Journal of Marine Research, 45, 829–841, 1987

The influence of planetary rotation on oceanic double-diffusive fluxes

by Dan Kelley[1]

ABSTRACT

Does planetary rotation inhibit double-diffusive fluxes of heat and salt in oceanic thermohaline staircases? The Taylor number Ta, which measures the effect of planetary rotation on convection, is $O(1)$ for laboratory diffusive staircases $O(10^4)$ for oceanic diffusive staircases. Does this mean that laboratory-based flux laws are inapplicable in the ocean? Lacking flux measurements in double-diffusive systems, these questions are addressed using an analogy between double-diffusive convection and thermal convection. It is shown that Ta is an inappropriate parameter, and that rotating thermal fluxes collapse onto a simple curve when a new Taylor number $Ta_{no\text{-}slip}$ is used instead of Ta. Fluxes are strongly inhibited for $Ta_{no\text{-}slip} > 10^3$. When extended to double-diffusion, this criterion implies that planetary rotation does not inhibit fluxes in either the laboratory or the ocean. There is no need to add a rotation correction to the laboratory-based flux laws before applying them to the ocean.

1. Introduction

Here the influence of planetary rotation on oceanic double-diffusion is addressed in a preliminary way, focussing attention on the inhibition of vertical heat and salt fluxes. The analysis is restricted to diffusive staircases,[2] being based on aspects of convection which are poorly understood for finger staircases.

The suppression of convection by planetary rotation has been measured in the laboratory for one-component (thermal) convection, but not for double-diffusive convection. The suppression is most easily understood in terms of the Taylor-Proudman theorem, which precludes vertical motion (and thus convection) in the limiting case of steady, linear, motion of rotating, homogeneous inviscid fluids (Batchelor, 1967). In the viscous, inhomogeneous case of interest here, the effect shows up in two ways: *delayed onset* of convection, and *reduced fluxes* in active convection.

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2. 'Diffusive' staircases have both salinity S and potential temperature T decreasing upward. 'Finger' staircases have both increasing upward. See reviews by Turner (1973, 1985) and Huppert and Turner (1981).

a. Delayed onset of convection in the laboratory. The onset of *thermal* convection occurs only after the Rayleigh number

$$Ra = \frac{g\alpha\Delta TH^3}{\nu\kappa_T}, \tag{1}$$

exceeds a critical value Ra_c. (Here g = gravitational acceleration, $\alpha = -\rho^{-1}\partial\rho/\partial T$, ΔT = temperature step across convecting layer, H = layer thickness, ν = molecular viscosity, and κ_T = thermal diffusivity.) The value of Ra_c is $\sim 10^3$ in nonrotating convection. It is larger in rotating convection. The rotation rate is traditionally measured by the Taylor number

$$TA = \frac{f^2H^4}{\nu^2}, \tag{2}$$

where f = the Coriolis parameter. Thus, the delayed onset of thermal convection is measured by an increase of Ra_c with Ta; this onset-of-convection boundary has been explored in the laboratory and with theory (Section 2).

For *double-diffusive* convection, a third parameter is involved, namely the density ratio

$$R\rho = \frac{\beta\Delta S}{\alpha\Delta T}, \tag{3}$$

where $\beta = \rho^{-1}\partial\rho/\partial S$ and ΔS = salinity step across the convecting layer. The onset-of-convection boundary $Ra_c(Ta, R\rho)$ has not yet been measured, but a linear theory has been developed by Pearlstein (1981).

b. Reduction of fluxes in active convection in the laboratory. For *thermal* convection, Rossby (1969) measured the (Ra, Ta)-dependence of the heat flux F^T, expressed nondimensionally in the Nusselt number

$$Nu = \frac{HF_T}{\kappa_T\Delta T}. \tag{4}$$

These measurements will be discussed at length below.

For *double-diffusive* convection, the dependence of Nu on $R\rho$—that is, the '4/3' flux laws—has been measured (Turner, 1965; Crapper, 1975; Marmorino and Caldwell, 1976; Newel, 1984) and modelled (Linden and Shirtcliffe, 1978) but the dependence on Ta has not been addressed until now.

c. Rotation and oceanic double-diffusive convection. The '4/3' flux laws, based on laboratory staircases with $f \sim 10^{-4}$ s^{-1}, $\nu \sim 10^{-6}$ m^{2}s^{-1} and $H \sim 10^{-1}$ m, typically have $Ta \sim 1$. Oceanic diffusive staircases have thicker layers ($\sim$1 m), so that $Ta \sim 10^4$.

Lacking laboratory measurements of rotating diffusive convection, we will use an analogy between thermal and diffusive convection to determine whether this large difference in Ta prevents application of the laboratory flux laws to the ocean. This allows us to develop a criterion for suppression of diffusive convection using Rossby's (1969) measurements of thermal convection. Unfortunately, the oceanic parameter values are outside the laboratory range, and direct extrapolation is prevented by poor understanding of the laboratory flux relationship (Section 2). These difficulties are overcome if a new rotation parameter, $Ta_{no\text{-}slip}$, is used instead of Ta. Recasting Rossby's $Nu = Nu(Ra, Ta)$ data as $Nu = Nu(Ra, Ta_{no\text{-}slip})$, a simple empirical rule becomes apparent: for $Ta_{no\text{-}slip} < 10^3$ the convective flux is barely affected; for $Ta_{no\text{-}slip} > 5 \times 10^3$ the flux is greatly reduced (Section 3). When applied to double-diffusive convection (Section 4), this $Ta_{no\text{-}slip}$ criterion suggests that rotation does not significantly affect fluxes in either the laboratory or the ocean.

2. The problem with the traditional Taylor number, T*a*

Figure 1 shows Rossby's (1969) measurements of $Nu = Nu(Ra, Ta)$ in rotating thermal convection in water. If the rotation rate Ta is increased while the heating rate Ra is held fixed, the heat flux Nu at first remains close to the nonrotating value $Nu_{nonrotating}$, until a critical value of Ta is exceeded, and then it decreases rapidly. This critical value of Ta depends on Ra—for example, it is 10^5 for $Ra = 10^4$ and 10^7 for $Ra = 10^5$.

The onset of convection is prevented for Ra below a certain boundary $Ra_c = Ra_c(Ta)$. This onset-of-convection boundary has been accurately explained theoretically. The deviation of Ra_c from the nonrotating value $\sim 10^3$ occurs at $Ta \sim 10^3$ for both thermal convection (Chandrasekhar, 1961: Chapter 3) and double-diffusive convection (Pearlstein, 1981: Table 1 and Figure 3). For high Ta, the linear theory predicts $Ra_c \propto Ta^{2/3}$ (Chandrasekhar, 1961). Two heavy lines on Figure 1 demonstrate that two theories predict $Ra_c(Ta)$ to within a factor of 2 over a range of 10^3.

On the other hand, there is at present no theory for the amount of flux reduction by rotation in active convection. In other words, the flux relationship $Nu = Nu(Ra, Ta)$ is not understood. Numerical models have been applied only to the weakly heated case, i.e. only for $Ra < 10Ra_c$ (Veronis, 1968; Somerville, 1971; Somerville and Lipps, 1973; Hathaway and Somerville, 1986), and so cannot be applied directly to Rossby's experiments, which have $Ra/Ra_c \sim 10^3$.

Rossby's data cannot reasonably be applied to oceanic diffusive staircases, for even if we knew the $R\rho$ dependence, the oceanic value of $Nu \sim 10^2$ (Kelley, 1984) is beyond the range of the data ($Nu < 12$). Lacking theoretical understanding of even the general characteristics of $Nu(Ra, Ta)$, it is unreasonable to extrapolate far beyond the boundaries of the data in Figure 1.

This extrapolation problem must be added to a list of more fundamental difficul-

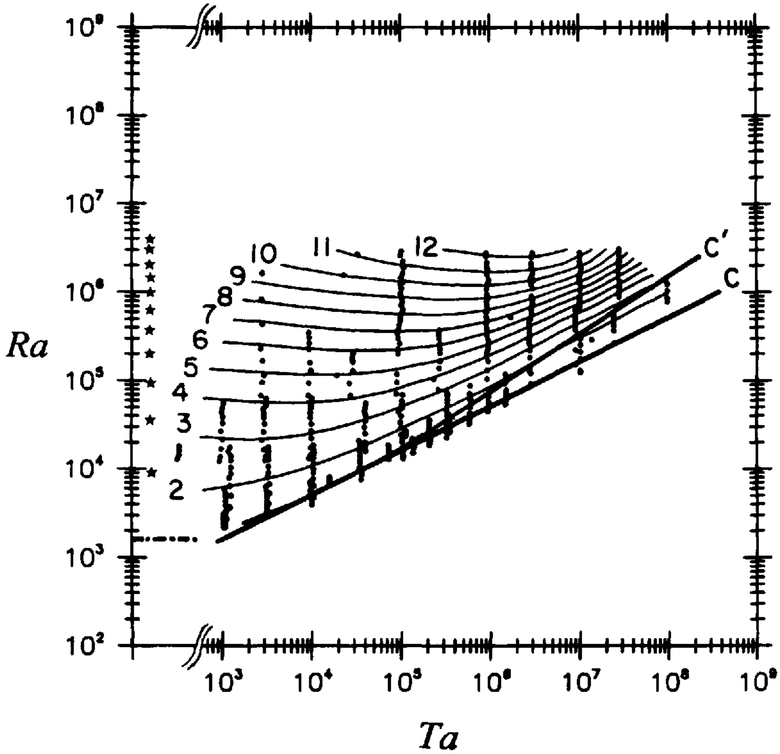

Figure 1. Rossby's (1969) measurements of heat flux in rotating thermal convection of water. The contours give the dimensionless heat flux $Nu(Ra, Ta)$; the dots indicate the data points. The nonrotating thermal fluxes $Nu_{nonrotating}(Ra)$ (=2, 3, . . . , 12) are indicated by the stars at the left. The two thick curves labeled C and C' are two formulations of the onset-of-convection stability boundary, $Ra_c(Ta)$ (C = Galdi and Straughan's (1985) "energy" method; C' = Chandrasekhar's (1961) linear method), and the thick horizontal dashed line at the left shows the nonrotating critical value $Ra_c(0) = 1700$.

ties:

- We do not understand the form of Rossby's $Nu(Ra, Ta)$ relationship, even though we know its limiting forms, namely the nonrotating flux law $Nu_{nonrotating}(Ra)$ and the onset-of-convection boundary $Ra_c(Ta)$.
- Ta is clearly not an appropriate parameter to describe the rotational inhibition of convection because the flux reduction depends not only on Ta but also on the rotation-*independent* parameter Ra (Fig. 1).

3. A new measure of planetary rotation, $\mathbf{Ta_{no\text{-}slip}}$

a. Interfacial boundary conditions. We wish to construct a new rotation parameter to measure the effect of rotation on convective fluxes. We will try to represent more

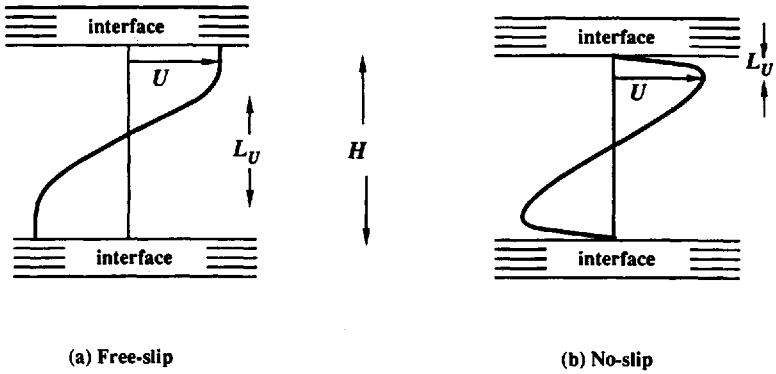

Figure 2. Schematic velocity profiles for convection between (a) free-slip interfaces and (b) no-slip interfaces. The canonical speed U, the thickness of the layer H and the thickness of high-shear boundary layer L_U are indicated.

accurately the ratio of $(Coriolis\ force)^2$ to $(friction\ force)^2$. This ratio cannot be calculated directly because it involves details of the velocity field not yet measured. Furthermore, computer constraints preclude adequate simulation of three-dimensional thermal convection at high Ra(Curry *et al.*, 1984), even without the addition of rotation and the salinity field. At this stage we must be satisfied with simple ideas and crude measurements.

Figure 2 (a) shows schematically the velocity field for convection with *free-slip* boundary conditions at the interfaces. The Coriolis term is $O(fU)$, where U is the velocity scale, and the friction term is $O(\nu U/H^2)$. Thus, the ratio of $(Coriolis\ force)^2$ to $(friction\ force)^2$ is Ta.

Figure 2 (b) shows the situation for *no-slip* boundary conditions. Again, the Coriolis term is $O(fU)$. The shear is concentrated in thin boundary layers of thickness $L_U \ll H$, so the friction term is $O(\nu U/L_U^2)$. Therefore the Taylor number is

$$Ta_{no\text{-}slip} = \frac{f^2 L_U^4}{\nu^2}. \tag{5}$$

We might expect $Ta_{no\text{-}slip}$ to be more appropriate than Ta, since no-slip boundary conditions apply to both thermal convection and double-diffusive convection. Some examples will illustrate this. *Thermal convection:* Rossby's apparatus had rigid boundaries, leading to no-slip conditions. *Diffusive convection:* Laboratory studies of diffusive staircases show that no-slip conditions hold at interfaces between layers. For example, tracer particles put in interfaces are motionless in time-lapse photographs (Huppert and Linden, 1979; Tsinober *et al.*, 1983). Dye inserted in convecting layers rapidly disperses, but dye inserted in the interfaces remains until it diffuses molecularly (Mancini *et al.*, 1975).

Figure 3 is a time-lapse history of an initially vertical dye streak in a 4-layer

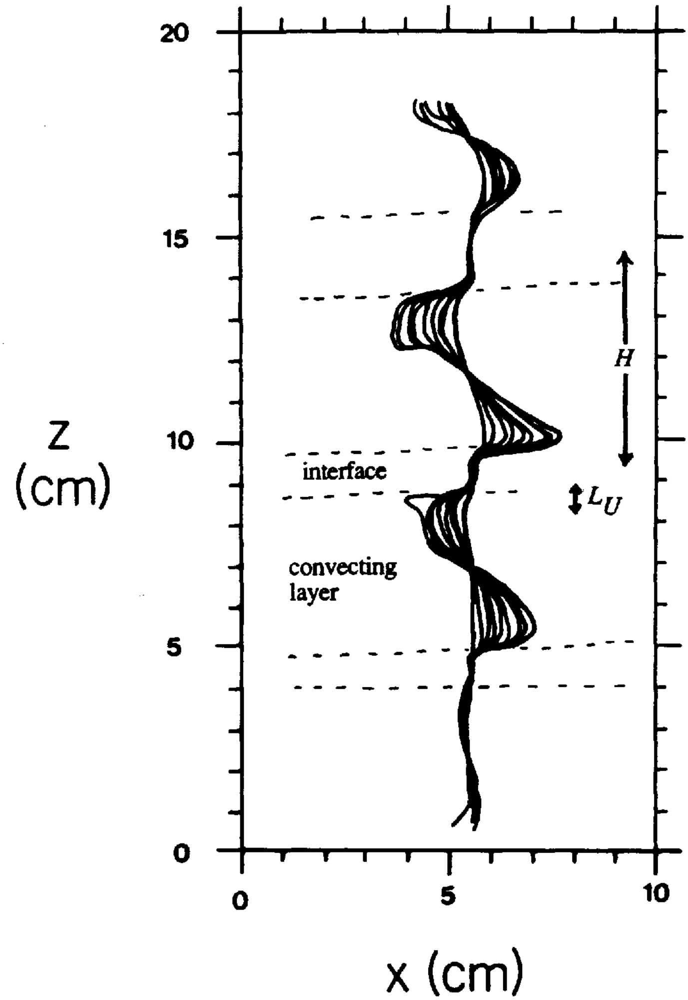

Figure 3. Tracings of successive side-view shadowgraph images of an initally vertical dye streak in a laboratory four-layer diffusive staircase. The interval between images is 2 s. The dashed lines roughly demark the lower and upper boundaries of the interface, as indicated by variations in light intensity on the shadowgraph. The layer-thickness H and the thickness of the high-shear boundary layer L_U are indicated.

diffusive staircase set up in the laboratory by the author. (The top and bottom layers should be ignored because each has only one double-diffusive interface, but the two middle layers should be a good model of layers in oceanic staircases.) The dye streak indicates:

- The motion near the centers of the interfaces is much slower than in the convecting layers; the interfacial velocity boundary condition is clearly no-slip.

- The shear near the interfaces is much greater than the shear in the interior of the convecting layer.

These points justify using $Ta_{no\text{-}slip}$ rather than Ta.

b. Formulation of $Ta_{no\text{-}slip}$. To formulate $Ta_{no\text{-}slip}$, we must express L_U in terms of known quantities, such as H and $Nu_{nonrotating}$.

According to Howard's (1964) model of one-component convection, as modified for diffusive convection by Linden and Shirtcliffe (1978), interfaces are composed of a motionless central core sandwiched between Rayleigh—unstable thermal boundary layers. The fluid in the boundary layers is continually or intermittently replaced by new fluid swept in by boundary currents, so that the velocity boundary layer thickness, L_U, is set by the temperature boundary layer thickness, L_T:

$$L_U = L_T \, . \tag{6}$$

To determine L_T, we will use the assumption of Howard (1964) and Linden and Shirtcliffe (1978) that the boundary-layer Rayleigh number,

$$Ra_{bl} = \frac{g\alpha\,(\partial T/\partial z)_{core} L_T^4}{\nu\kappa_T}\,, \tag{7}$$

is maintained at the critical value $\sim 10^3$. (Here $(\partial T/\partial z)_{core}$ is the temperature gradient in the central core of the interface.) Eliminating L_T between (6) and (7), and using (1), we get

$$\frac{L_U}{H} = \left(\frac{Ra_{bl}}{Ra}\right)^{1/4}\left(\frac{\Delta T}{H(\partial T/\partial z)_{core}}\right)^{1/4}; \tag{8}$$

this leaves us with the final step of determining $\Delta T/(\partial T/\partial z)_{core}$. Measurements in the laboratory (Shirtcliffe, 1973; Newel, 1984) and in the ocean (Melling *et al.,* 1984) verify another assumption of the Linden-Shirtcliffe model, namely that the conductive flux in the interface is equal to the convective flux in the layer:

$$F_T = \kappa_T(\partial T/\partial z)_{core} \, . \tag{9}$$

Combining (9) with (4), we get

$$\frac{\Delta T}{H(\partial T/\partial z)_{core}} = \frac{1}{Nu}\,, \tag{10}$$

so that (8) becomes

$$\frac{L_U}{H} = \left(\frac{Ra_{bl}}{Nu\,Ra}\right)^{1/4}. \tag{11}$$

Now L_U is determined if we can express Nu as a function of Ra, $R\rho$, and Ta. This cannot be done at present, for the dependence of Nu on Ta is not modelled. So, we will use the *nonrotating flux law* for Nu, which should predict L_U accurately for Ta sufficiently low that rotation does not affect the flux. $Nu_{nonrotating}$ will be used simply because it gives a *closed formulation;* inaccuracy at high rotation rates is not a roadblock to further progress since our goal is a criterion to distinguish between systems affected by rotation and systems not affected.

The '4/3' nonrotating flux formulation is (Turner, 1973),

$$Nu_{nonrotating} = C\,Ra^{1/3}. \tag{12}$$

For thermal convection $C = 0.08 \pm 0.01$. For diffusive convection C falls from 0.15 at $R\rho = 2$ to 0.05 at $R\rho = 5$; one empirical relationship out of several which disagree by at most a factor of 1.5 is (Marmorino and Caldwell, 1976)

$$C = 0.00859 \exp\left(4.6 \exp\left[-0.54(R\rho - 1)\right]\right). \tag{13}$$

Substituting $Ra_{bl} \sim 10^3$ and $0.05 < C < 0.15$ in (11) with (12), we find that

$$L_U = (0.6 \text{ to } 1.4) \times \frac{H}{Nu_{nonrotating}}. \tag{14}$$

Since C is not known to better than a factor of 1.5 for double-diffusive convection, the numerical factor may be reasonably taken as unity for our order-of-magnitude calculation of $Ta_{no\text{-}slip}$. Then (5) becomes:

$$Ta_{no\text{-}slip} = \frac{f^2 H^4}{\nu^2\, Nu^4_{nonrotating}}, \tag{15}$$

or, in terms of the traditional Taylor number,

$$Ta_{no\text{-}slip} = \frac{Ta}{Nu^4_{nonrotating}}. \tag{16}$$

This is the desired Taylor number for no-slip boundary conditions at interfaces, valid for thermal and diffusive convection.

c. Preference of $Ta_{no\text{-}slip}$ over Ta for thermal convection. Figure 4 shows Rossby's $Nu(Ra, Ta)$ thermal convection data converted $Nu(Ra, Ta_{no\text{-}slip})$ using (16) and the nonrotating flux law[3]

$$Nu_{nonrotating} = 0.138\, Ra^{0.296}. \tag{17}$$

3. This is a least-squares fit over for $2 \times 10^4 < Ra < 3 \times 10^6$. The fact that the 95% confidence limits on the exponent, 0.2961 ± 0.0002, firmly exclude the traditional ⅓ exponent is a matter for separate discussion (Kelley, 1987). Here, it is sufficient to note that departure from ⅓ is slight enough that our analysis is valid to within the uncertainties in flux.

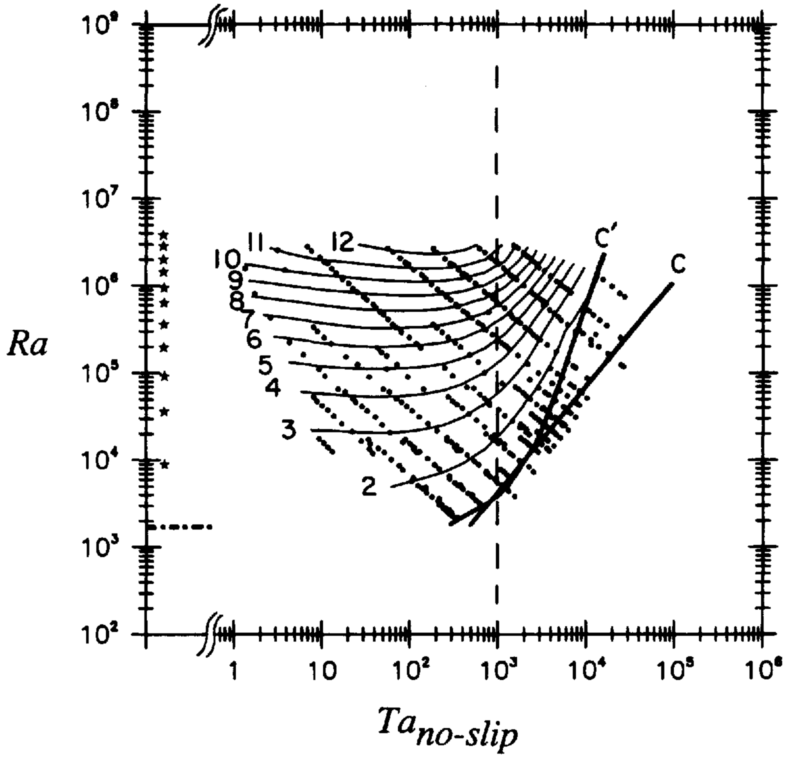

Figure 4. Rossby's heat fluxes, translated from the $Nu(Ra, Ta)$ form of Figure 1 into $Nu(Ra, Ta_{no\text{-}slip})$ form, using (16) with (17). The stability boundaries $Ra_c(Ta_{no\text{-}slip})$ and the nonrotating fluxes $Nu_{nonrotating} = Nu_{nonrotating}(Ra)$ have also been translated. The vertical dashed line at $Ta_{no\text{-}slip} = 10^3$ marks the bend of the stability curves, i.e., the beginning of flux inhibition.

The $Nu(Ra, Ta_{no\text{-}slip})$ relationship is simpler than the $Nu(Ra, Ta)$ relationship of Figure 1 in the sense that the variations with Ra and $Ta_{no\text{-}slip}$ are more nearly independent. Thus the Nu = constant curves are nearly parallel, offset by an amount given by the nonrotating flux law.

The flux remains close to the nonrotating value until a certain value $Ta_{no\text{-}slip}$ is reached, and then decreases rapidly. Since this critical value of $Ta_{no\text{-}slip}$ is only weakly dependent on Ra, flux inhibition is better indicated by $Ta_{no\text{-}slip}$ than by Ta.

This is further illustrated in terms of the relative reduction of the convective part of the flux (i.e., total flux − conductive flux):

$$F = \frac{Nu - 1}{Nu_{\text{nonrotating}} - 1}. \tag{18}$$

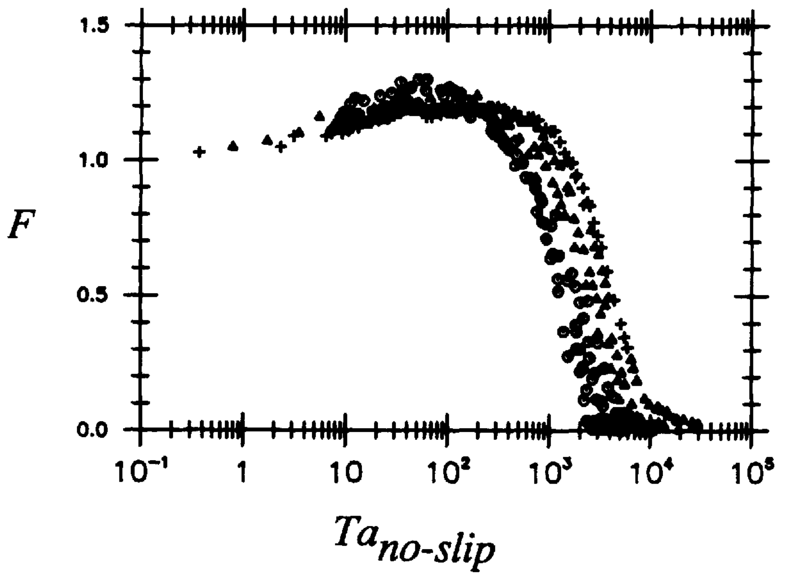

Figure 5. Ratio of the rotating convective flux to the nonrotating convective flux, F, for the thermal convection data of Figures 1 and 4. The text explains the slight (25%) increase of fluxes at moderate $Ta_{no\text{-}slip}$ (10 to 10^2) and the rapid cutoff of fluxes at high $Ta_{no\text{-}slip}$ ($>5 \times 10^3$). The symbols denote the Rayleigh numbers as follows: *circle* = $10^3 < Ra < 10^4$; *triangle* = $10^4 < Ra < 10^5$; *horizontal cross* = $10^5 < Ra < 10^6$.

The value of F ranges from 1 (when rotation has no effect on fluxes) to 0 (when rotation completely inhibits convection). Figure 5 shows $F(Ta_{no\text{-}slip})$ for Rossby's data. Note that F is determined mostly by $Ta_{no\text{-}slip}$, having only a weak dependence on Ra. For example, in the region of its rapid decrease, F is proportional to the square of $Ta_{no\text{-}slip}$ but only to the third-root of Ra.

d. Critical value of $Ta_{no\text{-}slip}$ *for flux inhibition.* According to Figure 5:

- For low $Ta_{no\text{-}slip}$, F remains within 25% of 1. The convective flux is barely altered by rotation for $Ta_{no\text{-}slip} < 10^3$. [The slight flux increase for $10 < Ta_{no\text{-}slip} < 10^2$, also seen in numerical simulations (Somerville and Lipps, 1973), is not understood. The increase is of little interest here, being smaller than the (50%) uncertainties in the nonrotating flux measurements.]
- At $Ta_{no\text{-}slip} \sim 10^3$, rotation begins to strongly inhibit convection; for $Ta_{no\text{-}slip}$ greater than 5×10^3, the convective fraction of the total flux is less than a tenth of the nonrotating value.

In other words, $Ta_{no\text{-}slip} \sim 10^3$ *is a critical value of the influence of planetary rotation on thermal convection.*

This critical value can be explained in terms of the onset-of-convection theory. Assuming that (12) can be roughly represented by[4]

$$Nu \sim \left(\frac{Ra}{Ra_c}\right)^{1/3}, \tag{19}$$

and using the high-Ta asymptotic relationship (Chandrasekhar, 1961: Chapter 2)

$$Ra_c \sim 10Ta^{2/3}, \tag{20}$$

we may rewrite (18) as

$$F \sim \frac{(0.1Ra/Ta^{2/3})^{1/3} - 1}{Nu_{nonrotating} - 1}. \tag{21}$$

Thus, the flux is strongly inhibited (i.e., $F \rightarrow 0$) for $Ta \rightarrow (0.1Ra)^{3/2}$. Eqs. (16) and (19) give $Ta \sim Ta_{no\text{-}slip}\,(Ra/Ra_c)^{4/3}$, so we can convert this into $Ta_{no\text{-}slip} \rightarrow 0.1^{3/2}Ra^{1/6} \cdot Ra_c^{4/3}$. Substituting canonical values ($Ra \sim 10^8$; $Ra_c \sim 10^3$) we find that the flux should be strongly inhibited for $Ta_{no\text{-}slip} \sim 10^4$, in good agreement with Rossby's data.

4. Application to oceanic staircases

According to Pearlstein's (1981) linear theory, the double-diffusive onset-of-convection boundary $Ra_c(Ta, R\rho)$ at fixed $R\rho$ has the same dependence on Ta as the thermal convection boundary $Ra_c(Ta)$, so $Ta_{bl} \sim 10^3$ should to be a critical value in diffusive convection as well as thermal convection. We therefore expect

- $Ta_{no\text{-}slip} < 1 \times 10^3$: rotation does not affect diffusive fluxes.
- $Ta_{no\text{-}slip} > 5 \times 10^3$: rotation strongly decreases diffusive fluxes.

We can now determine the effect of planetary rotation on oceanic diffusive fluxes. Calculating $Ta_{no\text{-}slip}$ using (16) with (12) and (13) we get:

- *Laboratory:* Typical values [$f \sim 10^{-4}$ s^{-1}; $\nu \sim 10^{-6}$ m^2 s^{-1}; $Nu = 15$ to match the oceanic case below; $H \sim 10^{-1}$ m] (Marmorino and Caldwell, 1976) yield $Ta \sim 1$ and $Ta_{no\text{-}slip} \sim 10^{-5}$. Rotation will not alter the fluxes.
- *Ocean:* Typical values [$f \sim 10^{-4}$ s^{-1}; $\nu \sim 10^{-6}$ m^2 s^{-1}; $H < 3$ m; $R\rho < 6$; $Ra < 2 \times 10^9$] yield $Ta < 8 \times 10^5$ and $Nu > 15$, so that $Ta_{no\text{-}slip} < 16$. Thus rotation should have little (<25%) effect on oceanic fluxes of heat and salt.

5. Summary

Rossby's (1969) measurements of the inhibition of convection by planetary rotation have been re-examined using a new measure of rotation, $Ta_{no\text{-}slip}$. This simplifies the

4. This states that the $Nu \propto Ra^{1/3}$ power-law can be extrapolated from high Ra down to Ra_c. Figure 1 shows that this is accurate to at best a factor of 2 in Nu.

flux relationship, leading to a simple empirical rule: for $Ta_{no\text{-}slip} < 10^3$ the fluxes are barely altered by rotation; for $Ta_{no\text{-}slip} > 5 \times 10^3$ rotation strongly inhibits convection. This rule was explained as a consequence of the $Ra_c(Ta)$ relationship at high Ta.

When applied to diffusive convection, the $Ta_{no\text{-}slip}$ criterion implies that rotation should not affect fluxes in either the laboratory ($Ta_{no\text{-}slip} \ll 1$) or the ocean ($Ta_{no\text{-}slip} \sim 10$). Thus, we needn't apply a rotational correction to the laboratory-based 4/3 flux laws before applying them to the ocean.

These predictions should be tested with experiments on a rotating table. Previously one would have presumed it necessary to measure the dependence of Nu on Ra, $R\rho$, and Ta. This would be arduous; to date only the $R\rho$ dependence has been mapped out adequately (i.e., to within a factor 1.5 in Nu). However, the present analysis suggests that these experiments could be greatly simplified. It may not be necessary to measure Nu at every combination of Ra, $R\rho$, and Ta, if it is first verified that the dependence of Nu on rotation rate obeys a simple rule, independent of Ra, when $Ta_{no\text{-}slip}$ is used instead of the traditional Taylor number Ta.

Acknowledgments. Most of this work was done while I was a student at Dalhousie University, supervised by Chris Garrett and Barry Ruddick and supported by the Natural Sciences and Engineering Research Council of Canada. I did some final analysis and wrote the paper while I was doing postdoctoral work at the Woods Hole Oceanographic Institution, supervised by John Toole and supported by OCE 84-00128. Sara Bennett and Eric Kunze carefully read an early version of the manuscript and made predictably helpful comments. Two reviewers were helpful in encouraging me to clarify the text, and a perceptive comment by George Veronis led to my explanation of the critical value for $Ta_{no\text{-}slip}$. This analysis was made possible by Tom Rossby's generosity in sending me his thermal convection measurements. Contribution number 6364 of the Woods Hole Oceanographic Institution.

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Received: *20 December, 1986;* revised: *20 August, 1987.*