H. (Note that this standard monopoly price can be a corner solution equal to $\underline{x}$ if this lower bound is high enough.) Then $\mathbb{E}[(X-p_H^M)_+]$ is the expected consumer surplus in the monopoly case.

To ensure the existence of a pure-strategy pricing equilibrium with an active market (i.e., consumers are willing to participate), we make the following assumptions:

Assumption 2. *(i) Both survival functions* $1-F$ *and* $1-G$ *are* -1*-concave (i.e., both* $1/(1-F)$ *and* $1/(1-G)$ *are convex).*
(ii)The search cost satisfies $s < \min_{i\in\{F,G\}} v_i^M$*, where* $v_i^M \equiv \mathbb{E}[(X_i - p_i^M)_+]$ *is the consumer surplus in the standard monopoly case with distribution* i*.*

The assumption of -1-concavity is equivalent to both $x - \frac{1-F(x)}{f(x)}$ and $x - \frac{1-G(x)}{g(x)}$ being increasing functions. As we will see, this ensures that a symmetric equilibrium exists and is determined by the first-order condition. Notice that the -1-concavity condition is weaker than the usual assumption that $1-F$ and $1-G$ are log-concave (i.e., both hazard rates $\frac{f}{1-F}$ and $\frac{g}{1-G}$ are increasing),[11] and it is satisfied by many often-used distributions.

[10]This can happen, for instance, when Y is an MPS of X_F and at the same time X_G is an FOSD of Y by truncating it from below.

[11]Log-concavity is 0-concavity, and ρ-concavity is more stringent than ρ'-concavity when $\rho > \rho'$. See, e.g., Caplin and Nalebuff (1991) for the concept of ρ-concavity. There are simple distributions whose survival functions are log-convex but -1-concave. For example, for the Pareto distribution with $F(x) = 1 - x^{-\beta}$ on $[1, \infty)$, $\frac{1-F(x)}{f(x)} = \frac{x}{\beta}$ is increasing (so $1-F$ is log-convex), but $x - \frac{1-F(x)}{f(x)}$ is also increasing (so $1-F$ is -1-concave) if $\beta > 1$.

As we will also see, the search cost condition ensures an active market both before and after information is improved.

3 The equilibrium

We now characterize the market equilibrium in the case of distribution F. (The analysis for the case of G is analogous.) Most of the results in this section are known in the literature under the log-concavity assumption, and here we prove them under the weaker -1-concavity assumption.

Let p_F denote the symmetric equilibrium price, and let r_F denote the *reservation match utility* which uniquely solves

$$\mathbb{E}[(X_F - r_F)_+] = \int_{r_F}^{\overline{x}_F} [1 - F(x)]dx = s \ . \tag{2}$$

When firms charge the same price, a consumer will then cease her search if and only if the best match utility so far is greater than r_F. Notice that under our search-cost condition in Assumption 2, we have $r_F > p_F^M \geq \underline{x}_F$, i.e., the reservation match utility must be interior. Therefore, in equilibrium some consumers will search beyond the first encountered firm.

To derive the equilibrium price p_F, suppose that a firm unilaterally deviates to price p. If a consumer comes to visit it, she will stop searching and buy its product immediately if its match utility is such that $X_F - p > r_F - p_F$, where the latter is the continuation surplus if the consumer chooses to search on (which is also the equilibrium consumer surplus). Hence, the firm's deviation profit will be proportional to $p[1 - F(r_F - p_F + p)]$. (In other words, in our model each firm acts as a local monopolist facing consumers who regard the continuation value of search as their outside option.) Its derivative with respect to p has the sign of

$$\frac{1 - F(r_F - p_F + p)}{f(r_F - p_F + p)} - p \ . \tag{3}$$

The first-order condition then implies

$$p_F = \frac{1 - F(r_F)}{f(r_F)} \ . \tag{4}$$

When $1 - F(x)$ is -1-concave, this first-order condition is also sufficient for defining the equilibrium price.

Lemma 1. *Under Assumption 2, the symmetric equilibrium price in search pool F is determined in (4) and the market is active in equilibrium.*

Proof. When $1-F$ is -1-concave, $x - \frac{1-F(x)}{f(x)}$ is an increasing function, and so (3) is positive for $p < p_F$ and negative for $p > p_F$. That is, a firm's deviation profit is single-peaked at $p = p_F$. Therefore, p_F defined in (4) is indeed the symmetric equilibrium price whenever the market is active.

Consumers are willing to participate into the market if and only if $r_F - p_F > 0$, or equivalently if

$$r_F - \frac{1-F(r_F)}{f(r_F)} > 0 \; . \tag{5}$$

If the monopoly price p_F^M is interior and so solves $p = \frac{1-F(p)}{f(p)}$, (5) is equivalent to $r_F > p_F^M$. From the definition of r_F, we know that this is the case if $s < \mathbb{E}[(X_F - p_F^M)_+] = v_F^M$, which is true under the search cost condition in Assumption 2. If the monopoly price p_F^M is a corner solution equal to the lower bound $\underline{x}_F$, then we must have $\underline{x}_F \geq \frac{1-F(\underline{x}_F)}{f(\underline{x}_F)}$. This implies (5) since $r_F > \underline{x}_F$ under Assumption 2 as we pointed out before.[12] □

When s goes up but the market is still active, the reservation match utility r_F decreases, and so the price rises if the hazard rate function $\frac{f}{1-F}$ is increasing (or if $1-F$ is log-concave), but drops if the hazard rate function is decreasing (or if $1-F$ is log-convex). Under the -1-concavity condition, however, an increase of s always lowers consumer surplus $r_F - p_F$, regardless of how price varies.

It is sometimes to convenient to denote by

$$\sigma_F \equiv F(r_F)$$

the probability that in equilibrium a consumer will continue to search after visiting a firm. We call it the "search propensity." By changing the variable in (2) from x to $t = F(x)$, we can also define the search propensity as the solution to

$$\int_{\sigma_F}^{1} \frac{1-t}{f(F^{-1}(t))} dt = s \; . \tag{6}$$

Then the equilibrium price p_F can also be expressed as a function of search propensity:

$$p_F = \frac{1-\sigma_F}{f(F^{-1}(\sigma_F))} \; . \tag{7}$$

This expression is useful in some subsequent analysis.

[12] This analysis implies that when $s \to v_F^M$, we have $r_F, p_F \to p_F^M$ if the monopoly price is an interior solution, in which case consumer surplus goes to zero, or $r_F \to \underline{x}_F$ and $p_F \to \frac{1}{f(\underline{x}_F)}$ if the monopoly price is a corner solution $\underline{x}_F$, in which case consumer surplus can be strictly positive.

An analogous analysis for the case of G applies under Assumption 2. The reservation match utility r_G there solves $\mathbb{E}[(X_G - r_G)_+] = s$ and the search propensity is $\sigma_G \equiv G(r_G)$. Then the equilibrium market price in search pool G is

$$p_G = \frac{1 - G(r_G)}{g(r_G)} = \frac{1 - \sigma_G}{g(G^{-1}(\sigma_G))} \ , \tag{8}$$

and consumer surplus is $r_G - p_G$.

4 Impact of information improvement

We now investigate how information improvement affects market performance. We first report general results in section 4.1, and then illustrate them by studying several examples in section 4.2.

4.1 General results

Search duration. We first examine how information improvement affects consumer search duration. Search duration is often the only observable related to consumer search behavior, and is also relevant to platform design if a platform, say, a search engine makes money from charging firms per-click fees.

Given X_G is greater than X_F in the increasing convex order, we have

$$\mathbb{E}[(X_G - u)_+] \geq \mathbb{E}[(X_F - u)_+] \text{ for any } u \tag{9}$$

as $(X - u)_+$ is an increasing and convex function of X.[13] That is, for any given best match utility so far, the expected benefit from one more search is greater in the case of G than in the case of F. From the definition of r_F and r_G, it is then immediate that consumers become choosier by setting a higher reservation match utility in the case of G, i.e., $r_G \geq r_F$.

This, however, does *not* mean that consumers necessarily search longer in the case of G as the distribution changes at the same time. For example, when G is higher than F in FOSD, it is also more likely for consumers to find a high match utility at each firm. Therefore, it is *ex ante* unclear whether consumers search longer or shorter after

[13] In fact (9) is an alternative definition of the increasing convex order, as any increasing convex function can be approximated by a linear combination of $(X - u)_+$ with different u's.

information is improved. More precisely, the (expected) consumer search duration is determined by the search propensity:

$$l_F \equiv \frac{1}{1-\sigma_F} \quad \text{and} \quad l_G \equiv \frac{1}{1-\sigma_G} \ . \tag{10}$$

Notice that given the measure of consumers per firm is one, search duration also measures each firm's traffic, and its reciprocal (i.e., $1-\sigma_i$) measures the conversion rate.

The following result reports conditions for a clear-cut comparison of search duration.

Proposition 1. *(i) Consumers search longer (and so each firm has more traffic but a lower conversion rate) in search pool G for any permissible search cost if X_G is greater than X_F in the "excess wealth order," i.e., if*

$$\mathbb{E}[(X_G - G^{-1}(\sigma))_+] > \mathbb{E}[(X_F - F^{-1}(\sigma))_+] \text{ for any } \sigma \in (0,1) \ . \tag{11}$$

This condition holds if $g(G^{-1}(\sigma)) < f(F^{-1}(\sigma))$ for any $\sigma \in (0,1)$.
(ii) Suppose $f(\overline{x}_F), g(\overline{x}_G) > 0$. Then there exists $\hat{s}$ such that for $s < \hat{s}$, consumers search less in search pool G if and only if $g(\overline{x}_G) > f(\overline{x}_F)$, which is the case if $\overline{x}_F = \overline{x}_G$ and $f(\overline{x}_F) \neq g(\overline{x}_G)$.

Proof. (i) From the definition of search propensity in (6) and its counterpart for G, we have

$$\int_{\sigma_F}^1 \frac{1-t}{f(F^{-1}(t))}dt = \int_{\sigma_G}^1 \frac{1-t}{g(G^{-1}(t))}dt \ .$$

Then $\sigma_F < \sigma_G$ (i.e., consumers search longer in the case of G) if

$$\int_{\sigma}^1 \frac{1-t}{f(F^{-1}(t))}dt < \int_{\sigma}^1 \frac{1-t}{g(G^{-1}(t))}dt \tag{12}$$

for any $\sigma \in (0,1)$. This is equivalent to (11) by changing variable from t to $x = F^{-1}(t)$ or $G^{-1}(t)$. It is evident that a sufficient condition for (12) is $g(G^{-1}(\sigma)) < f(F^{-1}(\sigma))$.

(ii) It suffices to show the result when s is close to 0. When s is close to zero, σ_F is close to 1. Using the (second-order) Taylor expansion and $f(F^{-1}(1)) = f(\overline{x}_F) > 0$, one can show that the left-hand side of (6) is approximately equal to $\frac{1}{2f(\overline{x}_F)}(1-\sigma_F)^2$. Then we have

$$1 - \sigma_F \approx \sqrt{2sf(\overline{x}_F)} \ . \tag{13}$$

Similarly, one can derive $1-\sigma_G \approx \sqrt{2sg(\overline{x}_G)}$ when s is close to zero. Then it is evident that $\sigma_F > \sigma_G$ if and only if $f(\overline{x}_F) < g(\overline{x}_G)$.

When F and G share the same upper bound $\overline{x} < \infty$, increasing convex order implies that $G(x) \leq F(x)$ for x sufficiently close to $\overline{x}$ and so $g(\overline{x}) \geq f(\overline{x})$. (This is ready to see from the alternative definition of increasing convex order that there exists a random variable Y which connects X_F and X_G via FOSD and MPS.) When these two densities are not equal, the strict inequality holds and so consumers search less in the case of G. $\square$

Intuitively consumers tend to search more when the match utility distribution is more dispersed. Excess wealth order is one location-free way to compare the degree of dispersion between two random variables, and it turns out to ensure an unambiguous comparison of search duration. (See Chapter 3.C in Shaked and Shanthikumar (2007) for a comprehensive discussion of this stochastic order concept.) The location-free property is desirable since merely shifting a distribution does not change consumer search incentive. Notice that $\mathbb{E}[(X_F - F^{-1}(\sigma))_+]$ is the expected benefit from one more search in search pool F when the best match utility so far has reached the $100\sigma_{\text{th}}$ percentile. So (11) means that consumers have a higher search incentive in this percentile sense in search pool G. A similar result has been shown in section 2.3.4. in Chateauneuf, Cohen, and Meilijson (2004). The simpler sufficient condition in result (i) actually defines that X_G is more dispersed than X_F in the sense of "dispersive order." [14] As we will see in section 4.2, when G is a truncation of F from below, they are typically ranked by dispersive order, and so a clear-cut search duration result is available.

MPS is another way to compare dispersion (when two random variables share the same mean), but unfortunately it is not enough for a clear-cut comparison result on search duration.[15] When X_F and X_G have the same mean, excess wealth order implies MPS and so is a stronger condition.

Result (ii) is intuitive to understand. When s is close to zero, consumers will cease their search only if they find a match utility close to the upper bound of the distribution. Then a higher density of match utility at the upper bound implies a higher likelihood of

[14] See, e.g., Chapter 3.B in Shaked and Shanthikumar (2007) for an introduction of this concept. Choi and Smith (2019) also show a similar dispersive-order result. One special case of dispersive order is that $\alpha X + \beta$ is greater than X in dispersive order for any $\alpha > 1$. Dispersive order has been used to study various economics problems. See, for example, Ganuza and Penalva (2010) for its application in information disclosure in auctions; Zhou (2017) and Choi, Dai, and Kim (2018) for oligopolistic price competition; Drugov and Ryvkin (2020) for effort choice in tournaments.

[15] Here is one counterexample: Suppose F has a triangle density on $[0, 1]$ with $F(x) = 2x^2$ for $x \in [0, \frac{1}{2}]$ and $1 - 2(1-x)^2$ for $x \in [\frac{1}{2}, 1]$. Suppose G is the uniform distribution on $[0, 1]$ and so it is an MPS of F. When $s < \frac{1}{12}$, one can check that $r_F = 1 - (\frac{3}{2}s)^{1/3}$ and $r_G = 1 - (2s)^{1/2}$. Then $l_F = [2(1-r_F)^2]^{-1} < l_G = [1 - r_G]^{-1}$ if and only if $s > \frac{1}{81}$. Hence, in this example, whether consumers search longer or not in the case of G depends on the magnitude of the search cost.

stopping search.

Price and consumer welfare. In our model total welfare is simply the reservation match utility. Hence, information improvement must enhance total welfare given G is greater than F in the increasing convex order. We then focus on how information improvement affects price and consumer welfare. (Profit comparison is the same as price comparison given all consumers buy in equilibrium.)

As we have seen in (4) and (8), the equilibrium price is the reciprocal hazard rate of the match utility distribution (which reflects the demand composition) evaluated at the reservation match utility (which captures consumer search incentive). A change of the match utility distribution often has opposite effects on these two components. For example, suppose information improvement leads to a higher distribution in the hazard rate order (i.e., $\frac{1-G(x)}{g(x)} \geq \frac{1-F(x)}{f(x)}$). It is evident from (4) that this increases the market price for a given reservation match utility. However, as this improvement is a case of FOSD, we also have $r_G \geq r_F$, an opposite force to lower the price whenever at least one of the reciprocal hazard rate functions is decreasing. The price expression in (7) helps illustrate a similar trade-off when information improvement leads to a more dispersed distribution in terms of dispersive order (i.e., $g(G^{-1}(t)) \leq f(F^{-1}(t))$). This increases the price for a given search propensity, but as this change induces a greater search propensity as shown in Proposition 1, there is also an opposite force to lower the price if at least one of the reciprocal hazard rate functions is decreasing. It is therefore often hard to obtain a clear-cut result on how a change of the match utility distribution affects the equilibrium price.[16]

For convenience, let

$$\tau_F(x) \equiv \frac{1-F(x)}{f(x)} \quad \text{and} \quad \tau_G(x) \equiv \frac{1-G(x)}{g(x)}$$

be the reciprocal hazard rate in the case of F and G, respectively. They are decreasing (increasing) functions if and only if $1-F$ and $1-G$ are log-concave (log-convex).

Consumer surplus equals the reservation match utility minus price, i.e., $v_i = r_i - p_i$, $i \in \{F, G\}$. Hence, information improvement must benefit consumers whenever it reduces the market price, and it can harm consumers only if it softens price competition. For

[16] Of course, if at least one of the reciprocal hazard rate functions is increasing (i.e., when $1-F$ or $1-G$ is log-convex), then in both cases the two forces work in the same direction and the market price unambiguously increases as information improves.

convenience, let us define two "virtual value" functions:

$$\eta_F(x) \equiv x - \frac{1-F(x)}{f(x)} \text{ and } \eta_G(x) \equiv x - \frac{1-G(x)}{g(x)} \ .$$

Then consumer surplus is $\eta_i(r_i)$ under distribution i. (Recall that both of the η functions are increasing given the -1-concavity in Assumption 2.)

Proposition 2. *(i) Suppose both of the reciprocal hazard rates τ_F and τ_G are monotonic, and at least one of them is decreasing. Then price is lower in search pool G for any permissible search cost if*

$$\mathbb{E}[(X_G - \tau_G^{-1}(p))_+] \geq \mathbb{E}[(X_F - \tau_F^{-1}(p))_+] \text{ for any } p \ . \tag{14}$$

(ii) Consumers are better off in search pool G for any permissible search cost if

$$\mathbb{E}[(X_G - \eta_G^{-1}(v))_+] \geq \mathbb{E}[(X_F - \eta_F^{-1}(v))_+] \text{ for any } v \ . \tag{15}$$

The proof in the appendix follows a similar logic as in the search-duration result. Notice that $\mathbb{E}[(X_i - \tau_i^{-1}(p))_+]$ ($\mathbb{E}[(X_i - \eta_i^{-1}(v))_+]$) is the expected benefit from one more search under distribution i when the best match utility so far has reached a certain level in terms of hazard rate (virtual value). Unfortunately, there are no simple stochastic order concepts which imply (14) or (15). An easy case where both (14) and (15) hold is when information improvement does not change the hazard rate of the match utility distribution. In that case, as we will study in detail later, information improvement has unambiguous impacts on price and consumer welfare.

More transparent conditions are available when the search cost is sufficiently small or large.

Proposition 3. *(i) Suppose $f(\overline{x}_F), g(\overline{x}_G) > 0$. Then there exists $\hat{s}$ such that for $s < \hat{s}$, price is lower in search pool G if and only if $g(\overline{x}_G) > f(\overline{x}_F)$, and consumers are better off in search pool G if and only if*

$$\overline{x}_G - 2\sqrt{\frac{2s}{g(\overline{x}_G)}} > \overline{x}_F - 2\sqrt{\frac{2s}{f(\overline{x}_F)}} \ .$$

Both are true if $\overline{x}_F = \overline{x}_G$ and $f(\overline{x}_F) \neq g(\overline{x}_G)$.
(ii) Suppose the monopoly price p_i^M, $i \in \{F, G\}$, is interior. If $v_F^M > v_G^M$, then there exists $\tilde{s}$ such that for $s \in (\tilde{s}, v_G^M)$, consumers are worse off (and so price must be higher) in search pool G; if $v_F^M < v_G^M$, then there exists $\tilde{s}$ such that for $s \in (\tilde{s}, v_F^M)$, consumers are better off in search pool G.

Proof. (i) It suffices to prove the result when s is close to 0. Notice that when σ is close to 1 and $f(\overline{x}_F) > 0$, we have

$$\frac{1-\sigma}{f(F^{-1}(\sigma))} \approx \frac{1-\sigma}{f(\overline{x}_F)}$$

by using the Taylor expansion. This, together with (13), implies that when s is close to zero, we can approximate the equilibrium price as

$$p_F = \frac{1-\sigma_F}{f(F^{-1}(\sigma_F))} \approx \sqrt{\frac{2s}{f(\overline{x}_F)}}\ .$$

Similarly,

$$p_G \approx \sqrt{\frac{2s}{g(\overline{x}_G)}}\ .$$

Then the desired price comparison result follows.

From the definition of r_i and using the Taylor expansion, one can also approximate the reservation match utilities as

$$r_F \approx \overline{x}_F - \sqrt{\frac{2s}{f(\overline{x}_F)}}\ ;\ r_G \approx \overline{x}_G - \sqrt{\frac{2s}{g(\overline{x}_G)}}\ .$$

Since consumer surplus is $r_i - p_i$, the desired result on consumer surplus follows.

(ii) Suppose first $v_F^M > v_G^M$. Then when the search cost approaches to v_G^M, consumers will have a zero surplus in search pool G if the monopoly price is an interior solution (which has been explained in the proof of Lemma 1), but a positive surplus in search pool F. As a result, consumers get worse off as information improves. The case of $v_F^M < v_G^M$ is similar. □

The first result for a small search cost is intuitive. When the search friction is small, consumers will not stop searching until finding an almost perfect match. In other words, for each firm their marginal consumers have a match utility close to the upper bound. The density of these marginal consumers essentially determines firms' pricing incentive. Therefore, price is lower in search pool G if there are more marginal consumers in that case (i.e., if $g(\overline{x}_G)$ is bigger). Together with result (ii) in Proposition 1, this result implies that when s is small, search duration and price tend to move in the *same* direction as information improves, which is opposite to the usual negative correlation between search duration and price when information quality is fixed but the search cost varies.

The first result also implies that when the search friction is small, if information improvement does not change the upper bound of match utility, it induces less search and a lower market price and so must benefit consumers. The second result is simply because when the search cost is high, how information improvement impacts consumers in a search market is the same as it does in a monopoly market.

Finally, notice that using (6) and (7), we have

$$p_F(1-\sigma_F) \geq \int_{\sigma_F}^{1} \frac{1-t}{f(F^{-1}(t))}dt = s$$

if the integrand is decreasing or if $1-F$ is log-concave. Then $p_F \geq s \times l_F$, where l_F is the search duration. That is, the price a consumer pays for the product is always greater than the expected search cost she incurs. (The opposite is true if $1-F$ is log-convex.) This observation implies that if both $1-F$ and $1-G$ are close to be log-linear (i.e., if both match utility distributions are close to be exponential), then p_i is approximately equal to $s \times l_i$ and so price is lower in search pool G if and only if it leads to less consumer search.[17]

4.2 Examples

In this section we study several examples to illustrate the above discussions. They also deliver some insights which are not transparent enough in the general analysis.

Truncation example. Suppose X_G is a truncation of X_F from below as in the motivating example for a more selective search pool. Then for any x in the support of G, we have $1-G(x) = k(1-F(x))$, where $k>1$ is a constant. Therefore,

$$\frac{1-G(x)}{g(x)} = \frac{k(1-F(x))}{kf(x)} = \frac{1-F(x)}{f(x)} \ , \tag{16}$$

i.e., the two distributions share the same hazard rate. (Since the reservation match utility in each case must be interior under the search cost condition in Assumption 2, we can indeed focus on the range of x in the support of G.) Given $r_G \geq r_F$, this special property immediately implies a clear-cut price comparison result: information improvement lowers (raises) the market price if $1-F$ is log-concave (log-convex). Property (16) also implies that $\eta_F(\cdot) = \eta_G(\cdot)$. Since the η function is increasing, information improvement then must benefit consumers regardless of how it affects the market price. Using Proposition 1, we can also show a clear-cut search duration comparison result.

[17] Consider the family of exponential distribution with CDF $1-e^{-x/\mu}$ on $[0,\infty)$. Its survival function is log-linear, and its reciprocal hazard rate is a constant μ. As μ increases, the distribution increases in the sense of both FOSD and dispersive order. One can check that search duration is $\frac{\mu}{s}$ and the market price is μ. Both increase in μ. Consumer surplus is $\mu(\ln \frac{\mu}{s} - 1)$ and it increases in μ as well.

Corollary 1. *When X_G is a truncation of X_F from below, consumers search less and price drops in search pool G if $1 - F$ is log-concave, and the opposite is true if $1 - F$ is log-convex; consumers always get better off in search pool G.*

The search duration result suggests that truncating a distribution does not always reduce its dispersion. When the distribution is log-convex (in which case it has a fat tail), a truncation from below actually increases its dispersion. It is also worth mentioning that when $1 - F$ is log-concave, the impact of information improvement in this example is qualitatively the same as when the search cost is small and the two distributions share the same upper bound.

As pointed out before, this truncation setup is used in de Corniere (2016) for studying targeted search advertising. That paper does not adopt the Wolinsky model but instead considers a Salop circular model where both a continuum of firms/advertisers and a continuum of consumers are uniformly, but independently, distributed on the circle. In the benchmark, when a consumer enters a query that reveals her taste location, the search engine displays all the firms randomly to her, and the consumer then conducts a sequential search in a random order. This is a spatial version of the Wolinsky model (and it was developed in Wolinsky (1983)). More precisely, since the disutility of buying a non-ideal product is assumed to be weakly convex in the distance between the consumer's taste location and the product location, the model is equivalent to the Wolinsky model with a weakly increasing match utility density function h (so that $1-H$ is concave). de Corniere is interested in the scenario where either the firms or a search engine can control the match precision. In particular, if a firm chooses a match broadness d, it will appear in a consumer's search pool only if it is within the distance of d from the consumer's location. This is the same as when a consumer sees a firm only if its match utility is above a threshold in the Wolinsky model. In this sense, our truncation example generalizes the model in de Corniere (2016) by allowing a more general match utility distribution. Our analysis also reveals the fundamental reason why de Corniere's model has an unambiguous prediction on how a more targeted search pool affects the market price (when the search cost is not too high).[18]

Another related paper is Eliaz and Spiegler (2011). It studies the quality control of a search engine by considering a variant of the Wolinsky model where each product is either a match or not for a consumer, and conditional on being a match its match utility is a ran-

[18]Zhong (2021) studies a similar search design problem in the Wolinsky model. He assumes that personalized recommendations lead to a distribution truncated from below.

dom draw from a common distribution with CDF $H(x)$. Products differ in their quality, denoted by q, in terms of their chance of being a match for a consumer, and the quality is unobservable to consumers. (This approach of modelling firm quality heterogeneity is adopted from Chen and He (2011).) A search engine can control the quality of firms displayed to consumers by setting a per-click fee. Since a higher-quality firm is more willing to join, only the products with a quality above a certain threshold, say, $\hat{q}$ will join and so be displayed to consumers. Consumers search in this pool sequentially and randomly, and when they encounter a matched product they also immediately discover its match utility. This model differs from ours as it has *ex ante* firm heterogeneity, but its feature of binary match outcomes ensures symmetric pricing across firms, so that it is essentially the same as our model with a match utility distribution $1-\mathbb{E}[q|q \geq \hat{q}]+\mathbb{E}[q|q \geq \hat{q}]H(x)$ which has a mass point at the lower bound. When $\hat{q}$ increases, the size of the mass point reduces, and so the distribution increases in the sense of FOSD. It increases not in the truncation form, but its hazard rate remains unchanged. That is why an information improvement in Eliaz and Spiegler (2011) has a similar impact as in de Corniere (2016).

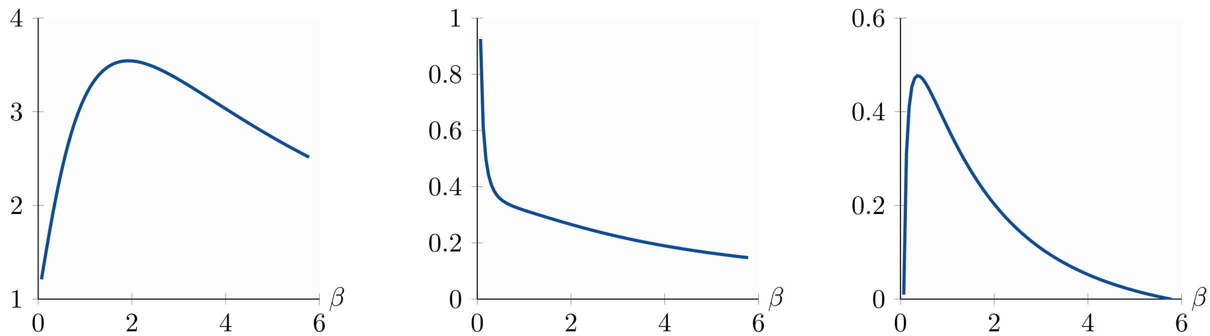
Beta distribution example. The above truncation example may give the impression that information improvement can raise price and harm consumers only in "irregular" cases such as when the distribution is log-convex. Here we report an example to show that this adverse effect of information improvement can also happen in a regular case.

Consider the family of distribution with CDF $1-(1-x)^{\beta}$. Its support is $[0,1)$ if $\beta \in (0,1)$ and $[0,1]$ if $\beta \geq 1$. (This belongs to the Beta distribution family.) Its survival function $(1-x)^{\beta}$ is log-concave, and its hazard rate function is $\frac{\beta}{1-x}$. The distribution increases in the sense of FOSD when β *decreases*: it has an increasing density if $\beta < 1$, is the uniform distribution if $\beta = 1$, and has a decreasing density if $\beta > 1$. One can check that the monopoly price in this example is $\frac{1}{1+\beta}$, and the search market is active if and only if $s \leq \frac{1}{1+\beta}(\frac{\beta}{1+\beta})^{1+\beta}$ (which never exceeds about 0.13).

Let $\xi(s,\beta) = (s(1+\beta))^{\frac{1}{1+\beta}}$. Then the reservation match utility in this example is $1-\xi(s,\beta)$, search duration is $\xi(s,\beta)^{-\beta}$, market price is $\frac{1}{\beta}\xi(s,\beta)$, and consumer surplus is $1-(1+\frac{1}{\beta})\xi(s,\beta)$. By logarithmizing each item, it is straightforward to show the following result.

Corollary 2. *In the Beta distribution example with match utility distribution* $1-(1-x)^{\beta}$, *as information improves (i.e., as* β *decreases), consumers search less if and only if* $s < \frac{1}{1+\beta}\exp(-\beta)$, *price drops if and only if* $s < \frac{1}{1+\beta}\exp(1-\frac{(1+\beta)^2}{\beta})$, *and consumer surplus* *improves if and only if* $s < \frac{1}{1+\beta}\exp(-\frac{1}{\beta})$.[19]

As we have known in the general analysis, if s is sufficiently small, information improvement decreases search duration and market price and benefits consumers. However, if s is relatively large, the impacts of information improvement can be completely reversed. For example, when $s = 0.05$, the search market is active for any β between about 0.07 and about 5.78. Figure 1 below depicts how β affects search duration, price and consumer surplus. In this example, price always increases as β decreases, and both search duration and consumer surplus vary with β non-monotonically. In particular, consumers suffer as β decreases (i.e., as information improves) in the range of about $[0.07, 0.37]$.



(a) search duration (b) price (c) consumer surplus

Figure 1: Impact of information improvement (as β *decreases*)
in a Beta distribution example

Truth-or-noise example. We finally revisit the truth-or-noise example for a more informative search pool. When the underlying match utility is random variable X and the precision parameter is θ_i, $i \in \{F, G\}$, the perceived match utility random variable is $X_i = \theta_i X + (1-\theta_i)\mu$, where μ is the mean of X. Then $X_G = \frac{\theta_G}{\theta_F}X_F + (1-\frac{\theta_G}{\theta_F})\mu$. When $\theta_G > \theta_F$, X_G is more dispersed than X_F in the dispersive order, and so consumers must search longer in search pool G according to Proposition 1.

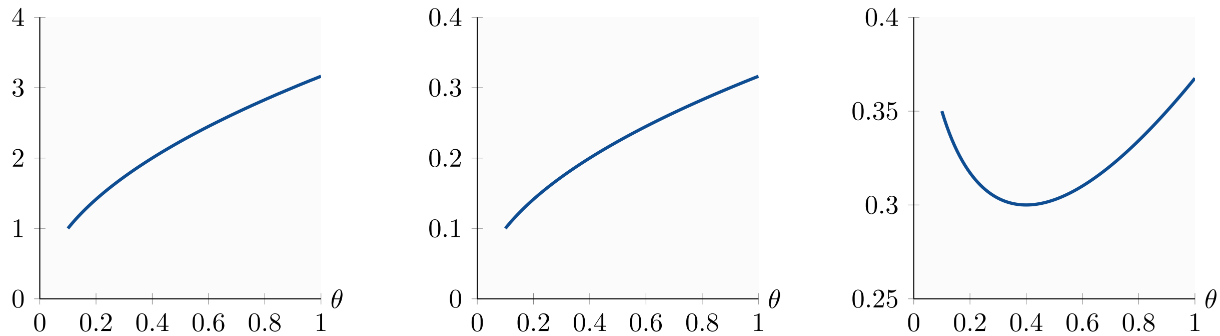
More generally, we can consider a linear transformation $X_G = kX_F + \phi(k)$, where $k > 1$ is a constant and $\phi(k)$ is a function of k and can be negative. (When $\phi(k) = 0$, we have the multiplicative case studied in Anderson and Renault (1999).)

Corollary 3. *Suppose* $X_G = kX_F + \phi(k)$ *with* $k > 1$ *is greater than* X_F *in increasing convex order (which includes the truth-or-noise example as a special case). Then consumers*

[19] Among the three thresholds, the one for price is the lowest and it is below the threshold for an active market.

always search longer in search pool G; if $f(\overline{x}_F) > 0$, price rises and consumers get better off in search pool G at least when the search cost is sufficiently small.

For a relatively large search cost, however, the rise of price can result in a lower consumer surplus. Figure 2 below depicts the impacts of information improvement in the truth-or-noise example when the underlying match utility distribution is uniform on $[0, 1]$ and $s = 0.05$. (In this case we need $\theta > 0.1$ to make the market active.) Here consumer surplus varies with θ non-monotonically.



(a) search duration (b) price (c) consumer surplus

Figure 2: Impact of information improvement (as θ *increases*)
in a truth-or-noise example

5 Concluding discussion

This paper has investigated how information improvement affects the performance of a search market. It nests a few specific ways to model information improvement such as targeted search advertising, personalized recommendations, and more informative product inspection into a unified framework. Conditions are derived for unambiguous impacts on search duration, market price, and consumer welfare. In particular, when the search friction is small and information improvement does not change the upper bound of match utility, or when information improvement truncates the match utility distribution from below, it typically induces consumers to search less, intensifies price competition and benefits consumers. More generally, however, it is also possible that information improvement raises the market price significantly so that consumers suffer from it.

Heterogeneous outside options. For simplicity, we have assumed that all consumers have the same outside option. As a result, all consumers enter and buy if the search market is active in equilibrium. The more realistic case is to consider heterogeneous outside options among consumers. Then a consumer will enter the search market only if the expected surplus from searching exceeds her outside option, but once she enters the market, she will behave as described in our model and her outside option no longer matters. Moreover, since price is unobservable before search, no firm's actual price choice will affect consumers' participation decision, and so each firm's pricing problem remains unchanged. This means that considering heterogenous outside options has no impact on how information improvement affects search duration and market price. The impact on profit, however, can now be different as industry profit is no longer simply equal to the market price. In particular, if information improvement induces a lower market price, it does not necessarily harm firms as it expands the demand. The impact on consumer surplus can now also differ due to the demand effect, but if information improvement reduces price, it still benefits consumers.

Alternative ways to model information improvement. Information improvement may take other formats that are not captured by our framework. One possibility, as adopted in Anderson and Renault (2000), is that some consumers become fully informed of the match utilities of all the available products before search. Since these consumers have no incentives to search beyond the best matched product which they already know, their presence relaxes price competition and harms other uninformed consumers. If all consumers become informed, due to Diamond (1971)'s argument each firm will act as a monopolist conditional on being the best matched supplier.[20]

Another possibility is to consider ranked products. If products are perfectly ranked for each consumer, we will end up with the aforementioned Diamond (1971) outcome. If products are imperfectly ranked, then a complication is the potential correlation of match utilities across products. This can make both the optimal stopping rule and the equilibrium pricing problem hard to characterize.[21]

Endogenous information improvement in search markets. This paper considers an exogenous information improvement. In practice, however, information improvement is often the outcome of strategic information provision by firms or platforms, as studied in

[20]More precisely, each firm will act as a multiproduct monopolist that sells all the products in the market. With an infinite number of firms, this will lead to a price equal to the maximum match utility and so the market will collapse unless the first search is free.

[21]A related work in this direction is Burguet and Petrikaite (2019). They consider targeted advertising in the Wolinsky model with a finite number of firms. They assume that each firm sends their ads only to the consumers who regard their product as one of the top two products, and consumers search only among the products from which they receive ads. But for simplicity they assume that the top two products for each consumer are not further ranked.

Eliaz and Spiegler (2011), de Corniere (2016) and Zhong (2021) though they restricted their attention to a particular type of information improvement as we have explained. Following the recent literature on Bayesian persuasion and information design, Dogan and Hu (2021) study how informative the inspection of each product should be in a search market if a platform wants to maximize consumer surplus.[22] More broadly, platform design in a search market can consider not only the informativeness of each product inspection, but also the disclosure of relative valuations across products or even the control of which products to display to consumers.

APPENDIX

Proof of Proposition 2. (i) Given $\tau_i(\cdot)$ is monotonic and $p_i = \tau_i(r_i)$, we have $r_i = \tau_i^{-1}(p_i)$. Then the definitions of r_F and r_G imply that

$$\mathbb{E}[(X_G - \tau_G^{-1}(p_G))_+] = \mathbb{E}[(X_F - \tau_F^{-1}(p_F))_+] \ .$$

When (14) holds, letting $p = p_G$ yields

$$\mathbb{E}[(X_G - \tau_G^{-1}(p_G))_+] \geq \mathbb{E}[(X_F - \tau_F^{-1}(p_G))_+] \ .$$

Then we have

$$\mathbb{E}[(X_F - \tau_F^{-1}(p_F))_+] \geq \mathbb{E}[(X_F - \tau_F^{-1}(p_G))_+] \ ,$$

or equivalently $\tau_F^{-1}(p_F) \leq \tau_F^{-1}(p_G)$. This implies $p_F \geq p_G$ if $\tau_F(x)$ is decreasing. If $\tau_G(x)$ is decreasing, letting $p = p_F$ in (14) yields the same result.

(ii) Let $v_i \equiv \eta_i(r_i)$ denote the consumer surplus under distribution i. Since $\eta_i(\cdot)$ is increasing given the -1-concavity assumption, we have $r_i = \eta_i^{-1}(v_i)$. The rest of the proof is similar to (i) and so omitted. □

Proof of Corollary 1. The results on price and consumer surplus have been explained in the main text. Here we prove the search duration result. We aim to show, when G is a truncation of F from below, G is less (more) dispersed than F in the dispersive order if $1 - F$ is log-concave (log-convex). Then the desired result follows immediately from Proposition 1.

[22]In the Bayesian persuasion framework, Board and Lu (2018) and Au and Whitmeyer (2021) study equilibrium product information disclosure by competing firms in a search market, but they assume away price competition.

Recall that G is less dispersed than F in the dispersive order if $g(G^{-1}(t)) > f(F^{-1}(t))$. Given $1 - G(x) = k(1 - F(x))$ where $k > 1$ is a constant, we have $g(x) = kf(x)$ which implies $g(G^{-1}(t)) = kf(G^{-1}(t))$, and

$$G^{-1}(t) = F^{-1}(1 - \frac{1-t}{k}) \ . \tag{17}$$

Notice that $kf(G^{-1}(t)) = f(F^{-1}(t))$ at $k = 1$, and one can check that the derivative of $kf(G^{-1}(t))$ with respect to k is

$$f(z) + k\frac{f'(z)}{f(z)}\frac{1-t}{k^2} = f(z) + \frac{f'(z)}{f(z)}[1 - F(z)] \ , \tag{18}$$

where $z = G^{-1}(t)$ and the equality used (17). When $1 - F$ is log-concave, we have $f^2 + (1 - F)f' \geq 0$ and so (18) is positive. Then $g(G^{-1}(t)) = kf(G^{-1}(t)) > f(F^{-1}(t))$ for $k > 1$, i.e., G is smaller than F in the dispersive order. The opposite is true if $1 - F$ is log-convex. □

Proof of Corollary 3. The search duration result is evident from Proposition 1 since X_G is more dispersed than X_F in the dispersive order. To prove the result on price and consumer surplus, notice that $G(x) = F(\frac{1}{k}(x - \phi(k)))$, and so

$$g(x) = \frac{1}{k}f(\frac{1}{k}(x - \phi(k))) \ .$$

Using $\overline{x}_G = k\overline{x}_F + \phi(k)$, we have $g(\overline{x}_G) = \frac{1}{k}f(\overline{x}_F) < f(\overline{x}_F)$, given $f(\overline{x}_F) > 0$. The price result when s is sufficiently small then follows from Proposition 3.

According to Proposition 3, the consumer-surplus result holds if

$$k\overline{x}_F + \phi(k) - 2\sqrt{\frac{2ks}{f(\overline{x}_F)}} > \overline{x}_F - 2\sqrt{\frac{2s}{f(\overline{x}_F)}} \ . \tag{19}$$

A necessary condition for X_G to be greater than X_F in increasing convex order is $k\mu_F + \phi(k) \geq \mu_F$, where μ_F is the mean of X_F. Given $k > 1$, this implies $k\overline{x}_F + \phi(k) > \overline{x}_F$. Therefore, (19) must hold when s is sufficiently small. □

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