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A one-dimensional model of meridional oceanic heat transport

by John A. T. Bye

ABSTRACT

A one-dimensional model for the global meridional oceanic heat transport flux distribution is established which indicates the dominant role of the Ekman heat transports between 35N and 30S, in which the depth averaged ocean temperature is approximately constant, and of the Ekman and diffusive transports at higher latitudes where the depth-average temperature decreases. The basic features of the distribution, for a coefficient of lateral diffusion of heat of 500 m²/s, are tropical poleward flux maxima of 2.6 EW in the northern and 3.9 EW in the southern hemisphere, and temperate equatorward flux maxima of 0.2 EW in the northern and 0.4 EW in the southern hemisphere. The heat fluxes due to the gyral circulations in the ocean basins in general add a smaller scale structure to the basic pattern.

A balance between the Ekman heat flux and the surface exchange flux, modified by diffusive processes leads to the production of oceanic fronts of typical width 150 km. These fronts occur in regions of significant small scale zonal wind structure, and at the latitudes of the coalescence of the ocean basins in the southern hemisphere.

1. Introduction

One of the major problems in air-sea interaction is the partition of the heat flux between the atmosphere and the ocean. There are basically two methods of estimating the oceanic meridional flux from meteorological data. First, one can estimate the components of meridional atmospheric heat flux and the divergence of the exchange heat flux with space at the top of the atmosphere and hence obtain the oceanic heat flux as a difference (Von der Haar and Oort, 1973; Oort and Von der Haar, 1976). Second, one can estimate the (net) surface heat fluxes at the ocean-atmosphere interface, and calculate the meridional oceanic heat flux by integration on the assumption of negligible heat flux crossing the coasts (Bryan, 1962; Sellers, 1965).

The resulting estimates of the meridional flux are subject to large errors owing to the lack of coverage of high quality data on surface fluxes at the air-sea interface, and the variability in estimates of the atmospheric fluxes. The errors arising in the latter case are fully discussed in Oort and Von der Haar (1976). It is therefore desirable to obtain independent results on oceanic heat flux from hydrological data.

1. The Flinders University of South Australia, Bedford Park, South Australia 5042.
Table 1. The maximum poleward meridional heat flux in the northern hemisphere obtained in various studies.

<table>
<thead>
<tr>
<th>Method of deriving estimate</th>
<th>Reference</th>
<th>Position (°N)</th>
<th>Magnitude (EW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface heat flux data</td>
<td>Sellers (1965)</td>
<td>20</td>
<td>1.6</td>
</tr>
<tr>
<td>Atmospheric fluxes</td>
<td>Von der Haar &amp; Oort (1973)</td>
<td>20</td>
<td>3.4 ± 1.2</td>
</tr>
<tr>
<td>Ocean model</td>
<td>Oort and Von der Haar (1976)</td>
<td>25</td>
<td>0.9</td>
</tr>
<tr>
<td>Atmospheric fluxes</td>
<td>Bryan et al. (1975)</td>
<td>20</td>
<td>2.9 ± 2</td>
</tr>
</tbody>
</table>

1. 1 EW = 10^{16}W

This program however has only just become feasible as high quality oceanographic data are required at frequent station intervals, and hence these results are not yet plentiful (Bennett, 1978).

Table 1 summarizes the estimates for the subtropical poleward flux maximum for the northern hemisphere obtained in these studies. The general form of the flux distribution is an increase in poleward flux from a small magnitude (of either sign) at the equator to the subtropical maximum, and then a decrease toward the pole, with a small equatorward flux to the north of 60°N. The resolution of the estimates is only 10°. For the southern hemisphere an estimate from hydrographic sections lies between 1.1 and 2.2 EW toward the equator at 30°S (Bennett, 1978). This result is of the opposite sign to that of 1.0 EW toward the pole obtained at this latitude from meteorological data by Sellers (1965).

Progress can also be made by setting up numerical models of ocean circulation from which the meridional heat fluxes can be extracted. However, these models are usually large, and it is often not possible to make a thorough investigation of the sensitivity of the estimates of the heat fluxes, the first results for which were obtained by Bryan et al. (1975), and are less than the estimates derived directly from data (Table 1). It is clear that there are considerable problems associated with these estimates in regard to both accuracy and resolution.

In order to try and circumvent these problems, a simple one-dimensional model has been established in which it is hoped that the major factors controlling the heat flux can be easily discerned and simple tests performed to determine the sensitivity of the results on these factors. The basic physical factors incorporated in the model were accordingly a representation of the Ekman transport, oceanic gyral circulation and the lateral diffusion of heat, and a simple exchange law for the surface heat flux. The geometry of the world ocean was also included in the model together with a zonal distribution of effective air temperature and surface wind stress. The only physical process omitted was mass exchange at the surface which would be crucial.
in predicting salinity distributions, but may play only a secondary role in oceanic heat transfer.

2. The momentum equations

The equations of motion for steady flow which is frictionless below the surface Ekman Layer, and is driven by a zonal wind, are

\[-\rho f v = \tau_s A(z) - \frac{\partial p}{\partial x}\]  
\[\rho f u = - \frac{\partial p}{\partial y}\]  
\[0 = - \frac{\partial p}{\partial z} - \rho g\]

in which \(u\) and \(v\) are respectively the velocity components along \(\partial x\) (toward the East) and \(\partial y\) (toward the North), \(\frac{\partial}{\partial x} = \frac{1}{a \cos \theta} \frac{\partial}{\partial \theta}\) and \(\frac{\partial}{\partial y} = \frac{1}{a} \frac{\partial}{\partial \theta}\) in which \(a\) is the radius of the earth, \(\theta\) is latitude, \(z\) is vertically upward, \(g\) is gravity, \(\rho\) is density, \(p\) is pressure, \(f = 2\Omega \sin \theta\) is the Coriolis acceleration in which \(\Omega\) is the angular velocity of rotation of the earth, \(\tau_s\) is the zonal wind stress, and \(A(z)\) is zero except in the surface Ekman layer and has the property that

\[\int_{-H}^{\eta} A(z) \, dz = 1\]  

where \(\eta\) is the surface elevation, and \(H\) is the undisturbed depth of the ocean.

On substituting the form, 

\[\rho = \rho_o (1 - \delta \rho_o)\]

in which \(\rho_o\) is a standard density, and \(\delta\) is the anomaly of density, in (3) and integrating we obtain,

\[p = p_B - \rho_o g (z + H) + \Delta D\]

in which \(p_B\) is the bottom pressure, and

\[\Delta D = \rho_o^2 g \int_{-H}^{z} \delta dz\]

is the dynamic height relative to the bottom. Equation (5) enables the pressure gradients in (1) and (2) to be determined; thus assuming a constant depth ocean in which \(H \sim H + \eta = H_o\), we obtain

\[-f \rho v = \tau_s A(z) - \left( \frac{\partial p_B}{\partial x} + \frac{\partial \Delta D}{\partial x} \right)\]  
\[f \rho u = - \left( \frac{\partial p_B}{\partial y} + \frac{\partial \Delta D}{\partial y} \right)\]
and using (4), and integrating between the surface and the bottom, we obtain,

$$-fV = r_s - H_0 \left( \frac{\partial p_B}{\partial x} + \frac{\partial \Delta D}{\partial x} \right)$$

(8)

$$jU = -H_0 \left( \frac{\partial p_B}{\partial y} + \frac{\partial \Delta D}{\partial y} \right)$$

(9)

in which

$$U = \int_{-H_0}^{0} \rho u dz$$

and

$$V = \int_{-H_0}^{0} \rho v dz$$,

and

$$\Delta D = \frac{1}{H_0} \int_{-H_0}^{0} \Delta D dz$$

For an incompressible ocean on a spherical earth, the continuity equation is

$$\frac{\partial u}{\partial x} + \frac{1}{\cos \theta} \frac{\partial}{\partial y} v \cos \theta + \frac{\partial w}{\partial z} = 0$$

(10)

in which \( w \) is the velocity along \( o_2 \), and the steady-state transport continuity equation is

$$\frac{\partial U}{\partial x} + \frac{1}{\cos \theta} \frac{\partial}{\partial y} V \cos \theta = 0$$

(11)

Thus (8) and (9) yield,

$$df = -\frac{1}{\cos \theta} \frac{d}{dy} \tau_s \cos \theta$$

(12)

On substituting (12) in (8) we have an expression for the depth-integrated zonal pressure gradient, thus

$$\frac{\partial}{\partial x} (\Delta D + p_B) = \frac{1}{H_0} \left\{ \tau_s - \frac{\sin \theta}{\cos^2 \theta} \frac{d}{d\theta} \tau_s \cos \theta \right\}$$

(13)

Suppose now that the equation of state for seawater has the quadratic form,

$$\delta = \alpha_1 T + \alpha_2 T^2$$

where \( \alpha_1 \) and \( \alpha_2 \) are constants, and \( T \) is in °C; then if at any point on the ocean surface the temperature profile is of the form,

$$T = T_0(x,y) F(y,z)$$

(14)

in which \( F(y,0) = 1 \) such that \( T_0 \) is the surface temperature, one obtains,

$$\frac{\partial}{\partial x} \Delta D = \rho_0^2 g \frac{\partial T_0}{\partial x} f_1(x,y,z)$$

(15)

and

$$\frac{\partial}{\partial x} \Delta D = \rho_0^2 g \frac{\partial T_0}{\partial x} f_2(x,y,H_0)/H_0$$

(16)

where

$$f_1(x,y,z) = \int_{-H_0}^{z} (\alpha_1 F + 2 \alpha_2 T_0 F^2) dz$$
and

\[ f_2(x,y,H_0) = \int_{-u}^{0} f_1(x,y,z) \, dz \]

To a high approximation, the small variation with \( x \) in \( f_1 \) and \( f_2 \) arising from the nonlinearity of the equation of state may be neglected, and \( T_0 \) estimated in these functions by the effective air-temperature, which is assumed to be the zonal function, \( T_{a*}(y) \). Thus,

\[ f_1(x,y,z) \rightarrow F_1(y,z) \]

and

\[ f_2(x,y,H_0) \rightarrow F_2(y,H_0) = \int_{-\eta_0}^{0} F_1(y,z) \, dz \]

where,

\[ F_1(y,z) = \int_{-\eta_0}^{z} (\alpha_1 F + 2\alpha_2 T_{a*} F^2) \, dz \]

On substituting (16) into (13) we now obtain,

\[
\frac{\rho_0^2 g F_2(y,H_0)}{H_0} \frac{\partial T_0}{\partial x} + \frac{\partial p_B}{\partial x} = \frac{1}{H_0} \left\{ \tau_s \sin \theta \cos^2 \theta \frac{d}{d\theta} \tau_s \cos \theta \right\} \tag{17}
\]

Suppose also that,

\[
\phi(x,y) = \frac{\rho_0^2 g F_2(y,H_0)}{H_0} \frac{\partial T_0}{\partial x} \left( \frac{\partial p_B}{\partial x} + \frac{\rho_0^2 g F_2(y,H_0)}{H_0} \frac{\partial T_0}{\partial x} \right)
\]

where \( \phi(x,y) \) is a meridional function such that,

\[
\phi(x,y) = 1 \quad \text{for a baroclinic ocean}
\]

\[
\phi(x,y) \rightarrow 0 \quad \text{for a barotropic ocean}
\]

then eliminating \( \frac{\partial p_B}{\partial x} \) in favor of \( \phi(x,y) \), (17) becomes,

\[
\frac{\partial T_0}{\partial x} = \phi(x,y) \left\{ \frac{\partial T_0}{\partial x} \right\}_0
\]

where

\[
\frac{\partial T_0}{\partial x} \left\{ \frac{\partial T_0}{\partial x} \right\}_0 = \frac{1}{\rho_0^2 g F_2(y,H_0)} \left\{ \tau_s \sin \theta \cos^2 \theta \frac{d}{d\theta} \tau_s \cos \theta \right\} \tag{18}
\]

Finally substituting (18) in (6) we obtain,

\[
v = g_1(\theta) A(z) + \phi g_2(\theta) F_1(y,z) + (1-\phi) g_3(\theta) \tag{19}
\]

where

\[
g_1(\theta) = -\tau_s/\rho f,
\]

\[
g_2(\theta) = \frac{1}{\rho f F_2} \left\{ \tau_s \sin \theta \cos^2 \theta \frac{d}{d\theta} \tau_s \cos \theta \right\}
\]
and
\[ g_3(\theta) = \frac{F_z}{H_0} g_2(\theta) \]

Equation (19) enables the meridional velocity \( v \) to be determined from the zonal wind field, and the density field provided that \( \phi(x,y) \) is known.

In each basin, there is no net meridional transport hence a return current \( v_B \) is present where,
\[ v_B = -\frac{L}{\epsilon} \left( \frac{g_1(\theta)}{H_0} + \frac{1}{L} \int_{L_1}^{L_2} \phi \, dx \right) g_3(\theta) \frac{F_1(y,z)}{g_3(\theta)} + \left( 1 - \frac{1}{L} \int_{L_1}^{L_2} \phi \, dx \right) g_3(\theta) \]

in which \( L(y) = L_2(y) - L_1(y) \) is the width of the ocean basin which has coastal boundaries at \( L_1(y) \) and \( L_2(y) \), and \( \epsilon \) is the width of the boundary current (\( \epsilon \ll L(y) \)).

3. The meridional heat transport

The conservation of heat equation in the steady-state is
\[ \frac{\partial}{\partial x} (C_v \rho u T + Q_1) + \frac{1}{\cos \theta} \frac{\partial}{\partial y} (C_v \rho v T + Q_2) \cos \theta + \frac{\partial}{\partial z} (C_v \rho w T + Q_3) = 0 \]

where \( C_v \) is the specific heat of seawater at constant volume, \( T \) is temperature and \( Q_1, Q_2 \) and \( Q_3 \) are the components of turbulent and diffusive heat flux in the \( x, y \) and \( z \) directions. On integrating (21) between the surface and the bottom we obtain,
\[ \int_{-H_0}^{0} \left( \frac{\partial}{\partial x} (C_v \rho u T + Q_1) + \frac{1}{\cos \theta} \frac{\partial}{\partial y} ((C_v \rho v T + Q_2) \cos \theta) \right) dz + Q_3) = 0 \]

where \( Q_3(\eta) \) is the surface heat flux, and on integrating along \( x \) between coastal boundaries at \( L_1(y) \) and \( L_2(y) \) through which the heat flux is zero, we obtain
\[ \frac{\partial}{\partial y} \int_{-H_0}^{0} \int_{L_1}^{L_2} (C_v \rho v T + Q_2) \, dx dz + \int_{L_1}^{L_2} Q_3(\eta) \, dx = 0 \]

Now on setting \( Q_2 = -\rho \kappa \frac{\partial T}{\partial y} \) in which \( \kappa \) is a coefficient of lateral diffusion of heat, and making the Boussinesq approximation with \( C_v \) constant, we obtain,
\[ \frac{\partial}{\partial y} \int_{-H_0}^{0} \int_{L_1}^{L_2} \left( v T - N \frac{\partial T}{\partial y} \right) \, dx dz + \frac{1}{\rho_0 C_v} \int_{L_1}^{L_2} Q_3(\eta) \, dx = 0 \]

where \( N = \kappa / C_v \). Next, on integrating the first term with respect to \( x \) in (22) using (14) we have,
\[ \int_{-H_0}^{0} \int_{L_1}^{L_2} v T \, dx \, dz = \int_{-H_0}^{0} \int_{L_1}^{L_2} v T \rho_b T_d F \, dx \, dz + \int_{-H_0}^{0} \int_{L_1}^{L_2} v T_0 F \, dx \, dz \]
On separating the baroclinicity $\phi(x,y)$ into a mean zonal function $\phi(y)$, and a longitudinal factor $B(x,y)$ such that,

$$\phi(x,y) = B(x,y) \phi(y) \quad (23)$$

where

$$\frac{1}{L} \int_{L_1}^{L_2} B(x,y) \, dx = 1$$

and assuming that the boundary current occurs near the western boundary such that,

$$T_0 = T_B + \int_{L_1}^{L_2} \frac{\partial T_0}{\partial x} \, dx$$

where $T_B$ is the surface temperature in the boundary current, and using (19) and (20), the heat flux integral reduces to,

$$\int_{-H_0}^{0} \int_{L_1}^{L_2} vT \, dx \, dz = \int_{-H_0}^{0} T_M L_1 g_1 \left( A(z) - \frac{1}{H_0} \right) \, dz$$

$$+ \left( \left( \frac{g_1}{H_0} + g_3 \right) B_1 + \frac{1}{2}(g_2 F_1 - g_3) \phi \right) L_2 \frac{\partial T_0}{\partial x} \int_{-H_0}^{0} F \, dz$$

where the overbar denotes a zonal average across the basin; $T_M = T_0$ is the mean zonal surface temperature,

$$\frac{\partial T_0}{\partial x} = \frac{T_E - T_B}{L} = \phi(y) \frac{\partial T_0}{\partial x} \bigg|_0$$

is the mean zonal temperature gradient, in which $T_B$ is the surface temperature on the eastern boundary, and $B_1(y) = \frac{T_M - T_B}{T_E - T_B}$. On integrating the heat flux integral over depth we obtain,

$$\int_{-H_0}^{0} \int_{L_1}^{L_2} vT \, dx \, dz = T_M L_1 g_1 \left( A_1 \frac{1}{H_0} \right) \, dz$$

$$+ L_2 \frac{\partial T_0}{\partial x} \left( \left( \frac{g_1}{H_0} + g_3 \right) F_0 B_1 + \frac{1}{2}(g_2 F_1 - g_3) \phi \right)$$

where

$$F_0 = \int_{-H_0}^{0} F(y,z) \, dz, \quad F_3 = \int_{-H_0}^{0} F_1 F(y,z) \, dz, \quad \text{and}$$

$$A_1 = \int_{-H_0}^{0} A(z) F \, dz.$$

Similarly, we have approximately that,

$$\int_{-H_0}^{0} \int_{L_1}^{L_2} N \frac{\partial T}{\partial y} \, dx \, dz = LN \frac{\partial}{\partial y} T_M F_0$$

The heat exchange term is approximated by the relation (Haney, 1971),
where $T_{v*}$ is the effective air temperature, and hence

$$\frac{1}{\rho_0 C_v} \int_{L_1}^{L_2} Q_3 \eta \, dx = L \mu \left( T_M - T_{v*} \right)$$

where $\mu = \lambda / \rho_0 C_v$, and will be taken to be constant. Thus the distribution of the mean zonal temperature satisfies the equation,

$$- N \frac{\partial}{\partial y} \left( L \left( F_0 \frac{\partial T_M}{\partial y} + \frac{\partial F_0}{\partial y} T_M \right) \right) + \frac{\partial}{\partial y} \left( \left( \frac{g_1}{H_0} + g_3 \right) F_0 B_1 + \frac{1}{2} (g_2 F_1 - g_3 F_0) \phi \right) + L \mu \left( T_M - T_{v*} \right) = 0$$

(27)

The first three terms on the left-hand side of the equation arise respectively from lateral diffusion, surface Ekman transport, and gyral circulations. Equation (27) has been solved for the particular density profile,

$$F = e^{z/h}$$

in which $h$ is the thermocline scale depth, and for which the integral functions are,

$$F_0(y,H_0) = - \frac{1}{\Lambda} (1 - e^{\Lambda H_0})$$

$$F_1(y,z) = - \frac{1}{\Lambda} \left( \alpha_1 (e^{-\Lambda z} - e^{\Lambda H_0}) + \alpha_2 T_{v*} (e^{-2\Lambda z} - e^{2\Lambda H_0}) \right)$$

$$F_2(y,H_0) = \frac{1}{\Lambda^2} \left( \alpha_1 (1 - e^{\Lambda H_0} (1 - \Lambda H_0)) + \frac{1}{2} \alpha_2 T_{v*} (1 - e^{2\Lambda H_0} (1 - 2\Lambda H_0)) \right)$$

$$F_3(y,H_0) = \frac{1}{\Lambda^2} \left( \frac{\alpha_1}{2} (1 - 2e^{\Lambda H_0} + e^{2\Lambda H_0}) + \frac{\alpha_2}{3} T_{v*} (1 - 3e^{2\Lambda H_0} + 2e^{3\Lambda H_0}) \right)$$

where $\Lambda = - 1/h$.

The Ekman function, $A(z)$, has also been given the exponential form,

$$A(z) = A_0 e^{z/h_E}$$

where $h_E$ is the Ekman layer scale depth, and $A_0 = 1 / (h_E (1 - e^{-H_0/h_E}))$ which satisfies the integral condition (4), and for which

$$A_1(y,H_0) = \left( \frac{\chi}{\chi + \Lambda} \right) \left( \frac{1 - e^{(\chi + \Lambda)H_0}}{1 - e^{\chi H_0}} \right)$$

(28)

where $\chi = - 1/h_E$. 

\[Q_3 \eta = \lambda (T_0 - T_{v*}) \]
4. Application to the world ocean

The world ocean can be divided into a series of ocean basins and a zonal channel, and the above theory applied without difficulty to each basin, except for those which include the equator. In these basins some representation of internal friction is essential, and in the vicinity of the equator we write,

\[ \nu = -\frac{1}{\rho} \frac{f}{(R^2 + f^2)} \left( \tau_s A(z) - \frac{\partial p}{\partial x} \right) \]

This result is exact for the system,

\[ -\rho f \nu = \tau_s A(z) - \frac{\partial p}{\partial x} - \rho Ru \]
\[ \rho f u = -\rho R \nu \]

in which \( \frac{\partial p}{\partial y} = 0 \), and \( R \) is a Goldberg-Mohn friction coefficient. The magnitude of \( R \) is chosen to give a prescribed equatorial temperature \( (T_M)_{E0} \), and \( \frac{\partial p}{\partial x} \) is estimated as in (1). In the circumpolar zonal channel (56S-63S) a purely geostrophic meridional current is also not possible since the surface wind stress cannot be balanced by the pressure gradient, \( \frac{\partial p}{\partial x} \), which is zero. Thus the surface Ekman transport is balanced by a deep meridional counter-current. We assume that this current is barotropic, hence in the circumpolar zonal channel,

\[ \nu = g_1 \left( A(z) - \frac{1}{H_0} \right) \]

and the term in \( \frac{\partial T_0}{\partial x} \) in (27) is omitted. The world ocean at any latitude in general consists of a series of basins. In the solution, these individual basins have been combined into a multiple basin with a representative central meridional temperature distribution, \( T_M \), by replacing \( L \) in (27) by \( \Sigma L_i \) and \( L^2 \) by \( \Sigma L_i^2 \), where \( L_i \) is the width of the \( i \)th individual basin.

The zonal variation of surface temperature in each basin is then given from (18) and (23) by the relation

\[ \frac{\partial T_0}{\partial x} = B \phi \left( \frac{\partial T_0}{\partial x} \right)_0 \]

from which we obtain, the coastal boundary temperatures,

\[ T_B = T_M - B_1 L \left( \frac{\partial T_0}{\partial x} \right)_0 \phi \]

and

\[ T_E = T_M + (1-B_1) L \left( \frac{\partial T_0}{\partial x} \right)_0 \phi \] (29)
Equations (29) have been used to obtain bounds on $\bar{\phi}(y)$; since necessarily

$$T_B \geq T_{FR} \text{ and } T_E \geq T_{FR}$$

where $T_{FR}$ is the freezing point of seawater. Since $0 < B_1 < 1$ and $\bar{\phi} > 0$, we have,

$$\bar{\phi}_u(y) = \frac{T_M - T_{FR}}{B_1 L_{Max} \frac{\partial T}{\partial x}} \frac{\partial T}{\partial x} \Bigg|_0 > 0$$

$$= \frac{T_M - T_{FR}}{(B_1-1) L_{Max} \frac{\partial T}{\partial x}} \frac{\partial T}{\partial x} \Bigg|_0 < 0$$

where $L_{Max}(y)$ is the width of the widest basin, and $\bar{\phi}_u(y)$ is an upper bound for $\bar{\phi}(y)$. From (20) and (23), we obtain the relation,

$$\bar{\phi}_L(y) = \frac{F_2(y,H_0)}{(F_2(y,H_0) - H_0 F_1(y,Z_L))}$$

where $\bar{\phi}_L(y)$ is the mean baroclinicity in the case of a level of no motion, $Z_L$, in the boundary layer (or in the zonal channel). $\bar{\phi}(y)$ is now determined by the relation,

$$\bar{\phi}(y) = \text{Min} \{\bar{\phi}_L(y), \bar{\phi}_u(y)\}$$

with $Z_L$ a prescribed function of $y$.

The coefficients in the equation of state were given the values $\alpha_1 = 0.0735/\rho_0^2$ kg/m$^3$K and $\alpha_2 = 0.00469/\rho_0^2$ kg/m$^3$K$^2$ where $\rho_0 = 1028$ kg/m$^3$, appropriate to seawater of salinity 35%o at atmospheric pressure (Mamaev, 1964). The thermocline scale depth was of the form,

$$h = C_1 + C_2 \theta^2,$$

where $\theta$ is latitude in degrees, and $C_1$ and $C_2$ are constants. This allows for a symmetrical variation with latitude, with a minimum depth at the equator.

The boundary conditions on the model are no meridional heat flux through the boundaries of the multiple basin.

5. The nature of the model solutions

Before considering the numerical solution of Equation (27) for the world ocean, it is useful to discuss the nature of the solution of a simplified version of the equation. Consider the following case of an infinitely deep uniformly baroclinic ocean ($H \to \infty$, $\phi = 1$, $h = \text{const}$) of constant total width consisting of $i$ basins of width $L_i$, (i.e. $L = \Sigma L_i = \text{const}$) with an infinitely thin Ekman layer ($h_E \to 0$) on the beta-plane with a linear equation of state ($\alpha_\delta = 0$). Then it is easily shown that the heat flux divergence equation reduces to the form,

$$-N \frac{d^2 T_M}{dy^2} - \frac{1}{f h} \frac{d}{dy} \left( \frac{\tau_s}{\rho_0} T_M \right) + \frac{1}{4} \frac{f L}{\beta^2 h^3 g L_\alpha \rho_0} \frac{d}{dy} \left( \frac{d \tau_s / \rho_0}{dy} \right)^2 \Sigma L_i^2$$
Bye: Meridional oceanic heat transport model

\[ + \frac{\mu}{h} (T_M - T_o^*) = 0 \]

where \( \beta = \frac{df}{dy} \)

On introducing the nondimensional variables, \( Y = y/L \) and \( W = \tau_y/\tau_0 \) in which \( \tau_0 \) is a reference wind stress, this equation can be expressed as follows,

\[
\frac{d^2T_M}{dY^2} + A W \frac{dT_M}{dY} + \left( C + A \frac{dW}{dY} \right) T_M = CT_0^* - E \frac{d}{dY} \left( \frac{dW}{dY} \right)^2 \left( \frac{\Sigma L_i^2}{L^2} \right)
\]

where

\[ A = \frac{\tau_0}{\rho_0} \]
\[ E = -\frac{1}{4} \frac{f}{h^3g\alpha_1\rho_0} \left( \frac{\tau_0}{\rho_0\beta} \right)^2 \]

and

\[ C = -\frac{\mu L^2}{hN} \]

The solution of the homogeneous equation

\[
\frac{d^2T_M}{dY^2} + A \frac{dT_M}{dY} + C T_M = 0
\]

in which it has been assumed that \( W = 1 \), has the roots,

\[ e^{\xi_1Y}, e^{\xi_2Y} \]

where \( \xi_1 = -\frac{A}{2} + \sqrt{A^2/4 - C} \), and \( \xi_2 = -\frac{A}{2} - \sqrt{A^2/4 - C} \). \( \xi_1, \xi_2 \) are both real since \( C < 0 \). The positive root (\( \xi_1 \)) corresponds to a northerly zonal boundary layer and the negative root (\( \xi_2 \)) corresponds to a southerly boundary layer. The form of the constant, \( A \), indicates that for a prescribed value of \( C \), the thickness of the boundary layer (\( |L/\xi_1| \) or \( |L/\xi_2| \)) is less for an Ekman flux toward, than away from the boundary. On taking the following typical values for the parameters in \( A \) and \( C \); \( \mu = 10^{-5} \text{ m s}^{-1}, f = 10^{-4} \text{s}^{-1}, \tau_0/\rho_0 = 10^{-4} \text{ m}^2 \text{s}^{-2} \) and \( Nh = 2.5 \times 10^5 \text{ m}^3 \text{s}^{-1} \) (say \( N = 500 \text{ m}^2 \text{s}^{-1} \) and \( h = 500 \text{ m} \)), we obtain \( A = 0.4 \times 10^{-5} \text{ L} \), and \( C = -0.4 \times 10^{-10} \text{L}^2 \), and hence \( |L/\xi_1|, |L/\xi_2| \) have the values 210 km and 120 km respectively. Thus the temperature boundary layers typically have a thickness of 150 km, and can be identified as oceanic frontal zones.

In addition to free boundary layers of this kind, the meridional structure of the zonal wind stress can induce a forced response in the ocean temperature.

This is evident from considering the approximate equation for the perturbation of sea surface temperature (sea surface temperature anomaly) produced by a perturbation of wind stress. On substituting \( T_M = T_M + T' \) and \( W + W_o + W' \), where
$T_M$ and $W_0$ are constants, and $T'$ ($y$) and $W'$ ($y$) are perturbations, in (30) and linearizing one obtains,

$$T_M = T_0$$

and

$$\frac{d^2T'}{dY^2} + AW_0 \frac{dT'}{dY} + CT' = - A \frac{dW'}{dY} T_M$$

(32)

The solution of (32) for a perturbation wind stress of the form, $W' = \Delta W \sin kY$, in which $k$ is a nondimensional wavenumber, is

$$T' = \Delta T \cos (kY + \gamma)$$

(33)

where

$$\Delta T = \frac{Ak \Delta W T_M}{((k^2-C)^2 + (AW_0k)^2)^{\frac{1}{2}}}$$

and

$$\gamma = \tan^{-1} \frac{AW_0k}{k^2-C}$$

The solution has the following properties.

(i) The relative amplitude of the temperature perturbation,

$$r = \frac{\Delta T}{\Delta W} = \frac{AkT_M}{((k^2-C)^2 + (AW_0k)^2)^{\frac{1}{2}}}$$

$r \to 0$ for $k \to 0$ and $k \to \infty$ and $|r|$ has a maximum of $T_M \left/ \left( -\frac{4C}{A^2} + W_0^2 \right)^{\frac{1}{2}} \right.$ for $k = \sqrt{-C}$. For the estimated magnitudes of $A$ and $C$, and $W_0 = 1$, $r_{MAX} = T_M/\sqrt{11}$. Thus a perturbation of wind stress of 0.02 N/m² with $T_M = 20^\circ$C yields a sea surface anomaly of amplitude 1.2°C. Anomalies of this magnitude often occur in long term sea surface temperature data.

(ii) The phase angle $\gamma \to 0$ for $k \to 0$ and $k \to \infty$, in which cases the temperature perturbation is $90^\circ$ out of phase with the wind stress perturbation. $|r|$ has a maximum value of $\tan^{-1} \left| \frac{1}{2} \frac{AW_0}{\sqrt{-C}} \right|$ for $k = \sqrt{-C}$. On substituting for $A,C$ and $W_0$, $\gamma_{MAX} = \tan^{-1} \frac{1}{2} \sim 18^\circ$.

(iii) The size $(k^{-1}L)$ of the wind stress perturbation which is optimal in producing the sea surface temperature anomaly is 150 km, which corresponds to a wavelength $\frac{2\pi L}{k}$ of 900 km.

(iv) At any wavelength as $Nh \to 0$, $r$ and $\gamma$ tend to limiting finite values, and for the conditions of maximum response we have the simple result that, $k_{MAX} \to \infty$, 
with,

\[ \tau_{\text{MAX}} \to \bar{T}_M / |W_0| \quad \text{and} \quad \tau_{\text{MAX}} \to \frac{\pi}{2} \]

In this case the sea temperature pattern is 90° out of phase with the zonal wind, and the intensity of the temperature fluctuations is equal to that of the fluctuations in the wind stress.

The physical nature of the solution is that the Ekman transports create convergences and divergences of oceanic heat flux which are balanced by exchange with the atmosphere, to the extent that they are not relaxed by lateral diffusion processes which do not play an essential physical role.

Consider now the situation at the latitude of the coalescence of two or more ocean basins i.e. at a jump in \( i \). Across this latitude \( \Sigma L_i^2 \) is discontinuous. Hence the term due to the gyral circulations in (27) gives rise to a delta function in the forcing of the heat divergence. This situation would give rise to an oceanic frontal zone of the form discussed in the solution of the homogeneous equation. The appropriate forcing term on the right-hand side of (30) due to the discontinuity is,

\[ E \left( \frac{dW}{dy} \right)^2 \nu \]

where \( \nu \Delta Y = \Delta L^2 / L^2 \) in which \( L \) is the total width of the basin at the front and \( \Delta L^2 \) is the change in \( \Sigma L_i^2 \) across the front, which occurs in a distance \( \Delta Y \). The maximum \( |\nu| \Delta Y = 1 \).

It is well known that the Sub-tropical Convergence occurs near such latitudes in the southern hemisphere, and the above reasoning is suggested as the fundamental cause. A more dramatic transition occurs across the northerly latitude of the circumpolar zonal channel in which the gyral circulation is absent (\( \Sigma L_i^2 = 0 \)), and this is indicated by the presence of the Polar Front. The processes which produce the zonal thermal boundary layers also produce thermal boundary layers adjacent to the western boundaries of the ocean basins. The east-west temperature structure is parameterized in the model through the function, \( B(x,y) \), which gives rise to \( B_i(y) \) in equation (27). The form of \( B(x,y) \) can be obtained from two-dimensional models of circulation in an ocean basin. A series of solutions from such a model have been obtained, and it is intended to report them in a separate paper.\(^2\) It was found that \( B(x,y) \) could be approximately represented by the exponential form,

\[ B(x,y) = \frac{1}{\delta} e^{-\frac{x}{\delta}} \quad \delta \to 0 \] (34)

in which

\[ X = \frac{x}{L}, \quad \text{and} \quad \delta = 1 - B_i \]

\( \text{2. These calculations were undertaken following a suggestion from Professor George Veronis, and a joint paper describing the results in detail is in preparation.} \)
The zonal wind stress ($\tau_s$) and the thermocline scale depth ($h$), and latitude.

The corresponding east-west temperature distribution is

$$T_0(x) = T_M + (T_B - T_E) (\delta - e^{-\frac{x}{\delta}})$$

(35)

At the central latitude of the gyre, the thermal boundary layer width, to a first approximation, was

$$\delta L = \frac{L}{\sqrt{-C}} = \sqrt{\frac{hN}{\mu}}$$

which is identical with the zonal boundary layer thickness in the absence of Ekman transport, which as discussed above typically has a value of 150 km. We take this estimate as being representative for each basin, and since it is generally much less than the basin width, the constant, $B_1$, which from (35) is,

$$B_1 = 1 - \delta$$

has everywhere a value just less than unity.

6. Results for the world ocean

The numerical solution of Equation (27) for the world ocean has been obtained by representing the differentials using finite-differences on a network with a spacing of $1^\circ$, with the wind stress and effective air-temperature distributions shown in Figures 1 and 4 respectively.

A series of solutions was obtained to determine the sensitivity of the derived heat fluxes and air-sea temperature differences on the seven solution parameters, $N$, $R$, $\chi$, $C_1$, $C_2$, $Z_L$ and $\lambda$.

It was found that the solutions were not critically dependent on a reasonable choice of the Ekman parameter $\chi$, the thermocline scale depth parameters $C_1$ and $C_2$, the level of no motion $Z_L$, or the surface exchange parameter, $\lambda$, and that a reasonable choice of the Goldberg-Mohn coefficient $R$ affected only conditions near the equator. The significant parameter was the coefficient of lateral diffusion of heat, $N$. 

Figure 1. The zonal wind stress ($\tau_s$) and the thermocline scale depth ($h$), and latitude.
Figure 2. The total meridional oceanic heat flux toward the north for \( N = 100, 500 \) and \( 2500 \) \( \text{m}^2/\text{s} \) and the other model parameters given in the text. \( 1 \text{ EW} = 10^{26} \text{ W}. \)

The discussion of the results will accordingly center around solutions in an ocean of depth 4000 m with the parameter values: with \( N = 100, 500 \) and \( 2500 \) \( \text{m}^2/\text{s} \) respectively, \( C_1 = 500 \text{m}, C_2 = 0.29, Z_L = -4h, R = 11 \times 10^{-6} \text{ s}^{-1}, h_E = 100 \text{m} \) and a surface exchange coefficient \( \lambda = 38 \text{ W/m}^2 \text{ K}. \) This model ocean has a thermocline scale depth of 500 m at the equator, which deepens to 850 m at 35° (Fig. 1), a level of no motion occurring at four times the thermocline scale depth, and a constant Ekman scale depth of 100 m. The equatorial sea surface temperatures \( (T_M)_{\text{Eq}} \) for \( N = 100, 500 \) and \( 2500 \) \( \text{m}^2/\text{s} \), were respectively 25.6° C, 25.6° C and 27.1° C.

The general form of the meridional heat flux at low values of the diffusion coefficient \( (N \lesssim 10^3 \text{ m}^2/\text{s}) \) is an antisymmetric damped oscillation centered on the equator (Fig. 2). Poleward flux maxima occur in the tropics, equatorward flux maxima occur at temperate latitudes, and small poleward flux maxima occur at high latitudes. For the \( N = 500 \) \( \text{m}^2/\text{s} \) case, which will be discussed in detail below, the magnitudes of the temperate flux maxima are 0.4 EW in the southern hemisphere and 0.2 EW in the northern hemisphere. A small northward heat flux of about 0.05 EW occurs across the equator.

The highly diffusive solution \( (N = 2500 \text{ m}^2/\text{s}) \) has a structure which is similar to the less diffusive cases at low latitudes, but in temperate latitudes the equatorward flux maxima are suppressed. The reason for this behavior can be seen in the component fluxes.

Figure 3 shows that between 25N and 30S the Ekman transport dominates, whereas all heat fluxes are important poleward of these latitudes. The diffusive flux
Figure 3. The components of the meridional heat flux toward the north for $N = 500 \text{ m}^3/\text{s}$.

is complementary with the gyral and Ekman fluxes in the sense that peaks in the latter generate collateral peaks in the diffusive flux, i.e. the temperature imbalances brought about by the structure in the Ekman and gyral circulations are relaxed by lateral diffusion. The diffusive flux is also proportional to the gradient of the mean temperature of the water column,

$$T = T_0 F_0 / H_0$$

Figure 4. The effective air-temperature ($T_*^a$) and the sea surface temperature ($T_m$) for $N = 100, 500$ and $2500 \text{ m}^3/\text{s}$.
and Figure 5 shows that $T$ is essentially constant at around $4^\circ$ C between 30N and 35S. Thus the effects of diffusion are not cumulative between the equator and the subtropics, and are only important on a global scale at high latitudes. In other words, the thermocline scale depth, which was chosen to correspond approximately with observations, is consistent with an ocean which maintains an approximately constant heat content between the equator and temperate latitudes (assuming a reasonable choice of the coefficient of lateral diffusion of heat).

The major peaks in the gyral heat transport in the northern hemisphere 1.1 EW at 27N can be identified with the subtropical gyre. In the southern hemisphere the gyral heat transport distribution is more complex owing to the wind stress structure (Fig. 1) and the coalescence of the ocean basins (Fig. 6). There are three well marked maxima (at 19S, 29S and 37S) which correspond with maxima of $\frac{\partial T_0}{\partial x}$ (Fig. 5), to the square of which the gyral heat transport is approximately porportional (cf. (18), (19) and (27)).
In addition the model indicates another maximum in the gyral heat transport at about 52S. This latitude is very close to the northern latitude of the circumpolar zonal channel, and it may be inappropriate to use the Sverdrup dynamics this far south (Gill, 1968). Thus a second solution was obtained with an artificially wide channel extending from 50S-63S. The heat flux and sea temperatures found in this solution only showed significant differences from the first solution south of 45S; indeed, in a solution in which the gyral circulations were omitted altogether a substantially similar flux and sea temperature distribution was obtained (Fig. 7). This solution demonstrates that the total heat flux pattern consists of a global signature mainly determined by the Ekman circulation, on which is superimposed a smaller scale structure determined by the gyral circulations, the pattern being smoothed by lateral heat diffusion. The air-sea temperature difference (Fig. 4) shows more structure than the flux distribution. The intensity of the extrema diminish as the coefficient of lateral diffusion is increased in such a manner that for $N = 2500 \text{ m}^2/\text{s}$, reversals of meridional temperature gradient only occur in low latitudes (20N-30S) where a series of three maxima and two minima, including the equatorial minimum, are predicted. The cause of this structure lies primarily with the zonal wind stress distribution (Fig. 1) as can be seen from Figure 7 in which the gyral circulations are omitted. The extrema occur approximately out of phase with the extrema in $\tau s/f$, and are of similar magnitude to that predicted by the perturbation equation (32).

On a global scale the air-sea temperature difference is negative poleward of 55° and between 15°-40°, positive near the equator and weakly positive in high temperate latitudes (40°-55°).

The magnitudes of the air-sea temperature differences are usually not more than 2°C except in the vicinity of the equator where it should be recalled that the effective air temperature is somewhat greater than the actual air temperature (Haney,
1971). At the poleward boundaries, the diffusion of heat in the form of a temperature tail is evident. Regions of high meridional temperature gradient occur outside the tropics in the southern hemisphere around 34S and 50S. The first is identified with the Southern Subtropical Convergence, and the second with the Polar Front, both of which are marked by discontinuities in $\sum L_i^2$ (Fig. 6).

The baroclinicity ($\phi$) of the circulation (Fig. 5) varies from $\phi = 1.08$ at the equator, where the level of no motion occurs within the water column at 2000 m and gives rise to supra-baroclinic conditions, to $\phi = 0$ in the polar regions where the freezing condition constrains conditions to be barotropic. Equations (23) and (35) enable the surface heat flux distribution over the ocean basins to be constructed. The form of equation (35) implies that the zonal distribution has a boundary layer form adjacent to the western coastlines, and almost uniform values in the interior of the basins. Figure 8 illustrates this behavior. The main features of the meridional distribution are the bands of heat flux from the atmosphere to the ocean (0°-15° and 40°-55°), and from the ocean to the atmosphere (15°-40°, with some small scale reversals, and 55°—polar boundaries). The largest exchanges ($\sim 300$ W/m²) occur.

Figure 8. The surface heat flux from the atmosphere to the ocean ($-Q_a$) for $N = 500$ m²/s. Contours are in W/m² × 10⁻². The hatched regions denote positive flux values, i.e. heat transfer from the atmosphere to the ocean.
near the equator and in the subtropics near the western boundary currents. An interesting sub-polar atmospheric heat sink is in the vicinity of the Falkland Island Current where exchanges of order 100 W/m² may occur, depending on the importance of the gyral circulation at this latitude.

Briefly, the effect of changing the other solution parameters was the following. An infinitely thin Ekman layer \( (h_E \rightarrow 0) \) increases somewhat the heat fluxes by Ekman transports, especially in the tropics, e.g. the poleward flux maximum is increased by about 20%. The deepening of the thermocline and/or level of no motion makes the gyral circulations more barotropic and vice-versa, thus changing the relative importance of the gyral heat flux. However the main features of the solution are unchanged.

The effect of changing the surface exchange coefficient, \( \lambda \), is to change the air-sea temperature differences approximately in inverse proportion. This behavior can be understood by reference to the solution of (32), in which for the large scale wind stress distribution, the dominant term in the denominator of the relative amplitude of the temperature perturbation \( (r) \) is \( C \), which is inversely proportional to \( \lambda \). The corollary of this result is that the surface heat fluxes are not very sensitive to changes in the surface exchange coefficient, and only show a small reduction as \( \lambda \) is decreased, and vice-versa. This is reflected in correspondingly small changes in the meridional heat fluxes. In an experiment in which the value of \( \lambda \) was doubled, it was found that the heat flux extrema were increased in magnitude by about 20%, with a corresponding decrease for the case in which \( \lambda \) was halved. In the experiments, the equatorial temperature \( (T_m)_{Eq} \) was retained at the same value by decreasing \( R \) to a value \( 8 \times 10^{-6}s^{-1} \) for \( \lambda = 76 \text{ W/m}_2\text{K} \), and increasing \( R \) to the value \( 17 \times 10^{-6}s^{-1} \) for \( \lambda = 19 \text{ W/m}_2\text{K} \).

7. Comparison with other studies

The model results for the maximum poleward meridional heat flux in the northern hemisphere of approximately 2.5 EW is in quite good agreement with the studies of Table 1, however the position of the maximum, which has two almost equal peaks at 5N and 13N (Fig. 2) is nearer the equator than in the other studies. The change in sign of the flux, which occurs at 33N is also further south than in the other studies, where it usually occurs near 55N. In the southern hemisphere an equatorward flux occurs between 39S and 47S (Fig. 2) to the south of the equatorial flux found by Bennett (1978) at 30S.

The distribution of surface heat flux (Fig. 7) shows more resolution than the distributions obtained directly from meteorological data; however, the magnitudes of the surface fluxes averaged over latitude deciles (Table 2) are of the same order of magnitude as the meteorological estimates. The results of Oort and Von der Haar (1976) show a heat flux from the atmosphere to the ocean from 0-20N, and from 60N to the pole, and a heat flux from the ocean to the atmosphere from 20-60N.
Table 2. The surface flux in latitude deciles.

<table>
<thead>
<tr>
<th>Latitude decile (°N)</th>
<th>Surface flux ((-Q_o)) (W/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Oort and Von der Haar (1976) ibid</td>
</tr>
<tr>
<td>0-10</td>
<td>52</td>
</tr>
<tr>
<td>10-20</td>
<td>58</td>
</tr>
<tr>
<td>20-30</td>
<td>-38</td>
</tr>
<tr>
<td>30-40</td>
<td>-9</td>
</tr>
<tr>
<td>40-50</td>
<td>-55</td>
</tr>
<tr>
<td>50-60</td>
<td>-107</td>
</tr>
<tr>
<td>60-70</td>
<td>15</td>
</tr>
<tr>
<td>70-80</td>
<td>6</td>
</tr>
<tr>
<td>80-90</td>
<td>23</td>
</tr>
</tbody>
</table>

The changes in sign of the surface flux in the model results occur at latitudes closer to the equator, with an additional band of heat flux from the ocean to the atmosphere at high latitudes. The near equator surface fluxes which individually attain values of over 200 W/m² are somewhat larger than the meteorological estimates. This appears to be due to the application of the effective air temperature \(T_{a*}\) in the surface flux relation (26), since at the equator \(T_{a*}\) has the value of 32°C which is four degrees Celsius larger than the air temperature \((T_a)\).

8. Conclusion

The model has demonstrated how a simple analytical representation of the ocean dynamics can lead to predictions of oceanic fluxes which can be easily examined for parameter sensitivity. The approach is capable of development in at least two ways. First, mass exchange at the sea surface, (cf. Bye, 1978), can be included in the model, and used to predict oceanic salinity distribution, and the modifications to the heat balance. Second, the model equation can be solved for individual basins, each with its own zonal distribution of wind stress and temperature, instead of for the multiple basin. It would also be more realistic to allow the coefficient of lateral heat diffusion to be determined by the solution using arguments based on large scale turbulence (e.g. Stone, 1978). It is probable however that this extension would lead to values of \(N\) generally in the range 100-500 m²/s, rather than 2500 m²/s. Thus it is thought that the 500 m²/s solution is a closer approximation to the real situation than the highly diffusive solution. A more important refinement may be the inclusion of a better representation of the gyral circulations than the simple model of Section 2.

It is possible also to consider time-dependent solutions of the meridional heat balance equation by assigning a scale depth for seasonal storage in the upper layers of the ocean and thus investigating the annual heat flux cycle of the ocean, which
from meteorological estimates (Oort and Von der Haar, 1976) has at least the same magnitude as the mean heat fluxes. In this connection, it should be noted that the use of an annual mean wind stress and air temperature omits the contribution to the Ekman heat flux arising from correlations in the fluctuations of wind stress and air temperature. However, an approximate calculation indicates that this is generally a small correction.

REFERENCES


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