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Silver and Gold and Liquidity

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A simple model with trade in gold is explored where the cost of liquidity is measured in terms of utility foregone by using the gold as a money or means of payment rather than for utilitarian purposes. We close with remarks on the use of both silver and gold.

As the model is extremely simple rather than strive for misplaced realism the analysis is carried out in terms of a simple playable game where gold is used as a means of payment and at the start of every period each individual can divide her holdings into gold used for money and jewelry. Jewelry can be converted to coin and vice-versa at the start of each period costlessly.

The full formulation of a general game requires considerable detail. However for simplicity in order to illustrate the properties of the supply of money, liquidity and price levels our comments are confined to a simple example.

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There are two types of traders and three goods. All traders have a utility function of the form

$$(1) \qquad \phi_i = \sum_{t=1}^{T} \beta^{t-1}\left\{\sqrt{x_t^i y_t^i} + k \log z_t^i\right\}, \quad i = 1, 2 ; \quad k > 0 .$$

The first two commodities are perishable, but are replenished each period. The third is infinitely durable and each trader is given an initial quantity denoted by A .

Traders of Type 1 (2) obtain at the start of each period an amount (2,0) (or (0,2)) of the first two goods.

By inspection the general equilibrium solution is that each period all traders consume $(1,1,A)$. Thus the payoff to each is

$$(2) \qquad \phi = \frac{1 - \beta^{T+1}}{1-\beta}(1 + k \log A) .$$

For $T \to \infty$ ϕ approaches

$$(3) \qquad \frac{1}{1-\beta}(1 + k \log A) .$$

We now compare outcome (3) with a strategic market game with two types of traders all of whom are small and consider a type symmetric noncooperative equilibrium.[1]

A strategy by a trader i is a vector $(\tilde{q}_1^i, \tilde{q}_2^i, \tilde{b}^i)$ where $\tilde{q}_j^i = q_{j1}^i$, q_{j2}^i, q_{j3}^i, ..., q_{jT}^i , $\tilde{b}^i = b_1^i$, b_2^i, ..., b_T^i where for a trader of type 1

[1]The assumption of a continuum of traders allows us to assume that an individual cannot influence price. Type symmetry is an assumption that there may exist a solution where players of the same type choose the same strategy.

$0 \le q^1_{1t} \le 2$, $q^1_{2t} = 0$, $0 \le b^1_t \le M^1_t$ where M^1_t is the amount of gold held by a trader of type 1 at the start of t . Similarly for a trader of type 2.

The payoff to a trader of type 1 is given by:

$$(4) \qquad \pi(\tilde{q}^1_1, \tilde{q}^1_2, \tilde{b}^1) = \sum_{t=1}^{T} \beta^{t-1}\left\{\sqrt{(2 - q^1_{1t})b^1_t/p_{2t}} + k \log(M^1_t - b^1_t)\right\}$$

where

$$(5) \qquad p_{2t} = \int b^1_t / \int q^2_{2t}$$

$$(6) \qquad p_{1t} = \int b^2_t / \int q^1_{1t}$$

and $M^1_1 = A$ and

$$(7) \qquad M^1_{t+1} = M^1_t - b^1_t + p_{1t}q^1_{1t} .$$

There are similar expressions for traders of type 2.

Let $V_i(A)$ be the value of the infinite horizon game to a trader of type i . If there is a solution it should satisfy the two functional equations:

$$(8) \quad V_1(A) = \max_{\tilde{q}^1_1, \tilde{b}^1} \left[\sqrt{(2 - q^1_{1t})(b^1_t/p_{2t})} + k \log(A - b^1_t) + \beta V(A - b^1_t + p_{1t}q^1_{1t})\right]$$

and

$$(9) \quad V_2(A) = \max_{\tilde{q}^2_2, \tilde{b}^2} \left[\sqrt{(b^2_t/p_{1t})(2 - q^2_{2t})} + k \log(A - b^2_t) + \beta V(A - b^2_t + p_{2t}q^2_{2t})\right]$$

Assuming that there is a stationary state solution and that there is enough money to finance trade[2] then we can guess that $q^i_{jt} = 1$ and by symmetry expect that traders of each type will have the same solution except for an interchange of the roles of goods 1 and 2.

Thus the solution would be of the form

$$(10) \qquad V(A) = \frac{1}{1-\beta}[\sqrt{b/p} + k \log (A-b)]$$

where b and p remain to be determined.

At equilibrium we have $q = 1$ and hence $b/q = b/1 = p$; and at equilibrium

$$(11) \qquad \frac{1}{2\sqrt{bp}} = \frac{k}{A-b} \quad \text{or} \quad A-b = 2kb$$

hence

$$(12) \qquad p = A/(1+2k)$$

thus

$$(13) \qquad V = \frac{1}{1-\beta}\left[1 + k \log \frac{2kA}{1+2k}\right]$$

in contrast with the C.E. at

$$(14) \qquad CE = \frac{1}{1-\beta}[1 + k \log A] \ .$$

We can call the commodity money equilibrium a <u>gold backed C.E.</u>

<u>Liquidity and Inefficiency</u>

The utility loss from using a commodity money to cover the full value

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[2]We return to this point later.

of trade is the difference between (14) and (13) or

$$(15)\qquad \frac{k}{1-\beta}\log\frac{2k}{1+2k}\ .$$

Enough Money

The total value of money in trade in any period at a stationary equilibrium is given by

$$(16)\qquad b\,\frac{d\varphi}{db}\ \text{ or in this example }\ b\,\frac{k}{A-b}\ .$$

This is bounded between 0 and k. There will always be enough money to support a gold backed C.E. if b is feasible (i.e. is in the range $0 \le b \le A$) and satisfies

$$(17)\qquad \text{Total purchases } \frac{A}{1+2k} = \frac{bk}{A-b} \text{ the value of money spent,}$$

or

$$(18)\qquad b = A^2/(A + k(1+2k)) \le A$$

hence in this instance there is always enough money.

This is not generally true as can be seen from the example with a linear separable money of the form kz^i_t rather than $k \log z^i_t$. Equation (17) becomes

$$(19)\qquad \frac{A}{1+2k} = bk \quad\text{or}\quad b = \frac{A}{k(1+2k)}$$

but for $k < 1/2$, $b > A$ hence the gold backed C.E. is not feasible.

The Rate of Interest

No money market or banking or loan facilities have been modeled here. There is one link which is the forward price of gold. In this example, the rate of interest

$$(20) \qquad 1+\rho = 1/\beta$$

keeps borrowing and lending inactive at the stationary state. A distinction can easily be made here between 100% inside gold backed lending and the creation of credit.

Uncertainty and Price Fluctuations

It is straightforward to modify the model to include randomness in the nondurables or in the loss or gain of gold. Constraining randomness to endowments of perishables we could imagine that nature moves first then informs all traders. For example with two states with probability (1/2, 1/2) and supplies (1,0), (0,1) in state 1 or (3,0), (0,3) in state 2 we have the

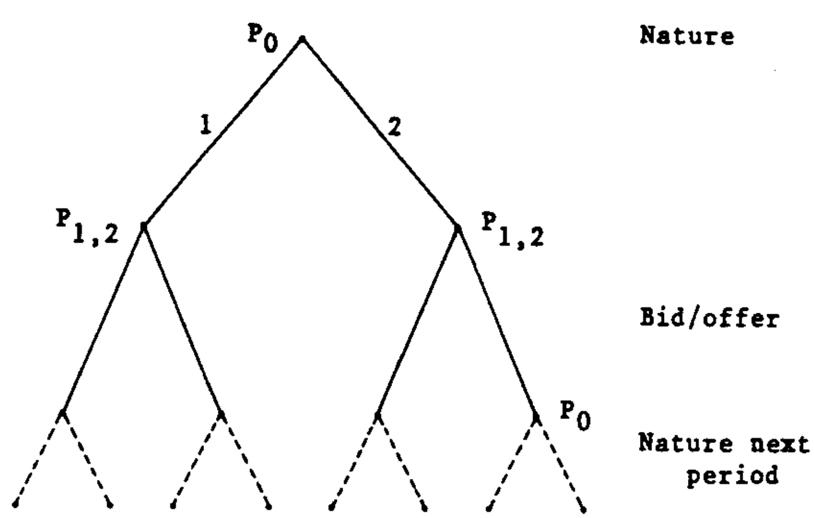

FIGURE 1

game in Figure 1. Qualitatively money provides insurance at the cost of liquidity.

Prices and Growth of the Money Supply

Suppose that the money supply grows at $\Delta\%$ per annum, then directly from (12) we have prices

$$p_t = \frac{A(1+\Delta)^t}{(1+2k)} . \tag{21}$$

The influence on price depends explicitly on the slope of the marginal utility for money.

On Velocity

A critique of models with the velocity of money of 1 or less in one shot trade is that this is unrealistic and that the "real world" velocity varies. A strategic mechanism view of exchange and production shows that there are many physical factors which leave little opportunity for great swings. Goods have to be produced, delivered and sold. Furthermore often if one set of traders changes payment timing another set will lose or gain. Thus to some extent variation of velocity is distributional in its influence on who pays for liquidity. It cannot be modeled in the example given above without the addition of considerably more detail.

Seasonals and Price Fluctuations

We can imagine instead of (0,2) and (2,0) as the endowments each period that there is a seasonal fluctuation such that in alternating periods supplies are (0,1), (1,0) and (0,3), (3,0). It is easy to modify (8) to (10) to see that a seasonal price fluctuation appears which also differentiates the one and two period interest rates.

Bankruptcy

In the simple models noted because no loan markets of any variety go active there is no need to model default as no one is a borrower or lender. This is no longer true with any credit even if at equilibrium there is no default. A partial discussion of the modeling of a bankruptcy penalty is given by Shubik and Wilson (1977) and Dubey and Shubik (1979) when there is no exogenous uncertainty an optimality penalty will be one which avoids strategic bankruptcy.

On Silver and Gold

We can easily modify equation (1) by adding another term say:

$$\phi_i = \sum_{t=1}^{T} \beta^{t-1}\left\{\sqrt{x_t^i y_t^i} + k_1 \log z_t^i + k_2 u(y_t^i)\right\} \tag{22}$$

where y_t^i is the amount of silver around and B is its initial supply. Separability is a first approximation for convenience. It is reasonably true that for both gold and silver world supplies are reasonably steady and grow over time with little if any losses. The problems with bimetallism can be illustrated by fixing the price between gold and silver.

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