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Measurements of Particle Motions in Ocean Waves

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ABSTRACT

Ducted impeller wave meters used aboard a light station to observe particle motions in wind waves at various depths beneath the free surface are described. The motions recorded display quasisinusoidal oscillations having an approximate $90^\circ$ phase shift between the horizontal and vertical velocity components that lie normal to the wave crests. The wave kinetic energy associated with the variances in the wave motions shows a strong depth attenuation, but this attenuation is not as great as the exponential energy decrease exhibited by a classical trochoidal wave. Integrals of the wave kinetic energy contained in the water column were approximately equal to the estimated potential energy of the waves.

Introduction. The majority of ocean-wave studies, aside from theoretical modeling, have involved measurement and analysis of the kinetic properties of sea-surface oscillations. Most often, free surface records have been obtained either directly with wave-staff devices or bobbing floats or indirectly through use of slope detectors, optical glitter patterns, or bottom-mounted pressure sensors. These records have then been subjected to statistical or spectral analysis. These methods, however, severely limit description of the internal dynamic motions of wind waves. The problem of direct observation of particle motions in ocean waves has, for the most part, been neglected or avoided, probably because of obvious instrumentation difficulties. However, it is likely that proper assessment of kinetic energy and momentum transfer processes associated with the motions can be achieved by only direct observation of motions in waves (and also the wind directly over the waves).

1. This paper is based on a Doctor of Science thesis submitted to the Department of Meteorology at Massachusetts Institute of Technology, June 1966.

2. Present address.
Figure 1. Orthogonally mounted ducted meters.
Figure 2. Collinearly mounted ducted meters.
A pilot investigation of particle motions in real ocean waves, using simple ducted impeller devices, has been described (Shonting 1964). The present paper describes the improved wave-meter systems and some recent field observations.

**Instrumentation.** Wave motions were measured with a modification of a previously described wave-meter system (Shonting 1964). The basic device is a ducted impeller, housed in a brass right-circular cylinder, 15 cm long and 8.5 cm in diameter. Fig. 1 shows two such cylinders orthogonally mounted. Axially positioned within each cylinder is a six-bladed impeller that spins in response to water flow through the cylinder. The impeller-shaft bearings, which consist of small carbide pins seated in quartz V-bearings, are mounted against neoprene cushions. The impeller-shaft bearings are supported by thin bronze struts at the ends of the rotor shaft; the impellers are fabricated of neutrally buoyant micarta (laminated phenol formaldehyde), providing minimum bearing load when immersed in seawater. The blades are designed so that the area projection parallel to the cylinder axis fills completely the cross-sectional area of the cylinder, providing a maximum torque-to-flow ratio.

Small magnets, weighing about 5 g, are mounted at the external edges of the blades. As the blades spin, the tiny magnets interact with a miniature induction coil potted with epoxy resin on the side of the cylinder (see Fig. 1). The cutting of the magnetic field of the magnets by the coil produces a 5 to 30 mv pulse across the coil leads, according to the angular velocity of the impeller. Because of the asymmetric placement of the coil with respect to the orbital path of the magnets on the blade tips, the sign of the voltage indicates the sense of impeller rotation. Thus, the frequency and sign of the pulses are indicative of the rate and direction of fluid flow through the cylinder. The voltage output, which needs no amplification, is relayed through a shielded water-tight cable to a recording oscillograph.

A detailed description of the instrument will be made available by the author. A slightly modified version of the instrument is to be manufactured by Braincon Corp., Marion, Massachusetts, USA.

**Calibration.** It was desired to utilize the ducted meters to measure the two orthogonal wave-motion components, $u$ (in the direction of the wave propagation, defined as the positive-x direction) and $w$ (the vertical direction, defined as z-positive upward). Also, it was of interest to measure a single-velocity component, either $u$ or $w$, simultaneously at two depths in the wave regime. For measurements of $u$ and $w$ at one depth, the orthogonal configuration (Fig. 1) was used, and for observing a single component at two depths, the collinear configuration was used (Fig. 2).

Calibration of the ducted meters required that the flow-response characteristics of each individually mounted meter and then of the orthogonally mounted system be determined.
The individual wave meters were towed in a water tank at various steady speeds parallel to the cylinder axis and at various off-angles (the angles subtended by the cylindrical axis and tow direction). The angular velocity of the impeller was proportional to fluid flow through the cylinder. The flow velocities derived from the individual ducted meters when towed at off-angles varied approximately as the cosine of the off-angle. Thus the meter approximatively resolved the flow component parallel to the cylinder. The calibration curves for two identically constructed cylinders were identical within the errors of measurement. The threshold of flow velocity at which the impellers commenced to spin was about 5 to 7 cm sec⁻¹.

When the cylinders were mounted orthogonally (as in Fig. 1), they provided approximate resolution of the horizontal \( (u) \) and vertical \( (w) \) velocity components lying in the vertical plane as defined by the axis of the \( u \) and \( w \) cylinders. A deviation from the cosine law existed in the case of the orthogonally mounted meters, which could produce up to 10% error in the estimate of the instantaneous velocity vector. This error was a definite function of the off-angle \( \theta \), but it was independent of the absolute speed of the flow (from 0 to 150 cm sec⁻¹) normally associated with wave-particle velocities.

It was desirable to try to correct the error in the velocity-component resolution caused by the deviation from the cosine law. Since the error in the individual-component estimate was known to be a function of \( \theta \), it was useful to determine \( \theta \) uniquely from the outputs of the orthogonal meters. The ratio of the output velocities \( (u/w) \) as a function of \( \theta \) was determined as shown in Fig. 3; the numbers in parentheses indicate the number of samples (each at a different towing speed), and the vertical bars indicate the spread of one standard deviation above and below the mean (±\( \sigma \)). The experimental points clearly follow the curve for \( \tan \theta \) (broken line), further suggesting that the impeller response follows closely the cosine law.

Since the output of the two orthogonal meters was a unique function of the off-angle and was independent of the towing speed, the ratio of the outputs of the two \( u \) and \( w \) meters (according to Fig. 3) determined a unique value of \( \theta \). The error correction could be applied to each of the meter outputs (i.e., one correction at \( \theta \) for \( u \) and another at \( 90^\circ - \theta \) for \( w \)). The error correction was applied to all time-series pairs of \( u \) and \( w \) obtained from the orthogonally mounted system.

The response time of the impellers was required in order to estimate the most rapid motional fluctuations that could be detected with the system. Two methods were used to estimate the frequency response. First, the orthogonal system was vertically oscillated in a water tank in approximately sinusoidal fashion at about 1.3 cps. An accelerometer was affixed to the wave meter to monitor the oscillations independently. In Fig. 4, a sample of the velocity record of the wave meter (obtained from the calibration curve for steady flow) is compared with the accelerometer output. Using a simple harmonic oscillator
model, the measured phase shift between the wave-meter record and the derivative of the accelerometer output indicated a response time of the order of 50 to 70 millisec.

For the second frequency-response evaluation, the NUWS wind tunnel was employed. The individual ducted meters were axially oriented with respect to the direction of moving air in the tunnel. A card placed over the upwind cylinder opening was suddenly removed. The voltage-pulse traces displayed an exponen-

Figure 3. Ratio of horizontal to vertical uncorrected velocities as a function of off-angle $\theta$. 
tial increase in frequency until equilibrium rotation was attained, and the time for the velocity to attain \((1 - e^{-t})\) of the tunnel wind speed was graphically estimated. Taking into account the dynamic similitude relationships associated with drag forces, both in air and water, the time response in water was estimated to be in the range of 40 to 70 millisec.

The apparent rapid response of the impellers demonstrates a unique feature of the ducted meter. A pressure perturbation imparted at the entrance of the cylinder is transmitted almost instantaneously to the impellers. This transmission is very efficient, since the impellers, when not rotating, completely block the flow, and the cylinder wall precludes divergent flow around the impellers (as can occur with an open or unshrouded impeller). This rapid response to motional perturbations indicates that this system has potential for measuring relatively high-frequency turbulence phenomena in the ocean.

Field Observations. The wave observations were made at the Buzzards Bay Entrance Light Station (BBELS) located at 41°23.8'N and 71°02.1'W, about 12 km south of the Massachusetts Coast. The tower, situated in 20 m of water, is a rigid structure having four legs (1 m in diameter) that offer little interference to the passing ambient wind waves and swell. The local wind waves are predominately generated by winds from the SSW to the NW direction, depending upon seasonal variations in the prevailing winds. The swell, when present, radiates from the open ocean to the south. For further description of this facility, see Shonting (1966).
To measure the two-dimensional wave motions \((u\) and \(w)\), the orthogonal wave-meter system was positioned at various depths beneath the free surface of the waves so that the axis of the \(u\) meter was aimed in the horizontal direction normal to the wave crests; the other cylinder was in a vertical position. The objective was to measure free flow through the cylinders in the vertical plane normal to the wave crests.

Ideally, the wave meter should have been rigidly attached to the tower. However, this was not practical because the working platform is about 22 m from the sea surface, and no staging was available below this level from which to fasten a rigid support structure. In lieu of a rigidly mounted wave-meter system, a wire suspension was used. The wave meter was supported by a pyramidal configuration of wire guys and counterweights, shown in Fig. 5. The wave meter was supported on a T-member having a cross bar 3 m long, the ends of which were fastened with guy wires to the north and south corners of the tower. These guys, when drawn taut, maintained a fixed azimuth of the \(u\) meter by preventing its rotation about the vertical axis.

The main guy, which served as the lowering cable, extended from a hand winch on the catwalk to the top of the T-bar support frame. A 20-kg weight was suspended on a 4-m wire pennant below the wave meter.
Vertical oscillations were completely damped because of the great vertical stability provided by the counterweight suspended below the meter. For added horizontal stability, a back guy and a taut vertical guy were anchored to 160 kg of lead. The T-frame was attached to the anchored guy by means of Nansen-bottle messengers fastened with steel hose clamps to the vertical support cable and attached to the T-frame above and below the wave meter. This not only allowed the meter to slide freely up and down the wire but severely damped the horizontal motion. The estimated horizontal swing of the meters had a maximum amplitude of about 5 to 10 cm when the wave meter was immediately beneath the free surface in the presence of large waves (i.e., waves 1.50 to 2.0 m in height). This motion was strongly damped, however, at greater depths because of the attenuated motion.

This suspended system allowed rapid changing of the depth to be measured and permitted the instrument to be suspended about 8 m away from the nearest legs and cross members—two features that would have been difficult to arrange with a rigidly mounted support system attached to a leg of the tower.

In practice, for the \( u \), \( w \) records, the \( u \) meter was aimed approximately normal to the wave crests and lowered to the desired level; the guys were then made taut.

The depth of the wave meter was defined relative to the level of the deepest observed wave trough, and this depth was monitored by a meter wheel from which the main guy was fairied from the hand winch on the catwalk.

Measurements were also made with the two cylindrical meters in a collinear configuration (Fig. 2). The rod was suspended on a single vertical guy, with a counterweight attached to a 4-m pennant leading to the base of the rod. For observations of \( w \) at two levels, the horizontal bar was not required, since slight rotation about the vertical axis would not affect the \( w \) measurement. For all measurements, the wave meter was placed on the upwind (i.e., the up-wave) side of the platform. During the periods of measurement, wind-velocity and sea-state conditions were monitored.

The electrical signal from each of the \( u \) and \( w \) pickup coils was cabled to a Sanborn 322 two-channel strip-chart recorder in the laboratory housing on the tower (see Fig. 5 insert). The records were read with a scanning tele-reader, which first converts the time interval of the voltage pulses and their sign into digital values and then places this information on punched cards. The card sequence was converted to velocity values as a function of time; then it was machine interpolated at equally spaced time intervals. The statistical analysis included computation of mean values, variances, auto-covariances, covariance functions, and auto-covariance and covariance spectra. The data processing was done on the Naval Underwater Weapons Station CDC 3200 digital computer.
Results and Discussion. The present review of wave-observation data that have actually been taken is not complete. Moreover, this discussion is intended merely to demonstrate the various types of wave phenomena that can be studied with wave meters. Only a few examples to highlight the general results are presented here.

Fig. 6 is a segment of a 5-minute record of the two components, $u$ and $w$, measured with the orthogonal system positioned at 2 m below the wave-trough level. During this observation the wind speed averaged 5.8 m sec$^{-1}$. The traces were made from the linear interpolation (every 0.2 sec) of the voltage-pulse output of the wave meter, transformed to corrected velocities from the calibration data.

Note the similarity in the oscillations of the velocity components with regard to both amplitude and frequency. Note also the approximate 90°-phase difference, as would appear with sine- and cosine-like wave components.

Fig. 7 shows a similar segment of a record of the vertical velocity, $w$, measured with the collinear ducted meters at the trough level and at a 2-m depth. There is a strong in-phase relationship between the two $w$ records; there is also a definite depth attenuation of amplitude of the oscillatory motion as displayed by the 2-m velocity trace with respect to the surface value. The wave records often display a slowly varying amplitude modulation of the velocity
components (as is shown in Fig. 7 at 5 to 25 sec). This phenomenon is generally observed in wave records of free-surface oscillations.

The portrayal of the velocity data in Figs. 6 and 7 indicates that, as with the free-surface elevation, the wave velocities measured at a point are not particularly sinusoidal. Note the steep slopes and jagged narrow troughs.

An important property of the time-series data of the wave motion is the variances of $u$ and $w$. These variances may be defined in terms of the Reynolds formulation of the mean velocities and fluctuation velocities of $u$ and $w$ (see Lamb 1945: art. 369):

$$u = \bar{u} + u' \quad \text{and} \quad w = \bar{w} + w';$$

(1)

here the barred terms are time averages of the quantity and the prime terms are deviations about the means so that $\bar{u}' = \bar{w}' = 0$. Normally, for surface waves, $\bar{w} = 0$ is assumed. The variances of $u$ and $w$ are thus written as $\sigma_u^2$ and $\sigma_w^2$ or $\sigma_u^2$ and $\sigma_w^2$. The terms $\rho \sigma_u^2$ and $\rho \sigma_w^2$, where $\rho$ is density, are equal to dynamic pressures (dyne cm$^{-2}$) or to the flow of momentum per unit area through the water associated with the fluctuations of $u$ and $w$, respectively. The above terms are proportional to the kinetic energy of the oscillatory or turbulent motions of the waves, so that evaluation of the variances of $u$ and $w$
can provide estimates of the distribution of wave kinetic energy in the water column.

Fig. 8 shows a plot of the variance $w'^2$ ($\text{cm}^2 \text{ sec}^{-2}$) measured as a function of depth for three different periods during which the wind (and hence the wave) conditions were similar. (The solid curves represent the variance as a function of depth of a classical trochoidal wave discussed below.)

The values of $w'^2$ at the surface range from 890 to 1360 cm$^2$ sec$^{-2}$. Beneath the surface, the variances, showing less scatter, decrease rapidly with depth, attaining approximately one-half the surface value at 2 m. The variance distribution falls off asymptotically, with a value of about 50 to 70 cm$^2$ sec$^{-2}$ at a depth of 10 m.

The variance appears to display an exponential decrease with depth. The relatively large-range surface values (above 2 m) are probably associated with the inability to position the wave meter for each measurement at precisely the same depth beneath the wave troughs. Thus, at (or near) the surface a slight error in vertical positioning of the instrument produces relatively large error in the variance.

It is of interest to compare the depth variations of the calculated variance with a theoretical wave model. Let us compare the observed behavior of the
variance with that obtained from the vertical velocity function of a simple irrotational wave of finite amplitude (see, for example, Proudman 1953: chap. 16). This function is given as a third-order approximation of a trochoidally shaped wave:

\[
\omega_i' = -\frac{2\pi A}{T} \left[ 1 - \frac{5}{2} \left( \frac{\pi A}{L} \right)^2 \right] \exp \frac{2\pi z}{L} \sin 2\pi \left( \frac{x_0}{L} - \frac{t}{T} \right),
\]

where \( \omega_i'(x_0, z, t) \) is the variation of the theoretical vertical-velocity component about a zero mean, \( A \) is the amplitude of the wave, \( T \) is the period, and \( L \) is the wavelength. The coordinate, \( x_0 \), is a constant indicating that the measurement of \( \omega_i' \) is made at a fixed horizontal position at various depths, \( z \). The corresponding horizontal component, \( u_i' \), is identical to relation (2) except for a plus sign and a cosine instead of a sine. Hence, the vorticity of this model, \( \omega = (\partial u/\partial z) - (\partial \omega/\partial x) \), is zero.

This model is derived with the assumption of a realistic ratio of wave amplitude to wavelength. The wave produced is somewhat unique in that, for any level, the time and depth variations of pressure and particle-velocity components are purely harmonic to the third order. However, the surface elevation is trochoidally shaped with steeper crests and flatter troughs than a pure sinusoidal wave form.

We define the theoretical variance function of \( \omega_i' \) in eq. (2) as:

\[
\overline{\omega_i'^2} = \frac{K}{T_s} \int_{-T_s/2}^{T_s/2} \sin^2 2\pi \left( \frac{x_0}{L} - \frac{t}{T} \right) dt,
\]

where

\[
K = \frac{4\pi^2 A^2}{T^2} \left[ 1 - \frac{5}{2} \left( \frac{\pi A}{L} \right)^2 \right] \exp \frac{4\pi z}{L},
\]

and \( T_s \) is the period over which the time average is made. We can integrate eq. (3), obtaining:

\[
\overline{\omega_i'^2} = \frac{K}{2} + \frac{KT}{2\pi T_s} \sin 4\pi \left( \frac{x_0}{L} - \frac{t}{T} \right) \bigg|_{-T_s/2}^{T_s/2}.
\]

Since \( T_s \gg T \), we may neglect the second term of the expression, obtaining:

\[
\overline{\omega_i'^2} = \frac{2\pi^2 A^2}{T^2} \left[ 1 - \frac{5}{2} \left( \frac{\pi A}{L} \right)^2 \right]^2 \exp \frac{4\pi z}{L}. \quad (5)
\]

In order to compare the depth attenuation of the theoretical variance of the trochoidal wave with that of the observed variance, we must attempt to evaluate the coefficient of the exponential term in eq. (5). This is difficult,
since the parameters $T$, $L$, and $A$ cannot be uniquely defined for a real sea. The variance calculated from (5) for zero depth is about $1100 \text{ cm}^2 \text{sec}^{-2}$, using the following values of wave parameters: period $T = 4.0 \text{ sec}$, wavelength $L = 18.0 \text{ m}$, and amplitude $A = 30 \text{ cm}$. The reasons for the choice of parameters are as follows. The autospectra of the near-surface time-series records (i.e., the records whose variances appear at, and above, 2 m in Fig. 8) display the dominant wave-energy peak at, or close to, 4.0 sec. Also, the periods observed by eye (using a stop watch to time crest passages) averaged from 3.5 to 4 sec. If we refer to the table of Marks (1964) and assume $T = 4.0 \text{ sec}$ to be a good “average” value of the wave periods, then we obtain an average wavelength of $L = 18.0 \text{ m}$ and an average wave amplitude of $A = 30 \text{ cm}$. Furthermore, the range of observed average wind speeds of BBELS - 5 and - 7 (indicated in Fig. 8) nearly coincides with the range of wind speeds in the Marks table (i.e., 5.6 to 8.2 m sec$^{-1}$) and is appropriate for the chosen wave parameters.

The variance curve is plotted as the solid line in Fig. 4. The surface value falls within the range of observed values. The curve is similar in shape to the observed variance pattern. However, the observed variances consistently remain to the right of the curve. Below 4 to 5 m the theoretical values of variance diminish faster than do the measured values. This seems plausible, since the sinusoidal oscillations of the irrotational classical waves transfer no momentum downward (due to the absence of any Reynolds stresses caused by turbulent diffusion), as the actual wave seems to do (Shonting 1964). On the other hand, the ocean waves cannot be completely irrotational, and by their very nature they serve to transfer turbulent energy statistically downward. This is suggested by the residual variance existing well below the 4 to 6 m depths. (This apparent turbulence at the deeper depths could also be attributed to the interaction of the local tidal currents with the sea bottom. If this is the case, then the content of the turbulent energy should vary measurably over the tidal cycle.)

It should be noted that the main difficulty with the comparison in Fig. 8 is that the real variances are associated with the motions of the whole spectrum of waves (detectable by the instrument) while the theoretical curve is for a single wave. It is obvious that the real variances, at least near the surface, should be larger than those attributed to a single class of waves. This is because of the contribution of the motions of the small waves, which are ever present with the larger waves. These small-wave motions would naturally diminish rapidly with depth.

Note also that the observed variance function essentially characterizes only the gross motional fluctuations in the waves. The lower limit of the size of eddies contributing to the variances in the velocity data is determined largely by the size of the wave meter; i.e., 15 to 18 cm. It is possible that the residual variance at deeper levels may be caused by interaction of the mean horizontal
Figure 9. The distribution of the variances, $\bar{w}^2$, with depth at various wind speeds for all BBELS observations.
Shonting: Measurements of Particle Motions

177

tidal flow with the meter, artificially generating turbulence about the meter that is then registered. The calibration tests, however, do not favor this possibility for steady flow or for slowly varying flows up to at least 115 cm sec$^{-1}$, since no response was detected in the impeller, whose axis was normal to the towing velocity.

It is evident that the comparisons in Fig. 8, however crude, do suggest similarities between theoretical and observed waves. Further comparisons should be made with more precisely obtained data.

To demonstrate the generality of the wave kinetic-energy attenuation with depth, the results of over 90 observations have been plotted in Fig. 9. Here the numbers represent the numerical value of the wind speed (m sec$^{-1}$), and their positions represent the variance, $\overline{w^2}$, as a function of depth. These values were obtained generally during fully developed seas.

The numerical wind speed is shown in an attempt to see whether the higher values produced higher variances for a given depth. The distribution of $\overline{w^2}$ is actually indicative of this, particularly at depths between 0 and 2.5 m. Thus, for a given depth, the wind speeds tend to increase upward on the ordinate. At depths beyond 5 m the relationship becomes uncertain.

Let us further examine the variances as they pertain to the vertical distribution of wave energy. Noting eq. (1), the total kinetic energy within a vertical column of water of depth $D$ and of unit cross-sectional area is given by:

\[
E_{KT} = \frac{1}{2} \rho \int_{-D}^{0} (\overline{\eta^2} + \overline{w^2}) \, dz + \frac{1}{2} \rho \int_{-D}^{0} (\overline{u^2} + \overline{w^2}) \, dz ,
\]

where $\overline{\eta}$ is the mean free surface and $E_{KT}$ is in units of ergs cm$^{-2}$. Integral $A$ is the kinetic energy associated with mean motion and $B$ is the kinetic energy of the turbulent or wave-induced oscillatory motions.

In our problem of two dimensional wave motion, $\overline{w} = 0$; and $\overline{u}$ is the mean current component in the direction of wave propagation. The upper limit of the integrals in eq. (6) is the mean free surface, which we assume is at $z = 0$. Thus, the turbulent kinetic energy associated with two-dimensional wave motion is given by:

\[
E_{KW} = \frac{1}{2} \rho \int_{-D}^{0} (\overline{u^2} + \overline{w^2}) \, dz .
\]

From our wave measurements we have available distributions of $\overline{u^2}$ and $\overline{w^2}$ as a function of depth; hence, by numerical integration of eq. (7), we can estimate $E_{KW}$.

Six sets of wave observations made from BBELS were used to calculate values of $E_{KW}$. The results are given in Table I, which lists the wind velocity, $V$ (m sec$^{-1}$), the depth of numerical integration, $D$ (m), and $E_{KW}$ (erg cm$^{-2}$).
Table I. Comparison of wave kinetic energy estimated by vertical integration of the variances of the wave motions with estimates of wave potential energy inferred from wave heights and wave lengths utilizing data from Marks (1964).

<table>
<thead>
<tr>
<th>Series components measured</th>
<th>Wind speed components measured</th>
<th>$E_{KW}$</th>
<th>$L$ (m)</th>
<th>$0.5H_m$</th>
<th>$E_p(0.5H_m)$</th>
<th>$H(L)$</th>
<th>$E_p(H_L)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBELS-5 $(u, w)$</td>
<td></td>
<td>6.8 WSW</td>
<td>1.82</td>
<td>10.5</td>
<td>38</td>
<td>1.77</td>
<td>37</td>
</tr>
<tr>
<td>(004-008)</td>
<td></td>
<td>6.9 WSW</td>
<td>1.32</td>
<td>10.5</td>
<td>38</td>
<td>1.77</td>
<td>37</td>
</tr>
<tr>
<td>BBELS-7 $(u, w)$</td>
<td></td>
<td>6.7 SSW</td>
<td>1.67</td>
<td>12.5</td>
<td>53</td>
<td>3.44</td>
<td>44</td>
</tr>
<tr>
<td>(009-016, 018-019)</td>
<td></td>
<td>9.5 ENE</td>
<td>4.10</td>
<td>12.5</td>
<td>55</td>
<td>3.71</td>
<td>65</td>
</tr>
<tr>
<td>BBELS-11 $(w)$</td>
<td></td>
<td>15 WNW</td>
<td>6.07</td>
<td>17.5</td>
<td>65</td>
<td>5.18</td>
<td>65</td>
</tr>
<tr>
<td>(032-047)</td>
<td></td>
<td>9.6 SSW</td>
<td>3.21</td>
<td>20</td>
<td>55</td>
<td>3.71</td>
<td>79</td>
</tr>
</tbody>
</table>

Note that, for BBELS-5 and -7, both $u$ and $w$ were obtained at various depths; hence, eq. (7) was used to estimate $E_{KW}$. However, with BBELS-11 and -14, only values of $w$ were obtained; thus the following integral was used:

$$E'_{KW} = \rho \int_0^\infty \overline{u^2}dz,$$

where it is assumed in eq. (7) that $\overline{u^2} \approx \overline{w^2}$, since these estimates of $E_{KW}$ are merely approximations used to indicate the orders of magnitude of the wave energy. It is noteworthy that the $E_{KW}$ values are similar in magnitude to wave energies tabulated by Stewart (1961) for similar wind conditions.

Let us compare further the measured kinetic energy with estimates of potential energy of the waves. In the classical theory of progressive surface waves (Lamb 1945: art. 230), the total energy per unit of horizontal area is given by

$$E_T = \rho gA^2,$$

where $g$ is the acceleration of gravity, $\rho$ and $A$ being defined as before. The potential energy, $E_p$, equals the kinetic energy, $E_K$, or:

$$E_K = E_p = \frac{E_T}{2}.$$
were estimated using values of $H$ and eq. (9). The values obtained are manifestly very subjective, and statistical interpretation of them is difficult. The wave-height value was estimated visually from the lower catwalk (shown in Fig. 5), about 18 m above the water. If these observed values had been averaged, they probably would have given the approximate highest waves, $H$ (see further Pearson et al. 1955 and Kinsman 1965). A somewhat arbitrary decision was made to definite the average wave height as $0.5 H_m$. These values of wave are listed in Table I.

The calculated potential energies range from 1.77 to 5.18 erg cm$^{-2}$ and are very similar in magnitude to the kinetic energy values $E_{KW}$. Note that in BBELS-5 and -11, $E_p(H/2)$ shows similar increases with the respective values of $E_{KW}$.

A reasonable criticism is that the choice of the average wave heights was arbitrarily adjusted so as to produce similar magnitudes of $E_K$ and $E_p(H/2)$. As a check on the validity of the estimated wave sizes, another method was used. The estimated wavelengths were probably more accurate than the estimated heights shown in Table I, since the distance between the main west legs of the platform (15 m) could be used as a reference at sea level. From observed values of $L$, interpolated corresponding values of average wave heights ($H_L$) were obtained from a table of wave properties constructed by Marks (1964). These values of $H_L$ are very similar to those of $H/2$; likewise, the corresponding calculated values of $E_p(H_L)$ are still very similar to those of $E_p(H/2)$.

It is thus shown that values of estimated potential energy of wind waves are very similar to the wave kinetic energy estimated from direct measurements of the wave-particle velocities. Obviously, the value of $E_p$ in eq. (9) is sensitive to the value of $A$, hence errors in choosing $A$ values could easily result in errors of $E_p$ by a factor of four. However, this seems not to have happened, and the important point is that $E_p(H/2)$ and $E_p(H_L)$ are of the same orders of magnitude as $E_{KW}$ estimated from measurements. These data not only lend credence to the utilization of wave meters as dynamic probes for waves but suggest further that an approximate equipartitioning of wave potential and kinetic energy actually occurs in the real ocean.

A forthcoming publication (Shonting, manuscript) presents a discussion of the wave-motion data in terms of energy spectra. Another paper is being prepared regarding the covariance and cospectral properties of the $(u, w)$ time-series data.

Tests are being conducted on an improved version of the above-described ducted meter that is slightly smaller, has a more rapid response time, and provides a velocity threshold of less than 2 cm sec$^{-1}$.

Conclusions. 1. A system has been described with which the dominant vertical and/or horizontal motions within ocean waves can be resolved and
recorded to frequencies as high as 3 to 5 cps. Depending upon the configuration, orthogonal wave components at one depth or similar wave components at two depths can be measured simultaneously.

2. Observations indicate the quasiharmonic relationship of the orthogonal horizontal downwind and vertical wave components, $u$ and $w$, these being roughly 90° out of phase.

3. The vertical wave-motion component, $w$, simultaneously observed at two depths, shows a strong in-phase relationship and the attenuation of the velocity amplitude with depth.

4. Observations on the variance of the vertical velocity display an exponential decrease in wave kinetic energy with depth.

5. A direct estimate of the wave kinetic energy has been obtained by integrating the variances of the wave motions over the water column. The values obtained are very similar to the potential energy of the waves estimated from their heights and wavelengths. The results indicate a partitioning of the total wave energy into equal amounts of kinetic and potential energy predicted by classical theory.

6. It is evident that further and more careful examination of wave motions, using the described type of ducted meter and perhaps more sophisticated versions of it, should provide unique information on the statistical properties of wave motions.

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