6-1-1988

Testing for a Unit Root in the Presence of a Maintained Trend

Sam Ouliaris

Joon Y. Park

Peter C.B. Phillips

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station

New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 880

TESTING FOR A UNIT ROOT IN THE PRESENCE OF A MAINTAINED TREND

by

Sam Ouliaris

Joon Y. Park

Peter C. B. Phillips

June 1988

Testing for a Unit Root in the Presence of a Maintained Trend

Sam Ouliaris
University of Maryland
and
School of Management
National University of Singapore

Joon Y. Park
Cornell University

Peter C. B. Phillips
Cowles Foundation for Research
Yale University

July, 1988

This paper develops statistics for detecting the presence of a unit root in time series data against the alternative of stationarity. Unlike most existing procedures, the new tests allow for deterministic trend polynomials in the maintained hypothesis. They may be used to discriminate between unit root nonstationarity and processes which are stationary around a deterministic polynomial trend. The tests allow for both forms of nonstationarity under the null hypothesis. Moreover, the tests allow for a wide class of weakly dependent and possibly heterogenously distributed procedures. We illustrate the use of the new tests by applying them to a number of models of macroeconomic behavior.

The first author gratefully acknowledges financial assistance from the College of Behavioral and Social Sciences at the University of Maryland.

1. Introduction

The purpose of this paper is to extend some existing statistical procedures for detecting a unit root in time series data. In the tests of Dickey and Fuller (1979) and Phillips and Perron (1988), the maintained hypothesis is that the time series is integrated with drift but with no trend. This paper extends these tests to allow explicitly for a deterministic polynomial time trend in the maintained hypothesis. An important feature of the new procedures is their invariance to the presence of drift and polynomial trend in the true data generation process. They should therefore be helpful in discriminating between the difference stationary and trend stationary specification. Our analysis is motivated in part by recent work by Bhargava (1986) which emphasizes the importance of developing tests of the unit root hypothesis which explicitly allow for trend in the maintained hypothesis. It is also motivated by the view that the linear time trend hypothesis is inappropriate for modelling the deterministic component of an economic time series. Perron (1987) has recently demonstrated the importance of using a flexible specification for the deterministic component. He shows that the results of Nelson and Plosser (1982), which provide support for the unit root hypothesis, may be reversed by using unit root tests which allow for a structural break in the deterministic component of a time series.

The issue of trend stationarity versus difference stationarity is critical in the ongoing debate on the nature of the business cycle. Most macroeconomic time series exhibit nonstationarity through the presence of a secular growth component. If a macroeconomic variable is trend stationary, then short term shocks (such as those arising from variations in government policy) have only a temporary impact on the long run evolution of the series. This behavior is consistent with traditional theories of the business cycle. However, if a macroeconomic variable is difference stationary, then short run shocks affect the level of the variable permanently. This is more compatible with real business cycle models of equilibrium output.

Prior to the work of Nelson and Plosser (1982) the prevailing view was that the secular component of macroeconomic time series was trend stationary and that the long term trend had little to do with the year to year variations in economic conditions. This led to the routine practice of detrending macroeconomic series in order to identify the cyclical component that was to be explained by business cycle theory. However, using the Dickey-Fuller (1979, 1981) procedures for detecting a unit root in time series models, Nelson and Plosser (1982) found strong evidence against the trend stationary model. Nelson and Plosser tested the null hypothesis of a unit root (with drift) for 14 macroeconomic time series and could not reject the unit root hypothesis in 13 cases. Perron (1986) has recently confirmed the findings of Nelson and Plosser using the statistical procedures developed in Phillips (1987) and Phillips and Perron (1988). Unlike the Dickey-Fuller statistics, these procedures allow for quite general weakly dependent innovation sequences. From the point of view of business cycle theory, the Nelson and Plosser results are more consistent with the implications of real business cycle theories since innovations in the stochastic trend apparently account for a significant portion of the short - as well as the long - run, variation in the time series. The interested reader is referred to Campbell and Mankiw (1987) for further discussion on the implications of the unit root hypothesis for modelling macroeconomic behavior.

The appropriate representation of nonstationarity in macroeconomic time series is also a vital issue from an econometric perspective. Durlauf and Phillips (1987) show that misspecification of a random walk as a stationary process evolving around a deterministic trend has major effects on the statistical analysis of the data. For example, it is well known that inappropriate detrending of a random walk produces spurious periodic behaviour at long lags, and this gives a misleading impression of persistence and high variance in the business cycle. (See Nelson and Kang (1982), and Chan, Hung and Ord (1977)). In addition, the theory of cointegration has emphasized the need to pre-test time series for unit roots. A cointegrated process is a linear combination of integrated variables which are stationary. In practical applications, it is important to determine whether each series (once purged of its deterministic part) possesses a unit root. Pre-testing guards against inadvertently mixing processes which are integrated of different orders (such as I(1) and I(0) processes, where the notation I(k) signifies a process whose k^{th} difference is stationary) since such processes are trivially cointegrated. Finally, a null hypothesis of no cointegration may itself be tested by applying unit root procedures to the residuals of the cointegrating regression (see Phillips and Ouliaris (1987)).

The organization of this paper is as follows. Section 2 develops Wald statistics for the null hypothesis that a time series has a unit root and possibily trend polynomials of an arbitrary order. The statistics are developed using the methodology in Phillips (1987). In section 3 we show how to incorporate a general polynomial time trend in the maintained hypothesis when the bounds procedure of Phillips and Ouliaris (1988) is applied. The new procedures are then applied to empirical models in the cointegration literature to see whether the original data stands up to the null hypothesis of a unit root.

2. Unit Root Tests with Deterministic Trend

Following the methodology in Phillips (1987), we begin by letting $\{y_t\}_0^\infty$ be a time series generated according to:

(1) $$y_t = \Sigma_o^{p-1}\beta_k t^k + y_{t-1} + \xi_t, \quad \beta_k \in \mathbb{R}.$$

(2) y_o = random with a distribution that is independent of n, the sample size.

Model (1) allows y_t to be an integrated process with a p^{th} order deterministic time polynomial in the null hypothesis. It encompasses all of the unit root models considered previously in the literature as special cases. For example, Phillips (1987) considers (1) under the assumption that $p = 0$ while Phillips and Perron (1988) allow $p = 1$.

In what follows, we assume that $\{\xi_t\}_o^\infty$ is a weakly stationary, zero mean innovation sequence with spectral density $f_\xi(\lambda)$. The partial sum process $X_n(r) = n^{-1/2}S_{[nr]} = n^{-1/2}\Sigma_1^{[nr]}\xi_t$, for $r \in [0,1]$, is required to satisfy an invariance principle for partial sums of weakly dependent innovations. Specifically, we require

$$(3) \qquad X_n(r) \xrightarrow{\mathcal{D}} B(r) \quad \text{as} \quad n \uparrow \infty.$$

The symbol " $\xrightarrow{\mathcal{D}}$ " here signifies weak convergence of the associated probability measure, while $B(r)$ is scalar Brownian motion with long run variance

$$\omega^2 = \lim_{n\to\infty} \frac{1}{n} E(S_n^2) = 2\pi f_\xi(0) = \sigma^2 + 2\lambda$$

where $\sigma^2 = E(\xi_1^2)$, $\lambda = \Sigma_{j=2}^{\infty} E(\xi_1\xi_j)$. We let $B(r) = \omega W(r)$ so that $W(r)$ is the standard Brownian motion. We sometimes refer to ω^2 as the long run variance because

$$n^{-1/2}\Sigma_1^n\xi_t = X_n(1) \Rightarrow B(1) \equiv N(0, \omega^2).$$

In what follows we represent $B(r)$ by B and $W(r)$ by W to simplify the presentation of the results.

Invariance principles such as (3) have been used extensively to analyze time series models with general integrated processes. They are known to apply for a very wide class of random sequences which are weakly dependent and possibily heterogeneous. In particular, following Hall and Hyde (1980), it may been shown that the invariance principle applies to all stationary and invertible ARMA models. Thus the maintained hypothesis given as (1) encompasses a very broad class of time series models.

Consider the least squares regression:

$$(4) \qquad y_t = \Sigma_0^p \hat{\beta}_k t^k + \hat{\alpha} y_{t-1} + \hat{\xi}_t$$

The hypotheses we are interested in testing are:

(I) $\alpha = 1$,

(II) $\alpha = 1$, and $\beta_p = 0$.

Let $h_p(\hat{\alpha}) = n(\hat{\alpha} - 1)$ represent the test statistic for (I) based on the estimated parameter for α derived from least squares estimation of (4). Similarly, let $t_p(\hat{\alpha})$ and $F_p(\hat{\alpha}, \hat{\beta}_p)$ denote the t and Wald statistics for (I) and (II), respectively. We assume $\beta_p = 0$ in (4) when the null hypothesis is true. If $\alpha < 1$ under the alternative, however, β_p may not be zero. We therefore maintain a p^{th} order polynomial trend both under the null and the alternative. The statistics are invariant with respect to β_k, $k = 0, \ldots, p - 1$.

Note that one must include a p^{th} order time polynomial in the fitted regression in order to test (I) and (II) satisfactorily. A regression model without this term would not discriminate between the trend/difference stationary specification since the regressor y_{t-1} in (4) would contain an unexplained time trend t^p which clearly dominates all the other components. In fact, the asymptotic power of t-type statistics for the null hypothesis $\alpha = 1$ using (4) without t^p would be zero.

The asymptotic distributions of the above statistics may be represented succinctly in terms of standardized Brownian motion. To facilitate the representation of the distributions, we define $W_k(r)$ to be the stochastic process on $[0,1]$ such that $W_k(r)$ is the projection residual of a Brownian motion $W(r)$ on the subspace generated by the polynomial functions $1, r, \ldots, r^k$ in $L^2[0,1]$. Here, $L^2[0,1]$ denotes the Hilbert space of square integrable functions on $[0,1]$ with the inner product $(f,g) = \int_0^1 fg$ for $f, g \in L^2[0,1]$. For explicit representations of W_k, $k = 0, 1$, see Park and Phillips (1986) and the review paper of Phillips (1987). We also define r_p to be the projection of r^p on the space spanned by the polynomials $1, r, \ldots, r^{p-1}$.

Theorem 2.1 represents the asymptotic distributions of these statistics in terms of the above notation.

Theorem 2.1: *Assume the time series* $\{y_t\}$ *is generated by* (1). *Then*

(a) $h_p(\hat{\alpha}) \xrightarrow{\mathcal{L}} (\omega^2\int_0^1 W_p dW + \lambda)(\omega^2\int_0^1 W_p^2)^{-1}$

(b) $t_p(\hat{\alpha}) \xrightarrow{\mathcal{L}} (1/\sigma)(\omega^2\int_0^1 W_p dW + \lambda)(\omega^2\int_0^1 W_p^2)^{-1/2}$

(c) $F_p(\hat{\alpha}, \hat{\beta}_p) \xrightarrow{\mathcal{L}} \frac{1}{\sigma^2}\left[(\omega^2\int_0^1 W_p dW + \lambda)^2(\omega^2\int_0^1 W_p^2)^{-1} + (\omega\int_0^1 r_p dW)^2(\int_0^1 r_p^2)^{-1}\right].$

The limiting distributions of the statistics are nonstandard. They depend on nuisance parameters through the presence of λ and ω^2. This hinders hypothesis testing, making the selection of appropriate critical values for statistical inference extremely difficult. However, we may define transformations of the statistics that eliminate the nuisance parameters asymptotically. In particular, we define:

$$(5)\quad K_p(\hat{\alpha}) = n(\hat{\alpha} - 1) - \frac{n^2(\hat{\omega}^2 - \hat{\sigma}^2)}{2s_o^2}$$

$$(6)\quad S_p(\hat{\alpha}) = \frac{\hat{\sigma}}{\hat{\omega}}t(\hat{\alpha}) - \frac{n(\hat{\omega}^2 - \hat{\sigma}^2)}{2\hat{\omega}s_o}$$

$$(7)\quad G_p(\hat{\alpha}, \hat{\beta}_p) = (\hat{\sigma}^2/\hat{\omega}^2)F_1 + \frac{n^2(\hat{\omega}^2 - \hat{\sigma}^2)^2}{4\hat{\omega}^2 s_o^2} - n(\hat{\alpha} - 1)(1 - [\hat{\sigma}/\hat{\omega}]^2)$$

where

s_o^2 = residual sum of squares from the regression of y_{t-1} on $1, t, \ldots, t^p$, and

$\hat{\omega}^2$ = any consistent estimator of ω^2.

The asymptotic distributions of these statistics are:

Theorem 2.2: *Assume the time series* $\{y_t\}$ *is generated by* (1). *Then*

$$K_p(\hat{\alpha}) \xrightarrow{\mathcal{D}} \left(\int_o^1 W_p dW\right)\left(\int_o^1 W_p^2\right)^{-1}$$

$$S_p(\hat{\alpha}) \xrightarrow{\mathcal{D}} \left(\int_o^1 W_p dW\right)\left(\int_o^1 W_p^2\right)^{-1/2}$$

$$G_p(\hat{\alpha}, \hat{\beta}_p) \xrightarrow{\mathcal{D}} \left[\left(\int_o^1 W_p dW\right)^2\left(\int_o^1 W_p^2\right)^{-1} + \left(\int_o^1 r_p dW\right)^2\left(\int_o^1 r_p^2\right)^{-1}\right].$$

These distributions are free of the nuisance parameters λ and ω^2. Monte-Carlo techniques can be used to simulate the distributions and thereby provide critical values for the purpose of hypothesis testing. The asymptotic distributions of the statistics are tabulated in Appendix 2 for $p = 2, 3, 4, 5$. Monte-Carlo evidence on the perfomance of the statistics for $p = 2$ is presented in Appendix 3.

In order to make the new procedures fully operational, we require a consistent estimator for the long run variance ω^2. It may be consistently estimated in a number of ways. Newey and West (1987) and Phillips (1987) recommend a class of estimators which can be written as

$$\omega_*^2 = \frac{1}{n}\Sigma_1^n \xi_t^2 + \frac{2}{n}\Sigma_{k=1}^{\ell} w_\ell(k)\Sigma_{t=k+1}^n \xi_t \xi_{t-k}$$

for a suitable weight function $w_\ell(k)$ which depends explicitly on the lag truncation parameter ℓ.

Since $\omega^2 = 2\pi f(0)$, the asymptotic variance may also be estimated by obtaining a consistent estimate of the spectrum at frequency zero. Let

$$I_n(\nu_j) = n^{-1}\left|\sum_1^n \xi_t e^{-it\nu_j}\right|^2$$

represent the periodogram of ξ_t evaluated at the frequencies, $\nu_j = \frac{2\pi j}{n} \in [-\pi, \pi]$. Estimates of ω^2 may be formed by smoothing the periodogram ordinates around frequency zero, namely:

$$\hat{\omega}^2 = \sum_{-k}^{+k} W_n(j) I\left(\frac{2\pi j}{n}\right), \qquad |j| \leq k,$$

where

$$\sum_{-k}^{+k} W_n(j) = 1, \qquad W_n(j) \geq 0 \text{ for all } j.$$

and k grows with n such that $k/n \longrightarrow 0$ and $\sum_{-k}^{+k} W_n^2(j) \longrightarrow 0$. The latter condition is required in order to ensure that the estimator is consistent for ω^2.

Note that there is always a trade off between bias and variance in choosing a weight function. A weight function which assigns equal weights to a very broad band of frequencies will produce an estimate of $f_\xi(0)$ which may have large bias because the estimate depends on values of the periodogram at frequencies which are distant from zero. On the other hand, a weight function which assigns most of its weight to a narrow frequency band centered at zero will yield an estimator of the spectrum with a relatively small bias, but a large variance.

The bias can be controlled by pre-whitening the series prior to estimating the spectrum. Pre-whitening serves to equalize the periodogram ordinates over a broad band of frequencies, thereby minimizing the role of the weight function. For example, suppose we fit the following ARMA(p,q) model to ξ_t:

$$\Phi(L)\xi_t = \Psi(L)v_t$$

where

$$\Phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \phi_3 L^3 - \ldots - \phi_p L^p$$

$$\Psi(L) = 1 - \psi_1 L - \psi_2 L^2 - \psi_3 L^3 - \ldots - \psi_q L^q$$

and v_t is a weakly stationary process with spectrum $f_v(\kappa_j)$. (All the roots of $\phi(L)$ are assumed to be outside the unit circle.) Then the spectrum of ξ_t at frequency κ_j is given by

$$f_\xi(\kappa_j) = f_v(\kappa_j)\frac{|1 - \Sigma_1^q \phi_j e^{-ij\kappa_j}|}{|1 - \Sigma_1^p \psi_j e^{-ij\kappa_j}|} .$$

Thus, for $\kappa_j = 0$

$$\hat{f}_\xi(0) = \hat{f}_v(0)\frac{|1 - \Sigma_1^q \hat{\phi}_j|}{|1 - \Sigma_1^p \hat{\psi}_j|} .$$

where $\hat{\phi}_j$ and $\hat{\psi}_j$ are consistent estimates of ϕ_j and ψ_j respectively.

The results presented in this paper are based on the Daniell estimator for $f_\xi(0)$. The Daniell estimator uses equal weights to smooth the periodogram. Thus $W(k) = (2k)^{-1}$ and

$$(8) \qquad \hat{\omega}_D^2 = \frac{1}{k}\Sigma_1^k \mathrm{Re}\left[I(\frac{2\pi j}{n})\right].$$

Since the Daniell estimator is best suited to models with "flat" periodogram values around frequency zero, the pre-whitening technique will be used in order to minimize the distortion arising from large periodogram ordinates distant from zero.

Finally, a word of caution must be given with regard to the estimation of the long run variance ω^2 used to construct the statistics. It is important to use the residuals from the regression (4) and not to incorporate the hypothesis $\beta = 1$ when estimating ω^2. Failing to do so has substantial effects on the power of tests and may result in the procedure being inconsistent. This problem has recently been pointed out by Phillips and Ouliaris (1987) in a related context of the residual based tests for cointegration. To look at the problem more closely, consider the simplest case of $p = 0$. The tests are therefore based on the regression,

$$(9) \qquad y_t = \hat{\alpha} y_{t-1} + \hat{\xi}_t .$$

Under the assumption of integration, we may estimate ω^2 using $\{\Delta y_t\}$ or $\{\hat{\xi}_t\}$ from (9). This would not affect the result since $\hat{\alpha} = 1 + O_p(n^{-1})$ under the null hypothesis. The two estimators, however, behave rather differently under the alternative hypothesis of no unit root. This occurs because $\{\Delta y_t\}$ has a moving average representation with a unit root when $\{y_t\}$ is stationary. If the estimation is based on $\{\Delta y_t\}$, then $\hat{\omega}^2 \xrightarrow{p} 0$. Furthermore, if some estimators such as a smoothed spectrum are used, we have $\hat{\omega}^2 = O_p(n^{-1})$ and the test becomes inconsistent (see Phillips and Ouliaris (1987)). Loosely put, the inconsistency is due to the fact that the behavior of the correction term mimics that of the leading term too closely when $\hat{\omega}^2$ is negligible. The proof is essentially the same as the one in Phillips and Ouliaris (1987) and will not be repeated here. The problem of inconsistency does not arise if the estimation of ω^2 is based on the regression residual in (9).

3. Testing for Unit Roots with General Deterministic Trends

The statistical procedures developed in the previous section enable us to detect a unit root in models with a deterministic time polynomial of an arbitary order. However, they cannot immediately deal with more general trend cycle models of the form:

$$(10) \qquad y_t = \mu + \varphi(t,\theta) + u_t$$

where $\varphi(t,\theta)$ is a deterministic function of time with parameter θ. Although it is in general possible to modify Wald statistics to account for general deterministic trends such as (10), the critical values of these statistics depend on the form of $\varphi(t,\theta)$. This is, of course, already clear from Theorem 2.1 (a) - (c) where these limit distributions depend on the projection of $W(r)$ on the orthogonal complement of the polynomial trend of order p, namely $W_p(r)$.

Equation (10) embodies a broad spectrum of stationary data generation processes and yields a very general alternative to the difference stationary specification. It is therefore desirable to have a method for directly testing (10) against a unit root specification. We now develop a nonparametric method for doing this. Our approach is based on the univariate bounds procedure for no cointegration developed in Phillips and Ouliaris (1988). This procedure exploits the fact that differencing a stationary series induces a negative unit root in its MA representation, resulting in a zero spectrum at the zero frequency. The bounds procedure provides a diagnostic for assessing whether or not the estimated spectrum at the zero frequency is sufficiently small to be negligible. It can easily be modified to deal with models such as (10).

The general approach is best introduced by way of example. Let $p = 0$ and

$$\xi_t = \Delta y_t = y_t - y_{t-1}$$

Under the null hypothesis of a unit root, $\{\xi_t\}$ is a stationary process having positive asymptotic variance. If, however, $\{y_t\}$ is stationary under the alternative, then $\{\xi_t\}$ has an MA(1) representation with a unit root and its spectrum will be zero at the zero frequency. Moreover, if the smoothed spectrum estimator is used to estimate $\omega^2 = 0$, the results in Phillips and Ouliaris (1987) imply that $\hat{\omega}^2$ is $O_p(n^{-1})$. This means that we may obtain diagnostic evidence in favor of the trend stationary specification by showing that the estimated spectrum at the zero frequency is negligible and thus consistent with the alternative hypothesis of $\omega^2 = 0$. This in turn may be done using the unit free (scalar) bounds procedure of Phillips and Ouliaris (1988).

To explain this procedure, let $\rho^2 = (\omega/\sigma)^2$, and $\hat{r}^2 = (\hat{\omega}/\hat{\sigma})^2$, be any consistent estimator of ρ^2. Also, we assume ω^2 is estimated by (8). We are therefore interested in the alternative hypothesis:

$$H_a = \rho^2 = \frac{\omega^2}{\sigma^2} = 0$$

According to the univariate bounds test, H_a is accepted if the upper limit for the true ρ^2 is "sufficiently" small. Phillips and Ouliaris (1988) point out that under the null hypothesis that $\rho^2 > 0$, $\hat{r}^2$

has an asymptotic normal distribution with mean ρ^2 and variance $\rho^2\left[\sum_{-k}^{+k} W_T(j)^2\right]$. Thus for the Daniell estimator given by (8) have

$$k^{1/2}(\hat{r}^2 - \rho^2)/\rho^2 \xrightarrow[d]{} N(0,1) \tag{11}$$

with a corresponding confidence interval for ρ^2 of:

$$\hat{r}^2/(1 + (z_\alpha/k^{1/2})) \le \rho^2 \le \hat{r}^2/(1 - (z_\alpha/k^{1/2})) \tag{12}$$

where z_α is the $(1 - \alpha)$ percentage point of the standard normal distribution. Similarily, H_a is rejected if the lower limit is above a preassigned level.

A maintained polynomial trend may be allowed for in a straight forward fashion. That is, when $p \ge 1$ we simply compute the regression residuals $\hat{\xi}_t$ from:

$$\Delta y_t = \Sigma_o^{p-1}\hat{\beta}_j t^j + \hat{\xi}_t$$

and mount the bounds test using $\hat{\xi}_t$. Of course, this approach can be generalized to allow for any form of deterministic trend in the maintained hypothesis, such as $\varphi(t,\theta)$ for example. For a given φ, the null hypothesis is specified as:

$$y_t = \mu + \varphi(t,\theta) + y_{t-1} + \xi_t$$

and the test is mounted using the least squares residuals $\hat{\xi}_t$ from the regression:

$$\Delta y_t = \hat{\mu} + \varphi(t,\hat{\theta}) + \hat{\xi}_t.$$

Again, we need to test whether ξ_t has an MA(1) representation with a unit root. Since the least squares estimators of $\hat{\mu}$ and $\hat{\theta}$ will be $O_p(n^{1/2})$ consistent under the null, we do not need to make any adjustments to the procedure.

In order to make the diagnostic procedure operational, guidelines must be set as to what constitutes a "sufficiently small" estimate for the upper bound. It is also necessary to set criteria for deciding when the lower bound is too large to accept the alternative hypothesis of $\rho^2 = 0$. These issues are complicated by the fact that the limit distribution (11) for $\hat{r}^2$ does not hold when $\rho^2 = 0$. Moreover, since our estimate of the lower bound is always greater than zero, the confidence interval for ρ^2 will never encompass $\rho^2 = 0$. This happens because we do not use the asymptotic distribution of $\hat{r}^2$ under the hypothesis $\rho^2 = 0$. The procedure is constructed so that this is the alternative.

Following Phillips and Ouliaris (1988), we recommend using 0.10 as the rejection point for the upper and lower bounds. If the upper bound for ρ^2 is less than 0.05, one could be fairly confident that the true value of the spectrum was sufficiently close to zero so as to be compatible with the alternative hypothesis of $\rho^2 = 0$. If the lower bound for ρ^2 is greater than 0.10, then one could be very confident that the true spectrum at the zero frequency is not zero.

In order to get some indication of how adequate such a decision rule may be, we simulated critical values for $\hat{r}^2$ using randomly selected processes under the alternative hypothesis of $\rho^2 = 0$ and the null hypothesis of $\rho^2 \neq 0$. The form of the data generation process was assumed to be ARMA(1,1). The series was differenced in order to induce a unit root in its moving average representation for the alternative $\rho^2 = 0$. The parameters of the ARMA(1,1) process were selected randomly from a uniform distribution over the interval [-0.6, 0.6], thereby restricting draws to ensure that the process possessed a unit root in its MA representation. Table 3 presents the values obtained for $\hat{r}^2$ by averaging the upper and lower percentile values of the empirical distributions of 50 processes. The simulations suggest that an upper bound of 0.10 would provide an extremely conservative decision rule for the upper bound since the 95 percentile point for the average distribution is 0.45.

The above analysis bears directly on recent papers by Cochrane (1986) and Campbell and Mankiw (1986,1987). These papers analyze the real per capita GNP trend/difference stationarity issue by considering the magnitude of the spectrum of real per capita GNP at the zero frequency. Campbell and Mankiw (1987) find that the long run variance of real per capita GNP is large, and thus argue that this is strong evidence in favor of the difference stationary model. In contrast, Cochrane (1986) estimated $\hat{r}^2$ for real per capita GNP and argued that since $\hat{r}^2$ was small (0.40) the random walk component of real per capita GNP was negligible. However, neither Cochrane (1986) nor Campbell and Mankiw (1987) compute upper bounds for $\hat{\rho}^2$. If the focus of attention is whether or not ρ^2 is small (and not necessarily zero), then the appropriate procedure is obviously to compute the upper bound of the confidence limit for ρ^2 using (12).

4. Empirical Applications

The new unit root procedures are particulary useful in applied work which utilizes the theory of cointegration to test steady state models of economic behavior. The steps involved in testing for cointegration may be outlined as follows. First, all the variables in the model should be pre-tested for a unit root, since a regression model involving a mixture of I(0) and I(1) variables is trivally cointegrated. Second, it is necessary to test whether the residuals of the model (or the deviations from the equilibrium condition) possess a unit root. If the residual vector has a unit root, the model is *not* a cointegrated system. When the cointegrating vector does not need to be estimated, standard unit root tests (such as those developed above) may be applied to the residual vector If the cointegrating vector needs to be estimated, unit root tests may also be used; however, different critical values apply (see Phillips and Ouliaris (1987)).

We now demonstrate the use of the new tests by applying them to the following standard economic models (all of which have recently been reformulated as cointegrated systems and all of which have known cointegrating vectors under the hypothesis of cointegration): (1) Spot and Forward Exchange Rates (Corbae and Ouliaris (1986), Corbae, Ouliaris and Zender (1987)); (2) Purchasing Power Parity (Corbae and Ouliaris (1987)) and (3) The Real Monetary Equation (Engle and Granger (1987)). In what follows, we are primarily interested in determining whether the existing results for these models are changed by using unit root tests which allow for polynomial trends in the maintained hypothesis. For completeness, we briefly review the theory underlying the above models in the context of the cointegration framework:

(1) Spot and Forward Exchange Rates

A necessary condition for market efficiency in the forward exchange market is that the difference between the spot and forward exchange rate is equal to the current risk premium plus a white noise error. When the spot and forward exchange rates are integrated processes, and the risk premium is stationary, this condition corresponds to the hypothesis that the spot and forward exchange rate are cointegrated with a known cointegrating vector of (1,-1). Moreover, since the unit root tests allow for innovation sequences which are in the ARMA class, we do not need to identify the risk premium in order to carry out the test. Thus the theory of cointegration provides a robust test for a necessary condition for market efficiency which does not require indentification of the risk premium.

We shall consider this hypothesis for six US dollar exchange rates: Canada, Germany, Switzerland, France, Japan, and the United Kingdom. In particular, we are particularily interested in determining whether the spot and forward rates can be modelled individually as integrated processes and whether the difference between the spot and forward exchange rates are stationary. The data are monthly, spanning the flexible exchange rate period (January 2, 1976 to January 2, 1985).

(2) Purchasing Power Parity

According to the absolute version of purchasing power parity, the dollar value of goods produced abroad and the dollar value of goods produced domestically should be equal in equilibrium. In stochastic versions of the standard model, this requirement would correspond to the statement that there should only be stationary fluctuations around the equation $P_t = S_t P_t^*$, which relates the level of domestic prices (P_t) to foreign prices (P_t^*) and the spot exchange rate. Moreover, when P_t, S_t and P_t^* are integrated variables, purchasing power parity is equivalent to the statement that $\log S_t$, $\log P_t^*$ and $\log P_t$ form a cointegrated system with a known cointegrating vector of (1,-1,-1). In other words, if purchasing power parity holds, the logarithm of the real exchange rate should be a stationary variable.

The PPP hypothesis will be tested for five countries: Canada, France, Italy, the United Kingdom, and West Germany. The data are quarterly, spanning the 1973(2) - 1986(4) period.

(3) The Real Monetary Equation

If prices, the money supply and real income are integrated processes, then the real monetary equation implies that these variables should form a cointegrated system. In particular, we require that the velocity of circulation displays only stationary fluctuations. Note that the real monetary equation is simply an example of a broader class of models which derive from equilibrium conditions implicit in steady state growth models.

We shall test the hypothesis that velocity is a stationary variable for four alternative definitions of the money supply: M1, M2, M3, and ML (Liquid Assests). The data are quarterly, and span the 1959(1) - 1987(3) period.

Table 4 presents the results of applying the new statistics to Models (1) - (3). The table contains the computed values of $S_p(\hat{\alpha})$, $G_p(\hat{\alpha}, \hat{\beta}_p)$ and $\hat{r}^2$ for a representative value of p (the order of the time polynomial).

The following conclusions may be drawn from the computed values of $S_p(\hat{\alpha})$, and $G_p(\hat{\alpha}, \hat{\beta}_p)$:

(a) The spot and forward exchange rates of Germany, Switzerland, France, Japan and the UK are integrated processes. The null hypothesis of a unit root in the level of these series cannot be rejected at the 5% level of significance using $p = 4$. This finding is not affected by including higher order polynomials in the fitted regression.

In contrast, the spot and forward exchange rates of Canada appear to be stationary. The null hypothesis of a unit root in these series may be rejected at the 5% level of significance using $G_4(\hat{\alpha}, \hat{\beta}_4)$ and at the 10% level of significance using $S_4(\hat{\alpha})$. We may therefore model the spot and forward exchange rates of Canada as a stationary process around a fourth-order polynomial trend. It is interesting to note that this finding depends on the order of the time polynomial which is included in the fitted regression. For example, when p = 3, $S_3(\hat{\alpha})$ = -2.9240 and $G_3(\hat{\alpha}, \hat{\beta}_3)$ = 8.5581, both of which are smaller than the 5% critical value. This result emphasizes the importance of including the fourth order polynomial term in the fitted regression. It also highlights the importance of adequately modelling the deterministic part of the time series when testing for a unit root. Interestingly, Corbae and Ouliaris (1987) find, using the Phillips-Perron (1988) unit root tests where p = 1, that the Canadian exchange rate is an integrated process. The results presented in Table 4 suggest that this may be due to the omission of higher order polynomial terms.

(b) The difference between the spot and forward exchange rates, or the implied risk premium, is stationary in the case of Canada, Germany, Switzerland, and France. We may reject the unit root hypothesis at the 5% level of significance using $S_p(\hat{\alpha})$ and $G_p(\hat{\alpha}, \hat{\beta}_p)$ for p = 2. It also holds for p = 0, and p = 1. The Canadian result is to be expected, since the spot and forward exchange rates are themselves stationary processes.

The results for Japan and the United Kingdom are not very favorable to the hypothesis that the implied risk premium is stationary. For these countries, one can reject the null hypothesis of no cointegration between the spot and forward exchange rate only at the 15% level of significance using p = 2.

(c) The results for the real exchange rate data do not yield any evidence in favor of purchasing power parity. We cannot reject the null hypothesis of a unit root in the real exchange rate data for any of the countries represented in the data. Moreover, the results are not affected by including higher order polynomials in the fitted regression. These findings are consistent with those reported in Corbae and Ouliaris (1987), which are based on the same tests with p set to zero.

(d) The money supply data does not provide any support for the real monetary equation. There is no evidence to suggest that velocity is a stationary variable. Moreover, this finding does not depend on the definition of the money supply, the value of p, and is consistent with the results reported in Phillips and Ouliaris (1988) using principal components methods.

Turning to the computed values of $\hat{r}^2$, the upper bounds for the true value of ρ^2 are all larger than 0.10, irrespective of the value of p used. Thus if we employ the 0.10 decision rule for the upper bound we would conclude that all the series in the data set possess a unit root - a result which is obviously in conflict with that suggested by the $S_p(\hat{\alpha})$ and $G_p(\hat{\alpha}, \hat{\beta}_p)$ statistics. Given that these tests are formal statistical procedures, the results for the bounds test raises doubts about the usefulness of the general approach.

The $\hat{r}^2$ procedure is best interpreted as a diagnostic tool rather than a formal statistical test. It should be evaluated with this qualification in mind. The empirical results suggest that the bounds procedure is quite good at detecting the presence of a unit root. The $\hat{r}^2$ procedure is clearly in agreement with the $S_p(\hat{\alpha})$ and $G_p(\hat{\alpha}, \hat{\beta}_p)$ statistics when these tests imply that a series possesses a unit root, since the corressponding point estimates for the upper bound are all very large. In contrast, the upper bound estimates for the series which $S_p(\hat{\alpha})$ and $G_p(\hat{\alpha}, \hat{\beta}_p)$ imply are stationary around a deterministic trend are uniformily less than 1.0. Thus there is some indication that a stationary series yields consistently smaller values for the upper bounds than a nonstationary series.

The above results suggest that the 0.10 cut off point in the $\hat{r}^2$ bounds procedure is too conservative for practical applications. A low cut off point ensures that the probability of a type 1 error will be small for all series except those which are nearly stationary. However, in the absence of a formal statistical procedure which allows for the null hypothesis to be $\rho^2 = 0$, it is obviously difficult to set an upper bound for ρ^2 which is not too conservative. Nevertheless, it is encouraging to find that the point estimates for the upper bound are quite large for those series where there is little evidence in favor of the stationarity hypothesis.

5. Conclusion

This paper has developed a number of procedures for detecting a unit root in a time series model. Unlike existing procedures for testing the unit root hypothesis, which take the null hypothesis to be the difference stationary model with/without drift but with no trend, the tests allow explicitly for polynomial trends and drift in the data generation process. Our aim was to develop tests which are invariant to the true values of the drift and trend parameters. Two classes of procedures were developed. The first class extended the Wald type tests of Phillips (1987) and Phillips and Perron (1988) to account explicitly for linear trend and drift in the maintained hypothesis. The second class extended the univariate bounds procedure for detecting no cointegration (or a unit root in univariate time series models) to very general (possibily) nonlinear trend/cycle models. These models incorporate the linear trend model as a special case.

The new procedures were illustrated using a number of interesting models in the applied cointegration literature. The results confirmed the importance of carefully modelling the deterministic component of a time series when testing for a unit root. We were able to show that some of the series can be modelled as stationary processes around a polynomial trend, in contrast to previous findings.

Appendix 1: Proofs

Proof of Theorem 2.1 The following results are needed in order to prove Theorem 2.1. Define $y_t^* = \Sigma_1^t \xi_j$ and

$$W_n(r) = (\sqrt{n}\,\hat{\omega})^{-1}\, y^*_{[nr]}$$

where $\hat{\omega}^2$ is a consistent estimate of the long run variance of $\{y_t\}$. Then we have $W_n \xrightarrow{\mathscr{D}} W$ and

$$\text{(A1)} \qquad n^{-2}\Sigma_1^n y_t^{*2} \xrightarrow{\mathscr{D}} \omega^2 \int_o^1 W^2$$

using the results in Phillips (1987).

Now let $f_i(r) = r^i$ and $f_{ni}(r) = \left(\frac{[nr]}{n}\right)^i$ for $r \in [0,1]$. Then $f_{ni} \longrightarrow f_i$ uniformly. Hence,

$$\text{(A2)} \qquad \frac{1}{n}\,\Sigma_1^n \left[n^{-(2i+1)/2}\; t^i\right] y_t^* = \frac{1}{n}\Sigma_1^n (\tfrac{t}{n})^i n^{-1/2} y_t^*$$

$$= \omega_n \int_o^1 f_{ni} W_n + o_p(1)$$

$$\xrightarrow{\mathscr{D}} \omega \int_o^1 f_i W$$

Also,

(A3) $$\frac{1}{n}\Sigma_1^n\left[n^{-(2i+1)/2}\ t^i\right]\left[n^{-(2j+1)/2}\ t^j\right] = \frac{1}{n}\Sigma_1^n(\frac{t}{n})^i(\frac{t}{n})^j \longrightarrow \int_o^1 f_i f_j$$

Define $g_{ni}(r) = n\left[\left(\frac{[nr]+1}{n}\right)^i - \left(\frac{[nr]}{n}\right)^i\right]$ for $r \in [0,1]$. It follows that $g_{ni} \longrightarrow \frac{d}{dr}f_i$ uniformly. We therefore have

(A4) $$\Sigma_1^n\ n^{-(2i+1)/2}\ t^i\xi_t = n^{-1/2}\ \Sigma_1^n\ (\frac{t}{n})^i\xi_t$$

$$= n^{-1/2}\ \Sigma_1^n\ \xi_t - n^{-1/2}\ \Sigma_1^n\left[(\frac{t}{n})^i - (\frac{t-1}{n})^i\right]\ \Sigma_1^{t-1}\xi_j$$

$$= \omega_n\left[W_n(1) - \int_o^1 g_{ni}W_n\right]$$

$$\xrightarrow{\mathcal{L}} \omega\left[W(1) - \int_o^1(\frac{d}{dr}f_i)W\right] = \omega\int_o^1 f_i dW.$$

Finally, we shall make use of the following result which is proved in Phillips (1988):

(A5) $$\frac{1}{n}\Sigma_1^n\ y_{t-1}^*\xi_t \xrightarrow{\mathcal{L}} \omega^2\int_o^1 WdW + \lambda, \qquad \lambda = \frac{1}{2}\ (\omega^2 - \sigma^2)$$

To prove part (a) of the theorem, we write

$$n(\hat{\alpha} - 1) = \left[\Sigma_1^n\ y_{t-1}^p y_{t-1}^p\right]^{-1}\left[\Sigma_1^n\ y_{t-1}^p\xi_t\right]$$

where

$$y_{t-1}^p = y_{t-1}^* - \left[\Sigma_1^n\ y_{t-1}^*\tau_t^{p\prime}\right]\left[\Sigma_1^n\ \tau_t^p\tau_t^{p\prime}\right]^{-1}\tau_t^p$$

and

$$\tau_t^{k\prime} = (n^{-1/2}, n^{-3/2}t, \ldots, n^{-(2k+1)/2} t^k) .$$

Part (a) is immediate from (A1) - (A3), (A5) and the continuous mapping theorem.

It is easy to prove part (b) since

$$\frac{\hat{\sigma}}{\hat{\omega}} t_p(\hat{\beta}) = \frac{1}{\hat{\omega}} \left[\Sigma_1^n y_{t-1}^p y_{t-1}^p\right]^{-1/2} \left[\Sigma_1^n y_{t-1}^p \xi_t\right]$$

To prove part (c), we use Lemma A1 in Park and Phillips (1986) and write

$$\frac{\hat{\sigma}^2}{\hat{\omega}^2} F_p(\hat{\beta}, \hat{\delta}_p) = \frac{1}{\hat{\omega}^2}\left[\Sigma_1^n y_{t-1}^p y_{t-1}^p\right]^{-1}\left[\Sigma_1^n y_{t-1}^p \xi_t\right]^2 + \frac{1}{\hat{\omega}^2}\left[\Sigma_1^n \eta_t^2\right]^{-1}\left[\Sigma_1^n \eta_t \xi_t\right]^2$$

where

$$\eta_t = t^p - \left[\Sigma_1^n t^p \tau_t^{p-1\prime}\right] \left[\Sigma_1^n \tau_t^{p-1} \tau_t^{p-1\prime}\right]^{-1} \tau_t^{p-1}$$

and τ_t^{p-1} is defined as above. The result follows from (A2) - (A5), and part (a) of the theorem.

Theorem 2.2 may be proved in a similar manner. The proof is therefore omitted.

Appendix 2: Critical Values

This appendix contains the critical values for the $K_p(\hat{\alpha})$, $S_p(\hat{\alpha})$, and $G_p(\hat{\alpha}, \hat{\beta}_p)$ statistics for p = 2, 3, 4, 5. The critical values are given in Table 1. They were estimated using Monte-Carlo techniques. The limiting distributions were tabulated using 500 observations and 25000 replications. The fundamental innovations were drawn from a standard normal random number generator.

The critical values reported in Table 1 may be used for models with a smaller number of observations. In preliminary runs, simulations were conducted with 50, 100 and 200 observations without much impact on the critical values.

Table 1

Critical Values for $K_p(\hat{\alpha})$, $S_p(\hat{\alpha})$ and $G_p(\hat{\alpha}, \hat{\beta}_p)$

Size	$K_2(\hat{\alpha})$	$S_2(\hat{\alpha})$	$G_2(\hat{\alpha}, \hat{\beta}_2)$
20.0	-19.854870	3.239486	11.339620
17.5	-20.657720	3.305651	11.775810
15.0	-21.549160	3.378969	12.284770
12.5	-22.671780	3.466916	12.842630
10.0	-23.890100	3.560110	13.493570
7.5	-25.443810	3.670237	14.362610
5.0	-27.477620	3.827886	15.606460
2.5	-31.167340	4.089778	17.592230
1.0	-36.045680	4.376567	19.954010

Size	$K_3(\hat{\alpha})$	$S_3(\hat{\alpha})$	$G_3(\hat{\alpha}, \hat{\beta}_3)$
20.0	-24.728810	3.590699	13.829130
17.5	-25.621250	3.661849	14.307780
15.0	-26.689450	3.735801	14.884850
12.5	-27.834770	3.821583	15.521640
10.0	-29.281110	3.922786	16.234650
7.5	-30.976720	4.045293	17.224380
5.0	-33.452020	4.206791	18.595130
2.5	-37.316560	4.446673	20.529310
1.0	-41.646470	4.739825	23.409930

Size	$K_4(\hat{\alpha})$	$S_4(\hat{\alpha})$	$G_4(\hat{\alpha}, \hat{\beta}_4)$
20.0	-29.498510	3.918707	16.283230
17.5	-30.491740	3.987659	16.825820
15.0	-31.665810	4.062522	17.439930
12.5	-33.055400	4.152921	18.128020
10.0	-34.532940	4.252258	18.928760
7.5	-36.504790	4.367366	19.940770
5.0	-38.715880	4.512972	21.305270
2.5	-42.764600	4.758404	23.591940
1.0	-48.285360	5.063203	26.649970

Size	$K_5(\hat{\alpha})$	$S_5(\hat{\alpha})$	$G_5(\hat{\alpha}, \hat{\beta}_5)$
20.0	-34.223020	4.215988	18.727650
17.5	-35.334720	4.286277	19.330270
15.0	-36.562040	4.367253	19.975390
12.5	-38.002490	4.453981	20.742790
10.0	-39.596630	4.552657	21.594930
7.5	-41.582940	4.662922	22.642760
5.0	-44.064410	4.824760	24.132100
2.5	-48.524830	5.059557	26.533940
1.0	-54.623880	5.389089	29.997240

Appendix 3: Monte-Carlo Evidence.

This appendix presents the results of a simple Monte-Carlo experiment designed to assess the power of the new statistics. In what follows we assume that the data generation process for $\{\xi_t\}_0^\infty$ is:

$$\xi_t = \psi_t + \phi\psi_{t-1}, \quad \psi = N(0,1)$$

That is, the data generation process for $\{\xi_t\}$ is MA(1) with a moving average parameter ϕ. The fundamental innovations, ψ, are normally distributed with mean zero and unit variance.

In what follows, we shall restrict our attention to the case $p = 2$. The size and power of the statistics may be assessed by varying the true parameters of the data generation process. Since the statistics are invariant to the true parameter values, size distortion (if any) may be evaluated by letting $\alpha = 1$ in the data generation function and varying the values of ϕ, β_o and β_1. To assess power, we need to consider $\alpha < 1$, for arbitrary values of ϕ, β_o and β_1.

Table 2 contains the results of the Monte-Carlo experiment. It tabulates the number of rejections of the null hypothesis for $0.0 \leq \beta_1 \leq 0.05$, $\alpha \leq 1.0$ expressed as a percentage of the number of iterations. The simulations were generated using 250 observations and 2500 iterations. The simulations indicate that tests are very powerful in detecting linear trend stationarity. The power of the three tests for $\alpha < 1$ is 100%, irrespective of the choice of values for β_1, β_o or ϕ.

Moreover, since the rejection rates for $\phi > 0.0$ and $\alpha = 1$ are close to 5% , the nominal size of the test, we can deduce that the tests do not possess any material size distortion for positive values of the moving average parameter. This is not the case for negative values of the MA(1) parameter. The empirical size of the test grows substantially as $\phi \longrightarrow -1.00$. From a practical standpoint, some size distortion for negative ϕ is not surprising in finite samples since the data generation process approaches stationarity as $\phi \longrightarrow -1.00$. (The interested reader is referred to Phillips and Perron (1988) for an analytical assessment of this issue using asymptotic expansions).

Overall, we conclude that the simulations suggest that the new procedures are useful tools for discriminating between the difference stationary and trend stationary specification.

Table 2
Rejections at the 95% Level
Innovations follow an MA(1) process: $\xi_t = \psi_t + \phi\psi_{t-1}$
True β_1 (maintained trend coefficient)

—— $S_2(\hat{\alpha})$ ——

Postive MA parameter

	0.00	0.01	0.02	0.03	0.04	0.05
α = 1.00, and						
ϕ = 0.00	7.25	7.40	7.00	7.40	7.75	7.50
ϕ = 0.10	4.80	5.95	5.05	5.60	5.60	5.50
ϕ = 0.20	3.80	5.05	4.20	4.85	4.80	4.75
ϕ = 0.30	3.60	4.15	3.70	4.10	4.15	4.30
ϕ = 0.40	3.45	3.70	3.45	3.80	3.90	3.80
ϕ = 0.50	3.15	3.45	3.35	3.60	3.60	3.60

Negative MA parameter

	0.00	0.01	0.02	0.03	0.04	0.05
α = 1.00, and						
ϕ = -0.10	10.55	10.80	10.15	10.30	10.45	10.50
ϕ = -0.20	16.20	18.30	16.10	16.95	16.90	17.10
ϕ = -0.30	27.30	30.15	27.05	28.75	28.75	29.50
ϕ = -0.40	47.10	48.35	47.80	46.60	49.30	50.00
ϕ = -0.50	73.65	73.30	72.90	74.30	73.50	74.70

——— $G_2(\hat{\alpha}, \hat{\beta}_2)$ ———

Postive MA parameter

α = 1.00, and

ϕ = 0.00	6.02	6.35	7.13	7.40	7.90	8.12
ϕ = 0.10	4.85	5.75	5.35	5.75	5.60	5.70
ϕ = 0.20	4.40	4.75	4.60	5.00	4.60	4.85
ϕ = 0.30	4.05	4.45	4.20	4.50	4.15	4.50
ϕ = 0.40	3.65	4.35	3.90	4.25	3.90	4.45
ϕ = 0.50	3.60	4.10	3.55	4.15	3.70	4.20

Negative MA parameter

α = 1.00, and

ϕ = -0.10	9.70	10.50	9.80	10.05	9.75	10.10
ϕ = -0.20	15.15	16.95	15.50	15.95	15.90	16.40
ϕ = -0.30	26.10	28.45	26.00	27.60	27.20	28.00
ϕ = -0.40	45.35	47.30	45.95	45.50	48.00	48.80
ϕ = -0.50	68.25	68.15	67.60	68.30	68.50	69.45

Power: $S_2(\hat{\alpha})$, $G_2(\hat{\alpha}, \hat{\beta}_2)$

α < 1.00, and

$0.0 \leq \phi \leq 0.50$

0.00	-+0.01	-+0.02	-+0.03	-+0.04	-+0.05
100.00	100.00	100.00		100.00	100.00

Table 3

Percentiles for ρ^2 under the Null and the Alternative

99.00%	97.50%	95.00%	92.50%	90.00%	87.50%	85.00%
Alternative hypothesis of Stationarity						
0.2250	0.2026	0.1863	0.1751	0.1670	0.1608	0.1556
Null hypothesis of a Unit Root						
0.8989	0.8416	0.7920	0.7618	0.7378	0.7185	0.7014
Average						
0.5618	0.5221	0.4891	0.4684	0.4524	0.4396	0.4285

Notes: (a) These values were obtained by averaging the lower and upper percentiles of the empirical distribution of $\hat{r}^2$ under the null hypothesis of a unit root and the alternative hypothesis of stationarity respectively. The data generation processes were drawn from an ARMA(1,1) process with randomly selected coefficients. The empirical distributions were simulated using 2500 iterations and 250 observations. Fifty data generation processes were drawn at random.

Table 4

Results for Models (1) - (3)

Series	p	$S_p(\hat{\alpha})$	$G_p(\hat{\alpha},\hat{\beta}_p)$	$\hat{r}^2$	$\hat{r}^2_L$	$\hat{r}^2_U$
1. Spot Exchange Rate						
- Canadian $	4	-4.6406	22.3890	0.5262	0.1364	0.9160[f]
- Deutsche Mark	4	-2.8141	8.2826	0.6119	0.1586	1.0652
- Swiss Franc	4	-3.0333	9.4625	0.8765	0.2272	1.5259
- French Franc	4	-2.7834	12.2638	0.5859	0.1518	1.0199
- Japanese Yen	4	-3.1699	10.1147	1.3141	0.3406	2.2876
- UK pound	4	-2.7493	11.9862	0.8018	0.2078	1.3958
2. Forward Rate						
- Canadian $	4	-4.7252	22.6782	0.4150	0.1075	0.7224
- Deutsche Mark	4	-3.1526	10.2546	0.6857	0.1777	1.1436
- Swiss Franc	4	-3.3083	11.1881	0.8918	0.2311	1.5525
- French Franc	4	-2.9697	11.6177	0.5772	0.1496	1.0049
- Japanese Yen	4	-3.2066	10.2988	1.3532	0.3507	2.3556
- UK pound	4	-3.1738	13.8568	0.7499	0.1943	1.3055[f]
3. Risk premium						
- Canadian $	2	-4.3334	19.0590	0.1842	0.0477	0.3207
- Deutsche Mark	2	-3.9186	15.6565	0.3291	0.0853	0.5730
- Swiss Franc	2	-4.1356	17.1039	0.4021	0.1042	0.7000
- French Franc	2	-4.3782	19.2336	0.4016	0.1041	0.6991
- Japanese Yen	2	-3.6850	13.5803	0.4611	0.1219	0.8125
- UK pound	2	-3.5058	12.5471	0.3980	0.1031	0.6929
4. Real Exc. Rate						
- Canadian/USA	2	-2.3318	6.2390	1.1964	0.3101	2.0828
- France/USA	2	-1.0546	2.4047	2.2337	0.5789	3.8885
- Italy/USA	2	-0.9441	2.2236	2.2262	0.5770	3.8755
- UK/USA	2	-1.1451	3.2189	2.4569	0.6368	4.2770
- West Ger/USA	2	-1.5366	3.6827	2.6262	0.6806	4.5717
5. Velocity						
- M1	4	-0.4670	12.7186	0.8014	0.2077	1.3951
- M2	4	-3.0827	10.1963	1.4662	0.3800	2.5524
- M3	4	-2.9562	9.0206	1.2585	0.3262	2.1908
- Liquid Assets	4	-2.9410	8.7984	0.6688	0.1733	1.1643

Notes to Table 4

1. Data:

   (a) Spot and Forward exchange Rates, measured in logarithms, January 2, 1976 to January 2, 1985. Number of observations = 458
   (b) Real exchange rate, measured in logarithms, July 1973 to December 1986. Source, CITIBANK databank, December 1987.
   (c) Velocity, measured in logarithms, March 1959 to December 1986

2. See Table 1 for the critical values of the statistics.

3. Some of the series were passed through an AR(3) filter in order to equalize the periodogram ordinates around frequency zero. These series are tagged by "f" in the table. The periodogram itself was estimated using the Daniell estimator with k = 7.

BIBLIOGRAPHY

Bhargava, A. (1986). "On the Theory of Testing for Unit Roots in Observed Time Series," *Review of Economic Studies*, 53, 9-384.

Campbell, J. Y. and N. G Mankiw (1986) "Are Output Fluctuations Transitory ?", National Bureau of Economic Research Working Paper No. 1916, May 1986.

Campbell, J. Y. and N. G. Mankiw (1987) "Permanent and Transitory Components in Macroeconomic Fluctiations," *American Economic Review, Papers and Proceedings*, 111-117.

Chan, K. Hung, J. C. and J. K. Ord (1977), "A Note on Trend Removal Methods: The case of Polynomial versus Variable Differencing, *Econometrica*, 45, 737 - 744.

Cochrane, J. H. (1986) "How Big is the Random Walk in GNP?", mimeo, University of Chicago.

Corbae P. D. and Ouliaris, S. (1986) "Robust Tests for Unit Roots in the Foreign Exchange Market," *Economics Letters*, Vol 22. pp. 375-80.

Corbae P. D. and Ouliaris, S. (1987) "Cointegration and Tests of Purchasing Power Parity", forthcoming *Review of Economics and Statistics*.

Corbae P. D., S. Ouliaris, and Zender, J. (1987) "Testing a Necessary Condition for Efficiency in the Forward Exchange Market,", University of Maryland Working Paper No. 25, College Park, MD.

Dickey D. A. and W. A. Fuller (1979) "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Association*, 74, 427-431.

______________________________, (1981) "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root", *Econometrica*, 49, 1057-1072.

Durlauf, S. and P. C. B. Phillips (1987) "Trends versus Random Walks in Times Series Analysis," forthcoming *Econometrica*

Engle, R. F. and C. W. J. Granger(1987). "Co-Integration and Error Correction: Representation, Estimation and Testing," *Econometrica*, Vol. 55, No. 2, 251 - 276.

Hall, P. and C. C. Heyde (1980). *Martingale Limit Theory and Its Applications*. New York: John Wiley.

Nelson, C. R. and H. Kang (1982), "Spurious Periodicity in Inappropriately Detrended Time Series", *Econometrica*, 49, 741-751.

Nelson, C. R. and C.I. Plosser (1982), "Trends and Random Walks in Macroeconomic Times Series," *Journal of Monetary Economics*, 10, 132-162.

Park, J. Y. and P. C. B. Phillips (1986). "Statistical Inference in Regressions with Integrated Processes: Part I," Cowles Foundation Discussion Paper, No. 811.

Park, J. Y. and P. C. B. Phillips (1987). "Statistical Inference in Regressions with Integrated Processes, Part II" Cowles Foundation Discussion Paper No. 819.

Perron, P. (1986) "Trend and Random Walks in Macroeconomic Time Series: Further Evidence from a New Approach," Department of Economics, University de Montreal, October, 1986.

Perron, P. (1987) "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis," Department of Economics, University de Montreal, October, 1987.

Phillips (1987). "Time Series Regression with Unit Roots," *Econometrica*, Vol. 55, No. 2, 277 - 302.

__________ (1987b) "Multiple Regression with Integrated Processes," IMS/SIAM Conference Volume (forthcoming)

__________ and S. N. Durlauf (1986) "Multiple Time Series with Integrated Variables," *Review of Economic Studies*, 53, 473-496.

__________________ and Ouliaris, S.(1987) "Asymptotic Properties of Residual Based Tests for Cointegration," Cowles Foundation Discussion Paper No. 847.

________ (1988). "Weak Convergence to the Matrix Stochastic Integral $\int_0^1 BdB'$," *Journal of Multivariate Analysis*, 24, 252-264.

__________________ and Perron (1988) "Testing for a Unit Root in Time Series Regression," *Biometrika*, 75, 335 - 346.

__________________ and Ouliaris, S. (1988) "Testing for Cointegration using Principal Components Methods," *Journal of Economic Dynamics and Control*, pp. 1-26.

Said, S. E. and D. A. Dickey (1984) "Testing for Unit Roots in Autoregressive-Moving Average Models of Unknown Order," *Biometrika*, 71, 599-607.