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DIAGNOSING HOUSING FEVER WITH AN ECONOMETRIC THERMOMETER

By

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Diagnosing Housing Fever with an Econometric Thermometer *

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Abstract

Housing fever is a popular term to describe an overheated housing market or housing price bubble. Like other financial asset bubbles, housing fever can inflict harm on the real economy, as indeed the US housing bubble did in the period following 2006 leading up to the general financial crisis and great recession. One contribution that econometricians can make to minimize the harm created by a housing bubble is to provide a quantitative 'thermometer' for diagnosing ongoing housing fever. Early diagnosis can enable prompt and effective policy action that reduces long term damage to the real economy. This paper provides a selective review of the relevant literature on econometric methods for identifying housing bubbles together with some new methods of research and an empirical application. We first present a technical definition of a housing bubble that facilitates empirical work and discuss significant difficulties encountered in practical work and the solutions that have been proposed in the past literature. A major challenge in all econometric identification procedures is to assess prices in relation to fundamentals, which requires measurement of fundamentals. One solution to address this challenge is to estimate the fundamental component from an underlying structural relationship involving measurable variables. A second aim of the paper is to improve the estimation accuracy of fundamentals by means of an easy-to-implement reduced-form approach. Since many of the relevant variables that determine fundamentals are nonstationary and interdependent we use the IVX (Phillips et al. (2009); Kostakis et al. (2015)) method to estimate the reduced-form model to reduce the finite sample bias which arises from highly persistent regressors and endogeneity. The recursive evolving test of Phillips, Shi, and Yu (2015a, PSY) is applied to the estimated non-fundamental component for the identification of speculative bubbles. The new bubble test developed here is referred to as PSY-IVX. An empirical application to the eight Australian capital city housing markets over the period 1999 to 2017 shows that bubble testing results are sensitive to different ways of controlling for fundamentals and highlights the importance of accurate estimation of these housing market fundamentals.

Keywords: Housing bubbles; periodically collapsing; unobservable; fundamentals; explosive; IVX; Australia housing markets.

JEL classification: C12, C13, C58

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1 Introduction

Housing fever arises when fundamental economic factors do not justify ongoing rises in house prices. Instead, prices are driven by repeated purchasing decisions by investors that are founded on expectations of higher housing prices in future (Stiglitz, 1990), leading to house price bubbles that can harm the real economy. During the expansion stage of a price bubble, there is a considerable inflow of resources into the real estate sector, which crowds out more productive investment (Hirano et al., 2015) and domestic saving (Caballero and Krishnamurthy, 2006), dampening corporation innovation and causing a shortage of liquid assets. The collapse of a housing market bubble can result in extensive reductions in household consumption (Skinner, 1996; Case et al., 2005) and unanticipated losses for lending institutions (Case et al., 2000) which exacerbate the negative economic shock and which may lead to wider economic decline and financial instability, as occured in dramatic fashion when the US housing bubble burst in 2006-2007.

Housing bubbles have attracted the attention of policymakers around the globe. For example, after the subprime mortgage crisis, Donald Kohn, former vice-chairman of the Federal Reserve Board, urged policymakers to "deepen their understanding about how to combat speculative bubbles to reduce the chances of another financial crisis."[1] One mechanism to achieve such understanding is to use empirical econometric models to assess whether a 'speculative bubble' is manifesting as a present and ongoing risk in the data. The key methodological requirements in doing so are threefold: (i) the capacity to identify a bubble in its expansionary phase, thereby distinguishing it from 'normal' periods of rising prices; (ii) this capacity in turn requires an explicit identification condition for a market experiencing such a phenomena; (iii) an empirical testing procedure that has power to detect deviations from 'normal' market behavior; (iv) quantification of economic fundamentals that drive markets during normal periods, so that abnormal periods are revealed by way of deviance from fundamentals; and (v) a real time process of assessment that enables early detection of speculative market behavior. In short, one contribution that econometricians can make to deepen policy maker understanding of potential speculative bubbles is to provide a 'thermometer' diagnostic for the presence of housing market fever (a.k.a. econometric bubble detection techniques) that meets these requirements. The aim is to detect the 'illness' (a.k.a. presence of speculations) accurately and promptly, thereby enabling effective policy actions.

This paper first provides a selective review of the literature on the econometric identification of housing bubbles. We start with a technical definition of housing bubbles, followed by discussions on significant challenges encountered and solutions proposed in the literature. By definition, the logarithmic price-to-rent (PR) ratio consists of a fundamental and a bubble component in the presence of speculations. The bubble component is explosive, periodically collapsing, and unobservable. We focus on challenges brought by the periodically collapsing feature (Blanchard, 1979; Blanchard and Watson, 1982) and the unobservable nature of bubbles.

The periodically collapsing feature results in low power in traditional approaches based on cointegration and unit root limit theory (Evans, 1991), which points to the need of either using nonlinear models

[1] See `https://www.federalreserve.gov/newsevents/speech/kohn20100103a.htm`.

or working with subsamples for bubble testing. Popular solutions to the low discriminatory power of standard methods are the recursive test proposed by Phillips, Wu, and Yu (2011) and the generalized version of this test suggested in Phillips, Shi, and Yu (2015a, PSY). The PSY test is based on a right-tailed unit root test (with an unit root null and an explosive alternative) and relies on a subsampling algorithm to overcome the challenge of low power and to assist in date stamping multiple bubble episodes in long historical time series. Since its development, this approach has enjoyed widespread use in empirical applications and as a real time diagnostic of the state of asset price markets. For example, using the PSY algorithm, the Federal Reserve Bank of Dallas provides quarterly exuberance indicators for 23 international housing markets.[2] The present paper provides an overview to the PSY approach with a practical illustration of its implementation to housing markets.

Several solutions have been proposed to address the challenge that bubbles are unobservable until they collapse. To be effective, solutions must meet they key methodological requirements (i)-(v) given above. The first proposal is to work with the raw data on house prices. According to the definition, house prices themselves have explosive characteristics during the expansive phase of a bubble. It has therefore become standard practice to apply bubble detection techniques to real house prices or log price/rental (PR) ratios,[3] where house rental data serves as a proxy for a market fundamental. The log PR ratios are often replaced by log price-to-income ratios when rent data are not available (e.g., Hu and Oxley 2018; Chen et al. 2019). The popularity of this solution is mostly due to its convenience. But the approach is not flawless as explosive root tests are primarily designed to capture the key time series feature of the bubble component, and this feature is only partly present as a component of house prices or log PR ratios.

An alternative solution, therefore, is to decompose house prices or log PR ratios into a fundamental and a non-fundamental component. The non-fundamental component comprises a bubble process and a random element during speculative episodes but constitutes just a random error element during normal market conditions. Campbell et al. (2009) propose a multi-step procedure to achieve this decomposition. Their procedure involves: 1) forecasting future streams of rent growth and interest rates using a VAR model; 2) calibrating model parameters from the data; and 3) 'assembling' the forecasted streams and calibrated parameters according to a structural economic definition of fundamentals. The non-fundamental component is then computed as the difference between the log PR ratios and the estimated fundamentals.

A second aim of the present paper is to enhance the estimation accuracy of the fundamental component in the above process by developing an improved estimation procedure. The new approach employs a 'reduced form' model, with the growth rate of PR ratios as the dependent variable and regressors that capture housing market demand and supply factors such as rent, interest rates, employment, population, GDP, and new housing completions. The goal is to separate variations that are driven by fundamentals from the remaining drivers of market prices. Given the endogeneity and nonstationarity of many of these

[2] See `https://www.dallasfed.org/institute/houseprice`.

[3] See, for example, Pavlidis et al. (2016); Gomez-Gonzalez et al. (2018); Anderson et al. (2011); Caspi (2016); Yiu and Jin (2013); Shi et al. (2016); Greenaway-McGrevy and Phillips (2016).

elements in the regression specification, we estimate the regression model by the IVX method (Phillips et al., 2009; Kostakis et al., 2015; Yang et al., 2020) to reduce finite sample bias in estimation, avoid non-standard limiting distributions of the estimated coefficients caused by highly persistent regressors and endogeneity, and so enhance inference. The fundamental component of market prices is then calculated as the cumulative sum of the fitted values. Unlike the existing multi-step approach to finding fundamentals, the new method is easy to implement by regression and has the advantage of allowing for a comprehensive set of fundamental driver variables, which thereby help to isolate the non-fundamental speculative component of prices.

There are two critical differences between the existing procedure and the new methods. The existing structural approach relies on a structural form specification of fundamentals which is a function of the return premium and the future streams of rent growth and interest rates. Implementation involves the estimation of a VAR model and calibrations of several unknown parameters. Both the estimation and calibration errors accumulate in the assembling step. By contrast, direct estimation of market fundamentals by the new reduced form regression approach is achieved in a single step with a large set of relevant regressor variables that affect fundamentals. The second key difference between the two approaches lies in their data requirements. The structural approach requires the availability of house prices and rents in dollar values (rather than indexes), which are sometimes hard to obtain in practice. Dollar values are not required in the reduced-form approach as either dollar values or indexes may be employed in the empirical regression.

With this new methodology for direct estimation of fundamentals, the non-fundamental time series may be computed as a residual and the PSY test can be applied to the resulting non-fundamental component, as in Shi (2017). This new approach to bubble testing is referred to as PSY-IVX testing. The method is illustrated here to investigate the presence of speculative bubbles in eight Australian capital city housing markets from 1999 to 2017. Australian housing markets experienced three significant booms over the sample period. The first occurred in the early 2000s after the announcement of the 50% capital gain tax discount; the second took place during the 2006-2007 commodity boom period; and the last episode started in 2013 and lasted until the end of the sample. The PSY-IVX procedure finds strong evidence of speculation in the early 2000s across multiple cities and over 2006-2007 in Perth and Darwin. Little evidence of speculation is detected during the most recent housing boom.

Our findings are compared with those reported in the recent work of Shi, Rahman, and Wang (2020, SRW), which computed the fundamental component using the structural procedure on the same data set. The two approaches are in agreement on the first and last episodes but disagree on the 2006-2007 episode. SRW find no evidence of speculation in both Perth and Darwin over the 2006-2007 period. Moreover, when applying the PSY test directly to the log PR ratios, longer and additional episodes of speculation are identified. These variations in the results indicate the important role that controlling for market fundamentals can play in housing bubble identification.

The remainder of the paper is organized as follows. Section 2 introduces technical definitions rele-

vant to the analysis of bubbles in time series data. Section 3 discusses challenges and solutions for the econometric identification of bubbles. The PSY-IVX approach for reduced-form determination of fundamentals is presented in Section 4. Section 5 reports the empirical application to Australian housing markets. Summary and conclusions are given in Section 6.

2 Bubble Definition

An important starting point in any analysis of bubbles is a quantitative definition that can be used in empirical research. A convenient and popular concept distinguishes between economic fundamentals and speculative factors which produce bubbles, leading to a model that separates these components (Blanchard and Watson, 1982). For housing market applications, let $p_t = \log(P_t)$ and $r_t = \log(R_t)$ where P_t denotes real housing prices and R_t real housing rents. The log price-to-rent (PR) ratio $(p_t - r_t)$ is then decomposed into fundamental (F_t) and bubble (B_t) components as follows:

$$p_t - r_t = F_t + B_t, \tag{1}$$

where

$$F_t = \frac{\kappa}{1-\rho} + \sum_{k=0}^{\infty} \rho^k (\Delta r_{t+1+k} - \gamma_{t+1+k}), \tag{2}$$

$$B_t = \lim_{j\to\infty} \rho^j p_{t+j}, \tag{3}$$

with $\bar{p}$ (respectively, $\bar{r}$) being the sample mean of p_t (r_t), $\rho = e^{\bar{p}}/(e^{\bar{p}} + e^{r})$, $\kappa = -log(\rho) + (1-\rho)(\bar{p} - \bar{r})$, $\gamma_t = \log(\Gamma_t)$, and Γ_t being the one-period gross return to housing. The decomposition (1) is obtained from the definition of the one period gross return to housing, i.e.,

$$\Gamma_{t+1} = \frac{P_{t+1} + R_{t+1}}{P_t}, \tag{4}$$

which can be rewritten in log form as $\gamma_{t+1} = \log(e^{p_{t+1}} + e^{r_{t+1}}) - p_t$. Applying a first order Taylor series expansion gives the approximate relationship

$$p_t - r_t = \kappa + \rho(p_{t+1} - r_{t+1}) + \Delta r_{t+1} - \gamma_{t+1}, \tag{5}$$

and recursive substitution leads to the model's equations (1)-(3). The infinite sums in (2) converge almost surely when $sup_t E(|\Delta r_t| + E|\gamma_t|) < \infty$ because $|\rho| < 1$.

The fundamental component F_t in (2) is a function of the future rent growth Δr_{t+1+k} and the future log gross return to housing γ_{t+1+k}. It is standard in the literature (e.g., Campbell et al. 2009; Sun and Tsang 2013; Shi 2017) to assume that the log gross return to housing equals the sum of the real interest

rate (i_t) and a time-varying risk premium (φ_t), i.e.,

$$\gamma_t = i_t + \varphi_t. \tag{6}$$

The risk premium is further assumed to take the simple form

$$\varphi_t = \varphi + \varepsilon_t, \tag{7}$$

where φ is the long-run risk premium and the zero mean error term ε_t captures short-term fluctuations brought by either market fundamentals or bubbles (Shi, 2017; Shi et al., 2020). Under these two assumptions, the fundamental component F_t can be written as

$$F_t = c + \mathcal{R}_t - \mathcal{I}_t - \mathcal{U}_t, \tag{8}$$

where

$$c = \frac{\kappa - \varphi}{1-\rho}, \mathcal{R}_t = \sum_{k=0}^{\infty} \rho^k \Delta r_{t+1+k}, \mathcal{I}_t = \sum_{k=0}^{\infty} \rho^k i_{t+1+k}, \text{ and } \mathcal{U}_t = \sum_{k=0}^{\infty} \rho^k \varepsilon_{t+1+k}.$$

The fundamental component F_t is now a function of the real interest rate, which could be stationary or non-stationary (Rose, 1988; Rapach and Weber, 2004). The quantity $\mathcal{R}_t$ (respectively, $\mathcal{I}_t$) is the aggregated discounted future values of housing rent growth (respectively, the real interest rate) based on perfect foresight. The expected value of F_t conditional on information at period t is then

$$E_t\left(F_t\right) = \frac{\kappa - \varphi}{1-\rho} + \sum_{k=0}^{\infty} \rho^k E_t\left(\Delta r_{t+1+k}\right) - \sum_{k=0}^{\infty} \rho^k E_t\left(i_{t+1+k}\right). \tag{9}$$

The bubble component B_t equals the present value of the asset price computed out to the infinite future. When the so-called 'transversality condition' is satisfied, we have $\lim_{j\to\infty} \rho^j p_{t+j} = 0$. Then $E_t\left(B_{t+1}\right) = 0$ and there is no speculative behaviour in the market. On the other hand, when the 'transversality condition' is violated ($\lim_{j\to\infty} \rho^j p_{t+j} \neq 0$), the bubble component follows a sub-martingale process such that

$$E_t(B_{t+1}) = \frac{1}{\rho} B_t. \tag{10}$$

By definition, the coefficient $1/\rho$ is greater than unity. This implies that the size of the bubble is expected to be larger in the next period, thereby representing speculative behaviour. Unlike the fundamental component, B_t is an explosive process.

Furthermore, in practice bubbles are typically not sustained and instead collapse periodically. So, an important empirical characteristic is that the explosive dynamic of a bubble does not prevail indefinitely, which leads to the common phenomenon that periods of both expansion and collapse may be experienced in the same sample period. Various mechanisms to model this phenonema have been suggested and

analyzed in the literature. For instance, under the data generating process proposed by Blanchard and Watson (1982), probabilities of a bubble collapse (π) and survival ($1-\pi$) are set *a priori* in the model, so that

$$B_{t+1} = \begin{cases} (\pi\rho)^{-1} B_t + \varepsilon_{t+1} & \text{with probability } \pi \\ \varepsilon_{t+1} & \text{with probability } 1-\pi \end{cases},$$

where $E_t(\varepsilon_{t+1}) = 0$. Similarly, Evans (1991) proposed a data generating process of the following form

$$B_{t+1} = \begin{cases} \rho^{-1} B_t \varepsilon_{B,t+1} & \text{if } B_t < b \\ \left[\zeta + (\pi\rho)^{-1} \theta_{t+1} (B_t - \rho\zeta)\right] \varepsilon_{B,t+1} & \text{if } B_t \geq b \end{cases}, \tag{11}$$

where $\varepsilon_{B,t+1} = \exp\left(v_{t+1} - \tau^2/2\right)$ with $v_{t+1} \sim i.i.d\, \mathcal{N}\left(0, \tau^2\right)$, θ_t follows a Bernoulli process which takes value 1 with probability π and zero otherwise, and ζ is the residual magnitude of the process when the bubble collapses. The rate of expansion depends on the size of the bubble. If the bubble size is smaller than b, the rate of expansion is $1/\rho$. However, when it exceeds b, the bubble expands at a faster rate $1/(\pi\rho)$ and faces a probability of collapsing (π). Importantly, both DGPs satisfy the condition in (10).[4].

3 Challenges and Solutions

The econometric identification of bubbles in time series data presents many challenges. In this Section we discuss some of the challenges that have arisen in econometric testing due to the fact that bubbles have multiple features and are typically short-lived and episodic. Speculative bubbles appear and subsequently collapse, leading to what has been called in the literature their periodically collapsing feature. This characteristic implies the presence of multiple structural changes in the generating mechanism that switch speculative behavior on and off. Such switches need to be detected and dated in the data if the historical course of a bubble is to be identified. Various solutions have been presented in the literature and the main approaches are described in what follows.

3.1 Periodically collapsing bubbles

Diba and Grossman (1988) studied the use of a right-tailed unit root test for bubble detection. The null hypothesis of this test is that there is no bubble in the market, so that the data follow a martingale process. The alternative hypothesis of interest is that speculative bubbles are present and the process is explosive. The effectiveness of this very simple test is severely compromised by the fact that a bubble may be present for only a short subperiod and it may periodically re-emerge and collapse. Evans (1991) showed that such right-tailed unit root tests have low discriminatory power when bubbles collapse periodically as they do in generating mechanisms of the form given in (11).

[4]Several papers have considered explicit data generating processes for $p_t - r_t$ that are designed for empirical work with data in the presence of bubbles. See, for example, Phillips et al. (2011); Phillips et al. (2015a); Phillips and Shi (2019).

To address these deficiencies in unit root testing algorithms, several approaches have been proposed. These methods typically seek to incorporate the switching mechanism or nonlinearity in the data generating process into the empirical model specification to assist in overcoming the low power of unit root testing. The approaches include Markov-switching ADF tests (Hall et al., 1999; Shi, 2013), regime-switching models (Brooks and Katsaris, 2005; Van Norden, 1996; Van Norden and Vigfusson, 1998), and methods based on subsampling techniques (Phillips et al., 2011; Homm and Breitung, 2012; Phillips et al., 2015a). The most popular solution is the recursive evolving test proposed by Phillips et al. (2015a,b) and various extensions of this procedure that have been developed to improve its performance in practical work. An overview of this PSY test is given in what follows.

Let y_t denote the data series of interest. Under the null hypothesis of no bubbles, y_t is assumed to follow a martingale process with an asymptotically negligible drift (Phillips et al., 2014) such that

$$y_t = cT^{-\eta} + y_{t-1} + \varepsilon_t, \text{ with constant } c \text{ and } \eta > 0.5, \tag{12}$$

where ε_t is a martingale difference sequence (m.d.s) and T is the sample size. The purpose behind the intercept $cT^{-\eta}$ in (12) is to allow for the presence of a small (or asymptotically negligible) drift in the data, which is induced by the unit autogregressive root in (12). This formulation is effective in capturing the mild drift that tends to occur in asset prices which grow over long periods. The alternative hypothesis is that y_t is a mildly explosive process with generating mechanism

$$y_t = \delta_T y_{t-1} + \varepsilon_t \text{ with } \delta_T = 1 + cT^{-\alpha}, c > 0 \text{ and } \alpha \in [0, 1). \tag{13}$$

In (13), the autoregressive coefficient $\delta_T = 1 + cT^{-\alpha}$ exceeds unity but by virtue of its specification is in the vicinity of unity and tends to unity as $T \to \infty$. The vicinity of unity for δ_T is deliberately prescribed to be wider than the commonly employed local to unity form (for which $\delta_T = 1 + cT^{-1}$). Such formulations were introduced by Phillips and Magdalinos (2007) and called mildly (or moderately) explosive. They are particularly useful in capturing periods of speculative exuberance in financial asset and real estate asset prices. This specification of the alternative hypothesis is consistent with the sub-martingale property of bubbles in (10).

The testing procedure is developed from a regression model of the form

$$\Delta y_t = \beta_0 + \beta_1 y_{t-1} + \sum_{i=1}^{K} \lambda_i \Delta y_{t-i} + \varepsilon_t, \tag{14}$$

where β_0, β_1, and λ_i are model coefficients, K is the lag order, and ε_t is the error term. The key parameter of interest is β_1. We have $\beta_1 = 0$ under the null and $\beta_1 > 0$ under the alternative. The model is estimated by Ordinary Least Squares (OLS) and the t-statistic associated with the estimated β_1 is referred to as the ADF statistic.

The construction of the PSY statistic requires computing the ADF statistic recursively from a subsam-

ple sequence. It is convenient for both exposition and asymptotics to measure time in sample fractions, in which case the total sample ranges fractionally from 0 to 1. Let r_1 and r_2 be, respectively, the start and end points of a certain subsample. We require a minimim window of r_0 to initiate the regression. So the full window subsample size r_w satisfies $r_w = r_2 - r_1 \geq r_0$. The ADF statistic computed from this subsample is denoted $ADF_{r_1}^{r_2}$. With this subsample arrangement, it is possible to investigate the presence of speculative bubbles using the PSY test for each observation falling between r_0 and 1. Suppose the observation of interest is some point $r \in [r_0, 1]$. In this case the end point of all the relevant subsamples is taken to be r (i.e., $r_2 = r$). In searching for the subsample with the greatest probability of being explosive, the PSY procedure allows the start point to vary within its feasible range so that $r_1 \in [0, r - r_0]$. The PSY test statistic itself is then defined as

$$PSY_r = \sup_{r_1 \in [0, r-r_0], r_2 = r} \left\{ ADF_{r_1}^{r_2} \right\}.$$

The PSY statistic corresponds to the largest ADF statistic in the sequence. Under the null hypothesis of (12), Phillips et al. (2015b) show that this statistic has the following limit distribution

$$PSY_r \to^d \sup_{r_1 \in [0, r-r_0], r_2 = r} \left\{ \frac{\frac{1}{2} r_w \left[W(r)^2 - W(r_1)^2 - r_w \right] - \int_{r_1}^{r} W(s)\, ds \left[W(r) - W(r_1) \right]}{r_w^{1/2} \left\{ r_w \int_{r_1}^{r} W(s)^2\, ds - \left[\int_{r_1}^{r} W(s)\, ds \right]^2 \right\}^{1/2}} \right\}, \tag{15}$$

where $W(\cdot)$ denotes standard Brownian motion. As is apparent from (15), the limit distribution is free of nuisance parameters and depends only on the process $W(\cdot)$, the start and end points (r_0, r) and the window width r_w. Critical values for the PSY statistic recursive sequence may be constructed by simulation from the asymptotic distribution (15) or by bootstrap methods, as discussed in Phillips et al. (2015a).

Another contribution of Phillips et al. (2015b) is to provide estimates of the bubble origination and termination dates (denoted by r_e and r_f, respectively) and to establish their consistency under a periodically collapsing bubble generating process. Let cv_r denote the critical value of the PSY statistic obtained from the above limiting distribution. The origination (resp. termination) of the bubble is estimated as the first chronological observation that the PSY_r statistic goes above (resp. below) the critical value. Denote the estimated origination and termination dates by $\hat{r}_e$ and $\hat{r}_f$, respectively. Under the given data generating process and some regularity conditions, the estimators converge to the true value as the sample size T goes to infinity, i.e.,

$$\hat{r}_e \to_p r_e \text{ and } \hat{r}_f \to_p r_f.$$

3.2 Unobservable bubble processes

Although B_t has the distinguishing time series feature of explosive submartingale behavior, it is not directly observable and may not even be present in a given series. This complicates inference. Asset prices are available and the fundamental component may be written in the form (8), leaving the bubble component as a residual. But the decomposition is not straightforward because the fundamental component must itself be measured. From (8) the fundamental component is defined in terms of its relation to the future income stream of the asset (rent growth) and the future cost for owning the property (the real rate of interest). Measurement of (8) therefore requires a methodology for computing these separate components. Different approaches have been suggested to perform the construction.

Solution I: use $p_t - r_t$ instead

The log price-to-rent ratio is composed as the sum of the fundamentals series F_t, which is at most I(1), and the explosive series B_t, if it is present. The log PR ratio is then I(1) in the absence of bubbles and explosive in the presence of bubbles. The problem of identifying the presence of a speculative bubble in an asset price series can therefore be reduced to detecting the presence of explosive dynamics in the log price-to-rent ratio. But this reduction brings complications for inference.

In particular, the presence of F_t in the log PR ratio introduces additional elements in testing explosiveness. Tests for an explosive root are designed for the unobserved B_t series not for series like log PR ratios with the additional fundamentals $I(1)$ component. To be specific, the null hypothesis of the PSY explosive root test is a martingale process with an asymptotically negligible drift (12). This specification matches well the bubble process B_t which has zero mean. But the log PR ratio is governed by F_t under the null and has a mean value of

$$c + (\mu_{\Delta r} - \mu_i) / (1 - \rho)$$

with $\mu_{\Delta r}$ and μ_i being the long run mean of Δr_t and i_t, according to (8). The drift value is nonzero and may be non-negligible. Moreover, the fundamentals component that affects asset prices is present in the PR ratio under the bubble alternative and therefore contaminates the pure bubble process B_t. Misspecifications such as these in both the null and alternative hypotheses may result in a different null limit distribution of the test statistic and false inferences concerning the presence of a bubble.

Solution II: decomposition

Another approach to tackle the unobservable bubble problem is to decompose $p_t - r_t$ into its fundamental and non-fundamental components based on (1) and (9). See, for example, Campbell et al. (2009); Sun and Tsang (2013); Shi (2017). The estimation of the fundamental component F_t requires unbiased forecasts of future streams of the rent growth rate and the real interest rate, as well as estimates of parameters κ, φ, and ρ in (9). The forecasting is accomplished using a VAR model, while the parameters are obtained

by calibration. Suppose the VAR is stationary with companion form

$$Z_{t+1} = \Pi_0 + \Pi Z_t + \varepsilon_{t+1}$$

where Z_t is a $K \times 1$ vector containing Δr_t and i_t (along with other related variables and their lags in the companion form) and ε_{t+1} is the error term. The expected value of Z_{t+k} conditional on information at period t is

$$E_t\left(Z_{t+k}\right) = \left(I_K - \Pi^k\right)\left(I_K - \Pi\right)^{-1}\Pi_0 + \Pi^k Z_t, \tag{16}$$

which provides forecasts for future rent growth rates and interest rates.

Let d_1 (respectively, d_2) be a $1 \times K$ row vector with all elements equal to zero, except that the one corresponding to Δr_t (respectively, i_t) is unity. The conditional expected value of the fundamental component is then

$$E_t\left(F_t\right) = c + \left(d_1 - d_2\right)\left(1-\rho\right)^{-1}\left(I_K - \rho\Pi\right)^{-1}\Pi_0 + \left(d_1 - d_2\right)\Pi\left(I_K - \rho\Pi\right)^{-1} Z_t, \tag{17}$$

by combining results in (16) and (9). The VAR model coefficients can be estimated by maximum likelihood and the other parameters (i.e., ρ, κ, φ) can be calibrated from the data. Denote the resulting estimated market fundamental by $\hat{F}_t$. The estimated non-fundamental component, denoted by $\widehat{NF}_t$, is the difference between $p_t - r_t$ and $\hat{F}_t$ and has the residual form

$$\begin{aligned}
\widehat{NF}_t &= p_t - r_t - E_t\left(F_t\right) - \left[\hat{F}_t - E_t\left(F_t\right)\right] \\
&= B_t - \mathcal{U}_t + \sum_{k=0}^{\infty}\rho^k\left[\Delta r_{t+1+k} - E_t\left(\Delta r_{t+1+k}\right)\right] - \sum_{k=0}^{\infty}\rho^k\left[i_{t+1+k} - E_t\left(i_{t+1+k}\right)\right] - \left[\hat{F}_t - E_t\left(F_t\right)\right].
\end{aligned}$$

The non-fundamental component includes not only the bubble component B_t but also the noise term $\mathcal{U}_t$, the forecast errors of Δr_t and i_t, and the estimation error of $E_t\left(F_t\right)$. Since the bubble component B_t can be expected to dominate $\widehat{NF}_t$, econometric methods such as the PSY test can be used to detect explosive bubble behaviour in $\widehat{NF}_t$ when it is present, as suggested in Shi (2017).

To summarize, the decomposition is based on fitting the structural form of F_t in (17) and involves the following steps in practice: (i) estimate a suitable VAR model for the components of the fundamental; (ii) calibrate the additional parameters ρ, κ, and φ from the data; (iii) compute the fundamental components according to equation (17); and (iv) calculate the non-fundamental component using (1).

Furthermore, this procedure requires house price and rent data in dollar values, which are, unfortunately, often available as indexes instead. Suppose P_t^0 and R_t^0 are, respectively, the real house price index and the real rent index. Provided that we know their dollar values for one period (i.e., P_i and R_i for any $i \in [1, T]$), one could recover the dollar values of the entire time series by re-standardization such that

$$P_t = c_1 P_t^0 \text{ and } R_t = c_2 R_t^0$$

with $c_1 = P_i/P_i^0$and $c_2 = R_i/R_i^0$ for all t. The values of c_1 and c_2 (hence P_i and R_i) affect both the left and right sides of equation (1) since

$$p_t - r_t = p_t^0 - r_t^0 + \log c_1 - \log c_2,$$
$$\rho = \frac{c_1 e^{\bar{p}^0}}{c_1 e^{\bar{p}^0} + c_2 e^{\bar{r}^0}}, \text{ and } \kappa = -log(\rho) + (1-\rho)(\log c_1 - \log c_2 + \bar{p}^0 - \bar{r}^0).$$

Following earlier convention we use lower case letters to denote logarithms of the correponding variables, i.e., $p_t^0 = \log\left(P_t^0\right)$ and $r_t^0 = \log\left(R_t^0\right)$. Clearly P_t and $R_t$cannot be simply replaced by P_t^0 and R_t^0 in the calculation of F_t and B_t, unless $c_1 = c_2$.

4 The PSY-IVX Approach

We introduce the new PSY-IVX approach in this section. Let $y_t = \Delta p_t - \Delta r_t$ be the growth rate of the price-to-rent ratio. We formulate the following regression model for y_t

$$y_t = \alpha + \beta x_t + \varepsilon_t, \tag{18}$$
$$x_t = \rho_x x_{t-1} + u_{xt} \text{ with } \rho_x = I_k + \frac{C_x}{T^\alpha}, \tag{19}$$

where $\alpha \in (0,1], C_x = \text{diag}(c_{x1}, ..., c_{xk})$, and $c_{xi} \leq 0$. Here x_t is a $k \times 1$ vector containing fundamental variables that capture housing demand and supply, and (ε_t, u_{xt}) are disturbances. The fundamental driver variables x_t are assumed to satisfy (19) with an autoregressive coefficient matrix ρ_x which is in the vicinity of the identity matrix I_k, so that the variables x_t may be $I(1)$, near integrated, or even mildly stationary, thereby allowing a great deal of flexibility in the characteristics that drive fundamentals. This flexibility is achieved by using the general setting for the coefficient matrix ρ_x given in (19). Mildly explosive components of x_t are excluded by requiring that $c_{xi} \leq 0$ for all $i = 1, .., k$.

In this formulation, the aim is to separate variations in the observed variable y_t that are due to the market fundamentals from those that are induced by speculative behaviour. It is therefore important in this regression not to include variables in x_t that may be contaminated by speculative thinking and behaviour, such as consumer sentiment and risk premia. For instance, in the subsequent application we consider two possible empirical settings for x_t. One contains only the real interest rate and the log of real housing rents. The second set includes four additional variables: log employment, log population, log real disposable income (approximated by real final demand in the relevant state), and log housing supply (proxied by new housing completions).

To accommodate potential serial correlation and conditional heteroskedasticity in the residuals, ε_t is

assumed to follow an AR(q)-GARCH(m, n) process as in Yang et al. (2020), so that

$$\varepsilon_t = \sum_{i=1}^{q} \phi_i \varepsilon_{t-i} + v_t, \text{ with } v_t = \sigma_t \eta_t \text{ and } \eta_t \overset{i.i.d}{\sim} (0,1), \tag{20}$$

$$\sigma_t^2 = \omega_0 + \sum_{i=1}^{m} \omega_i v_{t-i}^2 + \sum_{j=1}^{n} \gamma_j \sigma_{t-j}^2. \tag{21}$$

This specification reduces to the standard case with no serial correlation in ε_t and homoskedasticity when $\phi_i = \omega_i = \gamma_j = 0$.

As discussed below, the model (18)-(19) is estimated using the IVX-AR method. The fundamental component is then computed as the cumulative sum of the fitted values $\hat{y}_t$. Unlike the structural decomposition outlined in the previous section, this reduced-form approach is easy to implement and allows great flexibility in terms of the properties and numbers of the driver variables x_t, which is likely to improve estimation accuracy. Furthermore, this methodology does not need observations of house prices and rents in dollar values (only indexes are required), thereby aiding practical implementation.

4.1 The IVX Estimation Method

It is now well known that the OLS estimate of β suffers finite sample and second order asymptotic bias and standard t and Wald tests that are based on OLS estimation are invalid when the predictive variable x_t is highly persistent (i.e., when ρ_x is in the vicinity of the identity matrix). These characteristics likely apply in most predictive regressions, including the present model and real estate context. Due to omitted variables and joint determination of y_t and x_t there is usually contemporaneous and serial correlation between u_{xt} and ε_t, leading to regressor endogeneity. Even in predictive models with martingale difference equation errors ε_t, endogeneity is still present when the regressors are nonstationary due to potential feedforward correlation between ε_t and $u_{x,t+k}$ for $k \geq 1$.

Several approaches have been proposed to tackle these issues. One method is to use Bonferroni bounds (Campbell and Yogo, 2006). Another uses weighted empirical likelihood (Zhu et al., 2014). Yet another uses a general estimation procedure based on endogenous instrumentation known as IVX estimation. The IVX method has been shown to be valid under a broad spectrum of persistence and for multiple regressor cases. Phillips et al. (2009) gave the limit properties of IVX estimation and testing for settings where x_t is local-to-unity and mildly integrated in the stationary direction. The theory was extended to mixed models with mildly integrated and mildly explosive regressors by Phillips and Lee (2016).

The IVX method is a linear estimation procedure, its construction does not rely on any external information, and orthogonality (with respect to the errors) and relevance (to the regressors) are shown to hold asymptotically without further assumptions. A particularly attractive feature of this approach is that the Wald statistic for testing parameter restrictions has a limiting chi-square distribution and is therefore easy to implement in practical work. A recent modification to this method is designed to

provide effective accommodation of serially correlated and conditional heteroskedastic errors, as in (20) and (21). In particular, Yang et al. (2020) proposed a parametric adjustment of the IVX method which uses an autoregressive error correction to improve size properties of testing based on IVX estimation. This method, called IVX-AR, appears to be more effective in controlling size than the nonparametric adjustments that are built into standard IVX estimation (Phillips and Lee, 2016). The method is described below and is particularly useful when the equation errors are serially correlated rather than martingale differences.

Let $\phi = (\phi_1, \ldots, \phi_q)$, $y_{\phi,t} = y_t - \sum_{j=1}^{q} \phi_j y_{t-j}$ and $x_{\phi,t} = x_t - \sum_{j=1}^{q} \phi_j x_{t-j}$, based on the autoregressive error specification (20). The predictive regression can be rewritten after this Cochrane-Orcutt transform as

$$y_{\phi,t} = \alpha_\phi + \beta x_{\phi,t-1} + v_t,$$

where $\alpha_\phi = \alpha \left(1 - \sum_{j=1}^{q} \phi_j\right)$. The instrumental variables are denoted by $z_{\phi,t}$ and constructed from $x_{\phi,t}$ as

$$z_{\phi,t} = \sum_{j=1}^{t} \rho_z^{t-j} \Delta x_{\phi,j} \text{ with } \rho_z = I_k + \frac{C_z}{T^\beta} \text{ for some } C_z \text{ and } \beta \in (0,1)\,,$$

where $z_{\phi,0} = 0$ and k is the dimension of x_t. We set $C_z = -I_k$ and $\beta = 0.95$ in the empirical application, as suggested by KMS (2015). These settings ensure that the instruments $z_{\phi,t}$ are all mildly integrated processes, which is important in ensuring good asymptotic properties that include pivotal limit distributions for conventional t and Wald test statistics. By construction,

$$z_{\phi,t} = z_t - \sum_{j=1}^{q} \phi_j z_{t-j} \text{ with } z_t = \sum_{j=1}^{t} \rho_z^{t-j} \Delta x_j.$$

The instrumental variables are less persistent than the corresponding variables x_t from which they are derived when the variables in x_t are unit root or near unit processes.

Next, let $\underline{y}_{\phi,t} = y_{\phi,t} - \frac{1}{T}\sum_{t=1}^{T} y_{\phi,t}$ and $\underline{x}_{\phi,t} = x_{\phi,t} - \frac{1}{T}\sum_{t=1}^{T} x_{\phi,t}$ be demeaned versions of $y_{\phi,t}$ and $x_{\phi,t}$, and define the observation matrices $\underline{Y}_\phi = \left(\underline{y}_{\phi,1}, \ldots, \underline{y}_{\phi,T}\right)'_{T\times 1}$, $\underline{X}_\phi = \left(\underline{x}_{\phi,1}, \ldots, \underline{x}_{\phi,T}\right)'_{T\times k}$, and $Z_\phi = (z_{\phi,1}, \ldots, z_{\phi,T})'_{T\times k}$. The IVX-AR estimation algorithm is as follows.

Step 1: For a given ϕ, compute the IVX estimator

$$\hat{\beta}^{(\phi)} = \underline{Y}'_\phi Z_\phi \left(\underline{X}'_\phi Z_\phi\right)^{-1} = \sum_{t=1}^{T} \underline{y}_{\phi,t} z'_{\phi,t-1} \left[\sum_{i=1}^{T} \underline{x}_{\phi,i-1} z'_{\phi,i-1}\right]^{-1}$$

and obtain the residuals $\hat{v}_{\phi,t} = \underline{y}_{\phi,t} - \hat{\beta}^{(\phi)} \underline{x}_{\phi,t-1}$.

Step 2: Find the optimal ϕ by minimizing the sum of squared residuals

$$\phi^* = \arg\min_{\phi} \left(\sum_{t=1}^{T} \hat{v}_{\phi,t}^2 \right).$$

Step 3: Compute the IVX-AR estimator $\hat{\beta}_{IVX}$ using simple instrumental variables regression as

$$\hat{\beta}_{IVX} = \underline{Y}'_{\phi^*} Z_{\phi^*} \left(\underline{X}'_{\phi^*} Z_{\phi^*} \right)^{-1}.$$

The intercept is estimated by $\hat{\alpha}_{IVX} = \bar{y} - \hat{\beta}_{IVX} \bar{x}$.

HAC robust Wald test statistics are constructed as follows. Let $\theta = (\alpha, \beta')'$ be the $(k+1) \times 1$ parameter vector and $\hat{\theta}_{IVX}$ the corresponding IVX-AR estimate of θ. The null and alternative hypotheses to be tested are

$$H_0 : R\theta = r \text{ and } H_1 : R\theta \neq r,$$

where R is a $p \times (k+1)$ matrix imposing full rank restrictions on the coefficients, r is a $p \times 1$ vector, and p is the number of restrictions. Let $\hat{\varepsilon}_t = y_t - \hat{\alpha} + \hat{\beta} x_{t-1}$ and $\hat{u}_t = x_t - \sum_{j=1}^{q} \hat{\phi}_j x_{t-j}$, where $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\phi}_j$ are the OLS estimates of the coefficients. The HAC-robust wald statistic is defined as

$$\mathcal{W}_{IVX} = \left(R\hat{\theta}_{IVX} - r \right)' Q^{-1} \left(R\hat{\theta}_{IVX} - r \right),$$

where

$$\begin{aligned} Q &= R \left(Z'_{\phi^*} \underline{X}_{\phi^*} \right)^{-1} M \left(\underline{X}'_{\phi^*} Z_{\phi^*} \right)^{-1} R', \\ M &= Z'_{\phi^*} Z_{\phi^*} \hat{\sigma}_{\varepsilon}^2 - T \bar{z}_T \bar{z}'_T \hat{\Omega}_{FM}, \ \bar{z}_T = \frac{1}{T} \sum_{t=1}^{T} z_{t-1}, \\ \hat{\Omega}_{FM} &= \hat{\sigma}_{\varepsilon}^2 - \hat{\Omega}_{\varepsilon u} \hat{\Omega}_u^{-1} \hat{\Omega}'_{\varepsilon u}, \end{aligned}$$

with $\hat{\sigma}_{\varepsilon}^2 = \frac{1}{T} \sum_{t=1}^{T} \hat{\varepsilon}_t^2$. The estimated quantities

$$\hat{\Omega}_u = \hat{\Sigma}_u + \hat{\Lambda}_u + \hat{\Lambda}'_u \text{ and } \hat{\Omega}_{\varepsilon u} = \hat{\Sigma}_{\varepsilon u} + \hat{\Lambda}'_{u\varepsilon}$$

employ the following variance, covariance, long run variance and long run one-sided covariance matrix estimates $\hat{\Sigma}_u = \frac{1}{T} \sum_{t=1}^{T} \hat{u}_t \hat{u}'_t$, $\hat{\Sigma}_{\varepsilon u} = \frac{1}{T} \sum_{t=1}^{T} \hat{\varepsilon}_t \hat{u}'_t$, $\hat{\Lambda}_u = \frac{1}{T} \sum_{h=1}^{H} \left(1 - \frac{h}{H+1} \right) \sum_{t=h+1}^{T} \hat{u}_t \hat{u}'_{t-h}$, and $\hat{\Lambda}_{u\varepsilon} = \frac{1}{T} \sum_{h=1}^{H} \left(1 - \frac{h}{H+1} \right) \sum_{t=h+1}^{T} \hat{u}_t \hat{\varepsilon}'_{t-h}$, constructed in the usual fashion from regression residuals. In our empirical application, we use the Wald test to make inferences concerning the significance of β and thereby predictability of y_t using the regressors x_t. The bandwidth H in long run variance and covariance estimation is set to be $T^{1/3}$.

The estimated fundamentals $\hat{F}_t$ are formed as the partial sums of $\hat{y}_t$, where $\hat{y}_t = \hat{\alpha}_{IVX} + \hat{\beta}_{IVX} x_{t-1}$. That is, we set $\hat{F}_1 = p_1 - r_1$ and for $t \geq 2$ construct $\hat{F}_t = \hat{F}_1 + \sum_{i=2}^{t} \hat{y}_t$. The residual component, denoted by e_t, is $e_t = p_t - r_t - \hat{F}_t$.

4.2 Implementation of the PSY Procedure

We apply the PSY procedure to this calculated residual component $e_t = p_t - r_t - \hat{F}_t$ representing the data after removing the estimated fundamentals $\hat{F}_t$. Thus, to implement the PSY algorithm for bubble detection we set y_t in equation (14) equal e_t. By construction, the data series has a zero mean which is consistent with the specification of the null hypothesis (12). The minimum window size r_0 is set to $0.01 + 1.8/\sqrt{T}$, as suggested in Phillips et al. (2015a). The lag order of the ADF model (14) is selected by BIC with a maximum lag order of 4.

We employ the composite bootstrap procedure proposed in Phillips and Shi (2020) for the computation of critical values. This bootstrap algorithm is designed to improve finite sample size control in PSY testing in two ways: (i) by allowing for potential unconditional heteroskedasticity in the residuals; and (ii) by addressing the multiplicity issue that arises from multiple testing in recursive methods such as PSY.[5] This approach helps to control the probability of making false positive conclusions over a period of length T_b. If $T_b = T$, we allow the possibility of making at least one false positive conclusion over the entire sample period to be 5%. For real-time monitoring where T grows larger each period, T_b can be set to a fixed time period. The length of the time period is a matter of choice, taking into account that a more extended period leads to more conservative tests. In our empirical application below we set T_b to be one year. Results are similar when we set T_b to be two years. The full procedure is detailed below.

Step 1: Estimate the regression model (14) under the null ($\beta = 0$). The estimated coefficient and residuals are denoted, respectively, by $\hat{\beta}_0$ and $\hat{\varepsilon}_t$.

Step 2: Simulate a sample of $T_0 + T_b - 1$ observations from the equation

$$y_t^b = \hat{\beta}_0 + y_{t-1}^b + \varepsilon_t^b \tag{22}$$

with initial values $y_1^b = y_1$. The residuals $\varepsilon_t^b = w_t \hat{\varepsilon}_j$, where w_t is a random draw from the standard normal distribution and $\hat{\varepsilon}_j$ is bootstrapped from the residual sequence obtained in Step 1.

Step 3: Compute the PSY test statistic sequence from the simulated data series y_t^b, denoted by PSY_r^b, and calculate the quantity

$$\mathcal{M}^b = \sup_{r \in [r_0, 1]} \left(PSY_r^b \right).$$

[5] This bootstrap algorithm is based on the wild bootstrap procedure of Harvey et al. (2016). The asymptotic properties of this bootstrap procedure in the presence of conditional heteroskedasticity are presently unknown. But simulation evidence indicates that the method provides a considerable improvement in size control.

Step 4: Repeat Steps 2-3 $B = 2,999$ times.

Step 5: Take the 95% percentile of the $\left\{\mathcal{M}^b\right\}_{b=1}^{B}$ sequence as the critical value for use in the PSY bubble detection procedure.

5 Australian Housing Markets

There are eight capital cities in Australia: Sydney, Melbourne, Brisbane, Adelaide, Perth, Hobart, Darwin, and Canberra, with Sydney and Melbourne being the two largest cities. Figure 1 shows the location of each city. We investigate the presence of speculative bubbles in the eight capital city housing markets from 1999:Q1 to 2017:Q4 by applying the PSY-IVX approach to detection. Test results are compared with (i) those presented in SRW where housing market fundamentals are computed using the structural approach, and (ii) those obtained from the standard PSY procedure applied directly to log price-to-rent ratios with no adjustment for fundamentals.

Figure 1: Locations of the Australian eight capital cities. This map is sourced from the Australia Bureau of Meteorology.



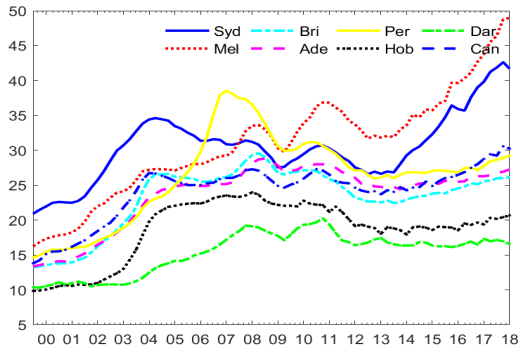
The sample period and data sources are identical to those of SRW for the ease of comparison. House prices are obtained from the SIRCA (2018) CoreLogic RP database at a monthly frequency, and rent indices are downloaded from the Australian Bureau of Statistics (ABS) at a quarterly frequency. The rent indexes are translated into dollar values using rent prices at 2014:Q4 provided by CoreLogic[6] and then converted to real data series with the state level CPI (excluding shelter) obtained from ABS. The real interest rate i_t is computed as the nominal mortgage rate less inflation expectations. The nominal mortgage rate is the monthly standard variable home loan rate offered by banks to owner-occupiers, and inflation expectations are proxied by taking the trimmed mean of 12-month ahead consumer inflation

[6]This is strictly unnecessary. With the present method, similar testing results should be obtained with the use of indexes, as explained earlier.

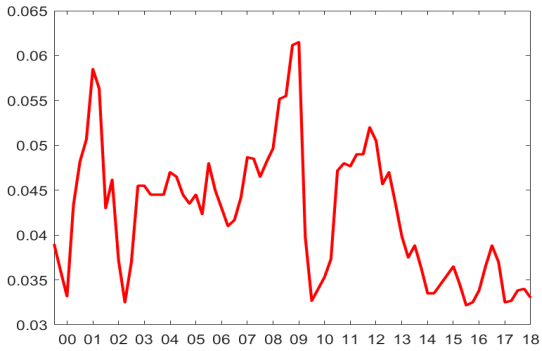
expectations, compiled by the Melbourne Institute at a quarterly basis. Both datasets are downloaded from the Reserve Bank of Australia. We convert monthly data into quarterly by averaging.

Figure 2: House price-to-rent ratios in the eight capital cities and the national real interest rate.

(a) Price-to-rent ratios



(b) Real interest rate



The price-to-rent ratios of the eight markets are displayed in Figure 2(a). There are several episodes of rapid expansion in the housing markets over the sample period. The first wave occurs in the early 2000s, which is after the introduction of a 50% capital gain tax discount in late 1999 and involves most cities in the study. We observe either a decrease or flattening of the PR ratios in all markets from 2004:Q2 onwards except Perth and Darwin where the momentum continues until early 2007. Another wave of dramatic rises in the PR ratios (particularly in Sydney and Melbourne) starts from 2013 and lasts until the end of the sample period. Figure 2(b) presents the real interest rate, which fluctuates throughout the sample period. There are two sharp drops in the real interest rate. One occurs in the early 2000s following the burst of the dot-com bubble, and the second occurs during the subprime mortgage crisis period 2008-2009. There are two relatively calm periods, which overlap with the periods of housing booms.

The dependent variable y_t in (18) is the growth rate of the price-to-rent ratio (i.e., $\Delta p_t - \Delta r_t$). The regressors are log real rents and the real interest rate. The lag order q in (20) is selected by the Bayesian information criteria (BIC) with a maximum order of 5 applied to the OLS residuals of (18). Table 1 shows the estimated coefficients from OLS and IVX. The significance of the OLS estimates are drawn from the standard t-statistic, while those of the IVX estimates are from the HAC constructed Wald statistic. Evidently, both variables are negative and highly significant in all cities (except r_t in Sydney when using IVX). There are some slight differences in the estimated coefficients between OLS and IVX.

The estimated market fundamentals of the eight cities, along with their log price-to-rent ratios, are displayed in Figure A.1. The two estimation methods (OLS and IVX) do not lead to obvious differences in $\hat{F}_t$. The discrepancy between the log price-to-rent ratio and the estimated fundamental is generally

Table 1: Estimated β using OLS and IVX

	Sydney		Melbourne		Brisbane		Adelaide	
	OLS	IVX	OLS	IVX	OLS	IVX	OLS	IVX
r_t	-0.05***	-0.03	-0.16***	-0.15***	-0.16***	-0.17***	-0.24***	-0.26***
i_t	-2.34***	-1.84***	-2.11***	-2.00***	-1.44***	-1.57***	-0.93	-1.30***
	Perth		Hobart		Darwin		Canberra	
	OLS	IVX	OLS	IVX	OLS	IVX	OLS	IVX
r_t	-0.15***	-0.15***	-0.28***	-0.30***	-0.09***	-0.09***	-0.14***	-0.13***
i_t	-2.15***	-2.24***	-1.34***	-1.56***	-0.98***	-1.26***	-1.45***	-1.60***

Note: *,**,*** to denote the 90%, 95%, and 99% significance, respectively. The significance of the OLS estimates are drawn from the standard t statistic, while those of the IVX estimators are based on the HAC-robust wald statistic.

more significant in the first half of the sample than the second half.

Figure 3 shows the estimated non-fundamental components from IVX (black line) and the identified bubble periods using the PSY procedure (shaded), which is referred to as PSY-IVX-2 (as the regression model for computing $\hat{y}_t$ includes two explanatory variables). The minimum window has 16 observations (4 years). Evidently from these results the early 2000s housing boom is found to be driven by speculation. More specifically, we find evidence of bubbles in five capital cities (Sydney, Brisbane, Adelaide, Hobart and Canberra) out of eight over this period. The identified bubble periods fall between 2003:Q1 and 2004:Q2. Interestingly, we do not find any evidence of housing speculation over the more recent period of 2013-2017, except in Sydney and that lasts only for one quarter (2015:Q3). This finding highlights the vital role played by market fundamentals (especially the real interest rate) in influencing tests for the presence of speculation over this period. Complementary to these findings, the tests show evidence of an additional substantial bubble period in the Perth and Darwin housing markets. These bubble periods coincide with the commodity boom (Ye, 2008), spanning from 2006 to 2007.

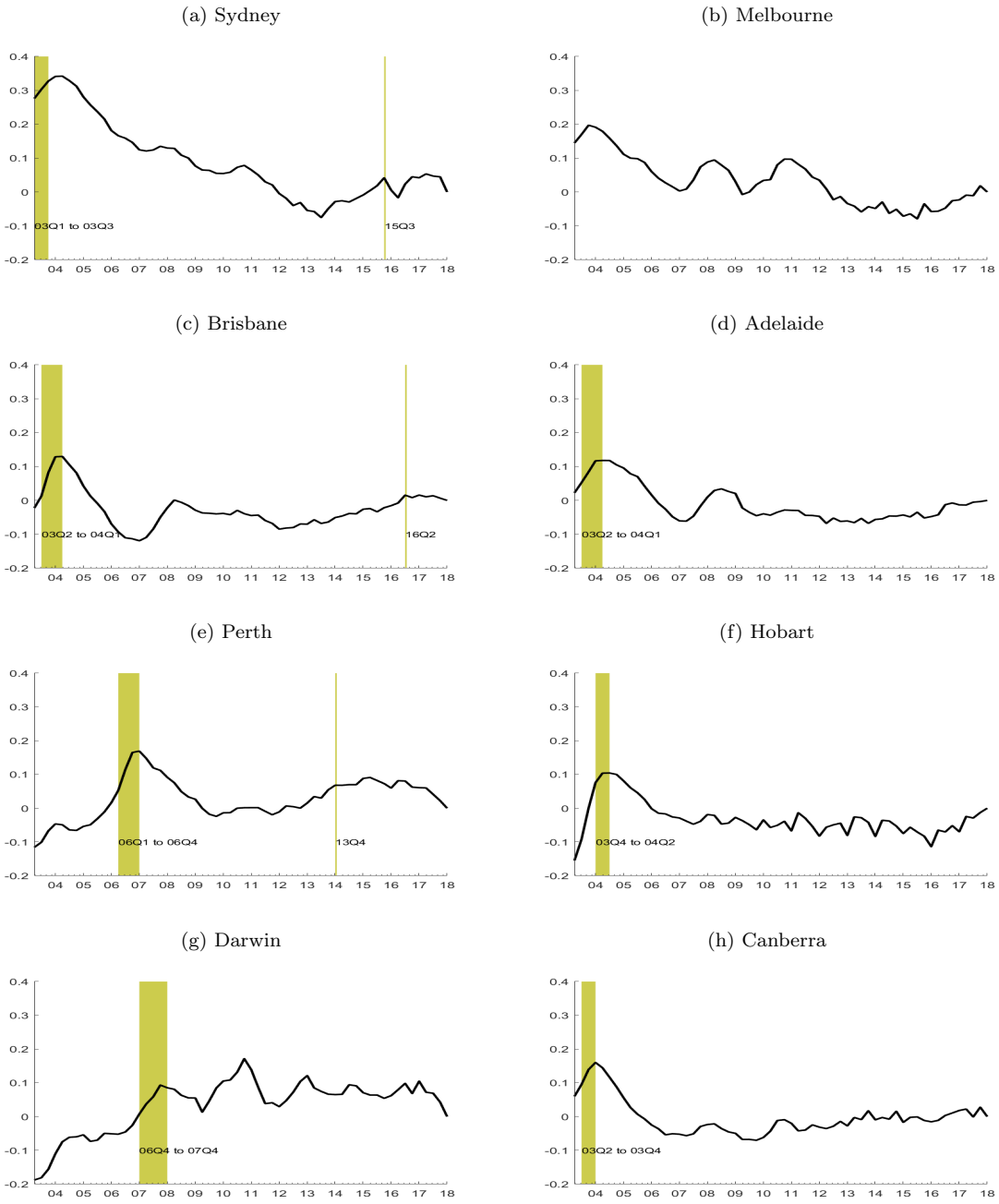
5.1 Controlling for Other Fundamental Factors

Next, we examine the impact of other potential fundamental factors on the price-to-rent ratios, as in SRW and Shi (2017). Variables considered include employment, population, state final demand, and housing supply (proxied by the one-year moving average of new housing completions).[7] Due to the unavailability of the city level data, we use data at the state level instead for those variables. Since most population are concentrated in the capital cities (Costello et al., 2011), this approximation is not expected to have significant impact on the results. All variables are downloaded from ABS at the quarterly frequency. Again, the data are identical to those used in SRW.

The regressor x_t in (19) now contains six variables: real rent growth, the real interest rate, and the four state level variables mentioned above. The additional fundamental variables are plotted in Figures

[7]State final demand is an estimate of the level of spending in the local economy by the private and public sectors. It serves as an alternative to disposable income which is unavailable at either the city or state level.

Figure 3: The estimated non-fundamental components (black line) and identified bubble periods from the PSY-IVX-2 method (shaded)



A.3 and A.4 and the estimated market fundamentals are in Figure A.2. The OLS and IVX estimators evidently provide similar results for all cities except Canberra. In particular, the IVX method reveals a stronger housing fundamental in Canberra than OLS over the period of 2006-2008. By comparing Figures A.1 and A.2, one can see that the discrepancy between the log PR ratios and the estimated fundamentals becomes visibly smaller for Sydney and Melbourne, larger in Canberra based on IVX, and remains roughly the same for other cities, after the consideration of the four additional variables.

The identified bubble periods are listed in Table 2 (labeled as PSY-IVX-6), along with results from PSY-IVX-2. There is a broad agreement between PSY-IVX-6 and PSY-IVX-2 in bubble testing results. First, there is substantial evidence of speculation between 2003 and 2004, involving four (respectively, five) cities according to the PSY-IVX-6 (respectively, PSY-IVX-2) method. Second, there is little evidence of bubbles during the 2013-2017 housing boom. The PSY-IVX-2 detects three one-quarter episodes (15:Q3 in Sydney, 16:Q2 in Brisbane, and 13:Q4 in Perth), while the PSY-IVX-6 only finds one short period in Perth (2016:Q1-Q2). Finally, both methods detect the episode related to the commodity boom in Perth and Darwin, although the identified period in Darwin is shorter with the PSY-IVX-6 method.

The consistency in findings between PSY-IVX-2 and PSY-IVX-6 suggests the dominating impact of rents and interest rates on the dynamics of housing prices, with exceptions in the Canberra and Darwin housing markets. We identify a significantly shorter bubble period between 2006 and 2007 in Darwin and no bubbles between 2003 and 2004 in Canberra, after controlling for the four additional fundamental factors.

We compare the testing results of PSY-IVX with those from SRW, listed in the fourth column of Table 2. SRW computes fundamental components using the structural procedure and applies the PSY test on the estimated residuals. As we can see, these two approaches lead to similar findings for the 2003-2004 period. Specifically, SRW finds evidence of speculation in three cities (Brisbane, Adelaide, and Hobart) over this period, while PSY-IVX-2 and PSY-IVX-6 identify bubbles in, respectively, four and five markets. For the 2013-2017 housing boom, SRW detects some short periods of speculation in both the Sydney and Melbourne housing markets, whereas the PSY-IVX approach suggests no bubble in the Melbourne housing market. Another difference between these two approaches occurs in the Perth and Darwin markets during the 2006-2007 commodity boom period. Unlike the PSY-IVX method, SRW concludes that the 2006-2007 housing boom in Perth and Darwin is purely driven by fundamentals.

Finally, the last column of Table 2 presents results obtained from the standard PSY test, which is applied directly to the log price-to-rent ratios. The PSY procedure identifies more substantial evidence of bubbles, with more cities and longer bubble periods identified for each housing boom episode. The variations in testing results of those approaches highlight the importance of an accurate market fundamental estimator.

Table 2: The identified bubble periods in the Australian housing markets

	PSY-IVX-2	PSY-IVX-6	SRW	PSY
Sydney	03:Q1 - Q3	03:Q4 - 04:Q1	16:Q3	03:Q2 - Q3
	15:Q3		17:Q1	14:Q3 - 15:Q4
			17:Q3	16:Q2 - 17:Q3
Melbourne	NA	NA	17:Q2-Q3	06:Q2 - 08:Q1
				15:Q3
				16:Q2 - 17:Q3
Brisbane	03:Q2-04:Q1	03:Q3 - 04:Q1	03:Q4	03:Q2 - 04:Q1
	16:Q2			07:Q3 - 08:Q1
				16:Q1 - Q2
				16:Q4 - 17:Q3
Adelaide	03:Q2 - 04:Q1	03:Q2 - 04:Q1	03:Q2 - 04:Q1	03:Q2 - 04:Q2
				07:Q3 - 08:Q2
Perth	06:Q1-Q4	06:Q1 - Q4	NA	03:Q2 - 07:Q1
	13:Q4	16:Q1 - Q2		
Hobart	03:Q4 - 04:Q2	03:Q4 - 04:Q1	03:Q2 - 04:Q3	03:Q2 - 04:Q4
Darwin	06:Q4 - 07:Q4	07:Q3	NA	03:Q4 - 08:Q1
Canberra	03:Q2 - Q4	NA	NA	03:Q2 - Q4
				07:Q3 - 07:Q4
				16:Q4 - 17:Q3

6 Conclusion

It has become standard in the econometrics literature to apply explosive root bubble tests directly to log prices or to log price-to-rent ratios in analyzing housing market exuberance. This approach is justified by the fact that the model specification underlying explosive root tests is consistent with the submartingale dynamic of a bubble process during its expansive phase. But log prices and log PR ratios involve fundamentals as well as the bubble process, at least when it is present, leading to a potential mismatch between the model for the data and typical empirical testing. Contamination of this type can lead to false conclusions on bubble detection. One solution to the mismatch is to apply the PSY explosive root tests to the non-fundamental component, which may be separately computed by regression. This is the approach that is adopted in the present paper.

One procedure for decomposing log PR ratios into a fundamental and non-fundamental component is based on a structural definition of fundamentals and involves multiple steps. That method requires data on house prices and rents in dollar values, which are often not available. Instead, this paper proposes a reduced-form approach for the estimation of housing market fundamentals, which is easy to implement and has general properties that improve estimation accuracy. Specifically, the reduced-form model is estimated by IVX-AR methodology, which is known to reduce the finite sample bias and non-standard limit theory

that originates in persistent regressors, endogeneity, serial correlation and conditional heteroskedasticity. Bubble testing is then performed on the non-fundamental component using the PSY procedure. The combined procedure is called PSY-IVX.

The PSY-IVX bubble detector is applied to the eight Australian capital city housing markets from 1999 to 2017 with two different sets of fundamental variables. The first set includes only rents and interest rates, while the second set is more extensive and has four additional variables (employment, population, state final demand, and new housing completions). Similar results from these two settings are obtained for most cities, except Canberra and Darwin. The PSY-IVX method finds strong evidence of bubbles in the early 2000s in half of the cities. Analysis leads to the conclusion that the 2006-2007 housing boom in Perth and Darwin was driven by speculation, whereas the 2013-2017 housing expansion was driven mainly by market fundamentals. Our findings disagree with earlier results in SRW for the period between 2006 and 2007, and with direct use of the PSY procedure on log price-to-rent ratios, which identifies more episodes and longer episodes of bubbles. These variations in the results of empirical tests reveal the importance of controlling for market fundamentals in assessing evidence for bubbles and the source of expansions in housing price data.

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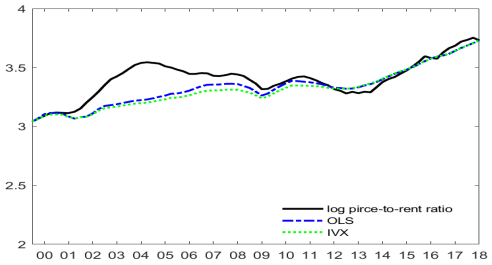
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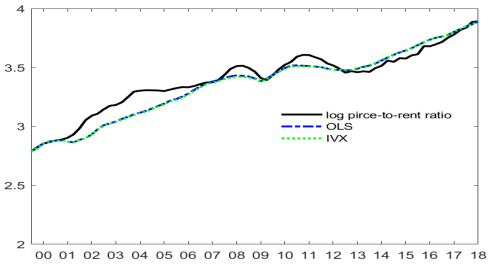
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Figure A.1: Log price-to-rent ratios and estimated market fundamentals from PSY-IVX-2

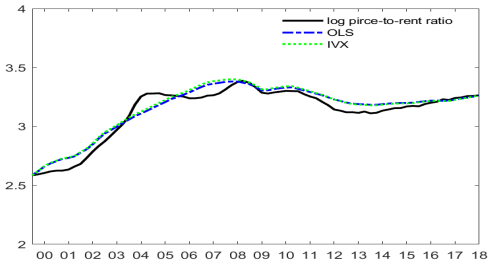
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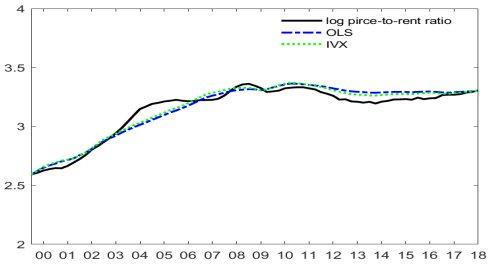
(b) Melbourne



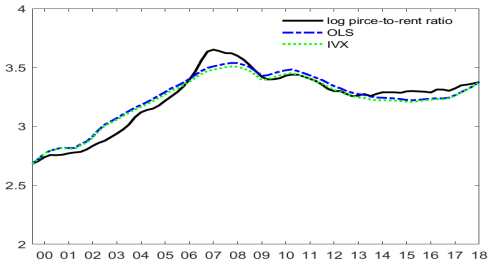
(c) Brisbane



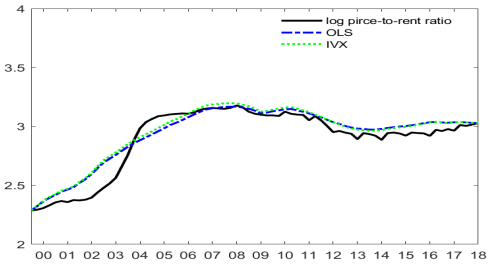
(d) Adelaide



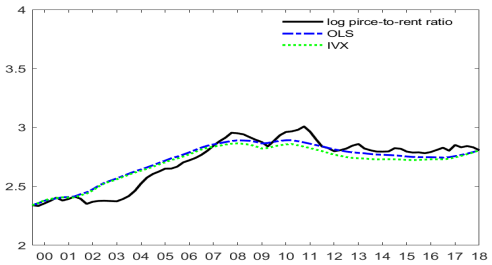
(e) Perth



(f) Hobart



(g) Darwin



(h) Canberra

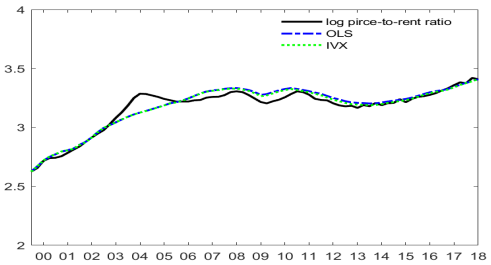
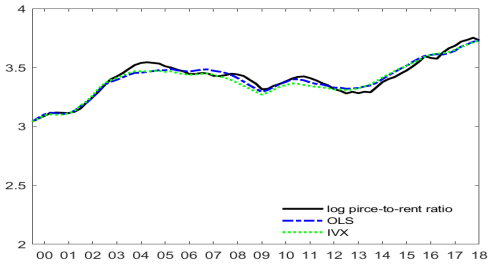
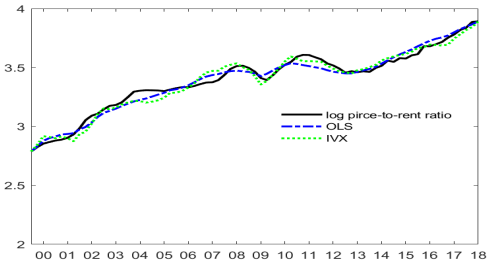


Figure A.2: Log price-to-rent ratios and estimated market fundamentals from PSY-IVX-6

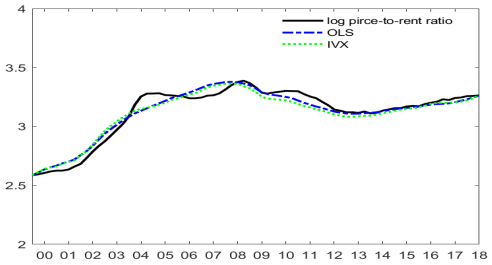
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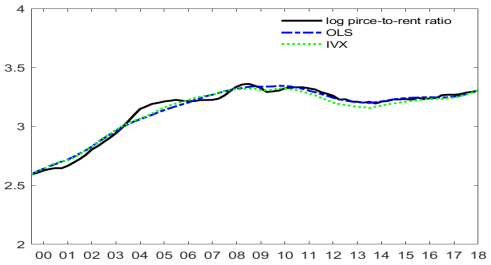
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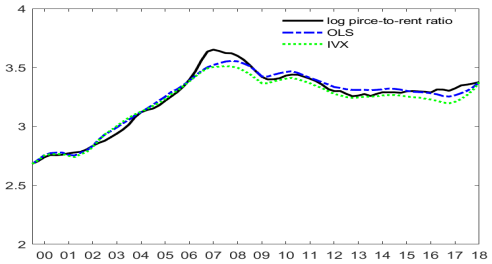
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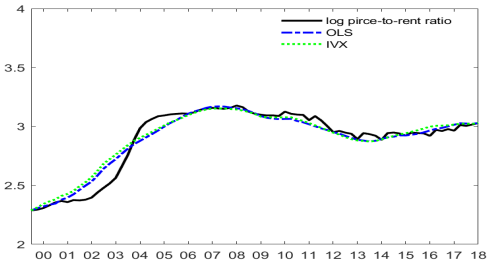
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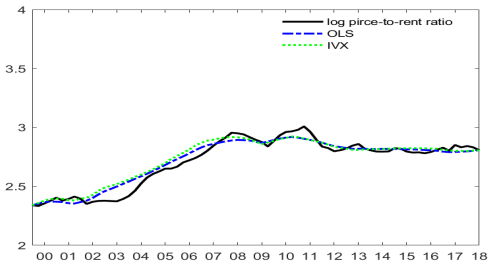
(e) Perth



(f) Hobart



(g) Darwin



(h) Canberra

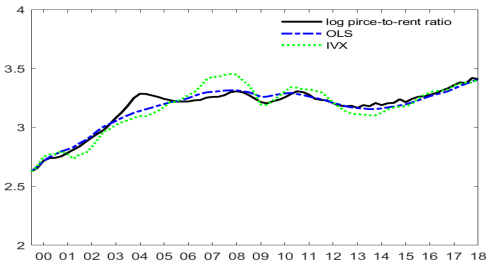
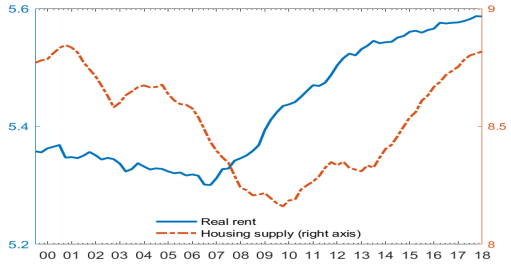
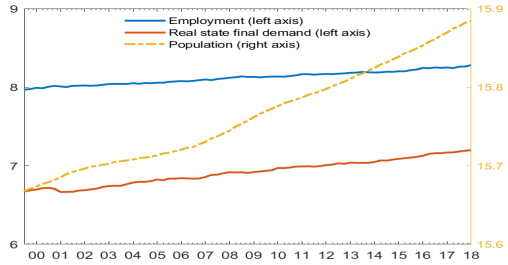


Figure A.3: Fundamental variables

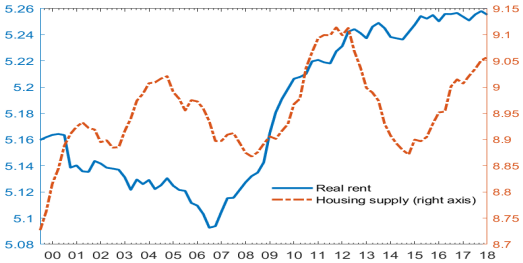
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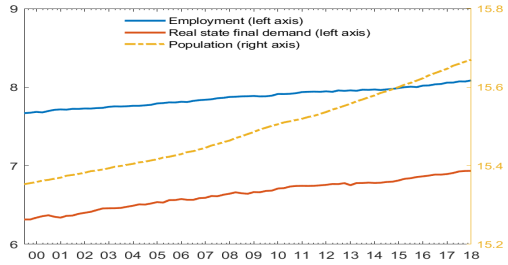
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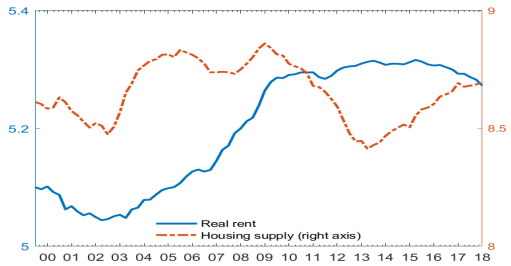
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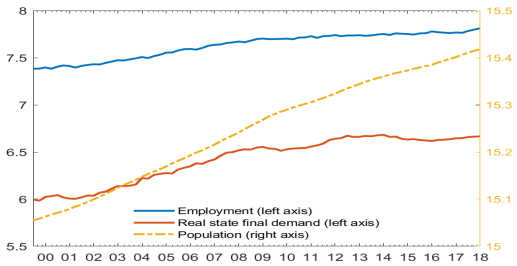
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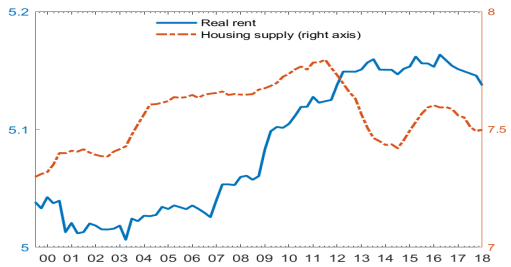
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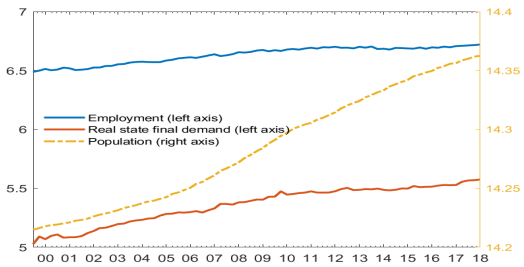
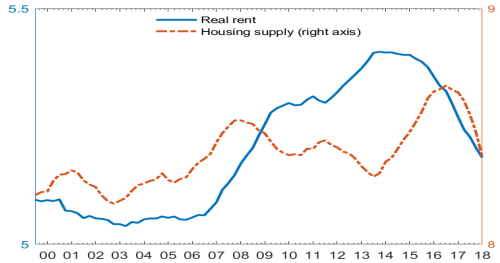
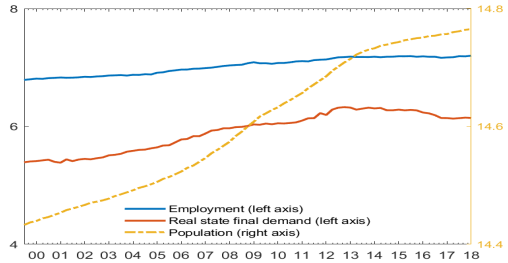


Figure A.4: Fundamental variables (cont.)

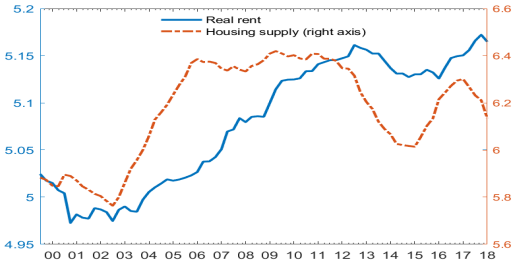
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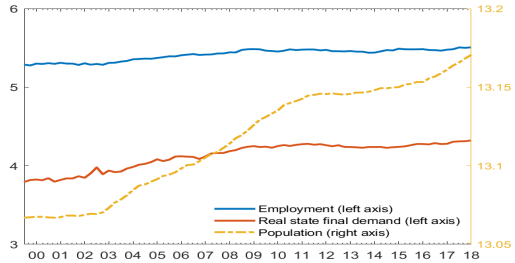
(b) Perth



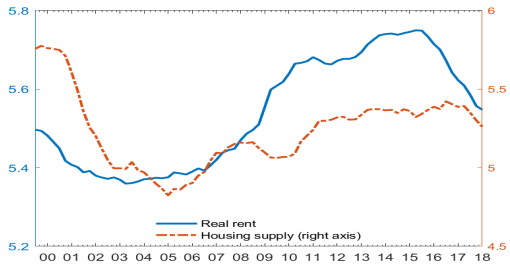
(c) Hobart



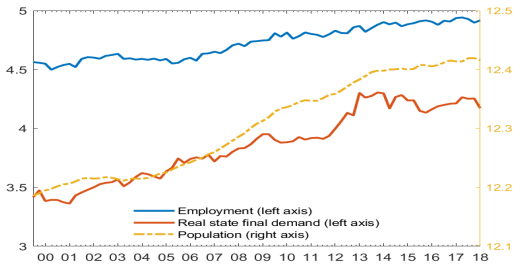
(d) Hobart



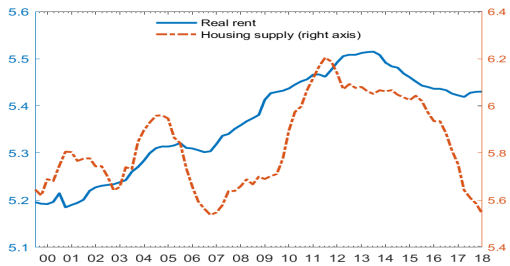
(e) Darwin



(f) Darwin



(g) Canberra



(h) Canberra

