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TRANSPARENCY AND DISTRESSED SALES UNDER ASYMMETRIC INFORMATION

By

William Fuchs, Aniko Öry, and Andrzej Skrzypacz

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Transparency and Distressed Sales under Asymmetric Information

By William Fuchs, Aniko Öry, and Andrzej Skrzypacz*

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We analyze price transparency in a dynamic market with private information and correlated values. Uninformed buyers compete inter- and intra-temporarily for a good sold by an informed seller suffering a liquidity shock. We contrast public versus private price offers. In a two-period case all equilibria with private offers have more trade than any equilibrium with public offers; under some additional conditions we show Pareto-dominance of the private-offers equilibria. If a failure to trade by the deadline results in an efficiency loss, public offers can induce a market breakdown before the deadline, while trade never stops with private offers.
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A public policy response to the recent financial crisis has been regulatory changes (some enacted, some still under consideration) aimed at improving the transparency with which financial securities are traded. For example, a stated goal of the Dodd- Frank Act of 2010 is to increase transparency in the financial system. The European Commission is considering revisions to the Markets in Financial Instruments Directive (MiFID), in part to improve the transparency of European financial markets. Such actions reflect a widely held belief that transparency is welfare enhancing because it is necessary for perfect competition, it decreases uncertainty, and it increases public trust. Yet, there are a number of nuances concerning transparency and the question of whether transparency enhances efficiency is correspondingly complicated. Indeed, as we show, in settings relevant to this public-policy debate, transparency of offers made can actually have negative welfare effects even in terms of Pareto.

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We consider a problem of an owner of an indivisible durable asset who suffers a liquidity shock and study the role of price transparency. Due to the liquidity shock, the seller's present value of the good drops to a lower level than the true value of the good. Hence, she would like to sell the asset to a buyer not facing a liquidity shock. The problem is that usually the owner of the asset is better informed about its quality. Any potential buyer therefore faces an adverse selection problem. As first stressed by Akerlof (1970), if there is only one opportunity to trade, competitive buyers are only willing to pay expected valuation of the asset. However, high seller types may not want to accept this price, if the adverse selection problem is sufficiently strong, even though there are positive gains from trade for all types. In a dynamic setting, in which sellers get several chances to sell their good, this logic of a lemons market leads to inefficient delay in trade. We show in this paper that while transparency of price offers has no impact on equilibrium outcomes in a static model, it affects the amount of inefficient delay, if the time between price offers is not too long.

More precisely, we examine a two-period model with a long-lived, privately-informed seller and a competitive market of buyers in every period (modeled as a number of short-lived buyers competing in prices in every period). We consider two opposite information structures: *transparent* (public offers), in which all buyers observe past price offers and *opaque* (private offers), in which every period new buyers make offers and they do not observe past rejected offers. First (see Theorem 1), we show that in an opaque market there is (weakly) more total trade with (weakly) higher prices in the second period. This implies that all the seller types that would have traded in the second period with a transparent market must be (weakly) better off. If in addition there are also weakly higher prices in the first period then the opaque market (weakly) Pareto dominates the transparent market. We show it is the case when not trading by a deadline imposes an efficiency loss and trading offers are frequent. We show Pareto ranking also when the gains from trade are linear in valuation and the distribution of valuations is uniform, even if missing the deadline would not additionally reduce welfare. Second, we show quite generally that the disclosure policy affects equilibrium prices if and only if discounting between offers is small (see Theorem 2). Third, if discounting between offers is small and past offers are not observable, buyers randomize between several price offers such that price realizations can be volatile.

In addition to motivating the gains from trade by assuming the seller is liquidity constrained, by allowing for a fraction of surplus being lost at the deadline we capture an additional notion of seller's distress. For example, when the deadline is reached, the opportunity to trade disappears or a profitable investment opportunity that the seller wants to finance by the proceeds from the sale of the asset vanishes. This can create a deadline effect in which the seller trades with a high probability just at the deadline. Thereby we illustrate an additional and novel difference between transparent and opaque markets: With public offers (see Proposition 1), the deadline effect endogenously leads to a trading impasse (illiq-

uidity) before the deadline. In contrast (see Proposition 2), with private offers there cannot be a trading impasse (i.e. there is trade with positive probability in every period).

What makes the markets operate differently in these two information regimes? In a transparent market, buyers can observe all previous price offers and thereby learn about the quality of the good through two channels: the number of rejected offers (time on the market) and the price levels that have been rejected by the seller. By rejecting a high offer, the seller can send a strong signal to future buyers that she is of a high type. For example, in transparent exchanges, sellers try to influence prices by taking advantage of the observability of order books. In contrast, in an opaque market, in which buyers cannot observe previously rejected prices, the seller signals only via delay. Intuitively then, private offers generate more trade because seller's continuation value increases in the public offer it rejects but is independent of it when it is private and thus, sellers are more reticent to accept public offers than to accept private offers (leading to less trade in the transparent market). [1]

Pure-strategy Perfect Bayesian Equilibria (PBE) with public offers always exist and they coincide with PBE with private offers if the discounting between two periods is large (that is, either if the interest rate is high or the price offers can be made infrequently). However, there are no pure-strategy PBE in the game with private offers, if the discounting between two periods is small enough. Intuitively, rejecting offers to signal a high valuation to tomorrow's buyers (driving up future prices) is more attractive if future profits are less discounted. Thus, the two information structures result in different trading patterns only in high-frequency markets.

Related Literature

The closest paper to ours in the economics literature is Hörner and Vieille (2009) (HV from now on). They are also interested in comparing the trading dynamics with public versus private offers. Our model differs from theirs in that while we assume intra-period competition, HV consider a model with a single buyer in each period. The lack of intra-period competition introduces Diamond Paradox effects (Diamond (n.d.)). As a result, the equilibrium in HV with public offers is, in their own words, quite paradoxical since the first offer is rejected with positive probability and all other offers are rejected with probability 1. Instead, in our model, the equilibrium with public offers has a positive probability of agreement in each period and slowly more and more types eventually trade. They do not provide an analysis about the relative efficiency between both information

[1] Comparing our paper to the existing literature yields that how price dynamics are affected by transparency depend on the microstructure of the market. For example, the claim that any pure-strategy equilibrium prices in a game with private offers are also supportable as equilibrium prices in a game with public offers is true because we have assumed intra-period competition. In Kaya and Liu (2012) there is one buyer per period and hence competition is only inter-period. In that case the games with private and public offers have different pure-strategy equilibria. The reason for the difference is that a monopolistic buyer would have a profitable deviation to a lower price if prices become transparent. In our model intra-period competition implies that a lower-than-equilibrium price is rejected for sure.

regimes. Although they show that private offers lead to more trade, in general it is not obvious if their model would lead to an efficiency ranking. It is possible that the endogenous trading impasse that arises with public offers in their model is actually valuable since it serves as a commitment device, where sellers know that they either trade in the first period or never again. Indeed, as shown in Fuchs and Skrzypacz (2015), efficiency is actually enhanced when the privately informed seller is exogenously restricted to only one opportunity to sell. Lastly, HV consider an infinite horizon model thus there are no counterparts in their work to our results regarding distress at the deadline.

Another interesting prior comparison between private and public offers goes back to Swinkels (1999). He looks at a dynamic version of the Spence signaling model where potential employers are allowed to make private offers to the "students" at any time. Swinkels shows that in this case the unique equilibrium outcome is a pooling equilibrium with all students being hired at time 0. This, he points out, is in direct contrast to the result in Nöldeke and Van Damme (1990), who show that, with public offers, the unique equilibrium to survive the NWBR refinement is a separating equilibrium where the high types go to school just long enough to credibly separate themselves from the low types. The main difference between both these papers and ours is similar to the difference between Spence and Akerlof. As in the latter, the adverse selection problem is stronger in our model and hence the buyers even with private offers would not be willing to buy at the price necessary to get all sellers to sell.[2]

Our result about non-existence of pure-strategy equilibria in the private offers case is related to the result in Kremer and Skrzypacz (2007) who study a dynamic version of the education signaling model with private offers, a finite horizon, and the type being (partially or fully) revealed in the last period (which endogenously creates adverse selection). They show that there do not exist fully separating equilibria in a game with a continuum of types or with a finite number of types if the length of periods is short enough. The intuition in their proof for why separation is not possible is similar to our intuition why pure-strategy equilibria do not exist. In particular, with private offers, sellers follow a reservation price strategy and the reservation prices are equal to the continuation payoffs which are independent of current prices. Kremer and Skrzypacz (2007) show that if the equilibrium was separating, in continuous time the reservation prices would have zero derivative at the lowest type resulting in a perfectly elastic supply. That in turn would lead to a profitable deviation for the buyers (who with a very small price increase could attract strictly better types). In our discrete-time model we show that if the candidate equilibrium of the game with private offers is in pure strategies, and the discount factor is high enough (or periods are short enough), then equilibrium supply is sufficiently elastic so to create similar profitable deviation for the buyers.

More recently, Kim (2012) compares three different information structures in a

[2]This is also what causes delays in trade in the bargaining model by Deneckere and Liang (2006).

continuous time setting in which many sellers and buyers, who arrive over time at a constant rate, match randomly. In every match, the buyer makes a price offer that the seller can accept or reject. The type space of the seller is binary. Instead of looking at observability of past offers, he compares steady state equilibria in settings in which buyers do not observe any past histories to settings in which the time on the market or the number of past matches can be observed by buyers. The welfare ordering is not as clear cut as in our paper. It is shown that with small frictions, it is optimal if only the time on the market is observable while with large frictions the welfare ordering can be reversed. For repeated first-price auctions, Bergemann and Hörner (2010) consider three different disclosure regimes and they show that if bidders learn privately about their win, welfare is maximized and information is eventually revealed.

Besides our contribution regarding the implications of transparency, our paper also contributes to the literature on dynamic lemons markets in general. One of the most recent works by Deneckere and Liang (2006) considers an infinite horizon bargaining situation, i.e., one long-lived buyer and one long-lived seller, with correlated valuations. They show that even in the limit as the discount factor goes to one, there can be an inefficient delay of trade unlike predicted by the Coase conjecture.[3] Janssen and Roy (2002) obtain similar results with a dynamic competitive lemons market with discrete time, infinite horizon and a continuum of buyers and sellers. While in their model both market sides compete, we assume that there is only one seller. Unlike most previous papers that consider slightly different market structures, we are able to provide a more complete characterization of equilibria in mixed strategies with private offers. This makes it possible to understand these kinds of equilibria in more detail. For example, we show that "non-offers," i.e. offers never accepted, are always part of an equilibrium in the first period if offers are private and offers are frequent.

A number of recent papers work directly in continuous time and, rather than modeling buyers as strategic, they assume there is some competitive equilibrium price path. This paper is a complement to those papers. For example, one can understand the No Deals Condition in Daley and Green (2012) as arising from private offers and the Market Clearing condition in Fuchs and Skrzypacz (2015) as arising in a setting with public offers.[4]

The finance literature has also looked at transparency questions. In particular, our model is more directly related to what is referred to as pre-trade transparency. Most of the theoretical and empirical work has focused on order book transparency.[5] The two main trade-offs regarding transparency within this literature are the "Advertising" and the "Information" effects. The former refers to the notion that when the desire to trade is made public then it is beneficial because more potential counter-parties become aware and might participate. The

[3]See also Fuchs and Skrzypacz (2013*b*).

[4]We have benefited from discussions with Brett Green on these issues.

[5]See for example Buti and Rindi (2011), Flood et al. (1999), Boehmer, Saar and Yu (2005), Madhavan, Porter and Weaver (2005), Pancs (2011)

latter effect refers to the information revealed about the underlying asset that the poster of the offer has. Importantly it leads to less trade with a public order book since traders do not want to reveal private information to the market. Neither of these effects are present in our model since the size of the market is fixed and all the information is on the hands of the seller who does not make any offers. This allows us to highlight the novelty of the dynamic signaling effect we uncover in our paper.

I. Model and Preliminaries

A. General Setup

A seller has an asset that she values at c which is her private information and distributed on $[0,1]$ according to a cumulative distribution function F. One can think of the asset giving an expected cash flow each period and c being its present value for the seller.[6] There are two opportunities to trade with two short-lived buyers arriving in each period $t \in \{1,2\}$.[7] They make simultaneous price offers to purchase the asset.[8] The value of the asset for the buyers is given by $v(c)$ with $v'(c) > 0$, $v(1) = 1$, and gains from trade $v(c) - c$ strictly positive for all $c \in [0,1)$.[9] The game ends as soon as the good is sold. If trade has not taken place by the end of the second period, then the seller obtains a fraction of the gains from trade: $\alpha(v(c) - c)$ with $\alpha \in [0,1]$. One can think of α as a measure of distress at the deadline. If $\alpha = 1$ there is no efficiency loss beyond delay from reaching the deadline. If $\alpha < 1$ there is additional efficiency loss if trade does not take place before the deadline. When there is no opportunity to trade after period 2 we have $\alpha = 0$.

The seller discounts payoffs between the two periods according to a discount factor $\delta \in (0,1)$. All players are risk neutral. Given the seller's type is c and agreement over a price p is reached in period t, the seller's (period 1 present value) payoff is $(1-\delta^{t-1})c + \delta^{t-1}p$; a buyer's payoff is $v(c) - p$ if he gets the good and 0 otherwise. If there is no trade, the seller's payoff is $c + \delta^2\alpha(v(c) - c)$.

Without loss of generality, we restrict prices to be in $[0, v(1)]$, since it is a dominant strategy for the seller to reject any negative price, and for any buyer it is a dominated strategy to offer any price higher than $v(1)$.

We explore two different information structures. In the *public offers* case, period 2 buyers observe rejected offers from period 1. In contrast, with *private offers*, period 2 buyers are aware that the seller has rejected all offers in period 1 but do not know what those offers actually were.

[6] Alternatively, and mathematically equivalently, c can be thought as the cost of producing the asset.

[7] In Section III we extended some results to more than two periods.

[8] The analysis is the same if there are more than two buyers since the buyers compete in a Bertrand fashion.

[9] In most of the paper we assume $v(1) = 1$ to rule out the possibility of trade ending before the last period. This allows us to avoid making assumptions about out-of-equilibrium-path beliefs if the seller does not sell by t even though in equilibrium she is supposed to. If $v(1) > 1$ but δ is small enough so that not all types trade in equilibrium, our analysis still applies.

Period 2 buyers' belief about the seller types they are facing, is characterized by a cumulative distribution function (cdf) denoted by $F_2(c)$. Without loss of generality, we assume that the seller responses are independent of buyer identity. That is, conditional on receiving the same price offer, she treats both buyers equally.

B. *Equilibrium Notion*

We are interested in characterizing perfect Bayesian equilibria (PBE) of the two games. A PBE of a given game is given by (possibly mixed) pricing strategies for the two buyers in each period, a sequence of acceptance rules of the seller, and the buyers' beliefs F_2 at the beginning of period 2, satisfying the following three conditions:

1) Any price offer in the support of a buyer's strategy must maximize the buyer's payoff conditional on the seller's acceptance rule, the other buyer's strategy and his belief $F_t(c)$, where $F_1(c) = F(c)$ is the common prior.

2) Buyers' beliefs F_2 are updated (whenever possible) according to Bayes' rule taking the seller's and the other buyers' strategies as given. In the public-offers game beliefs are updated conditional on the offered prices in period 1.

3) The seller's acceptance rule maximizes her profit taking into account the impact of her choices on the agents' updating and the future offers she can expect to follow as a result.

In the game with private offers, equilibrium strategies and beliefs depend only on the calendar time. In the game with public offers, period 2 strategies and beliefs depend also on the publicly observed prices offered in period 1. With public offers, deviations from equilibrium price offers are observed by future buyers and induce different continuation play. With private offers, off-equilibrium price offers do not change the continuation play.

C. *Preliminaries*

As in other similar dynamic games, in equilibrium, the seller's acceptance rule can be characterized by a cutoff strategy. More precisely, given any history and maximal price offer p , there exists a cutoff $k_t(p)$ such that sellers with valuations above a cutoff $k_t(p)$ reject a price offer p in period t while sellers with valuations less than $k_t(p)$ accept it. In the bargaining literature, it is the better types that accept first and this property is known as the Skimming property. Since here it is the worse types that trade first, we call it Reverse-skimming instead.

LEMMA 1: *(Reverse-skimming property) In any continuation equilibrium with either type of information structure, the following must hold: For any highest price offer p in period t, there exists a cutoff type $k_t(p)$ so that a seller of type c accepts p if $c < k_t(p)$ and rejects p if $c > k_t(p)$.* [10]

[10]Note that $k_2(p)$ is independent of the price history.

See the Appendix for a formal proof. This lemma holds independently of the information structure in place (although the cutoffs may differ). The intuition for the lemma is straightforward. If a type-c seller is willing to accept a price that, if rejected, would induce a given future price path, then all lower-type sellers would also be willing to accept that price rather than wait for a price on that path because their flow payoff from possessing the asset is smaller.

A buyer's expected profit *conditional on having the higher offer* is given by [11]

$$\Pi_t(p; F_t) = \int_0^{k_t(p)} (v(c) - p)\, dF_t(c). \tag{1}$$

Thanks to the Reverse-skimming property, if past prices are observed publicly, the belief about the remaining seller types in period 2 is given by a single cutoff $k_1(p)$. Therefore, with public offers, if p is the highest price offer observed in period 1, then F_2 is just F_1 truncated to $[k_1(p), 1]$. In contrast, with private offers, if period 1 buyers play mixed strategies, period 2 buyers have non-degenerate beliefs over the possible cutoffs induced by period 1 prices. In that case, we denote the cdf representing the distribution of cutoffs after period 1 from period 2 buyers' point of view by $K_1 : [0, 1] \to [0, 1]$. The pdf of the equilibrium belief, $f_2(c)$, is then by Bayes rule:

$$f_2(c) = \int_0^c \frac{1}{1 - \tilde{k}} dK_1(\tilde{k}).$$

We make a regularity assumption that

$$\frac{f(c)}{F(c)}(v(c) - c) \tag{2}$$

is strictly decreasing and that

$$v''(c) \geq 0. \tag{3}$$

This implies that a one-shot game would have a unique equilibrium and that a zero profit condition must be satisfied in both periods for both information structures.[12]

LEMMA 2: *(Zero profit) In any equilibrium, buyers must make zero profits in both periods with both information structures.*

We prove this lemma together with the following lemma which shows that in equilibrium each cutoff can only be induced by a single price.

[11] The expected profit of the buyer is the probability that he has the higher offer, or that he wins in case of a tie, times $\Pi_t(p; F_t)$.

[12] If $\alpha = 0$, assumption (3) can be dropped.

LEMMA 3: *(Inverse supply)*
(i) (Private offers) With private offers, on equilibrium path, there exists a unique price $p_t(k)$ that results in a given cutoff seller type k. $p_t(\cdot) = k_t^{-1}(\cdot)$ is increasing and continuous. It is given by

$$p_1(k) = \underbrace{\delta\left[\left(\int_k^1 p_2(\tilde{k})dK_2(\tilde{k})\right) + K_2(k)p_2(k)\right]}_{\text{continuation payoff}} + \underbrace{(1-\delta)k}_{\substack{\text{utility from} \\ \text{keeping the good}}} \tag{4}$$

where K_2 represents the cdf of the distribution of period 2 equilibrium cutoffs and

$$p_2(k) = \delta\alpha v(k) + (1-\alpha\delta)k. \tag{5}$$

(ii) (Public offers) Consider an equilibrium with public offers. After any history, there is a unique price $p_t(k)$ at which the type-k seller is the highest type accepting the price. Let $\kappa_2(k)$ be the period 2 cutoff of the continuation equilibrium given the period 1 cutoff is believed to be k (which we show is unique). Then, $p_t(k)$ is increasing and given by

$$p_1(k) = \delta p_2(\kappa_2(k)) + (1-\delta)k \tag{6}$$

and

$$p_2(k) = \delta\alpha v(k) + (1-\alpha\delta)k. \tag{7}$$

The formal proof of the two lemma is presented in the Appendix, but we provide some intuition here. From now on we call $p_t(\cdot)$ the inverse supply function defined in this lemma. It is derived from the seller's indifference condition in each period as described in this lemma.

In period 2, the unique price that results in cutoff type k is the same for both information structures (since the seller continuation payoff is independent of the history). However, in period 1, the seller's strategy and hence, $p_1(k)$ are different across information structures.

With private offers, period 1 prices do not affect F_2 or the continuation play. As a result, the continuation payoff in (4) is independent of past cutoffs. The first part of the continuation payoff $\int_k^1 p_2(k_2)dK_2(k_2)$ is nothing but the expected price the seller can get if she sells the asset in period 2. $p_2(k)$ is the expected payoff that a type-k seller can expect if she does not sell tomorrow either, which happens with probability $K_2(k)$. $(1-\delta)k$ represents the payoff of a type-k seller if she held on to the good for exactly one more period. This total expected benefit from waiting must correspond to the payoff from selling today (at $p_1(k)$).

With public offers, however, period 1 prices can affect period 2 price offers, which makes the argument more evolved. We show that the period 2 cutoff of the continuation game, given that period 2 buyers believe the cutoff type after

period 1 is k, increases in k. As a result, $k_t(p)$ is increasing and an inverse supply function exists.

As a consequence of Lemma 3, one can think of buyers essentially choosing cutoffs instead of prices given the seller's cutoff strategy $k_t(\cdot)$. More precisely, we can write a buyer's expected profit conditional on having the higher offer, if he bids a price $p = k_t^{-1}(k)$, and given that buyers believe that current cutoffs are distributed according to a cdf K, as

$$\pi_t(k; K) = \int_0^k \int_0^c \frac{1}{1-\tilde{k}} dK(\tilde{k}) \cdot (v(c) - p_t(k)) \cdot f(c) dc. \tag{8}$$

If K has its entire mass on a singleton ℓ (which is always the case with public offers), then we write $\pi_t(k; \ell)$ instead of $\pi_t(k; K)$, abusing notation slightly. In particular, in period 1, $\pi_1(k; 0) = \int_0^k (v(c) - p_1(k)) \cdot f(c) dc$ where $p_1(k)$ varies across the two information structures.

II. Distress, Transparency, and Welfare

In this section we present all our main results. We are interested in two types of questions. First, how do the two information structures compare in terms of welfare (Theorem 1, Corollary 1, and Theorem 4) and second, how do equilibria differ with the two information structures (Theorems 2 and 3).

A. General Results

A full characterization of equilibria, in particular with private offers, is difficult because, as we show, buyers play mixed strategies and the equilibrium is generally not unique.[13] Nevertheless, even without an explicit characterization of equilibria with private and public offers, we can show that all equilibria with private offers result in more trade than all equilibria with public offers.

THEOREM 1: *Consider an arbitrary equilibrium with public offers and an equilibrium with private offers. Then, the following hold:*
(i) In expectation, there is (weakly) more total trade in the equilibrium with private offers.
(ii) All types that trade in the second period with public offers are (ex-ante, weakly) better off when offers are made privately.
(iii) Expected second period prices are (weakly) higher with private offers.
(iv) If the expected price in the first period with private offers is weakly higher than the expect price with public offers, then the private-offers equilibrium Pareto dominates the public-offers equilibrium.

[13] An explicit characterization of equilibria if valuations are linear and costs are uniformly distributed is presented in Section II.C.

PROOF. First, note that with public offers, we can restrict attention to pure-strategy equilibria because for any mixed-strategy equilibrium, one can construct a pure-strategy equilibrium that Pareto dominates it. Such pure-strategy equilibrium can be constructed as follows. The period 1 price p_1^* is the largest price in the support of period 1 prices in the mixed-strategy equilibria and must also result in the largest period 1 cutoff k_1^*. By the regularity assumption that (2) is decreasing and (3), there is only one price p_2 that satisfies the period 2 zero profit condition, given the period 1 cutoff k_1. Then, given any period 1 cutoff $k > k_1$ profits are greater than zero at p_2. Hence, the period 2 price p_2^* following the period 1 cutoff k_1^* must be the largest period 2 price that is chosen with positive probability in the mixed equilibrium. Hence, the pure-strategy equilibrium given by price offers $p_1^* = p_1(k_1^*)$ and $p_2^* = p_2(k_2^*)$ exists and Pareto-dominates the mixed equilibrium.

Let us consider a public-offers equilibrium with cutoff types k_1^* and k_2^*. If offered the equilibrium price p_1^* in period 1, the continuation payoff of type c after rejecting the offer is given by

$$V(c; p_1^*) \equiv \delta \max\{p_2^*, p_2(c)\} + (1-\delta)c.$$

Note that by definition $V(k_1^*; p_1^*) = p_1^*$. In a private offers equilibrium, the continuation payoff of type c after rejecting an offer in period 1 is independent of the price in period 1 and given by

$$W(c) \equiv p_1(c) = \delta \left[\left(\int_c^1 p_2(\tilde{k}) dK_2(\tilde{k}) \right) + K_2(c) p_2(c) \right] + (1-\delta)c.$$

Assuming $W(k_1^*) < V(k_1^*; p_1^*) = p_1^*$ can be shown to lead to a contradiction.[14] We can thus focus on the case $W(k_1^*) \geq V(k_1^*; p_1^*)$. In this case, for all $c \leq k_2^*$, since the equilibrium with private offers might involve mixing in the second period and might result in the seller deciding not to sell (if the second period realized offer is low), the derivative of the continuation value with respect to type is higher:

$$\frac{\partial}{\partial c} W(c) = 1 - \delta + \delta K_2(c) \left(1 - \alpha\delta + \alpha\delta v'(c)\right) \quad \geq \quad 1 - \delta = \frac{\partial}{\partial c} V(c; p_1^*).$$

Hence, all seller types $k \in [k_1^*, k_2^*]$ have a better outside option with private offers when rejecting the period 1 price which implies that all types $k \in [k_1^*, k_2^*]$ are better off with private offers. Sellers with $k \geq k_2^*$ wait until the deadline with public offers. They always have this option with private offers as well and can even be better off if they see a preferable price before. This proves (i). Note that (ii) then follows as well since the buyers break even and surplus for the seller is

[14] A lower continuation value with private offers would imply acceptance by types higher than k_1^* in the first period, which in turn must imply higher prices in period 1 and , therefore also in period 2. This would imply $W(k_1^*) > V(k_1^*; p_1^*)$.

derived from trade. Thus, more surplus can only be achieved with more trade. Given that the seller's reservation price in the second period is independent of information structure, more trade can only be achieved with higher average prices, proving (iii). If expected period 1 prices are higher with private offers than with public offers, then all seller types $k < k_1^*$ are also better off with private offers. This proves (iv).

□

REMARK 1: *A noteworthy consequence of Theorem 1 is that there can exist at most one pure-strategy equilibrium with private offers. This follows because any private-offers pure-strategy equilibrium corresponds to a public-offers pure-strategy equilibrium and all public-offers pure-strategy equilibria can be ranked in terms of the amount of trade. Hence, only the pure-strategy equilibrium with most trade is a candidate for a private-offers equilibrium outcome.*

We have shown that if equilibria differ in the two information structures, then there is more trade with private offers. Next, we show that equilibria with private and public offers do not always coincide. In particular, they must differ when the seller discounts future periods only a little. A high discount factor can alternatively be interpreted as frequent opportunities to trade as discussed in Section III.A. We show that pure-strategy equilibria seize to exist with private offers.

THEOREM 2: *(i) With public offers, a pure-strategy equilibrium always exist.*
(ii) With private offers, there exists a δ^* such that for all $\delta > \delta^*$ no pure-strategy equilibria exist.

We present most of the proof here, but defer technical calculations to the Appendix. First, the existence of public offer equilibria follows by backward induction. A buyer's expected period 2 profit conditional having the higher offer is given by

$$\pi_2(k_2; k_1) = \frac{1}{1 - F(k_1)} \int_{k_1}^{k_2} \left(v(c) - \underbrace{(1-\delta)k_2 + \delta v(k_2)}_{=p_2(k_2)} \right) f(c)dc.$$

Since in equilibrium buyers must make zero profits (Lemma 2), any equilibrium cutoff of the continuation game $\kappa_2(k_1)$ must satisfy

$$\pi_2(\kappa_2(k_1); k_1) = 0. \tag{9}$$

Note that such a continuation cutoff $\kappa_2(k_1)$ always exists and is smaller than 1 because $v(1) = 1$. In order to attract a cutoff-type k_1 in the first period buyers

need to bid at least $p_1 = (1-\delta)k_1 + \delta p_2(\kappa_2(k_1))$. Hence, buyers' profits in period 1 can be written as

$$\pi_1(k_1;0) = \int_0^{k_1} \left(v(c) - \underbrace{((1-\delta)k_1 + \delta p_2(\kappa_2(k_1)))}_{p_1(k_1)} \right) f(c)dc.$$

Then,

$$k_1^* = \sup\{k \in [0,1] | \pi_1(k;0) > 0\}$$

(with $k_1^* = 0$ if the set is empty) and $k_2^* = \kappa_2(k_1^*)$ supports an equilibrium. From now on we denote the equilibrium cutoffs in the game with public offers by k_t^*.

What can we say about equilibria of the game with private offers? First, recall that period-1 buyers' profits with private offers

$$\pi_1(k_1;0) = \int_0^{k_1} \left(v(c) - ((1-\delta)k_1 + \delta p_2(k_2^*))\right) f(c)dc$$

are differentiable in k_1 and the zero-profit condition

$$\mathbb{E}\left[v(c) | c \in [0, k_1]\right] = p_1(k_1)$$

must be satisfied for all k_1 in the support of the equilibrium strategy of period 1 buyers (Lemma 2). Similarly, profits must be equal to zero in period 2 and buyers must have correct beliefs about the period 1 cutoff.

Suppose the game with private offers has a pure-strategy equilibrium that induces the same cutoffs k_t^* that we found in the game with public offers. Consider the incentives of buyers in the first period. With private offers, if buyers deviate to a higher price, to induce a marginally higher cutoff than k_1^*, we can compute using (II.A) that the net marginal benefit (NMB) of that deviation is

$$\frac{\partial}{\partial k_1}\pi_1(k_1;0)\big|_{k-1=k_1^*} \quad = \quad F(k_1^*) \cdot \left[\frac{\partial}{\partial k_1}\mathbb{E}[v(c)|c \in [0,k_1]]\big|_{k_1=k_1^*} - \underbrace{\frac{\partial}{\partial k_1}p_1(k_1)}_{=1-\delta} \right].$$

Now, as $\delta \to 1$, it follows from the seller's indifference conditions that $k_1^* \to 0$. When we consider the limit $k_1^* \to 0$, we can apply L'Hopital's rule to obtain:

$$\lim_{k_1^* \to 0} \frac{\partial}{\partial k_1}\mathbb{E}[v(c)|c \in [0,k_1]]|_{k=k_1^*} \quad = \quad \frac{v'(0)}{2}.$$

Thus,

$$\lim_{k_1^* \to 0} \frac{1}{F(k_0^*)} \cdot \frac{\partial}{\partial k_1} \pi_1(k_1; 0)|_{k_1 = k_1^*} \quad = \quad \frac{v'(0)}{2} - (1 - \delta).$$

is strictly positive for large enough δ as long as $v'(0) > 0$. Hence, there exists a δ^* such that no pure-strategy equilibrium can be sustained with private offers for all $\delta > \delta^*$.

Intuitively, the difference in the two information structures can be seen as follows. With public offers, the seller has a stronger incentive to reject high price offers in period 1 than if the offer had been made privately: Suppose one of the buyers made an out-of-equilibrium high offer. With public offers the seller gains additional reputation of her type being high by rejecting this offer, the strength of her signal being endogenously determined by the amount of money she left on the table. Consequently, her continuation value increases upon a rejection of the higher price. Instead, with private offers, she cannot use the out-of-equilibrium higher offer as a signal, so her continuation value remains constant. Thus, she has stronger incentives to accept the higher offer if it is private. Formally, this is reflected by different period 1 supply functions $p_1(k)$ in the two information structures. In particular, for all $k > k_1^*$, the price that makes k indifferent with public offers is greater than the price with private offers:

$$\underbrace{(1-\delta)k + \delta p_2(\kappa_2(k))}_{p_1(k) \text{ with public offers}} > \underbrace{(1-\delta)k + \delta p_2(k_2^*)}_{p_1(k) \text{ with private offers}}.$$

The effect is large enough to break down pure strategy equilibria with private offers if the discount factor is large enough because the seller's value of signaling to future buyers is higher as the next period starts sooner.

B. Distress and Market Breakdown

Recall that we assumed that if the seller rejects offers at $t = 2$, she captures $\alpha(v(c) - c)$ of the continuation surplus. $\alpha < 1$ can be interpreted as a measure of distress. In the following we discuss how it affects equilibria in the two information regimes and show that for $\alpha < 1$ and large enough discount factors it follows as a corollary of Theorem 1 that private-offers equilibria Pareto-dominate all public-offers equilibria.

To this end, consider a game with public offers. We show that trade in period 1 can break down if $\alpha < 1$ and δ is large. If some surplus is lost after the deadline, there is an extra incentive to trade in period 2, right before the deadline. The lower the α, the more types trade at the deadline. This leads to quiet period (i.e., no trade) in period 1 if δ is high. In search of a contradiction, suppose there was trade in period 1 and let us denote the largest seller type trading in period 1 by $\hat{k}$. The highest price at which he could possibly be trading is $v(\hat{k})$. Since the mass

of types trading in period 2 is uniformly bounded from below for all δ, the price at $t = 2$ must be strictly greater than $v(\hat{k})$. Thus, if δ is close to 1, the cost of waiting in order to trade at the higher price the next period is negligible relative to the benefit and thus $\hat{k}$ should not trade. Formally:

PROPOSITION 1: *(Quiet Period) With public offers, for any $\alpha < 1$ there exists a $\delta^{**} < 1$ such that if $\delta > \delta^{**}$ in equilibrium there is no trade in the first period.*

This logic can be extended to multiple periods as we show in the Appendix in Proposition 6. The reason this logic does not apply when $\alpha = 1$ is that in that case as δ increases to 1, while probability of trade in period 2 is positive, it is not uniformly bounded away from zero. In fact, it converges to zero and the period 2 price converges to $v(\hat{k})$ and there can be trade in both periods along the sequence, as we have shown in the previous section. Thus, in contrast to HV who find that with public offers there is trade only in the first period, we find that without distress with public offers there is trade in every period and with distress there is no trade in the first period.

In contrast, with private offers, an equilibrium cannot have quiet periods (i.e. periods with zero probability of trade). To see this, suppose that in the current period there was to be no trade but in the next period there would be some trade at a price p. The buyers could offer a price p in the current period attracting all sellers and some higher types that would have accepted p in the next period. They accept because the offer is private and thus, does not change the continuation game for the seller if she were to reject it. Such a deviation is profitable for buyers because buyers in the next period would have made non-positive profit. Thus there cannot be quiet periods in equilibrium. We summarize this observation as:

PROPOSITION 2: *(No Quiet Periods) With private offers, for all α and δ there must be a strictly positive probability of trade in every period.*

In particular, in period 1 there must be a positive probability of trade. This establishes another important difference in the equilibrium behavior across information structures. This difference allows us to easily argue that when $\alpha < 1$ for high δ the opaque environment Pareto-dominates the transparent one because we already know from Theorem 1 that all types that would sell in period 2 with public offers are better off with the private information structure.

COROLLARY 1: *If $\delta > \delta^{**}$ so that the game with public offers has no trade in period 1, then any equilibrium in the private-offers game Pareto-dominates the equilibrium of the public-offers game.*

C. The Linear and Uniform Case

With linear valuation $v(c) = Ac + B$, $alpha = 1$, and c being uniformly distributed on $[0, 1]$, we can fully characterize the set of private-offers equilibria and

use this in order to show that the private information structure Pareto-dominates the public one. To this end, we first present a stronger version of Theorem 2 in the linear-uniform environment.

THEOREM 3: *Let $v(c) = Ac + B$ be linear, $\alpha = 1$ and c uniformly distributed on $[0, 1]$.*

(i) With public offers, there is a unique equilibrium which is in pure strategies.

(ii) With private offers, there exists a

$$\delta^* = \quad 1 - \tfrac{A}{2} \quad \in (0, 1)$$

such that the following hold:

1) *For all $\delta < \delta^*$, the equilibrium is unique and the equilibrium outcome coincides with the equilibrium outcome with public offers.*

2) *For all $\delta > \delta^*$ no pure-strategy equilibria exist. Instead, there are multiple mix-strategy equilibria. For any equilibrium with private offers, the expected price in the first period is strictly higher than the expected price with public offers.*

In the following, we present most of the proof and the intuition of Theorem 3. The proofs of Propositions 3 and 4, as well as the construction of a mixed-equilibrium with private offers (which conclude the proof of Theorem 3 part (ii) 2) are deferred to the Appendix.

The unique public offer equilibrium can be calculated using backward induction. It is given by the period 1 cutoff

$$k_1^* = \frac{2B \cdot (1-\delta)^2(2-A)}{2(1-\delta)(1-A)\left(A\delta - 2\delta + 2\right) + A^2} \tag{10}$$

and the period 2 cutoff

$$k_2^* = \frac{2B \cdot (2(1-\delta)^2 + A\delta(1-\delta))}{2(1-\delta)(1-A)\left(A\delta - 2\delta + 2\right) + A^2}. \tag{11}$$

This fails to be an equilibrium with private offers if

$$\frac{\partial}{\partial k_1}\pi_1(k_1; 0)|_{k_1 = k_1^*} = k_1^* \cdot \big(\frac{A}{2} - \underbrace{(1-\delta)}_{\frac{\partial p_1}{\partial k_1}(k_1)}\big) > 0$$

Hence, for high discount factors, $\delta > \delta^* = 1 - \frac{A}{2}$, it is profitable for a buyers to deviate to higher prices. The reason is that by rejecting a higher price, the seller cannot send such a strong signal to tomorrow's buyers about her type, and hence she would accept such a price. Consequently, there is no pure-strategy

equilibrium with private offers if $\delta > \delta^*$. If $\delta \leq 1 - \frac{A}{2} \equiv \delta^*$, then buyers in period 1 do not have an incentive to deviate because their profit

$$\pi_1(k_1; 0) = k_1 \cdot \left(\frac{A}{2} k_1 + B - p_1(k_1) \right)$$

is a quadratic function with a null at $k_1 = 0$ and $k_1 = k_1^*$ and negative slope at k_1^*.

The discussion above establishes that if $\delta > \delta^*$ there can be only mixed-strategy equilibria in the game with private offers. We further claim that if $\delta < \delta^*$ the private-offers game has only a pure-strategy equilibrium with outcome that coincides with the public equilibrium outcome, no mixed-strategy equilibrium exists. To establish this result, we fist argue that mixing cannot occur in period 1. Period 1 prices $p_1(k)$ with private offers are given by (4) (and allows for mixing in period 2). Substituting (4), we get that $\pi_1(k_1; 0)$ is piece-wise quadratic and the coefficient in front of the quadratic component k^2 is always smaller than $\frac{A}{2} - 1 + \delta$. For $\delta < \delta^*$ this is negative and hence buyers in period 1 must play a pure-strategy in equilibrium. Consequently, buyers in period 2 must have a degenerate belief K_2 and by the arguments in the public-offers case, the continuation equilibrium is unique and in pure strategies.

Mixed-strategy equilibria for $\delta > \delta^* = 1 - \frac{A}{2}$ are characterized by the following proposition.

PROPOSITION 3: *Suppose $\delta > \delta^* = 1 - \frac{A}{2}$. In any mixed-strategy equilibrium with private offers, the following hold:*
(i) In period 2, buyers mix between exactly two prices that result in the two cutoffs given by

$$\underline{k}_2 = \frac{B(1-\delta)}{A\delta - \delta + 1 - \frac{A}{2}}, \qquad \overline{k}_2 = \frac{B(1-\delta^2)}{A\delta^2 - \delta^2 + 1 - \frac{A}{2}},$$

where $\underline{k}_2$ is chosen with probability $q_2 \equiv \frac{\frac{A}{2} - (1-\delta)}{\delta(A\delta + 1 - \delta)}$.
(ii) In period 1, buyers mix between prices that induce cutoffs 0 and cutoffs that lie in $(\underline{k}_2, \overline{k}_2)$. Cutoff 0 is induced on the equilibrium path with a positive probability.

How does the mixing help resolve the problem of non-existence of equilibrium? Consider any cutoff $k_1 > 0$ consistent with the equilibrium outcome in period 1. It must be that $\pi_1(k; 0) = 0$ and $\frac{\partial}{\partial k}\pi_1(k; 0) \leq 0$ at that cutoff. As we argued above, the sign of $\frac{\partial}{\partial k}\pi_1(k; 0)$ depends on the sign of $\frac{\partial}{\partial k}(\mathbb{E}[v(c)|c \leq k] - p_1(k))$. Mixing in period 2 changes the derivative of $p_1(k)$. In particular, if k_1 trades in period 2 if the price offer is high and does not trade in period 2 if the offer is low then $p_1(k) = \delta \mathbb{E}[\max\{p_2(\tilde{k}), p_2(k)\}] + (1-\delta)k$ where $p_2(k)$ (defined in equation (5)) is the seller's continuation payoff if she rejects period 2 prices and $\tilde{k}$ is the equilibrium period 2 cutoff distributed according to K_2. Mixing in period 2 makes

the seller's continuation payoff in period 1 more sensitive to her type and hence the supply function $p_1(k)$ becomes less elastic. If the probability of k_1 not trading in period 2 is high enough, then period 1 buyers have no incentive to increase prices.

In equilibrium buyers must mix over period 1 offers for two reasons. First, if the posterior belief in period 2 were a truncation of the uniform prior, there would be a unique continuation equilibrium price. Mixing in period 1 is needed to induce a posterior such that mixing in period 2 is indeed a continuation equilibrium. Second, and more generally, note that the lowest type in the support of F_2 trades in period 2 for sure (recall $v(c) > c$). If the lowest cutoff induced in period 1 were strictly positive then for that type $\frac{\partial}{\partial k} p_1(k) = 1 - \delta$. As discussed above, that would imply $\frac{\partial}{\partial k}\pi_1(k;0) > 0$ for $\delta > \delta^*$ and buyers would have a profitable deviation. Therefore in equilibrium buyers in period 1 must make with positive probability a non-offer, i.e. offer a low price that is rejected by all types.[15] At the same time, it cannot be that no type trades in period 1. If so, buyers could deviate to the highest price offered in equilibrium in period 2 and make a strictly positive profit (since that price would be accepted by types better than those that trade in period 2).

Even though the private-offers equilibrium strategy in period 1 is not unique, all equilibrium strategies have some properties in common. In particular, the expected cutoff type is constant across equilibria and on average higher than with public offers:

THEOREM 4: *(Welfare) If $\delta > \delta^*$, the following holds:*
(i) The expected cutoff in period 1 is constant across all equilibria with private offers.
(ii) Denoting the expected equilibrium cutoff in period 1 with private offers by $\mathbb{E}^{K_1}[k_1]$, it is higher than the equilibrium cutoff with public offers:

$$\mathbb{E}^{K_1}[k_1] > k_1^*.$$

Because the expected period 1 cutoff is constant across equilibria, we can simply calculate the expected period 1 cutoff with private offers and show that it is greater than k_1^*. Hence, the reserve price of any type that trades in period 1 is at least:

$$p_1(k) \geq (1-\delta)k + \delta \mathbb{E}^{Private}[p_2]$$

because that type has the option not to sell in period 2. Integrating the reserve prices over the equilibrium distribution of the period 1 cutoff types we get the

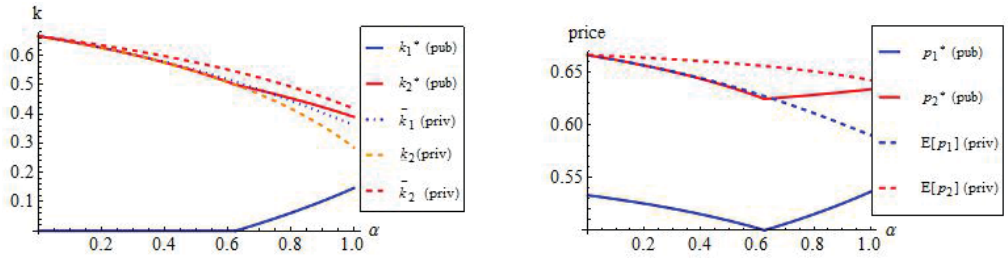
[15] In equilibrium the lowest on-path period 1 cutoff is $k_1 = 0$. While at that cutoff $\frac{\partial}{\partial k}(\mathbb{E}[v(c)|c \leq k] - p_1(k)) > 0$, the reservation prices of the low types are sufficiently high so that for all cutoffs $k \in (0, \underline{k}_2)$, $\pi_1(k;0) < 0$. In particular, $p_1(0) > v(0)$.

average transaction price in period 1:[16]

$$\mathbb{E}^{Private}[p_1] \equiv \int_0^1 p_1(\tilde{k})dK_1(\tilde{k}) \geq (1-\delta)\underbrace{\int_0^1 \tilde{k}dK_1(\tilde{k})}_{>k_1^*} + \delta\underbrace{\mathbb{E}^{Private}[p_2]}_{>p_2(k_2^*)}$$

where K_1 is the cdf of the equilibrium distribution of period 1 cutoffs with private offers. This is greater than the period 1 public offer price $p_1(k_1^*) = (1-\delta)k_1^* + \delta p_2(k_2^*)$ because the average cutoff in period 1 is higher (as we show in the Appendix) and the average price in period 2 is also higher (as we showed in Theorem 1). Thus, all seller types $c < k_1^*$ are better off with private offers - they either sell in the first period at a higher expected price or chose to sell in the second period which must give them higher profits by revealed preference. Consequently, we have established the Pareto ranking of equilibria, i.e., all seller types are ex-ante better off with the private information structure than with the public information structure.

For the uniform-linear case, we can also calculate numerically some equilibria for different levels of distress cutoffs and prices as the level of distress α changes and they are illustrated in Figure 1.



(a) Cutoffs for $\delta = 0.8$, $A = B = 0.5$

(b) Expected prices for $\delta = 0.8$, $A = B = 0.5$

FIGURE 1. ROLE OF DISTRESS

Indeed there is more trade the more distress is faced at the deadline (i.e., as α decreases). However, trade breaks down in period 1 with high level of distress (i.e., α less than ≈ 0.63) if offers are public. In contrast, this effect is almost completely alleviated with private offers. Hence, if distress is a severe issue, the benefit of opaque environments is potentially even higher than without distress at the deadline.

[16]Recall that in the private offers equilibrium buyers make a non-offer with positive probability. That price is unbounded from below, but the equilibrium payoffs of all types can be computed as if the price offered in that case is equal to the reserve price of the lowest type, as we do in this expression.

III. Robustness and Generalizations

A. More than two periods

We now generalize Theorem 2 by allowing more opportunities to trade before the deadline with each period $\{0, \Delta, \ldots, 1-\Delta\}$ having length Δ and the discount rate being r, i.e., $\delta = e^{-r\Delta}$.

The proof of Lemma 3 can be extended by induction to show that an inverse supply function exists in every period even with more opportunities to trade, albeit it does not have to be defined on the entire cutoff space. The following analysis is independent of what happens at the deadline and therefore holds true for all $\alpha \in [0, 1]$.

THEOREM 5: *(i) With public offers, there exists a pure-strategy equilibrium for all $0 < \delta < 1$.*
(ii) Equilibrium cutoffs (and prices) in any pure-strategy equilibrium with private offers correspond to equilibrium cutoffs (and prices) in a pure strategy equilibrium with public offers.
(iii)There exists a $\Delta^* < 1$ such that if $\Delta < \Delta^*$ there is no pure-strategy equilibrium with private offers.

We discuss the main steps of the proof here and provide additional details in the Appendix. The existence of a pure-strategy equilibrium with public offers can be shown by construction in two steps. First, we define for each period a mapping that maps today's cutoff to tomorrow's cutoff by backward induction. Then, the cutoff in the first period is this function realized at cutoff zero and all other cutoffs can inductively found (for formal construction see the Appendix). In general, multiple pure-strategy equilibria can coexist.

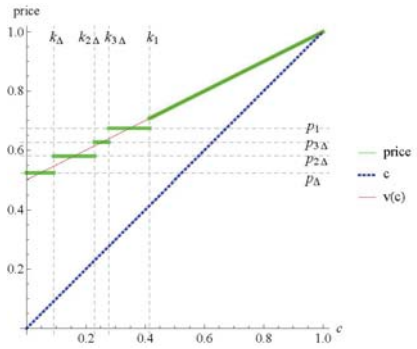
Figure 2 illustrates prices at which different seller types trade for $v(c) = 0.5 + 0.5c$, $r = 0.5$, and $\Delta \in \left\{\frac{1}{4}, \frac{1}{8}\right\}$, as well as $\Delta \to 0$. p_t denotes the price and k_t denotes the equilibrium cutoff in period t.

With private offers, there must be trade with positive probability in each period because buyers can always mimic the strategies of future buyers as we have already discussed in Proposition 2. Hence, in any pure-strategy equilibrium with cutoffs $(k^*_\Delta, \ldots, k^*_{1-\Delta})$, the zero-profit condition

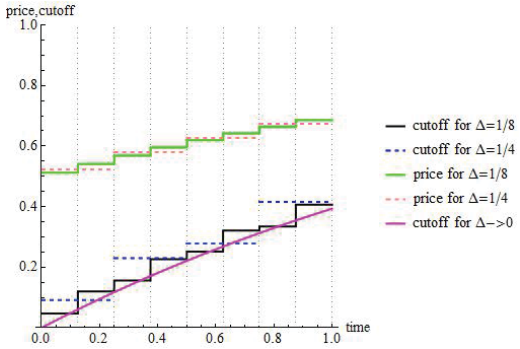
$$\mathbb{E}[v(c)|c \in [k^*_{t-\Delta}, k^*_t]] = p_t(k^*_t) \tag{12}$$

must be satisfied for all t. Moreover, a buyer's expected period 1 profit conditional on having the higher bid is given by

$$\pi_0(k; 0) = F(k) \cdot \underbrace{[\mathbb{E}[v(c)|c \in [0, k]] - p_0(k)]}_{=0 \text{ at } k=k^*_0}. \tag{13}$$



(a) Prices as a function of seller types for $\Delta = \frac{1}{4}$



(b) Prices and cutoffs over time for $\Delta \in \{\frac{1}{4}, \frac{1}{8}\}$

FIGURE 2. PURE-STRATEGY EQUILIBRIA WITH $r = 0.5$, $v(c) = \frac{c+1}{2}$

Analogously to Theorem 2, one can show that the NMB of a deviation is given by:

$$\frac{\partial}{\partial k}\pi_0(k;0)|_{k=k_0^*} = F(k_0^*) \cdot \left[\frac{\partial}{\partial k}\mathbb{E}[v(c)|c \in [0,k]]|_{k=k_0^*} - \underbrace{\frac{\partial}{\partial k}p_0(k)}_{=1-\delta} \right]. \tag{14}$$

By the same argument we used to provide intuition for Theorem 2, it follows that for large δ (i.e., small Δ), period 1 buyers can profitably deviate by offering a higher price, establishing part (iii) of Theorem 5.

B. Gap at the top $v(1) > 1$

Throughout the paper, we have assumed that $v(1) = 1$. This assumption together with continuity and monotonicity of $v(c)$ guarantees that in any equilibrium, a positive mass of high type sellers do not trade before the deadline. The reason is that the expected value of buyers is always smaller than 1, so that the highest type $c = 1$ never trades before information is revealed. Hence, we did not have to worry about off-equilibrium beliefs of buyers if they see a seller rejecting even though on equilibrium path all sellers should have traded. The freedom in choice of off-equilibrium beliefs could lead to additional multiplicities of equilibria.

Nevertheless, all results can easily be generalized to settings with $v(1) > 1$ if we assume that the lemons problem is severe enough so that trade does not end before the deadline or if we make some out-of-equilibrium belief assumptions. For example, if buyers' beliefs remain unchanged (or become more pessimistic) after the last period of trade, the game can still be solved by backward induction and

the same arguments can be applied as in the proofs of the theorems.

C. No Gap at the bottom ($v(0) = 0$)

We made the assumption that $v(0) > 0$ to make sure that there is always some trade before time 1. If we have no gap at the bottom then it is possible for trade to completely unravel in all periods if the lemons condition is satisfied. For $v(c) = Ac + B$, the lemons condition is $\frac{A+B}{2} < 1$.

PROPOSITION 4: *If $B = 0$ and $A < 2$, there always exists a pure-strategy equilibrium with private and public offers. In that equilibrium, there is no trade before the deadline.*

D. Two types

Several of the recent papers that look at dynamic adverse selection consider only two possible types (e.g., Daley and Green (2012), Camargo and Lester (2011), and Nöldeke and Van Damme (1990)) or use the two type case as examples (e.g., Swinkels (1999), HV, and Deneckere and Liang (2006)). Hence it is interesting to explore equilibria of our games with two types. In order to be closer to the HV setup we assume there is no opportunity of trade after the last period (i.e., $\alpha = 0$).

Consider a situation with $v_H \geq c_H$ and $v_L > c_L = 0$ where the seller's cost is c_L with probability ϕ. Let ϕ satisfy the static lemons condition:

$$\phi v_L + (1 - \phi)v_H < c_H.$$

Then, we show that equilibria in both information structures coincide.

PROPOSITION 5: *With two seller types and two opportunities to trade, equilibria with private and public offers coincide for every δ.*

This points out that in a dynamic setup it can be important to have a rich enough type space. The differences in results are driven by the inability of sellers to have a rich signal space because Bertrand competition only allows for two prices that make one of the two seller types indifferent between buying and waiting. The formal analysis can be found in the Appendix.

IV. Conclusion

The Pareto ranking of information structures, suggests that when designing policy it might be worth considering if it is possible to obtain some of the other benefits from price transparency while limiting its negative effects, in particular when trade takes place frequently. Our analysis also shows that when thinking about policy and how to best resolve these trade-offs the details of the market structure, such as the timing of competition, and the level of distress are likely to play an important role.

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APPENDIX

Proofs: Model and Preliminaries

Proof of Lemma 1.

In both information structures, the seller accepts a price p_2 if and only if $p_2 - c$ is greater than $\delta \cdot \alpha(v(c) - c)$. Since $\delta\alpha < 1$, if a seller type c weakly prefers to accept p_2, then all types $c' < c$ strictly prefer to accept the it. Similarly, in period 1, p_1 is accepted by the seller if and only if $p_1 - c$ is less than accepting the period 2 price $\delta \cdot (\mathbb{E}[p_2] - c)$ and than waiting for the deadline $\delta^2\alpha(v(c) - c)$. p_2 is independent of c. Since the derivative of the continuation payoff with respect to c is less than 1, again if a seller type c prefers to accepts p_1, then all types $c' < c$ strictly prefer to accept it. (Note that it is irrelevant that, with public offers, p_2 is a function of p_1.)

Generalization of this logic to more than two periods is straightforward.

□

Proof of Lemma 2 and 3.

(i) (Private offers) With private offers, beliefs of buyers are independent of price histories. Hence, the continuation game in an equilibrium is unaffected by past offers. We argue by backward induction.

In period 2, a seller of type c accepts an offer p if and only if $p \geq \delta v(c)+(1-\delta)c \equiv p_2(c)$. p_2 is increasing and continuous. Consequently, $p_2(k) = k_2^{-1}(k)$ is the unique price that results in a cutoff k in period 2.

In period 1, the continuation payoff of a seller c who rejects is given by

$$\begin{aligned} W(c) &= \delta \cdot \int_c^1 \left(p_2(\tilde{k}) - c\right) dK_2(\tilde{k}) + \delta \cdot K_2(c) \cdot \alpha \cdot (v(c) - c) + c \\ &= \delta \left[\left(\int_c^1 p_2(\tilde{k}) dK_2(\tilde{k})\right) + K_2(c)p_2(c)\right] + (1 - \delta)c. \end{aligned}$$

Since p_2 is increasing, $W(c)$ is increasing and continuous. $p_1(c) = \mathcal{V}(c)$ defines the inverse supply function. Note that $p_t(k)$ is the unique price that results in a cutoff k and hence $p_t = k_t^{-1}$.

An analogous argument with backward induction can be made for more than two periods.

In period 2, the profits must be zero because, buyers' continuation profits are continuous in period 2 prices (and cutoffs). Since in equilibrium, period 2 prices are not affected by period 1 prices, period 1 profits are also continuous in period 1 prices and hence, the zero profit condition must hold.

(ii) (Public offers) With public offers, it follows analogously to the private offers case that $p_2(k) = \alpha\delta v(k)+(1-\alpha\delta)k$. Hence, period 2 profits of buyers (conditional on offering the highest price) are given by

$$\pi_2(k; k_1) = \frac{1}{1 - F(k_1)} \cdot \int_{k_1}^{k} (v(c) - p_2(k)) f(c) dc$$

which is continuous. Hence, by the Bertrand-competition logic, any period 2 cutoff in a continuation equilibrium $\kappa_2(k_1)$ must satisfy the zero profit condition and for all $k > \kappa_2(k_1)$ it must hold that $\pi_2(k; k_1) \leq 0$. Moreover $\kappa_2(k_1)$ The assumption that (2) is decreasing and (3) guarantees that $\kappa_2(k_1)$ is unique because the assumption implies that the first order condition defining $\kappa_2(k_1)$ which can be written as

$$F(c) \cdot (1 - \alpha\delta) \cdot \left[\frac{f(c)}{F(c)}(v(c) - c) - \left(\frac{\alpha\delta v'(c)}{1 - \alpha\delta} + 1 \right) \cdot \left(1 - \frac{F(k_1)}{F(c)} \right) \right] \quad = \quad 0$$

has a unique solution. Thus, by continuity of $\pi_2(\cdot;\cdot)$, $\kappa_2(\cdot)$ must be continuous. By the zero profit condition, for $k_1 > k_1'$, $\kappa_2(k_1) \geq \kappa_2(k_1')$ because for all $k \geq \kappa_2(k_1)$, $\pi_2(k; k_1) \leq 0$ and therefore the same must hold for all $k_1 > k_1'$. Hence, $\kappa_2(\cdot)$ must be increasing and

$$p_1(k) = (1 - \delta) \cdot k + (1 - \delta) \cdot p_2(\kappa_2(k))$$

is increasing, well defined, and continuous. Finally, the zero profit condition must be satisfied in period 1 because profits of buyers

$$\pi_1(k; 0) = \int_0^k (v(c) - p_1(k)) f(c) dc$$

are continuous in k.

□

Proofs: Distress, Transparency, and Welfare

Proof of Theorem 2. See the more general proof of Theorem 5.

□

Proof of Theorem 3 and Proposition 3.

Before we do the actual construction of equilibrium cutoffs, we need to show some properties about the type of mixing that can occur in an equilibrium. In Lemma 5 and 6 we show that for $\delta < \delta^*$ buyers at most mix between countably many prices and that with $\delta > \delta^*$ there is only countable mixing after the first period. In order to prove these statements the following lemma is useful.

LEMMA 4: *With private offers, $p_1(k)$ is differentiable almost everywhere and differentiable from the right everywhere. The derivative*

$$\frac{\partial}{\partial k} p_1(k) = 1 - \delta - \delta^2 K_2(k) (1 - A) \, (> 0)$$

is nondecreasing.

The proof of this lemma follows immediately from the fact that K_2 is a cdf. It is worth noting that this lemma generalizes to a multi-period setup by induction.

LEMMA 5: *If $\delta < 1 - \frac{A}{2}$, buyers in period 1 mix at most between countably many cutoffs. If $\delta > 1 - \frac{A}{2}$ and expected period 1 profit $\pi_1(k;0) = 0$ for all $k \in (a,b)$, then any $k \in (a,b)$ cannot be in the support of K_2 since it must hold that $K_2(k) = \frac{\delta - 1 + \frac{A}{2}}{\delta \frac{\partial}{\partial k} p_2(k)}$.*

Proof of Lemma 5. In period 1, expected buyers' profits are given by

$$\pi_1(k;0) = k \cdot \left[\frac{A}{2} k + B - p_1(k) \right].$$

If buyers mix between all cutoffs $k \in (a,b)$ at time 0, then they must make zero profits for all such cutoffs, i.e., for all $k \in (a,b)$

$$\delta \left(\int_k^1 p_2(\tilde{k}) dK_2(\tilde{k}) + K_2(k) p_2(k) \right) + k(1-\delta) = \frac{A}{2} k + B$$

or equivalently

$$\delta \left(\int_k^1 p_2(\tilde{k}) dK_2(\tilde{k}) + K_2(k) p_2(k) \right) = \left(\delta - \left(1 - \frac{A}{2} \right) \right) k + B.$$

Note that the left hand side of the identity must be nondecreasing in k, so if $\delta < 1 - \frac{A}{2}$, then there cannot be mixing on (a,b) in the first period. If $\delta \geq 1 - \frac{A}{2}$, then the left hand side is differentiable, so the right hand side must be differentiable, so that

$$K_2(k) = \frac{\delta - \left(1 - \frac{A}{2}\right)}{\delta \frac{\partial}{\partial k} p_2(k)}$$

on $k \in (a,b)$. Since K_2 is a cdf, $\frac{\partial}{\partial k} p_2(k)$ cannot be increasing on (a,b), so that by Lemma 4 $\frac{\partial}{\partial k} p_2(k)$ must be constant on (a,b). This implies that the support of K_2 is disjoint from (a,b) and because $\frac{\partial}{\partial k} p_2(k)$ must be constant on (a,b).

□

LEMMA 6: *With private offers and if $\delta > \delta^*$, all mixed-strategy equilibria must satisfy the following properties.*
(i) In period 2, buyers mix between at most countably many prices.
(ii) If buyers in period 1 mix continuously between prices that result in cutoffs in an interval (a,b), then buyers in periods 2 never choose a price that results in a cutoff in (a,b).

Proof of Lemma 6. Assume there exists and interval (a, b) such that buyers in period 2 mix between all cutoffs, i.e., for all $k \in (a, b)$

$$\pi_2(k; K_1) = \int_0^k \int_0^c \frac{1}{1-\tilde{k}} dK_1(\tilde{k}) \left(Ac + B - p_t(k)\right) dc = 0.$$

After applying integration by parts and setting

$$H(k) \equiv \int_0^k \left(\int_0^c \int_0^x \frac{1}{1-\tilde{k}} dK_1(\tilde{k}) dx \right) dc,$$

one can see that this is equivalent to the ordinal differential equation

$$AH'(k)k - AH(k) = H'(k) \left(p_2(k) - B\right).$$

Thus, we can conclude that

$$H(k) \equiv \int_0^k \left(\int_0^c \int_0^x \frac{1}{1-\tilde{k}} dK_1(\tilde{k}) dx \right) dc \quad = \quad const \cdot \exp\left(\int_0^k \frac{1}{z - \frac{p_2(z)-B}{A}} dz \right)$$

and by Fubini's Theorem $H(k) = \int_0^k \frac{k-\tilde{k}}{2(1-\tilde{k})} dK_1(\tilde{k})$ which is increasing because $\frac{k-\tilde{k}}{2(1-\tilde{k})} > 0$ for $0 < \tilde{k} < k$. Thus, the cdf $K_1(\cdot)$ must be strictly increasing everywhere on (a, b). Hence, if buyers mix on (a, b) in period 2, then they must mix in period 1 which is a contradiction by Lemma 5. Hence, there cannot be mixing on an interval in period 2.

□

Now, the proof of Proposition 3 follows in three steps. First, we show in step 1 that buyers in period 2 mix between exactly two prices and we show the first part of (ii). Step 2 discusses the second part of (ii), i.e. that there must be non-offers with positive probability in period 1. Finally, in step 3 we can pin down the exact values of $\underline{k}_2$ and $\overline{k}_2$.

Step 1: In period 2*, buyers mix between exactly two prices resulting in cutoffs* $\underline{k}_2, \overline{k}_2$ *and period* 1 *cutoffs must be in* $\{0\} \cup [\underline{k}_2, \overline{k}_2]$.

First, note that buyers in both periods must mix between at least two cutoffs. The reason is that if buyers in period 1 would play pure strategies, then there is a unique price at which period 2 buyers make zero profits, i.e. the unique Bertrand equilibrium in that period contains only pure strategies of the buyers. If period 2 buyers played pure strategies in equilibrium, then the same argument holds for expected profits in period 1. Since we have already established in Theorem 2 that if $\delta > 1 - \frac{A}{2}$ there cannot be pure-strategy equilibria, there must be mixing in both periods.

Let us first consider the continuation game in period 2 given beliefs about the current cutoffs represented by the cdf K_1. Buyers' profits are then given by

$$\pi_2(k_2; K_1) = \int_0^{k_2} \left(\int_0^c \frac{1}{1-\tilde{k}} dK_1(\tilde{k}) \right) (Ac + (1-\delta)B - k_2(\delta A + 1 - \delta)) dc.$$

π_2 is continuous and at the smallest element $k_1 < 1$ in the support of K_1, for all ϵ small enough we have

$$\begin{aligned}
\frac{\partial \pi_2}{\partial k_2}(k_2; K_1)\big|_{k_2 = k_1+\epsilon} &= \int_0^{k_1+\epsilon} \frac{1}{1-\tilde{k}} dK_1(\tilde{k})(1-\delta)\left(B + (k_1+\epsilon)(A-1)\right) \\
&\quad - \int_{k_1}^{k_1+\epsilon} \left(\int_0^c \frac{1}{1-\tilde{k}} dK_0(\tilde{k}) \right) dc (A\delta + 1 - \delta) \\
&> \int_0^{k_1+\epsilon} \frac{1}{1-\tilde{k}} dK_1(\tilde{k}) \cdot \\
&\quad \left[(1-\delta) \left(B + k_1 \underbrace{(A-1)}_{=-B} \right) - \epsilon(2(A\delta + 1 - \delta) - A) \right] \\
&> 0,
\end{aligned}$$

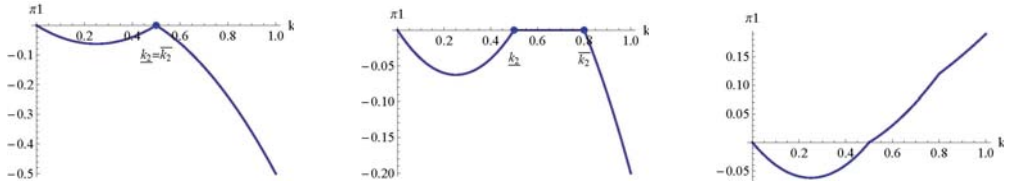
so in equilibrium, buyers in period 2 do not choose prices that result in a cutoff type smaller or equal to k_1 with positive probability since if they did increasing the price a little bit would be a profitable deviation for any buyer. In particular, in any equilibrium, seller types close to zero trade in period 2, so that $K_2(k_2) = 0$ for small k_2.

By Proposition 6, the support of K_2 is discrete and $p_1(\cdot)$ is piecewise linear, continuous and by Lemma 4, it is also weakly convex. Hence, buyers' expected profit in period 1

$$\pi_1(k; 0) = k \cdot \left(\frac{A}{2} k + B - \underbrace{\left(\delta \left[\left(\int_k^0 p_2(\tilde{k}) dK_2(\tilde{k}) \right) + K_2(k) p_2(k) \right] + k(1-\delta) \right)}_{p_1(k)} \right).$$

is continuous, piecewise quadratic and at any cutoff in the support of K_2 it has a "downward" kink (that is the slope is dropping discontinuously) because of the convexity of p_1. Hence, in equilibrium, expected period 1 profits must qualitatively look like one of the graphs in figure B1. Note that for small k, $p_1(k) = \delta \int_k^1 p_2(\tilde{k}) dK_2(\tilde{k}) + k(1-\delta)$ because $K_2(k) = 0$ for small k. Hence, the parabola most to the left must be open above because $\frac{A}{2} - (1-\delta) > 0$. We have already argued that buyers must mix between at least two prices in every period, so we can exclude the possibility of the expected profit function in period

1 having a shape as in figure B1 (c). Hence, there exist cutoffs $0 < \underline{k}_2 \leq \overline{k}_2 < 1$ such that period 1 buyers choose only prices with positive probability that are in $\{0\} \cup [\underline{k}_2, \overline{k}_2]$.



(a) Mixing between two cutoffs (b) Mixing on an interval (c) No mixing

FIGURE B1. POSSIBLE SHAPES OF BUYERS' PROFITS IN PERIOD 1

Using these insights about π_1, we can conclude that $\pi_2(\cdot; k_1)$ is piecewise quadratic on $[0,1] \setminus [\underline{k}_2, \overline{k}_2]$ where the coefficient in front of the quadratic term is negative as a multiple of $\frac{A}{2} - (1-\delta) - \delta A < 0$. Hence, all pieces of π_2 are open below. At every cutoff that is chosen with positive probability in period 1, π_2 has a kink. Hence, period 2 expected profits are qualitatively as in figure B2. Note however, that π_2 does not have to be piecewise quadratic in $[\underline{k}_2, \overline{k}_2]$ as in figure B2.

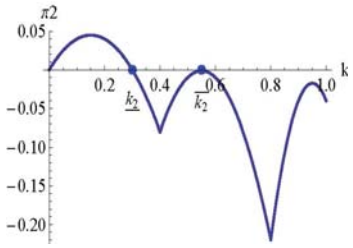


FIGURE B2. QUALITATIVE SHAPES OF BUYERS' EXPECTED PROFITS IN PERIOD 2

Next, we argue that π_1 must look like in figure B1 (b). Let us first assume that none of the pieces of π_2 is constant and equal to zero as is the case in figure B1 (a). Then, in a mixed-strategy equilibrium, buyers in period 1 mix between exactly two prices that result in cutoff types 0 and $k_1 = \underline{k}_2 = \overline{k}_2$, respectively. Moreover, k_1 must be a cutoff type in period 2, because it corresponds to a kink of π_1. We can conclude $\pi_1(\underline{k}_2) = \pi_2(\overline{k}_2) = 0$ and $\pi_2(k) \leq 0$ for all $k \geq \overline{k}_2$. In addition, π_2 has its only kink at $\overline{k}_2$, so buyers do not mix between prices in period 2, but choose a price with probability one that results in a cutoff $\overline{k}_2$. This cannot be an equilibrium as argued before. Hence, there cannot be an equilibrium where none of the pieces of π_1 is constant and equal to zero.

Finally, period 2 buyers must mix between exactly two cutoffs $\{\underline{k}_2, \overline{k}_2\}$. This can be seen as follows: One can infer directly from Lemma 6 (ii) that period 2 buyers do not choose prices that result in cutoffs in $(\underline{k}_2, \overline{k}_2)$. Moreover, because $\pi_1(k;0) = 0$ on $\{0\} \cup [\underline{k}_2, \overline{k}_2]$ only, π_2 can have kinks in that region only. Hence, $\pi_2(\underline{k}_2) = \pi_2(\overline{k}_2) = 0$, $\pi_2(k; k_1) \leq 0$ for $k \geq \underline{k}_2$ and the fact that π_2 is piecewise quadratic on $[0, \underline{k}_2] \cup [\overline{k}_2, 1]$ with parabolas that are open below imply that $\pi_2(k) > 0$ for $k \in (0, \underline{k}_2)$ and $\pi_2(k; k_1) < 0$ for $k \in (\overline{k}_2, 1]$.

Thus, in any equilibrium the support of K_1 is a subset of $\{0\} \cup [\underline{k}_2, \overline{k}_2]$ and the support of K_2 is $\{\underline{k}_2, \overline{k}_2\}$ for some $\underline{k}_2, \overline{k}_2 \in (0, 1]$. Let $K_2(\underline{k}_2) = q_2$ and $K_1(0) = q_1$, noting that we already know from Lemma 5 that $q_1 = \frac{\delta - 1 + \frac{A}{2}}{\delta(1-\delta+\delta A)} \neq 0$.

Step 2: In any mixed-strategy equilibrium, there must be non-offers with positive probability in period 1, i.e. $q_1 > 0$.

Let us assume $q_1 = 0$ and let us denote the smallest element in the support of K_1 by $\underline{k} < 1$. Note that $Ak + (1-\delta)B - k(\delta A + (1-\delta)) = (1-\delta)(k(A-1)+B) \geq (1-\delta)B(1-k)$ which is strictly positive for $B > 0$ and $k < 1$. Hence, there exists an $\epsilon > 0$ such that $A\underline{k} + (1-\delta)B - (\underline{k} + \epsilon)(\delta A + (1-\delta)) > 0$. Then, $\pi_2(\underline{k} + \epsilon) > 0$ which is a contradiction to $\underline{k}_2 < \underline{k}$ being in the support of K_2.

Step 3: $\underline{k}_2 = \frac{B(1-\delta)}{A\delta - \delta + 1 - \frac{A}{2}}$ and $\overline{k}_2 = \frac{B(1-\delta^2)}{A\delta^2 - \delta^2 + 1 - \frac{A}{2}}$

In equilibrium, it must hold that $\pi_2(\underline{k}_2; k_1) = 0$, that is

$$\int_0^{\underline{k}_2} Ac + (1-\delta)B - \underline{k}_2(\delta A + 1 - \delta)dc =$$
$$\underline{k}_2 \cdot \left(\frac{A}{2}\underline{k}_2 + (1-\delta)B - \underline{k}_2(\delta A + 1 - \delta) \right) = 0$$

which is equivalent to $\underline{k}_2 = \frac{B(1-\delta)}{A\delta - \delta + 1 - \frac{A}{2}}$. For $\overline{k}_2$, we use that $\pi_1(\overline{k}_2; 0) = 0$ since this is equivalent to

$$\overline{k}_2 \cdot \left(\frac{A}{2}\overline{k}_2 + (1-\delta^2)B - \delta(\delta(A-1)+1)\overline{k}_2 - (1-\delta)\overline{k}_2 \right) = 0$$

because $K_2(\overline{k}_2) = 1$. Hence, $\overline{k}_2 = \frac{B(1-\delta^2)}{A\delta^2 - \delta^2 + 1 - \frac{A}{2}}$.

Using the insights from Proposition 3, we can construct an equilibrium in which buyers in period 1 mix between exactly two cutoffs which completes the proof of Theorem 2.

If period 1 buyers mix between exactly two cutoffs 0 and $\overline{k}_1$ with $K_1(0) \equiv q_1$ and $K_1(\overline{k}_1) = 1 - q_1$ for some $q_1 \in (0, 1)$. Then the expected profit in period 2 is

given by

$$\pi_2(k;k_1)=\begin{cases} q_1\cdot k\cdot\left(\frac{A}{2}k+(1-\delta)B-k\left(1-\delta+\delta A\right)\right) & \text{if } k<\overline{k_1}\\ (q_1-1)\,\frac{\overline{k_1}}{1-\overline{k_1}}\left(\frac{A}{2}\overline{k_1}+(1-\delta)B-k\left(1-\delta+\delta A\right)\right) & \\ +k\left(\frac{A}{2}k+(1-\delta)B-k\left(1-\delta+\delta A\right)\right)\cdot & \text{if } k>\overline{k_1}.\\ \left(q_1+\frac{1}{1-\overline{k_1}}(1-q_1)\right) & \end{cases}$$

Note that both parts are quadratic in k and that π_2 is continuous everywhere. Moreover, in both parts the coefficient in front of k^2 is negative. The first part is equal to zero if $k\in\left\{0,\frac{B(1-\delta)}{1-\delta+\delta A-\frac{A}{2}}\right\}$ and the second part must only have one zero in equilibrium, i.e., it must hold that the discriminant is zero.

$$\left[(1-\delta)B\left(q_1+\tfrac{1}{1-\overline{k_1}}(1-q_1)\right)-(q_1-1)\,\tfrac{\overline{k_1}}{1-\overline{k_1}}\left(1-\delta+\delta A\right)\right]^2=$$
$$(2A-4+4\delta(1-A))\left(q_1+\tfrac{1}{1-\overline{k_1}}(1-q_1)\right)(q_1-1)\,\tfrac{\overline{k_1}}{1-\overline{k_1}}\left(\tfrac{A}{2}\overline{k_1}+(1-\delta)B\right)$$

and the null must be at

$$\overline{k_2}=-\frac{(1-\delta)B\left(r+\frac{1}{1-\overline{k_1}}(1-q_1)\right)-(q_1-1)\,\frac{\overline{k_1}}{1-\overline{k_1}}\left(1-\delta+\delta A\right)}{(A-2+2\delta(1-A))\left(q_1+\frac{1}{1-\overline{k_1}}(1-q_1)\right)}.$$

We can solve these two equations for $\overline{k}_1, q_1$ and get

$$\overline{k}_1=\frac{2B(1-\delta)(-2\delta^3+2\delta^3A+4\delta^2A-2\delta^2+2\delta-A+2)}{(-2\delta^2+2\delta^2A+2\delta A-A+2)(-2\delta^2+2\delta^2A+2-A)}$$

such that $0\leq\underline{k}_2<\overline{k}_1<\overline{k}_2\leq 1$.

Figure B3 illustrates for $v(c)=\frac{1+c}{2}$ the expected profit functions π_1 and π_2 in the equilibrium. It highlights how cutoffs in period 1 must correspond to kinks of π_2 and cutoffs in the second period must correspond to kinks of π_1. Other equilibria, in which period 1 buyers 1 mix between $\{0\}$ and several prices in $(\underline{k}_2,\overline{k}_2)$, can coexist.

□

Proof of Theorem 4.

We first show that the expected cutoffs in period 1 are constant across all mixed-strategy equilibria with private offers and equal to

$$\int_0^{\overline{k}_2}\tilde{k}dK_1(\tilde{k})=\frac{(1-\overline{k}_2)\left(1-\frac{\delta}{1+\delta}\overline{k}_2\right)}{1-\overline{k}_2\cdot\frac{(1+\delta)(1-\delta+A\delta)-\frac{A}{2}}{(1+\delta)(1-\delta+A\delta)}}+\frac{1+2\delta}{1+\delta}\overline{k_2}-1. \tag{B1}$$

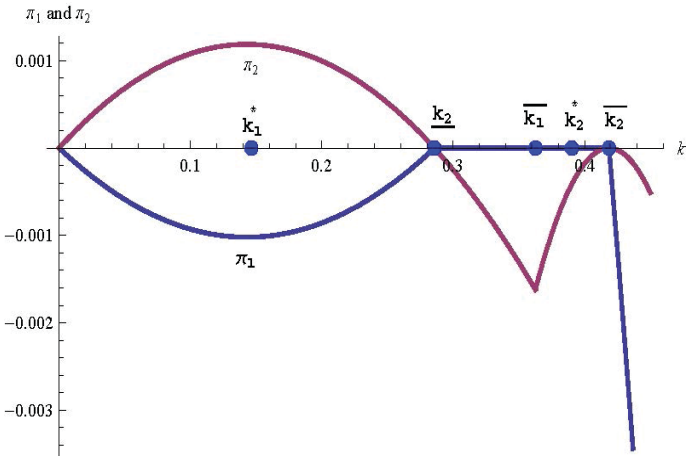


FIGURE B3. BUYER'S PROFIT IN PERIOD 1 AND 2 ON EQUILIBRIUM PATH

Moreover, the following must hold

$$\int_0^{\overline{k}_2} \frac{1}{1-\tilde{k}} dK_1(\tilde{k}) = \frac{1}{1-\overline{k}_2 \cdot \frac{(1+\delta)(1-\delta+A\delta)-\frac{A}{2}}{(1+\delta)(1-\delta+A\delta)}}. \tag{B2}$$

To prove this, note that in any equilibrium it must hold that $\pi_2(\overline{k}_2; K_1) = 0$ and for all $d > \overline{k}_2$, $\pi_2(k; K_1) \leq 0$, i.e.,

$$\int_0^{\overline{k}_2} \int_0^c \frac{1}{1-\tilde{k}} dK_1(\tilde{k})(Ac + B - ((1-\delta+A\delta)\overline{k}_2 + \delta B))dc = 0$$
$$\int_0^d \int_0^c \frac{1}{1-\tilde{k}} dK_1(\tilde{k})(Ac + B - ((1-\delta+A\delta)d + \delta B))dc \leq 0 \ \ \forall\, d > \overline{k}_2.$$

Let us first simplify the first equality. By applying Fubini's Theorem and then, noting that $\frac{\overline{k}_2-k}{1-k} = 1 + \frac{\overline{k}_2-1}{1-k}$ and $\frac{\overline{k}_2^2-k^2}{1-k} = 1 + k + \frac{\overline{k}_2^2-1}{1-k}$, we can deduce

$$\begin{aligned}
&\int_0^{\overline{k}_2}\int_0^c \frac{1}{1-\tilde{k}} dK_1(\tilde{k})(Ac + B - ((1-\delta+A\delta)\overline{k}_2 + \delta B))dc \\
= \;& \frac{A}{2}\int_0^{\overline{k}_2} \frac{\overline{k}_2^2 - \tilde{k}^2}{1-\tilde{k}} dK_1(\tilde{k}) + ((1-\delta)B - (1-\delta+A\delta)\overline{k}_2)\int_0^{\overline{k}_2} \frac{\overline{k}_2 - \tilde{k}}{1-\tilde{k}} dK_1(\tilde{k}) \\
= \;& \frac{A}{2} + (1-\delta)B - (1-\delta+A\delta)\overline{k}_2 + \frac{A}{2}\int_0^{\overline{k}_2} \tilde{k} dK_1(\tilde{k}) \\
& + \int_0^{\overline{k}_2} \frac{1}{1-\tilde{k}} dK_1(\tilde{k}) \left(\left(\overline{k}_2^2 - 1\right)\frac{A}{2} + (\overline{k}_2 - 1)((1-\delta)B - (1-\delta+A\delta)\overline{k}_2) \right) \\
= \;& \frac{A}{2} \cdot \left(1 - \frac{1+2\delta}{1+\delta}\overline{k}_2 + \int_0^{\overline{k}_2} \tilde{k} dK_1(\tilde{k}) + (\overline{k}_2 - 1)\left(1 - \frac{\delta}{1+\delta}\overline{k}_2\right)\int_0^{\overline{k}_2} \frac{1}{1-\tilde{k}} dK_1(\tilde{k}) \right).
\end{aligned}$$

Thus, in equilibrium, the following must hold

$$\text{(B3)}\quad 1 - \frac{1+2\delta}{1+\delta}\overline{k}_2 + \int_0^{\overline{k}_2} \tilde{k} dK_1(\tilde{k}) = (1-\overline{k}_2)\left(1 - \frac{\delta}{1+\delta}\overline{k}_2\right)\int_0^{\overline{k}_2} \frac{1}{1-\tilde{k}} dK_1(\tilde{k}).$$

In order to simplify the second inequality, we can use that $\pi_2(\overline{k}_2; k_1) = 0$, and see that for $d > \overline{k}_2$

$$\begin{aligned}
&\int_0^{d}\int_0^c \frac{1}{1-\tilde{k}} dK_1(\tilde{k})(Ac + B(1-\delta) - (1-\delta+A\delta)d)dc \\
= \;& \int_0^{\overline{k}_2}\int_0^c \frac{1}{1-\tilde{k}} dK_1(\tilde{k})dc(\overline{k}_2 - d)(1-\delta+A\delta) \\
& + \int_{\overline{k}_2}^{d}\int_0^{\overline{k}_2} \frac{1}{1-\tilde{k}} dK_1(\tilde{k})(Ac + B(1-\delta) - (1-\delta+A\delta)d)dc \\
= \;& (d - \overline{k}_2)\left[\int_0^{\overline{k}_2} \frac{1}{1-\tilde{k}} dK_1(\tilde{k}) \left(\frac{A}{2}\overline{k}_2 + B(1-\delta) - \underbrace{\left(1 - \delta + A\delta - \frac{A}{2}\right)}_{>0} d \right) \right. \\
& \left. - \int_0^{\overline{k}_2}\int_0^c \frac{1}{1-\tilde{k}} dK_1(\tilde{k})dc(1-\delta+A\delta) \right]
\end{aligned}$$

is quadratic in d and the parabola is open below. The parabola has a zero at $\overline{k}_2$ and we will show in the following that it cannot have another zero. If $\pi_2(k'; k_1) = 0$ for a $k' > \overline{k}_2$, then π_2 is positive on $(\overline{k}_2, k')$ which cannot hold in equilibrium.

If the parabola (if it was extended to values smaller than $\overline{k}_2$) has a zero at a $k' < \overline{k}_2$ and if the support of K_1 does not contain $(\overline{k}_2 - \epsilon, \overline{k}_2)$ for a $\epsilon > 0$, then $\pi_2(k; k_1) > 0$ for $k \in (\overline{k}_2 - \epsilon, \overline{k}_2)$ which leads to a contradiction. Finally, if there is continuous mixing on some $(\overline{k}_2 - \epsilon, \overline{k}_2)$, then since the slope from the right of π_2 is negative at π_2, the slope from the left must also be negative because

$$\begin{aligned}
\frac{\partial}{\partial k_2}\pi_2(k_2) &= \frac{\partial}{\partial k_2}\int_0^{k_2}\int_0^c \frac{1}{1-\tilde{k}}dK_1(\tilde{k})(Ac-(1-\delta+A\delta)k_2+B(1-\delta))dc \\
&= \int_0^{k_2}\frac{1}{1-\tilde{k}}dK_1(\tilde{k})(1-\delta)(Ak_2-k_2+B) \\
&\quad -(1-\delta+A\delta)\int_0^{k_2}\int_0^c\frac{1}{1-\tilde{k}}dK_1(\tilde{k})dc
\end{aligned}$$

and $k_2(A-1)+B > 0$. This again cannot hold in equilibrium. As a result, the parabola can only have one zero $\overline{k}_2$ and it follows from by plugging in the value of $\overline{k}_2$ calculated in Proposition 3 that

$$\begin{aligned}
&\frac{\int_0^{\overline{k}_2}\frac{1}{1-\tilde{k}}dK_1(\tilde{k})\left(\frac{A}{2}\overline{k}_2+B(1-\delta)\right)-\int_0^{\overline{k}_2}\int_0^c\frac{1}{1-\tilde{k}}dK_1(\tilde{k})dc(1-\delta+A\delta)}{\int_0^{\overline{k}_2}\frac{1}{1-\tilde{k}}dK_1(\tilde{k})(1-\delta+A\delta-\frac{A}{2})} = \overline{k}_2 \\
\Leftrightarrow\quad &\frac{A}{2}\overline{k}_2+B(1-\delta)-\frac{\int_0^{\overline{k}_2}\int_0^c\frac{1}{1-\tilde{k}}dK_1(\tilde{k})dc(1-\delta+A\delta)}{\int_0^{\overline{k}_2}\frac{1}{1-\tilde{k}}dK_1(\tilde{k})} \\
&= \frac{B(1-\delta^2)\left(1-\delta-\frac{A}{2}+A\delta\right)}{\left(1-\delta^2-\frac{A}{2}+A\delta^2\right)} \\
\Leftrightarrow\quad &(1-\delta)B\cdot\frac{\frac{A}{2}}{\left(1-\delta^2-\frac{A}{2}+A\delta^2\right)(1-\delta+A\delta)} = \frac{\int_0^{\overline{k}_2}\int_0^c\frac{1}{1-\tilde{k}}dK_1(\tilde{k})dc}{\int_0^{\overline{k}_2}\frac{1}{1-\tilde{k}}dK_1(\tilde{k})} \\
\Leftrightarrow\quad &\overline{k}_2\cdot\frac{\frac{A}{2}}{1-\delta^2+A\delta+A\delta^2} = \frac{1+(\overline{k}_2-1)\int_0^{\overline{k}_2}\frac{1}{1-\tilde{k}}dK_1(\tilde{k})}{\int_0^{\overline{k}_2}\frac{1}{1-\tilde{k}}dK_1(\tilde{k})} \\
\Leftrightarrow\quad &\int_0^{\overline{k}_2}\frac{1}{1-\tilde{k}}dK_1(\tilde{k}) = \frac{1}{\frac{\frac{A}{2}\overline{k}_2}{1-\delta^2+A\delta+A\delta^2}-\overline{k}_2+1}
\end{aligned}$$

This proves (B2). Plugging (B2) into (B3), shows (B1).

We can now easily calculate the difference between the expected period 1 cutoff with private offers and the period 1 cutoff with public offer using (10), (11),

Proposition 4 and Proposition 3 and see that it is positive:

$$\int_0^{\overline{k}_2} \tilde{k} dK_1(\tilde{k}) - k_1^* =$$
$$\frac{(1-\frac{B(1-\delta^2)}{A\delta^2-\delta^2+1-\frac{A}{2}})\left(1-\frac{\delta}{1+\delta}\frac{B(1-\delta^2)}{A\delta^2-\delta^2+1-\frac{A}{2}}\right)}{1-\overline{k}_2\cdot\frac{(1+\delta)(1-\delta+A\delta)-\frac{A}{2}}{(1+\delta)(1-\delta+A\delta)}}+\frac{1+2\delta}{1+\delta}\frac{B(1-\delta^2)}{A\delta^2-\delta^2+1-\frac{A}{2}}-1$$
$$-\frac{2B\cdot(A\delta-2\delta+2-A)\cdot(1-\delta)}{2(1-\delta)(1-A)(A\delta-2\delta+2)+A^2}=$$
$$\frac{(1-A)(1-\delta)(-4+4A-A^2+8\delta-6A\delta+2A^2\delta-4\delta^2+2A\delta^2+2A^2\delta^2)}{(2-A-2\delta^2+2A\delta^2)(4-4A+A^2-8\delta+10A\delta-2A^2\delta+4\delta^2-6A\delta^2+2A^2\delta^2)}=$$
$$\frac{1}{2}\overline{k}_\Delta\cdot\frac{2A(1-\delta)-(A-1)^2(1-\delta)^2-3(1-\delta)^2+\delta^2A}{(4-4A+A^2)(1-2\delta+\delta^2)+2A\delta-2A\delta^2+A^2\delta^2}\geq$$
$$\frac{1}{2}\overline{k}_\Delta\cdot\frac{(1-\delta)^2-(A-1)^2(1-\delta)^2+\delta^2A}{(4-4A+A^2)(1-2\delta+\delta^2)+2A\delta-2A\delta^2+A^2\delta^2}>0.$$

for $\delta > 1-\frac{A}{2}$ and $A+B=1$.

□

Proofs: Robustness and Generalizations

Proof of Theorem 5.

(i): We show that, if all buyers choose pricing strategies that result in a cutoff seller $\kappa_t(k_{t-\Delta})$ (defined below) given they believe the current cutoff is $k_{t-\Delta}$, this constitutes an equilibrium. To this end, define $\kappa_t(\cdot)$ inductively for $t = 0, \Delta, \ldots, 1-\Delta$ as follows. First, using $p_{1-\Delta}(k) = \alpha\delta v(k) + (1-\alpha\delta)k$, it follows that

$$\text{(C1)} \qquad \begin{array}{l}\kappa_{1-\Delta}(k_{1-2\Delta}) = \\ \sup\left\{k\in[k_{1-2\Delta},1]\middle|\frac{1}{1-F(k_{1-2\Delta})}\int_{k_{1-2\Delta}}^{k}\left(v(c)-p_{1-\Delta}(k)\right)f(c)dc>0\right\}.\end{array}$$

is left-continuous (we define $\kappa_{1-\Delta}(k_{1-2\Delta}) = k_{1-2\Delta}$ if the set over which we compute the sup is empty). Then,

$$p_{1-2\Delta}(k) = \delta p_{1-\Delta}(c_{1-\Delta}(k_{1-2\Delta})) + (1-\delta)k$$

is left-continuous.

Next, we show that given left-continuous $\kappa_{t+\Delta}(k)$, it follows that for $t < 1-\Delta$

and $p_t(k) = \delta p_{t+\Delta}(\kappa_{t+\Delta}(k)) + (1-\delta)k$,

$$\kappa_t(k_{t-\Delta}) = \sup\left\{k \in [k_{T-2\Delta}, 1] \middle| \frac{1}{1-F(k_{T-2\Delta})} \int_{k_{T-2\Delta}}^{k} (v(c) - p_t(k))\, f(c)dc > 0\right\} \tag{C2}$$

(with $\sup \emptyset = k_{t-\Delta}$) is left-continuous.

Step 1: If $\pi_t(k; k_{t-\Delta})$ is left-continuous in k, then κ_t is increasing
Because of left-continuity of $\pi_t(\cdot; k_{t-\Delta})$, we either have $\pi_t(\kappa_t(k_{t-\Delta}), k_{t-\Delta}) > 0$ or $\pi_t(\kappa_t(k_{t-\Delta}), k_{t-\Delta}) = 0$. Moreover, note that $\pi_t(k; k_{t-\Delta})$ is always differentiable in $k_{t-\Delta}$. Let us consider an arbitrary $k_{t-\Delta}$ and an infinitesimal increase in $k_{t-\Delta}$. If $\pi_t(\kappa_t(k_{t-\Delta}), k_{t-\Delta}) > 0$, there exists an $\epsilon > 0$ so that $\pi_t(\kappa_t(k_{t-\Delta}), k_{t-\Delta} + \gamma) > 0$ for all $\gamma < \epsilon$. Hence, $\kappa_t(k_{t-\Delta} + \gamma) > \kappa_t(k_{t-\Delta})$ for all $\gamma < \epsilon$. On the other hand, if $\pi_t(\kappa_t(k_{t-\Delta}), k_{t-\Delta}) = 0$, then

$$\begin{aligned}
& \frac{\partial}{\partial k_{t-\Delta}} \pi_t(k; k_{t-\Delta})\big|_{k=\kappa_t(k_{t-\Delta})} \\
= \; & \frac{f(k_{t-\Delta})}{1-F(k_{t-\Delta})} \cdot \\
& \left[\frac{1}{1-F(k_{t-\Delta})} \int_{k_{t-\Delta}}^{\kappa_t(k_{t-\Delta})} (v(c) - p_t(\kappa_t(k_{t-\Delta})))f(c)dc - (v(k_{t-\Delta}) - p_t(c_t^*(k_{t-\Delta})))\right] \\
= \; & -\frac{f(k_{t-\Delta})}{1-F(k_{t-\Delta})}(v(k_{t-\Delta}) - p_t(\kappa_t(k_{t-\Delta}))) > 0.
\end{aligned}$$

This is the case because if we had $v(k_{t-\Delta}) - p_t(\kappa_t(k_{t-\Delta})) \geq 0$, then $\int_{k_{t-\Delta}}^{\kappa_t(k_{t-\Delta})} (v(c) - p_t(\kappa_t(k_{t-\Delta})))f(c)dc > 0$ v being increasing. This is a contradiction to the zero-profit assumption $\pi_t(\kappa_t(k_{t-\Delta}), k_{t-\Delta}) = 0$. Hence, $\kappa_t(\cdot)$ is increasing at $k_{t-\Delta}$.

Step 2: $\kappa_t(\cdot)$, $p_t(\cdot)$ and $\pi_t(\cdot; k_{t-\Delta})$ are left-continuous
We argue by backward induction in t. $p_{1-\Delta}(\cdot)$ is left-continuous because v is continuous and hence, $\pi_{1-\Delta}(k_{1-2\Delta}; k)$ is left-continuous in k. (It is even continuous.) Let $k_{1-2\Delta}^{(n)} \uparrow k_{1-2\Delta}$. Then, $\kappa_{1-\Delta}^*(k_{1-2\Delta}^{(n)}) \leq \kappa_{1-\Delta}(k_{1-2\Delta})$ for all n and $\kappa_{1-\Delta}(k_{1-2\Delta}^{(n)})$ is an increasing sequence by step 1. Hence, $\lim_{n\to\infty} \kappa_{1-\Delta}^*(k_{1-2\Delta}^{(n)})$ exists. We will show next that $\lim_{n\to\infty} \kappa_{1-\Delta}(k_{1-2\Delta}^{(n)}) = \kappa_{1-\Delta}(k_{1-2\Delta})$. Therefore, consider an arbitrary sequence $k^{(m)} \uparrow \kappa_{1-\Delta}(k_{1-2\Delta})$ such that $\pi_{1-\Delta}(k^{(m)}; k_{1-2\Delta}) > 0$ (which must exist by definition of $\kappa_{1-\Delta}$). Then, for any m, there exists an $n(m)$ such that $\pi_{1-\Delta}(k^{(m)}; k_{1-2\Delta}^{(n)}) > 0$ for all $n \geq n(m)$ because $\pi_{1-\Delta}(k; \cdot)$ is continuous for all k. Hence, $k^{(m)} \leq \kappa_{1-\Delta}(k_{1-2\Delta}^{(n(m))}) \leq \kappa_{1-\Delta}(k_{1-2\Delta}) = \lim_{m\to\infty} k^{(m)}$. Hence, $\lim_{n\to\infty} \kappa_{1-\Delta}(k_{1-2\Delta}^{(n)}) = \lim_{m\to\infty} \kappa_{1-\Delta}(k_{1-2\Delta}^{(n(m))}) = \kappa_{1-\Delta}(k_{1-2\Delta})$ and thus, $\kappa_{1-\Delta}(\cdot)$, $p_{1-2\Delta}(\cdot)$ and $\pi_{1-2\Delta}(\cdot; k_{1-3\Delta})$ are left-continuous.

Let now assume that $c_{t+\Delta}(\cdot)$, $p_t(\cdot)$ and $\pi_t(\cdot; k_{t-\Delta})$ are left-continuous. Hence, $\kappa_t(\cdot)$ is increasing by step 1. The rest of the argument works analogously to above,

so that $\kappa_t(\cdot)$, $p_{t-\Delta}(\cdot)$ and $\pi_{t-\Delta}(\cdot; k_{t-2\Delta})$ are left-continuous for all t.

Hence, buyers do not make negative expected profits because

$$k \mapsto \frac{1}{1-F(k_{t-\Delta})} \int_{k_{t-\Delta}}^{k} \left(v(c) - \left(\delta p_{t+1}(\kappa_{t+\Delta}(k)) + (1-\delta)k\right)\right) f(c)dc$$

is left-continuous. The equilibrium cutoffs $(k_0^*, \ldots, k_{1-\Delta}^*)$ are then, given by $k_0^* = \kappa_0(0), \ldots, k_{1-\Delta} = \kappa_{1-\Delta}(\kappa_{1-2\Delta}(\ldots \kappa_0(0)))$. None of the buyers has an incentive to deviate from this equilibrium, since by increasing the price offer, buyers will either make zero or negative expected profits by definition of $\kappa_t(\cdot)$ and by decreasing the price they will not receive the good and make zero expected profits. Note that for some $v(c)$ and F_1 there could be multiple equilibria because there can be several prices that result in zero expected profits for the buyers.

(ii) and (iii) are proven in the main part of the paper.

□

Proof of Proposition 4.

Let $B = 0$ and $A < 2$. The zero-profit condition implies that the cutoff at time 0 must satisfy

$$p_0 = v\left(\frac{k_0}{2}\right) = \frac{A}{2}k_0.$$

Moreover, it must hold that

$$p_0 \geq (1-\delta)k_0 + \delta p_1 \geq k_0.$$

However, this can never hold simultaneously for $A < 2$ except if $k_0 = 0$. Hence, in the unique pure-strategy equilibrium (with private and public offers), there is no trade before the deadline.

□

Proof of Proposition 5.

Consider a situation with $v_H \geq c_H$ and $v_L > c_L = 0$ where the seller's valuation is v_L with probability ϕ. The static lemon's condition (LC) is satisfied if

$$\phi v_L + (1-\phi)v_H < c_H.$$

Finally, denote the fraction of v_L-sellers such that the lemons condition is just satisfied by ϕ^*, i.e.,

$$\phi^* v_L + (1-\phi^*)v_H = c_H.$$

We solve the game by backward induction. Given the belief ϕ_2 about the fraction of v_L-sellers in the market, buyers' expected period 2 profits are given by

$$\pi_2(p) = \begin{cases} \phi_2 v_L + (1-\phi_2)v_H - p & \text{if } p \geq c_H \\ v_L - p & \text{if } c_L < p < c_H \\ 0 & \text{otherwise} \end{cases}$$

if they sell at a price p. Since buyers compete in a Bertrand fashion, the equilibrium price is

$$p_2^*(\phi_2) = \begin{cases} \phi_2 v_L + (1-\phi_2)v_H & \text{if } \phi_2 < \phi^* \\ \{\phi_2 v_L + (1-\phi_2)v_H, v_L\} & \text{if } \phi_2 = \phi^* \\ v_L & \text{if } \phi_2 > \phi^* \end{cases}.$$

If (LC) is satisfied with ϕ_2, only low types trade and $p_2^* = v_L$.

The price in the continuation equilibrium is as in figure C1. If (LC) is satisfied,

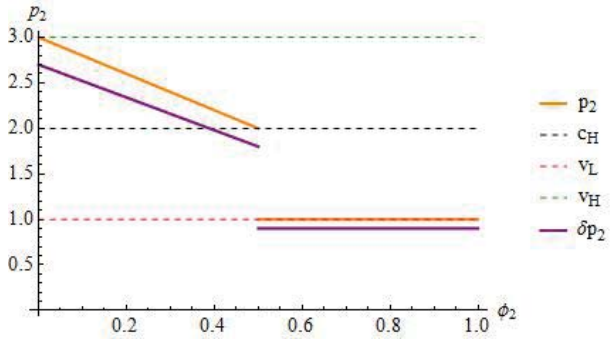


FIGURE C1. PERIOD 2 PRICE

then the period 1 price is always $p_1 = v_L$. Moreover, the following holds:

1) If $\delta \leq \frac{v_L}{v_H}$, then all v_L-sellers trade in period 1 and $p_2 = v_H$.
2) If $\frac{v_L}{v_H} < \delta < \frac{v_L}{c_H}$, then in period 1 enough v_L-sellers trade such that in period 2
$$\phi_2 v_L + (1-\phi_2)v_H = \frac{v_L}{\delta}.$$
Note that $\phi_2 < \phi$, such that in period 2, $p_2 = \phi_2 v_L + (1-\phi_2)v_H$.
3) If $\frac{v_L}{c_H} < \delta$, then in period 1 enough v_L-sellers trade such that in period 2 $\phi_2 = \phi^*$, such that in period 2, buyers are indifferent between bidding

$c_H = \phi^* v_L + (1-\phi^*)v_H$ and v_L. They mix between the two such that

$$\mathbb{E}[p_2] = \frac{v_L}{\delta}.$$

These are by construction all equilibria with both private and public offers.

□

The Role of Distress with Many Periods

The role of distress ($\alpha < 1$) can also be generalized in a setup with more than two trading opportunities with linear valuations and uniformly distributed costs. In particular, we can show that even as $\Delta \to 0$, there must be a positive mass of trade at the deadline. This is formalized in the following lemma.

LEMMA 7: *For any $\alpha < 1$, with public offers, as $\Delta \to 0$, trade at time $1-\Delta$ is strictly bounded away from zero.*

Proof of Lemma 7.
At time $1-\Delta$, prices are given by

$$p_{1-\Delta}(k) = (1-\alpha\delta)\underbrace{k}_{<v(k)} + \alpha\delta v(k)$$

Thus, there must be positive trade at time $1-\Delta$ because

$$\pi_{1-\Delta}(k_{1-2\Delta}+\epsilon; k_{1-2\Delta}) = \int_{k_{1-2\Delta}}^{k_{1-2\Delta}+\epsilon} (v(c) - p_{1-\Delta}(k_{1-2\Delta}+\epsilon))f(c)dc$$

and for small ϵ,

$$\lim_{\Delta\to 0} v(k_{1-2\Delta}) - p_{1-\Delta}(k_{1-2\Delta}+\epsilon) > 0.$$

Consequently, trade in period $1-\Delta$ is bounded away from zero as $\Delta \to 0$. This shows that with public offers, as $\Delta \to 0$, trade at time $1-\Delta$ is strictly bounded away from zero.

□

PROPOSITION 6: *(Quiet Periods) With public offers, for any $\alpha < 1$ there exists a $\delta^{**} < 1$ such that if $\delta > \delta^{**}$ there will be no trade in at least one period preceding the deadline, and possibly no period but the last period.*

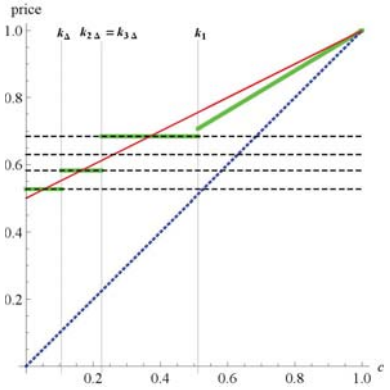
Proof of Proposition 6. By Lemma 7, $p_{1-\Delta}(k_{1-\Delta})$ is greater and bounded away from $v(k_{1-2\Delta})$. If there was trade in period $1-2\Delta$, then the highest type trading in that period $k_{1-2\Delta}$ can at most get a price $v(k_{1-2\Delta})$. On the other hand, the period after, he can buy at a price $p_{1-\Delta}(k_{1-\Delta})$ which is strictly greater

than and bounded away (for all Δ) from $v(k_{1-2\Delta})$. Hence, for small enough Δ, there cannot be trade in period $1-2\Delta$. In other words, for large δ, there must be a quiet period before the deadline.

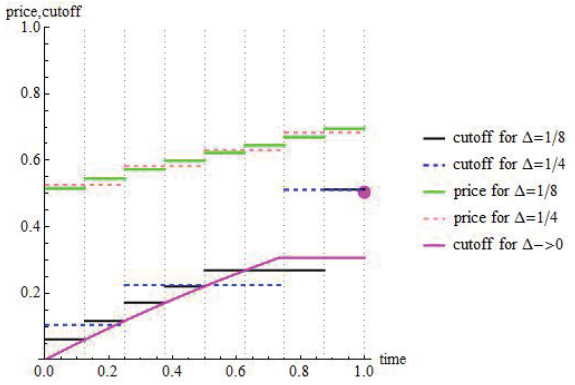
By the same logic, for a given fixed N, when δ^N is large, there will be N quiet periods before the deadline.

□

Note that this is in stark contrast to the no trade result in HV. Recall they have trade only in the first period with public offers while potentially we have no trade but in the last period. The differences in outcomes are caused by two differences in the models: first, we have intra- and inter-period competition while HV have only the latter; second we have a short horizon with a destruction of continuation surplus at the deadline.



(a) Prices as a function of seller types for $\Delta = \frac{1}{4}$



(b) Prices and cutoffs over time for $\Delta \in \{\frac{1}{4}, \frac{1}{8}\}$

FIGURE D1. PURE-STRATEGY EQUILIBRIA WITH $r = 0.5$, $v(c) = \frac{c+1}{2}$, $\alpha = 0.8$

In Figure D1 we plot the unique pure-strategy equilibria with public offers using $v(c) = \frac{c+1}{2}$ for $\Delta \in \left\{0, \frac{1}{4}, \frac{1}{8}\right\}$, $r = 0.5$, and $\alpha = 0.8$.[17] Indeed, in the limit, as $\Delta \to 0$, with public offers, there is a mass of trade at time 1 and some "quiet periods" in which no trade takes place. In particular, in the last period it must hold that

$$p_1 = (1-\alpha)k_1 + \alpha v(k_1) = \mathbb{E}[v(c)|[k_{1-}, k_1]]$$

where at time 1 the mass of seller types $[k_{1-}, k_1]$ trades (where k_{1-} is the limiting cutoff as time approaches 1 from the left). Moreover, before the quiet period,

[17] See Figure 2 for a comparison when $\alpha = 1$.

there must be continuous trading over time. In particular, for $A = B = 0.5$ the cutoff is given by

$$k_t = 1 - e^{-rt}.$$

Finally, the condition that seller k_{1-} must be indifferent between buying just before the quiet period starts and waiting until time 1 pins down the evolution of cutoffs over time. It turns out the quiet period before deadline is caused not only by distress but also by the market structure. In particular, we can contrast equilibrium dynamics in our model with intra- and inter-period competition to a monopoly case. As shown in Fuchs and Skrzypacz (2013*a*), in a model with one long-lived buyer the distress at deadline also induces an atom of trade at the end, but the quiet period does not arise there.