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### Growth and Distribution: A Neoclassical Kaldor-Robinson Exercise

James Tobin

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GROWTH AND DISTRIBUTION:  
A NEOCLASSICAL KALDOR-ROBINSON EXERCISE

by

James Tobin

January, 1990

GROWTH AND DISTRIBUTION: A NEOCLASSICAL KALDOR-ROBINSON EXERCISE  
(Corrected and revised December 1989)

James Tobin  
Yale University

This paper is a corrected version of a paper of the same title published in the Cambridge Journal of Economics Volume 13, Number 1, March 1989, pp. 37-45. Offprints were distributed as Cowles Foundation Paper No. 730. I am indebted to Professor Ian Steedman of Manchester University for pointing out to me a serious error in the original paper. His comment will be published in the Journal. I apologize for this error, and I especially regret making the error in an issue of the Journal devoted wholly to celebrating the contributions to economics of the late Nicholas Kaldor.

My mistake was to state factor-price frontiers as relations of the wage to the rental price of a unit of capital rather than to the profit rate, i.e. to  $rp$  rather than to  $r$ , where  $r$  is the profit rate and  $p$  is the price of capital goods in terms of consumption. This would be relatively innocuous within a specification of a single technology and type of capital, where the two measures of capital cost would be uniquely related. But -- as Professor Steedman points out and as I once understood but forgot -- it makes no sense in comparing the frontiers for two technologies, which differ as to the nature of capital and as to the relation of  $p$  to  $r$ .

For anyone who is interested, the present version of the paper corrects the error.

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ABSTRACT

Kaldor's capital/labor income distribution theory relied on differential saving propensities from profits and wages. Robinson's growth models typically specified constant-coefficient technologies in which marginal productivities cannot determine distribution. Here these two insights are combined in a two-sector (capital goods, consumption goods) economy. Two technologies are available, but only as either-or alternatives. The choice of technology and the income distribution depend on the saving propensities. Steady-state consumption need not be greater when the economy is more capitalized and profit rates are lower.

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Retrospect: Kaldorian Distribution Theory

In 1956 Nicholas Kaldor published his "Keynesian" theory of the distribution of output between labor and property incomes, and in 1960 I published a short spoof of his article.<sup>1</sup> I was a brash young American. In reprinting that note in a collection of my essays in 1971, I wrote:

"Chapter 7 is an irreverent spoof of a distribution theory advanced by Nicholas Kaldor and others....[It] is a footnote to the running controversy between neoclassical growth theory and its opponents. Neoclassical theory would have the division of full employment output between investment and consumption depend on the society's propensity to save. If property owners and wage earners differ in their saving behavior, the distribution of income between them would help to determine the share of investment in national output. The income distribution, in turn, depends, in neoclassical theory, on the marginal productivities of capital and labor. Kaldor rejected marginal productivity theory and needed an explanation of factor shares in its place. He regarded the investment share of total output as independently determined by technology and entrepreneurship -- something to which the national saving propensity must adapt, rather than vice versa....I would like to record here my judgment, which the reading lists of my courses confirm, that Mr. Kaldor has made many outstanding contributions to economic theory. He should be excused this aberration."<sup>2</sup>

Nicky Kaldor was unperturbed by my note, although he did bother to reply.<sup>3</sup> Fortunately, we subsequently became good friends and were usually on the same side of macroeconomic controversies. I had the opportunity to

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<sup>1</sup> N. Kaldor, "Alternative Theories of Distribution", Review of Economic Studies, Volume XXIII 1955-56, 83-100.

J. Tobin, "Toward a General Kaldorian Theory of Distribution: A Note", Review of Economic Studies, Volume XXVII 1960, 119-20. Reprinted in Tobin, Essays in Economics: Volume 1, Macroeconomics, Chicago: Markham Publishing Co., 1971, Chapter 7.

<sup>2</sup> Tobin, Essays in Economics: Volume 1, Macroeconomics, p. 3.

<sup>3</sup> N. Kaldor, "A Rejoinder to Mr. Atsumi and Professor Tobin", Review of Economic Studies, Volume XXVII 1960, 121-23.

express my esteem for him in his presence both at the celebration of the centenary of Keynes's birth at Kings in 1983 and at Yale when Lord Kaldor gave the first set of Okun Memorial Lectures in 1983.<sup>4</sup>

For this symposium in his memory, I return to the subject of our disagreement three decades ago.

I was not criticizing the proposition that saving propensities might differ for incomes of different types, as well as for incomes of different magnitudes. After all, one important strand of mainstream saving theory, the life cycle model, also focuses on the difference between human and nonhuman wealth. More important, businesses, especially corporations, may not be acting just as agents of convenience for individual shareowners when they plow back profits. They may be instead institutionalizing that compulsion for accumulation which Marx -- and Joan Robinson, Kaldor, and other post-Keynesians -- have regarded as central to capitalism.

In this institutional spirit, Kaldor himself applied his differential propensities to sources of income, labor or property, rather than to classes of persons, workers and capitalists. For this reason he could not get excited about the long-run implications of recognition that both classes save and accumulate wealth, the discussion triggered by the Pasinetti process.<sup>5</sup> Empirically, it has not been possible to prove that business

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<sup>4</sup> See my "Comment" on Lord Kaldor's paper "Keynesian Economics after Fifty Years" at the 1983 conference, in D. Worswick and J. Trevithick, editors, Keynes and the Modern World, Cambridge: Cambridge University Press, 1983. See also my Preface to the Okun Lectures, N. Kaldor, Economics without Equilibrium, Armonk, N.Y.: M.E. Sharpe Inc., 1985.

<sup>5</sup> L.L. Pasinetti, "Rate of Profit and Income Distribution in Relation to the Rate of Economic Growth, Review of Economic Studies, Volume, XXIX, 1962, 267-79. In the course of the debate provoked by this seminal article, Kaldor, in "Marginal Productivity and Macroeconomic Theories of Distribution", Review of Economic Studies, Volume XXXIII, 1966, 309-19, said

saving is just a one-for-one substitute for household saving.

Kaldorian saving propensities can easily be built into Swan-Solow-type neoclassical growth models where, like other saving functions, they help to determine a stable steady-state capital intensity and corresponding stable values of other variables. Indeed the classical saving function, a popular extreme form of the Kaldorian hypothesis -- nothing is saved from wages and nothing is consumed from profits -- leads to the "Golden Rule" optimum, the steady state with maximum consumption per worker. In that equilibrium investment equals profits and, in a Swan-Solow model, the marginal productivity of capital is equal to the growth rate. However, the weight of evidence is against the view that national saving and investment are as large as capital incomes.<sup>6</sup>

Moreover, if Kaldorian saving propensities are built into a one-good neoclassical growth model, they will help to determine the distribution of income and wealth. The steady-state capital stock is endogenous, dependent on saving behavior. Therefore the marginal productivities of capital and labor and, except in the special case of the Cobb-Douglas production function, the relative shares of labor and capital incomes are likewise endogenous and dependent on saving behavior. But differential saving propensities are not, except in the special case of the Leontief production

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"...I have always regarded the high savings propensity out of profits as something which attaches to the nature of business income, and not to the wealth (or other peculiarities) of the individuals who own property. It is the enterprise, not the particular body of individuals owning it at any one time, which finds it necessary .... to plough back a proportion of the profits earned.....Hence the high savings propensity attaches to profits as such, not to capitalists as such."

<sup>6</sup> A. Abel, N. Mankiw, L. Summers, and R. Zeckhauser, "Assessing Dynamic Efficiency: Theory and Evidence", National Bureau of Economic Research Working Paper No. 2097, 1986.

function, necessary to determine distributive shares. Almost any saving function, for example the primitive assumption that a constant fraction of income of all kinds is saved, will determine the steady-state capital/labor ratio and thus also marginal productivities and factor shares.

What did bother me thirty years ago? First, I found it hard to believe that factors' returns had nothing to do with their productivities, and my note made fun of some implications of that belief. Second, in relation to macroeconomic theory, my problem was this: If marginal productivity is dropped as an explanation of income shares and the consumption function is drafted to replace it, how is aggregate output to be explained? I was shocked to see a "Keynesian" model that apparently assumed output to be independent of aggregate demand even in the short run. And given full employment, I thought, the role of the consumption function is to help to determine investment as equal to saving, which it cannot do if it is assigned the burden of determining wages and profits. To me, a model with investment wholly exogenous was both unKeynesian and unpalatable.

Of course, marginal productivities are indeterminate, within limits, when factors are fully employed and technology requires them to be used in constant proportions. Maybe differential saving propensities can help in these circumstances. What determines investment and output remains a problem. Animal spirits? Perhaps, in short run business cycles. In those circumstances there is no mechanism to insure that capital capacity and labor supply stand in the correct proportions to each other. Whether capacity is and is expected to be short relative to labor supply or redundant is surely important in investment decisions. In the long run, capital capacity is adjusted to the requirements of exogenous growth in



effective labor supply and of technology.

I take my cue from Joan Robinson:

"The rate of investment ...can be accounted for in two ways which do not seem to be connected with each other. Investment is determined, in one sense, by profit expectations, the 'animal spirits' of entrepreneurs which incline them to take the risks of investment, and the state of supply of finance, which may be subsumed under the head of the level of interest rates.

"In another sense, the rate of investment that can be maintained over the long run depends on technical conditions and the supply of labour. According to this view, the rate at which the effective supply of labor is growing...limits the rate at which capital can accumulate, because there would be no point in bringing capital goods into existence when there is not going to be labour to operate them."

#### Factor Shares and Saving in a Growth Model with Leontief Technologies

I provide here a simple example of a model in which the distribution of income between wages and returns to capital ownership cannot be explained by marginal productivities, because they are not determinate. The reason is that the technological input/output coefficients are constant a la Leontief. Kaldorian differential saving propensities are shown to be a natural way to close the model, to determine factor shares, and to equate aggregate saving to technologically required investment. But other consumption/saving functions may also do the trick. The primitive uniform constant propensity to save, the same for wages and profits, can make excess consumption demand a function of the profit rate. The reason is that the relative price of capital goods depends on the profit rate and therefore so does aggregate net income in terms of consumption goods.

Of course, a workable neoclassical alternative is to equate the profit

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<sup>7</sup> J. Robinson, "The Theory of Distribution", in her Collected Economic Papers, Volume II, Oxford: Blackwell, 1960, 146. The author says that the paper is "an amended version of a paper published in French in Economie Applique, Oct.-Nov. 1957."

(interest) rate to a constant rate of time preference (augmented in a non-stationary model by a term for decline in the marginal utility of growing per capita consumption.) But the specification that saving is supplied perfectly elastically with respect to the interest rate depends on infinitely long horizons for consumption and saving decisions and other implausible assumptions.

A technology consists of two activities; one produces consumption goods, the other capital goods. The two goods are not the same; the price of capital goods in terms of consumption goods is endogenous. Each activity uses labor and capital services. I shall analyze steady states in which total quantities of labor and capital are fully employed in the two activities. The labor supply is exogenous, growing in effective units at a constant rate, determined by natural increase and/or Harrod-neutral progress. The steady-state capital stock, relative to the labor force, is determined by the technology. The output of capital goods -- gross investment -- is what is needed to offset depreciation and to equip the increment in labor supply. Capital goods are used in both activities. In production, the capital goods used in the consumption goods activity are the same as those used in the capital goods activity itself. In use, they are different, both in the labor required to operate them and in their speed of depreciation.

Available to the economy are two or more technologies, each defined by the four input/output coefficients describing the consumption and investment activities and by the two depreciation rates. The economy may choose one among whole technologies, but it cannot mix activities. That is, for example, the consumption activity of technology I cannot co-exist with

the investment goods activity of technology II. The nature of the investment goods produced and used might determine the differential productivities of those goods and of labor in making the two kinds of goods. I believe this assumption is in the spirit of some of Joan Robinson's representations of technology.<sup>8</sup> The all-or-nothing choice of technology gives rise to the possibility that different technologies will be chosen at high and low profit rates.

I assume that the economy will be on its factor-price frontier, along which the activities in use break even and it is not possible to increase either wages or rates of return to saving, both measured in consumption goods, without reducing the other factor's earnings. The break-even conditions for the two activities determine two prices in terms of consumption goods: the wage and the price of capital goods. They determine these prices given the profit rate. The equations do not determine the profit rate. That is the essential indeterminacy, traceable to the fixed-

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<sup>8</sup> See J. Robinson, The Accumulation of Capital, London: Macmillan, 1966, especially Chapter 10, "The Spectrum of Techniques". Here she introduces discrete constant-proportion techniques for labor and fixed capital. However, she does not explicitly model technologies with distinct activities for consumption goods and investment goods.

J. Robinson and J. Eatwell, An Introduction to Modern Economics, London: McGraw-Hill, 1973, 183-195 sets forth a multi-sector Sraffa type input/output model and derives from it a wage/profit-rate frontier. A uniform mark-up rate is applied to the cost of every intermediate input in the pricing of all intermediate and final outputs. Evidently this corresponds to a uniform one-period lag between inputs and proximate outputs. But no such lag and markup apply to labor inputs and wages. In any case, this model does not handle fixed capital, or even inventories other than those implicit in the work in progress during the one-period lag.

The general model of alternative "blueprints", each involving an indivisible technology using different kinds of capital goods that produce together with labor both the capital goods themselves and consumption goods has been discussed in, for example, L.L. Pasinetti, "Switches of Technique and the 'Rate of Return' in Capital Theory", Economic Journal, Volume 79, 1969, 508-531. The general model is so complex that points of interest depend greatly on simple illustrations, like the one in my text.

coefficient technology, the vacuum that the Kaldorian saving function may fill.

The two factor prices are the wage (in consumption goods per worker per period) and the profit rate net of depreciation (pure number per period). Total net capital income, in consumption goods, is the profit rate multiplied by the value of the capital stock. This capital income and the wage bill are the factor incomes that add up to the value of total net output. And it is those factor incomes to which the Kaldorian consumption or saving coefficients apply. Within each technology, the price of capital goods, and therefore the value of the capital stock, depend endogenously on the profit rate.

A neoclassical theorist might argue that the consumption/saving decision depends on the profit rate as well as, or even instead of, aggregate profits measured in consumption goods. This dependence would represent the intertemporal substitution effect, Irving Fisher's interest incentive for saving. This is certainly not what Kaldor had in mind. I have not allowed for it in the present model.

For each technology, the steady-state input balance equations determine outputs of consumption goods and investment goods (per effective worker). These are technologically determined, independent of wages, capital goods prices, and profit rates.

"Reverse switching" is quite possible. That is, a lower-consumption technology can be on the factor-price frontier at higher wages and lower profits. Of course, the other direction of switching, which seems more normal, is also possible.

I find this "switching" implication preferable to the usual examples,

which involve curiously rigid alternative sets of time lags between labor inputs and outputs.<sup>9</sup> In the present example, there are no such lags (although they could be added) and the emphasis is on fixed rather than working capital. Another advantage of the present model is that the steady-state output of investment goods is determined quite naturally to meet the requirements of technology and growth.

#### The Formal Model

Here is the model: First, the equations for a technology for the outputs of consumption goods C (activity a) and investment goods I (activity b):

$$a_L C + b_L I = 1 \quad (\text{Labor demand} = \text{supply, normalized to 1}) \quad (1)$$

$$a_K C = K_C, \quad b_K I = K_I \quad (\text{Capital in each industry})$$

$$a_K(n+d)C + b_K(n+d+s)I = I \quad (\text{Steady state gross investment})$$

$$a_K(n+d)C + (b_K(n+d) - (1 - sb_K))I = 0 \quad (\text{Investment goods demand} = \text{supply}) \quad (2)$$

Here  $n$  is the growth rate, and  $d, d+s (\geq 0)$  are the depreciation rates in the consumption goods and investment goods activities. Equations (1) and (2) may be solved for the steady-state outputs  $C$  and  $I$ . Let  $A$  be the determinant of the input/output coefficients,  $a_L b_K - a_K b_L$ , and let  $v = (1 - sb_K)$ . Then:

$$C = (b_K(n+d) - v) / (A(n+d) - va_L), \quad I = -a_K(n+d) / (A(n+d) - va_L) \quad (3)$$

The factor-price frontier can be found from the "dual" of the system (1)-(2). The price of consumption goods, the numeraire, is normalized to 1. The wage rate is  $w$ . The price of investment goods is  $p$ . The profit rate is

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<sup>9</sup> In her Chapter 10 of The Accumulation of Capital, "The Spectrum of Techniques", Mrs. Robinson discusses switching as "A Curiosum", pp. 109-113. See P.A. Samuelson, "A Summing Up", Quarterly Journal of Economics, Volume 80, 1966, 568-583, for a review of reswitching possibilities in models with input/output lags.

r. Thus the gross rental cost of capital services is  $p(r+d)$  in the consumption goods activity and  $p(r+d+s)$  in investment goods production. The break-even equations for the two activities are:

$$a_L w + a_K p(r+d) = 1 \quad (\text{Consumption goods activity}) \quad (4)$$

$$b_L w + (b_K(r+d) - (1-sb_K))p = 0 \quad (\text{Investment goods activity}) \quad (5)$$

These two equations are to be solved for  $w$  and  $p$ , given  $r$ . The solutions are:

$$w = (b_K(r+d) - v) / (A(r+d) - va_L) , \quad p = -b_L / (A(r+d) - va_L) \quad (6)$$

By inspection, comparing (3) and (6) gives the standard "Golden Rule" result that  $w = C$  when  $r = n$ .

The wage equation in (6) is the factor-price frontier. Its slope is:

$$\partial w / \partial r = -va_K b_L / (A(r+d) - va_L)^2 \quad (7)$$

This slope is negative. If  $A$  is negative -- the investment goods activity is more capital-intensive than the consumption-goods activity -- the frontier is concave to the origin. This is also true if  $A$  is positive but the expression  $(A(r+d) - va_L)$  is negative. The frontier is convex to the origin if  $A$  and that expression are both positive. If  $A$  is zero -- both activities are equally capital-intensive -- the frontier is a line with slope  $-b_K/va_L$ . In that case the wage for  $r=0$  is  $1/a_L - b_K d/va_L$  and for  $r=-d$  is just  $1/a_L$ , the productivity of labor in the consumption goods activity; and the profit rate for  $w=0$  is  $(1-(d+s))/b_K$ , the net productivity of capital in capital goods production.

The value of the capital stock is:

$$pK = a_K b_L v / [(A(r+d) - va_L)(A(n+d) - va_L)] \quad (8)$$

The slope of a  $(w-r)$  frontier is supposed to be a measure of capital intensity. For example, in the Swan-Solow model -- one product, two

Figure 1: Factor Prices and Consumption

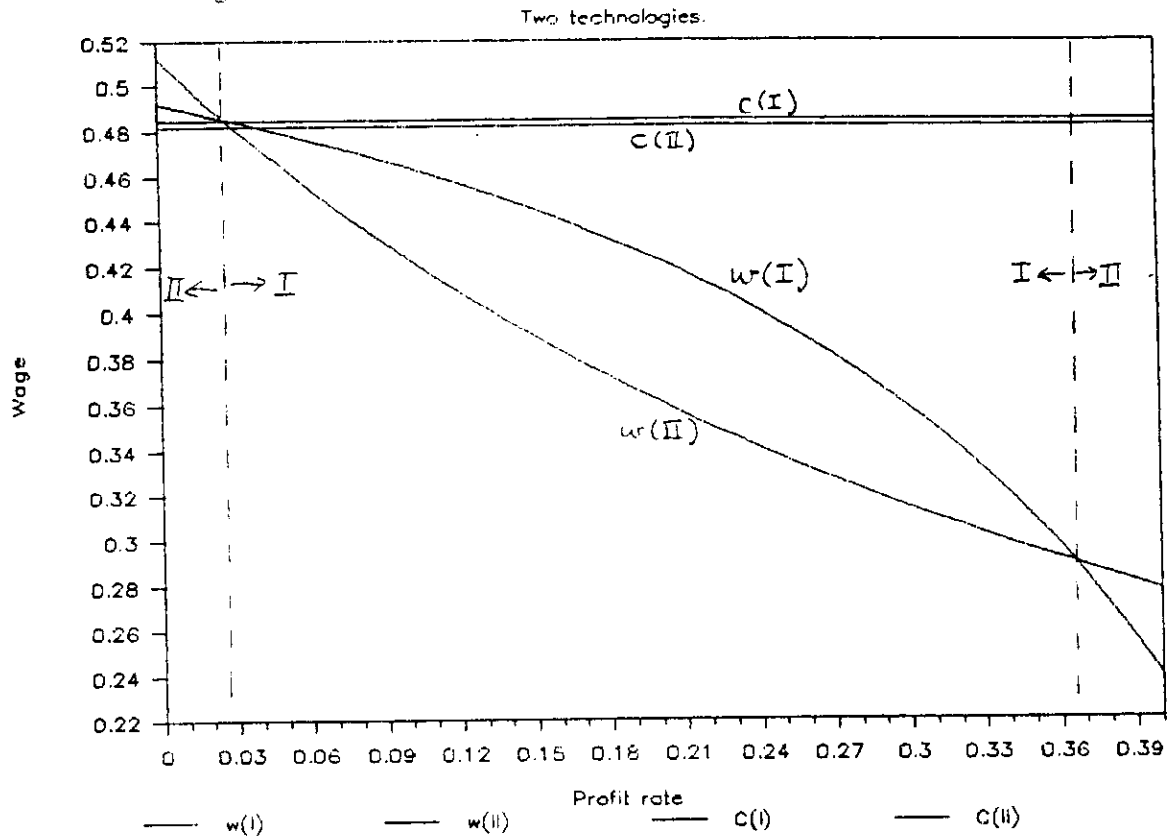


Fig. 1. Factor prices and consumption: two technologies. The factor-price frontier for Technology, I  $w(I)$ , is concave to the origin; the frontier for Technology II  $w(II)$ , is convex to the origin. Switch points are at profit rates of 0.026 and 0.366. Consumptions for the two technologies, each independent of the profit rate, are shown as  $C(I)$  and  $C(II)$ .  $C(I)$  is the larger. In each technology wage and consumption are equal when the profit rate is equal to 0.03, the assumed growth rate.

Fig. 1a: Factor Prices and Consumption

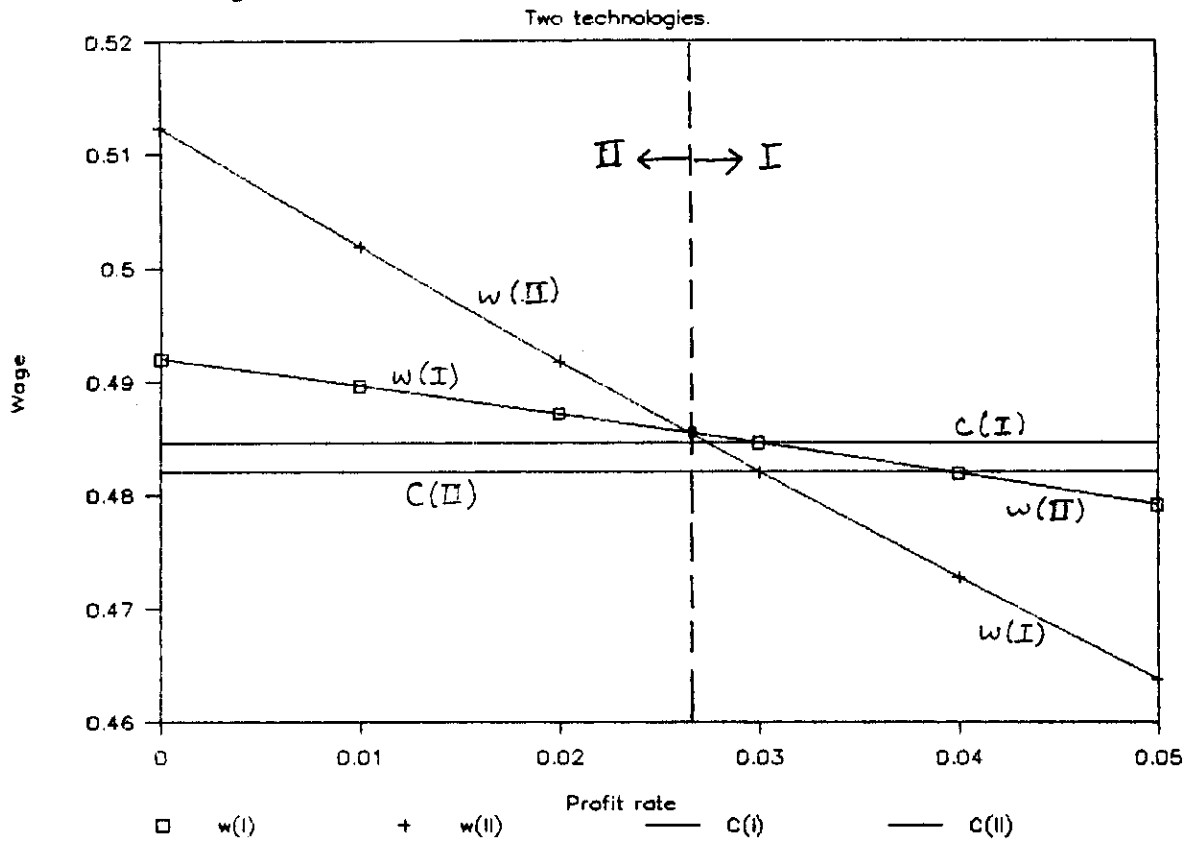


Fig. 1a. The upper left hand corner of Fig. 1, the vicinity of the first switch point.



factors, etc. -- the slope is equal to the negative of the capital/labor ratio. Here too, as can be seen from (7) and (8), the two are closely related, and are equal when  $r=n$ .

The same calculations can be made for a second technology, indeed for every available technology, each one defined by values of  $a_L$ ,  $a_K$ ,  $b_L$ ,  $b_K$ ,  $d$ , and  $s$ . The growth rate  $n$  is also exogenous, but it is assumed to be independent of technology. Here a two-technology numerical illustration is presented.

Table 1 tells the numbers assumed in this illustration and reports some of the calculations.

Table 1

Assumed Parameter Values and Key Results in Illustration  
(Labor supply normalized to equal 1)

	Technology I	Technology II
$a_L$	2	0.05
$a_K$	1	2
$b_L$	0.9	2
$b_K$	2	0.02
$d$	0.035	0.47
$s$	-0.025	0.1
$n$	0.03	0.03
$C$	0.485	0.482
$I$	0.034	0.488
$K$	0.55	0.97
price when $r=n$	0.47	0.97
first switch pt, $r$	0.026	0.026
first switch pt, $w$	0.486	0.486
second switch pt, $r$	0.366	0.366
second switch pt, $w$	0.289	0.289

Figures 1 and 1a show illustrative factor-price frontiers for two technologies. One is convex to the origin, the other concave. The example

Fig. 2: Value of Capital Stock

Related to  $r$  in two technologies

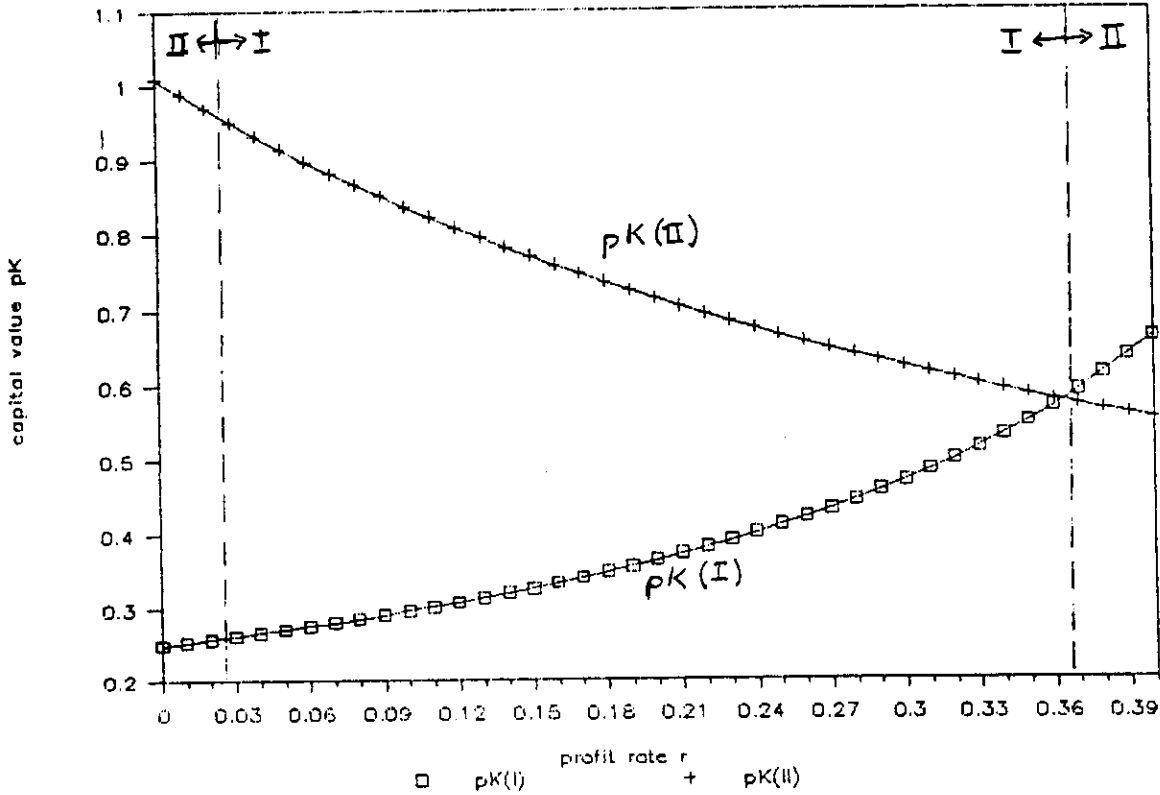


Fig. 2. Value of capital stock, related to  $r$  in two technologies. In each technology the steady-state capital stock is constant in units of that technology's capital goods. But the price of capital goods varies with the profit rate, positively in Technology I, negatively in Technology II. At the two points where technologies are switched, the value of capital stock jumps. At the second switch point capital stock values in the two technologies differ very little, but they are not equal.

has been contrived to exhibit switching. There are two switch points. Both are shown in Figure 1, but only the first one in Figure 1a. Technology II has the lower steady-state consumption, but is used in preference to Technology I at very low and very high profit rates. The "reverse" switch shown in Figure 1a occurs at a profit rate lower than the growth rate. It is not surprising that wasteful over-capitalization occurs at such low profit rates.

Indeed it is not hard to see that a reverse switch can occur in this model only at profit rates below the growth rate. A switch point is a profit rate  $r^*$  at which  $w(I) - w(II) = 0$ . From the wage formula in (6), it is clear that the difference  $w(I) - w(II)$  is a quadratic function of  $r$ , call it  $Q(r)$ . Likewise, because  $C=w$  at  $r=n$ , the difference  $C(I) - C(II)$  is  $Q(n)$ . As in the example, suppose that  $Q(n)>0$ . Consider a reverse switch point, such that  $Q(r)<0$  for  $r<r^*$ , while  $Q(r^*)=0$ . Clearly  $n$  cannot be smaller than, or even equal to,  $r^*$ . But  $r^*$  could be smaller than  $n$ . Of course a reverse switch point may not occur at any non-negative profit rate.

The second switch in the example is "normal" in the sense that a higher profit rate entails lower consumption as well as a lower wage.

Figure 2 shows the capital stocks, valued in terms of consumption goods, in relation to profit rates in the two technologies. Technology II is relatively capital-intensive, in particular in the consumption goods activity. However, Technology II capital depreciates rapidly.

All that is needed now is to superimpose a consumption function on Figure 1 or 1a. If nothing is saved from wages and nothing is consumed from capital income, the profit rate is equal to the growth rate, and the wage is equal to the consumption afforded by the technology in use at that profit

Fig 3: Determining Wage and Profit Rate

Using Kaldor consumption function.

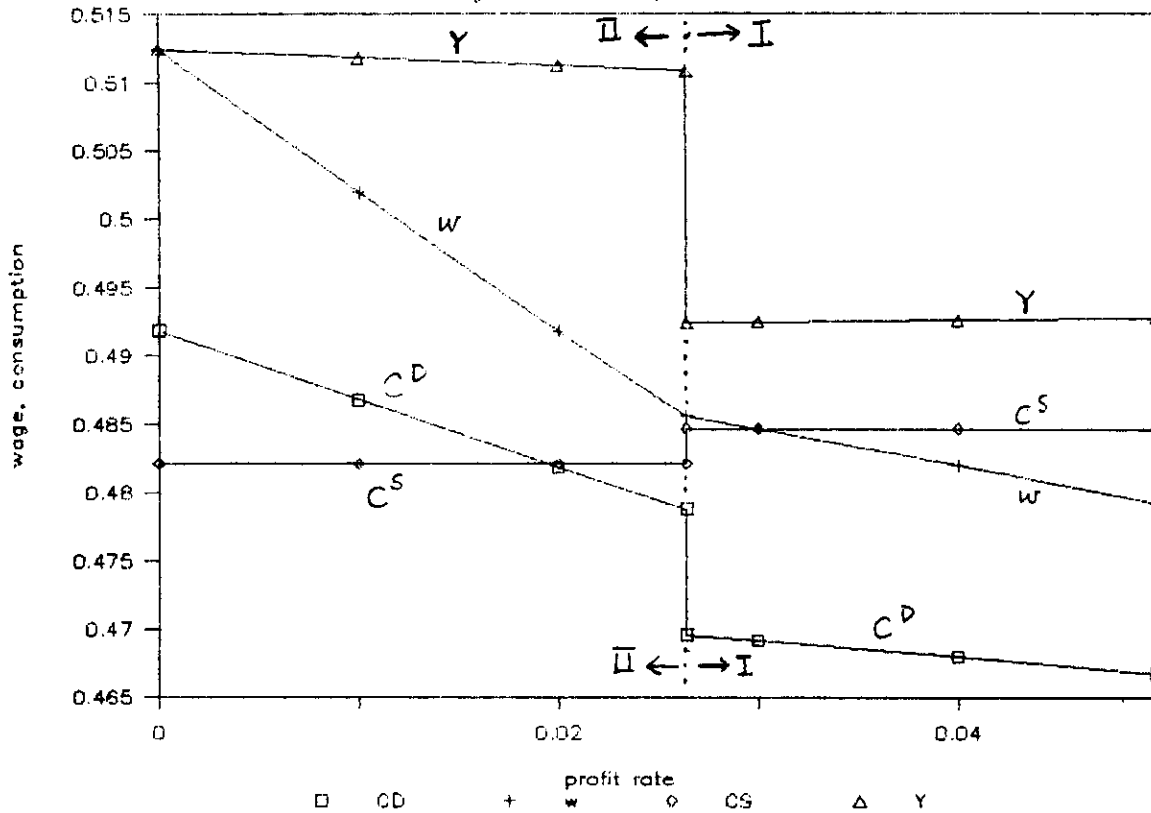


Fig. 3. Determining wage and profit rate using Kaldor consumption function.

As in Fig. 1a, the region in the vicinity of the first switch point is shown. The downward-sloping  $w$  curve (+), kinked at the switch point, is the frontier. Consumption supply  $C^S$  ( $\diamond$ ) jumps from  $C(II)$  to  $C(I)$  at that point.

Consumption demand  $C^D$  (squares), with propensities of 0.96 for wages and 0.5 for profit incomes, can be equal to supply only at the switch point. The top locus is aggregate net income  $Y$  ( $\Delta$ ), equal to  $w+rpK$ , which would be consumption demand if the propensity were 1.0 from both kinds of income. Because of the dependences of  $p$  on  $r$  shown in Fig. 2, this locus declines before the switch point, jumps down, and then rises slowly.

rate. In general, consumption demand  $C^D$  is equal to  $c_L w + c_K r p K$ , where  $c_L$  and  $c_K$  are the propensities to consume from labor income and capital income respectively. In equilibrium consumption demand must equal consumption supply  $C^S$ , the consumption corresponding to the dominant technology.

In the two technologies of the example, net investment  $npK$  is small relative to net output  $w+rpK$  (equal to  $C+npk$  -- remember that  $p$  depends on  $r$ ), between 1.5 and 7 percent for profit rates between 0 and 0.5. Likewise capital incomes never exceed 6 percent. Therefore, the average propensities to consume in aggregate and from wage incomes have to be quite close to 1 in order to have any equilibrium at all.

Figure 3 continues the illustration of Figure 1a.  $C^S$  is shown, jumping at the profit rate where the two factor-price frontiers cross in Figure 1a. The downward-sloping continuous but kinked curve, designated by the wage  $w$ , is the factor-price frontier, taking account of both technologies and the switch from one to the other. The curve for  $C^D$ , consumption demand, takes consumption propensities to be 0.96 and 0.5 for labor and capital incomes respectively. In equilibrium consumption demand must equal consumption supply. In the example depicted in Figure 3, this happens to occur to the left of the first switch point, at a profit rate of 0.02. With higher consumption propensities the intersection of  $C^D$  and  $C^S$  could be moved to or beyond the switch point.

At the switch point itself the "intersection" would generally be an overlap of the vertical jumps of  $C^D$  and  $C^S$ . This would not be a full equilibrium, because in neither of the two possible technologies would the value of  $C^D$  at the common prevailing wage and profit rate be equal to the corresponding value of  $C^S$ . At the same time, it would be true that to the

left there would be excess consumption demand, and to the right excess supply. A way to obtain a switch-point equilibrium, in which both the zero-profit conditions and the consumption equation are satisfied, would be to allow the operation of both technologies simultaneously in suitable proportions.

Note that the wage frontier itself would be consumption demand if workers spent all their wages and capitalists saved all their profits. The equilibrium would be at a profit rate equal to the growth rate 0.03, with technology I and its consumption.

Note also that it is not strictly essential to have different propensities for the two types of income. The reason is that the variations of  $r$  and of  $p$  bring some variation in the functional distribution of income even in these fixed-coefficient technologies. In Figure 3, the top locus is  $Y$ , aggregate net income per worker, equal to  $w+rp_k$ . It is initially declining with  $r$ , then jumps down at the switch point, where the price of capital goods falls drastically. Thereafter net income rises with  $r$ , almost imperceptibly in the figure. The changing distribution of income can be seen by comparing net income and the wage as the profit rate varies. A consumption function with the same propensity for each type of income would be just the net income locus shifted down proportionately. For example, a common propensity of 0.984 would support the "golden rule" equilibrium.

However, that equilibrium would be unstable by Kaldor's criterion, in the sense that if the profit rate were lower consumption demand would be less than supply, and the excess supply would cause the profit rate to fall further. That is because in Technology I net income per worker, wage plus profit, measured in consumption goods, rises with the profit rate  $r$ .

However, stable equilibria with common consumption propensities are also possible. In this exercise they could occur in the domains of Technology II, where net income falls with  $r$ .

In this exercise, I have tried to place certain insights of Lord Kaldor and Joan Robinson in a context where their purpose and relevance may be understood and appreciated by a wider audience. Are there morals to the story? Competitive markets are most likely to exist and to perform well when local incremental decisions are possible. However, societies frequently face all-or-nothing decisions, choices among lumpy alternatives, often difficult or impossible to reverse. The ordinary tools of neoclassical economics are much less useful for the second class of problems than for the first.