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Abstract

We review the theory of leverage developed in collateral equilibrium models
with incomplete markets. We explain how leverage tends to boost asset prices,
and create bubbles. We show how leverage can be endogenously determined
in equilibrium, and how it depends on volatility. We describe the dynamic
feedback properties of leverage, volatility, and asset prices, in what we call the
Leverage Cycle. We also describe some cross-sectional implications of multiple
leverage cycles, including contagion, flight to collateral, and swings in the
issuance volume of the highest quality debt. We explain the differences between
the leverage cycle and the credit cycle literature. Finally, we describe an
agent based model of the leverage cycle in which asset prices display clustered
volatility and fat tails even though all the shocks are essentially Gaussian.

Keywords: Leverage, Leverage Cycle, Volatility, Collateral Equilibrium,
Collateral Value, Liquidity Wedge, Flight to Collateral, Contagion, Adverse
selection, Agent Based Models.

1   Introduction

Before the great financial crisis of 2007-09, mainstream macroeconomics regarded
interest rates and technology shocks as the most important drivers of economic ac-
tivity and asset prices. The Federal Reserve, charged with maintaining stable prices

and employment, subscribed to this view, relying almost exclusively on its power to influence interest rates: when activity was too low, it typically lowered interest rates, when activity got too frenetic, it raised interest rates. The recent financial crisis, however, has challenged this view. During the Great Moderation of the late 1990s and up through 2006, volatility was low, and asset prices and leverage were soaring. During the crisis of 2007-09 and its aftermath, now called the Great Recession, volatility soared and asset prices and leverage plummeted. Interest rates do not seem to have played a pivotal role in the transition, nor do there seem to have been any remarkable technology shocks. Certainly lowering interest rates to zero, as the Fed did to short rates in 2008, did not put an end to the recession.

The purpose of this Review is to describe an alternative view in which the key driver of asset prices, and therefore of economic activity, is endogenous leverage and volatility, rather than interest rates. The leverage cycle theory was developed in Geanakoplos (1997, 2003), and extended to multiple leverage cycles in Fostel and Geanakoplos (2008), before the financial crisis. Parts of these papers are described in what follows through a sequence of simple variations of one baseline example.

At most times, agents cannot borrow as much as they would like at the going riskless interest rates because lenders are afraid they might default. In order to protect their loans against default, lenders often insist that borrowers secure them by posting collateral. How much collateral is required per dollar borrowed depends in part on the insecurity of the lender, which to a great degree depends on the volatility of the value of the collateral itself, especially out in the bad tail. Credit markets get tighter when volatility rises. And when credit gets tight, asset prices fall, even with constant interest rates. These observations are the basis for the leverage cycle, first developed in Geanakoplos (2003), which we now describe.

When volatility is low for an extended period of time, leverage rises, both because lenders feel more secure, and because Wall Street innovates to stretch the available scarce collateral. As shown in Figure 1, at the beginning of the Great Moderation, borrowing $86 or less on a $100 house was normal. By the end of the Great Moderation, leverage had risen so much that by late 2006 it was normal to borrow $97 on a $100 house.[1]

When leverage rises throughout the economy, and not just for one borrower, asset

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[1]Think of leverage not as the amount borrowed, but as the ratio of the value of the collateral to the cash downpayment used to buy it. A similar number is the ratio of the amount borrowed to the price of the collateral, or loan to value (LTV).

Housing Leverage Cycle
Margins Offered (Down Payments Required) and Housing Prices

Down Payment for Mortgage -- Reverse Scale

0% / 2% / 4% / 6% / 8% / 10% / 12% / 14% / 16% / 18% / 20%

Case Shiller National HPI

190 / 170 / 150 / 130 / 110 / 90

2000 2001 2002 2003 2004 2005 2006 2007 2008 2009

— Avg Down Payment for 50% Lowest Down Payment Subprime/AltA Borrowers
— Case Shiller National Home Price Index (right axis)

Observe that the Down Payment axis has been reversed, because lower down payment requirements are correlated with higher home prices.

Note: For every AltA or Subprime first loan originated from Q1 2000 to Q1 2008, down payment percentage was calculated as appraised value (or sale price if available) minus total mortgage debt, divided by appraised value. For each quarter, the down payment percentages were ranked from highest to lowest, and the average of the bottom half of the list is shown in the diagram. This number is an indicator of down payment required: clearly many homeowners put down more than they had to, and that is why the top half is dropped from the average. A 13% down payment in Q1 2000 corresponds to leverage of about 7.7, and 2.7% down payment in Q2 2006 corresponds to leverage of about 37.
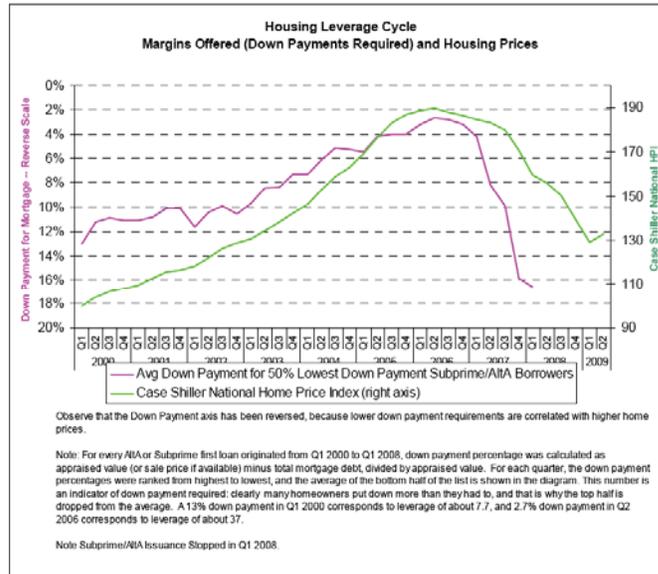
Note Subprime/AltA Issuance Stopped in Q1 2008.

Figure 1: Leverage Cycle.

prices rise. More people can afford to buy, buyers can purchase more units, and they are willing to spend more for the collateral because they can use it to borrow. Borrowing therefore rises for compounded reasons: it is a higher percentage (higher LTV) of a higher number (higher collateral prices). At the ebullient stage, when leverage is at its highest, the economy appears to be in wonderful shape: prices and investor's profits are high and stable, and economic activity is booming.

However, this is precisely the phase at which the economy is most vulnerable. A little bit of bad news that causes asset prices to fall has a big impact on the most enthusiastic and biggest buyers because they are the most leveraged.[2] Most importantly, if the bad news increases uncertainty or volatility, lenders will tighten credit. In 2006 the so called $2.5 trillion of toxic mortgage securities that later threatened the whole financial system could have been purchased with a downpayment of about $150 billion, with the remaining $2.35 trillion spent out of borrowed money (LTV of 93%). In 2008, those same securities required a downpayment of 75%; at 2006 prices, that would have meant a downpayment of almost $2 trillion cash, and just $600 billion borrowed ($LTV$ of 25%). Within two years, leverage for these assets

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[2] An investor who is leveraged 30 to 1 loses 30% of his investment when the asset price falls only 1%.

fell from about 16 to less than 1.4. As Figure 1 shows, the normal downpayment for housing financed by non-government mortgages fell from 14% in 2000, to 2.7% in 2006, then rose to 16% in 2007.

As we will see, leverage cycle crashes always occur because of a coincidence of three factors. The bad news itself lowers the prices. But it also drastically reduces the wealth of the leveraged buyers, who were leveraged the most precisely because they are the most optimistic buyers. Thus the purchasing power of the most willing buyers is reduced. And most importantly, if the bad news also creates more uncertainty and volatility, then credit markets tighten and leverage will be reduced, just when the optimists would like to borrow more, making it much harder for the optimists and any potential new buyers to find funding.

Geanakoplos (1997) and Geanakoplos and Zame (1997) developed a collateral general equilibrium model and proved that under the usual convexity and continuity conditions, collateral equilibrium always exists.[3] Geanakoplos (1997) showed how leverage can be determined endogenously in equilibrium. At first glance this seems a difficult problem: how can one supply equals demand equation for loans determine two variables, the interest rate and the LTV? In collateral equilibrium models the puzzle is solved by postulating that equilibrium prices consist of a menu, with a different interest rate for each LTV. Geanakoplos (1997, 2003) show in some special cases that all agents would choose the same contract from the menu. Fostel and Geanakoplos (2013) proved that in all binomial economies with financial assets, one and only one contract is indeed chosen. This leads to a simple formula for leverage: the LTV of each asset is its worst case return divided by the riskless rate of return. Thus leverage is determined by bad tail risk. In some special cases this formula can be recast to say that equilibrium margins are proportional to the volatility of a dollar's worth of the asset. As volatility goes up, leverage goes down, and as volatility goes down, leverage goes up.

These early collateral equilibrium models also provide a framework to study asset pricing implications of leverage. The key insight is that in collateral equilibrium investors do not set the marginal utility of an asset's dividends equal to its price; rather, they set the marginal utility of the asset's dividends net of the loan repayments equal to its downpayment. Assets can thus sell for more than the marginal utility of their dividends. Based on this insight, Fostel and Geanakoplos (2008) de-

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[3]This is not completely obvious; it is not true for the general equilibrium model with incomplete markets.

velop a formal theory of asset pricing, that links liquidity and collateral to asset prices. First, they defined the liquidity wedge as the extra interest that collateral constrained borrowers would be willing to pay for loans if they did not have to put up the collateral. They showed that constrained borrowers would then use the very same liquidity wedge as the factor with which to discount all future cash flows. The liquidity wedge rises when consumption today falls (if agents have declining marginal utility of consumption) and also rises as the opportunities to invest increase. Second, an asset's price reflects its future returns, but also its ability to be used as collateral to borrow money. This second role is priced in equilibrium by a premium called Collateral Value: asset prices are equal to their payoff value plus their collateral values. Finally, they showed that collateral value is ultimately equal to the liquidity wedge multiplied by the size of the loan the collateral backs.

The leverage cycle of Geanakoplos (2003) is an example of how this pricing theory can have interesting dynamic implications. As we mentioned before, the price crashes because of three factors: the bad news, the capital loss of the most optimistic buyers, and the collapse of market leverage or LTV. The price falls a little because every agent's original opinion of the value of the asset is worsened by the bad news. It falls a lot either because the original optimist is forced to sell the asset to new buyers who valued the asset a lot less even before the bad news (the crisis economy), or because the original optimists manage to retain all the assets but value them much less on account of their higher liquidity wedge. When the capital loss and plunging LTV are large enough, and when the willingness of investors to pay for the assets is sufficiently heterogeneous, the price fall can be a big multiple of what anybody in the economy thinks is warranted by the bad news itself.

Fostel and Geanakoplos (2008) extended the model to include multiple leverage cycles over different asset classes. Collateral equilibrium pricing theory explains cross-section properties like flight to collateral, contagion and the enormous volatility in the volume of trade of high quality assets.

First, when bad news comes, some assets fall more in value than others in what is traditionally called a flight to quality. Fostel and Geanakoplos (2008) argue that the increasing spread between the assets is actually due to the increasing spread in their collateral values: when the liquidity wedge rises, it magnifies the difference in collateral values because good collateral backs bigger loans than bad collateral. They call this pricing behavior flight to collateral.

Second, they give a collateral equilibrium explanation for contagion. Bad news for one asset class (like Asian debt in 1997 or subprime loans in 2007) sometimes causes widespread losses in other asset classes with completely independent dividends. They note that the declining stage of the leverage cycle in one asset market raises the liquidity wedge (and raises it more as the depressed prices in that market look like a special opportunity). But the liquidity wedge is part of the discount rate universal to all asset classes, and so the prices of all other assets bought by the same investors must fall. Interestingly this explanation of contagion does not rely on extreme sell-offs. Leverage cycle price declines migrate to unrelated asset classes even when optimistic agents keep buying.

Finally, they give a collateral equilibrium explanation for the precipitous drop in debt issuance by the highest type borrowers during down phases of the leverage cycle when there is informational noise. Because of the widening spread of collateral values, good issuers drastically curtail their issuance in order to prevent bad issuers from imitating them. Flight to collateral combined with informational asymmetries generate violent swings in high quality debt even though bad news is equally bad for all asset classes.

Recently, Thurner, Farmer, and Geanakoplos (2012) reexamined the leverage cycle from an agent based modeling perspective. In their model, fluctuations in volatility are entirely endogenous, rather than driven by shocks to asset dividends. It is assumed that the agents who leverage have a more stable opinion of the value of assets than the cash buyers. When asset prices rise toward the value these leveraged buyers think is correct, their bets pay off and they become relatively richer and come to control more of the market. Prices therefore become more stable, that is, volatility declines, so lenders permit them to leverage more, driving volatility further down and their leverage further up. At that point a little bit of bad news leads to margin calls and forced selling, which leads to rapid price declines and a spike in volatility. This causes lenders to toughen margin requirements, creating more margin calls, more selling, and more volatility. It turns out that in this agent based model of the leverage cycle, asset prices display clustered volatility and fat tails even though all the shocks are essentially Gaussian.

Before the crisis, another branch of non-mainstream macroeconomics, led by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), also investigated collateral and what Kiyotaki-Moore (1997) called the credit cycle. The credit cycle featured the multiplier-accelerator feedback from good news about asset dividends, to higher asset

prices, to more borrowing, to more investments improving asset values. Nevertheless, the credit cycle literature missed some important elements of the leverage cycle. The credit cycle ignored leverage. The multiplier-accelerator story would work with a constant LTV; in fact, in Kiyotaki and Moore (1997), leverage falls when asset prices are rising, dampening the cycle instead of driving it. Volatility plays no role in the credit cycle. Endogenous leverage, and certainly changing leverage, is not really a focus of the credit cycle models. To the extent that endogenous leverage was considered at all, it was in a corporate finance context, in which assets cannot be fully levered because lenders want to see that the borrowers have skin in the game to incentivize them to work harder to improve the dividends of the assets (as in Holmstrom and Tirole (1997)). However, the holders of mortgage securities (and to a great extent the owners of houses), which formed the bulk of the collateral that fueled the 2000-2009 leverage cycle, had no control over the dividends or value of those securities. Leverage changed as lenders (perhaps led or misled by rating agencies) got more or less nervous about the future value of the assets. Finally, the credit cycle literature emphasized the view that collateral constraints depress the value of assets, and prevent investors from finding the money to invest as much as they wish. But that misses the collateral value of assets. When the only way to borrow is by holding certain kinds of collateral, the good collateral will rise, not fall, in price, leading to over investment and even bubbles. The credit cycle literature missed the bubble of the leverage cycle, as well as the speedy collapse brought on by rapidly falling leverage.

The review is organized as follows. Section 2 develops a static collateral equilibrium model and all the theoretical results regarding leverage and asset pricing. Section 3 analyzes the leverage cycle. Section 4 studies multiple leverage cycles. Section 5 presents an agent based model of the leverage cycle.

2   A Binomial Model of Endogenous Leverage

2.1   The Model

Time and Uncertainty

Consider a finite-horizon general equilibrium model, with time $t = 0, \ldots, T$. Uncertainty is represented by a binomial tree of date-events or states $s \in S$, including a

root $s = 0$. Each state $s \neq 0$ has an immediate predecessor s^*, and each nonterminal node $s \in S \setminus S_T$ has a set $S(s) = \{sU, sD\}$ of immediate successors. We denote the time of s by the number of nodes $t(s)$ on the path $(0, s]$ from 0 to s, not including 0. We stick with binomial trees in this review because they are the simplest models in which one can study the role of uncertainty in shaping leverage, and because one can prove general theorems about leverage and default in such models.

Goods and Assets

There is a single perishable consumption good c and $K = \{1, ..., K\}$ assets k which pay dividends d_s^k of the consumption good in each state $s \in S \setminus \{0\}$. The dividends d_s^k are distributed at state s to the investors who owned the asset in state s^*.

We take the consumption good as numeraire in every state and $p_s \in R_+^K$ denotes the vector of asset prices in state s.

We will assume that all assets are *financial assets*, that is, they give no direct utility to investors, and pay the same dividends no matter who owns them. Financial assets are valued exclusively because they pay dividends. Houses are not financial assets because they give utility to their owners. Nor is land if its output depends on who owns it and tills it.

Debt and Collateral

The heart of our analysis involves contracts and collateral. In Arrow Debreu equilibrium the question of why agents repay their loans is ignored. We suppose from now on that the only enforcement mechanism is collateral.

A *debt contract* $j \in J$ is a one-period non contingent bond issued in state $s(j) \in S$ that promises $b(j) > 0$ units of the consumption good in each immediate successor state $s' \in S(s)$, using one unit of asset $k(j) \in K$ as collateral. We denote the set of contracts with issue state s backed by one unit of asset k by $J_s^k \subset J$; we let $J_s = \bigcup_k J_s^k$ and $J = \bigcup_{s \in S \setminus S_T} J_s$.

The price of contract j in state $s(j)$ is π_j. An investor can borrow π_j at $s(j)$ by selling contract j, promising $b(j)$ in each $s' \in S(s(j))$, provided he holds one unit of asset $k(j)$ as collateral. Let φ_j be the number of contracts j traded at $s(j)$. There is no sign constraint on φ_j. A positive φ_j indicates the agent is selling $|\varphi_j|$ contracts

j or borrowing $|\varphi_j|\pi_j$; a negative φ_j indicates the agent is buying $|\varphi_j|$ contracts j or lending $|\varphi_j|\pi_j$.

We shall assume that the most a borrower can lose is his collateral if he does not honor his promise, as is the case with "no-recourse" collateral. Hence the actual delivery of contract j in each state $s' \in S(s(j))$ is

$$min\{b(j), p_{s'k(j)} + d_{s'}^k\} \tag{1}$$

The rate of interest promised by contract j in equilibrium is $(1+r_j) = b(j)/\pi_j$. If the promise is small enough that $b(j) \le p_{s'k(j)} + d_{s'}^k, \forall s' \in S(s(j))$, then the contract will not default. In this case its price defines a riskless rate of interest. In equilibrium, all one period contracts j that do not default and are issued at the same state $s = s(j)$ can be priced so as to define the same riskless rate of interest which we call r_s.

The *Loan-to-Value LTV_j* associated to contract j in state $s(j)$ is given by

$$LTV_j = \frac{\pi_j}{p_{s(j)k(j)}}. \tag{2}$$

The *margin m_j* associated to contract j in state $s(j)$ is $1 - LTV_j$. *Leverage* associated to contract j in state $s(j)$ is the inverse of the margin, $1/m_j$, and moves monotonically with LTV_j.

We define *leverage for asset k in state s, LTV_s^k*, as the trade-value weighted average of LTV_j across all actively traded debt contracts $j \in J_s^k$ by all the agents $h \in H$

$$LTV_s^k = \frac{\sum_h \sum_{j \in J_s^k} \max(0, \varphi_j^h)\pi_j}{\sum_h \sum_{j \in J_s^k} \max(0, \varphi_j^h)p_{sk}}. \tag{3}$$

Finally, *leverage for investor h in state s, LTV_s^h*, is defined analogously as

$$LTV_s^h = \frac{\sum_k \sum_{j \in J_s^k} \max(0, \varphi_j^h)\pi_j}{\sum_k \sum_{j \in J_s^k} \max(0, \varphi_j^h)p_{sk}}. \tag{4}$$

Investors

Each investor $h \in H$ is characterized by a utility, u^h, a discount factor, δ_h, and subjective probabilities γ_s^h denoting the probability of reaching state s from its pre-

decessor s^*, for all $s \in S \backslash \{0\}$.[4] We assume that the utility function for consumption in each state $s \in S$, $u^h : R_+^L \to R$, is differentiable, concave, and monotonic. The expected utility to agent h is

$$U^h = u^h(c_0) + \sum_{s \in S \backslash 0} \delta_h^{t(s)} \bar{\gamma}_s^h u^h(c_s). \tag{5}$$

where $\bar{\gamma}_s^h$ is the probability of reaching s from 0 (obtained by taking the product of γ_σ^h over all nodes σ on the path $(0, s]$ from 0 to s).

Investor h's endowment of the consumption good is denoted by e_s^h in each state $s \in S$. His endowment of the assets in state s is $a_s^h \in R_+^K$. We assume that the consumption good is always present, $\sum_{h \in H}(e_s^h + d_s \cdot \sum_{\{\sigma : \sigma < s\}} a_s^h) > 0, \forall s \in S$. We suppose agents start with no debts, $J_{0^*} = \emptyset$.

Given asset prices and contract prices (p, π), each agent $h \in H$ choses consumption, c, asset holdings, y, and contract sales/purchases φ in order to maximize utility (4) subject to the budget set defined by

$$B^h(p, \pi) = \{(c, y, \varphi) \in R_+^S \times R_+^{SK} \times (R^{J_s})_{s \in S \backslash S_T} : \forall s$$

$$(c_s - e_s^h) + p_s \cdot (y_s - y_{s^*} - a_s^h) \leq$$

$$\leq \sum_{k \in K} d_s^k y_{s^*k} + \sum_{j \in J_s} \varphi_j \pi_j - \sum_{k \in K} \sum_{j \in J_{s^*}^k} \varphi_j min(b(j), p_{sk} + d_s^k);$$

$$\sum_{j \in J_s^k} max(0, \varphi_j) \leq y_s^k, \forall k\}.$$

In each state s, expenditures on consumption minus endowments, plus total expenditures on assets net of asset holdings carried over from the previous period and asset endowments, can be at most equal to total asset deliveries plus the money borrowed selling contracts, minus the payments due at s from contracts sold in the past. Finally, those agents who borrow must hold the required collateral.

Collateral Equilibrium

A *Collateral Equilibrium* in this economy is a vector of financial asset prices and contract prices, consumption decisions, and financial decisions on assets and contract holdings $((p, \pi), (c^h, y^h, \varphi^h)_{h \in H}) \in (R_+^K \times R_+^{J_s})_{s \in S \backslash S_T} \times (R_+^S \times R_+^{SK} \times (R^{J_s})_{s \in S \backslash S_T})^H$ such that

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[4]Of course, $\gamma_{sD}^h = 1 - \gamma_{sU}^h, \forall s \in S \backslash S_T$.

1. $\sum_{h \in H} (c_s^h - e_s^h) = \sum_{h \in H} \sum_{k \in K} y_{s*k}^h d_s^k, \forall s.$

2. $\sum_{h \in H} (y_s^h - y_{s*}^h - a_s^h) = 0, \forall s.$

3. $\sum_{h \in H} \varphi_j^h = 0, \forall j \in J_s, \forall s.$

4. $(c^h, y^h, \varphi^h) \in B^h(p, \pi), \forall h$

   $(c, y, \varphi) \in B^h(p, \pi) \Rightarrow U^h(c) \leq U^h(c^h), \forall h.$

Markets for consumption, assets and promises clear in equilibrium and agents optimize their utility in their budget set. Geanakoplos and Zame (1998) showed that equilibrium in this model always exists.

2.2 A First Example

In order to fix ideas and motivate our main theoretical results let us first consider a simple static example, similar to one in Geanakoplos (1997). Suppose $T = 1$, $S = \{0, U, D\}$ and suppose that there is only one asset, Y, that pays dividends $d_U = 1, d_D = .2$. Suppose the set of contracts $J = \{1, 2, ..., 1000\}$, where $b(j) = j/100$ for all j.

Suppose there are two types of agents $H = \{O, P\}$, with logarithmic utilities, who do not discount the future. Agents differ in their beliefs and wealth. Optimists assign a probability $\gamma_U^O = .9$ to the good state whereas pessimists assign a probability of only $\gamma_U^P = .4$ to the same realization. Both agents are endowed with a single unit of the asset at the beginning, $a_0^h = 1, h = O, P$, and are endowed with consumption goods: $e_0^O = e_D^O = 8.5, e_U^O = 10$ and $e_s^P = 100, s = 0, U, D$.

Table 1 describes the essentially unique equilibrium in this economy. The price of the asset is $p = 0.708$. Optimists hold all the assets in the economy and use them all as collateral to borrow money from the pessimists. It turns out that the only contract traded in equilibrium is $j^* = 20, b(j^*) = d_D = .2$, which sells for the price $\pi_{j^*} = .199$. Optimists use two units of the assets as collateral to sell two units of the contract that promises to pay $b(j^*) = .2$, avoiding default in equilibrium. Thus they borrow $2\pi_{j^*} = .398$. The resulting asset leverage is $LTV = \frac{\pi_{j^*}}{p} = \frac{.199}{.708} = .282$.

In equilibrium all contracts are priced, even those that are not traded. For $b(j) \leq .2$, $\pi_j = b(j)/1.001$. One can borrow on these contracts at the same riskless rate of

interest of 0.1%. For $b(j) > .2, \pi_j = \frac{.4min\{b(j),1\}+.6min\{b(j),.2\}}{1.001} < b(j)/1.001$. Since these contracts involve default, the associated interest rate $1 + r_j = b(j)/\pi_j$ is much higher than the riskless rate. For example for $b(j) = .3, \pi_j = 0.239$, and the interest rate is $r_j = 25.12\%$.

Table 1: Equilibrium Static Economy.

States	$s = 0$	$s = U$	$s = D$
Prices and Leverage			
p	0.708		
$b(j^*)$	0.2		
π_{j^*}	0.199		
r_{j^*}	0.1%		
LTV	0.282		
Asset Y			
Optimists	2		
Pessimists	0		
Debt Contracts φ_j			
Optimists	2		
Pessimists	-2		
Consumption			
Optimists	8.2	11.6	8.5
Pessimists	100.3	100.4	100.4

The prices of the contracts in the equilibrium described above correspond to the marginal utilities of the pessimist, so he is indifferent between lending or not. The optimists strictly prefer not to take any contract with $b(j) \neq .2$. The asset on the other hand is priced according to the marginal utilities of optimists.[5] The key equations to calculate the equilibrium are

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[5]Thus the equilibrium in the table has asset and contract prices that cannot be determined by state prices. However, this is not literally the unique equilibrium. One can modify the prices of contracts that are not traded without disturbing market clearing. Fostel-Geanakoplos (2013) showed that in binomial trees with debt contracts and one financial asset, there is always an equilibrium with unique state prices explaining the asset price and all the contract prices. In this example, the state price for U is .635 and for D is .363.

$$\frac{1}{c_0^O}(p - \pi_{j^*}) = \gamma_U^O \frac{1}{c_U^O}(1 - d_D) + \gamma_D^O \frac{1}{c_D^O}(d_D - d_D) \qquad (6)$$

$$\frac{1}{c_0^P}\pi_{j^*} = \gamma_U^P \frac{1}{c_U^P}d + \gamma_D^P \frac{1}{c_D^P}d_D \qquad (7)$$

Observe that the first equation requires that the marginal utility to the optimist of the downpayment for the asset is equal to his marginal utility of the asset dividends net of the j^* loan deliveries. The usual requirement that the marginal utility of the asset price is equal to the marginal utility of the asset dividends does not hold in collateral equilibrium.

There are several important ideas coming out of this simple example that we will discuss now and further formalize in Sections 2.3 to 2.7.

2.2.1 Endogenous Leverage

First, in equilibrium there is not just one interest rate but a menu of interest rates depending on the promise per unit of collateral. A problem when calculating equilibrium is to know which contract is actively traded. Agents have access to a whole menu of contracts J_0, all of which are priced in equilibrium. But because collateral is scarce (there are only two units of collateral in the economy, and in Arrow Debreu equilibrium, promises would be much bigger), only a few contracts will be traded. As Geanakoplos (1997) explained, all contract types are not rationed equally; instead most will be rationed to zero trade, and just a few, possibly just one type, will be actively traded in equilibrium. The example shows that only one contract is traded, the *max min* contract j^* satisfying $b(j^*) = d_D = .2$. This is the maximum amount optimists can promise while guaranteeing they will not default in the future. One might have thought that optimists would be so eager to borrow money in order to buy the asset from pessimists (who they believe undervalue the asset), that they would want to promise more than .2 per asset, happily paying a default premium in order to get more money at time 0. According to the equilibrium, this is not the case.

Second, the reason they do not borrow more is that they are constrained from borrowing more at the going interest rate. The equilibrium interest rate is $r = 0.1\%$, and at that rate optimists would be willing to borrow much more than .398. Indeed, in order to get a tiny additional loan they would be willing to pay a rate of 37%, even

if they were forced to deliver in full (out of their future endowments) under punishment of death.[6] But no lender will be willing to lend them more money, even at that rate, because there is no punishment. There is only collateral to enforce the delivery, and promises beyond .2 per asset will default in state D. The only way to borrow more, while using the same collateral, is to sell a contract j with $b(j) > .2$. But as we explained before, such contracts are priced by the market at much higher interest rates than 0.1%. Of course the borrowers are also aware that they will default and not actually have to pay everything they owe in state D, and as a result they are willing to pay a default premium. But this default premium is not enough to satisfy the lenders: the equilibrium implied interest rate on j with $b(j) > .2$ is still higher, so borrowers will not borrow more. The threat of default is so strong, it causes the lenders to constrain the borrowers. More precisely, the offered interest rate rises too fast as a function of $b(j)$ for the borrowers to be willing to take on more debt.

Third, leverage could be anything, but equilibrium endogenously chooses leverage according to a simple formula. Each contract has a leverage associated to it. When only one contract is traded in equilibrium, this uniquely pins down the leverage in the economy. Moreover, since there is no default, the price of the only traded contract is given by $\pi_{j^*} = \frac{d_D}{1+r_j^*}$. Leverage can then be characterized by the simple formula $LTV = \frac{\pi_{j^*}}{p} = \frac{d_D/p}{1+r_{j^*}}$. Thus LTV is given by the ratio between the worst case rate of return on the asset and the riskless rate of interest.

There is a growing literature on leverage. Some of the papers such as Acharya and Viswanathan (2011), Adrian and Shin (2010), Brunnermeier and Sannikov (2013) and Gromb and Vayanos (2002), focus on investor-based leverage Other papers, such as Acharya, Gale and Yorulmazer (2011), Brunnermeier and Pedersen (2009), Fostel and Geanakoplos (2008, 2012a and 2012b, 2013), Geanakoplos (1997, 2003 and 2010) and Simsek (2013), focus on asset-based leverage.

Not all these models actually make room for endogenous leverage. Often an ad-hoc behavioral rule is postulated. To mention just a few, Brunnermeier and Pedersen (2009) assume a VAR rule. Gromb and Vayanos (2002) and Vayanos and Wang (2012) assume a maxmin rule that prevents default. Some other papers like Garleanu and Pedersen (2011) and Mendoza (2010) assumed a fixed LTV. In some of the papers, such as Brunnermeier and Sannikov (2013), leverage is endogenous, but

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[6]The marginal utility of one unit of consumption at time 0 is 1.37 times bigger than the expected marginal utility of consumption at time 1.

borrowers are not constrained. They are borrowing all they would like to at the going riskless rate of interest.

In other papers leverage is endogenous, and borrowers are constrained, but the modeling strategy is different from the one we are emphasizing in this review. In the corporate finance approach of Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Holmstrom and Tirole (1997), Acharya and Viswanathan (2011) and Adrian and Shin (2010) the endogeneity of leverage relies on asymmetric information and moral hazard problems between lenders and borrowers. Asymmetric information is important in loan markets for which the borrower is also a manager who exercises control over the value of the collateral. Lenders may insist that the manager puts up a portion of the investment himself in order to maintain his skin in the game. The recent crisis, however, was centered not in the corporate bond world, where managerial control is central, but in the mortgage securities market, where the buyer/borrower generally has no control or specialized knowledge over the cash flows of the collateral.

2.2.2 Leverage Raises Asset Prices

The collateral equilibrium asset price $p = .708$ is much higher than its price in Arrow-Debreu equilibrium. With complete markets, the pessimists are so rich that they can insure the optimists without greatly disturbing their marginal utilities. These in turn will determine the Arrow prices. The Arrow prices are $p_U = .427$ and $p_D = .556$ which gives an asset price of $p = p_U 1 + p_D .2 = .539$. Thus leverage can dramatically raise asset prices above their efficient levels.

One might think that short-sale constraints would suffice to explain high asset prices. Pessimists would like to short the asset but cannot. What would happen if we dropped leverage, but still prohibited short selling? In the ensuing equilibrium optimists would buy all of the asset and therefore indeed their marginal utilities alone would determine the asset price. Nevertheless, the asset price would only be $p = .609$. One reason the no leverage price is much lower than the leverage price is that the optimists need to give up so much consumption at time 0 to buy the assets (since they cannot borrow), that their marginal utilities of the asset payoffs relative to the marginal utility of consumption at date 0 becomes low.

Second, the asset price in collateral equilibrium is higher than its marginal utility to every agent, even to the agents who buy it. In this example, the payoff value of the asset for the optimist is $PV^O = \frac{\sum_{s=U,D} \delta^O \gamma_s^O d_s du^O(c_s^O)/dc}{du^O(c_0^O)/dc} = .655$. For the pessimist the

payoff value is much lower. Yet the price is $p = .708 > PV^O = .655$. The reason the optimists are willing to pay more for the asset than its payoff value to them is that holding more of the asset enables them to borrow more money. This is what Fostel and Geanakoplos (2008) called Collateral Value.

Harrison and Kreps (1978) emphasized that short-sale constraints could raise the price of assets. Geanakoplos (2003) showed that leverage could raise the price of assets substantially more. Fostel-Geanakoplos (2012, 2013) showed that for one family of economies, the leverage price is always higher than the Arrow Debreu price and higher than the no short selling no leverage price. They showed that for a subfamily, the no leverage no short selling price is in turn always higher than the Arrow Debreu price. [7]

2.2.3 Collateral Value and Bubbles

Harrison and Kreps (1978) defined a bubble as a situation in which an asset trades for a price which is above every agent's payoff value. They showed a bubble could emerge in equilibrium if there were at least three periods, because the buyer in the first period could sell it in the second period to somebody who valued it more than he did from that point on. As the example shows, in collateral equilibrium a static two period model is enough to generate a bubble. The crucial idea, coming from the equilibrium equation (6) from Geanakoplos (1997), is that the buyer of an asset that can serve as collateral does not equate the marginal utility of a dollar of the asset to a dollar of consumption, but rather he equates the marginal utility of what comes from a dollar's worth of downpayment to a dollar's worth of consumption.

Collateral value was missed in other early work on collateral, such as in Kiyotaki-Moore (1997), because in their model consumption effectively was driven to zero. [8] It seemed in their example as if collateral was undervalued because its marginal utility was greater than its equilibrium price, while the marginal utility of consumption was equal to its price. But when consumption is zero, this is a meaningless comparison. In their model (as always in collateral equilibrium when the borrowing constraint

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[7]None of these price rankings is universally true. For example, if the utilities are linear, then the collateral equilibrium price does not depend on the future endowments, but the Arrow Debreu price does. Thus by manipulating future endowments one could make the Arrow Debreu price higher or lower than the leverage price.

[8]Farmers consume the "bruised fruit" in equilibrium, but what is crucial is that this bruised fruit is not marketable.

is binding), a dollar's worth of collateral gives less marginal utility than a dollar can bring if it is spent optimally (which would not be on consumption, but on the downpayment for still more collateral). Measuring the marginal utility of a dollar properly, even in the Kiyotaki-Moore example, collateral is overvalued. Other papers have developed the concept of collateral value. Garleanu and Pedersen (2011) define collateral value in a CAPM economy with agents with different risk aversion. Lagos (2010) generate liquidity premiums in a search and matching setting. An early study of the subject in a partial equilibrium model can be found in Hindy (1994).

2.3 Absence of Default

In the example we saw that despite the fact that agents had access to a whole menu of contracts, in equilibrium optimists borrow only through the maximum contract that prevents default. This is a general property of this class of models. The Binomial No-Default Theorem states that in binomial economies with financial assets serving as collateral, every equilibrium is equivalent (in real allocations and prices) to another equilibrium in which there is no default. Thus in binomial economies with financial assets, potential default has a dramatic effect on equilibrium, but actual default does not.

Binomial No-Default Theorem:

Suppose that S is a binomial tree, that is $S(s)=\{sU,sD\}$ for each $s \in S\backslash S_T$. Suppose that all assets are financial assets and that every contract is a one period debt contract.

Let $((p,\pi),(c^h,y^h,\varphi^h)_{h\in H})$ be an equilibrium. Suppose that for any state $s \in S\backslash S_T$ and any asset $k \in K$, the maxmin contract $j^*(s,k)$ defined by $b(j^*(s,k)) = min\{p_{sUk}+ d_{sU}^k, p_{sDk} + d_{sD}^k\}$ is available to be traded, i.e. $j^*(s,k) \in J_s$. Then we can construct another equilibrium $((p,\pi),(c^h,\bar{y}^h,\bar{\varphi}^h)_{h\in H})$ with the same asset and contract prices and the same consumption choices, in which only the max min contracts are traded.

Proof: See Fostel-Geanakoplos (2013).[9]

According to the Binomial No-Default Theorem, in searching for equilibrium in our example of section 2.2, we never needed to look beyond the max min promise

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[9]The Binomial No-Default Theorem is valid in a more general context than the one considered in this paper. It is valid with arbitrary preferences and endowments, contingent and non-contingent promises, many assets, many consumption goods, multiple periods, and production.

$b(j^*) = .2$, for which there is no default. The promise per unit of collateral is unambiguously determined simply by the payoffs of the underlying collateral, independent of preferences or other fundamentals of the economy. Agents will promise as much as they can while assuring their lenders that the collateral is enough to guarantee delivery.[10]

The theorem provides a hard limit on borrowing. Therefore, it shows that there must be a robust class of economies in which agents would like to borrow more at going riskless interest rates but cannot, even when their future endowments are more than enough to cover their debts.

The hard limit on borrowing is caused by the specter of default, despite the absence of default in equilibrium. If the asset payoff in the down state were to deteriorate, creating a clearer and more present danger of default, lenders would tighten credit.

The hard limit is endogenous. Lenders willingly offer contracts on which there would be default, but they charge such high interest rates that borrowers never choose them. One might have thought that the volume of trade in loans that default and loans that do not default could be the same. The defaulting loans would simply trade at higher interest rates reflecting a default premium. However, the theorem shows that this is not the case.

Binomial economies and their Brownian motion limit are special cases. But they are extensively used in finance. They are the simplest economies in which one can begin to see the effect of uncertainty on credit markets. With multiple states, default could emerge in equilibrium. Moreover, some borrowers might use collateral to take loans that would default while other borrowers might use the same collateral to take out loans in which delivery is fully guaranteed. Thus, the no-default and maxmin uniqueness properties do not extend beyond binomial economies. However, even in more general economies, borrowers would still be constrained, and as default becomes more likely, lenders would tighten credit. The binomial case is the simplest and starkest setting in which one can clearly connect default and the tightness of credit markets.[11]

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[10]The Binomial No-Default Theorem does not say that equilibrium is unique, only that each equilibrium can be replaced by another equivalent equilibrium in which there is no default. However, as Fostel and Geanakoplos (2013) also showed, among all equivalent equilibria, the maxmin equilibria (which never involve default) use the least amount of collateral. These collateral minimizing equilibria would naturally be selected if there were the slightest transactions cost in using collateral or handling default.

[11]Even in binomial economies, we would observe default in equilibrium if we were to consider

Geanakoplos (2003, 2010) and Fostel-Geanakoplos (2008, 2012a and 2012b, 2013) all work with binomial models of collateral equilibrium with financial assets, showing in their various special cases that, as the Binomial No-Default Theorem implies, only the *VaR= 0* contract is traded in equilibrium. Many papers have given examples in which the No-Default Theorem does not hold. Geanakoplos (1997) gave a binomial example with a non-financial asset (a house, from which agents derive utility), in which equilibrium leverage is high enough that there is default. Geanakoplos (2003) gave an example with a continuum of risk neutral investors with different priors and three states of nature in which the only contract traded in equilibrium involves default. Simsek (2013) gave an example with two types of investors and a continuum of states of nature with equilibrium default. Araujo, Kubler, and Schommer (2012) provided a two period example of an asset which is used as collateral in two different actively traded contracts when agents have utility over the asset. Fostel and Geanakoplos (2012a) provide an example with three periods and multiple contracts traded in equilibrium.

2.4   Endogenous Leverage

In our static example we saw that leverage was characterized by a very simple formula. As the following result shows, this is a general characterization for leverage in the class of binomial economies with financial assets.

Binomial Leverage Theorem:

Under the assumptions of the Binomial No-Default theorem, equilibrium leverage can always be taken to be

$$LTV_s^k = \frac{d_{sD}^k/p_{sk}}{1+r_s} = \frac{worst\ case\ rate\ of\ return}{riskless\ rate\ of\ interest}.$$

Proof: See Fostel-Geanakoplos (2013).

Equilibrium leverage depends on current and future asset prices, and the riskless rate of interest, but is otherwise independent of the utilities or the endowments of the agents. The theorem shows that in binomial models, it makes sense to use the Value at Risk equals zero rule, assumed by many other papers in the literature.

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non-financial asset as collateral. But it would still be the case that borrowers are constrained.

Though simple and easy to calculate, the binomial leverage formula provides interesting insights. First, it explains why changes in the bad tail can have such a big effect on equilibrium even if they hardly change expected payoffs: they change leverage. The theorem suggests that one reason leverage might have plummeted from 2006-2009 is because the worst case return that lenders imagined got much worse. Second, the formula also explains why (even with rational agents who do not blindly chase yield), high leverage historically correlates with low interest rates. Finally, it explains which assets are more leveraged: the asset whose future return has the least bad downside will be leveraged the most.

Collateralized loans always fall into two categories. In the first category, a borrower is not designating all the assets he holds as collateral for his loans. In this case he would not want to borrow any more at the going interest rates even if he did not need to put up collateral (but was still required, by threat of punishment, to deliver the same payoffs he would had he put up the collateral). His demand for loans is then explained by conventional textbook considerations of risk and return. In the second category, the borrower is posting all of his assets as collateral. In this case of scarce collateral, he is constrained by the specter of default: in order to borrow more, he may be forced to pay sharply higher interest rates. In binomial models with financial assets, the equilibrium LTV can be taken to be the same easy to compute number, no matter which category the loan is in, that is whether it is demand or supply determined.

The distinction between plentiful and scarce capital all supporting loans at the same LTV suggests that it is useful to keep track of a second kind of leverage that we call *diluted* leverage:[12]

$$DLTV_s^k = \frac{\sum_h \sum_{j \in J_s^k} \max(0, \varphi_j^h)\pi_j}{\sum_h y_{sk}^h p_{sk}} \leq LTV_s^k. \tag{8}$$

Similarly one can define *diluted investor* leverage

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[12]Consider the following example: if the asset is worth $100 and its worst case payoff determines a debt capacity of $80, then in equilibrium we can assume all debt loans written against this asset will have LTV equal to 80%. If an agent who owns the asset only wants to borrow $40, then she could just as well put up only half of the asset as collateral, since that would ensure there would be no default. The LTV would then again be $40/$50 or 80%. Hence, it is useful to consider diluted $DLTV$, namely the ratio of the loan amount to the total value of the asset, even if some of the asset is not used as collateral. The diluted $DLTV$ in this example is 50%, because the denominator includes the $50 of asset that was not used as collateral.

$$DLTV_s^h = \frac{\sum_k \sum_{j \in J_s^k} \max(0, \varphi_j^h)\pi_j}{\sum_k y_{sk}^h p_{sk}} \leq LTV_s^h. \tag{9}$$

It is often said that leverage should be related to volatility: the lower the volatility the higher the leverage. It turns out that this is the case in binomial economies with only one financial asset.

Binomial Leverage-Volatility Theorem:

Under the assumptions of the Binomial No-Default theorem, for each state $s \in S \backslash S_T$, and each asset $k \in K$, there are risk neutral pricing probabilities $\alpha = p_U(s,k)$ and $\beta = 1 - \alpha = p_D(s,k)$ such that the equilibrium price p_{sk} and equilibrium margin $m_s^k = 1 - LTV_s^k$ can be taken equal to

$$p_{sk} = \frac{\alpha(p_{sUk} + d_{sU}^k) + \beta((p_{sDk} + d_{sD}^k)}{1 + r_s}$$

$$m_s^k = \sqrt{\frac{\alpha}{\beta}} \frac{Vol_{\alpha,\beta}(k)}{(1 + r_s)p_{sk}}$$

where $Vol_{\alpha,\beta}(k) = \sqrt{\alpha\beta}(p_{sUk} + d_{sU}^k - p_{sDk} - d_{sD}^k)$.

Proof: See Fostel-Geanakoplos (2013).

The theorem says that equilibrium margin on an asset is proportional to the volatility of a dollar's worth of the asset. The trouble with this theorem is that the risk neutral pricing probabilities α and β depend on the asset k. If there were two different assets k and k' co-existing in the same economy, we might need different risk neutral probabilities to price k and k'. Ranking the leverage of assets by the volatility of their payoffs would fail if we tried to measure the various volatilities with respect to the same probabilities.

2.5 Liquidity Value and Credit

In our example in Section 2.2, we saw that agents were not able to borrow as much as they would like to at the going interest rate. They were willing to pay a much higher interest in order to get their hands on extra money today. We will now introduce concepts that help us precisely quantify the tightness of credit markets. Let us begin

by defining the marginal utility to agent h associated to trading contract j at state s, assuming that consumption is positive at s, sU and sD.[13]

Definition 1: The *Payoff Value* of contract j to agent h at state s is

$$PV_{sj}^h = \frac{\sum_{\sigma \in \{U,D\}} \delta_h \gamma_{s\sigma}^h min\{b(j), p_{s\sigma k(j)} + d_{s\sigma}^{k(j)}\} du^h(c_{s\sigma}^h)/dc}{du^h(c_s^h)/dc} \tag{10}$$

Definition 2: The *Liquidity Value* LV_{sj}^h associated to contract j to agent h at s is

$$LV_{sj}^h = \pi_j - PV_{sj}^h. \tag{11}$$

The liquidity value represents the surplus a borrower can gain by borrowing money today selling a contract j backed by collateral k.

2.6 Liquidity Wedge and Discount Rate

The liquidity value gives an expression of how much less a borrower would take and still be willing to sell the same promise. Another way of saying that he finds the loan beneficial on the margin is by defining

Definition 3: The *Liquidity Wedge* ω_{sj}^h associated to contract j for agent h at state s is

$$1 + \omega_{sj}^h = \frac{\pi_j}{PV_{sj}^h} \tag{12}$$

In the case that contract j fully delivers, ω_{sj}^h defines the extra interest a potential borrower would be willing to pay above the going riskless interest rate if he could borrow an additional penny and was committed (under penalty of death) to fully deliver. This extra interest is called the liquidity wedge; it gives a measure of how tight the contract j credit market is. We have seen that in binomial economies, agents only take out riskless loans. It is obvious that there cannot be two riskless loans actively trading for different interest rates, for that would mean the lender who got the lower interest rate had made a mistake. Hence we can unambiguously define the state s liquidity wedge ω_s^h for any agent h who actively borrows there.

The liquidity wedge can be given another very important interpretation as shown in the following theorem.

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[13]The definition of Payoff Value becomes more complicated if consumption, say c_s, is zero.

Discount Theorem:

Define the risk adjusted probabilities for agent h in state s by

$$\mu_{sU}^h = \frac{\gamma_{sU}^h du^h(c_{sU}^h)/dc}{\gamma_{sU}^h du^h(c_{sU}^h)/dc + \gamma_{sD}^h du^h(c_{sD}^h)/dc},$$

$$\mu_{sD}^h = \frac{\gamma_{sD}^h du^h(c_{sD}^h)/dc}{\gamma_{sU}^h du^h(c_{sU}^h)/dc + \gamma_{sD}^h du^h(c_{sD}^h)/dc} = 1 - \mu_{sU}^h.$$

If agent h is taking out a riskless loan in state s, then his payoff value in state s for a tiny share of cash flows consisting of consumption goods $x = (x_{sU}, x_{sD})$ is given by

$$PV^h(x) = \frac{\mu_{sU}^h x_{sU} + \mu_{sD}^h x_{sD}}{(1 + r_s)(1 + \omega_s^h)}.$$

Proof: See Fostel and Geanakoplos (2008).

This result is important because it shows that in evaluating assets that he might purchase, an agent who is borrowing constrained will discount the cash flows by a spread above the riskless rate; this spread is the same for all cash flows. As he becomes more liquidity constrained, in the sense of having a higher liquidity wedge, his willingness to pay for all assets will decline. The only exception might be for some assets that can serve as such good collateral that they bring an additional collateral value of enabling their owner to issue more loans.

2.7 Collateral Value and Asset Pricing

An asset's price reflects its future returns, but also its ability to be used as collateral to borrow money. Consider a collateral equilibrium in which an agent h holds an asset k at state $s \in S$, and suppose h consumes a positive amount in each state. As we saw in the example of Section 2.2, when the asset can be used as collateral and the collateral constraint is binding, the asset price can exceed the agent's asset valuation given by the Payoff Value defined as follows:

Definition 4: The *Payoff Value* of asset k to agent h at state s is

$$PV_{sk}^h \equiv \frac{\sum_{\sigma \in \{U,D\}} \delta_h \gamma_{s\sigma}^h (p_{s\sigma k} + d_{s\sigma}^k) du^h(c_{s\sigma}^h)/dc}{du^h(c_s^h)/dc} \tag{13}$$

Definition 5: The *Collateral Value* of asset k in state s to agent i is

$$CV_{sk}^h \equiv p_{sk} - PV_{sk}^h \qquad (14)$$

The collateral value stems from the added benefit of enabling borrowing that some durable assets provide. Collateral values distort pricing and typically destroy the efficient markets hypothesis, which in one of its forms asserts that there are risk-adjusted state probabilities that can be used to price all assets. Some assets may bring lower returns to investment, even accounting for the riskiness of the returns, because their prices are inflated by their collateral values.

Collateral Value = Liquidity Value Theorem:

Suppose that $y_{sk}^h > 0$ and $\varphi_j^h > 0$ for some agent h and some $j \in J_s^k$. Then, in equilibrium the following holds,

$$LV_{sj}^h = CV_{sk}^h$$

The liquidity value associated to any contract j that is actually issued using asset k as collateral equals the collateral value of the asset.

Proof: See Fostel-Geanakoplos (2008) and Geanakoplos-Zame (2013).

The collateral value is the additional cost an agent is willing to pay above the payoff value due to the fact that he can use the asset as collateral. The liquidity value is the benefit of borrowing through a contract that uses the asset as collateral. In equilibrium, these two are the same. No agent will overpay for the collateral unless he can gain at least as much liquidity value. If the liquidity value were more, then the agent would not be content and would buy more collateral in order to issue still more loans. The equality demonstrated in the theorem is the key equation in computing collateral equilibrium. It is equivalent to equation (6), that asserted that the difference in payoff value between the collateral and the loan had to be equal to the downpayment on the collateral.

2.8   Liquidity and Endogenous Contracts

Since one collateral cannot back many competing loans, the borrower will always select the loan that gives the highest liquidity value among all loans with the same

collateral. This leads to a theory of endogenous contracts in collateral equilibrium, and in particular, to a theory of endogenous leverage, as we saw in Sections 2.3 and 2.4. From definitions 2 and 3 it is clear that the liquidity value and liquidity wedge satisfy the following for any contract j:

$$LV_{sj}^h = PV_{sj}^h \omega_{sj}^h \tag{15}$$

All loans that deliver for sure will have the same liquidity wedge. If this wedge is positive, the borrower will naturally choose the biggest loan, since that has the highest payoff value and therefore the highest liquidity value. That explains why in binomial economies, the borrower always prefers the maxmin contract to all contracts that promise strictly less. Borrowers could also gain a surplus from contracts that promise more and default. But the lenders require a sharply higher interest, and so the liquidity wedge declines rapidly as loans default more. As a result, the borrowers voluntarily choose to trade only the maxmin contract.[14]

3   Leverage Cycle

We will now study the dynamic implications of the results presented in Section 2. We will see how, in a dynamic context, leverage and asset prices engage in a positive feedback, rising together then falling together, to create something we call the Leverage Cycle.

We extend the static example of Section 2 to a three period economy, so $T = 2$, and $S = \{0, U, D, UU, UD, DU, DD\}$. There is one financial asset Y which pays dividends only in the final period. We follow the idea from Geanakoplos (2003) which had a continuum of risk neutral agents, as adapted in Fostel-Geanakoplos (2008) to two risk averse agents.

The tree of asset payoffs has the property that good news reduces uncertainty about the payoff value and bad news increases uncertainty about the payoff value of the asset. We assume, as shown in Figure 2, that after good news at $s = U$ the asset

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[14]It is quite possible that a contract has a very high liquidity wedge associated to it, and therefore might be very useful to introduce into an economy in which agents could be counted on to deliver without posting collateral, but is not chosen in collateral equilibrium because it is small and therefore has a low payoff value and thus a low liquidity value. Such a promise might be useful in a GEI economy, but not in a collateral economy, because it uses up too much collateral.

payoff is equal to $d_{UU} = d_{UD} = 1$ with certainty. However, after bad news at $s = D$, the future payoff volatility increases. We assume that $d_{DU} = 1$ and $d_{DD} = .2$. The coincidence of bad news and increased volatility is the hallmark of the leverage cycle. We have seen that volatility tends to reduce leverage. Thus the bad news in the leverage cycle will reduce expected payoffs at the same time it reduces leverage.[15]
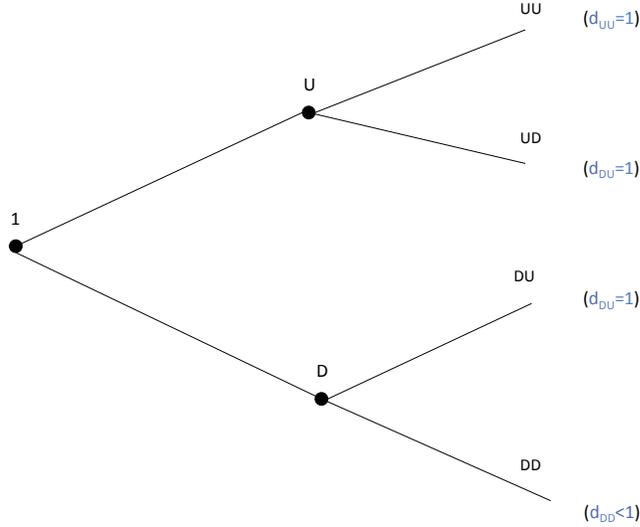

Figure 2: Asset Y Payoffs.

As before, there are two types of agents $H = \{O, P\}$ with logarithmic utilities who do not discount the future. Agents differ in their beliefs and wealth. Optimists assign a probability $\gamma_{sU}^O = .9$ of moving up from any state $s \in S \setminus S_T$ whereas pessimists assign a probability for moving up of only $\gamma_{sU}^P = .4$ for all $s \in S \setminus S_T$. Both agents are endowed with a single unit of the asset at the beginning, $a_0^h = 1, h = O, P$, and endowment of the consumption good in each state as follows: $e_0^O = e_D^O = 8.5, e_s^O = 10, s \neq 0, D$ and $e_s^P = 100, \forall s$.

Table 2 describes the essentially unique equilibrium in this economy.[16] By the No-Default Theorem, we know that the only contract traded in each node is the maxmin

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[15]For a more general treatment of volatility and the leverage cycle, see Fostel and Geanakoplos (2012a).

[16]To calculate the equilibrium we use the same logic as in equations (6) and (7) for each node. Detailed equations and programs for all the examples in the review are available upon request.

contract that prevents default. Since after good news there is no remaining uncertainty, the equilibrium decisions at that node are simple: there is no borrowing (since debt and the asset are perfect substitutes), and agents just trade the asset against the consumption good. At the initial node 0 and after bad news D the situation is more subtle.

Table 2: Equilibrium Leverage Cycle.

States	$s = 0$	$s = U$	$s = D$	$s = UU$	$s = UD$	$s = DU$	$s = DD$
			Prices, Leverage and Liquidity Wedge				
p	0.909	0.982	0.670				
j^*	0.670		0.2				
π_{j^*}	0.664		0.201				
LTV	0.730		.299				
PV	0.850	0.98	0.602				
CV	0.059	0	0.068				
ω	0.097	0	0.518				
			Asset Holdings				
Optimists	2	.408	2				
Pessimists	0	1.591	0				
			Debt Contract Trades				
Optimists	2		2				
Pessimists	−2		−2				
			Consumption				
Optimists	8.92	10.22	7.56	10.41	10.41	11.6	10
Pessimists	99.58	99.78	100.94	101.59	101.59	100.4	100.4

The equilibrium portfolios at 0 and D are of the same type as in the static example. Optimists hold all the assets in the economy and use them as collateral to borrow money from the pessimists. They buy the asset on margin selling the maxmin contract at each node: at 0 they promise the price of the asset p_D after bad news, and at D they promise .2 per each unit of the asset.

3.1 Ebullient Times

Collateral is usually scarce; borrowing is usually constrained. But when volatility is low, as at $s = 0$, the existing scarce collateral can support large amounts of borrowing

to buy assets that are acceptable collateral. If there is sufficient heterogeneity among agents in their enthusiasm for holding the asset, and short selling is not allowed, a bubble can emerge in which the prices of the assets that can be used as collateral rise to levels far above their "Arrow-Debreu" Pareto efficient levels, even though all agents are rational. In this example, leverage at time 0 is almost 4 to 1 ($LTV = .73$), and the asset price at time 0 is .91. In Arrow-Debreu equilibrium, the asset price would only be .71. The price is so high in the leverage equilibrium because the pessimists have no way to express their opinion about the asset except by selling. The optimists not only can buy out of their endowments, they can also borrow and buy more, leveraging their opinion. On top of all that, the optimists are willing to pay a collateral value of .06 above and beyond the asset payoff value of .85 to them, because holding it enables them to borrow more money.

The combination of high prices and low volatility creates an illusion of prosperity. But in fact the seeds of collapse are growing as the assets get more and more concentrated in the hands of the most enthusiastic and leveraged buyers. When bad news that creates more uncertainty occurs, the bubble can burst.

3.2 The Crash

Leverage cycle crashes always occur because of a coincidence of three factors. The bad news itself lowers the prices. But it also drastically reduces the wealth of the leveraged buyers, who were leveraged the most precisely because they are the most optimistic buyers. Thus the purchasing power of the most willing buyers is reduced. And most importantly, if the bad news also creates more uncertainty, then credit markets tighten and leverage will be reduced, just when the optimists would like to borrow more, making it much harder for the optimists and any potential new buyers to find funding.

The price of the asset in our example goes down from .91 at 0 to .67 at D after bad news, a drop of 24 points. At both 0 and D, the optimists are the only agents holding the asset, and in their view the expected payoff of the asset drops only 7 points, from .99 to .92, after the bad news. So there is something much more important than the bad news which explains the drop in asset price. This is the downward path of the leverage cycle.

First notice that the optimists, though still buying all the asset in the economy, lose wealth after bad news. At 0 they started with 8.5 units of the consumption good

and half the assets. In order to maintain high consumption and to buy up the rest of the assets, which they regard as a good investment, they become so leveraged at 0 that they owe the value of all their assets at D; after paying they are left with only their initial endowment of 8.5 consumption goods. So they get poorer at D, and are forced to consume less if they want to repurchase all their assets. Second, the higher volatility at D reduces the amount they can leverage. Leverage plummets from 4 at 0 to 1.4 at D (equivalently, the LTV goes from .73 to .29). Optimists are forced to drastically scale back their consumption at D if they want to continue holding all the assets. In fact they do want to continue, because they regard their opportunity at D as an even greater than at 0. Indeed, the disagreement between optimists and pessimists over the value of the assets is higher at D than at 0. As a result of their decreased consumption and their perception of a greater opportunity, their liquidity wedge, which is a measure of how much they are willing to pay above the riskless interest rate, increases dramatically, from 0.1 to 0.52. Curiously, they are able to borrow the least just when they feel the greatest need. By the Discount Theorem, they then discount all future cash flows at a much higher rate than the riskless rate, and it is this extra discounting of the future that reduces the value of the assets so much more than the bad news. On account of the bigger discount, the payoff value of the assets sinks all the way to .60. Of course there is still a collateral value of .07. But despite the high liquidity wedge, the collateral value of the assets is limited by the small amount of borrowing they support.

In summary, it is the combination of bad news, loss of current wealth (liquidity scarcity) and lower leverage that makes the crash in prices really dramatic. This evolution from low volatility and rising leverage and asset prices, to high volatility and declining leverage and asset prices is the Leverage Cycle.

3.3 Margin Calls

The most visible sign of the crash is the margin call. After the bad news at D starts to reduce asset prices, optimists who want to roll over their loans need to put up more money to maintain the same LTV on their loans. They could do that either by selling assets or by reducing their consumption. In our example here, they choose to reduce their consumption. They then effectively get a second margin call because the new LTV is much lower than before, forcing them to reduce consumption further. The reduction in consumption increases the rate at which they discount future cash

flows, and it is this more than the bad news which causes asset prices to crash. In his original model of the leverage cycle, Geanakoplos (2003) developed an alternative model of the leverage cycle in which the initial endowment of consumption goods of optimists is lower at D than at 0. When the margin call comes they are too poor to hold the assets by cutting down on consumption and are forced to sell instead. The new buyers are less enthusiastic or optimistic about the assets than the original optimists, and so the price crashes because the marginal buyer is a different and more pessimistic agent. The mechanism is analogous, whether the loss in value comes from the same agents discounting more, or from new agents who value the assets less.

Brunnermeier and Pedersen (2009) provide a theory of margin calls which they call margin spirals. Margins in their theory are exogenously set by the Value at Risk equal zero rule. Margin calls arise in a context of multiple equilibria.

3.4 Maturity Mismatch

If the optimists had borrowed for two periods instead of for one, they would not be forced to reduce their consumption (or to sell) at D. One might have thought that in order to reduce this margin call risk at D, optimists would prefer to take out long term debt instead of short term debt at 0. Geanakoplos (2010) examined this question in a similar model and observed that even if they were given the choice of long term debt, they would choose the short term debt. In our current model all debts are by hypothesis for one period. We could augment the current model by allowing non contingent two period debt as well as the short term debt. If long term debt could not be re traded in the middle periods, then the binomial no default theorem could be immediately extended to long term debt when the collateral only takes on two values across all the states of nature at which the bond payments come due. In this example, the collateral is worth either 1 or .2 across the four terminal states at time two. Hence we could conclude that among all long term debt contracts, only the debt contract collateralized by one unit of the asset and promising .2 units of consumption in every terminal state might be traded in equilibrium. But the optimists would not want to borrow on that contract, since they could raise .67 instead of .199 by borrowing on the one period contract and risking the unlikely (from their point of view) margin call at D.

3.5 Crisis Economy vs Anxious Economy

When the crash comes at D, the optimists still feel things will turn around, and think on average the asset will pay .92 in the end. Buying at D is an opportunity for them, since the asset has gone down very little in expected payoff but has a much lower price. Fostel and Geanakoplos (2008) distinguished between the case where they are forced to sell at D, which they called the crisis economy, and the case where they have enough liquid wealth at D to maintain their assets and perhaps buy new ones, which they called the anxious economy. In the current example, the optimists are not forced to sell, but they do not buy more either. It is thus on the borderline between a crisis economy and anxious economy.

3.6 Volatility

The signature of the leverage cycle is rising asset prices in tandem with rising leverage, followed by falling asset prices and leverage. But the underlying cause of the change in leverage is a change in volatility, or more generally, in some kind of bad tail uncertainty. In our example, the volatility of the asset's value is .126 at time 0, when leverage is almost 4, and increases to .394 at D, when leverage plummets to 1.4. The sharp increase in volatility is mostly due to a technology shock. In the standard real business cycle literature, there are technology shocks that increase or decrease productivity, but there is not much attention paid to shocks that increase volatility. Leverage can also rise for endogenous reasons. After the optimists lose income at D, their expenditure on assets becomes much more sensitive to their wealth.

Many recent papers have assumed a link between leverage and volatility (see for example Brunnermeier and Pedersen (2009), Thurner et.al., 2012, and Adrian and Boyarchenko, 2012). Geanakoplos (2003, 2010) and Fostel and Geanakoplos (2008, 2012a) derive this link from first principles, as special cases of the binomial leverage theorem. In Brunnermeier and Sannikov (2013) leverage is also derived endogenously from first principles, but it is determined not by collateral capacities but by agents' risk aversion; it is a "demand-determined" leverage that would be the same without collateral requirements. The time series movements of LTV come there from movements in volatility because the added uncertainty makes borrowers more scared of investing, rather than from reducing the debt capacity of the collateral or making lenders more scared to lend.

3.7 Smoothing the Leverage Cycle

Asset prices are much too high at 0 (compared to Arrow Debreu first best prices) and then they crash at D, rising and falling in tandem with leverage. If we added investment and production of the asset into the model, we would find overproduction at 0 and then a dramatic drop in production at D. Macroeconomic stability policy has concentrated almost entirely on regulating interest rates. But interest rates over the cycle in the Leverage Cycle example barely move. The leverage cycle suggests that it might be more effective to stabilize leverage than to stabilize interest rates.

Optimists have a higher marginal propensity to buy the asset at 0 and D than pessimists because they are more enthusiastic about the asset. Thus regulating leverage to lower levels at 0 will not only lower the asset prices at 0, but also raise the asset price at D because it will leave optimists less in debt. This will smooth the leverage cycle and move prices closer to Arrow Debreu levels. In a slightly more complicated model it will lead to Pareto improvements. It will not, however, lead to a Pareto improvement in this example, and for an instructive reason.

In collateral equilibrium, borrowers are constrained from borrowing as much as they like, but lenders are not. If an increase in borrowing and lending could be arranged it could make both borrowers and lenders better off, assuming that borrowers could be coerced into delivering fully out of their future endowments. Forcing a small reduction in credit is positively harmful to borrowers, and has little effect on lenders, assuming that future prices do not change. This probably explains why government policy has been almost exclusively geared to expanding credit rather than reigning it in.

But in collateral equilibrium, insurance markets are often missing, as in the Leverage Cycle example. Curtailing credit will lead to price changes in the future, which have redistributive consequences that may be beneficial. Geanakoplos-Polemarchakis (1986) proved that when insurance markets are missing, there is almost always an intervention in financial markets at 0 that will induce future price changes that are Pareto improving. But when there is a positive liquidity wedge, the future Pareto improvement that might come from curtailing leverage must overcome the immediate effect of limiting an already constrained credit market.

In the Leverage Cycle example, optimists sell assets at U. But optimists and pessimists have identical utilities at U because there is no remaining uncertainty, and they both have discount rates of 1. Thus curtailing leverage at 0 does not affect

prices at U. Curtailing leverage at 0 does raise the price of assets at D, but there is no trade in assets at D, since the optimists buy them all at 0 and retain them all at D. Thus the increase in asset prices at D does not redistribute wealth and has a negligible effect on welfare.

Table 3: Smoothing the Leverage Cycle.

	Unrestricted Leverage	Leverage Restricted $j = .58$	Leverage Transfer $j = .58, t = 0.0004$
Price at $s = 0$	0.820423	0.819622	0.819613
Price at $s = U$	0.925092	0.925097	0.925097
Price at $s = D$	0.590795	0.591873	0.591873
Utility Optimists	60.0274	60.0279	60.0275
Utility Pessimists	1311.6860	1311.6858	1311.6862

We are thus led to consider a modified Leverage Cycle example in which pessimists have a discount rate of .95 and in which they are endowed with an additional 1.5 assets at both U and D, but which is otherwise the same as the Leverage Cycle example. The equilibrium is described in Table 3, as is the equilibrium after leverage is regulated to a smaller level at 0. In the modified Leverage Cycle example, curtailing leverage at 0 not only raises the price of assets at D, but also raises the price of assets at U, because now optimists are more patient than pessimists and so will invest more of their extra money at U into assets than pessimists withdraw when they receive smaller debt payments. Since optimists are selling the asset at U, this price rise helps optimists and hurts pessimists. Since optimists care more about U than pessimists do, this increases the sum of utilities (normalized so that the marginal utility of consumption at 0 is 1 for all agents). At D the optimists are buying the extra endowment of assets, and so the price rise hurts optimists and helps the pessimist sellers. But pessimists care more about D than optimists, and so the price change again raises total utility.

The increase in future total utility is more than the loss in total utility from curtailing the already rationed borrowing. But curtailing leverage has one more effect. It lowers the price of assets at 0, thereby helping optimists and hurting pessimists. In order to make all agents better off, the policy intervention should reduce leverage and transfer some consumption at time 0 from optimists to pessimists. Both of these objectives could be achieved by taxing borrowing and then redistributing the revenue to all

agents (and not back to those who paid the tax). Table 3 shows that such an intervention is indeed Pareto improving.

The most important benefit from curtailing leverage is not captured by the modified Leverage Example, because in binomial economies with financial assets there is no default. Geanakoplos and Kubler (2013) constructs a multi state example with common beliefs in which there is heterogeneity because optimists get utility from housing. They are thus led to borrow so much on their mortgages that some of them will default in some of the states. Curtailing leverage has the extra benefit that by raising the future price of houses, it reduces default, since whether a homeowner defaults depends on how far underwater he is. Though the lender rationally anticipates that by curtailing the loan, he can reduce the chances of his own borrower defaulting, he does not take into account that by lending less he can help increase future housing prices and thus reduce other borrowers' chances of defaulting. If defaulting homeowners neglect repairs on their houses, curtailing leverage can lead to Pareto improvements.

3.8   Agent Heterogeneity

The leverage cycle relies crucially on agent heterogeneity. In the example, heterogeneity was created by differences in beliefs. But there are many other sources of heterogeneity. Some agents are more risk tolerant than others. Some agents can use assets more productively than others. Some households like living in houses more than others. And some agents need assets to hedge more than others. It is very important to understand that the connection between leverage and asset prices does not rely on differences in beliefs.

To see this, consider a variant of the our leverage cycle example in which agents have the same log utilities and identical beliefs, so that $\gamma_{sU}^{O} = \gamma_{sU}^{P} = .5$ for all $s \in S \setminus S_T$. Endowments of the consumption good for the O group are: $e_0^O = 8.5, e_U^O = 5.5, e_D^O = 38.8, e_{UU}^O = e_{UD}^O = 5.4, e_{DU}^O = 30.6$ and $e_{DD}^O = 250$, and for the P group are: $e_0^P = 100, e_U^P = 125, e_D^P = 83.2, e_{UU}^P = e_{UD}^P = 125.4, e_{DU}^P = 104.2$ and $e_{DD}^P = 69.3$. For the O group, the asset is a natural hedge to their endowments; for the P group, the asset is not so useful. Starting with the same endowments of the asset as in our leverage cycle example, equilibrium asset prices and portfolio trades are identical to those in the leverage cycle example displayed in Table 2.[17]

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[17]In the static example of Section 2 we could have given both agents the same beliefs $\gamma_U^h = .5$,

3.9 Lessons from the Leverage Cycle

The lessons from the leverage cycle are first, that increasing leverage on a broad scale can increase asset prices. Second, that leverage is endogenous and fluctuates with the fear of default. Third, that leverage is therefore related to the degree of uncertainty or volatility of asset markets. Fourth that the scarcity of collateral creates a collateral value that can lead to bubbles in which some asset prices are far above their efficient levels. Fifth, that the booms and busts of the leverage cycle can be smoothed best not by controlling interest rates, but by regulating leverage. Sixth, that the amplitude of the cycle depends on the heterogeneity of the valuations of the investors.

3.10 Credit Cycle vs Leverage Cycle

Our final observation is that the Leverage Cycle is not the same as a Credit Cycle. A Leverage Cycle is a feedback between asset prices and leverage, whereas a Credit Cycle is a feedback between asset prices and borrowing. If LTV is fixed at a constant, then borrowing and asset prices rise and fall together. But leverage is unchanged. Of course a leverage cycle always produces a credit cycle. But the opposite is not true. Classical macroeconomic models of financial frictions such as Kiyotaki and Moore (1997) produce credit cycles but not leverage cycles. In all those models leverage is counter-cyclical despite the fact that borrowing goes down after bad news. The reason for the discrepancy is that to generate leverage cycles, uncertainty is needed, and a particular type of uncertainty: one in which bad news is associated with an increase in future volatility. The literature on credit cycles has traditionally not been concerned with volatility. In our example above, leverage is the most important quantitative driver of the change in asset prices over the cycle. If LTV were held to a constant, the cycle would be considerably dampened.

provided that we gave them different endowments. If the beliefs are homogeneous and endowments for the O group are: $e_0^O = 8.5, e_U^O = 4.85, e_D^O = 42.5$ and for the P group are: $e_0^P = 100, e_U^P = 125.1, e_D^P = 83.26$, then we get the same equilibrium prices, collateral values and liquidity wedge as we did in our example with different beliefs.

4 Multiple Leverage Cycles

Many kinds of collateral exist at the same time, hence there can be many simultaneous leverage cycles. Collateral equilibrium theory not only explains how one leverage cycle might evolve over time, it also explains some commonly observed cross sectional differences and linkages between cycles in different asset classes.

The main lesson of this section is to show how multiple co-existing leverage cycles can explain flight to collateral, contagion and drastic swings in the volume of trade of high quality assets.

4.1 Multiple Leverage Cycles and Flight To Collateral

When similar bad news hits two different asset classes, one asset class often preserves its value better than another. This empirical observation is traditionally given the name Flight to Quality, because it is understood as a migration toward safer assets that have less volatile payoff values. Fostel and Geanakoplos (2008) emphasized a new channel which they called Flight to Collateral: After volatile bad news, collateral values widen more than payoff values, thus giving a different explanation for the diverging prices.

Consider the same economy we described in Section 3, except that now there are two financial assets. Asset Y pays $d_s^Y = 1, s = UU, UD, DU$ and $d_s^Y = .2, s = DD$. Asset Z is perfectly correlated with asset Y and pays $d_s^Z = 1, s = UU, UD, DU$ and $d_s^Z = .1, s = DD$. Y pays more than Z, but we shall see that its most important difference is that it can be leveraged more than Z. Agents start with asset endowments of .5 units of each asset, $a_0^h = (.5, .5), h = O, P$ at the beginning.

Equilibrium is described in Table 4. Portfolio regimes in equilibrium are as in the example in Section 3. After good news, when uncertainty is completely resolved, agents share the assets (which are then perfect substitutes). At 0 and D, optimists hold all the assets in the economy and use them as collateral to borrow money from pessimists. By the No-Default theorem, they leverage by using each asset to back the maxmin contract, and there is no default in equilibrium.

Each asset experiences a leverage cycle. Prices for both assets go down after bad news by more than their expected values decline. There is a severe wealth redistribution

away from optimists, which increases the liquidity wedge, at the same time that borrowing conditions deteriorate.

However, something interesting happens when we look at the cross section variation of all the variables. The gap between asset prices widens after bad news by more than the gap in expected payoffs. The price of Y falls from .906 at 0 to .664 at D, while the price of Z falls from .897 to .621 (a drop of almost 27% vs 31%). After bad news, the payoff value of Y goes down from .844 to .593, while the payoff value of Z goes down from .838 to .586. However, their collateral values move in opposite directions. The collateral value of Z goes down from 0.059 at 0 to 0.035 at D, falling in tandem with its payoff value, amplifying its leverage cycle. Interestingly, the collateral value of Y increases from 0.063 to 0.071, mitigating the gravity of its leverage cycle. In short, the spread in asset prices went from .009 at 0 to .043 at D, a widening of .034. The spread in collateral values went from .004 at 0 to .036 at D, a widening of .032. Thus the widening spread in prices is almost entirely explained by the widening of collateral values.

Table 4: Equilibrium Flight to Collateral.

States	$s=0$	$s=U$	$s=D$	$(0-D)/0\%$	$s=0$	$s=U$	$s=D$	$(0-D)/0\%$
			Asset Y				Asset Z	
p	0.906	0.982	0.664	26.71	0.897	0.982	0.621	30.76
PV	0.844	0.982	0.593	29.73	0.838	0.98	0.586	30.07
CV	0.063	0	0.071	-12.69	0.059	0	0.035	40.67
π_{j^*}	.658		0.201		0.615		0.100	
LTV	0.726		0.303		0.686		0.162	
				Asset Holdings				
Opts	1	.4367	1		1	.4367	1	
Pess	0	.5632	0		0	.5632	0	
				Debt Contract Trades				
Opts	1		1		1		1	
Pess	-1		-1		-1		-1	

States	$s=0$	$s=U$	$s=D$	$s=UU$	$s=UD$	$s=DU$	$s=DD$
ω	0.105		0.541				
				Consumption			
Opts	8.87	10.25	7.51	10.43	10.43	11.7	10
Pess	99.63	99.75	100.98	101.56	101.56	100.3	100.3

Flight to collateral occurs when the liquidity wedge is high and the dispersion of LTVs is high. During a flight to collateral, investors would rather buy those assets that enable them to borrow money more easily (higher LTVs). The other side of the coin is that investors who need to raise cash get more by selling those assets on which they borrowed less money because the sales revenues net of loan repayments are higher.

Flight to collateral is related to what other papers have called Flight to Liquidity. Flight to liquidity was discussed by Vayanos (2004) in a model where an asset's liquidity is defined by its exogenously given transaction cost. In Brunnermeier and Pedersen (2009), market liquidity is the gap between fundamental value and the transaction price. They show how this market liquidity interacts with funding liquidity (given by trader's capital and margin requirements) generating Flight to liquidity. In our model an asset's liquidity is given by its capacity as collateral to raise cash. Hence, our flight to collateral arises from different leverage cycles in equilibrium and their interaction with the liquidity wedge cycle.

4.2   Multiple Leverage Cycles and Contagion

In this section we show how multiple coexisting leverage cycles can explain contagion. When bad news hits one asset class, the resulting fall in its price can migrate to other assets, even if their payoffs are statistically independent from the original crashing assets. To illustrate this consider the same economy as in Section 4.1, except that now both asset payoffs at the end are independent.

For that we need to assume that $S = \{0, U, D, UU, UD, DUU, DUD, DDU, DDD\}$, where the last four states are immediate successors of D. According to each agent $h \in H$, the transition probability from D to each of its immediate successors $D\alpha\beta$, $\alpha, \beta \in \{U, D\}$, is $\gamma_\alpha^h \gamma_\beta^h$. Asset Y pays $d_s^Y = 1$ in terminal states in which the second to last letter is U, $s = UU, UD, DUU, DUD$ and $d_s^Y = .2, s = DDU, DDD$. Asset Z is independent from asset Y and pays $d_s^Z = 1$ in all terminal states in which the last letter is U, $s = UU, DUU, DDU$, and $d_s^Z = .1, s = UD, DUD, DDD$. After U, asset Y pays 1 for sure. But Z pays 1 or .1 after U and after D. Hence, U is good news for Y and D is bad news for Y, but nothing new is learned about asset Z in the middle period. Figure 3 shows asset payoffs.
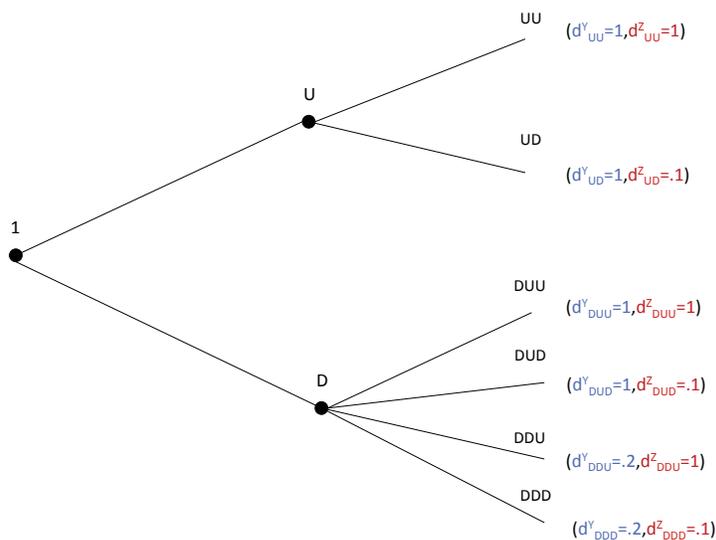
Figure 3: Assets Y and Z Independent Payoffs.

In this example bad news is about Y only. So we should expect the price of Y to go down after bad news due to a deterioration of its expected payoff value. But we should not expect the price of asset Z to go down after bad news about Y.

Equilibrium is described in Table 5. As in all the previous examples, optimists hold all the assets at 0 and D and use them as collateral to back the maxmin contract. After good news, optimists now hold all of asset Z and none of Y, about which uncertainty is completely resolved. This is because at U optimists see a special opportunity to invest in Z, which they think pessimists under-value, but no advantage in holding the sure asset Y.

As expected, asset Y experiences a leverage cycle: Its price rises from .925 to .991 after good news and crashes after bad news by more than its payoff value, going down from .925 to .667. Surprisingly, the price of Z also goes up from .789 to .827 after good news about Y, and goes down by more than 20% from .789 to .624 after bad news about Y. The leverage cycle on Y migrates to asset class Z, producing a leverage cycle on this market as well. In short, we see contagion in equilibrium.

Fostel-Geanakoplos (2008) showed that contagion is generated by a change in the liquidity wedge. The Y leverage cycle lowers the liquidity wedge at U and sharply increases the liquidity wedge at D, as we have seen in our previous examples. A

leverage cycle in one asset class alone can move the liquidity wedge. But, the liquidity wedge is a universal factor in valuing all assets, as we saw in Section 2. An increase in the optimists' liquidity wedge at D reduces their valuation of all assets. There is also another factor that can be seen clearly in this example, which Fostel and Geanakoplos (2008) called the Portfolio Effect, amplifying the movements of the liquidity wedge at U and D. At U optimists do not see any advantage in giving up consumption to invest in Y. At D they buy all of Y, reducing their consumption, further increasing the liquidity wedge.

Table 5: Equilibrium Contagion.

States	$s=0$	$s=U$	$s=D$	$(0-D)/0\%$	$s=0$	$s=U$	$s=D$	$(0-D)/0\%$
			Asset Y				Asset Z	
p	0.925	0.991	0.667	27.89	0.789	0.827	0.624	20.91
π_{j^*}	0.660		0.201		0.617	0.099	0.100	
LTV	0.721		0.299		0.792	0.119	0.160	
				Asset Holdings				
Opts	1	0	1		1	1	1	
Pess	0	1	0		0	0	0	
				Debt Contract Trades				
Opts	1		1		1	1	1	
Pess	-1		-1		-1	-1	-1	

States	0	U	D	UU	UD	DUU	DDU	DUD	DDD
ω	0.054		0.544						
					Consumption				
Opts	8.92	9.79	7.51	10.90	10.00	11.70	10.80	10.90	10
Pess	99.57	100.20	100.98	101.10	101.10	100.3	100.3	100.3	100.3

There is a vast literature on contagion. Despite the range of different approaches, there are mainly three different kinds of models. The first blends financial theories with macroeconomic techniques, and seeks international transmission channels associated with macroeconomic variables. Examples of this approach are Corsetti, Pesenti, and Roubini (1999), and Pavlova and Rigobon (2008). The second kind models contagion as information transmission. In this case the fundamentals of assets are assumed to be correlated. When one asset declines in price because of noise trading, rational traders reduce the prices of all assets since they are unable to distinguish declines due to fundamentals from declines due to noise trading. Examples

of this approach are King and Wadhwani (1990), Calvo and Mendoza (2000) and Kodres and Pritsker (2002). Finally, there are theories that model contagion through wealth effects, as in Kyle and Xiong (2001). When some key financial actors suffer losses, they liquidate positions in several markets, and this sell-off generates price comovement. Our model shares with the last two approaches a focus exclusively on contagion as a financial market phenomenon. But our model further shows how leverage cycles can produce contagion in less extreme but more frequent market conditions: the anxious economy, where there is no sell-off. The leverage cycle causes contagion even when trade patterns differ from those observed during acute crises.

4.3 Multiple Leverage Cycles, Adverse Selection and the Volume of Trade.

Following Dubey-Geanakoplos (2002) and Fostel-Geanakoplos (2008), we now extend the model to encompass asymmetric information: owners of the assets know their quality, but investors do not. We will show that multiple leverage cycles can generate violent swings in the volume of trade. The volume of sales of good assets will always be diminished because their owners need to curtail their sales in order to signal buyers they are good. In normal times the quantity reduction will be small, but after volatile bad news the quantity reduction can be severe. The endogenous quantity signals are modeled in the same way endogenous leverage was modeled in Section 2.

Consider the same economy of Section 4.1, except that now investors own no assets. We suppose that assets Y and Z are owned by a new class of agents we call issuers. At each state $s \in \{0, U, D\}$ there is one class of issuers that are young at s and that in aggregate are endowed with one unit of Y and another class of issuers that are young at s and are endowed with one unit of Z. We assume that issuers have logarithmic utility for consumption in the state $s \in \{0, U, D\}$ in which they are born and in states $t \in S_T(s)$, where $S_T(s)$ is the set of terminal nodes that follow s. Issuers are endowed with 6.5 units of the consumption good when young and 10 units of the consumption good in each of their terminal states. Their beliefs are the same as the optimists. Finally, we suppose that issuers trade only when young.

Investors cannot distinguish the assets, but their issuers can. Investors judge how good the assets are according to how much an issuer sells, figuring that if the assets were so good their owner wouldn't want to sell so many of them. Rothschild and

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Stiglitz (1976) showed that owners of good assets cut down on the volume of their sales to signal they are good.

We can combine perfect competition with quantity signaling by defining equilibrium in terms of a quantity-price schedule. In each state $s \in S$, there are many different markets, each characterized by a quantity limit (which a seller in that market cannot exceed) and its associated market clearing price:

$$\vec{p}_s = \{(x_s, p_s(x_s)); x_s \in (0, 1], p_s \in \Re_+\}. \tag{16}$$

The quantity-price schedule $\vec{p}_s$ is taken as given and issuers and investors decide in which of these markets to participate. We assume exclusivity, i.e., issuers can only sell in one quantity market. So they must choose a quantity x_s to sell and then take as given the corresponding market clearing price $p_s(x_s)$. This is exactly analogous to how we modeled debt contracts when we described endogenous leverage in Section 2.

Given the price schedule $\vec{p}_s$, each young issuer decides consumption and issuance in order to maximize his utility subject to the budget set defined by

$$\begin{aligned}
B(\vec{p}_s) = \{(\mathbf{c}, x) &\in R_+^{1+S_T(s)} \times R_+ : \\
c_s &\leq 6.5 + p_s(x)x \\
x &\leq 1 \\
\forall t &\in S_T(s) : c_t = 10 + (1-x)d_t\}
\end{aligned}$$

where $d_t = d_t^Y$ or $d_t = d_t^Z$ depending on whether the issuer is of type Y or Z.

Investors who buy assets in market $(x_s, p_s(x_s))$ get a pro rata share of the deliveries of all assets sold in that market. Investors are assumed to be rational and to have the correct expectation of deliveries from each market $(x_s, p_s(x_s))$. Thus, if only one issuer type is choosing to sell at the quantity x_s, then it reveals its type, and from then on, its asset payoffs are known to be the corresponding type.

With this interpretation there is room for *signaling* as well as *adverse selection* without destroying market anonymity. Firms may (falsely) signal more reliable deliveries by publicly committing to (small) quantity markets where the prices are high because the market expects only good types to sell there. The quantity limit characterizing each asset market is exogenous and the associated price is set endogenously as in any

traditional competitive model. However, it may occur that in equilibrium only a few asset quantity markets are active, even when all the quantity markets are priced in equilibrium. In this sense, the active quantities are set endogenously as well, without the need of any contract designer. Market clearing and optimizing behavior are enough. Dubey-Geanakoplos (2002) proved that in models like the one considered in this section, there is a unique equilibrium that is robust to perturbations. This is a separating equilibrium of the kind Rothschild-Stiglitz (1972) studied in which the Z owners sell all they want at the price $p = p_s(x_s^Z)$ they receive, thereby revealing they are the bad type, while the Y owners sell less than they would like at the price $p' = p_s(x_s^Y)$ they receive, thereby revealing they are the good type; the Z firms are indifferent between their choice, of selling a large quantity at a small price, and imitating the Y firms and selling a smaller quantity at a larger price.

Table 6 presents the unique separating equilibrium. The leverage cycle price behavior described in Section 4.1 is still present here: there are two coexisting leverage cycles and flight to collateral. The new thing in this simulation comes from the supply side. In order to signal that their assets are good (so that investors will pay more for them and be able to borrow more using them as collateral), the Y owners always sell less than they would if their types were common knowledge. However, after bad news at D, the drop in volume of their sales is huge. The bad Z type issuance goes down 22% from $x_0^Z = 1$ at 0 to $x_D^Z = .78$ at D, whereas the good type Y issuance goes down 67% from $x_0^Y = .92$ all the way to $x_D^Y = .30$.

It is not surprising that with the bad news and the corresponding fall in prices, equilibrium issuance falls as well, because issuers are optimists and do not want to sell at such low prices. The interesting thing is that flight to collateral combined with informational asymmetries generates such a big drop in good issuance, even though the news is almost equally bad for both assets. The explanation is that the bigger price spread between types caused by the flight to collateral requires a smaller good type issuance for a separating equilibrium to exist. Unless the good issuance level x_D^Y becomes onerously low, bad types would be more tempted by the bigger price spread to mimic good types and sell at the high price. The good types are able to separate themselves by choosing low enough quantities since it is more costly for the bad type to rely on the payoff of its own asset for final consumption than it is for the good type.

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Table 6: Equilibrium Adverse Selection.

States	$s=0$	$s=U$	$s=D$	$(0-D)/0\%$	$s=0$	$s=U$	$s=D$	$(0-D)/0\%$
			Asset Y				Asset Z	
p	0.878	0.981	0.665	24.25	0.866	0.981	0.622	28.17
π_{j^*}	0.654		0.201		0.612		0.100	
LTV	0.745		0.303		0.707		0.162	
			Issuance					
	0.923	1	0.300	67.49	1	1	0.78	22.00
			Asset Holdings					
Opts	0.923	0.426	0.300		1	0.426	0.78	
Pess	0	0.573	0		0	0.573	0	
			Debt Contract Trades					
Opts	0.923		0.300		1		0.78	
Pess	-0.923		-0.300		-1		-0.78	
States	$s=0$	$s=U$	$s=D$	$s=UU$	$s=UD$	$s=DU$	$s=DD$	
ω	0.208		0.536					

There is a growing literature that tries to model asymmetric information within general equilibrium, like Gale (1992), Bisin and Gottardi (2006), and Rustichini and Siconolfi (2008). Our model combines asymmetric information in a general equilibrium model with a model of endogenous credit constraints and leverage.

5 Leverage and Agent Based Models

In collateral equilibrium, agents are hyper rational, correctly anticipating what prices will be in all future states. This makes it clear that while the leverage cycle depends on agent heterogeneity, it does not depend on irrational exuberance. But there is a cost to working with general equilibrium models: equilibrium is difficult to compute.

Thurner-Farmer-Geanakoplos (2013) describes the leverage cycle in a so-called agent based model in which agents follow behavioral rules that determine their actions as a function of their past observations, rather than as a function of their expectations about the future. The long run dynamic (spanning tens of thousands of periods) of the model is easy to compute, since it involves forward iteration without the need for finding a fixed point in which expectations match what actually happens. The

agent based model is also able to represent the key features of the leverage cycle, in an even richer setting than the general equilibrium model of Section 2, but at the cost of introducing ad hoc behavioral rules that cannot be justified as fully rational.

There is one asset Y of unit size in the economy in addition to money. Agent heterogeneity is represented by two classes of investors, noise traders and funds. Noise traders "believe" the asset is worth what it sold for the previous period plus some Gaussian noise, with a tiny bias toward a fixed value V. Their behavioral rule is that every period they are willing to hold $\eta(t)$ dollars worth of assets, where $log\eta(t) = \rho log\eta(t-1) + (1-\rho)log(V) + \sigma\chi(t)$ and $\chi(t)$ is Gaussian with mean 0 and variance 1, and ϱ is just barely less than 1. Fund managers "believe" the asset price will revert to V, and are willing to bet on that by leveraging. Their behavioral rule is that every period they are willing to hold $\mu(t)$ dollars worth of assets, where $\mu(t)$ is 0 when the current asset price $p(t)$ is greater than long run value V, and $\mu(t)$ increases linearly at rate β as the undervaluation $V - p(t)$ grows above zero.

One period loans are always available from the bank at a fixed rate r, but they must be collateralized by the asset holdings. Let $\delta(t)$ be the amount borrowed at time t; a negative $\delta(t)$ signifies money deposited in the bank at interest rate r. A crucial assumption is that the asset holdings of a fund manager can never imply a leverage exceeding a hard upper bound. More precisely, suppose that fund managers hold only assets, and give or take one period loans from the bank. Their wealth at time t is then $W(t) = \frac{\mu(t-1)}{p(t-1)}p(t) - (1+r)\delta(t-1)$ and their budget constraint is $\mu(t) = W(t) + \delta(t)$. The leverage constraint is that $\mu(t)/W(t) \leq \lambda$. Observe that if $\lambda = 5$, then a \$1 increase in wealth permits the fund manager to buy \$5 more assets, and a \$1 decrease in wealth forces him to hold \$5 less in assets if he was up against his leverage constraint. A drop in asset prices makes every fund manager eager to put more money into the asset, but may force these willing buyers to become big sellers.

Thurner et al (2013) suppose that there are many fund managers with different aggressiveness β and different leverage constraints λ. They suppose that every manager begins at time 0 with a stock of cash and a memory of past asset prices. They suppose that the bank sets leverage limits λ for each fund as a decreasing function of past volatility of asset prices (computed over a fixed window of 100 periods). Finally they suppose there is a household sector that has income which it allocates to fund managers based on their recent returns (computed over the same window of 100 periods), with more money allocated to more successful funds.

The model shows that if the leverage limits λ are held low, then prices very nearly follow a random walk. If the leverage limits are high (around 5 or 7), then price movements display a fat tail and clustered volatility. The agent based model also allows us to understand the leverage cycle as a process with a slow build up and a violent crash that also might take many periods.

The leverage cycle does not arise from a once and for all exogenous shock to asset payoffs. On the contrary, the leverage cycle is a process crucially depending on the heterogeneity of investors. Some investors are more aggressive (higher β) than others, or more able to leverage (higher λ) and buy than others. When the market is going up, these investors do well and via their increased relative wealth and their superior adventurousness, a relatively small group of them will come to hold a disproportionate share of the assets. When the market is controlled by a smaller group of agents who are more homogeneous than the market as a whole, their commonality of outlook will tend to reduce the volatility of asset prices. Fund managers always believe the value of the asset is V, while the noise traders randomly fluctuate all over in their opinions. The bigger the wealth of fund managers, the more stable the asset prices. The large Wall Street investment banks tend to have similar computer models and managers trained at the same schools, and thus similar, stable valuations of assets. But if λ is a decreasing function of volatility, as will arise in any endogenous model of leverage and as is mandated by Basel III rules, their own success in dominating the market will enable the fund managers to leverage more, which will give them a still more disproportionate share of the assets, and reduce volatility still further. Despite the leverage restrictions intended from Basle III, the extremely low volatility still gives room for very high leverage. After a run of good luck, the market will appear to be at its best: A zenith of prices, a low of volatility, and a flood of profits for the biggest funds. But in fact the market is poised for its biggest fall.

At this point some exogenous bad luck, causing noise traders to reduce their asset demand, will reduce asset prices and have a disproportionate effect on the wealth of the most adventurous buyers. Of course they will regard the situation as an even greater buying opportunity, but in order to maintain their prior leverage levels they will be forced to sell instead of buying. At this point volatility will rise and the Basle III lending rules will force them to reduce leverage and sell more. The next class of buyers will also not be able to buy much because their access to leverage will also suddenly be curtailed. The assets will cascade down to a less and less willing group of buyers. After awhile, the household sector will reallocate its money away from the

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most aggressive funds, again causing them to sell. In the end, the price of the assets will fall not so much because of the exogenous shock, but because the marginal buyer will be so different from the marginal buyer before the shock.

The recent crash in home and mortgage prices, and the ensuing global recession, has brought forth numerous proposals for the regulation of leverage. As Poledna et al (2013) say, the trouble is that many of these proposals ignore the mechanism of the leverage cycle, and thus might unwittingly do more harm than good. Under some conditions, Basle II not only would fail to stop the leverage build up, but it would make the de leveraging crash much worse by curtailing all the willing buyers simultaneously.

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