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A preliminary comparison of selected numerical eddy-resolving general circulation experiments with observations

by William J. Schmitz, Jr.¹ and William R. Holland²

ABSTRACT

Results from several numerical experiments based on one class of eddy-resolving gyre-scale models have been compared kinematically with observations. The model oceans are forced by a simplified steady wind stress distribution in basins with idealized coastlines and bottom topography. The two-layer numerical simulations considered include new experiments as well as previously existing runs, totaling approximately 20 cases. The basic model results examined were time mean flow fields and time averages of second order properties of the eddy field (essentially fluctuation kinetic energy, and its distribution in a few frequency bands). Results for the model lower layer were compared with observations at depths near 4000 m from the western North Atlantic. Although the investigation concentrated on the abyssal depth range due to the nature of the oceanic data base available, it was possible to give some consideration to the relation between model upper layer and observed thermocline depth characteristics.

The apparently most realistic simulation relative to abyssal data was selected for detailed examination. This numerical experiment is comparable in basin size and wind-forcing amplitude with the mid-latitude North Atlantic. Several problems are identified, one of the most significant being inadequate zonal penetration from the west coast for the basic model fields. However, a variety of observed characteristics are approximately reproduced as long as comparisons are made at analogous relative locations in simulated and observed mean gyres and associated eddy fields. An extension of this run, containing idealized but variable bottom topography, was also examined. Relative to the flat-bottom case, smaller abyssal kinetic energy levels in better agreement with observation are found in the interior of the model ocean, along with relevant spectral intercomparisons in MODE-like regions. The apparently most realistic two-layer simulation available does not compare well with observations of eddy kinetic energy at thermocline depths in the vicinity of the Gulf Stream, the Mid-Atlantic Ridge, and the North Equatorial Current. The southern flank of the subtropical gyre is the location of the only major discrepancy in abyssal kinetic energy.

All model runs examined were characterized at abyssal depths by zonal penetration from the west coast of their mid-latitude jet (simulated Gulf Stream) and associated eddy kinetic energy distribution that is too short, by a factor of two or so. We tentatively conclude that this problem is associated with a lack of vertical resolution in a two-layer formulation. This may,
to some extent, be associated with the Rossby number being too small when the upper layer depth is chosen to represent the vertical scale of the main thermocline. Choosing a shallower upper layer in a two-layer formulation, in any event a rather contrived representation of the observed general circulation, leads to an unrealistic radius of deformation. It would appear that at least three layers are needed to adequately model these zonal scales of the oceanic system.

1. Introduction

The development and analysis of numerical eddy-resolving gyre-scale general ocean circulation models has been pursued extensively in the last several years. The first experiments of this kind (Holland and Lin, 1975a, b) indicated that instabilities in intense currents (i.e. the model Gulf Stream and associated recirculation) led to eddy energy production, and also that the eddies modified the time-averaged flow in a fundamental way. The results of this preliminary work have been followed by additional and generally more complex numerical calculations (Holland, 1978; Robinson et al., 1977; Semtner and Mintz, 1977; Semtner and Holland, 1978).

In this study, one class of models described by Holland (1978) has been run over an extended parameter range and results compared with selected segments of the existing North Atlantic data base (Fu et al., 1981; Luyten, 1977; Owens et al., 1982; Schmitz, 1976, 1977, 1978, 1980). Some earlier eddy-resolving general circulation studies (Holland, 1978; Holland and Rhines, 1980) indicated that certain prominent observed features of a generalized subtropical gyre circulation and associated eddy field could be reproduced qualitatively to some extent. Our efforts here have been directed toward a more specific comparison with data, although our goal is to ascertain the relevance of the major properties of the models as opposed to the details, consistent with the state of the art. Abyssal kinetic energies and their frequency distribution are probably the most basic and well established characteristics of the eddy field that can be readily intercompared, and the present study is focused accordingly. All model runs considered have their largest eddy kinetic energies in the vicinity of a mid-latitude jet or simulated Gulf Stream, a general property of the data base. Initially, the results from several numerical experiments were scanned to find those which produced relevant abyssal kinetic energy amplitudes, and then the potentially most promising cases were examined in more detail.

Evaluation of models relative to data typically leads to judgmental difficulties and is therefore controversial. This investigation is no exception, and we are keenly aware of our limitations. Far from the least of the problems involved is the practical question of matching the highly idealized geography/geometry of the model relative to that appropriate to the ocean, simply in order to actually implement an intercomparison, and we concentrate to some extent on this question. The model runs by Holland (1978) were made for areas that are generally smaller than any ocean basin. New numerical experiments in larger domains (not precisely as large as the North Atlantic, but close) are a focal point in this investigation. Another conceptual
issue is associated with the weight to be allocated to a numerical experiment whose results compare favorably (or not) with data kinematically, since such agreement (or disagreement) could be fortuitous. Our position is that kinematical compatibility with observation is a necessary but insufficient test of any model intended to be used to study the ocean.

The organization of this presentation is discussed in the next section, where we describe how we have proceeded in this intercomparison, and consider some of the reasons for doing so. Model and observational background material are contained in the Appendix, along with a general discussion of procedural questions.

2. Method of intercomparison

In this section we discuss those properties of the data base and of the numerical experiments under consideration that are fundamental to this intercomparison, and then the approach or method used is outlined. Attention is confined to the numerical model class (see Appendix) using "bottom friction" in (symmetric) "double gyre" basins as described by Holland (1978). The observations used were selected on the basis of accessibility and relative ease of intercomparison with model results. All of our efforts involving both observations and models have been essentially dictated by practicality.

Model results are based upon quasi-geostrophic equations applied to a two-layer, wind-driven ocean in a closed rectangular basin. Figure 1 exhibits the vertical and horizontal configuration of the model schematically. The wind-forcing is steady, trigonometric, and a symmetric function of latitude only (Fig. 1b; symbols are defined in detail in the Appendix). A few eddy-resolving general circulation experiments have been run with variable bottom topography. Its inclusion in the quasigeostrophic context is straightforward (see Appendix) as long as the amplitude of the bathymetry is small compared to the mean depth of the lower layer (Bretherton and Karweit, 1975). To first order one might consider bottom topographic variations to be made up of large scale features (continental slopes, Mid-Atlantic Ridge) upon which are superimposed "smaller scale roughness." In the following, a numerical experiment containing the latter type of topography will be shown to possess some potentially relevant features.

All bottom friction experiments are called members of model class 5 in the following, in analogy with the identification by Holland (1978) of his experiment (or case) 5 as the "basic bottom friction experiment." Each individual run has been assigned a second numerical identity (in general: \(5.n\); \(n = 0,1, \ldots\)). Experiment 5 described by Holland (1978) is here called 5.0. Starting from 5.0, we have explored the results of changing a number of important model properties (Table 1) such as basin dimensions (\(L\) is east-west extent, \(L'\) north-south), wind stress amplitude (\(\tau_o\)), and bottom friction coefficient (denoted by \(\epsilon\), see Appendix). For all
but the last of these, values appropriate to a given ocean basin are known approximately. Experiment 5.13 is nearest in basin size (4000 x 4000 km) to the North Atlantic at mid-latitude. A wind stress amplitude of about 1 dyne cm$^{-2}$ seems reasonable. We concentrate in the following on three numerical experiments, with limited attention to six others (the nine cases in Table 1). The remaining model runs were examined quickly, and set aside either as kinematically unrealistic, or as runs to be considered in more detail at a later date.

Previous analyses of the numerical model class under consideration have focused on investigations of energy (Holland, 1978) or vorticity balances (Holland and Rhines, 1980). Very preliminary comparisons with observations were made by Hol-
Table 1. Model parameters (symbols are defined in the text). In all cases, \( f_0 = 9.3 \times 10^{-9} \text{s}^{-1}, \beta = 2.0 \times 10^{-3} \text{m}^{-1} \text{s}^{-1}, g' = 0.02 \text{ ms}^{-2}, \ A_s = 8.0 \times 10^9 \text{ m}^2 \text{s}^{-1} \), and the layer depths were \( H_1 = 1000 \text{ m} \) and \( H_2 = 4000 \text{ m} \).

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>Description</th>
<th>( L ) (km)</th>
<th>( L' ) (km)</th>
<th>( \tau_0 ) ( (10^{-4} \text{ m}^2 \text{s}^{-2}) )</th>
<th>( \epsilon ) (s(^{-2}))</th>
<th>Bottom topography amplitude (m)</th>
<th>Special remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>Basic case</td>
<td>1000</td>
<td>2000</td>
<td>1</td>
<td>( 1 \times 10^{-7} )</td>
<td>0</td>
<td>Standard small basin case with a flat bottom.</td>
</tr>
<tr>
<td>5.2</td>
<td>Strong winds</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td>Small basin with double-strength winds.</td>
</tr>
<tr>
<td>5.9</td>
<td>Weak winds</td>
<td></td>
<td></td>
<td></td>
<td>0.5</td>
<td></td>
<td>Small basin with half-strength winds.</td>
</tr>
<tr>
<td>5.12</td>
<td>Mid-size basin</td>
<td>2000</td>
<td>2000</td>
<td></td>
<td>0.5 ( \times 10^{-7} )</td>
<td>Double the width of 5.0</td>
<td></td>
</tr>
<tr>
<td>5.13</td>
<td>Large basin</td>
<td>4000</td>
<td>4000</td>
<td></td>
<td>0.5 ( \times 10^{-7} )</td>
<td>Double the size of 5.12</td>
<td></td>
</tr>
<tr>
<td>5.16</td>
<td>Small basin</td>
<td></td>
<td></td>
<td></td>
<td>0.5 ( \times 10^{-7} )</td>
<td>Small friction version of 5.0</td>
<td></td>
</tr>
<tr>
<td>5.17</td>
<td>Mid-size basin</td>
<td>2000</td>
<td>2000</td>
<td></td>
<td>0.5 ( \times 10^{-7} )</td>
<td>Small friction version of 5.12.</td>
<td></td>
</tr>
<tr>
<td>5.18</td>
<td>Large basin</td>
<td>4000</td>
<td>4000</td>
<td></td>
<td>0.5 ( \times 10^{-7} )</td>
<td>Small friction version of 5.13.</td>
<td></td>
</tr>
<tr>
<td>5.20</td>
<td>Large basin; random topography</td>
<td>4000</td>
<td>4000</td>
<td></td>
<td>100</td>
<td>Like 5.13 but with random bottom bumps.</td>
<td></td>
</tr>
</tbody>
</table>
land (1978) and for a somewhat different type of model by Robinson et al. (1977). For present purposes, the model results have in addition been processed, analyzed and organized much as if the model data had been obtained by observation, in order to facilitate kinematical intercomparison. Time series at order daily intervals for order years (see Appendix) were generated at various model locations and standard averages and Fourier transforms calculated.

Each numerical experiment yields results at horizontal grid points (grid size $D_s = 20$ km) labeled $I = 2$, $N$ for the zonal ($x = (I-2)D_s$) coordinate and $J = 1$, $M$ for the meridional ($y = (J-2)D_s$) coordinate, with 1 and 3 subscripts referring to upper and lower layer respectively (Fig. 1). Horizontal labels $(I, J) = 1$ and vertical identifiers 0, 2, 4 are used for computational convenience. We deal primarily with $K_s'$, the horizontal eddy kinetic energy (per unit mass, hereafter understood) for the lower layer of the model, and its frequency distribution. Some attention will also be paid to $K_1'$ (upper layer). By definition,

$$K_i' = \frac{(u_i')^2 + (v_i')^2}{2}.$$  

An overbar denotes a time-average, and a primed quantity represents the deviation from that average. $u_i$ and $v_i$ are the zonal and meridional velocity components for layer $i$. The model results were listed for present purposes every 5th or 10th grid point (100-200 km spacing), so that plots will appear overly smooth relative to a more highly sampled procedure. When referring to similar data-based quantities, the terms abyssal and thermocline $K_E$ are used in correspondence to $K_s'$ and $K_1'$ respectively, with analogous definition. Model mean flow components are for example $\bar{u}_i$, and a similar symbol is used for observations with an indication of depth other than a subscript.

We will focus on 5.0 and 5.13 (especially the latter), experiments for the smallest ($1000 \times 2000$ km) and largest ($4000 \times 4000$ km) basins run to date, along with a few variations relative to features other than basin size (Table 1). Essentially, we have scanned all available runs, and 5.13 and its topographic extension 5.20 appear to be the most realistic, as described in the following. 5.0 is of some special interest because it is probably the most thoroughly discussed numerical experiment in the literature. Examination of several numerical experiments leads to some clarification of the significance of the detailed intercomparison of 5.13. The effects of varying forcing and dissipation amplitudes only are brought out by comparing 5.2, 5.9 and 5.16 with 5.0. Three other runs (listed in Table 1) are also used. 5.17 and 5.18 provide additional information on the variation of the amplitude of lower layer kinetic energy with the friction coefficient ($\epsilon$). Results from 5.12 (a $2000 \times 2000$ km basin, intermediate in size between 5.0 and 5.13) are useful in helping to rationalize a major discrepancy (inadequate zonal penetration of model eddy and mean kinetic energies at abyssal depths).
We next describe selected kinematic properties of the runs that were examined in some detail, in order to orient the comparison. The $K_3'$ map for 5.0 (Fig. A.1), when compared with its corresponding mean circulation in Figure 2, demonstrates clearly that the strongest abyssal eddies are found in the vicinity of the simulated Gulf Stream. The mean abyssal gyres in Figure 2(b) penetrate about 600 km from the west coast, nearly coincident with the zonal penetration of the 30 cm$^2$s$^{-2}$ contour in Figure A.1, after which $K_3'$ decays sharply towards the east coast. This type of relationship between the abyssal mean flow pattern and $K_3'$ appears to be a general model property. The maximum values of $K_3'$ for 5.0 in Figure A.1 are low by a factor of 3-4 relative to the order 100 cm$^2$s$^{-2}$ grossly typical (Schmitz, 1976, 1977, 1978) of observations of abyssal kinetic energy near the Gulf Stream (west of the Grand Banks of Newfoundland). 5.2 and 5.16, with larger forcing and smaller dissipation than 5.0 respectively (Table 1), are more realistic in $K_3'$ amplitude (Fig. 3).

The streamline patterns for the mean flow from model run 5.13 (Fig. 4) are similar to those for 5.0 in Figure 2 in that the same basic configuration exists. The abyssal gyres (Fig. 4b) and associated $K_3'$ distribution (Fig. 5) for 5.13 extend somewhat less than twice as far from the west coast as is the case for 5.0 A small basin run could not be quantitatively realistic in this regard, but neither is 5.13, when compared with the abyssal gyre according to Worthington (1976), where the general circulation for potential temperatures less than 4°C is taken to be approximately 2500 km in zonal extent. The maximum in $K_3'$ for 5.13 in the vicinity of the model Gulf Stream is near 100 cm$^2$s$^{-2}$ in amplitude (Fig. 5), as is realistic in a general way for the North Atlantic (Schmitz, 1977, 1978), and in contrast to 5.0. Figure 6 is a composite latitudinal plot for $K_3'$ for model 5.13, demonstrating a 3-lobe structure in the westernmost part of the basin and a single maximum thereafter, a peak at $I = 32$ or 42 (a distance, $x$, of 600 to 800 km from the western boundary), a fairly sharp decay to $I = 62$ ($x = 1200$ km) and 82 ($x = 1600$ km), with a weak and flat field at larger model longitudes.

Figure 7(a) is a $K_3'$ map for 5.20, and 7(b) a superposition of selected $K_3'$ contours for 5.13 and 5.20. There are three general effects associated with the type of bottom topography (described in the Appendix) added to 5.13 to obtain 5.20, as brought out by Figure 7(b). First, topographic scales are introduced; second, the maximum value of $K_3'$ near the model Gulf Stream (near field) is somewhat enhanced; and third, the eddy field for the lower layer in 5.20 is less intense than for 5.13 at locations (the far field) away from the energetic segment of the system. The main purpose of Figure 7(b) is to illustrate the latter characteristic; the 1 cm$^2$s$^{-2}$ contour for 5.20 approximately overlays the 5 cm$^2$s$^{-2}$ contour for 5.13.

Since all model runs were made in basins smaller than the North Atlantic, our initial task was to figure out how to compare model results with data from specific
Figure 2. Mean streamline patterns for model run 5.0: contour intervals are variable, being selected to bring out the major properties of the model general circulation: (a) upper layer, (b) lower layer. Solid lines indicate anticyclonic circulation, dashed lines cyclonic.

latitudes and longitudes. This difficulty led to the new experiments in basins much larger than that for 5.0. We first compare briefly the scales and amplitudes of the mean flow (Section 3), with emphasis on the apparently most realistic numerical experiment available (5.13). Kinetic energy distributions are considered in Section 4. Since all maps of $K_s'$ yield maxima in the vicinity of the western segment of the model Gulf Stream, initial efforts to find realistic amplitudes were devoted to this region (Section 4a). The way in which this is done leads to a concurrent examination of zonal scales, and a comparison of the properties of different model runs. We
also briefly examined the frequency distribution of abyssal $K_B$ in this region, to find out whether or not we were comparing results across common time scales [see Schmitz and Owens (1979) for a discussion of this point]. Latitudinal distributions of kinetic energy (Section 4b) are examined next, using both thermocline and abyssal data. For the thermocline intercomparisons model values from the upper layer are compared against data at 500-600 m depths, which is the approximate mid-point of the upper layer in the model. Kinetic energy maps are discussed in Section 4c. Frequency distributions at locations (away from the immediate vicinity of the Gulf Stream) in the recirculation and in the interior are considered in Section 4d. Section 5 is a summary, 6 conclusions.
3. The Mean field

The streamline patterns of the time-averaged flow fields for models 5.0 and 5.13 in Figures 2 and 4 are typical in a qualitative sense for all class 5 models. A generalized streamline pattern is outlined schematically in Figure 8. There is an eastward "jet" at the mid-latitude of the basin in each layer, taken to be the simulated Gulf Stream. There are two scales of time-averaged circulation for the upper layer in Figure 8. These patterns are labeled $I_1$ and $O_1$ to denote inner and outer gyres. The outer gyre in the model southern half-basin may be analogous to the observed subtropical gyre, and the inner model gyre to the recirculation associated with the Gulf Stream System. Each half-basin contains two gyres in the lower layer, both of the smaller horizontal scale type. The deep gyres closest to the center of the basin are labeled $I_s$ in Figure 8 and are approximately coincident with the $I_1$ recirculations. This type of inner and outer gyre mean flow pattern (for the southern half-basin, and except for the deep gyre(s) labelled $A_3$) is grossly consistent with
Worthington's (1976) picture of the North Atlantic circulation, and with the type of result found from existing moored instrument data (Schmitz, 1977, 1978, 1980). Since the deep gyres to the south (north) of the $I_3$ recirculations are relatively new elements of any scheme for the general circulation (see, however, Wunsch, 1978), they are labelled $A_3$ or additional deep gyres [Holland and Rhines (1980) discuss their origin]. There is some support for a weak abyssal flow immediately south of the recirculation with an eastward component in the data presented by Schmitz (1980) and by Owens, Luyten and Bryden (1982). It is to be understood that the flow patterns in the model northern half-basin are due to idealized symmetry and geometry (Appendix).

Although the mean circulation patterns in Figure 8 have qualitatively realistic features, there are quantitative problems. The deep model gyres for numerical experiments 5.0 and 5.13 [in Figs. 2(b) and 4(b)] penetrate only 600-800 and 1000-1200 km respectively from the western boundary (the most intense eddies extend essentially the same distance east into the model ocean, reference Figs. A.1 and 5). If we were to try to compare, for example, mean flows along 55W (a principle locus of the long-term averages available, see Appendix) with model results at a distance of around 1500 km from the model west coast (roughly the range between 55 and 75-80W, at 35 to 40N), then there would be no result for run 5.0, which is only 1000 km in zonal extent. For run 5.13, the abyssal zonal current components 1500 km from the coast are 1 cm s$^{-1}$ or less, much lower than observed at 55W (Schmitz, 1977, 1978, 1980). To proceed further, we scanned the results for 5.13 to see if there were model zonal mean currents at a smaller value of $x$ that resembled the data at 55W, as described in the next paragraph.

Figure 9 contains the meridional distribution of mean zonal velocity components from run 5.13 at $x = 600$ km ($I = 32$). Also plotted are the observed time-averaged zonal velocity components along 55W, taken from Figure 3 by Schmitz (1980). The upper panel shows the upper layer or thermocline depth flows (for the model representing an average over the upper 1000 m; the observations are from 600 m); similarly the lower panel shows the abyssal or lower layer flows (for the model representing an average over the depth range 1000 m to 5000 m; the observations are from 4000 m). The meridional scales in Figure 9 were lined up to demonstrate some correspondence in gross amplitude and meridional structure between model and observed mean currents. There is a latitudinal shift between the observations and the model when drawn relative to the axis of the (mean) Gulf Stream, observed and simulated, as indicated in Figure 10. In the model, the eastward flow in the deep ocean near mid-basin directly underlies the surface Stream, whereas the observations show a decided shift to the south. Model $u_3$ values at $x = 800$ km ($I = 42$) are lower in amplitude than the $I = 32$ data in Figures 9 and 10 by about a factor of 2, and values at $I = 22$ are a factor of about 2 larger.
Figure 4. Mean streamline patterns for model run 5.13: contour intervals are variable, being chosen to bring out the major properties of the model general circulation: (a) upper layer, (b) lower layer. Solid lines indicate anticyclonic circulation, dashed lines, cyclonic.

We have seen in this section that the mean field for model 5.13 has some relevant features but also some defects, in particular, that its zonal penetration from the west coast is too short, by a factor of about 2 to 3. That is, the deep gyre for 5.13 extends east about 1000-1200 km, as opposed to 2500 km according to Worthington (1976), and this model's mean zonal flows at a distance from the model west coast of 600-800 km are comparable in amplitude to the observed mean flows at a distance of about 1700 to 2100 km (55W) from the west coast (located at 75 to 80W). In Section 4, it will be demonstrated that essentially the same general conclusions hold for kinetic energy distributions.
4. The Eddy field

We will next compare simulated and observed distributions of fluctuation kinetic energy in four ways, as constrained by the data available (see Appendix): (a) For maximum amplitude and zonal scales near the Gulf Stream, including a key frequency distribution; (b) For latitudinal scales at both thermocline and abyssal depths; (c) For two dimensional abyssal patterns; and (d) For a few frequency distributions at locations removed from the Gulf Stream.

a. Abyssal $K_E$ in the vicinity of the Gulf Stream. We first consider results from the model run (5.13) in the largest basin (4000 by 4000 km). $K'_E$ (G.S.M.) is defined to be the maximum value of $K'_E$ near the model Gulf Stream (G.S.M. denotes
Gulf Stream maximum). Figure 11 is a plot of $K_{3}'$ (G.S.M.) as a function of $x$, with analogous data from 70 and 55W superimposed. The length of a degree of longitude in Figure 11 is 85 km (appropriate to about 40N), the west coast is taken to be at 75W, and half of the model basin is shown. $K_{3}'$ (G.S.M.) at an $x$ of about 1700 km in Figure 11 is more than an order of magnitude lower than the observed $K_B$ at 55W, although $K_{3}'$ (G.S.M.) at an $x$ ($\sim 450$ km) equivalent to 70W is realistic. The prominent result is that 5.13 has a zonal penetration scale (from the west coast) for abyssal kinetic energy that is significantly smaller than observed. The $K_B$ amplitude for 5.13 in Figure 11 at $x = 600$ to 800 km is that closest to the 55W data. This situation is nearly identical to the dilemma encountered in comparing mean flows in Section 3.
The penetration length of the mid-latitude jet from the west coast for model 5.13 is roughly 1000 km (Fig. 4), about the same as that at which the abyssal eddy field in Figure 5 begins to decay abruptly to the east. That is, the scales for both mean and eddy fields are similar, a property of most (or all) class 5 models. The observed abyssal gyre extends (Worthington, 1976) to roughly 50W or 40W (that is, to the Grand Banks of Newfoundland or the Mid-Atlantic Ridge), where the evidence available (Luyten, 1977), although short-term, suggests that the abyssal eddy field decays to a value of 50 cm^2s^-2 or so in $K_B$. That is, the observed eddy and mean fields have at least qualitatively similar scales. This implies that if we compare results at equivalent locations in model and observed gyres, then the distributions of abyssal $K_B$ near the Gulf Stream should be more nearly the same. This was done for the mean flow in Section 3 by asking if there was a model longitude anywhere with data similar to that at 55W, and in the previous paragraph it was noted that a similar argument would hold for $K_s'$ (G.S.M.). We proceed toward the same goal in the next paragraph, but in slightly different fashion, to show how this could be done somewhat more quantitatively.

One way of visualizing how this works is shown in Figure 12(a), where the zonal coordinate for both model and observations is non-dimensionalized by a particular choice of jet-penetration or inner gyre scale. Other characteristics of Figure 12 are similar to those for Figure 11. The quantitative significance of the results in Figure 12(a) should not be over-stated. Different plausible choices of longitude of the west coast and of gyre size [Fig. 12(b)] lead to less precise graphical coincidence than in Figure 12(a). Choice of the model jet penetration scale of 1000 km in Figure 12 is also subjective. We have, in addition, explored the results of non-dimensionalizing by basin width for a variety of model runs; at this time it appears to us that the
fundamental zonal scale is the size of the inner gyre (or penetration length of the model mid-latitude jet).

Even though an intercomparison of other model properties such as latitudinal scales and frequency distribution is not clearly useful at corresponding longitudes, there may be some potential in intercomparison at analogous positions in model and observed gyres. The following examination of the frequency distribution of $K_s$ at abyssal depths near the Gulf Stream at 55W is an example of the procedure that will be followed in intercomparing other results at similar relative positions in model and observed gyres. It also lets us find out whether or not we have a fundamental
time-scale match between model and data; the significance of intercomparison across common frequencies was demonstrated in the context of another model class by Schmitz and Owens (1979). $K_3$ (G.S.M.) of about 125 cm$^2$s$^{-2}$ is found for model 5.13 at an $x$ of 600-800 km (Fig. 11) ($I = 32$ or 42). Since this amplitude is consistent with the observed abyssal $K_B$ (G.S.M.) at 55W, and does occur at a position in the model gyre equivalent to a location near 55W in Figure 12, we intercompare the frequency distribution of $K_B$ [Fig. 13; Table 2] at these locations. The agreement between model and observation in Figure 13 is remarkable if it is not accidental. Frequency distributions for sites on either side of PM210 (location labels PM2XX are defined in the Appendix) are included in Table 2 as an indication of the variability of data-based estimates. At site PM211 the effect of the continental rise is seen as a shift in $K_B$ toward higher frequencies (Schmitz, 1978).
Figure 8. Schematic mean streamline pattern for model class 5. $I_1$ denotes inner gyre; $O_1$, outer gyre; $A_3$, "additional gyre."

The ranges of time scales spanned by each frequency band in Table 2 are indicated by the symbol $K_E(T_1, T_2)$, where $T_1, T_2$ are respectively the maximum, minimum periods contained in the indicated estimate of $K_E$. The low frequency cutoff was located at one cycle per 720 days due to limitations on the length of the time series of observations available; this same choice is made for all intercomparisons of the frequency distribution of $K_E$ in this article. The mesoscale period range was taken to be roughly 50 to 150 days (actually 50.7 to 156.7 days for coincidence with the natural frequencies for the model time series). In all frequency distribution figures (as in Fig. 13), model locations will be referred to as 5.$n$ ($K, I, J$), where $n$ is the numerical experiment identifier and $K, I, J$ are model vertical, zonal, and meridional grid point coordinates respectively. Perhaps the most remarkable feature of Figure 13 and Table 2 is the behavior at periods longer than the mesoscale. Model forcing is steady, and the model time-dependence resulting from instabilities does contain significant realistic energy beyond eddy scales. There do not appear to be significant problems associated with the existence of energetic, very long period variability in comparing two year and five year time series, as demonstrated by the last two columns in Table 2, where $K_E(T)$ denotes the total $K_E$ (i.e., including $K_E$ associated with periods longer than 720 days) for each source. For all cases in Table 2, including both model and data, the fraction of $K_E$ at periods longer than 720 days is bounded by 15 percent. It will be shown in the following that some of
the favorable features of Figure 13 and Table 2 are typical in the broadest sense.

We now examine results from the smallest (1000 × 2000 km) basins. This will help demonstrate the significance of having selected 5.13, and in particular these results are used to help rationalize a major failure of 5.13; that is, the inadequate zonal penetration noted previously. As noted above, 5.0 has previously been examined more extensively (Holland, 1978; Holland and Rhines, 1980; Rhines and Holland, 1979; Harrison and Holland, 1981) than any other numerical experiment. Construction of a graph for 5.0 in direct analogy to that for 5.13 in Figure 11 is not possible because the 70 and 55W data are separated by a distance larger than the 1000 km zonal extent of the model 5.0 basin. To proceed, we define $\bar{K}$...
Figure 10. Mean zonal velocity component ($\bar{u}$) for model 5.13 at a distance of 600 km from the west coast, along with observations along 55W. The model latitude ($y$) origin has been chosen (approximately) to line up with the axis of the Gulf Stream at the southern end of its observed range (denoted by the bar so labeled).

(O.G.S.M.) as the average of the 70 and 55W $K_B$ values in Figure 11 [O.G.S.M. is used as an abbreviation for observed Gulf Stream maximum; in analogy to our previous use of $K_{3'}$ (G.S.M.)]. $\bar{K}$ (O.G.S.M.) is plotted without regard to longitude as a dashed line in Figure 14. Results from the small basin runs with forcing and dissipation amplitudes other than those used for 5.0 are also included in Figure 14. It is clear that 5.0 is low in $K_{3'}$ (G.S.M.) relative to observation by a factor of 3 to 4, and that 5.16 (like 5.0 but with bottom friction coefficient half as large, Table 1) and 5.2 (double the wind forcing relative to 5.0, Table 1) are more realistic. 5.9 (1/2 valued wind forcing relative to 5.0, Table 1) is totally (order of magnitude) unrealistic with respect to $\bar{K}$ (O.G.S.M.). We could now scale the results for the
small basins, case 5.2 say, like we did for 5.13 in Figure 12. This has been done but no new significant information emerges. We also looked for a spectral intercomparison at a location in any small basin run (5.2 was chosen) nearest in energy level to the 55W data at PM210. One finds a nearly identical result (Fig. 15; Table 2) to that in Figure 13. The agreement between model data points 5.13 (3,32,100) and 5.16 (3,22,100) is better than the intercomparison of either with PM210 (4000).

The amplitude of $K_a'$ is expected to increase by increasing forcing or decreasing dissipation. We find that $K_a'$ (G.S.M.) varies like $\tau_0^2$ [Fig. 16(a)] with some precision (other parameters held constant). Also, $K_a'$ (G.S.M.) goes roughly as $\epsilon^{-1}$ [Fig. 16(b)]. Since the range of $\epsilon$ for the numerical experiments in Table 1 spans only two values, Figure 16(b) is plotted for the three run pairs (other parameters held constant) available, with the ratio of $\epsilon$ multiplied by the maximum $K_a'$ for one of the run pairs. $K_a'$ (G.S.M.) is approximately proportional to $\tau_0^2 \epsilon^{-1}$ times some function of $L$ and $L'$ (other model properties held constant). We are in the process of examining this latter result in more detail, and our intention is to consider such questions in a later study.

In Figure 14, the decay in $K_a'$ (G.S.M.) starts about halfway across the basin for model runs 5.0 and 5.16, and somewhat farther for 5.2. The only difference between 5.2 and 5.0 is in the choice of $\tau_0$, and the only pertinent non-dimensional quantity changed is the Rossby number, defined by Holland (1978) to be $R_o = \pi \tau_0 /$
Figure 12. Maximum abyssal eddy kinetic energy ($K_E$, cm$^2$s$^{-2}$) near the Gulf Stream, as a function of distance from the west coast, non-dimensionalized by the zonal extension of the model mid-latitude jet or of the model inner gyre. Model run 5.13 results are plotted with solid dots connected by lines. 70W data are denoted by an open square and 55W data by an open circle surrounding small dots. Two plausible choices of origin of the west coast and distance of eastward penetration of the Gulf Stream are shown: (a) origin at 80W, with the Stream extending to the Grand Banks (50W), (b) origin at 75W, with the Stream extending to the Mid-Atlantic Ridge (40W).

($H, \beta^2L^3$). Note that with this definition, $R_o$ can only be compared directly for runs with the same ratio of $L$ to $L'$. $R_o$ for 5.2 is twice that for 5.0. $R_o$ is identical for 5.0 and 5.16, and they have essentially the same pattern in $K_E'$ (G.S.M.) and mean streamlines. The small basins are a bit short to explore the relation between $R_o$ and zonal penetration scale in any detail because the eastern boundary may start to play some role.
Comparison of 5.2 with 5.16 and/or 5.0 provides a hint but not a clear cut example of the relation of \( R_0 \) to the zonal penetration scale of the recirculation and energetic eddy field. In order to determine if \( R_0 \) is indeed affecting the scale in question, we compared 5.12 (a 2000 \( \times \) 2000 km basin) with 5.13; \( R_0 \) for 5.12 is 8 times as large as for 5.13. The penetration scale of the abyssal recirculation and intense eddy field for 5.12 (Fig. 17) is approximately 1500 km, about 300 to 500 km larger than 5.13, and relative to basin width roughly twice as large as that for 5.13. We need to be cautious here. In moving from model run 5.0 to 5.2 only \( R_0 \) was modified and therefore only \( R_0 \) was affected. However, in moving from model run 5.12 to 5.13, \( R_0 \) was not the only non-dimensional number changed (the others were the non-dimensional radius of deformation and the non-dimensional frictional

Table 2. Abyssal kinetic energies (cm\(^2\)s\(^{-2}\)) for model and data for selected period ranges (in days). Notation is defined in the text.

<table>
<thead>
<tr>
<th>Source</th>
<th>( K_E(720, 156.5) )</th>
<th>( K_E(156.5, 50.7) )</th>
<th>( K_E(50.7, 4) )</th>
<th>( K_E(\Sigma) )</th>
<th>( K_E(T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM209 (4000)</td>
<td>27</td>
<td>52</td>
<td>55</td>
<td>134</td>
<td>140</td>
</tr>
<tr>
<td>PM210 (4000)</td>
<td>23</td>
<td>42</td>
<td>48</td>
<td>113</td>
<td>127</td>
</tr>
<tr>
<td>PM211 (4000)</td>
<td>12</td>
<td>38</td>
<td>80</td>
<td>130</td>
<td>135</td>
</tr>
<tr>
<td>5.13 (3, 32, 102)</td>
<td>29</td>
<td>46</td>
<td>37</td>
<td>112</td>
<td>125</td>
</tr>
<tr>
<td>5.13 (3, 42, 102)</td>
<td>20</td>
<td>51</td>
<td>47</td>
<td>118</td>
<td>124</td>
</tr>
<tr>
<td>5.2 (3, 22, 102)</td>
<td>29</td>
<td>46</td>
<td>36</td>
<td>111</td>
<td>125</td>
</tr>
</tbody>
</table>
Figure 14. Maximum abyssal eddy kinetic energy ($K_E$, cm$^2$s$^{-2}$) near the Gulf Stream as a function of zonal distance from the west coast, for model runs identified by the label [5.n] and plotted as dotted lines. The dashed line labeled $\bar{K}$ (O.G.S.M.) is an average of observations from 70 and 55W.

parameters). In physical space, the zonal scales for 5.12 and 5.13 differ by about 30 percent and the amplitude of the abyssal eddy field for 5.12 is less than that for 5.13 by about the same percentage.

b. **Latitudinal distributions of $K_E$, $K_3'$** values for model run 5.13 near zonal grid points $I = 32$ and 42 ($x = 600$ to 800 km) have been demonstrated to be com-

Figure 15. Kinetic energies for model and observation in selected frequency bands at abyssal depth near the Gulf Stream. The same method is employed for all such plots in this article, reference Figure 13 and accompanying text.
Figure 16. Maximum lower-layer eddy kinetic energy \( [K_3' (G.S.M.)] \) near the Gulf Stream:
(a) as a function (other parameters held constant) of model wind stress amplitude \( [\tau_0, \text{ dynes cm}^{-2}] \); the dashed line is a least squares fit of \( K_3' (G.S.M.) \) as a function of \( \tau_0 \), (b) for model runs with friction coefficient \( \varepsilon_1 \) (other parameters held constant) as a function of relative frictional parameter \( \alpha = \varepsilon_1/\varepsilon_2 \) multiplied by \( K_3' (G.S.M.) \) for the numerical experiment with friction coefficient \( \varepsilon_1 \) [the dashed line is equivalent to an \( \varepsilon^{-1} \) fit for \( K_3' (G.S.M.) \)]. Results from model runs are identified by the label \( \text{[5.]} \text{n}\). The model runs entered above the dashed line are associated with \( \varepsilon_1 \); the model runs entered below the dashed line are associated with \( \varepsilon_1 \), and are plotted along the abscissa with their \( K_3' (G.S.M.) \) multiplied by \( \alpha \). The ordinate is \( K_3' (G.S.M.) \) for the model runs associated with \( \varepsilon_1 \).
Figure 17. $K_s'$ map for model run 5.12. Contours selected in order to indicate only the scale of zonal penetration of the energetic segment of the abyssal eddy field.

Comparable in amplitude (Fig. 12) and also in frequency distribution (Fig. 13) to 55W data near the Gulf Stream. We now examine the compatibility of model latitudinal distributions near these values of model longitude with the 55W data. The general trend of the intercomparison of abyssal kinetic energies in Figure 18 is favorable. In this Figure, the southern latitude of the range of positions appropriate to the mean axis of the Gulf Stream is lined up with the model mid-basin.

Comparatively new data along 55W near 15N are now available from cluster C of PM3 (Keffer et al., 1979; symbols and some discussion are contained in the Appendix) in the vicinity of the North Equatorial Current (NEC). An intercom-
Figure 18. Abyssal kinetic energies ($K_E$, cm$^2$s$^{-2}$) as a function of model meridional grid point and observed latitude: solid squares denote data along 55W; dots are used for model run 5.13 results along $I = 32$, X's for $I = 42$.

Comparison with this data, although inhibited because the model meridional coordinate doesn’t penetrate as far south as 15N, can be perceived quickly. Even though 5.13 does have a NEC equivalent in the sense of a similarly-directed flow at southerly latitudes (Fig. 4), $K_3'$ there is quite low (a factor of 5-10) relative to the PM3 data [exhibited in detail in Section 4(c) below].

$K_1'$ for 5.13 decays more abruptly (Fig. 19) to the east (near $I = 32, 42$) than $K_3'$, but an intercomparison with the 500-600 m depth 55W data is not seriously

Figure 19. Upper layer kinetic energies as a function of meridional grid point for model 5.13, at indicated zonal grid points.
impaired as a consequence because this decay in $K'_1$ is most pronounced near the Gulf Stream where observations are presently sparse. In Figure 20(a), there is a tendency for agreement on the general decay scale, but $K'_1$ values begin to exceed observed values approaching the Gulf Stream, as is more clearly demonstrated on
the expanded scale in Figure 20(b). One can identify a significant difference between model and observed thermocline level kinetic energies at only one data point. There is some additional evidence that $K'_t$ for model 5.13 is significantly larger than that observed in the immediate vicinity of the Gulf Stream. $K'_t$ for run 5.13 reaches approximately 1600 cm$^2$s$^{-2}$ in the model Gulf Stream near $l = 32$ (Fig. 19). Yet Wyrtki et al. (1976) reports values of only 700 cm$^2$s$^{-2}$ (approximately) for the surface near this location. Estimates of thermocline (700 m) depth $K_B$ based on SOFAR float data (Schmitz et al., 1981) are near 450 cm$^2$s$^{-2}$. Nishida and White (1981) found maximum $K_B$ values for roughly 100 m depth (relative to 1000 m) in the Kuroshio Extension of about 200-300 cm$^2$s$^{-2}$.

Zonal and meridional contributions to $K_B$ were intercompared using the data in Figure A.2 in a similar fashion to the way the $K_B$ data in Figure A.2 were used in developing Figures 17 and 19. Although the model results were less consistent with the data than was found in the comparison with $K_B$, no new information of fundamental significance to the present investigation was detected.

c. Abyssal $K_B$ maps. Observed values of abyssal $K_B$ have been superimposed (Fig. 21) on the $K'_s$ maps for model runs 5.13 and 5.20 [Figs. 5 and 7(a)]. Figure 21(a) is to some extent a summary of the zonal and meridional distributions discussed above, but with some additional data (from 70W and PM3). In Figure 21, the observations are shown (reference abscissa scale at top of graph) at longitudes scaled (Section 4) by the estimate of jet penetration or inner gyre scale as in Figure 12(a), and also with their latitude fixed in relation to the mean axis of the Gulf Stream, except for PM3 cluster C. The PM3 data are set apart by dark borders around circles inside squares, partly to call attention to the fact that the value of 9 cm$^2$s$^{-2}$ for $K_B$ near the southern boundary of the model along roughly 55W is entered at an arbitrary equivalent latitude. The value for abyssal $K_B$ of 1 cm$^2$s$^{-2}$ from PM3 results near 28N (1250 km south of axis) is entered at an average equivalent longitude for clusters A and B. The length of a degree of longitude in Figure 21 is fixed at 85 km, appropriate to 40N. A dotted line is shown in Figure 21 at about $l = 95$, with dashed lines pointing to the 55W data. If non-dimensionalization were not employed, this dotted line is where the 55W data would be located on Figure 21; that is, in a region where there is no resemblance between model and data. This is perhaps the most graphical example we can produce of the necessity for getting the zonal scales right in the model. The area of maximum disagreement (except for PM3 cluster C) for 5.13 vs. data in Figure 21(a), near the southern edge of the model 5 cm$^2$s$^{-2}$ contour, is considerably improved for model run 5.20 [Fig. 21(b)] where bottom roughness has reduced $K'_s$ in the far field. However, the agreement near maximum values of $K'_s$ is not as good for 5.20 as for 5.13.

d. Frequency distributions of $K_B$ away from the Gulf Stream. Selected frequency distributions at locations in the recirculation and near the interior were intercom-
Figure 21. $K'_s$ maps with data superimposed: (a) for model run 5.13, (b) for model run 5.20. The contours are model values; the numbers in boxes are observations. The data inside rectangles are from 70 and from PM1 and PM2 along 55W, the squares with bordered circles are from PM3. The dotted line referenced to 55W by dashed lines and arrows denotes where data from this longitude would be located if plotted in physical space rather than against zonal coordinate non-dimensionalized by jet penetration scale.

pared for both model layers. The search for MODE-like energy levels for both thermocline and abyssal depths for $l$'s less than 32, in order to be in the vicinity of the model equivalent longitude for MODE, was successful for model run 5.20 but not 5.13. The resulting frequency distributions in Figure 22 compare reasonably well, but perhaps less so than for abyssal depths near the Gulf Stream.
The frequency distributions in Figure 23 are from locations between the Gulf Stream and MODE-like regions. The agreement between model and observation is good for equivalent energy levels. In Figure 23(b), we have also plotted the frequencies of an additional model location close to that shown in Figure 23(a). This is another example (along with Figs. 13, 15 and Table 2) of agreement between model and observed frequency distributions with similar overall energy levels.

5. Summary

Recent eddy-resolving gyre-scale models have begun to give some new insight into the dynamical nature of the oceanic general circulation. Model studies of this type
have been carried out by Holland and Lin (1975 a, b), Robinson et al. (1977), Semtner and Mintz (1977), and Holland (1978). These numerical investigations, along with a variety of studies of the processes occurring in the simulations (Haidvogel and Holland, 1978; Harrison and Robinson, 1978, 1979; Harrison and Holland, 1981; Holland and Rhines, 1980; Rhines and Holland, 1979; Semtner and Holland, 1978), have proceeded in parallel with observational-based efforts (Dantzler, 1976, 1977; Fu et al., 1981; Luyten, 1977; Nishida and White, 1982; Owens et al., 1982; Richman et al., 1977; Schmitz, 1976, 1977, 1978, 1980; Wyrtki

Figure 22. Kinetic energies for model and observation in selected frequency bands at MODE-like locations: (a) thermocline depths, (b) abyssal depths. The same method is employed for all such plots in this article, reference Figure 13 and accompanying text.
et al., 1976) which are beginning to identify the major characteristics of the statistical properties of ocean flows. In the present investigation, detailed kinematical intercomparisons between model and observation were initiated in an attempt to gain added insight into both.

Here we have been concerned with a single numerical model class, based on experiment 5 from Holland (1978). We have carried out an intercomparison between a subset of the observations available and around 20 runs of model class 5, many of these numerical experiments being new. The data base used leads to a concentration on abyssal kinetic energies and their frequency distribution in the
western North Atlantic, with some secondary attention to thermocline-level kinetic energies; these were compared with corresponding results from the model lower and upper layers respectively.

As basin size is made more realistic, the intercomparison becomes in a sense more practical. We concentrated on the largest basin case, 5.13 (and its derivative 5.20, characterized by the addition of idealized topography to 5.13). Although many properties of 5.13 compare favorably with observation when examined at equivalent positions in the general circulation, the zonal scale of this circulation and its associated eddy field is too small. The zonal penetration of the model mid-latitude jet and associated eddy field does not become realistic for the larger basins. Examination of a number of cases points to the influence (as a first approximation) of processes typified by the Rossby number, defined by Holland (1978) to be: 

$$ R_o = \frac{\pi \tau_o}{H_1 \beta^3 L^3}. $$

In order to achieve a sufficiently high $R_o$ with $\tau_o$, $\beta^3$ and $L$ appropriate to the North Atlantic, $H_1$ would have to be made relatively small, leading to an unrealistic radius of deformation in the two-layer formulation and a contrived representation of the large scale density field. Therefore, we conclude that more vertical resolution is required. Preliminary results from a three-layer simulation suggest that the zonal scale desired may be achievable. Semtner and Holland (1978) identified discrepancies between two-layer quasi-geostrophic and five-level primitive equation experiments that could be resolved with three-layer quasi-geostrophic simulations.

Abyssal kinetic energies from 5.13 and 5.20 agree favorably with the data base in some regions of the western North Atlantic (with the comparison made in approximately equivalent locations relative to the mean flow). The distributions of abyssal kinetic energy with frequency (in a few bands) for these models also resembles those for the data examined, again, in certain regions. These model runs become less relevant in upper level kinetic energies near the Gulf Stream and in some sections of the interior, although there is better agreement at locations in between. $K_E$ in both layers is low by a factor of 2-5 in the vicinity of the North Equatorial Current.

PM208 is a unique site in the data base, and does not have a clear-cut analogue in the results for any model run. This is the location closest to the Gulf Stream where data are available at both thermocline and abyssal depths. At this site, the most energetic yet occupied, $K_E$ is observed to be weakly depth-dependent (Schmitz, 1978). However, in all model runs examined, $K_E$ is much more depth-dependent at sites close to or in the model-equivalent Gulf Stream than anywhere else. PM208 is the location of maximum discrepancy with $K_E$ in Figure 20 (Section 4). There is also no analogue in the models for the type of mean flow pattern observed at PM208, where both thermocline and abyssal zonal means are eastward and 5 to 10 cm s$^{-1}$ (Schmitz, 1980). The only eastward flow in the models near the Gulf
Stream is under the Gulf Stream. In fact, the data at both 70 and 55W show mean westward flow at abyssal depths under the mean position of the axis of the Gulf Stream (Schmitz, 1977, 1980), with an eastward mean displaced south.

There are also potentially significant divergences between model and observation at locations in the vicinity of the NEC, between the NEC and the recirculation, and near the Mid-Atlantic Ridge along 28N (in the vicinity of PM3 clusters A and B). In the vicinity of the NEC, model run 5.13 kinetic energies are low relative to the PM3 data from cluster C (16N, 54W). Model $K_s'$ and $K_i'$ values are roughly 1-5 cm$^2$s$^{-2}$ but the data (Keffer et al., 1979) yield abyssal values around 10 cm$^2$s$^{-2}$ and 500 m values of about 40 cm$^2$s$^{-2}$. That is, the model results are both too low and too barotropic. This discrepancy may be due to differences in geometry between the model and the ocean at NEC latitudes, and the proximity of the southern boundary of the model to cluster C. More relevant (local) wind forcing and choices of parameters appropriate to NEC latitudes may be needed to model the NEC; no model run considered was designed to do so. On the other hand, the low energies near the two-layer model NEC may be associated with lack of vertical structure with consequent loss of a local baroclinic instability mechanism; there is preliminary evidence that this is so from initial three-layer simulations. For locations that are intermediate in position between the NEC and the recirculation, the prototype being the MODE area, $K_s'$ for model run 5.13 is quite close to that observed, but $K_i'$ ($\sim 20$) is low by a factor of 2-3. This is a vertical structure situation similar to that of the NEC, that is, the model is too barotropic [see also, Schmitz and Owens (1979)] here as well. A similar situation applies to the relation of the models to the data from PM3 clusters A and B. $K_s'$ for model run 5.20 is roughly in line with observation ($\sim 1$ cm$^2$s$^{-2}$) in the relevant scaled longitude range but $K_i'$ is around 1-5 cm$^2$s$^{-2}$, as opposed to the observed 35 cm$^2$s$^{-1}$ (Fu and Wunsch, 1979).

6. Conclusions

A major discrepancy occurs in attempting to simulate the zonal scale of penetration for the Gulf Stream (and its associated eddy field). With an upper layer depth chosen to match the vertical scale of the main thermocline ($\sim 1000$ m), non-linear effects are not appropriately modeled. Another general problem, not understood at this time, occurs in the comparison of model upper level results with thermocline depth data in the Gulf Stream, and near the Mid-Atlantic Ridge. The model also fails to account for the observed level of eddy activity at any depth in the North Equatorial Current.

Results from two-layer numerical experiments based on the model class evaluated are consistent with several properties of the data base in the North Atlantic, perhaps surprisingly so, as long as the comparison is undertaken at analogous positions relative to the distribution of eddy and mean fields. For example, model distribu-
tions of eddy kinetic energy with frequency are consistent to within 20 percent or so with observation in some areas.

The essential implication of this investigation relative to the future of practically-oriented model studies is to focus on numerical experiments with higher than two-layer vertical resolution in realistically-sized basins. In addition, further exploration of other model characteristics (i.e., frictional mechanism, bottom topography, gyre symmetry) is needed.

APPENDIX

Background and approach

a. Modeling considerations. The numerical experiments discussed are derivatives of experiment 5 in Holland (1978), except that one case includes variable bottom depth. The basic equations are

\[ \frac{\partial}{\partial t} \nabla^2 \psi_i = J(f + \nabla^2 \psi_i, \psi_i) - (f_0/H_1)w_2 + F_1 + H_1^{-1} \text{curl}_z \tau, \]  

(1)

\[ \frac{\partial}{\partial t} \nabla^2 \psi_2 = J(f + \nabla^2 \psi_2, \psi_2) + (f_0/H_2)w_2 + (f_0/H_3) J(h_i, \psi_3) + F_2 + B_3, \]  

(2)

\[ \frac{\partial}{\partial t} (\psi_3 - \psi_2) = J(\psi_3 - \psi_3, \psi_3) - (g'/f_0)w_2, \]  

(3)

where \( g' = g\Delta \rho/\rho_0 \) is "reduced gravity" and \( f = f_0 + \beta(y - L'/2) \) is the Coriolis parameter. \( \rho_0 \) is the average density of the two layers and \( \Delta \rho \) the density difference. \( h_i \) (Fig. 1) is the deviation of the bottom relief from its mean depth, and \( H_1 \) and \( H_2 \) are the constant mean depths of the layers. The subscript scheme is indicated in Figure 1a. \((u,v)\) and \((x,y)\) represent zonal and meridional speeds and positions. The \( \psi_i \) are streamfunctions at the various levels and \( w_z \) the vertical velocity at the interface. By definition \( u_i = -\partial \psi_i/\partial y \) and \( v_i = \partial \psi_i/\partial x \), and \( J \) is the Jacobian operator. The interfacial streamfunction \( \psi_2 \) is evaluated as the weighted average of \( \psi_1 \) and \( \psi_3 \), i.e., \( \psi_2 = (H_1 \psi_1 + H_2 \psi_3)/H_3 \). The width of the ocean basin is \( L \) and its north-south extent \( L' \); \( f_0 \) corresponds to the mid-latitude of the basin. Curl, \( \tau \) is the vertical component of the wind stress curl. No interfacial stresses are included. All notation used here relative to models is identical to that employed by Holland (1978).

The vorticity equations are supplemented by boundary conditions that depend upon the particular parameterization of friction chosen [the \( F_i \) and \( B_i \) in (1) and (2)]. In earlier experiments two kinds of "lateral friction" were used, Laplacian and biharmonic. These have the form

\[ F_i = A_i \nabla^2 \psi_i \]  

(Laplacian friction)  

(4)

or

\[ F_i = -A_i \nabla^2 \psi_i \]  

(biharmonic friction).  

(5)

When bottom friction is included, the particular form for lateral friction has a relatively less pronounced role (Holland, 1978) as long as the horizontal eddy coefficient is small. All of the experiments discussed in any detail here use the Holland (1978) experiment 5 parameterization, i.e., bottom friction

\[ B_i = -\epsilon \nabla^2 \psi_i \]  

(6)
plus a small lateral biharmonic friction (5), with $A$, chosen to be as small as possible consistent with grid size. The lateral boundary conditions used with (5) and (6) are, on the boundaries: $\psi = \text{constant (a function of time only)}$, and $\nabla^2 \psi = \nabla^2 \psi = 0$ (Holland, 1978).

The details of the techniques used in numerical integration have been discussed elsewhere (Holland, 1978). Horizontal grid spacing was chosen to be 20 km. Each experiment is spun up from rest until statistical equilibrium, as determined by monitoring energy levels, is reached. Time series are then formed at two-day intervals at each model spatial grid point, with desired variables averaged over the next 1800 days, and these results are used for analysis and evaluation of the model. The statistical reliability of time averages for various choices of averaging techniques have been discussed for one model run by Holland and Rhines (1980) and by Harrison and Holland (1981).

Holland (1978) described several numerical experiments made in order to compare runs in single with those in double gyre basins; examination of the effects of dissipative mechanism(s) was a prominent secondary goal. Variable bottom topography was not considered. A preference for double-gyre basins emerged, along with the choice for modeling friction [as in equations (5) and (6)]. Attention to the double gyre configuration has been essentially dynamically motivated; the mid-basin jet becomes free rather than strongly constrained by the northern boundary [Holland and Lin (1975a, b); Holland (1978)], with correspondingly more diversified behavior. However, it should also be noted that there is observational motivation for this choice as well; that is, the frequency distribution of kinetic energy for double-gyre runs is less monochromatic than for single-gyre runs, a realistic tendency. All runs examined in any detail in this investigation use bottom friction and are double-gyre cases. Asymmetric basins would be more realistic, but attempts to model this feature are not yet sufficiently developed or understood. A basic effect of using equations (5) and (6) relative to (4) is a more limited horizontal spread of lower layer kinetic energy away from mid-basin (Fig. A.1), for the parameters used. Bottom roughness also inhibits the horizontal spread of lower layer kinetic energy.

Random variable bottom topography was generated for use in experiment 5.20 in order to examine in a preliminary fashion the role of bottom roughness. The topography used in 5.20 filled the entire basin and is best described as a field of small scale, small amplitude bumps. The root mean square height deviation (from a total basin mean depth of 5000 m) was 100 m and the spectral distribution was white for horizontal scales in the range of 100 to 500 km.

b. The Data base. There are only a few sites in any ocean basin where time-averages and spectral distributions comparable to those that can be calculated from the model results are available. The similarity is imprecise even at these sites, due to the limited length of the time-series of data thus far obtained, relative to that capable of being generated in the model runs. In both data sets there is evidence for energy associated with secular (or climatological) time scales which lie in frequency between a plausible identification of a mesoscale or eddy frequency band (periods of, say, 50-150 days) and means based on the longest time series available, the latter identified here as the general ocean circulation. Of course, climatological variations of this definition of the general circulation are also possible. It turns out that the model and observed time scales are compatible. There is a relative shortage of the desired data at shallow depths, particularly in the vicinity of the Gulf Stream, again for logistical reasons. The data base under consideration is, simply stated, sparse relative to what is needed for a thorough intercomparison or even for a complete description.

The observations cited most extensively in this study are reasonably high quality time-series of horizontal currents and temperatures from the vicinity of 55W, in the latitude range 31.5 to 41.5N. These time-series were obtained from three deployments of a group of moored current-
temperature meters collectively called POLYMODE Array 2 (abbreviated PM2). Twelve mooring sites were maintained for approximately two years. Eight of the twelve long-term PM2 mooring sites had current-temperature meters at nominal depths of 600, 1000, 1500 and 4000 m, and a single standard depth of 4000 m was maintained at four moorings near the Gulf Stream. Three other sites were maintained for nine months only. Specific locations will be referred to as PM2A(z) where A is the site number (1-12) and z the depth; i.e., PM210(4000) is site ten at a depth of 4000 m. These data and results have been examined in some detail previously (Schmitz, 1977, 1978, 1980; Tarbell et al., 1978; see also Owens and Hogg, 1980; and Bryden, 1980). 55W is the only longitude in the North Atlantic where such data are
available, although there are other more restricted observations, described in the following, that are also used in this investigation.

Data from an exploratory array of roughly nine month duration called POLYMODE Array 1 (called PM1) are also relevant. This array consisted of seven moorings with instruments at nominal depths of 500, 1000, 2000, and 4000 m. Data and results have been described by Schmitz (1976, 1977, 1978, 1980) and by Spencer et al. (1979). A key location in PM1, containing three moorings in the vicinity of 28N and 55W, is called PM1 Δ. Its significance for present purposes is that \( K_E \) averaged horizontally and in time at both thermocline and abyssal depths was the minimum that had been observed in the western North Atlantic; this result still holds for thermocline depths.

More recent data are available from clusters (labeled A, B, and C) of 4-5 moorings each, collectively called (PM3) POLYMODE Array 3 (Fu and Wunsch, 1979; Fu et al., 1981; Keffer et al., 1979; Tarbell, 1980). Clusters A and B were deployed on opposite sides of the Mid-Atlantic Ridge, centered at about 28N. Cluster C was deployed near 16N, 54W, in the vicinity of the North Equatorial Current. Observations were made at both thermocline and abyssal depths.

Long term moored instrument data have also been obtained from the vicinity of the center of the MODE area (nominal location 28N, 70W). The earlier segments of this data base were described by Gould et al. (1974), further properties by Richman et al. (1977) and by Schmitz (1978). Two data reports exist (Chausse and Tarbell, 1976; Tarbell and Spencer, 1978). Results from MODE will be referred to as MODE (\( z \)) where \( z \) is the depth. Very recent results near 31N along 70W are now available (Owens et al., 1982). Some of the earliest long-term moored instrument time-series were obtained near the Gulf Stream along 70W, at abyssal depths only. Properties of these observations have been discussed by Schmitz (1976, 1977) and Luyten (1977). Relevant data sources are Spencer (1979) and Tarbell et al. (1979).

The most complete horizontal coverage of any time mean or time-averaged eddy field properties are those based on ship-drift or on XBT observations along ship tracks. The maps of near-surface kinetic energies constructed by Wyrtki et al. (1976) and potential energy charts for the upper thermocline by Dantzler (1977) are the most prominent examples. Various charts of time and space averaged surface currents (and temperatures) exist in atlas or report form. Intercomparisons of this surface data with models seems a likely subject for future investigation. At present, the method of intercomparison with a two-layer model is unclear.

Data whose distribution with frequency can readily be matched to the models, and that are available to us, are clearly consistent with the surface maps mentioned in the preceding paragraph although indicating that horizontal scales of variability of time-averages are depth-dependent (Schmitz, 1978). All available data at all depths yield maximum \( K_E \) in the vicinity of the Gulf Stream. The most complete latitudinal section available containing data at thermocline as well as abyssal depths is along 55W; \( K_E \) values are shown in Figure A.2, along with zonal and meridional variances separately. The solid box labeled axis on Figure A.2 denotes a range of mean positions of the axis of the Gulf Stream along 55W following Schmitz (1978).

c. Procedural considerations. We began this intercomparison without a specific plan, as a consequence of the embryonic status of both data base and model development. The major initial consideration was a purely practical one: how does one identify location in a model with idealized geometry relative to the specific latitudes, longitudes, and depths of oceanic data.

This is, as labeled, a preliminary investigation; the initial detailed direct comparison of recent eddy-resolving general circulation model results with observation. To the best of our knowledge, the type and level of examination in this investigation are unique in the history of
Figure A.2. POLYMODE Array 2 data as a function of latitude along 55W at indicated depths: (a) Eddy kinetic energies ($K_e$), (b) zonal ($u''^2$) and meridional variances ($v''^2$). A range of mean positions of the axis of the Gulf Stream is indicated by solid rectangles.

Ocean general circulation research. We do not at this time carry out a systematic evaluation of all numerical experiments available, but make a quick search for basic general compatibilities and most apparent problems. One of our main motivations has been to produce results that would assist in guiding the future development of numerical models, and perhaps suggest new field observations. The former goal has been met here in essentially two ways. First, several new model runs were motivated in response to the initial stages of this intercomparison. Second, our tentative conclusion that greater than two-layer vertical resolution is needed to realistically account for the zonal scale of both the general circulation and its associated eddy field has led to preliminary and encouraging three-layer numerical experiments [see also Semtner and Holland (1978)]. In fact, an embryonic attempt at integrating a two-layer model based on the detailed geometry/geography of the North Atlantic that is in progress has encountered even more severe difficulties with zonal penetration of its simulated Gulf Stream. Efforts are being made to determine if increased vertical resolution will help to rectify this situation.

The sparsity of the data base should be apparent, even without comparison with models possessing orders of magnitude more information. One key requirement for more complete model evaluation would be the acquisition of additional thermocline depth data, particularly in the vicinity of the Gulf Stream. Data at any depth from key longitudes in the North Atlantic are also needed; the intercomparisons completed suggest that 60-65W and 40-50W are in this category. A compatible data set from another ocean basin could be critical in permitting a
check on model characteristics that are supposed to be the same for all ocean basins, and in establishing those properties which may be different.

Finally, the limitations of the particular numerical model and the experiments thus far carried out with it should be emphasized. While the model has high horizontal resolution (20 km), it is not clear that even this can completely represent the fine scale turbulent processes that affect the large-scale circulation. The energetic eddies are fairly well resolved but their "cascade" processes to even finer scales may not be. While much better horizontal resolution has been achieved compared to "pre-eddy" models the vertical resolution still is minimal. In these experiments the vertical structure has a resolution consistent with resolving only the barotropic and first baroclinic modes in the ocean. While there are indications that large-scale and mesoscale motions have much of their energy in these modes it is also well known that some phenomena will require much more vertical resolution to adequately represent the physical phenomena involved. In addition to these resolution questions the geometrical and physical simplifications made in these studies must be examined in future experiments. The physical basis of the model, the quasigeostrophic equations, should be tested against more elaborate formulations.

Acknowledgments. During this investigation WJS was supported by the Office of Naval Research under contract N00014-76-C-0197, NR 083-400, and WRH by the National Science Foundation at the National Center for Atmospheric Research. Acquisition of the data base utilized was also supported in part by the International Decade of Ocean Exploration Office of the National Science Foundation under grant OCE75-03962. J. McWilliams, B. Owens and J. Price gave helpful advice. This study was considerably aided by the technical proficiency of M. Zemanovic, K. Bohr, and J. Chow. This is contribution number 4697 from the Woods Hole Oceanographic Institution and number 146 from the Open Ocean Dynamics Experiment, POLYMODE.

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Received: 22 September, 1980; revised: 31 July, 1981.