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Anat Bracha

Donald J. Brown

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COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
YALE UNIVERSITY
Box 208281
New Haven, Connecticut 06520-8281

http://cowles.econ.yale.edu/

Affective Decision Making: A Behavioral Theory of Choice*

Anat Bracha[†] and Donald J. Brown[‡]

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Abstract

Affective decision-making is a strategic model of choice under risk and uncertainty where we posit two cognitive processes – the "rational" and the "emotional" process. Observed choice is the result of equilibirum in this intrapersonal game. As an example, we present applications of affective decision-making in insurance markets, where the risk perceptions of consumers are endogenous. We then derive the axiomatic foundation of affective decision making, and show that, although beliefs are endogenous, not every pattern of behavior is possible under affective decision making.

JEL classification numbers: D01, D81, G22

Keywords: Affective Choice, Endogenous Risk Perception, Insurance, Variational Preferences.

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†The Eitan Berglas School of Economics, Tel Aviv University

‡The Economics Department, Yale University

1 Introduction

Many of our everyday decisions – such as working on a project, taking the flu shot, or buying insurance – often require probability assessment of future events: probability of a project's success, of getting sick, or the probability of being involved in an accident. In assessing these probabilities, we tend to be optimistically biased, where optimism biased is defined as the tendency to overstate the likelihood of desired future outcomes and understate the likelihood of undesired future outcomes (Irwin 1953; Weinstein 1980; Slovic et al 1982; Slovic 2000). A young man drinking at a bar thinking it would be safe for him to drive home is an example; an entrepreneur who starts a new business, confident that she is going to make it and succeed where others have failed, is another. Indeed, one can argue that although statistical data exist, each person has their own alcohol resistance or ability. Hence, the young man may have good reasons to believe overall empirical frequencies does not apply to him; similarly, the entrepreneur can be justified in being confident that she is going to make it (and we can be justified in being confident this paper will be published at a top journal). The common feature to these examples is that decision makers have some freedom in constructing their beliefs, and they are optimistic – they seem to be choosing the beliefs that are convenient to them. This is true even for events that are purely random, and in fact, studies in psychology show that optimism bias is associated with mental health (Alloy and Ahrens 1987, Taylor and Brown 1988)

Interestingly, optimism bias is, by definition, inconsistent with the independence of weights and payoffs found in most individual choice models such as expected utility, subjective expected utility, prospect theory, maxmin expected utility and variational preferences. In this paper we accommodate for optimism bias by suggesting an alternative model of choice under risk where decision weights– perceived risk – are endogenized.

We achieve this by considering two distinct psychological processes that mutually determine choice, an approach that is inspired by Kahneman (2003) and corresponds to a widely discussed idea in psychology, with a sizable evidence in neuroscience.

Psychology distinguishes between two systems, such as analytical and intuitive, or deliberate and emotional processing (Chaiken and Trope 1999), and neuroscience suggests different brain modules that specialize in different activities. For instance, the amygdala is associated with emotions while the prefrontal cortex is associated with higher level, deliberate thinking (e.g. Reisberg, 2001). Behavior is then thought to be a result of the different systems interacting (e.g., Sacks 1985; Damasio 1994; Epstein 1994; LeDoux 2000; Gray et al, 2002, Camerer, Loewenstein and Prelec 2004; Pessoa 2008). For example, Gray et al (2002) conclude that "at some point of processing, functional specialization is lost, and emotion and cognition conjointly and equally contribute to the control of thought and behavior", and recently, Pessoa (2008) adds by arguing that "...emotions and cognition not only strongly interact in the brain, but that they are often integrated so that they jointly contribute to behavior", a point made also in the specific context of expectation formation.

Although the evidence on modular brain and the dual-processes theory cannot typically be pinned down to the formation of beliefs, given that beliefs formation is partially affected by the beliefs we would like to have, i.e., affective considerations, decision making under risk maps naturally to the interplay between two cognitive processes as suggested by Kahneman. That is, decision making under risk can be modeled as a *deliberate* process choosing an optimal action, and *emotional* cognitive process forming risk perception. That is, the deliberate process chooses a lottery or a sure amount, chooses insurance, or a spouse, while the emotional process considers the probability of winning a lottery, being in a car accident, or getting married. Choice is, then, the result of the interaction of the two processes.

In our model we call these systems of reasoning the *rational process* and the *emotional process*. The rational process coincides with the expected utility model. That is, for a given risk perception, i.e., perceived probability distribution, it maximizes expected utility. The emotional process is where risk perception is formed. In particular, the agent selects an optimal risk perception to balance two contradictory impulses: (1) affective motivation and (2) a taste for accuracy. This is a definition of motivated reasoning, a psychological mechanism where emotional goals motivate agent's beliefs, e.g., Kunda (1990), and is a source of psychological biases, such as optimism bias. Affective motivation is the desire to hold a favorable personal risk perception — optimism — and is captured by the expected utility term. The desire for accuracy is the mental cost incurred by the agent for holding beliefs other than her *base rate,* given her desire for favorable risk beliefs. The base rate is the belief that minimizes the mental cost function of the emotional process. This is the agent's *correct* risk belief, if her risks are objective such as mortality tables.

To reach a decision, the two processes interact to achieve consistency. This interaction is modeled as a simultaneous-move *intrapersonal* game, and consistency between the two processes, which represents the candidate for choice, is characterized by the pure strategy Nash equilibria of the game.

Indeed, recent literature in economic theory recognize the possibility that agents might choose their beliefs in a self-serving or optimistic way, such as Akerlof and Dickens (1982) , Bodner and Prelec (2001) , Bénabou and Tirole (2002) , Yariv (2002) , Caplin and Leahy (2004) , Brunnermeier and Parker (2005) , and Kőszegi (2006). The dual processes hypothesis, as well, was recently recognized in economic modeling. Specifically, in models of self-control and addiction such as Thaler and Shefrin's (1981), Bernheim and Rangel's (2004) , Loewenstein and O'Donoghue (2004), Benhabib and Bisin's (2005) , Fudenberg and Levine (2006), and Brocas and Carrillo (2008).

Existing models are restricted in that choice of beliefs and action is not done in tandem and assume that an agent chooses beliefs in a strategic manner to resolve a trade-off between a standard instrumental payoff and some notion of psychologically based belief utility[1], while the existing models of dual processes are restricted in that the two systems, or decision modes, are conceived as mutually exclusive.

The first contribution of our paper is modeling optimism bias in a novel approach – using a simultaneous choice of action and beliefs, where the trade-off is done by an intrapersonal *game* between two processes – a formulation that may be viewed as an attempt to capture specialization and integration of brain activity. More specifically, in the intrapersonal game the emotional process chooses optimal beliefs (for a given action) to maximize mental profit from those beliefs. The rational process chooses optimal action (for a given belief) to maximize expected utility. Choice is an equilibrium outcome determined by the two accounts. Hence, the two processes in affective decision making (ADM) are simultaneously active, and mutually determine choice as opposed to the common dual-processes formulation. While there are cases where a descriptive model seems to require mutually exclusive systems, as is the case of self-control and addiction, there are other cases where a descriptive model seems to require several different processes that together determine choice. We provide such a formulation– one process chooses action while the other forms perceptions, and both are necessary for decision making.

As an application of affective decision-making, we present an example of the demand for insurance in a world with two states of nature: Bad and Good. The relevant probability distribution in insurance markets is personal risk, hence the demand for insurance may depend on optimism bias. Affective choice in insurance markets is defined as the insurance level and risk perception which constitute a pure strategy Nash Equilibrium of the ADM intrapersonal game.

The systematic departure of the ADM model from the expected utility model allows for both optimism and pessimism in choosing the level of insurance, and shows, consistent with consumer research (Keller and Block 1996), that campaigns intended to educate consumers on the loss size in the bad state can have the unintended consequence that consumers purchase less, rather than more, insurance. Hence, the ADM model suggests that the failure of the expected utility model to explain some data sets may be due to systematic affective biases. Furthermore, the ADM intrapersonal game is a potential game – where a (potential) function, of a penalized SEU form, captures the best response dynamic of the game. This property has the natural interpretation of the utility function for the composite agent, or integration of the two systems, and allows for the second contribution of this paper, next.

Deviations from the basic models of rational choice often raise the concern that the theory lacks the discipline imposed by a clear paradigm, and that, as a result, 'anything goes'. ADM is no exception. Once we allow agents to choose their beliefs, one might suspect that any mode of behavior, as well as any specific choice, can be rationalized by appropriately chosen beliefs, rendering the theory vacuous. Is this the case with ADM?

An answer to this question may be found in an axiomatic approach. A major role

[1] The axiomatic foundation for this is provided by Caplin and Leahy (2001) and Yariv (2001).

of axiomatic derivations, relating a theory to observations, is precisely this: to tell us under which circumstances the theory will have to be abandoned, and to guarantee that such circumstances exist, namely, that the theory is refutable.

The second contribution of our paper is in deriving axiomatization of the ADM model, and showing that ADM is indeed refutable.

We start with a decision maker's preference relation over acts, as in Savage (1954) or (more precisely) Anscombe-Aumann (1963) and provide a set of axioms, stated in terms of observable behavior, that are equivalent to the ADM model. We follow Maccheroni, Marinacci and Rustichini's [MMR] (2006) variational preferences, a general class of preferences that rationalize ambiguity-averse choices. Variational preferences subsume both maxmin preferences (Gilboa and Schmeidler 1989) and multiplier preferences (Hansen and Sargent 2000), where the decision maker is playing a sequential game against a malevolent nature who moves last. Hence the solution concept is maxmin. In the affective decision making (ADM) model proposed in this paper the rational and the emotional process of the decision-maker are engaged in a simultaneous move, potential game, where the solution concept is Nash equilibrium. Both classes of models are penalized SEU models. In the variational preferences models the penalty reflects the decision maker's uncertainty that her "subjective" beliefs about the states of the world are the correct state probabilities. In the ADM model, the penalty reflects the mental cost of her "optimistic" beliefs about preferred outcomes. This formal resemblance is reflected in a corresponding similarity between the set of axioms.

More specifically, variational preferences are characterized by six axioms, where axiom 5, due to Schmeidler (1989), is the axiom for ambiguity aversion. This axiom has the simple geometric interpretation that the preference relation over acts is quasi-concave. Moreover, if axiom 5 is replaced by axiom $5^{'}$ where the preference relation over acts is quasi-linear, then axioms :$1-4$, $5^{'}$ and 6 characterize the subjective expected utility (SEU) model. Both of these results are proven in MMR (2006).

We show that the third and final possibility is that the preference relation over acts is quasi-convex gives rise to the ADM formulation. That is, preferences over acts can be quasi-concave, quasi-linear or quasi-convex. If in addition preferences satisfy axioms $1-4$ and axiom 6 in MMR (2006), then the corresponding classes of preferences over acts are: variational preferences, SEU preferences and ADM preferences.

The resemblance between variational preferences and ADM seems at first glance to be mostly formal: models of uncertainty aversion, allowing for a set of probabilities, do not typically suggest that the decision maker derives utility from her beliefs or chooses them. Indeed, if she did choose her beliefs, there would be little point in choosing the worst possible ones. Rather, the common interpretation is that the decision maker cannot pin down a unique probability distribution, and, as a result, has the entire set as her "belief". Then, a behavioral assumption of uncertainty aversion is added to explain the choice of the minimal expected utility (plus a "cost" factor) rather than, say, the maximal one. By contrast, in our model the decision maker is assumed to choose specific beliefs, and the maximum is not taken to be an attitude towards uncertainty, but rather an aspect of rational choice. Yet, the equivalence to ambiguity-seeking preferences is interesting. It suggest that optimism bias may serve as an internal mechanism for uncertainty and suggest that motivation may explain ambiguity-seeking attitudes along the lines of the competence hypothesis of Heath and Tversky (1991) or the status quo bias of Roca et al (2006). Hence, the insight arising from the axiomatic foundation is an alternative behavioral interpretation of ADM as ambiguity-seeking choice.

The axiomatic foundation and its suggested alternative interpretation of the ADM model is an additional difference between our paper and the existing literature. In particular, the closest model to the ADM is Optimal Expectations by Brunnermeier and Parker (2005) . Optimal expectations consider an agent who chooses both beliefs and actions in a dynamic model, where beliefs are chosen at period one for all future periods trading off greater anticipated utility with cost of poor decisions due to optimistic beliefs. Hence, optimal expectations are optimistic beliefs not constrained by reality. Our model, in contrast, is a static model where beliefs and actions mutually determine choice, and where beliefs trade off greater anticipated utility with mental cost of distorting beliefs, costs that are function of reality. That is, the further away costs are from objective probabilities, the greater are the mental costs. Having a simultaneous framwork where costs are based solely on beliefs is a simpler framework, consistent with attention effect, cognitive dissonance and, more importantly, the static framework we suggest is a potential game. The potential has the natural interpretation of the utility function for the composite agent, it allows for future welfare analysis and for us to characterize the ADM choice by six axioms. This, in turn, allows an interpretation of the ADM model as a model of ambiguity-seeking choice behavior. An interpretation not shared by Brunnermeier and Parker (2005) and the other dual process models

The remainder of the paper is organized as follows. In section 2 we present the application of the ADM intrapersonal game to insurance markets, and in section 3, we present the axiomatic foundation of ADM and discuss its natural separation between risk and ambiguity attitudes. All proofs are in the Appendix.

2 The ADM Intrapersonal Game

Affective decision-making (ADM) is a theory of choice, which generalizes expected utility theory by positing the existence of two cognitive processes – the *rational* and the *emotional process*. Observed choice is the result of their simultaneous interaction. This theory accommodates endogenity of beliefs, probability perceptions and tastes. In this paper, we present a model of affective choice in insurance markets, where probability perceptions are endogenous.

Consider an agent facing two possible future states of the world, Bad and Good with associated wealth levels ω_B and ω_G , where $\omega_B < \omega_G$. The agent has a strictly increasing, strictly concave, smooth utility function of wealth, $u(W)$, with $\lim_{w\to-\infty} Du(W)=\infty$, $\lim_{w\to\infty} Du(W)=0$.[2] Risk perception is defined as the perceived probability $\beta\in[0,1]$ of the Bad state occurring. To avoid (perceived) risk, the agent can purchase or sell insurance $I\in(-\infty,\infty)$ to smooth her wealth across the two states of the world. The insurance premium rate, $\gamma\in(0,1)$ is fixed for all levels of insurance.

The *rational process* chooses an optimal insurance (I^*) to maximize expected utility given a perceived risk β. Specifically, the rational process maximizes the following objective function:

$$\underset{I}{max}\left\{\beta u(\omega_B+(1-\gamma)I)+(1-\beta)u(\omega_G-\gamma I)\right\}.$$

The *emotional process* chooses an optimal risk perception (β^*) given an insurance level I, to balance affective motivation and taste for accuracy. Specifically, the emotional process maximizes the following objective function:

$$\underset{\beta}{max}\left\{\beta u(\omega_B+(1-\gamma)I)+(1-\beta)u(\omega_G-\gamma I)-c(\beta;\beta_0)\right\}.$$

Affective motivation is captured by the expected utility term – the agent would like to assign the highest possible weight to her preferred state of the world. Taste for accuracy is modeled by introducing a mental cost function $c(\beta;\beta_0)$ that is a nonnegative, and smooth function of β. It is strictly convex in β, and reaches a minimum at $\beta=\beta_0$, where β_0 is the objective probability. See Figure 1.

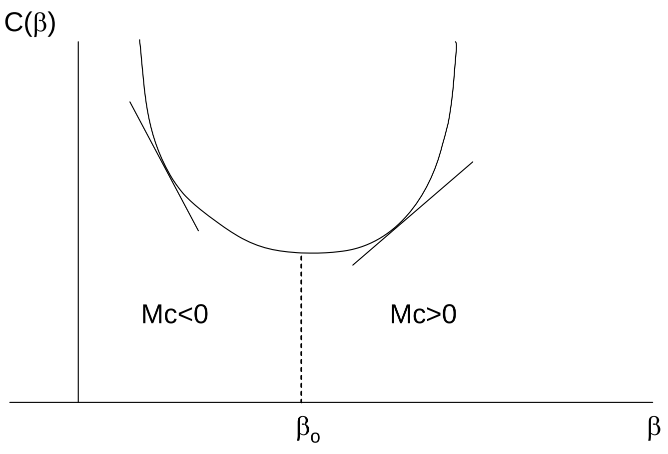

Figure 1

[2] All qualitative results remain the same for the case of $\lim_{W\to0}Du(W)=\infty$, $\lim_{W\to\infty}Du(W)=0$.

The farther away β is from β_0, the greater are the psychological cost. This is because to justify favorable beliefs agents need to use strategies such as the availability heuristic, which can be unconsciously manipulated to arrive at the desired beliefs. Such mental strategies, or justification processes, are likely to be costly and are captured by the cost function. We assume that biased recall becomes increasingly more costly as the distance between desired beliefs β and the objective odds β_0 increases. We will assume that $c(\beta; \beta_0)$ is a smooth function of β_0. It is well-known that agents attribute a special quality to situations corresponding to the extreme beliefs $\beta \in \{0, 1\}$ (Kahneman and Tversky 1979). Hence we assume that there exist limits $\underline{\beta}$, $\overline{\beta} \in (0, 1)$ such that for $\beta \in (\underline{\beta}, \overline{\beta})$, $c(\beta; \beta_0)$ is finite, and $lim_{\beta \to \underline{\beta}} c(\beta; \beta_0) = lim_{\beta \to \overline{\beta}} c(\beta; \beta_0) = +\infty$.

The interaction of the two processes in decision-making is modeled using an intrapersonal simultaneous-move game. Modeling the interaction of the two processes as a simultaneous move game reflects a recent view in cognitive neuroscience; namely, both processes mutually determine the performance of the task at hand (Damasio 1994).

Definition 1 *An intrapersonal game is a simultaneous move game of two players, namely, the rational and the emotional processes. The strategy of the rational process is an insurance level,* $I \in (-\infty, \infty)$*, and the strategy of the emotional process is a risk perception ,* $\beta \in (\underline{\beta}, \overline{\beta})$*. The payoff function for the rational process* $g : (\underline{\beta}, \overline{\beta}) \times (-\infty, \infty) \to R$ *is* $g(\beta, I) \equiv \beta u(\omega_B + (1-\gamma)I) + (1-\beta)u(\omega_G - \gamma I)$*. The payoff function for the emotional process* $\psi : (\underline{\beta}, \overline{\beta}) \times (-\infty, \infty) \to R$ *is* $\psi(\beta, I) \equiv g(\beta, I) - c(\beta; \beta_0)$*, where* $c(\cdot)$ *is the mental cost function of holding belief* β*, which reaches a minimum at* β_0*.*

The pure strategy Nash equilibria of this game, if they exist, are the natural candidates for the agent's choice, as they represent mutually determined choice and reflect consistency between the rational and emotional processes. Excluding the case of tangency between the best responses of the two processes, we have the following existence theorem (see Figure 2 for illustration).

Proposition 2 *The ADM intrapersonal game has an odd number of pure strategy Nash equilibria. The set of Nash equilibria is a chain in* R^2*, under the standard partial order on points in the plane.*[3]

[3] The existence of a pure strategy Nash equilibrium also can be derived for the case of a logarithmic utility function, in which the agent's income in each state is not negative.

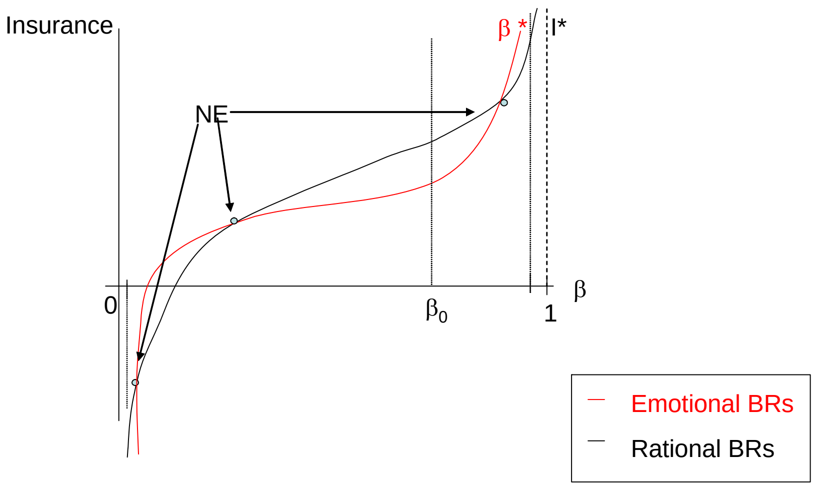

Figure 2

Note that with the ADM model one captures differences in report and choice tasks, as reported in different studies (in the context of normal form games see Costa-Gomes and Weizsäcker 2008). In the insurance context, when asked to report the probability of, say, an accident with no action to subsequently take the agent activates the emotional process and tends to report low chances. However, when asked to choose an action both processes are activated and together determine choice – hence the chosen action will generally be inconsistent with the reported beliefs.

Interestingly, the intrapersonal game defined above is a potential game, where the potential function can be interpreted as the utility function of the composite agent.

Proposition 3 *The intrapersonal game is a potential game, in which the emotional process's objective function is the potential function for the game. Because the potential function is strictly concave in each variable (risk perception and insurance), its critical points are the pure strategy Nash equilibria of the game.*

The potential function allows us to find sufficient condition for uniqueness, conduct welfare analysis, and make predictions about future behavior. A sufficient condition for uniqueness follows:

Proposition 4 *A sufficient condition for a unique pure strategy Nash equilibrium of* *the intrapersonal game is:*

$$\frac{\partial^2 c(\beta;\beta_0)}{\partial\beta^2} > -\frac{[Du(\omega_B+(1-\gamma)I)(1-\gamma)+Du(\omega_G-\gamma I)\gamma]^2}{[\beta D^2u(\omega_B+(1-\gamma)I)(1-\gamma)^2+(1-\beta)D^2u(\omega_G-\gamma I)\gamma^2]},$$
$$\forall\,(I,\beta)\in\left[I^*(\underline{\beta}^{'}),I^*(\overline{\beta}^{'})\right]\times\left[\underline{\beta}^{'},\overline{\beta}^{'}\right],$$

where $\underline{\beta}^{'}\equiv\beta^*\left(I^*\left(\underline{\beta}\right)\right)$ *and, similarly,* $\overline{\beta}^{'}\equiv\beta^*\left(I^*\left(\bar{\beta}\right)\right)$.

Hence, for large mental costs, the equilibrium is unique (think of $\lambda>0$, $\hat{c}(\cdot)=\lambda c(\cdot)$). Moreover, for very large mental costs, the ADM model reduces to the expected utility model[4].

However, considering the general case, where the mental costs are not very large, risk perceptions are endogenous and the ADM model systematically departs from the expected utility model. This suggests that the failure of the expected utility model to explain some data sets may be due to systematic affective biases. How exactly does affective choice in insurance markets differ from the demand for insurance in the expected utility model? Proposition 5 below shows that the expected utility outcome in the case of an actuarially fair insurance market (full insurance) falls within the choice set of the ADM agent. However, if the insurance market is not actuarially fair, then this is no longer the case.

Proposition 5 *If* $\gamma=\beta_0$, *there exists at least one Nash equilibrium* (β^*,I^*) *with* $\beta^*=\beta_0=\gamma$, *and* $I^*=$ *full insurance.*

If $\gamma>\beta_0$, *there exists at least one Nash equilibrium* (β^*,I^*) *with* $\beta^*<\beta_0$ *and* $I^*<I^*(\beta_0)$

If $\gamma<\beta_0$, *there exists at least one Nash equilibrium* (β^*,I^*) *with* $\beta_0<\beta^*$ *and* $I^*>I^*(\beta_0)$.

To understand the intuition behind these results, consider a standard myopic adjustment process where the processes alternate moves. If $\gamma>\beta_0$, at β_0 the rational process, similar to the expected utility model, prescribes buying less than full insurance. The emotional process, in turn, leads the decision maker to believe "this is not going to happen to me" and determines that she is at a lower risk. This effect causes a further reduction in the insurance purchase, with a result of less than full insurance, even less than what the expected utility model would predict. Considering such adjustment process, the ADM model captures two widely discussed phenomena: cognitive dissonance and attention effects. Cognitive dissonance is when one holds two contradicting beliefs at the same time. Hence if one thinks of the adjustment process as a process of reaching a decision, in this process the agent suffers cognitive dissonance and choice represents a resolution of it. As for attention effects – if

[4] As $c\to\infty$, $\beta^*\to\beta_0$ for all values of I. As a result, the ADM model converges to the expected utility model.

one's attention is manipulated to first think of an action, or first think of risk beliefs, generally he or she will end up with different choices. In particular, according to our model, thinking first of probabilities of adverse events leads to greater optimism and lower insurance purchased than if the agent's attention is given to thinking of insurance first.

Note that proposition 5 also implies that, from the viewpoint of an outside observer, both optimism and pessimism (relative to β_0) are possible. This is due to the characteristics of insurance: if an agent purchases more than full insurance, then the "bad" state becomes the "good" state, and vice versa. Consequently, if there is no effective action, i.e., one cannot change the bad state to a good state, we would observe optimism and less-than-optimal insurance.

Here is another example of the difference between affective choice and the demand for insurance in the expected utility model. In the expected utility model, if people realize that they face a higher potential loss, due to educational campaigns that make them aware of the possible catastrophe, then they purchase more insurance. In the ADM model, if an agent realizes she faces higher possible loss, then she might purchase less insurance. Because the increased loss size affects both the emotional and the rational processes in different directions; the rational process prescribes more insurance, the emotional process prescribes lower risk belief to every insurance level (due to greater incentives to live in denial). If the emotional effect is stronger the agent will buy less insurance than previously. That is, if the loss is great, agents might prefer to remain in denial and ignore the possible catastrophes altogether, which will lead them to take fewer precautions such as buying insurance. This is consistent with consumer research showing that high fear arousal in educating people on the health hazards of smoking leads to a discounting of the threat (Keller and Block 1996). Proposition 6 and Figure 3 below summarize the conditions for educational campaigns to produce the counter-intuitive affective result.

Proposition 6 *An educational campaign result in less insurance if*

$$\frac{r(\omega_B - \gamma I)}{Du(\omega_B - \gamma I)} > \frac{r(\omega_G + (1-\gamma)I)}{Du(\omega_G + (1-\gamma)I)},$$

where $r(\cdot)$ *is the absolute risk aversion property of the utility function* $u(\cdot)$

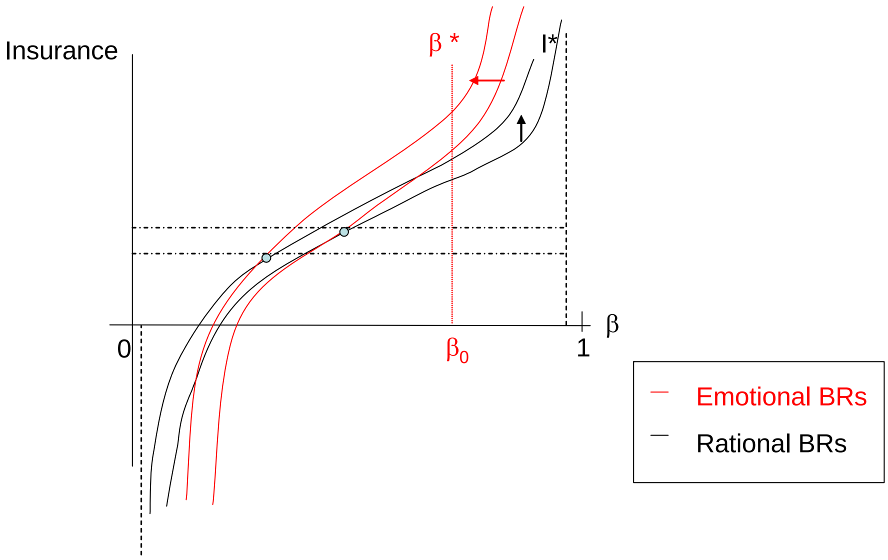

Figure 3

In Proposition 6, if the utility function $u(\cdot)$ exhibits constant or increasing absolute risk aversion, educational campaigns will lead to higher insurance purchase if and only if initially the agent buys more than full insurance. Insurees who initially buy less than full insurance will buy even less after the educational campaign. Hence, for such utility functions, educational campaigns divide the insurance market into a set of agents who purchase more insurance – the intended consequence – and a set of agents who purchase less insurance – the unintended consequence.

3 Axiomatic Foundation of ADM

This section addresses the question: What preferences over risky or uncertain acts are represented by the ADM model? The axioms over acts that give rise to an ADM representation are suggested by a duality property of the ADM potential function. This duality is analogous to the dual relationship between the cost and profit functions of a price taking, profit maximizing firm producing a single good. That is, the profit function, $\phi(p) = \sup_{y \geq 0}\{py - c(y)\}$ where p is the price of output, y is the output, and $c(y)$ is the continuous, convex cost function. As is well known $c(y) = \sup_{p \geq 0}\{py - \phi(p)\}$. In convex analysis $\phi(p)$ is called the Legendre-Fenchel conjugate of $c(y)$, and $c(y)$ is the biconjugate of $\phi(p)$, where we have invoked the theorem of the biconjugate. Returning to the ADM model, we note that the potential function

$\Pi(f,p) = \int u(f)dp - c(p)$ where $c(p)$ is the smooth, convex cost function of the emotional process. The Legendre-Fenchel conjugate of the ADM potential function and the theorem of the biconjugate suggest axioms on preferences over acts that admit an ADM representation. Moreover, these axioms allow an additional interpretation of affective decision-making as ambiguity-seeking choice behavior model.

The axiomatic foundation of the ADM model follows the setup in MMR, where: S is the set of states of the world; Σ is an algebra of subsets of S, the set of events; and X, the set of consequences, is a convex subset of some vector space. F is the set of (simple) acts, i.e., finite-valued Σ-measureable functions $f : S \longrightarrow X$. $B(\Sigma)$ is the set of all bounded Σ -measureable functions, and endowed with the sup-norm it is an AM -space with unit, the constant function 1. $B_o(\Sigma)$ the set of Σ -measureable simple functions is norm dense in $B(\Sigma)$. The norm dual of $B(\Sigma)$ is $ba(\Sigma)$, finitely additive signed measures of bounded variation on Σ (see Aliprantis and Border 1999 for further discussion).

The potential function for the ADM intrapersonal game is $\Pi(f,p) = \int u(f)dp - c(p)$. To make our notation consistent with the notation in convex analysis, we define $J^*(p) = c(p)$, and write the potential function as $\Pi(f,p) = \{\langle u(f), p\rangle - J^*(p)\}$. If $W(f) = \max_{p\in\Delta}\{\langle f, p\rangle - J^*(p)\}$, and the decision-maker maximizes $W(f)$ over her choice set K, then $\max_{f\in K} W(f) = \max_{f\in K}\max_{p\in\Delta}\{\langle u(f), p\rangle - J^*(p)\} = \max_{f\in K, p\in\Delta}\Pi(f,p)$. It follows from the Envelope theorem that $\arg\max_{f\in K, p\in\Delta}\Pi(f,p)$ is the set of pure strategy Nash equilibria of the ADM intrapersonal game, i.e., ADM choice. Below we present the axioms:

$A.1$ (Weak Order): If $f,\ g,\ h \in F$, (a) either $g \succsim f$ or $f \succsim g$, and (b) $f \succsim g$ and $g \succsim h \Longrightarrow f \succsim h$.

$A.2$ (Weak Certainty Independence): If $f,\ g \in F$, $x,\ y \in X$ and $\alpha \in (0,1)$,then
$\alpha f + (1-\alpha)x \succsim \alpha g + (1-\alpha)x \Longrightarrow \alpha f + (1-\alpha)y \succsim \alpha g + (1-\alpha)y$.

$A.3$ (Continuity): If $f,\ g,\ h \in F$, the sets$\{\alpha \in [0,1] : \alpha f + (1-\alpha)g \succsim h\}$ and $\{\alpha \in [0,1] : h \succsim \alpha f + (1-\alpha)g\}$are closed

$A.4$ (Monotonicity): If $f,\ g \in F$ and $f(s) \succsim g(s)$ for all $s \in S$,the set of states, then $f \succsim g$.

$\widehat{A.5}$ (Quasi-Convexity): If $f,\ g \in F$ and $\alpha \in (0,1)$, then
$f \sim g \Longrightarrow \alpha f + (1-\alpha)g \precsim f$

$A.6$ (Nondegeneracy): $f \succ g$ for some $f,\ g \in F$

These axioms where $\widehat{A.5}$ is replaced by $A.5$ (quasi-concavity) are due to MMR (2006)

Theorem 7 *Let $\succsim$ be a binary order on F. The following conditions are equivalent:*

(1) The relation $\succsim$ satisfies axioms $A.1 - A.6$

(2) There exists a nonconstant affine function $u : X \longrightarrow R$ and a continuous, convex function $J^* : \Delta \longrightarrow [0,\infty]$ where for all $f,\ g \in F$, $f \succsim g \iff W(f) \geq W(g)$ and $W(h) = \max_{p\in\Delta}\{\langle u(h), p\rangle - J^*(p)\}$ for all $h \in F$

Let $V(f) = \sup_{p\in(R^k)^*_{++}} \Pi(f,p) = \sup_{p\in(R^k)^*_{++}} \{\langle u(f),p\rangle - J^*(p)\}$. Then $J(u(f)) = V(f)$ for some convex function J, by definition of the Legendre-Fenchel conjugate. In other words, we present axioms on preferences over risky and uncertain acts that characterize $J(u(f))$. It follows from the theorem of the biconjugate that $J(u(f)) = \sup_{p\in(R^k)^*_{++}} \{\langle u(f),p\rangle - J^*(p)\}$. That is, $J(\cdot) = (J^*)^*$.

As mentioned, the resemblance of ADM to variational preferences seems to be only formal. In variational preferences the decision maker does not derive utility from her beliefs or chooses them, and indeed, in such a case there is little point in choosing the worst possible belief. Nevertheless, the extreme nature, or "paranoiac" interpretation of maxmin has prodded researchers to look at other models (e.g., Klibanoff Mukerji Marinacci 2005, 2009, and Gajdos, Hayashi, Tallon, Vergnaud 2008). By contrast, when the maximization is viewed as a part of an optimal decision process, albeit involving the choice of beliefs, maximization appears rather natural.

Still, thinking of the alternative interpretation of the ADM model as ambiguity-seeking choice, suggested by its formal resemblance to the variational preference model, we find a useful separation of ambiguity and risk attitudes for the multiple priors models similar to the capacities in the non-additive probability measure literature[5]. To see this take an example of the multiple priors approach – maxmin expected utility of Gilboa and Schmeidler (1989). MMR (2006) show that maxmin expected utility can be represented by variational preferences where the decision maker maximizes $V(x) = \inf_{p\in(R^k)^*_{++}} \{\langle \vec{u}(x),p\rangle - J^*(p)\} = J(\vec{u}(x))$, and $J(\cdot)$ is concave. Follmer and Shied (2004) show that this representation of preferences over acts exhibits ambiguity aversion–see example 2.75 on page 88.

In contrast to maxmin, we show that the ADM model is an example of a multiple prior model consistent with ambiguity-seeking choice behavior. It follows from the axiomatic foundation of the ADM model that preferences over acts in the ADM model can be represented by $W(h) = \max_{p\in\Delta} \{\langle u(h),p\rangle - J^*(p)\}$, where $J^*(\cdot)$ is convex. Hence the general representation of preference over acts for the ADM model is $J(\vec{u}(x)) = W(h) = \max_{p\in\Delta} \{\langle u(h),p\rangle - J^*(p)\}$, where $J(\cdot)$ is convex. Hence if $-J(\vec{u}(x))$ is concave and represents an ambiguity-averse decision maker a convex

[5]Examples of the non-additive probability measure literature are Choquet expected utility (CEU) and the special case of rank-dependent expected utility (RDEU). Choquet expected utility of an act x over n states $w_1,...,w_n$ such that $u(x(w_1)) \geq u(x(w_2)) \geq \cdots \geq u(x(w_n))$ is $CEU(x;c) = \sum_{i=1}^{n}[u(x(w_i)) - u(x(w_{i+1}))]c(w_1,...,w_n) + u(x(w_n)))$, where $c(\cdot)$ is a capacity. A capacity $c(\cdot)$ is convex, or sub-additive, if for all events A,B, $c(A)+c(B) \leq c(A\cap B)+c(A\cup B)$. A capacity $c(\cdot)$ is concave, or super-additive, if for all events A,B $c(A)+c(B) \geq c(A\cap B)+c(A\cup B)$. A CEU defined with a convex capacity is ambiguity-averse, an affine capacity is ambiguity-neutral, and a concave capacity is ambiguity-seeking. Similarly, RDEU is defined as $CEU(x;c) = \sum_{i=1}^{n}[u(x(w_i)) - u(x(w_{i+1}))]\phi(p(w_1,...,w_n)) + u(x(w_n)))$, where $\phi(\cdot)$ is a distortion with $\phi(0)=0$, $\phi(1)=1$, and $p(\cdot)$ is an additive probability measure. If $\phi(\cdot)$ is convex, affine, or concave, RDEU represent ambiguity-aversion, ambiguity-neutral, or ambiguity-seeking, respectively. Hence, if capacities are convex or concave, behavior of CEU and $RDEU$ agent exhibits ambiguity aversion, or ambiguity seeking respectively.

$J(\cdot)$, $J(\vec{u}(x))$ represents an ambiguity-seeking decision maker.

To sum, the axiomatic foundation of the ADM can be seen as an additional interpretation of the model as ambiguity-seeking model, and a nice separation between risk attitudes and ambiguity attitudes. That is, risk attitudes are contained in the shape of the utility function $u(\cdot)$, as usual, and ambiguity attitudes are represented in the shape of $J(\cdot)$. When $J(\cdot)$ is concave, affine, or convex, the agent is ambiguity averse, neutral, or seeking, respectively.

4 Appendix

Proof of Proposition 3. Denote the rational process's payoff function as (R) and the emotional process's payoff function as (E). A necessary and sufficient condition for the intrapersonal game to have a potential function (Monderer and Shapley 1996) is $\frac{\partial^2 R}{\partial\beta\partial I} = \frac{\partial^2 E}{\partial\beta\partial I}$. This condition clearly is satisfied in the ADM model. The potential function $P(\beta, I)$ is a function such that (Monderer and Shapley, 1996): $\frac{\partial P}{\partial\beta} = \frac{\partial E}{\partial\beta}$, $\frac{\partial P}{\partial I} = \frac{\partial R}{\partial I}$. Because $\frac{\partial E}{\partial I} = \frac{\partial R}{\partial I}$, (E) can serve as a potential function. The critical points of the potential function are $\frac{\partial P}{\partial\beta} = \frac{\partial E}{\partial\beta} = 0$, $\frac{\partial P}{\partial I} = \frac{\partial R}{\partial I} = 0$. The potential function is strictly concave in each variable, so at each critical point, each process is maximizing its objective function, given the strategy of the other process. Therefore, the critical points of the potential function are the pure strategy Nash equilibria of the intrapersonal game, and all pure strategy Nash equilibria are critical points of the potential function. ■

Proof of Proposition 2. By the boundaries on risk perception, $0 < \underline{\beta} < \overline{\beta} < 1$, $\beta^* \in \left(\underline{\beta}, \overline{\beta}\right)$, and insurance $I^* \in \left[I^*(\underline{\beta}), I^*(\overline{\beta})\right]$. Hence, all Nash equilibria will have perceived probabilities in the interval $\left[\beta^*\left(I^*(\underline{\beta})\right), \beta^*\left(I^*(\overline{\beta})\right)\right]$ where $0 < \underline{\beta} < \beta^*\left(I^*(\underline{\beta})\right) < \beta^*\left(I^*(\overline{\beta})\right) < \overline{\beta} < 1$. Define $\beta^*\left(I^*(\underline{\beta})\right) \equiv \underline{\beta}'$, $\beta^*\left(I^*(\overline{\beta})\right) \equiv \overline{\beta}'$; because all the Nash equilibria of the intrapersonal game for $\beta \in \left(\underline{\beta}, \overline{\beta}\right)$ are $\in \left[\underline{\beta}', \overline{\beta}'\right]$ the focus can remain on the latter probability space.

The existence and chain results can be shown by defining a restricted intrapersonal game in which the insurance pure strategy space is restricted to $\left[I^*(\underline{\beta}), I^*(\overline{\beta})\right]$ and the perceived probabilities are restricted to $\beta \in \left[\underline{\beta}', \overline{\beta}'\right]$, such that the equilibria points of the intrapersonal game are not altered. The restricted game is a supermodular game, and thus, these results follow from the properties of this class of games (see Topkis 1998). To Show that the game admits odd number of equilibria, think of the geometry of the game. As $\beta \to \overline{\beta}$, the best response of the emotional process is above the best response of the rational process, while this relationship is reversed for $\beta \to \underline{\beta}$. Since the best responses are monotonically increasing, it follows that there exists odd number of Nash equilibria. ■

Proof of Proposition 4. The emotional process's objective function $\beta u(\omega_B + (1-\gamma)I) + (1-\beta)u(\omega_G - \gamma I) - c(\beta; \beta_0)$ is the potential function of the game. The maximization of (P) with respect to a pair (I, β) gives rise to a pure strategy Nash equilibria of the game.$\beta \in \left[\underline{\beta}', \overline{\beta}'\right]$ and $I \in \left[I^*(\underline{\beta}'), I^*(\overline{\beta}')\right]$ (see proof of Proposition 2), hence only the restricted intrapersonal game in which both players' strategy spaces are compact need be considered. Neyman (1997), proved that a potential game with a strictly concave, smooth potential function, in which all players have compact, convex strategy sets, has a unique pure strategy Nash equilibrium. That is, the Hessian of the potential function is negative definite, as follows from the condition given above. ■

Proof of Proposition 5. Consider the case in which $\gamma = \beta_0$. At full insurance, there is no mental gain for holding beliefs $\beta \neq \beta_0$ but there exists mental cost. Therefore, at full insurance, the mental process's best response is $\beta = \beta_0$. Given that $\gamma = \beta_0 = \beta$, the rational process's best response is full insurance. Consequently, full insurance and $\beta = \beta_0$ is a Nash equilibrium of this case. Next, consider the case $\gamma > \beta_0$; because the insurance premium is higher than β_0, $I^*(\beta = \beta_0) < z$. Also, $\beta^* = \beta_0$ only at full insurance, where $I = z$. Therefore, at $\beta = \beta_0$ the mental process's best response falls above the rational process's best response. This relationship is reversed at the limit $\beta \to \underline{\beta}$, and both the mental and the rational best responses increase; therefore, there exists a Nash equilibrium with $\beta < \beta_0$ and less insurance than predicted by the expected utility model. A similar argument can be used to prove the result when $\gamma < \beta_0$. ■

Proof of Proposition 6. Define $\tilde{I}(\beta; \beta_0)$ as the inverse function β^{*-1}. Define $\Pi(\beta; \beta_0) = I^*(\beta) - \tilde{I}(\beta; \beta_0)$, $\Pi : \left[\underline{\beta}', \overline{\beta}'\right] \to R$

Educational campaigns on impending catastrophes increase the loss size, z. Because $\Pi(\beta; \beta_0) = 0$ is a NE, $\frac{\partial \Pi}{\partial z} < 0$ represent the unintended consequence of such campaigns.

$$\frac{\partial \Pi}{\partial z} < 0 \Leftrightarrow \frac{\frac{\partial \tilde{I}}{\partial z}}{\frac{\partial I^*}{\partial z}} > 1$$

.

$$\frac{\partial I^*}{\partial z} = \frac{\left[u''(w_2 - z + (1-\gamma)I^*)\right]\left[u'(w_2 - \gamma I^*)\right]^2}{\left[u'(w_2 - \gamma I^*)\right]\left[\begin{array}{c} u''(w_2 - z + (1-\gamma)I^*)u'(w_2 - \gamma I^*)(1-\gamma) \\ +u'(w_2 - z + (1-\gamma)I^*)u''(w_2 - \gamma I^*)\gamma \end{array}\right]};$$

$$\frac{\partial \tilde{I}}{\partial z} = \frac{\left[u'(w_2 - z + (1-\gamma)\tilde{I})\right]}{\left[u'(w_2 - z + (1-\gamma)\tilde{I})(1-\gamma) + u'(w_2 - \gamma \tilde{I})\gamma\right]} \Rightarrow \frac{\partial \Pi}{\partial z} < 0$$

$$\Leftrightarrow \frac{r(w_2 - \gamma I)}{u'(w_2 - \gamma I)} > \frac{r(w_1 + (1-\gamma)I)}{u'(w_1 + (1-\gamma)I)}, \text{where } r(x) = -\frac{u''(x)}{u'(x)}$$

■

Proof of Theorem 7. Axioms $1-4$ are used in MMR to derive a nonconstant affine utility function, u, over the space of consequences, X. u is extended to the space of simple acts, F, using certainty equivalents.That is, $U(f) = u(x_f) \in B_o(\Sigma)$ for each $f \in F$, where x_f.is the certainty equivalent of f. This is lemma 28 in MMR, where $I(f) = U(f)$ is a niveloid on $\Phi = \{\varphi : \varphi = u(f) \text{ for some } f \in F\}$. Niveloids are functionals on function spaces that are monotone: $\varphi \leq \eta \implies I(\varphi) \leq I(\eta)$ and vertically invariant: $I(\varphi + r) = I(\varphi) + r$ for all φ and $r \in R$–see Dolecki and Greco (1995) for additional discussion. Φ is a convex subset of $B(M)$ and by Schmeidlers's axiom 5, I is quasi-concave on Φ. We also assume axioms1-4, so lemma 28 in MRR holds for the niveloid J in the ADM representation theorem. By axiom $\widehat{5}$, J is quasi-convex on Φ.

MMR show in lemma 25 that I is concave if and only if I is quasi- concave. Hence J is convex if and only if J is quasi-convex, since J is convex(quasi-convex) if and only if $-J$ is concave(quasi-concave). MMR extend I to a concave niveloid $\widehat{I}$ on all of $B(\Sigma)$– see lemma 25 in MMR. Epstein, Marinacci and Seo [EMS] (2007) show in lemma $A.5$ that niveloids are Lipschitz continuous on any convex cone of an AM-space with unit and concave(convex) if and only if they are quasi-concave(convex). Hence, since $B(\Sigma)$ is a convex cone in an AM-space with unit, $\widehat{I}$ is Lipshitz continuous. It follows from the theorem of the biconjugate for continuous, concave functionals that $I(\varphi) = \inf_{p\in ba(\Sigma)} \left\{ \int \varphi dp - \widehat{I}^*(p) \right\}$, where $\widehat{I}^*(p) = \inf_{\varphi \in B_o(\Sigma)} \left\{ \int \varphi dp - \widehat{I}(\varphi) \right\}$ is the concave, conjugate of $\widehat{I}(\varphi)$– see Rockafellar (1970), pg 308.for finite state spaces. MMR show on pg. 1476 that we can restrict attention to Δ, the family of positive, finitely additive measures of bounded variation in $ba(\Sigma)$. Hence $I(\varphi) = \min_{p\in\Delta} \left\{ \int \varphi dp - \widehat{I}^*(p) \right\} = \min_{p\in\Delta} \left\{ \int u(f)dp + c(p) \right\}$, where $\varphi = u(f)$ and

$c(p) = -\widehat{I}^*(p)$. $c(p)$ is convex since $\widehat{I}\,^*(p)$ is concave.

Extending -J to -$\widehat{J}$ on $B(\Sigma)$, using lemma 25 in MMR, it follows from the theorem of the biconjugate for continuous, convex functionals that

$J(\varphi) = \max_{p\in ba(\Sigma)} \left\{ \int \varphi dp - \widehat{J}^*(p) \right\}$ where $\widehat{J}^*(p) = \max_{\varphi\in B_o(\Sigma)} \left\{ \int \varphi dp - \widehat{J}(\varphi) \right\}$ is the convex, conjugate of $\widehat{J}(\varphi)$ –see Rockafellar (1970), pg 104 for finite state spaces and Zălinescu (2002), pg 77 for infinite state spaces.

Again it follows from MMR that $J(\varphi) = \max_{p\in\Delta} \left\{ \int \varphi dp - \widehat{J}^*(p) \right\} = \max_{p\in\Delta} \left\{ \int u(f)dp - c(p) \right\} = W(f)$, where $\varphi = u(f)$ and $c(p) = \widehat{J}^*(p)$. $c(p)$ is convex since $\widehat{J}\,^*(p)$ is convex.

$f \succsim g \iff J(u(f)) \geq J(u(g)) \iff W(f) \geq W(g)$.Hence $\arg\max_{f\in F} J(u(f)) \subseteq$ set of pure strategy Nash equilibria of the ADM intrapersonal game, where $u(\cdot)$ is the Bernoulli utility function of the rational process and $\widehat{J}^*(\cdot)$ is the cost function of the emotional process. ■

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