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On the Influence of the Peripheral Antarctic Water Discharge on the Dynamics of the Circumpolar Current

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ABSTRACT

The discharge of water caused by melting, ablation, and glacial run-off along the coastline of Antarctica produces a westward zonal flow that could partly counteract the eastward wind-driven current.

1. Introduction. The Antarctic Ocean is singled out from the other world oceans by its unusual geometry as well as its dynamics. In particular, Munk and Palmén (1951) have shown that unless an unrealistically high value is assigned to the kinematic eddy viscosity, the eastward current that would be produced by the surface wind stress is much stronger than the observed current. Therefore, any theory of the Antarctic Circumpolar Current must include some mechanism that is capable of reducing the strong wind-driven current. Munk and Palmén have suggested that the bottom topography could produce such an effect. In the present paper it is shown that the peripheral water

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discharge due to melting, ablation, and glacial run-off could provide an additional "slow-down mechanism."

2. Laboratory Model. Consider a laboratory model of the Antarctic Ocean that consists of an annular region bounded by two vertical, concentric, porous, circular cylinders mounted on a rotating table. A uniform influx of water along the inner surface would model the peripheral water discharge from Antarctica. The outer cylinder, through which the water is discharged, can be thought of as the Antarctic Convergence belt, across which water characteristics are known to be discontinuous. Assuming that the Rossby number is small, the flow induced by the source-sink driving can be considered independently from that due to the wind stress. Finally, neglecting both baroclinic and $\beta$ effects, we can assume that the fluid filling the annular region is homogeneous and that the bottom surface is horizontal. We shall use this model to derive a formula for the zonal transport (Fig. 1).

Figure 1. Schematic diagram of the laboratory model. The sense of rotation is clockwise as in the southern hemisphere. The zonal current produced by the peripheral water discharge flows in the counterclockwise direction, i.e., in the direction opposite to that of the wind-driven current.
Most of the above assumptions, made for the sake of simplicity, could be relaxed considerably. In particular, the inner and outer cylinders can have arbitrary cross sections, and the discharge can have both peripheral and vertical variations without affecting our conclusions (Barcilon 1966). This would also be true of the $\beta$ effect, which can be easily accounted for by a slope of the bottom surface.

3. Westward Zonal Transport. Within the framework of this linear theory, the net zonal transport is equal to the algebraic sum of the transports due to the wind stress and the peripheral water influx. We shall derive a formula for only the latter, since an order-of-magnitude estimate of the former is given by Munk and Palmén (1951) and Hidaka and Tsuchiya (1953).

With all the foregoing assumptions, the equations of motion for an incompressible fluid in a frame rotating with a clockwise angular velocity $\Omega$ are

$$-2\Omega v' = -\frac{1}{\Omega} \frac{\partial p'}{\partial r'} + v_H \left\{ \frac{\partial^2}{\partial r'^2} + \frac{1}{r'} \frac{\partial}{\partial r'} - \frac{1}{r'^2} \right\} u' + v_v \frac{\partial^2 u'}{\partial z'^2}, \quad (1)$$

$$-2\Omega u' = v_H \left\{ \frac{\partial^2}{\partial r'^2} + \frac{1}{r'} \frac{\partial}{\partial r'} - \frac{1}{r'^2} \right\} v' + v_v \frac{\partial^2 v'}{\partial z'^2}, \quad (2)$$

$$0 = -\frac{1}{\Omega} \frac{\partial p'}{\partial z'} + v_H \left\{ \frac{\partial^2}{\partial r'^2} + \frac{1}{r'} \frac{\partial}{\partial r'} \right\} w' + v_v \frac{\partial^2 w'}{\partial z'^2}, \quad (3)$$

$$\frac{\partial u'}{\partial r'} + \frac{u'}{r'} + \frac{\partial w'}{\partial z'} = 0, \quad (4)$$

where $u'$, $v'$, $w'$ are the velocity components in the radial, zonal, and vertical directions ($r'$, $\theta$, $z'$), and $v_H$, $v_v$ are the horizontal and vertical eddy coefficients. We have also implicitly assumed that the motion is axially symmetrical, i.e., $\partial/\partial \theta = 0$. If $a$ and $b$ are the radii of the inner and outer cylinders, $d$ the depth of the fluid, and $Q$ the mass flux across the inner porous wall, we can introduce dimensionless variables as follows:

$$r' = ar, \quad z' = dz, \quad (u', v') = \frac{Q}{2\pi Q da} (u, v),$$

$$w' = \frac{Q}{2\pi Q da} \cdot \frac{d}{a}, \quad (w', p') = \frac{Q}{2\pi ad} \cdot \Omega a \cdot p.$$

The equations of motion (1)–(4) can be rewritten thus:
The Ekman number $E$ and the geometrical ratio $\lambda$ are defined thus:

$$E = \frac{v_v}{\Omega d^2},$$

and

$$\lambda = \frac{d}{a}.$$

We have also assumed that $v_v = \lambda^2 v_H$.

Equations (6) together with the following boundary conditions,

$$u - 1 = v = w = 0 \quad \text{at} \quad r = 1 \quad \text{and} \quad \alpha \quad \text{(where} \quad \alpha = b/a),$$

$$u = v = w = 0 \quad \text{at} \quad z = 0 \quad \text{(rigid bottom surface)},$$

$$u_z = v_z = w = 0 \quad \text{at} \quad z = 1 \quad \text{(stress-free upper surface)},$$

constitute a well-posed boundary-value problem. Since the complete solution of a very similar boundary-value problem can be found in Lewellen (1965), we shall omit most of the intermediate calculations.

If we focus our attention solely on the flow outside the top and bottom Ekman layers, we are led to consider a simpler boundary-value problem (see e.g., Barcilon 1966), viz.

$$+ 2v = -p_r + E \left( \nabla^2 - \frac{1}{r^2} \right) u,$$

$$- 2u = E \left( \nabla^2 - \frac{1}{r^2} \right) v,$$

$$0 = -p_z + \lambda^2 E \nabla^2 w,$$

$$u_r + \frac{u}{r} + w_z = 0,$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}.$$
with

\[
\begin{align*}
    u - 1 &= v = w = 0 \quad \text{at } r = 1, \ \alpha, \\
    w &= \frac{E^{1/2}}{2} \left( \frac{v_r + \frac{v}{r}}{u_r + \frac{u}{r}} \right) \quad \text{at } z = 0, \\
    w &= -\frac{E}{2} \frac{\partial}{\partial z} \left( \frac{v_r + \frac{v}{r}}{u_r + \frac{u}{r}} \right) \quad \text{at } z = 1.
\end{align*}
\]

(11)

By taking advantage of the smallness of \( E \), we can make use of the standard boundary-layer arguments and, in particular, consider that the flow is made up of interior and boundary-layer fields. Denoting the interior fields with capitals, we can easily deduce from equations (10), in which the \( E \) terms have been neglected, and equations (11) that

\[
\begin{align*}
    U &= W = 0, \\
    V &= \frac{A}{r},
\end{align*}
\]

(12)

where \( A \) is an undetermined constant whose value is obtained from a consideration of the side-wall boundary layers. These boundary layers have a rather complicated double structure, with sublayers of thickness \( E^{1/3} \) and \( E^{1/4} \). Since these layers have been extensively discussed elsewhere (see e.g., Lewellen 1965, Barcilon 1966), we shall write down only the expressions for the radial and zonal velocity fields that are necessary to determine \( A \), viz.,

\[
\begin{align*}
    u_{b,1} &= B \exp \left\{ (1 - r) E^{-1/4} \right\} + O \left( E^{1/12} \exp \left\{ (1 - r) E^{-1/3} \right\} \right), \\
    v_{b,1} &= -2BE^{-1/2} \exp \left\{ (1 - r) E^{-1/4} \right\} + O \left( E^{-1/4} \exp \left\{ (1 - r) E^{-1/3} \right\} \right),
\end{align*}
\]

(13)

where \( B \) is another undetermined constant. Making use of the boundary conditions at \( r = 1 \), we deduce that:

\[
\begin{align*}
    B &= 1, \\
    A &= 2E^{-1/2}.
\end{align*}
\]

(14)

In other words, the zonal velocity is:

\[
V = -\frac{2E^{-1/2}}{r};
\]

or in dimensional units:
This zonal current is flowing in a direction opposite to that of the basic rotation; i.e., this is a westward current in the southern hemisphere. The zonal mass transport due to this current is:

\[
V_{\text{discharge}} = -\frac{Q^{1/2} Q}{\pi v_v^{1/2}} \frac{1}{r'}.
\]  

(15)

\[
M_{\text{discharge}} = -\frac{Q^{1/2} Q}{\pi v_v^{1/2}} \int_a^b \frac{d r'}{r'};
\]

that is,

\[
M_{\text{discharge}} = -\frac{Q^{1/2} Q d}{\pi v_v^{1/2}} \ln \frac{b}{a}.
\]  

(16)

The radial mass transport is entirely effected via the bottom Ekman layer.

Maksimov (1965), relying solely on the “smallness” of \(Q\), came to the conclusion that the influence of the peripheral water discharge on the dynamics of the Circumpolar Current is negligible. However, because of the factor \(E^{-1/2}\), which enters in the expression for \(M_{\text{discharge}}\), a small \(Q\) can produce a sizeable zonal transport.

4. Conjectures. The crucial parameter of this model is obviously \(Q\), the rate of water flux due to peripheral ablation, melting, and discharge. Although a great number of investigations have been concerned with the ice budget of Antarctica (Wexler 1961), there are no reliable measurements of this quantity at the present time. In order to obtain an order-of-magnitude estimate, we shall assume that the net amount of water on the Antarctic Continent is roughly constant, and therefore we assume that \(Q\) is comparable to the rate of snow precipitation. Using mainly surface and stratigraphic data, Giovinetto (1964) has calculated that the average annual snowfall over Antarctica is about \(2.1 \times 10^{18}\) g yr\(^{-1}\). However, data obtained from oxygen-isotope ratios (Epstein et al. 1963) suggest that surface and stratigraphic measurements tend to underestimate the snow accumulation by as much as a factor of 2 and that yearly variations could be as large as a factor of 4. Keeping these remarks in mind and substituting the following numerical values in (16), viz., \(Q = 10^{11}\) g sec\(^{-1}\) (this figure is actually very close to that used by Maksimov 1965), \(v_v = 10\) cm\(^2\) sec\(^{-1}\), \(d = 5 \times 10^5\) cm, \(\Omega = 0.7 \times 10^{-4}\) sec\(^{-1}\), we find that the mass transport is of the order of \(10^{13} - 10^{14}\) g/sec. This figure is encouraging and suggests that, even if the water flux from Antarctica were not the only process responsible for slowing the strength of the wind-driven current, it could still play an important role.
Another peculiar feature of the Antarctic Ocean is the reversal in direction of the current at high latitudes. In a recent review article, Deacon (1963) has described it as follows:

"A low-pressure belt extends round the continent in about 65°S ... North of the low-pressure belt the prevailing wind is west and the water moves east ... South of the low-pressure trough the winds are rather more variable but the current appears to set mainly to the west following the trend of the Antarctic coastline."

Since $V_{\text{discharge}}$ is largest along the coastline, it is tempting to conjecture that in this region the peripheral water discharge is the dominant driving mechanism and is responsible for this so-called "east-wind drift." However, because of both the crudeness of our model and the speculations concerning the magnitude of the water discharge, this is just a conjecture.

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