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Abstract

This paper analyzes the entry of new products into vertically differentiated markets where an entrant and an incumbent compete in quantities. The value of the new product is initially uncertain and new information is generated through purchases in the market. We derive the (unique) Markov perfect equilibrium of the infinite horizon game under the strong long run average payoff criterion.

The qualitative features of the optimal entry strategy are shown to depend exclusively on the relative ranking of established and new products based on current beliefs. Superior products are launched relatively slowly and at high initial prices whereas substitutes for existing products are launched aggressively at low initial prices.

The robustness of these results with respect to different model specifications is discussed.

JEL CLASSIFICATION: C72, C73, D43, D83.

KEYWORDS: Entry, Duopoly, Quantity Competition, Vertical Differentiation, Bayesian Learning, Markov Perfect Equilibrium, Experimentation, Experience Goods.

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1 Introduction

1.1 Motivation

In this paper, we analyze the optimal entry strategies for different types of experience goods in a dynamic Cournot duopoly with vertically differentiated buyers. Our main goal is to obtain a characterization of the features of the new product that lead to qualitatively different entry strategies. We show that a new product that represents a certain improvement to an existing product is launched in the market at prices above the static equilibrium level and sales quantities below the static level. A new product that has a positive probability of being the leading brand in the market, but also a positive probability of being revealed inferior to the current product, is launched with a more aggressive strategy where the initial prices are low and initial sales exceed the static equilibrium quantities.

The firms compete in a continuous time model with an infinite horizon. The uncertainty about the new product is common to all buyers in the market. Additional information about the quality of the new product is generated only through experiments, i.e. through purchases in the market. The information generated is assumed to be public and while the exact mechanism of information transmission is left unmodelled, it is motivated by considerations such as word of mouth communication between the buyers and consumer report services. As a consequence, all buyers have identical beliefs about the new product, and we can represent the stage game as a vertically differentiated quantity game parametrized by the common belief about the new product. Examples of markets where the assumptions of common value (aside from the aspect of vertical differentiation) and common information may be valid include the entry in service markets such as new airlines carriers or new providers of communications services. Here the common value is the (expected) performance or reliability of the new service. The new information is the sum of all idiosyncratic experiences by the buyers. Since an individual buyer decides based on the expected performance, all idiosyncratic experiences are of equal value in providing information.

We have chosen a model of quantity competition as the stage game. With this choice, we extend the scope of viable new products. In particular, quantity competition allows for the possibility of launching an innovation which brings the two competitors closer to each other without change in the leadership. In a model of price competition, such innovations would never be profitable and as a result, improved substitutes would never be observed. In those models, the profits of both firms vanish as the substitutability of the two products increases and as a result, the static profit functions of the two firms are nonmonotonic in the level of differentiation. We believe that a model where each firm's profit is increasing in its own quality is better suited for a dynamic investigation of a market with vertical differentiation.

In order to simplify the analysis, we assume that there is no discounting. As we want to stay close to the model with small discounting, we use the strong long run average criterion as defined in Dutta (1991) as the intertemporal evaluation criterion. This criterion can be justified as the limit of models where the discount rate is tending to zero, and it retains the recursive formulation of standard discounted dynamic programming. Under the assumptions of no discounting and quantity competition, it is surprisingly simple to examine the Markov perfect equilibria of the model. In section 5, we show that for quite general demand structures, the comparisons between static and dynamic equilibrium policies can be based exclusively on information about static payoff functions. It is sufficient to calculate the static equilibrium profits for a given belief and compare that with the long run average profits for the same belief. Whenever the long run average profit for the entrant exceeds the myopic profit, there is an incentive to generate additional information and as a result, the dynamic equilibrium quantity exceeds the myopic equilibrium price. If the myopic equilibrium profit exceeds the long run profit, the entrant is best served with a slow generation of new information, and as a result, the equilibrium quantity falls below the myopic level. It is hoped that the simplicity of the technique of undiscounted dynamic programming as used here will prove useful in other applications beyond the scope of this paper.

In section 4, we assume that the underlying stage game is the standard linear model used in the literature on vertical differentiation. This allows us to interpret the dynamic equilibria in an economically intuitive manner. Using the curvature properties of the stage game profit functions, we show that aggressive entry corresponds to relatively low (current) expected quality of the entrant's product while cautious entry corresponds to high expected quality. In the linear model, we can also solve the dynamic equilibrium policies explicitly and as a result, we get a set of empirically testable predictions for the model.

The paper proceeds as follows. Section 2 introduces the basic model and the learning environment. Section 3 derives the benchmark results of the static duopoly game. The main results are then presented for the standard linear demand specification in Section 4 where we derive the Markov perfect equilibrium of the intertemporal game. In Section 5, we extend the model beyond the linear specification and show that the qualitative conclusions extend to much more general demand structures. All the proofs are relegated to an appendix.

1.2 Related Literature

Our model is related to a number of branches in the literature on imperfect competition. The model of vertical differentiation was first developed in the context of a duopoly model by Gabszewicz & Thisse (1979), (1980), and Shaked & Sutton (1982), (1983). The emphasis in those models was on the optimal choices of product qualities for competing producers. The product characteristics were commonly known to all the participants in the market, and the quality choices by the firms were followed by a second stage price competition. Gal-Or (1983) and Bonnano (1986) first considered quantity competition in a model of vertical differentiation. Our primary interest in this paper is in explaining the observed differences in the qualitative features of initial pricing. To allow for a wide range of possibilities, we want to have the flexibility in the demand structure afforded by vertical differentiation.

The recent literature on experimentation and strategic experimentation has considered models closely related to the one analyzed here. Early models such as Rothschild (1974) and McLennan (1984) consider the learning problem of a monopolist facing a fixed demand curve with unknown parameters.[1] Aghion, Espinosa & Jullien (1993), Harrington (1995) and Keller & Rady (1998) analyze a duopolistic market where two competitors learn about the substitutability between their products. In these models, useful information becomes available whenever either of the firms makes a sale. The main difference between these papers and the current paper is that here the actual demand curve, and not only the beliefs about the demand, depends on past sales. Bergemann & Välimäki (1997) considers the entry problem of a new product in a situation where the buyers are horizontally differentiated. Even though the analysis in that paper uses tools similar to the current paper, the economic findings in the two papers are quite different reflecting

[1] The monopoly learning problem is further analyzed, among others, in Prescott (1972), Kihlstrom, Mirman & Postlewaite (1984), Easley & Kiefer (1988), Aghion, Bolton, Harris & Jullien (1991), Mirman, Samuelson & Urbano (1993), and Treffler (1993).

the differences between vertical and horizontal differentiation. A strategic learning model in continuous time without discounting appeared also in an early version of Bolton & Harris (1999).

To our knowledge, the current paper is the first model of entry with vertical differentiation and uncertain demand.[2] In the absence of vertical differentiation, the previous models of entry cannot generate qualitatively different predictions for the speed of entry for different types of new products. The public observability of utility signals is central to some recent models of word-of-mouth communication such as McFadden & Train (1996).

Finally, conditions for initially high prices have been obtained in asymmetric information models of entry. In those papers, the monopolist is assumed to know the true value of the product, and the prices chosen serve as signals of the true quality. A prominent example of such models is Bagwell & Riordan (1991) where high and declining prices serve as signals of high product quality. Judd & Riordan (1994) consider a model with initially symmetric information where private signals are received by the monopolist and the buyers after first period choices. The firm then faces a signalling problem in the second period. The results in these models depend on the details of the information revelation mechanism and the cost structure. In our model, the results depend only on the quality difference between the products which can in principle be inferred directly from the realized prices.

2 Model

In this section we first describe the preferences of the buyers and then introduce the stochastic environment in which the game is played. The incumbent I and the entrant E compete in quantities in a market with vertically differentiated products. Time is continuous and the time horizon is infinite, with $t \in [0, \infty)$. The incumbent is well established in the market and its product characteristics are common knowledge at the beginning of the game. The entrant has a new product whose value is initially uncertain and whose value can be learned over time through experience. The marginal cost of production is normalized to zero for both firms.

[2] A recent paper by Ching (1999) provides structural estimations for a model with very similar features to the one considered here. He estimates the entry behavior for generic drugs in model of vertical differentiation with market learning about a common uncertain parameter (attributes of the generic drug).

The preferences of the buyers are described by a model of vertical differentiation. The buyers are characterized by a parameter θ which is assumed to be distributed on the interval $[0,1]$ according to a twice continuously differentiable density function $f(\theta)$. In much of the paper, we will assume that $f(\theta)$ is uniform. The parameter θ_i of buyer i can be interpreted as her willingness to pay (or the inverse of the marginal utility of income). Each buyer has a unit demand at each instant of time. The incumbent's product, also called the established or safe product, has quality s with $s > 0$ to all buyers. The value of the safe product to buyer i is then the product of his willingness to pay and the value of the product, or

$$\theta_i s.$$

Symmetrically, the value of the uncertain product for individual i is given by

$$\theta_i \mu.$$

The value, μ, of the new product is initially unknown to *all* parties. It can be either low or high:

$$\mu \in \{\mu_L, \mu_H\},$$

with $0 \leq \mu_L < \mu_H < \infty$. Initially all market participants have a common prior belief α_0 that the new product has a high valuation, or

$$\alpha_0 = \Pr(\mu = \mu_H).$$

The expected value given a belief $\alpha(t)$ in period t is denoted by $\mu(\alpha(t))$, where

$$\mu(\alpha(t)) \triangleq \alpha(t)\,\mu_H + (1 - \alpha(t))\,\mu_L.$$

Since the buyers are nonatomic, they have no individual effect on prices and quantities and as a result, they choose according to their myopic preferences at each stage.[3] To complete the description of the stage game payoffs, we need to specify the profit functions for the two firms. The profits resulting from a vector of quantities $(Q_I(t), Q_E(t))$ are given by $P_i(t)\,Q_i(t)$ for $i = I, E$, where the $P_i(t)$ are obtained from static market clearing conditions. The dynamic game payoffs for the two firms are discussed in section 4.

[3] We are implicitly assuming that the firms' information sets consist of all past market observations, i.e. all past prices and quantities.

The uncertainty about the new product can only be resolved over time by experience with the new product. The information contained in any individual purchase is noisy and thus can only provide an improved but not perfect estimate of the new product. We assume that the evolution of the belief about the quality of the new product is governed by the following diffusion process:

$$d\alpha(t) = \sqrt{\frac{q_E(t)\,\alpha(t)\,(1-\alpha(t))\,(\mu_H - \mu_L)}{\sigma^2}}dB(t)\,, \tag{1}$$

where $B(t)$ is the standard Wiener process and $q_E(t)$ is the sales quantity of the entrant in period t. In the appendix, we provide a microfoundation for this particular form of the evolution of the beliefs. There we derive the diffusion process $\alpha(t)$ from a discrete time model with a finite number of buyers, where each buyer is sampling from a normal distribution with known variance σ^2 and unknown mean μ, which is either μ_L or μ_H. As we take the limit both with respect to the number of buyers and the time elapsed between any two periods, the resulting evolution of the posterior belief is given by the stochastic differential equation above.

Observe that being a posterior belief, $\alpha(t)$ follows a martingale, i.e. has a zero drift. The variance of the process is at its largest when $\alpha(t)$ is away from its boundaries as the marginal impact of new information is at its largest when the posterior is relatively imprecise. The economic assumption behind the form of this particular process is that the variance in the posterior belief is linear in the quantity of sales by the entrant. In other words, the informativeness of the market experiment grows linearly in the sales of the entrant. The remaining term in the expression, $\frac{\mu_H - \mu_L}{\sigma^2}$, is sometimes referred to as the signal to noise ratio as it measures the strength of the signal $\mu_H - \mu_L$ to the inherent noise in the observation structure, σ^2. For notational convenience we define

$$\Sigma(\alpha(t)) \equiv \sqrt{\frac{\alpha(t)\,(1-\alpha(t))\,(\mu_H - \mu_L)}{\sigma^2}}.$$

From equation1, we see that as long as $q_E(t)$ is bounded away from 0 for all t, $\alpha(t)$ converges to $\alpha^* \in \{0, 1\}$ almost surely. In fact, the convergence is fast enough to make the following limit finite almost everywhere:

$$\lim_{T\to\infty} \mathbb{E}_{\alpha_0}\left[\int_0^T [\phi(\alpha(t)) - \phi(\alpha^*)]\,dt\right],$$

where $\phi(\cdot)$ is an arbitrary continuous and piecewise smooth function of α. This result allows us to use the strong long run average as the intertemporal evaluation criterion in our model.

3 Static Equilibrium

In this section, we derive some of the basic equilibrium properties in a static model for the case where $f(\theta)$ is the uniform density. The value of the safe product is s and of the new product is $\mu(\alpha)$ for a given α. In the description of the equilibrium conditions we shall assume that $\mu(\alpha) \leq s$. The corresponding results for $\mu(\alpha) > s$ are symmetric and stated in the relevant proposition as well. Define α_m as the belief at which the expected value of the new product is equal to the established one:

$$\mu(\alpha_m) = s \Leftrightarrow \alpha_m = \frac{s - \mu_L}{\mu_H - \mu_L}.$$

The static prices are denoted by P_E and P_I for the entrant and the incumbent respectively. The quantities are denoted by Q_E and Q_I. The *equilibrium* prices and quantities are denoted by $P_i(\alpha)$ and $Q_i(\alpha)$ as we are interested in the comparative static behavior of the equilibrium variables as a function of the belief α. Naturally, the evolution of the belief α is of no relevance in the static model.

The equilibrium conditions are given by the profit maximization conditions of the firms and the indifference conditions of the marginal buyers. The latter can be stated as

$$(1 - Q_I)\, s - P_I = (1 - Q_I)\, \mu(\alpha) - P_E.$$

and

$$(1 - Q_I - Q_E)\, \mu(\alpha) - P_E = 0.$$

The first indifference condition implies that at the equilibrium prices, buyers with valuations $\theta \in [1 - Q_I, 1]$ prefer the incumbent. The second indifference condition implies that buyers with valuations $\theta \in [1 - Q_I - Q_E, 1 - Q_I)]$ prefer the entrant. It also follows that all buyers get a nonnegative expected utility from their purchases, but the segment with the lowest valuations may not buy at all. The market clearing prices for given quantities $\{Q_E, Q_I\}$ are:

$$P_E = \mu(\alpha)(1 - Q_I - Q_E)$$

and

$$P_I = s(1 - Q_I) - \mu(\alpha)\, Q_E.$$

The static profit functions of the firms can then be written as functions of the quantities $\{Q_E, Q_I\}$:

$$\pi_E \left(Q_E, Q_I \left| \alpha \right.\right) = \mu\left(\alpha\right) Q_E \left(1 - Q_I - Q_E\right)$$

and

$$\pi_I \left(Q_E, Q_I \left| \alpha \right.\right) = Q_I \left(s \left(1 - Q_I\right) - \mu\left(\alpha\right) Q_E\right).$$

For $\mu(\alpha) = s$, the profits coincide with those in the homogenous goods Cournot model. The Nash equilibrium of the duopoly is obtained by solving simultaneously for profit maximizing $\{Q_E(\alpha), Q_I(\alpha)\}$.[4] We observe first that the entrant's best response function is independent of the level of differentiation. The quantity set by the incumbent firm, Q_I determines the size of the market for the entrant, and the entrant behaves as a monopolist on the residual demand and sets

$$Q_E\left(Q_I\right) = \frac{1}{2}\left(1 - Q_I\right). \tag{2}$$

The best response function of the incumbent is given by:

$$Q_I\left(Q_E\right) = \frac{1}{2} - \frac{\mu\left(\alpha\right)}{2s} Q_E. \tag{3}$$

For every $\mu(\alpha)$, the optimal reaction to $Q_E = 0$ is given by the monopoly quantity $Q_I = \frac{1}{2}$. For $\mu(\alpha) = s$, the effect of the entrant's decisions on the optimal reactions of the incumbent is at its strongest. As $\mu(\alpha)$ decreases, the best response function of the incumbent becomes flatter, eventually converging to a constant on the monopoly quantity. In a sense, the incumbent firm becomes strategically independent of the entrant as $\mu(\alpha)$ declines. We may rewrite the best response function as a convex combination:

$$Q_I\left(Q_E\right) = \frac{\mu\left(\alpha\right)}{s + \mu\left(\alpha\right)} Q_M + \frac{s}{s + \mu\left(\alpha\right)} Q_I\left(Q_E\left(\alpha_m\right)\right),$$

so that the best response function of the incumbent is a weighted average of the monopoly best response $Q_M = \frac{1}{2}$ and the Cournot best response at zero differentiation, $\alpha = \alpha_m$. The equilibrium quantities are derived explicitly in the appendix, here we merely state the monotonicity properties in the belief α.

[4] With the linear demand specification, the profit function of each firm is concave in its own quantity, and therefore first order conditions are also sufficient for optimality.

Proposition 1 (Static Policies)

1. *$P_E(\alpha), Q_E(\alpha)$ and $P_E(\alpha) Q_E(\alpha)$ are increasing in α.*

2. *$P_I(\alpha), Q_I(\alpha)$ and $P_I(\alpha) Q_I(\alpha)$ are decreasing in α.*

Proof. See appendix. ■

As expected, the quantity and the price of the entrant are increasing in α. The entrant can increase both his sales as well as his margins as the quality is improved. The incumbent responds to an increase in the value of the competing product both by lowering his sales as well as his margins. It is worthwhile to point out that the monotonicity result extends over the entire range of posterior beliefs, and holds also around the point α_m where the leadership between the two firms is changing. This is one instance where the model with quantity competition behaves in a more regular manner than the one with price competition, which displays nonmonotonicities in the prices and quantities around the switching point α_m.

The experience gained by the buyers may change the posterior belief α either upwards or downwards. By the law of iterated expectation, the mean of the change is zero, but the variance is positive. As a result, the curvatures of the profit functions of the two firms in α are going to play a central role in the analysis of the intertemporal competition.

Proposition 2 (Curvatures)

1. *For $\mu(\alpha) < s$,*

   (a) *$P_E(\alpha), Q_E(\alpha)$ and $P_E(\alpha) Q_E(\alpha)$ are convex in α,*

   (b) *$P_I(\alpha), Q_I(\alpha)$ and $P_I(\alpha) Q_I(\alpha)$ are concave in α.*

2. *For $\mu(\alpha) > s$,*

   (a) *$P_E(\alpha), Q_E(\alpha)$ and $P_E(\alpha) Q_E(\alpha)$ are concave in α,*

   (b) *$P_I(\alpha), Q_I(\alpha)$ and $P_I(\alpha) Q_I(\alpha)$ are convex in α.*

Proof. See appendix. ■

An intuition for the curvature as well as the change in the curvature of the policies and revenues can be given as follows. For low posterior beliefs where $\mu(\alpha) < s$, a marginal increase in α increases the profit of the entrant through two channels. First, it increases the price for fixed quantities. This direct effect is the same at all levels of α as long as the quantities supplied are unchanged. There is also the indirect effect from a stronger competitive position of the entrant and the corresponding reduction in the quantity of the incumbent. This effect is strongest when α is close to α_m, and vanishes for very low values of α. The combination of these two effects leads to a convex profit function as long as $\mu(\alpha) < s$. As α increases beyond α_m, the position of the entrant resembles increasingly one of a monopolist. The indirect effect then becomes weaker and it is only the ability of the new firm to increase its prices which increases its profits.

This general argument already indicates that the curvature properties continue to hold for more general class of densities over the space of the vertical differentiation parameter θ than the uniform density assumed here.

4 Dynamic Equilibrium

We define the Markov perfect equilibrium for the duopoly game in subsection 4.1, where we also introduce the familiar dynamic programming equations with discounting. In subsection 4.2 we consider the limit case of no discounting under the optimality criterion of the strong long run average. In subsection 4.3 we then characterize the unique equilibrium and associated equilibrium policies.

4.1 Equilibrium

In the equilibrium analysis, we restrict ourselves to Markovian policies to focus on the interaction between the information revealed in the market and the policies adopted by the buyers and sellers. The equilibrium prices and quantities in the dynamic setting are denoted by $p_i(\alpha)$ and $q_i(\alpha)$ to distinguish them from their static counterparts. The value functions of the firms can then be described by the Hamilton-Jacobi-Bellman equations[5]

$$rV_E(\alpha) = \max_{q_E(\alpha)} \left\{ p_E(\alpha)\, q_E(\alpha) + \frac{1}{2} q_E(\alpha)\, \Sigma^2(\alpha)\, V_E''(\alpha) \right\} \text{ and} \tag{4}$$

$$rV_I(\alpha) = \max_{q_I(\alpha)} \left\{ p_I(\alpha)\, q_I(\alpha) + \frac{1}{2} q_E(\alpha)\, \Sigma^2(\alpha)\, V_I''(\alpha) \right\} \tag{5}$$

for the entrant and the incumbent respectively. In these equations, the first term on the right hand side represents the flow payoff resulting from the equilibrium sales at the equilibrium quantities. The second term represents the flow value of information which is composed of the variance of the posterior belief weighted by the second derivative of the value functions, which measures the marginal value of information. The variance term is equal for both firms as information is symmetric and only sales by the entrant generate more information about the value of the new product. The attitude towards information is represented by the curvature of the value function of each firm. As we will see shortly, the (sign of the) curvature may differ and even change in equilibrium as a function of the posterior belief α.

Since every buyer is of negligible size, her decision doesn't influence the market experiment and hence her value of information is independent of her decision. The purchase decision of each buyer is therefore exclusively determined by the current payoff offered by the various alternatives. In consequence, the sorting of buyers in the intertemporal equilibrium will display the same structure as in the static equilibrium. Buyers with high valuations will buy from the alternative with the myopically superior payoff and buyers with low valuations will buy from the alternative with the myopically inferior payoff. Thus the equilibrium prices are determined again by the supplied quantities and the indifference condition of the marginal buyers as in the static equilibrium:

$$\begin{aligned} p_E(\alpha) &= \mu(\alpha)\,(1 - q_I(\alpha) - q_E(\alpha))\,, \\ p_I(\alpha) &= s\,(1 - q_I(\alpha)) - \mu(\alpha)\, q_E(\alpha)\,, \end{aligned} \tag{6}$$

[5] See Dixit & Pindyck (1994) or Harrison (1985) for a complete derivation of the dynamic programming equation in continuous time when uncertainty is represented by a Brownian motion. The derivation is based on a discrete time version with a random walk where the limit is taken as the time elapsed between any two periods converges to zero.

if $\mu(\alpha) \leq s$, and symmetrically:

$$\begin{aligned} p_E(\alpha) &= \mu(\alpha)(1 - q_E(\alpha)) - s q_I(\alpha), \\ p_I(\alpha) &= s(1 - q_I(\alpha) - q_E(\alpha)), \end{aligned} \tag{7}$$

if $\mu(\alpha) > s$. The difference with the static equilibrium is that the value function of the sellers now contains the intertemporal considerations represented by the second derivative of the value function.

Definition 1 (Markov perfect equilibrium) *A Markov perfect equilibrium is given by* $\{q_E(\alpha), q_I(\alpha)\}$ *such that the equations (4)-(7) are satisfied for all* $\alpha \in [0,1]$.

The first order conditions for the firms differ in the intertemporal problem only through the added variance term. As the sales quantities today affect the variance only linearly through the sales of the new firm, the equilibrium quantities can be given explicitly as a function of $s, \mu(\alpha)$ and $\Sigma^2(\alpha) V_E''(\alpha)$. But after inserting the equilibrium quantities and prices into the value functions of buyers and sellers, they become a set a nonlinear differential equations, which can only be approximated via numerical techniques. For this reason we consider the limiting case as the discount rate vanishes or $r \to 0$ and then derive the equilibrium policies under the strong long-run average criterion.

4.2 Optimization without discounting

In establishing the equilibrium policies under no discounting, the *strong* long run average criterion has the important property that the optimal policies under this criterion are the unique limits to the associated policies under discounting. The equilibrium policies to be derived therefore maintain all the qualitative properties of the equilibrium with small, but positive, discount rates $r > 0$. In particular, they preserve the intertemporal trade-off of the experimentation policies under discounting.[6]

We start by fixing the policies of all other players to a set of arbitrary (Markovian) policies and consider the decision problem of the entrant. In the next subsection, we return to the full

[6] See Dutta (1991) for a detailed analysis of the link between optimality criteria under discounting and no discounting.

equilibrium problem. The reformulation for the incumbent only requires the obvious substitutions. The long run average payoff for the entrant under an initial belief α_0 is given by:

$$v_E(\alpha_0) = \sup_{q_E(\alpha(t))} \lim_{T\to\infty} \frac{1}{T} \mathbb{E}_{\alpha_0} \left[\int_0^T q_E(\alpha(t))\, p_E(\alpha(t))\, dt \right].$$

If $\alpha(t)$ converges almost surely to zero or one, then the long run average starting at any arbitrary belief α_0 is equal to

$$v_E(\alpha_0) = (1-\alpha_0)\, v_E(0) + \alpha_0 v_E(1).$$

As $v_E(0)$ and $v_E(1)$ are simply the full information payoffs associated with the static payoffs at $\alpha = 0$ or $\alpha = 1$, the long-run average can be computed exclusively on the basis of the static problem. In contrast, the *strong* long-run average is defined through the following optimization problem:

$$V_E(\alpha_0) = \sup_{q_E(\alpha(t))} \lim_{T\to\infty} \mathbb{E}_{\alpha_0} \left[\int_0^T \left(q_E(\alpha(t))\, p_E(\alpha(t)) - v_E(\alpha(t)) \right) dt \right]. \tag{8}$$

Thus the strong long run average criterion maximizes the expected return net of the long-run average. The limit as $T \to \infty$ in (8) is well-defined and finite. The strong long-run average hence discriminates between policies based on finite time intervals as well. The infinite horizon problem (8) can be represented by a dynamic programming equation as follows:

$$v_E(\alpha) = \max_{q_E(\alpha)} \left\{ q_E(\alpha)\, p_E(\alpha) + \frac{1}{2} q_E(\alpha)\, \Sigma^2(\alpha)\, V_E''(\alpha) \right\}. \tag{9}$$

The difference between the dynamic programming equation under discounting (4) and no discounting (9) is simply that the flow payoff, $rV_E(\alpha)$, is replaced by the long-run average payoff, $v_E(\alpha)$, whereas the right hand side of the equation remains identical. However as the long-run average is independent of the current policy $q_E(\alpha)$, we can rewrite (9) to read:

$$0 = \max_{q_E(\alpha)} \left\{ q_E(\alpha)\, p_E(\alpha) - v_E(\alpha) + \frac{1}{2} q_E(\alpha)\, \Sigma^2(\alpha)\, V_E''(\alpha) \right\}.$$

After dividing the entire expression through $q_E(\alpha)$ (assuming that $q_E(\alpha) > 0$ can be guaranteed), the optimality equation can be rewritten as

$$0 = \max_{q_E(\alpha)} \left\{ p_E(\alpha) - \frac{v_E(\alpha)}{q_E(\alpha)} \right\} + \frac{1}{2} \Sigma^2(\alpha)\, V_E''(\alpha). \tag{10}$$

Here we can now detect the advantage of the undiscounted program relative to the discounted one. The first-order conditions for entrant, and in fact for all agents in the game don't involve the second derivative of the value function any more. The only modification relative to the static program is the introduction of the long-run average but as we saw above, it can be computed on the basis of the static equilibrium as well. It is this replacement of the value function by the long-run average in the limit as discounting goes to zero which makes the undiscounted problem much more accessible.

4.3 Equilibrium Analysis

Consider now the entire set of equilibrium conditions under no discounting. The dynamic programming equation for the entrant is

$$0 = \max_{q_E(\alpha)} \left\{ p_E(\alpha) q_E(\alpha) - v_E(\alpha) + \frac{1}{2} q_E(\alpha) \Sigma^2(\alpha) V_E''(\alpha) \right\}, \tag{11}$$

and for the incumbent it is by extension:

$$0 = \max_{q_I(\alpha)} \left\{ p_I(\alpha) q_I(\alpha) - v_I(\alpha) + \frac{1}{2} q_E(\alpha) \Sigma^2(\alpha) V_I''(\alpha) \right\}, \tag{12}$$

where $v_I(\alpha)$ and $v_E(\alpha)$ are the long-run average payoffs of the sellers. The behavior of the buyers is again given by the static indifference conditions of the marginal buyers. The indifference conditions of the buyers (see (6) and (7)) determine the equilibrium prices as a function of the supplied quantities. A Markov perfect equilibrium is then a solution to the dynamic programming equations of the firms after inserting the equilibrium prices into (6) and (7):

$$0 = \max_{q_E(\alpha)} \left\{ (\mu(\alpha)(1 - q_E(\alpha)) - m(\alpha) q_I(\alpha)) q_E(\alpha) - v_E(\alpha) + \frac{1}{2} q_E(\alpha) \Sigma^2(\alpha) V_E''(\alpha) \right\}, \tag{13}$$

and for the incumbent:

$$0 = \max_{q_I(\alpha)} \left\{ (s(1 - q_I(\alpha)) - m(\alpha) q_E(\alpha)) q_I(\alpha) - v_I(\alpha) + \frac{1}{2} q_E(\alpha) \Sigma^2(\alpha) V_I''(\alpha) \right\}. \tag{14}$$

As suggested earlier, we divide both equations through $q_E(\alpha)$ and rewrite (13) and (14) as:

$$0 = \max_{q_E(\alpha)} \left\{ \frac{(\mu(\alpha)(1 - q_E(\alpha)) - m(\alpha) q_I(\alpha)) q_E(\alpha) - v_E(\alpha)}{q_E(\alpha)} \right\} + \frac{1}{2} \Sigma^2(\alpha) V_E''(\alpha), \tag{15}$$

and

$$0 = \max_{q_I(\alpha)} \left\{ \frac{\left(s\left(1 - q_I\left(\alpha\right)\right) - m\left(\alpha\right) q_E\left(\alpha\right)\right) q_I\left(\alpha\right) - v_I\left(\alpha\right)}{q_E\left(\alpha\right)} \right\} + \frac{1}{2}\Sigma^2\left(\alpha\right) V_I''\left(\alpha\right). \tag{16}$$

The unique equilibrium quantities are then derived by standard first-order condition and given by

$$q_E\left(\alpha\right) = \sqrt{\frac{v_E\left(\alpha\right)}{\mu\left(\alpha\right)}}, \tag{17}$$

and

$$q_I\left(\alpha\right) = \frac{1}{2} - \frac{1}{2}\frac{m\left(\alpha\right)}{s}\sqrt{\frac{v_E\left(\alpha\right)}{\mu\left(\alpha\right)}}, \tag{18}$$

where

$$m\left(\alpha\right) \triangleq \min\left\{s, \mu\left(\alpha\right)\right\}.$$

It can be verified that the quantity $q_I\left(\alpha\right)$ is continuous at $\alpha = \alpha_m$, but not differentiable. The equilibrium prices $p_E\left(\alpha\right)$ and $p_I\left(\alpha\right)$ follow from the indifference conditions (6) and (7) of the marginal buyers. The monotonicity properties of quantities and prices, which we observed in the static equilibrium (as a comparative static result) are preserved in the dynamic model indicating how the equilibrium variables evolve as a function of the posterior belief α:

Proposition 3 (Prices and Quantities)

1. *$p_E\left(\alpha\right)$, $q_E\left(\alpha\right)$ and $p_E\left(\alpha\right) q_E\left(\alpha\right)$ are increasing in α.*
2. *$p_I\left(\alpha\right)$, $q_I\left(\alpha\right)$ and $p_I\left(\alpha\right) q_I\left(\alpha\right)$ are decreasing in α.*

Proof. See appendix. ■

As the expectation about the quality of the new product improves, the entrant can achieve higher sales at higher prices. Conversely, the incumbent lowers its sales and yet has to suffer a decrease in the equilibrium price. However the changes in the responsiveness of the equilibrium variables to changes in the posterior belief is modified by the intertemporal considerations. A local argument may suffice here to give the intuition. We saw earlier that the static revenue of the new firm is convex if $\mu\left(\alpha\right) < s$. This means that the entrant would prefer a lottery with expected value of α rather than the expected value itself. The dynamic effect is that the entrant seeks to increase the level of experimentation in the market by increasing it sales. As α becomes sufficiently large so that $\mu(\alpha) > s$, the local argument points in the other direction, as the entrant now becomes averse to more information and this tends to lower the quantity it is supplying to market relative to the myopic policy. As a consequence, the entrant acts the most aggressively for low posterior values. This leads to the following results:

Proposition 4 (Curvatures)

1. *$q_E(\alpha)$ is concave in α,*

2. *$p_E(\alpha)$ is convex if $\mu(\alpha) \leq s$, and concave if $\mu(\alpha) > s$,*

3. *$q_I(\alpha)$ is convex in α,*

4. *$p_I(\alpha)$ is convex in α.*

Proof. See appendix. ■

In response to the aggressive attitude of the new firm for low posterior beliefs, the incumbent reacts most strongly by lowering its sales for low posterior beliefs as well and hence the quantity offered by the incumbent becomes globally convex. While a lower quantity can partially help the incumbent to prevent a decline in its price, the equilibrium price of the incumbent is subject to the same factors as its supply. It suffers the biggest decline for low posterior values and gradually becomes less affected by a change in the posterior belief. The only exception is the equilibrium price of the new firm which maintains the curvature properties from the static equilibrium. As the new firm becomes less aggressive in terms of its supply response with an increase in the posterior belief, the equilibrium price absorbs a larger share of competitive advantage as α increases towards α_m, the point of symmetric competition. However as α increases beyond α_m, the new firm behaves gradually more like a monopolist and any price increase is then due to a direct increase in the value of the product rather than the indirect effect of the improved competitive position.

The curvature properties of quantities and prices can be directly translated into a time series profile by exploiting the martingale properties. Due to the aggressive initial sales the expected future sales of the entrant are falling and expected future prices are rising over time. Conversely, the quantity and the price of the incumbent are expected to recover over time from the entry of the new firm and hence increase over time. These properties are due to $q_E(\alpha)$ being a submartingale, whereas the latter is due $q_I(\alpha)$ and $p_I(\alpha)$ being supermartingales.

The change in the curvature properties has already indicated how the behavior of the firms in the dynamic equilibrium are modified relative to their myopic best response functions. Next we show that we can strengthen the distinction between aggressive and defensive behavior of entrant (and incumbent) by partitioning the state space into two intervals, identical for entrant and incumbent, where static and intertemporal policies can be ranked unambiguously.

To illustrate this point, it is helpful to go back to the value functions of both firms before any modifications:

$$0 = \max_{q_E(\alpha)} \left\{ (\mu(\alpha)(1 - q_E(\alpha)) - m(\alpha) q_I(\alpha)) q_E(\alpha) - v_E(\alpha) + \frac{1}{2} q_E(\alpha) \Sigma^2(\alpha) V_E''(\alpha) \right\}, \quad (19)$$

and for the incumbent:

$$0 = \max_{q_I(\alpha)} \left\{ (s(1 - q_I(\alpha)) - m(\alpha) q_E(\alpha)) q_I(\alpha) - v_I(\alpha) + \frac{1}{2} q_E(\alpha) \Sigma^2(\alpha) V_I''(\alpha) \right\}. \quad (20)$$

Suppose the value of information to the entrant is zero at some critical posterior belief α_c, or $V_E''(\alpha_c) = 0$, then its dynamic best response function at α_c is identical to the static one. As intertemporal considerations in terms of $V_E''(\alpha)$ or $V_I''(\alpha)$ enter the best response function of the incumbent only indirectly through the choices of the entrant (see (20)), it follows that if $V_E''(\alpha_c) = 0$, then necessarily $q_i(\alpha_c) = Q_i(\alpha_c)$ and $p_i(\alpha_c) = P_i(\alpha_c)$ for both firms $i = E, I$. Moreover since the dynamic programming equation (19) has to hold it follows that at α_c, the flow revenues (static or intertemporal) have to be equal to the long-run average value $v_E(\alpha_c)$. We recall that the long-run average $v_E(\alpha)$ at α_c is the expected value of the static equilibrium revenues at $\alpha = 0$ and $\alpha = 1$ weighted with $1 - \alpha_c$ and α_c respectively. Thus even if we don't know $V_E(\alpha)$ or $V_I(\alpha)$, we can find the points where static and intertemporal values coincide through a comparison of static values with the long-run average. Conversely, at all points where static revenues and long-run average diverge we can expect to see discrepancies between static and intertemporal policies.

Proposition 5 (Single Crossing)

1. *The difference* $p_E(\alpha)\, q_E(\alpha) - v_E(\alpha)$ *crosses zero at most once and then from below.*

2. *The critical point* α_c *satisfies* $\alpha_c > \alpha_m$.

3. *A necessary condition for crossing to occur is* $\mu_L < s < \mu_H$.

4. *A necessary and sufficient condition for crossing to occur is:*

$$[p_E(0)\, q_E(0)]' - v_E'(0) < 0,$$

and

$$[p_E(1)\, q_E(1)]' - v_E'(1) < 0.$$

Proof. See appendix. ■

The proof proceeds by establishing the above properties first for static revenues $P_E(\alpha)\, Q_E(\alpha)$ and then extending them to intertemporal flow revenues $p_E(\alpha)\, q_E(\alpha)$. Thus there is at most one critical point where the value of information for the entrant is zero. As the equilibrium policies we derived earlier as well as the long-run average are continuous, it follows that the preference of the entrant towards information represented by $V_E''(\alpha)$ changes signs at most once. As the sign of the term $V_E''(\alpha)$ predicates the bias in the intertemporal policy, the proposition shows that this bias changes at most once, and in fact a necessary condition is that there is uncertainty about the ranking of the alternatives, or

$$\mu_L < s < \mu_H.$$

The evolution of equilibrium revenue and long-run average for the entrant are displayed below for the case that the necessary and sufficient condition is satisfied.

Insert Figure 1 here

By the Bellman equation (19), the sign of the second derivative, $V_E''(\alpha)$, of the value function is the opposite of the difference $p_E(\alpha)\, q_E(\alpha) - v_E(\alpha)$ and hence the difference also represents the evolution of the value of information for the new firm. The relationship between the flow revenues, long-run average and value of information is less clearly connected for the incumbent. This is due to the fact that the value of information, $V_I''(\alpha)$, doesn't enter the first order conditions of the incumbent, as the value of information is a function of the sales of the new firm, but not of the established firm.

Proposition 6 (Value of Information)

1. *$V_E(\alpha)$ is convex in α for all $\alpha \leq \alpha_c$ and concave in α for all $\alpha > \alpha_c$.*

2. *$V_I(\alpha)$ is convex in α.*

Proof. See appendix. ■

Finally we can establish how the presence of market learning affects the equilibrium policies of the firms on either side of the critical value α_c. For all posterior beliefs $\alpha < \alpha_c$, the value of more information is positive to the entrant. In consequence, his entry strategy is to supply the market aggressively. As we formally state below, this leads to lower than myopic prices for the incumbent as well as the entrant. The incumbent is trying partially to offset the increase in supply by the entrant through a decrease in his own supply. As the value of information becomes negative for $\alpha > \alpha_c$, the entrant lowers its supply, and this sets off an adjustment of prices and quantities in the expected direction. Both equilibrium prices increase and the incumbent gains a relatively larger market share.

Proposition 7 (Static vs. Dynamic Strategies)

1. *For $\alpha < \alpha_c$:*

   (a) *$q_E(\alpha) > Q_E(\alpha)$ and $p_E(\alpha) < P_E(\alpha)$,*

   (b) *$q_I(\alpha) < Q_I(\alpha)$ and $p_I(\alpha) < P_I(\alpha)$.*

2. *For $\alpha > \alpha_c$:*

   (a) *$q_E(\alpha) < Q_E(\alpha)$ and $p_E(\alpha) > P_E(\alpha)$,*

   (b) *$q_I(\alpha) > Q_I(\alpha)$ and $p_I(\alpha) > P_I(\alpha)$.*

Proof. See appendix. ■

The behavior of the equilibrium prices and quantities are displayed (in their differences) below for the same environment as in Figure 1.

Insert Figure 2 here

When we combine the time series behavior of the equilibrium together with the properties of the equilibrium policies relative to their static counterparts, a rather complete picture regarding the entrance and deterrence behavior emerges. As the policies depend essentially on the current position of the firms in the quality spectrum, it might be helpful for the interpretation to consider the two polar cases relative to the intermediate case where $\mu_L < s < \mu_H$. If $\mu_L < \mu_H \leq s$, we refer to the new product as a *substitute* and if $s \leq \mu_L < \mu_H$, then we refer to it as an *improvement*. A substitute is at best equal to the established product, whereas an improvement is at least as good as the established product. The first scenario may represent the introduction of a generic pharmaceutical or a no name product, whereas the second may represent a new version of a current product with additional features whose (positive) contribution is yet uncertain.

With a substitute entry is aggressive, and the equilibrium price of the entrant is below the static price. Over time, the expected equilibrium price of the entrant is increasing and the expected supply is decreasing as the entrant becomes more established and less aggressive. The effect of entry with uncertain valuations on the incumbent is that both sales as well as prices are uniformly lower for the incumbent. But the submartingale property of both equilibrium variables then shows that sales and prices are expected to increase over time. The entry strategy with an improved product is substantially different. The supply is at all times lower than with a static equilibrium, as the new firm will lose more through a (gradual) decrease in the posterior than a (gradual) increase. In consequence, the new firm will start with lower than myopic quantities and is essentially cream-skimming. Over time, its expected price is decreasing and the expected sales and revenues of the incumbent are increasing. Thus the aggressiveness of the strategy is almost entirely predicated by the relative position of the new firm to the established firm.

5 Robustness

In this section we discuss in some detail how robust our equilibrium results are to different modelling assumptions. In Subsection 5.1 we remove the assumption of a uniform distribution on θ and extend the analysis to more general inverse demand functions. In Subsection 5.2 we discuss how our qualitative results would be changed by considering price rather than quantity competition.

5.1 Quantity Competition and General Distributions

We begin the analysis by considering a general distribution $F(\theta)$ over the unit interval. Associated with any given distribution $F(\theta)$ and any given belief α is a static profit function $\pi_i(Q_E, Q_I \,|\alpha)$ for firm i. In addition, denote by $\pi_E(Q_E \,|\alpha)$ the profit function of the entrant when he faces a competitive fringe with quality s rather than a single competitor. We make the following three assumptions on the behavior of the static profit functions for the remainder of this section:

1. $\pi_i(Q_E, Q_I \,|\alpha)$ is concave in Q_i for all i and all α.
2. $\pi_E(Q_E \,|\alpha)$ is concave in Q_E for all α.
3. The static best response functions satisfy the stability condition: $-1 < Q_i'(Q_j) < 0$, $\forall i$.

As our main interest is in the dynamic aspects of the competition model, we do not attempt to present the most general conditions on $F(\theta)$ which would guarantee that the fairly standard assumptions above on the static profit functions are met. Yet it can be verified that a sufficient condition for all three assumptions jointly is that the distribution function $F(\theta)$ is convex, which includes the uniform density model analyzed so far.

We proceed to show that the qualitative properties of the entry and deterrence behavior can be derived in this general setting based exclusively on the interaction between static profit functions and long-run average values.

As before, the following dynamic programming equations characterize the Markov perfect equilibria:

$$0 = \max_{q_E} \left\{ \pi_E(q_E, q_I \,|\alpha) - v_E(\alpha) + \frac{1}{2} q_E \Sigma^2(\alpha) V_E''(\alpha) \right\},$$

and

$$0=\max_{q_I}\left\{\pi_I\left(q_E,q_I\left|\alpha\right.\right)-v_I\left(\alpha\right)+\frac{1}{2}q_E\Sigma^2\left(\alpha\right)V_I''\left(\alpha\right)\right\},$$

where $v_E(\alpha)$ and $v_I(\alpha)$ are the long-run average revenues under the static profit functions $\pi_E(Q_E,Q_I|\alpha)$ and $\pi_I(Q_E,Q_I|\alpha)$. For $q_E>0$, we may divide the above equations by q_E to obtain:

$$0=\max_{q_E}\left\{\frac{\pi_E\left(q_E,q_I\left|\alpha\right.\right)-v_E\left(\alpha\right)}{q_E}\right\}+\frac{1}{2}\Sigma^2\left(\alpha\right)V_E''\left(\alpha\right),$$

$$0=\max_{q_I}\left\{\frac{\pi_I\left(q_E,q_I\left|\alpha\right.\right)-v_I\left(\alpha\right)}{q_E}\right\}+\frac{1}{2}\Sigma^2\left(\alpha\right)V_I''\left(\alpha\right).$$

In order to facilitate the comparison with the static equilibrium which is a solution to

$$\max_{q_E}\left\{\pi_E\left(Q_E,Q_I\left|\alpha\right.\right)-v_E\left(\alpha\right)\right\},$$

and

$$\max_{q_I}\left\{\pi_I\left(Q_E,Q_I\left|\alpha\right.\right)-v_I\left(\alpha\right)\right\},$$

we consider the first order conditions to the dynamic programming equations. These can be written as:

$$q_E\frac{\partial}{\partial q_E}\pi_E\left(q_E,q_I\left|\alpha\right.\right)=\pi_E\left(q_E,q_I\left|\alpha\right.\right)-v_E\left(\alpha\right),\tag{21}$$

and

$$\frac{\partial}{\partial q_I}\pi_I\left(q_E,q_I\left|\alpha\right.\right)=0.\tag{22}$$

We observe that the first order condition of the incumbent leads to the same best response function as his static one. Moreover if the right hand side in (21) vanishes, then the equations (21) and (22) reduce to the static equilibrium conditions. Hence we know that the dynamic equilibrium conditions are satisfied at the static equilibrium values of $\{Q_E(\alpha),Q_I(\alpha)\}$ if and only if $\pi_E(Q_E(\alpha),Q_I(\alpha)|\alpha)=v_E(\alpha)$. Thus the coincidence of static and dynamic equilibrium policies is in general linked to the equality of static equilibrium profit and long-run average for the *entrant*.

Denote by $Q_i(q_j)$ the myopic best response of firm i to the firm j's quantity, where we omit the dependence of Q_i on α for notational simplicity. As indicated by equation (22), the static and the dynamic best response are identical for the incumbent, or $q_I(q_E) = Q_I(q_E)$. All dynamic equilibria must therefore lie on the reaction curve of the incumbent: $\{q_E, q_I(q_E)\}$. Assumptions 1 and 3 guarantee that there is a single stable static equilibrium, and thus we know that for all $q_E > Q_E(\alpha)$, $q_E > Q_E(Q_I(q_E))$ and hence by the strict concavity of $\pi_E(q_E, q_I)$ in q_E,

$$\frac{\partial \pi_E(q_E, q_I(q_E)|\alpha)}{\partial q_E} < 0, \tag{23}$$

for all $q_E > Q_E(\alpha)$. A similar argument can be made for the case where $q_E < Q_E(\alpha)$ to show that

$$\frac{\partial \pi_E(q_E, q_I(q_E)|\alpha)}{\partial q_E} > 0. \tag{24}$$

As the first order condition of the entrant in the dynamic equilibrium requires that

$$\operatorname{sgn}\left(\frac{\partial \pi_E(q_E, q_I(q_E)|\alpha)}{\partial q_E}\right) = \operatorname{sgn}(\pi_E(q_E, q_I(q_E)|\alpha) - v_E(\alpha)), \tag{25}$$

a local argument around the static equilibrium quantities $\{Q_E(\alpha), Q_I(\alpha)\}$ based on (23) and (24) seems to suggests the direction in which dynamic quantities deviate from static ones. In fact, the argument is facile for the case that

$$\pi_E(Q_E(\alpha), Q_I(\alpha)|\alpha) < v_E(\alpha),$$

and requires more care for the case of

$$\pi_E(Q_E(\alpha), Q_I(\alpha)|\alpha) > v_E(\alpha),$$

only to the extent that we want to guarantee that *all* equilibria have the desired property. Assume therefore initially the following relation between the static equilibrium profits and the long-run average:

$$\pi_E(Q_E(\alpha), Q_I(\alpha)|\alpha) < v_E(\alpha).$$

All we have to do to determine the location of the dynamic equilibrium is to find out how $\operatorname{sgn}(\pi_E(q_E, q_I(q_E)|\alpha) - v_E(\alpha))$ behaves on the locus designated by $\{q_E, q_I(q_E)\}$. The claim is that every quantity q_E which satisfies the dynamic equilibrium conditions must display $q_E > Q_E(\alpha)$. To see this we observe that either we have

$$\pi_E(q_E, q_I(q_E)|\alpha) < v_E(\alpha) \text{ for all } q_E, \tag{26}$$

or

$$\pi_E(q_E, q_I(q_E)|\alpha) \geq v_E(\alpha) \Rightarrow q_E > Q_E(\alpha). \tag{27}$$

In the first case, the static profit function remains for all pairs $\{q_E, q_I(q_E)\}$ below the long-run average, and the equilibrium condition (25) together with the stability condition (23) implies that $q_E(\alpha) > Q_E(\alpha)$. Consider next the case of (27). If there exist values $\{q_E, q_I(q_E)\}$ such that the static profit exceeds the long-run average, then the stability condition (23) informs us that the equilibrium condition can not possibly hold at $q_E > Q_E(\alpha)$. Hence we can conclude that whenever $\pi_E(Q_E, Q_I|\alpha) < v_E(\alpha)$, the dynamic equilibrium quantity sold by the new firm exceeds the static equilibrium quantity, or $q_E(\alpha) > Q_E(\alpha)$. To establish this argument we only used the stability condition of the static best response function. The complementary results for

$$\pi_E(Q_E(\alpha), Q_I(\alpha)|\alpha) > v_E(\alpha),$$

are proven in the appendix under the additional concavity assumptions.

Proposition 8

Suppose that assumptions 1-3 hold. Then:

1. $\pi_E(Q_E(\alpha), Q_I(\alpha)|\alpha) < v_E(\alpha) \Rightarrow q_E(\alpha) > Q_E(\alpha)$,
2. $\pi_E(Q_E(\alpha), Q_I(\alpha)|\alpha) > v_E(\alpha) \Rightarrow q_E(\alpha) < Q_E(\alpha)$.

Proof. See appendix. ■

Under assumptions 1-3, the predictions for the dynamic model are then straightforward. To determine whether the equilibrium quantities of the new firm exceed or fall short of the myopic quantities, all we need to do is to compare the myopic equilibrium profits to the long-run average profits. If the static equilibrium profits are below the long run average profits, then the new firm will adopt an aggressive sales policy and by the property of the best response function, the incumbent will adopt a more defensive stance. In contrast, if the static equilibrium profits are above the long-run average revenues, the entrant will proceed cautiously with the introduction of the new product and the incumbent will increase his supply to the market. The dynamic programming equations also inform us that the entry strategies are always associated with $V_E''(\alpha) > 0$ and $V_E''(\alpha) < 0$, respectively. The change in the entry strategy can therefore generally be located as in the uniform model analyzed earlier at the intersection $\pi_E(Q_E(\alpha), Q_I(\alpha)|\alpha) = v_E(\alpha)$, where all the necessary data can be computed on the basis of the static profit function alone.

5.2 Price Competition

Finally, we sketch how the qualitative results would be affected by a model of price competition. We show that despite some fundamental differences in the static equilibrium profit functions, the dynamic equilibria of the two models share very similar properties.

The most important change in terms of the static equilibria of the two models is that the equilibrium profits are no longer monotone in the quality of the new product. As emphasized in the literature on vertical differentiation, the competitor with lower quality product doesn't want to increase the quality of his product if this brings the inferior product too close to the superior product. In consequence, the equilibrium prices and revenues are not monotone in α either, rather they display a global minimum at $\alpha = \alpha_m$. At the point α_m, price competition with identical products leads to the Bertrand outcome with marginal cost pricing. The static equilibrium profit functions display a kink at α_m, but on the intervals $[0, \alpha_m)$ and (α_m, ∞) they are concave for the entrant as well as the incumbent.

In the dynamic model, the strategic interaction is more complex with price competition. With quantity competition, the only variable which affects the evolution of future states, i.e. the level of sales by the entrant, is directly a decision variable of the entrant. In obtaining the dynamic best response of the incumbent, we can therefore ignore the impact of his current decision on future states. But this implies that the best response of the incumbent to any output decision by the entrant is the same in the static and the dynamic model. As a result, all comparisons can be carried out by analyzing the shifts in the best response function of the entrant. In a model with price competition, the price decisions by the firms *jointly* determine the sales level of the entrant. In consequence, we have to analyze the joint effects of changes in the two best response functions on the dynamic equilibrium. To see how this interaction is resolved in equilibrium, we check how the static policies are modified by intertemporal considerations. The graphic below illustrates the static equilibrium revenues as well as the long run average revenues for the case that the value of the new product can either be lower or higher than the established product.

Insert Figure 3 Here

Due to the local minimum at $\alpha = \alpha_m$, the long-run average is always above the static revenues. Thus if the static policies were in fact the dynamic equilibrium policies, then the respective Bellman's equations would indicate that $V_E''(\alpha) > 0$ as well as $V_I''(\alpha) > 0$. But this would imply that both firms would like to see more sales by the entrant relative to the static equilibrium. We can therefore conjecture that the entrant will lower and the incumbent will raise its price relative to the static equilibrium price. In consequence, sales by the entrant must be larger (and the incumbent's sales must be lower) than in the static equilibrium. If on the other hand, the product is an improvement, and $s < \mu_L < \mu_H$, then the long-run average revenue is lower than the static equilibrium revenue as the following graphic illustrates.

Insert Figure 4 Here

By the same intuition as above we can then infer from the value functions that if the static policies were indeed equilibrium policies in the dynamic model, then it would have to be that $V_E''(\alpha) < 0$ as well as $V_I''(\alpha) < 0$. But this implies that both firms perceive sales by the entrant as carrying a negative value of information. The strategic response relative to the static solution for the new firm is to raise its price, and for the incumbent to decrease its price. This leads to lower quantities for the entrant and higher quantities for the incumbent. Thus the qualitative behavior of entrant and incumbent are similar in a model for quantity competition.

The only difference between the two models arises when $\mu_L < \mu_H < s$. By the concavity of the static profit function, the static revenues of the new firm are always below the long-run average. Observe, however, that a marginal improvement actually brings the new firm closer to its competitor in the quality spectrum and this leads to lower profits. If we interpret the random product quality as reflecting the uncertain value of some new features in the product, it would be unlikely that these features would be included in the product if $\mu_L < \mu_H < s$.

6 Conclusion

This paper analyzed the entry game in a model with vertical differentiation. The precise location of the new product relative to the existing product was initially uncertain and was learned over time through experience. We derived the optimal entry and deterrence strategies for the competitors. It was shown that their qualitative properties depend on the current position of the new product in the quality spectrum. This allowed particularly sharp characterization results for the polar cases of a substitute or an improvement respectively. By focusing on the Markov perfect equilibrium of the game, we derived a set of time series implications which may be amenable to empirical tests.

The current analysis faced some restrictions by the very nature of the model. First, we assumed that the value *and* the uncertainty about the new product was common to all buyers, after controlling for the element of vertical differentiation. It may be interesting to pursue how the equilibrium strategies would be affected if the experience by the buyers would contain an idiosyncratic element (see Milgrom & Roberts (1986) for a simple monopoly model). The second limitation is the "once and for all" nature of the innovation presented by the new product. This was reflected in the model by the fact the posterior beliefs converged to either of the absorbing states $\alpha \in \{0, 1\}$ almost surely.

The techniques employed in this paper, however, generalize beyond the present model. The use of the undiscounted optimization criterion, and in particular the notion of the long-run average, allowed us to make a series of predictions based almost exclusively on the static equilibrium behavior. While the long-run average here was computed on the basis of the absorbing and mutually exclusive posterior beliefs, the technique extends naturally to ergodic distributions of the state variables. This should make the methodology used in this paper an attractive candidate for a much richer class of strategic models such as investment games and models of industry evolution, for which there are very few explicit solutions currently known (e.g. Ericson & Pakes (1995)). In particular, it would seem feasible to combine dynamic competition models such as the one analyzed here with an ongoing process of innovation.

7 Appendix

We first present a derivation of the Bayesian filtering equation (1) based on a discrete time model with a finite number of buyers. The limiting behavior of the discrete learning model will lead to the Brownian motion depicted in (1) as the number of buyers becomes large and the time elapsed between any two periods converges to zero. Suppose therefore in an economy with N buyers, each individual experiment with the new product by buyer i is an independent and identically distributed random variable $\widetilde{x}_i$ with a normal distribution of unknown mean $\mu_N = \mu/N$ and known variance $\sigma_N^2 = \sigma^2/N$. The parameter μ can take on the values μ_L or μ_H. Notice that mean as well as variance of the individual experiment are scaled with resect to the total number of buyers N. The utility for buyer i is then given by $\theta_i x_i$, where x_i is a sample realization of $\widetilde{x}_i$. Based on the individual experiences of all buyers, we can describe the aggregate or market experience in every period. As the informational content in every realization x_i is independent of the willingness to pay θ_i of individual i, we take the market experience to be the sum of the individual random variables while omitting the weights θ_i:

$$\widetilde{x}(N) = \sum_{i=1}^{N} \widetilde{x}_i$$

As mean and variance of the random variable $\widetilde{x}_i$ are normalized by the number of buyers in the market, aggregate mean and aggregate variance of the market experiment $\widetilde{x}(N)$ is independent of the number N of buyers and given by (μ, σ^2). If only a number k of buyers experiment with the new product, where $k \leq N$, then the aggregate experiment is given by the random variable:

$$\widetilde{x}(k) = \sum_{i=1}^{k} \widetilde{x}_i,$$

which is again normally distributed with mean $\frac{k}{N}\mu$ and variance $\frac{k}{N}\sigma^2$. If we take the limit as N goes to infinity, the distribution of an aggregate experiment with a fraction n of the buyers, where

$$n = \frac{k}{N}$$

is given by

$$\widetilde{x}(n) \sim N\left(n\mu, n\sigma^2\right).$$

Next we take the limit as the time between any two periods converges to zero. In the continuous time limit the market experiment then becomes a Brownian motion which can be described by the stochastic differential equation

$$dx\left(n\left(t\right)\right)=n\left(t\right)\mu dt+\sigma\sqrt{n\left(t\right)}dB\left(t\right),\quad t\in[0,\infty).$$

The flow realization in period t is given by the true mean μ weighted by the fraction of buyers participating in the experiment and the random term of the standard Brownian motion $dB\left(t\right)$ weighted by the standard deviation $\sigma\sqrt{n\left(t\right)}$.

Based on the evolution of the market experiment the market can update the prior belief α_0 to the posterior belief $\alpha\left(t\right)$. Based on standard result for Bayesian updating in continuous time, it can be shown that the posterior belief $\alpha\left(t\right)$ also evolves as a Brownian motion.[7] It can be represented by

$$d\alpha\left(t\right)=\sqrt{\frac{n\left(t\right)\alpha\left(t\right)\left(1-\alpha\left(t\right)\right)\left(\mu_H-\mu_L\right)}{\sigma^2}}dB\left(t\right).$$

and as in equilibrium $n\left(t\right)=q_E\left(t\right)$, equation (1) follows.

All proofs to the propositions in the text are collected below.

Proof of Proposition 1. The unique solution to the best response functions (2) and (3) yield the following equilibrium quantities:

$$Q_E\left(\alpha\right)=\frac{\mu\left(\alpha\right)+M\left(\alpha\right)-m\left(\alpha\right)}{4M\left(\alpha\right)-m\left(\alpha\right)}\text{ and }Q_I\left(\alpha\right)=\frac{s+M\left(\alpha\right)-m\left(\alpha\right)}{4M\left(\alpha\right)-m\left(\alpha\right)},\tag{28}$$

where $m\left(\alpha\right)$ and $M\left(\alpha\right)$ are defined as follows:

$$m\left(\alpha\right)\triangleq\min\left\{s,\mu\left(\alpha\right)\right\},\ M\left(\alpha\right)\triangleq\max\left\{s,\mu\left(\alpha\right)\right\}.$$

The equilibrium prices are given by:

$$P_E\left(\alpha\right)=\mu\left(\alpha\right)Q_E\left(\alpha\right)\text{ and }P_I\left(\alpha\right)=sQ_I\left(\alpha\right).\tag{29}$$

The monotonicity properties follow directly from the relevant derivatives. ■

[7] See Liptser & Shiryayev (1977), Chapter 9, for the derivation of the filtering equation for the continuous time, Brownian motion model.

Proof of Proposition 2. The curvature properties follow directly from the second derivatives of (28) and (29), respectively. ■

The next lemma records the construction of the long-run averages for the firms.

Lemma 1 (Long-run averages)

The long-run averages are given by:

$$v_E(\alpha) = (1-\alpha)\frac{\mu_L(\mu_L + M(0) - m(0))^2}{(4M(0) - m(0))^2} + \alpha\frac{\mu_H(\mu_H + M(1) - m(1))^2}{(4M(1) - m(1))^2}, \tag{30}$$

and

$$v_I(\alpha) = (1-\alpha)\frac{s(s + M(0) - m(0))^2}{(4M(0) - m(0))^2} + \alpha\frac{s(s + M(1) - m(1))^2}{(4M(1) - m(1))^2}.$$

Proof. The long-run average values $v_i(\alpha)$ are equal to the expected full-information payoffs:

$$v_i(\alpha) = (1-\alpha)v_i(0) + \alpha v_i(1),$$

if $q_E(\alpha)$ is bounded away from zero for all α. It can be verified from (17) that this indeed guaranteed in equilibrium. But $v_i(0)$ and $v_i(1)$ are simply the values to the full information static equilibrium problems when the value of the new product is known to be either μ_L or μ_H. The composite values are then computed immediately. ■

Next we record without proof some properties of ratios and products of $\mu(\alpha)$ and $v_E(\alpha)$.

Lemma 2

1. *The ratios* $\frac{v_E(\alpha)}{\mu(\alpha)}$ *and* $\sqrt{\frac{v_E(\alpha)}{\mu(\alpha)}}$ *are increasing and concave in* α.

2. *The product* $v_E(\alpha)\mu(\alpha)$ *is increasing and convex in* α.

3. *The product* $\sqrt{v_E(\alpha)\mu(\alpha)}$ *is increasing and concave in* α.

Proof of Proposition 3. The first order conditions associated with (16) and (15) deliver the solutions for $q_E(\alpha)$ and $q_I(\alpha)$ given in (17) and (18). The market clearing conditions (6) and (7) lead to the equilibrium prices:

$$p_E(\alpha) = \mu(\alpha)\left(1 - \sqrt{\frac{v_E(\alpha)}{\mu(\alpha)}}\right) - m(\alpha)\left(\frac{1}{2} - \frac{1}{2}\frac{m(\alpha)}{s}\sqrt{\frac{v_E(\alpha)}{\mu(\alpha)}}\right),$$

and

$$p_I(\alpha) = \frac{s}{2} - \frac{m(\alpha)}{2}\sqrt{\frac{v_E(\alpha)}{\mu(\alpha)}}.$$

Next we prove the monotonicity properties. Consider first $\mu(\alpha) \geq s$, or $m(\alpha) = s$. A necessary and sufficient condition for $q_E(\alpha)$ to be increasing is that:

$$v_E(1)\,\mu(0) \geq v_E(0)\,\mu(1),$$

which is equivalent to

$$\frac{\mu_L + M(0) - m(0)}{4M(0) - m(0)} \leq \frac{\mu_H + M(1) - m(1)}{4M(1) - m(1)},$$

which can be shown to hold for all values of μ_L, μ_H and s. It then follows directly that $q_I(\alpha)$ and $p_I(\alpha)$ are decreasing in α. It remains to show that $p_E(\alpha)$ is increasing. Suppose initially that $\mu_L, \mu_H \geq s$. It is sufficient to show that:

$$\mu(\alpha) - \sqrt{\mu(\alpha)\, v_E(\alpha)}$$

is increasing in α. As $\mu(\alpha) > v_E(\alpha)$ for all α, it suffices to show that

$$\frac{\mu'(\alpha)}{v_2'(\alpha)} \geq \sqrt{\frac{\mu(\alpha)}{v_E(\alpha)}}.$$

By Lemma 2, the rhs is convex and decreasing, and evaluating the inequality at $\alpha = 0$ is sufficient as $\mu'(\alpha)$ and $v_2'(\alpha)$ are constant. We then obtain

$$\frac{\mu_H - \mu_L}{\frac{\mu_H(2\mu_H - s)^2}{(4\mu_H - s)^2} - \frac{\mu_L(2\mu_L - s)^2}{(4\mu_L - s)^2}} \geq \frac{4\mu_L - s}{2\mu_L - s}. \tag{31}$$

As the lhs is increasing in μ_H, it is sufficient to evaluate it as $\mu_H \downarrow \mu_L$, in which case the inequality reads as

$$(4\mu_L - s)^2 \geq 8\mu_L^2 - 2\mu_L s + s^2,$$

which is satisfied by hypothesis of $\mu_L \geq s$. Suppose next that $\mu_L < s < \mu_H$. Then at (31), the argument changes only slightly as $v_E(\alpha)$ has a different form, or:

$$\frac{\mu_H - \mu_L}{\frac{\mu_H(2\mu_H - s)^2}{(4\mu_H - s)^2} - \frac{\mu_L(s)^2}{(4s - \mu_L)^2}} \geq \frac{4s - \mu_L}{s}.$$

As the lhs is now decreasing in μ_H, it is sufficient to evaluate it in the limit as $\mu_H \to \infty$, where the inequality reads

$$4 \geq \frac{4s - \mu_L}{s},$$

which completes the argument.

Consider next $\mu(\alpha) \leq s$. The price is then given by:

$$p_E(\alpha) = \frac{1}{2}\mu(\alpha) - \sqrt{\mu(\alpha) v_E(\alpha)} + \frac{\mu(\alpha)}{2s}\sqrt{\mu(\alpha) v_E(\alpha)}.$$

Suppose initially that $\mu_L, \mu_H < s$. It is now sufficient to show that

$$\frac{1}{2}\mu(\alpha) - \sqrt{\mu(\alpha) v_E(\alpha)} \tag{32}$$

is increasing in α. By the multiplication rule this is equivalent to showing that

$$\mu'(\alpha)\sqrt{\mu(\alpha) v_E(\alpha)} \geq \mu'(\alpha) v_E(\alpha) + \mu(\alpha) v_E'(\alpha).$$

As the term in (32) is concave in α, it remains to show that the inequality holds at $\alpha = 0$ or after cancelling some terms:

$$\frac{\mu_H - \mu_L}{4s - \mu_L} \geq \frac{(\mu_H - \mu_L)s}{(4s - \mu_L)^2} + \frac{\mu_H s}{(4s - \mu_H)^2} - \frac{\mu_L s}{(4s - \mu_L)^2}.$$

As the rhs term is increasing faster in μ_H than the lhs, it is sufficient to evaluate it at $\mu_H = s$, or

$$\frac{s - \mu_L}{4s - \mu_L} \geq \frac{(s - \mu_L)s}{(4s - \mu_L)^2} + \frac{1}{9} - \frac{\mu_L s}{(4s - \mu_L)^2},$$

which is satisfied for all $\mu_L \leq s$. Suppose now that $\mu_L < s < \mu_H$, it is then sufficient to show that $p_E'(\alpha) > 0$ at $\alpha = 0$ by Lemma 2, which is equivalent to showing that at $\alpha = 0$:

$$\mu'(\alpha)\sqrt{\mu(\alpha) v_E(\alpha)} + \frac{1}{2s}\left(3\mu(\alpha)\mu'(\alpha) v_E(\alpha) + (\mu(\alpha))^2 v_E'(\alpha)\right) \geq \mu'(\alpha) v_E(\alpha) + \mu(\alpha) v_E'(\alpha).$$

Since the lhs is increasing faster in μ_H than the rhs it is sufficient to evaluate the inequality at $\mu_H = s$, and again it can be verified that the inequality holds for all $\mu_L \leq s$.■

Proof of Proposition 4.(1.) By Lemma 2.

(2.) It follows directly from Lemma 2 that $p_E(\alpha)$ is convex for $\mu(\alpha) < s$ and concave for $\mu(\alpha) > s$.

(3.) By Lemma 2.

(4.) By Lemma 2.■

The proof of Proposition 5 relies on the following two lemmata. The first states that the difference $P_E(\alpha) Q_E(\alpha) - v_E(\alpha)$ satisfies identical single crossing properties as $p_E(\alpha) q_E(\alpha) - v_E(\alpha)$. The second shows that the crossing point is necessarily identical for the two differences. Denote by A_c the crossing point for the static revenue function.

Lemma 3

1. *The difference* $P_E(\alpha) Q_E(\alpha) - v_E(\alpha)$ *crosses zero at most once and then from below.*

2. *The critical point* A_c *satisfies* $A_c > \alpha_m$.

3. *A necessary condition for crossing is* $\mu_L < s < \mu_H$.

4. *A necessary and sufficient condition for crossing to occur is:*

$$[P_E(0) Q_E(0)]' - v_E'(0) < 0 \text{ and } [P_E(1) Q_E(1)]' - v_E'(1) < 0.$$

Proof. (1.) Observe initially that $P_E(0) Q_E(0) - v_E(0) = 0$ and $P_E(1) Q_E(1) - v_E(1) = 0$. We first show that if $\mu_L < \mu_H \leq s$, or $s \leq \mu_L < \mu_H$, then $P_E(\alpha) Q_E(\alpha) - v_E(\alpha)$ never crosses at any $\alpha \in (0, 1)$. By Lemma 1, $v_E(\alpha)$ is linear in α, and by Proposition 2 $P_E(\alpha) Q_E(\alpha)$ is either convex or concave respectively. This together with the behavior at the end points excludes an interior crossing point. Consider next $\mu_L < s < \mu_H$, then the revenue function $P_E(\alpha) Q_E(\alpha)$ changes curvature behavior exactly once at $\alpha = \alpha_m$. As the curvature changes from convex to concave, the boundary behavior then implies that $P_E(\alpha) Q_E(\alpha) - v_E(\alpha)$ has to cross from below and can cross zero at most once.

(2.) It is easily verified that at $\alpha = \alpha_m$, $P_E(\alpha_m) Q_E(\alpha_m) - v_E(\alpha_m) < 0$.

(3.) The necessary condition follows from the arguments given for (1).

(4.) The necessary and sufficient conditions follow from the curvature and boundary behavior of the static and long-run revenue functions. ■

Lemma 4 $\alpha_c = A_c$.

Proof. As $p_E(\alpha) q_E(\alpha)$ and $v_E(\alpha)$ are continuous a change in sign for $p_E(\alpha) q_E(\alpha) - v_E(\alpha)$ requires a point $\alpha = \alpha_c$ at which

$$p_E(\alpha_c) q_E(\alpha_c) - v_E(\alpha_c) = 0. \tag{33}$$

At such a point α_c, either

$$q_E(\alpha_c) = Q_E(\alpha_c) \tag{34}$$

or

$$q_E(\alpha_c) \neq Q_E(\alpha_c). \tag{35}$$

Suppose first (34) were to hold, then it follows by the equilibrium conditions (16) and (6)-(7) that $p_E(\alpha_c) = P_E(\alpha_c)$ as well. But then it is has to be the case that $\alpha_c = A_c$. Suppose to the contrary that (35) would hold, then we show that (33) can't hold. Since $q_E(\alpha_c) \neq Q_E(\alpha_c)$, it has to be the case that $V_E''(\alpha_c) \neq 0$, by the first-order conditions from the Bellman equation (13). But then the hypothetical policies at α_c don't satisfy the Bellman equation and hence cannot be equilibrium conditions. Thus if $\alpha_c \in (0,1)$ it has to be that $\alpha_c = A_c$. It remains to show that if $P_E(\alpha) Q_E(\alpha) - v_E(\alpha)$ changes sign, then $p_E(\alpha) q_E(\alpha) - v_E(\alpha)$ necessarily changes signs as well. This is established easily as at α_c, $q_E(\alpha_c) = Q_E(\alpha_c)$ is a solution to the first order condition (15) and as the solution is unique, the claim follows. ■

Proof of Proposition 5. (1.)-(3.) By Lemma 3, the difference $p_E(\alpha) q_E(\alpha) - v_E(\alpha)$ shares the single-crossing behavior with the difference $P_E(\alpha) Q_E(\alpha) - v_E(\alpha)$. By Lemma 4, they also share the crossing point.

(4.) As the myopic and intertemporal policies are identical at the endpoints, or $q_E(\alpha) = Q_E(\alpha)$ and $p_E(\alpha) = P_E(\alpha)$ for $\alpha \in \{0,1\}$, it follows that the gradient of the flow revenues at the endpoints are necessary and sufficient conditions as well.■

Proof of Proposition 6. (1) By the Bellman equation (11) the sign of the second derivative is the opposite of the sign of $p_E(\alpha) q_E(\alpha) - v_E(\alpha)$. The result then follows from Proposition 5.

(2.) The flow revenues $p_I(\alpha) q_I(\alpha)$ are convex in α and $v_I(\alpha)$ is linear in α. It then follows after observing that $p_I(0) q_I(0) = v_I(0)$ and $p_I(1) q_I(1) = v_I(1)$, that $p_I(\alpha) q_I(\alpha) < v_I(\alpha)$ for all $\alpha \in (0,1)$. By the Bellman equation (12), this implies that $V_I''(\alpha) > 0$ for all $\alpha \in (0,1)$.■

Proof of Proposition 7. We first observe that the asymmetry in the relationship between myopic and intertemporal quantities for the sellers follows directly from the best response function based on (16). It is therefore sufficient to consider the relationship between $q_E(\alpha)$ and $Q_E(\alpha)$. It follows from the first order condition (13) of the entrant, that $q_E(\alpha) > Q_E(\alpha)$ if and only if $V_E''(\alpha) > 0$. Likewise $q_E(\alpha) < Q_E(\alpha)$ if and only if $V_E''(\alpha) < 0$. The results concerning the equilibrium quantities follow then directly from Proposition 6.

For the equilibrium prices consider first the interval $\alpha \in [0, \alpha_m]$. As the inequality $q_E(\alpha) > Q_E(\alpha)$ leads to $q_I(\alpha) < Q_I(\alpha)$, the best response function based on (16) implies together with market clearing condition (6) that $p_E(\alpha) < P_E(\alpha)$, which in turn leads to $p_I(\alpha) < P_I(\alpha)$. Consider next the interval $\alpha \in [\alpha_m, \alpha_c]$. The inequality $q_E(\alpha) > Q_E(\alpha)$ leads to $q_I(\alpha) < Q_I(\alpha)$. The best response function based on (16) implies together with market clearing condition (7) that $p_I(\alpha) < P_I(\alpha)$, which in turn leads to $p_E(\alpha) < P_E(\alpha)$. In the remaining interval $\alpha \in [\alpha_c, 1]$, the inequality $q_E(\alpha) < Q_E(\alpha)$ leads to $q_I(\alpha) > Q_I(\alpha)$. The best response function (16) together with market clearing condition (7) implies that $p_I(\alpha) > P_I(\alpha)$, which in turn leads to $p_E(\alpha) > P_E(\alpha)$.

Proof of Proposition 8. The case of $\pi_E(Q_E(\alpha), Q_I(\alpha) | \alpha) < v_E(\alpha)$ was argued in the text. Suppose now that $\pi_E(Q_E(\alpha), Q_I(\alpha) | \alpha) > v_E(\alpha)$. Suppose first that $q_E < Q_E(\alpha)$, then we want to show that at q_E, $\pi_E(q_E, q_I(q_E) | \alpha) > v_E(\alpha)$. The argument is by contradiction. Suppose not, then it would follow from the Bellman equation that $V_E''(\alpha) > 0$, but then $q_E < Q_E(\alpha)$, cannot be an equilibrium, as the entrant would have an incentive to deviate and increase the quantity. By a similar argument, we can exclude the possibility of $q_E > Q_E(\alpha)$ where $\pi_E(q_E, q_I(q_E) | \alpha) > v_E(\alpha)$ holds simultaneously.

Finally we present sufficient conditions to rule out possible equilibria in the region where $q_E > Q_E(\alpha)$ and $\pi_E(q_E, q_I(q_E) | \alpha) < v_E(\alpha)$. Observe first for all q_E sufficiently close to $Q_E(\alpha)$, we have:

$$q_E \frac{\partial \pi_E(q_E, q_I(q_E) | \alpha)}{\partial q_E} < \pi_E(q_E, q_I(q_E) | \alpha) - v_E(\alpha). \tag{36}$$

A sufficient condition to rule out equilibria with $q_E > Q_E(\alpha)$ is therefore that the derivative of the lhs is always below the derivative of the rhs for $q_E > Q_E(\alpha)$. Using the fact that

$$\pi_E(q_E, q_I(q_E) | \alpha) = p_E(q_E, q_I(q_E))\, q_E$$

we may rewrite the inequality (36) as

$$q_E^2 \frac{\partial p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_E} < -v_E\left(\alpha\right).$$

The sufficient condition can then be written as

$$2\frac{\partial p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_E} + q_E\left(\frac{\partial^2 p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_E^2} + \frac{\partial^2 p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_I \partial q_E} q_I'\left(q_E\right)\right) < 0. \tag{37}$$

As the first term is always strictly negative independent of the $F\left(\theta\right)$, it follows that it is sufficient to show that

$$\frac{\partial^2 p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_E^2} + \frac{\partial^2 p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_I \partial q_E} q_I'\left(q_E\right) \leq 0.$$

Consider first $\mu\left(\alpha\right) \leq s$, then the equilibrium price of the entrant can be written as:

$$p_E = \mu\left(\alpha\right) F^{-1}\left(1 - q_E - q_I\right),$$

and hence

$$\frac{\partial^2 p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_E^2} = \frac{\partial^2 p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_I \partial q_E}.$$

The sufficient condition (37) can then be written as

$$2\frac{\partial p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_E} + q_E \frac{\partial^2 p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_E^2}\left(1 + q_I'\left(q_E\right)\right) < 0. \tag{38}$$

By the assumption of concavity of the profit function of the duopolist:

$$2\frac{\partial p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_E} + q_E \frac{\partial^2 p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_E^2} < 0.$$

Now if

$$\frac{\partial^2 p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_E^2} > 0,$$

then (38) holds since $q_I'\left(q_E\right) < 0$ by the stability of the best response. On the other hand, if

$$\frac{\partial^2 p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_E^2} < 0,$$

then (38) holds since

$$\frac{\partial p_E\left(q_E, q_I\left(q_E\right)\right)}{\partial q_E} < 0,$$

and $1 + q_I'(q_E) > 0$.

Next suppose that $\mu(\alpha) > s$. Then the price of the entrant is given by:

$$p_E = \mu(\alpha) F^{-1}(1 - q_E) + s\left[F^{-1}(1 - q_E - q_I) - F^{-1}(1 - q_E)\right]$$

Let $H(\cdot)$ be the inverse function of F, or $H = F^{-1}$, and let h be the first derivative of H. The condition (37) can be written as:

$$\begin{aligned} -2\left[h(1 - q_E)(\mu(\alpha) - s) + h(1 - q_E - q_I)s\right] + \\ q_E\left[h'(1 - q_E)(\mu(\alpha) - s) + \left(1 + q_I'(q_E)\right)h'(1 - q_E - q_I)s\right] < 0. \end{aligned} \tag{39}$$

By concavity of the profit function of the monopolist, we know that

$$-2h(1 - q_E - q_I)(\mu(\alpha) - s) + q_E h'(1 - q_E - q_I)(\mu(\alpha) - s) < 0 \tag{40}$$

and also

$$-2h(1 - q_E)(\mu(\alpha) - s) + q_E h'(1 - q_E)(\mu(\alpha) - s) < 0. \tag{41}$$

But since $0 < 1 + q_I'(q_E) < 1$,

$$\left(-2h(1 - q_E - q_I)(\mu(\alpha) - s) + q_E h'(1 - q_E - q_I)(\mu(\alpha) - s)\right)\left(1 + q_I'(q_E)\right) < 0,$$

and therefore

$$-2h(1 - q_E - q_I)s + \left(1 + q_I'(q_E)\right)q_E h'(1 - q_E - q_I)s < 0. \tag{42}$$

Finally adding (41) and (42) yields (39).■

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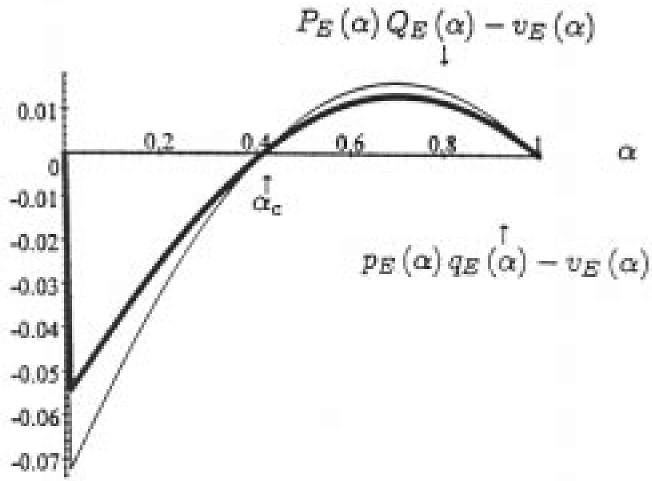

$p_E(\alpha)\,q_E(\alpha) - v_E(\alpha)$ and $P_E(\alpha)\,Q_E(\alpha) - v_E(\alpha)$.

Figure 1:

Equilibrium revenue minus long-run average for entrant:

for $\mu_L = \frac{99}{100}, s = 1, \mu_H = 2$.

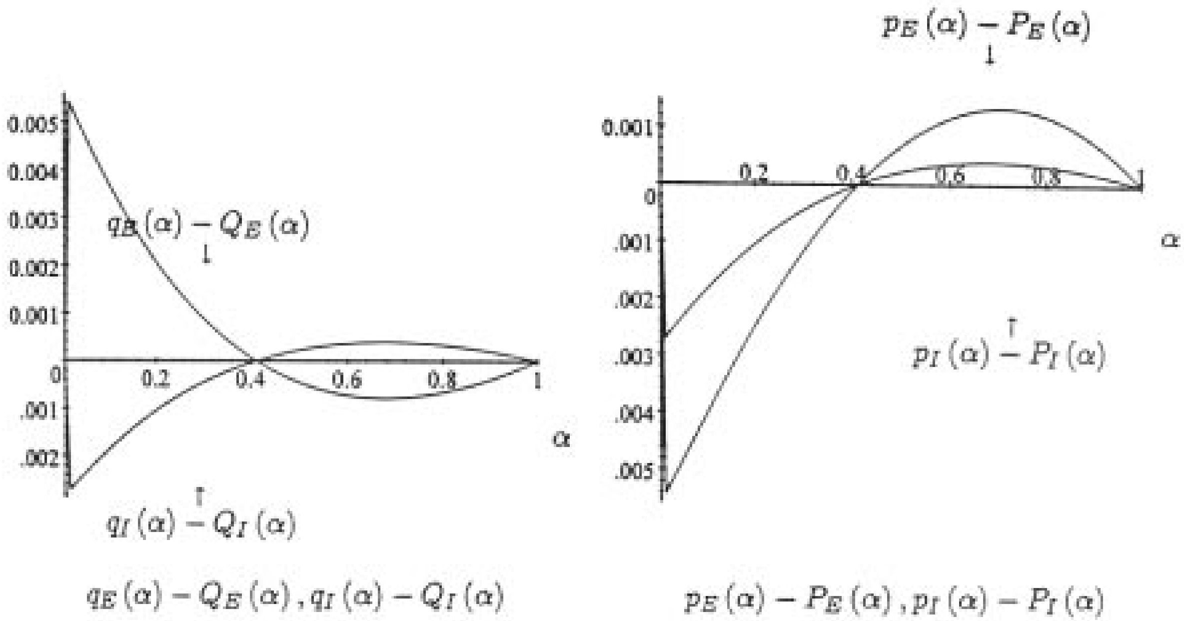

FIGURE 2:

Static and dynamic equilibrium policies

for $\mu_L = \frac{99}{100}$, $s = 1$, and $\mu_H = 2$.

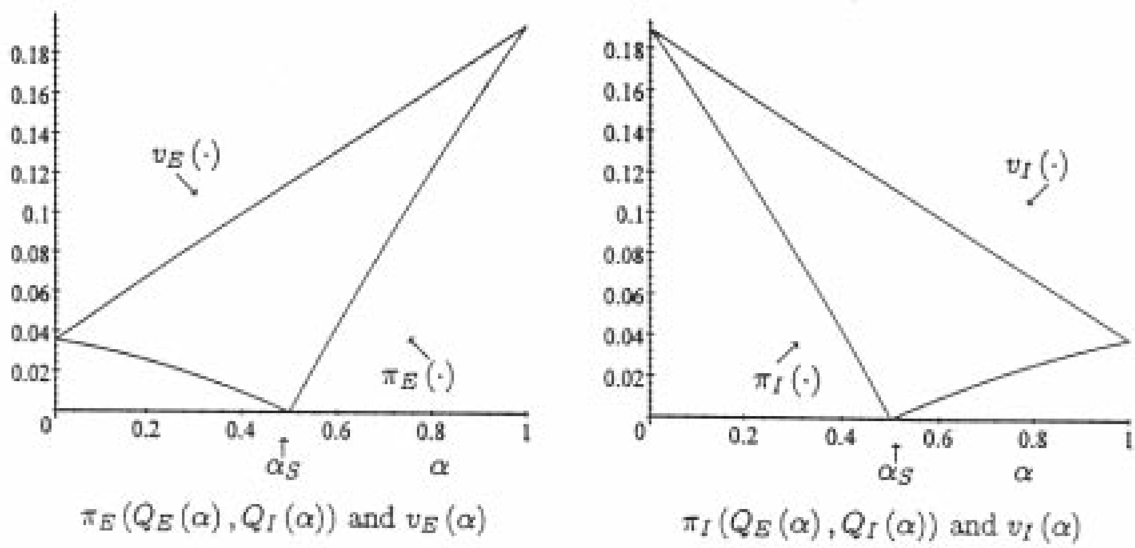

FIGURE 3:

Static equilibrium profits and long-run average (I),

with $\mu_L = \frac{3}{2}$, $s = 2$, and $\mu_H = \frac{5}{2}$.

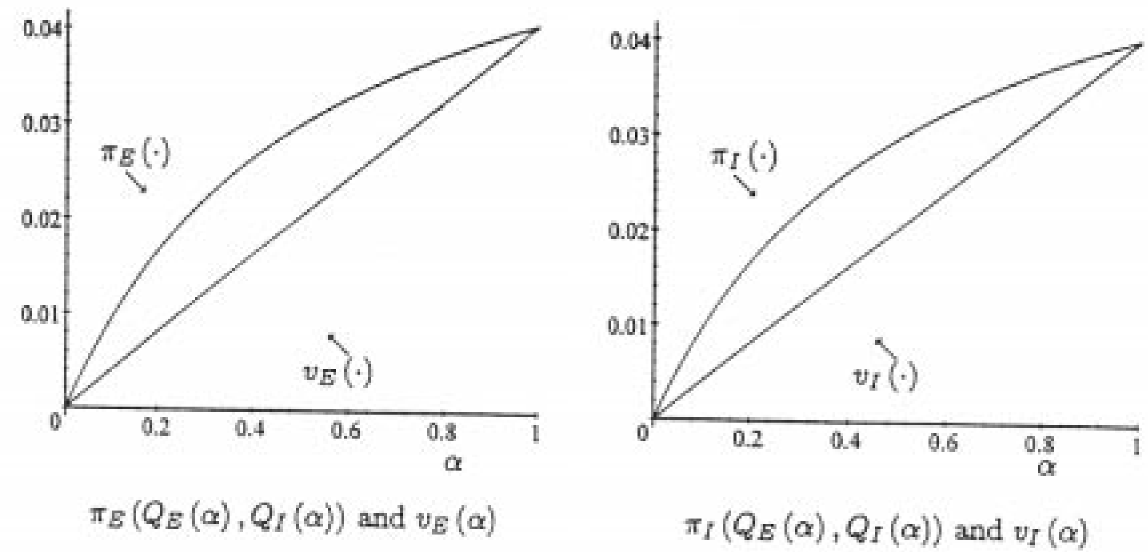

FIGURE 4:

Static equilibrium profits and long-run average (II),

with $s = \mu_L = 1$ and $\mu_H = 2$.