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THE DYNAMIC DEMAND FOR CAPITAL AND LABOR

by

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January 1985

THE DYNAMIC DEMAND FOR CAPITAL AND LABOR*

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I. Introduction

There is a significant gap between the theory and empirical work in the standard models of investment. This paper offers some plausible estimates of investment dynamics that are both consistent with a structural model and useful for policy analysis. Because the choice of capital stock is inherently connected with the choice of other factors, labor in particular, the model is of capacity choice in general instead of investment in particular. When a firm purchases or hires factors of production, it determines its productive capacity. Short-run variation in output comes principally from variation in factors that are adjustable at little or no cost. Investment responds relatively slowly to shocks because of adjustment costs.

In this paper I provide structural estimates of demand for factors--capital in particular. These estimates can be used for analyses of tax policy that are immune from the Lucas critique. The dynamic response of investment implied by the estimates is more plausible than that found in previous research. I use a model where the firm maximizes the present discounted value of profits subject to a technology with adjustment costs. Such a specification has a long history in the investment literature. Moreover, it is a generalization of Tobin's q-model, which is equivalent to an adjustment cost model where only capital is costly to adjust. The firm chooses its capital stock, its number of production and non-production workers, and hours of production workers.

I estimate the first-order conditions, or Euler equations, of the problem rather than the closed-form decision rules. The estimation technique is Hansen's [1982] generalized method of moments. To evaluate the performance of the model, I consider its dynamic properties. My estimates

of the dynamic response of the capital stock do not have the implausibly long adjustment lags found in other research.

It is well known that empirical investment equations behave poorly in the 1970s as Clark [1979] demonstrates. He divides the models of investment into five categories, which I simplify to two. The first class of models takes output as the explanatory variable for investment (accelerator and cash-flow models). The second takes factor prices as the driving variables (neo-classical, cost-of-adjustment, and q models). Clark concludes that output-based equations forecast better than factor price equations in the short run.

Models that take output as exogenous do not fit into the standard optimizing models of investment. First, the models based on output, if they are based on an optimizing problem at all, assume, contrary to fact, that factor prices are fixed. Second, the models based on output are typically non-structural and are therefore subject to the Lucas [1976] critique. Third, and perhaps most important, the policy questions about investment often involve changes in factor prices through changes in tax policy. One cannot address such questions in a context of a model excluding factor prices. Moreover, a model that includes factor prices but also has output as an exogenous variable implies changes in factor prices change only the capital to labor ratio without expanding output.

The second class of models, from which the theory of this paper descends, has a long history. Factor prices determine investment in the neo-classical models of Jorgenson and Hall (see Jorgenson [1963] and Hall and Jorgenson [1967]), the cost-of-adjustment models (see Lucas [1967], Treadway [1969], and Lucas and Prescott [1971]), and the q -models (see Brainard and Tobin [1968] and Tobin [1969]). Abel [1979, 1982, 1983] and

Hayashi [1982] show that the cost-of-adjustment models and the q -models are essentially equivalent. I use a generalization of the cost-of-adjustment model that takes into account the choice of number of employees and their hours of work as well as the choice of the stock of capital.

The q -approach has not yielded satisfactory estimates of the parameters of the investment function. Abel and Blanchard [1983] resort to a sales-investment specification because of their difficulty in usefully explaining investment with q (see their 1981 paper for the estimates with q). Their dissatisfaction with the q -approach does not arise from an a priori defect in the theory but in its empirical performance. Although the market value of publicly traded corporate capital is easy to measure, it is difficult to measure its replacement cost. Moreover, the appropriate concept in a q -equation is marginal rather than average q .¹ Marginal q cannot be directly measured, although it can in principle be constructed. Indeed, a q is implied in the calculations in this paper. Nonetheless, the implementation of the model in this paper does not depend on constructing a series for q .

Summers's [1981] paper highlights the problem with the estimates based on q -theory. In particular, he finds extremely slow adjustment of the capital stock to changes in factor prices. The estimates offered in this paper do not have this shortcoming. By examining the adjustment problem directly, rather than through a summary statistic such as q , I obtain more reasonable results than Summers and others.

II. The Firm's Decision Problem

The representative firm maximizes the present discounted value of cash flow. The choice variables of the firm are the purchases of capital, the net hires of employees, and the number of hours that the employees work. I distinguish between production and non-production workers. These choices determine output; thus, output is endogenous. The firm takes factor prices and the investors' ex ante required rate of return as exogenous.

Stocks of factors are costly to adjust, so lagged factor stocks enter the decision rule of the firm. That is, the firm in the short run does not hire the long-run, profit-maximizing level of inputs. Because the exogenous variables change over time, the firm takes into account their expected path in making its current decisions. Since the time path of factors determines the time path of output, no separate choice of capacity utilization is made.

I now consider the individual components of real cash-flow: real output, real labor cost, and real capital cost. I combine these below to obtain the expression for discounted cash-flow.

II.1. Production Function

The real output of the firm is given by the production function

$$(1) \quad y_t = f(K_t, L_t, N_t, H_t, K_t - dK_{t-1}, L_t - q_{t-1}L_{t-1}, N_t - q_{t-1}N_{t-1}, H_t - H_{t-1}, X_t)$$

where K_t is the stock of capital, L_t the number of production workers, N_t the number of non-production workers, and H_t the average hours of production workers. The parameter d is one minus the rate of depreciation of capital and q_{t-1} is one minus the quit rate, that is, the rate at which

the stock of workers depreciates. The vector X_t represents unobserved factors in the production function. Trend productivity and shocks to the production function are the obvious examples of such unobservables. Measured output, y_t , is value-added so intermediate inputs are neglected.²

Output depends on the level and gross rate of change of the factors of production. Output is allowed to depend on the rates of change of the inputs to allow for adjustment costs. Brechling and Mortenson [1969] and Brechling [1975] characterize in detail the properties of a production function such as (1). The adjustment costs are internal to the production process. That is, the cost is output lost when the factors of production are varied. This cost does not represent specific payment to a factor. External adjustment costs (such as the purchase cost of capital) are accounted for elsewhere.

In adjusting the stock of labor, output is lost through the inexperience of new workers and the time taken to readjust the schedule and pattern of production. Adjustment costs for the number of employees are likely to be much more important than that for the average hours worked. Output is lost when capital is adjusted through the lost production time during installation, the difficulty of incorporating new machines into the production process, and the labor input diverted to install the new capital.

The production function must be parameterized in order to render the theory empirically useful. Some researchers estimate closed-form decision rules rather than Euler equations. Examples are Sargent [1978], Kennan [1979], Meese [1980], and Hansen and Sargent [1980, 1982]. They specify the production functions so the problems are linear quadratic. This specification yields closed-form decision rules under rational expectations.

Epstein and Denny [1983] offer a general functional form and then

approximate it for estimation. They assume static expectations, which is a serious shortcoming given the explicitly dynamic nature of the problem. Indeed, they do not specify the source of the randomness in the firm's environment. As I discuss in Section IV, the structure of the error term in the factor demand equations determines what instruments are valid to identify the parameters. Therefore, it is difficult to evaluate the identification of their parameters in the absence of explicit assumptions about the stochastic environment facing the firm. Epstein and Denny do discuss the restrictions on the shape of the production function. In particular, they test whether it is concave. An exception to the practice of using quadratic approximations is Pindyck and Rotemberg [1983], who derive conditional factor demands on the basis of trans-log cost functions.

I estimate the Euler equations themselves, rather than an approximated decision rule, that is, the solution of the Euler equation. This procedure allows important complications to be introduced. These include non-quadratic specifications of the production function, a non-linear wage bill, and a variable rate of discount. Only efficiency of estimation is lost by estimating the Euler equation instead of their solutions. In particular, the technique of Hansen and Sargent [1980, 1982] exploits the restrictions between the demand equations and the stochastic processes of the variables on the other side of the market. These restrictions arise only if the autoregressive representation of the stochastic process followed by those variables is greater than first order. Otherwise, estimating the Euler equation and the closed-form solution are equivalent. The gain in efficiency seems a high price to pay for restricting the rate of return to be constant and for having to make strict assumptions about the technology. In this research I need not impose the extreme restrictions

on the structure of the problem which are made by Hansen and Sargent to yield closed-form, rational expectations solutions.

I choose the Cobb-Douglas production function for tractability and ease of interpretations. Nothing in the method of this paper depends on the use of the Cobb-Douglas form. The estimates and solution could easily be carried out with a quadratic or CES function. The Cobb-Douglas function has the advantage of simplicity so one can inspect the parameters for the plausibility of their magnitudes as well as for statistical significance. The results of other studies which report production function parameters in terms of the steady-state values of derivatives (that is, the quadratic specification commonly used in the rational expectations literature), are difficult to evaluate. In particular, assumptions of convexity and constant returns to scale are often neither imposed nor tested.³

Many authors assume that employees and hours enter multiplicatively in the production function. Both Sargent [1978] and Meese [1980] make this assumption. Indeed, Meese makes the much stronger assumption that firms choose man-hours alone instead of separately choosing number of employees and hours of work. (Sargent handles this issue by distinguishing between straight-time and overtime hours.) Employees and hours have different marginal costs. Adding an hour may entail an overtime premium; adding an employee may involve fixed costs such as health insurance. Employees and hours also have different adjustment costs. Hence, assuming that the decision variable is man-hours rather than hours and employees separately distorts the firm's choice problem even if hours and employees enter the production function multiplicatively. Treating the decision variable as man-hours instead of employees and hours separately may also lead to an incorrect linearization of the production function even if

those variables enter it multiplicatively. Additionally, straight-time and average overall wages vary insubstantially over the business cycle. Overtime hours do vary substantially causing the marginal cost of an hour worked to be higher than the basic data suggest. The dynamic factor demand models rely on variation in factor prices to explain the dynamics of demand; neglecting this substantial variation in cost is a mistake.

Some authors, such as Fair [1969], Nadiri and Rosen [1969], and Bernanke [1983], stress that hours and employment may not enter multiplicatively in the production function. In this paper, I also let them enter separately.

The production function I use is

$$\begin{aligned}
 (2) \quad \log y_t &= \log[f(K_t, L_t, N_t, H_t, K_t - dK_{t-1}, L_t - q_{t-1}L_{t-1}, \\
 &\quad N_t - q_{t-1}N_{t-1}, H_t - H_{t-1}, X_t)] \\
 &= a_0 + a_K \log(K_t) + a_L \log(L_t) + a_N \log(N_t) + a_H \log(H_t) \\
 &\quad - (1/2) [g_{KK}(K_t - dK_{t-1})^2 + g_{LL}(L_t - q_{t-1}L_{t-1})^2 \\
 &\quad + g_{NN}(N_t - q_{t-1}N_{t-1})^2 + g_{HH}(H_t - H_{t-1})^2] \\
 &\quad - [g_{KL}(K_t - dK_{t-1})(L_t - q_{t-1}L_{t-1}) \\
 &\quad + g_{KN}(K_t - dK_{t-1})(N_t - q_{t-1}N_{t-1}) \\
 &\quad + g_{KH}(K_t - dK_{t-1})(H_t - H_{t-1}) \\
 &\quad + g_{LN}(L_t - q_{t-1}L_{t-1})(N_t - q_{t-1}N_{t-1}) \\
 &\quad + g_{LH}(L_t - q_{t-1}L_{t-1})(H_t - H_{t-1}) \\
 &\quad + g_{NH}(N_t - q_{t-1}N_{t-1})(H_t - H_{t-1})] \\
 &\quad + a_1 t + v_t^f .
 \end{aligned}$$

For the sake of concreteness, I have parameterized X_t to be a trend in the log, $a_1 t$, and a shock, v_t^f . As will be shown below, these terms drop out of the estimated Euler equations as long as they are additive in logs in the production function.

I impose the constant-returns-to-scale restriction on output gross of adjustment costs so that $a_K + a_L + a_N = 1$. The form of the function implies convexity. There is no natural CRS restriction on the cost of adjustment parameters. That part of the function is convex as long as the matrix of the g parameters is positive definite. I do impose the requirement that the matrix of the g parameters is symmetric. It is useful to note that I do not restrict the coefficient of hours, H_t , when I impose constant returns to scale. It is natural to leave a_H unrestricted: a replication of capital and labor would presumably leave hours per worker constant. The difference $a_H - a_L$ measures the departure of employees and hours from entering the production function multiplicatively.

I assume that capital and labor have adjustment costs related to the change in the stock. The adjustment cost for capital is a function of the square of gross investment, $K_t - dK_{t-1}$. The depreciation rate is taken to be parametric so the capital series can be derived from the gross investment series. An alternative specification would be to use net investment. The rationale for using net investment would be that replacement investment is somehow more routine and therefore less costly than net investment. Machines are rarely replaced one-for-one; consequently, there is not a clear distinction between replacement and new investment. Therefore, there is not a strong case of using net investment.

Likewise, the adjustment cost for employees is proportional to the

gross change in the number of employees, $L_t - q_{t-1}L_{t-1}$ and $N_t - q_{t-1}N_{t-1}$, where q_{t-1} is one minus the quit rate. As with the depreciation rate, the firm takes the quit rate as given. Unlike the depreciation rate, the quit rate is published data and varies across time. The choice variable of the firm is the current stock of employees. In arriving at the figure it takes into account the probability that a fraction of the workers will quit each period.

I also allow there to be interrelated adjustment costs in (2). These arise if the lost production time through adjusting one factor is greater or less when the firm adjusts other factors.

I expect there to be little or no cost to adjusting hours. Firms can adjust hours merely by extending the length of the shift. (Diminishing marginal product is, of course, captured in the production function.) Such an adjustment is likely to be much easier than adding a worker. In particular, extending the operation of the plant for an hour requires no realignment of workers and machines. An extra worker, on the other hand, must be incorporated into the pattern of production.

II.2. Cost of Labor

Now I consider the cost of the labor to the firm. Labor cost is a function of the number of employees, the hours they work, and rate of compensation of employees not sensitive to hours worked. Compensation not sensitive to hours worked includes all payments to salaried employees and certain non-wage payments to hourly workers.

I assume only hours of production workers are variable and that all non-production workers are salaried. The average wage is an increasing function of hours worked due to the overtime premium. Suppose that the

wage bill for production workers is given by

$$(3) \quad w_t^* L_t H_t = w_t L_t [w_0 + H_t + w_1 (H_t - 40) + w_2 (H_t - 40)^2] + v_t^W$$

where w_t^* is the average wage, w_t the straight-time wage, and v_t^W is a measurement error. (Abel [1979] gives a theoretical discussion of such a wage bill in a similar context.) The coefficient w_1 is the overtime premium. I include parameters w_0 and w_2 to allow a more general specification.⁴

Labor cost is the sum of the wage bill for the production workers, their non-wage compensation, and the compensation of non-production workers. Summing these gives

$$(4) \quad \text{total labor cost} \\ = w_t L_t [w_0 + H_t + w_1 (H_t - 40) + w_2 (H_t - 40)^2] + s_t^L L_t + s_t^N N_t + v_t^W,$$

where s_t^L is the non-wage compensation of production workers per worker and s_t^N is the total compensation of non-production workers per worker. Non-wage compensation includes pension contributions and health insurance. It should include only payments not a function of hours worked. Since I assume a firm cannot choose the hours of non-production workers their compensation is a fixed rate per employee.

11.3. Cost of Capital

The purchase price of capital is

$$(5) \quad p_t^K = p_t (1 - t_t^K \text{PVCCA}_t - \text{ITC}_t)$$

where p is the price of new capital relative to the price of output (the ratio of the deflators from the National Accounts), PVCCA is the present

discounted value of depreciation allowances, and ITC is the effective investment tax credit rate weighted for the composition of investment. The present discounted value of depreciation figures are weighted statutory depreciation rates discounted by the term structure of interest rates. Jorgenson and Sullivan [1981] outline the method for producing such estimates.⁵

Meese [1980] uses the rental cost of capital rather than the purchase cost for the price of capital in a model similar to (1). The rental cost (excluding tax terms for ease of exposition) is

$$p(\rho + \delta)$$

where ρ is the required return of investors and δ is the rate of depreciation. In the problem of maximizing discounted cash flow the investors' required rate of return enters through R_t , the discount rate. Meese's use of the rental rate rather than purchase cost allows him to vary the required rate of return while assuming R_t is constant. Such an assumption allows for closed-form solution, but it makes it difficult to interpret the discount rate; the same required rate of return should enter in both the rental rate and the discount rate.

II.4. The Firm's Objective

The problem of the representative firm is to maximize expected present discounted value of cash flow. The expected value of real, discounted, after-tax cash flow is

$$\begin{aligned}
(6) \quad E_t \sum_{i=0}^{\infty} R_{t+i} \{ & f(K_{t+i}, L_{t+i}, N_{t+i}, H_{t+i}, K_{t+i} - dK_{t+i-1}, \\
& L_{t+i} - q_{t+i-1}L_{t+i-1}, N_{t+i} - q_{t+i-1}N_{t+i-1}, \\
& H_{t+i} - H_{t+i-1}, X_{t+i}) (1 - \tau_{t+i}^K) - P_{t+i}^K (K_{t+i} - dK_{t+i-1}) \\
& - [w_{t+i}L_{t+i}(w_0 + H_{t+i} + w_1(H_{t+i} - 40) + w_2(H_{t+i} - 40)^2) \\
& + s_{t+i}^L L_{t+i} + s_{t+i}^N N_{t+i}] (1 - \tau_{t+i}^K) \} ,
\end{aligned}$$

where E_t denotes expectation conditional on information available at time t and where

K_t = capital stock

L_t = employees, production workers

N_t = employees, non-production workers

H_t = hours per production worker

f = production function

$d = 1 - \delta$, where δ = depreciation rate

q_t = one minus the quit rate

P_t^K = after-tax purchase price of capital (equation (5))

w_t = straight-time wage

s_t^L = fringe benefits per production worker

s_t^N = compensation per non-production worker

τ_t^K = corporate tax rate

$R_{t+i} = \prod_{j=t+1}^{t+i} r_{j-1}$, where $r_t = 1/(1+\rho_t)$ and ρ_t = required rate of return from periods t to $t+1$.

The previous sections discuss the individual components of (6) in detail. I estimate the Euler equations implied by (6) together with the equation for the wage bill (4).

The firm's decision variables at time t are its capital stock, number of production workers and non-production workers, and average hours of production workers. Substituting the production function (2) into (6) and differentiating yields the following four first-order conditions:

$$(7a) \quad E_t \{ [a_K/K_t - g_{KK}(K_t - dK_{t-1}) - g_{KL}(L_t - q_{t-1}L_{t-1}) - g_{KN}(N_t - q_{t-1}N_{t-1}) - g_{KH}(H_t - H_{t-1})] y_t (1 - \tau_t^K) + [g_{KK}(K_{t+1} - dK_t) + g_{KL}(L_{t+1} - q_t L_t) + g_{KN}(N_{t+1} - q_t N_t) + g_{KH}(H_{t+1} - H_t)] y_{t+1} (1 - \tau_{t+1}^K) dr_t - p_t^K + dr_t p_{t+1}^K \} = 0$$

$$(7b) \quad E_t \{ [a_L/L_t - g_{KL}(K_t - dK_{t-1}) - g_{LL}(L_t - q_{t-1}L_{t-1}) - g_{LN}(N_t - q_{t-1}N_{t-1}) - g_{LH}(H_t - H_{t-1})] y_t (1 - \tau_t^K) + [g_{KL}(K_{t+1} - dK_t) + g_{LL}(L_{t+1} - q_t L_t) + g_{LN}(N_{t+1} - q_t N_t) + g_{LH}(H_{t+1} - H_t)] y_{t+1} (1 - \tau_{t+1}^K) q_t r_t - [w_t(w_0 + H_t + w_1(H_t - 40) + w_2(H_t - 40)^2) + s_t^L] (1 - \tau_t^K) \} = 0$$

$$(7c) \quad E_t \{ [a_N/N_t - g_{KN}(K_t - dK_{t-1}) - g_{LN}(L_t - q_{t-1}L_{t-1}) - g_{NN}(N_t - q_{t-1}N_{t-1}) - g_{NH}(H_t - H_{t-1})] y_t (1 - \tau_t^K) + [g_{KN}(K_{t+1} - dK_t) + g_{LN}(L_{t+1} - q_t L_t) + g_{NN}(N_{t+1} - q_t N_t) + g_{NH}(H_{t+1} - H_t)] y_{t+1} (1 - \tau_{t+1}^K) q_t r_t - s_t^N (1 - \tau_t^K) \} = 0$$

$$(7d) \quad E_t \{ [a_H/H_t - g_{KH}(K_t - dK_{t-1}) - g_{LH}(L_t - q_{t-1}L_{t-1}) - g_{NH}(N_t - q_{t-1}N_{t-1}) - g_{HH}(H_t - H_{t-1})] y_t (1 - \tau_t^K) + [g_{KH}(K_{t+1} - dK_t) + g_{LH}(L_{t+1} - q_t L_t) + g_{NH}(N_{t+1} - q_t N_t) + g_{HH}(H_{t+1} - H_t)] y_{t+1} (1 - \tau_{t+1}^K) r_t - w_t L_t [1 + w_1 + 2w_2(H_t - 40)] (1 - \tau_t^K) \} = 0$$

where

$$y_t = f(K_t, L_t, N_t, H_t, K_t - dK_{t-1}, L_t - q_{t-1}L_{t-1}, N_t - q_{t-1}N_{t-1}, H_t - H_{t-1}, X_t) .$$

I estimate these equations together with the equation for the wage bill (4) with quarterly data for U.S. manufacturing. Thus, I am assuming that the manufacturing sector can be modeled as a representative firm.

III. Data

The previous dynamic factor demand studies use annual or higher frequency data depending on whether or not they use output to explain demand. Consistent output, employment, and investment figures for manufacturing are available in the national income and product accounts only on an annual basis. Examples of studies using such data are Berndt and Morrison [1979], Pindyck and Rotemberg [1983], and Epstein and Denny [1983]. Indeed, the latter two pair of authors use data supplied by Berndt. On the other hand, authors using only factor prices as forcing variables choose data of higher frequency. Examples of such studies are Sargent [1978], Kennan [1979], and Meese [1980].

The data here are quarterly data for manufacturing from 1955 through 1980. I use manufacturing as a compromise between wide coverage and homogeneity of the underlying firms. Output data appear in the estimating stage to reduce the nonlinearity of the estimation problem. The dynamic demands are functions of factor prices and rates of return. Output is determined endogenously in the model. The output data are the quarterly index of manufacturing output produced by the Federal Reserve Board scaled to equal actual output in 1967.

The quarterly data for investment is from the Survey of Current Business.

Structures and equipment are aggregated. I construct the capital stock data using a fixed depreciation rate of 0.0175 per quarter and a benchmark net capital stock of 311.8 billion 1972 dollars at the end of 1981 (see the Survey of Current Business [October 1982, p. 33]).

In the model, the discount rate varies across time. This feature strongly distinguishes the results from those based on closed-form solution, which require a fixed discount rate. The discount rate, R_t , is defined above. In these estimates, I take the required rate of return to be the after-tax, real return on three-month Treasury bills plus a constant risk premium of two percent per quarter.

I calculate the premium by taking a weighted average of the return in excess of the return on Treasury bills of the stock market and of corporate bonds. The weight for equity is 0.8. The excess return of the stock market is 6.7 percent; the excess return of corporate bonds is 0.6. (See Ibbotson and Sinquefeld [1982, p. 15].) Therefore, the premium is about eight percent at annual rate or about two percent at quarterly rate.

The data for employment are the BLS establishment survey figures for the number of production workers and non-production workers. The wage series is the average straight-time rate per hour for the production workers. The hours series are total average weekly hours. Hours are multiplied by the number of weeks in the quarter in the marginal cost expressions in the Euler equations to express cash flow at quarterly rate. To construct the fixed cost of employing a worker, I divide the compensation minus wages, salaries, and contribution to social insurance into the number of workers using annual national income and product accounts data.⁶

The expression for the price of capital is given in equation (5). The purchase price is the implicit deflator from the BEA. The quarterly

series for the present value of depreciation allowances and the investment tax credit are those computed by Data Resources, Inc.

IV. Estimation and Results

To estimate the first-order conditions (7), I replace the conditional expectations with actual values and use instrumental variables. Moreover, I make the substitution that $y_t = f(\cdot)$. The equations I estimate are then the same as (7) except that the zeros on the right-hand sides are replaced with a vector of error terms u_t . If the equations are specified correctly, the error term u_t equals only a forecast error e_t . I consider a more general error term $u_t = e_t + v_t$. The added component v_t is either measurement error or specification error or both. What instruments are valid depends on whether u_t is serially uncorrelated.

To estimate the system of Euler equations together with the equation for the wage bill (5), I use Hansen's [1982] generalized method of moments (GMM).⁷ The procedure is essentially three-stage least squares with a covariance matrix that allows for general conditionally heteroskedastic and moving average errors. Under the maintained hypothesis that the model is exactly correct, the errors are serially uncorrelated. Any instrument known at time t is valid. In the case of misspecification, the errors may be serially correlated. Details of how the moving average error term may arise are given in Hansen and Sargent [1980]. The intuition for it is that v_t , the measurement or specification error is part of the information set, and therefore contributes to the forecastability of e_t . If the error term is a first-order moving average, then only instruments dated at time $t-1$ are valid. I present estimates using both time t and time $t-1$ instruments.

To carry out the estimation, I substitute y_t for $f(\cdot)$. This procedure has several advantages. First, it makes what would otherwise be a highly nonlinear system linear in parameters. Second, it brings output data to bear on the problem without estimating demand functions that are conditional on output; that is, it imposes equation (2). Third, it eliminates the need to explicitly parameterize the shock to technology and the rate of technological progress, λ_t . The substitution reduces the amount of noise in the estimated system by eliminating v_t^f , the error term in equation (2).

Garber and King [1983] criticize methodology of the type used in this paper. In particular, they note that if unobserved shocks move the production function (factor demands) the estimated curves will be factor supplies. This paper makes a substantial advance in addressing the problem raised by Garber and King. The specification explicitly allows for a productivity shock without losing identification. The productivity shock, v_t^f in equation (2), can have arbitrary serial correlation, but must enter additively in logs. Given the parameterization of the shock, output data can be used to make v_t^f observable. Hence, the Garber and King critique does not apply. Indeed, using output data accommodates a wide range of productivity shocks without compromising identification.

A criticism of this approach to identification is that the system will not be identified if the shock is not additive in logs, or more generally, separable from the function. The appropriate rejoinder is that the separability is typical of identifying restrictions needed in any econometric application. It is much weaker than assuming no shock at all. Moreover, most of the minimum distance estimators generally used in econometrics have separable errors (see Amemiya [1983]).

Hence, the approach used in this paper may open the way to more plausible parameterizations of stochastic Euler equations when both inputs and outputs are observed. Unfortunately, the approach can not be readily used in the consumption or labor supply literature (e.g., Mankiw, Rotemberg, and Summers [forthcoming]) because utility is not observable.

The instruments used in the estimates for Table 1 are the factor prices (p_t^K , w_t , s_t^L , and s_t^N), the factor stocks (K_t , L_t , N_t , and H_t), their logs, the tax rate, t_t^K , the required rate of return r_t , the quit rate, a constant, and a trend. (The required rate of return is not known at time t because of uncertainty about the inflation rate. Therefore, even in estimates with current instruments, r_t is lagged.)

Table 1 presents estimates of system (7) and equation (4). The cross-equation restrictions are imposed and the production function gross of adjustment costs is constrained to be constant returns to scale. I report both the standard three-stage least squares (3SLS) and the Hansen generalized method of moments (GMM) standard errors. The GMM errors are substantially larger than the 3SLS ones. The 3SLS errors are inconsistent unless the errors are homoskedastic and serially uncorrelated. If the errors are serially correlated, only lagged instruments are valid.

Table 1 presents estimates with and without interrelated adjustment costs. The estimated coefficients are plausible and significant. Consider the elasticities in the Cobb-Douglas part of the production function. The elasticity of production workers (L_t) is about 0.46 and the elasticity of non-production workers (N_t) is about 0.27 implying a total labor elasticity of about three-quarters, which is broadly consistent with its share in national income. The implied value of a_K is 0.27. The coefficients are estimated very precisely and change little when the instrument list

or specification is varied.

The estimate of the a_H , the hours elasticity, is substantially greater than a_L . Thus, labor input should be treated as hours and workers separately and not just as man-hours. The coefficients are consistent with the theory developed above where short-run variation in the utilization of capital comes from variation in the number of man-hours worked at a fixed plant. That is, increasing average hours increases the work week of capital as well as adding labor input.

The estimates of the cost of adjustment parameters are also plausible. They all have the correct sign except two of four of the estimates of g_{LL} and one of the four estimates of g_{HH} , all of which differ unimportantly and insignificantly from zero. Capital has important adjustment costs. They are significant with the 3SLS standard errors but not so with the GMM standard errors. In any case, the estimated coefficient is substantial. Varying non-production workers induces important and significant adjustment costs. Varying production workers or their hours induces small and insignificant adjustment costs. Such a finding is not surprising. It is likely that hours are not costly to adjust because of the ease of lengthening shifts. The result that production workers are not costly to adjust is more difficult to rationalize. Perhaps given the institutionalization of the temporary layoff, such a result should not be too surprising. The interrelated adjustment costs are never significant with the GMM standard errors; g_{KL} is significant with 3SLS standard errors. Even though there are no adjustment costs for production workers alone, when capital is varied, there is an added cost to varying production workers.

It is difficult to evaluate the magnitude of the adjustment cost based on the coefficients alone. Therefore, I calculate the marginal cost of

adjustment for typical values of the variables when the gross change in the other variables is zero. The marginal reduction in output from adjustment cost due to investment is

$$-g_{KK}y_t(K_t - dK_{t-1})$$

and the reduction from changing the number of non-production workers is

$$-g_{NN}y_t(N_t - q_{t-1}N_{t-1}) .$$

The average level of output for the period is 58 billion 1972 dollars at a quarterly rate. The average gross investment is 8.2 billion 1972 dollars. The estimate of g_{KK} from column (b) in Table 1 is 0.0014. Therefore, the marginal cost of adjustment from a representative amount of investment is about one percent of the output for the quarter or about eight percent of the cost of the investment. Previous estimates of the marginal cost of investment are implausibly high. For example, Summers [1981] finds very high marginal costs of adjustment from estimates based on the q approach. This problem with his result is seen as a major barrier to their use for practical, policy-oriented discussion (see Tobin and White [1981]). In particular, the high adjustment costs imply extremely long lags in adjusting to permanent changes in factor prices or the required rate of return.

Consideration of the data on which the q -model is estimated demonstrates why it produces such high estimates of adjustment costs. The stock market is much more variable than investment. The q -theory as Summers implements it would have investment respond to these changes except for adjustment costs. Therefore, estimated adjustment costs must be very high to rationalize the relatively small response of investment to changes in the stock

market. The model used in this paper takes the price of and required return to capital per se as its data. Therefore, it has the potential to produce more plausible estimates such as the ones presented here.

The adjustment costs for non-production workers are also plausible. The average stock of non-production workers is 4.5 million. Consider a five percent change in the number of employees. An estimate of g_{NN} of 0.081 implies an adjustment cost of 1.8 percent of output for the quarter. Similar arguments establish that the costs of adjusting production workers, L_t , and their hours, H_t , are insubstantial.

The estimates of the wage function (4) are highly plausible. The estimated overtime premium is 0.43, a value close to 0.5, the typical premium in contracts. It differs significantly from 0.5 with the 3SLS but not with the GMM standard errors.

The overtime premium may not be paid symmetrically. I estimate separate w_1 coefficients depending on whether $H_t - 40$ is greater or less than zero. The t-statistic for the hypothesis that the coefficients are equal is 0.3, so one cannot reject the hypothesis that the premium is symmetric.

The last line of Table 1 gives J , the value of the minimized objective function in the GMM estimation. It gives a test of the overidentifying restrictions of the model which Hansen [1982] discusses. J is distributed as chi-squared with $N-k$ degrees of freedom where N is the number of instruments and k the number of parameters estimated. The number of instruments here is 85 (17 times 5 equations). The number of parameters is either 10 or 16 depending on the specification. The overidentifying restrictions are rejected in the estimates without the

interrelated adjustment costs (columns a and b) at the five percent level but not at the one percent level. They are rejected at the one percent level in the estimates with interrelated adjustment costs (c and d). The value of the statistic is about the same for all the equations, but more parameters are estimated in (c) and (d). Given that there is no well-defined alternative model, it is difficult to see what direction such a rejection implies for further research. Moreover, the rejection notwithstanding, the estimated parameters and the dynamic response they imply are plausible.

The coefficients change very little when lagged instruments are used to allow for a moving-average error (a versus b and c versus d). Consequently, one cannot reject the hypothesis that the current instruments are valid.⁸

The GMM standard errors exceed the 3SLS ones by a factor of two or three. None of the adjustment cost parameters are significant at the customary levels with the GMM standard errors but some are strongly significant with the 3SLS standard errors. One could draw several conclusions from these findings. The first is that these estimates--and many past estimates using least squares--need to be reevaluated in light of Hansen's covariance estimator. If the magnitude of the change in the standard errors from 3SLS to GMM in this paper is typical, many estimates previously believed to be significant may indeed be insignificant. A second conclusion is that the small sample properties of GMM are not well understood and therefore some weight should be given to the 3SLS estimates. Moreover, the parameter values are the best point estimates. Finally, these signs and magnitudes accord with economic theory and with priors about their size.

I report both standard errors. I discuss the economic interpretation

of the point estimates with the reservation that they may be subject to substantial error. It is difficult to evaluate Euler equation estimates using traditional diagnostics. In particular, R^2 and SEE are irrelevant because there is no dependent variable per se. Instead one can appeal to the plausibility of the parameter estimates and of the dynamics that the estimates imply.

To quantify the rates of adjustment of the capital stock implied by the estimated Euler equations, I study their dynamic properties. Specifically, the lower the root of the capital equation, the more rapidly the capital stock will adjust to steady state following a change in the cost of capital. The root of the Euler equation for capital is 0.75.⁹ The root of 0.75 implies a rapid rate of adjustment. The speed of adjustment greatly exceeds that estimated by others. Summers [1981] estimates that after twenty years just over half the adjustment to a shock in the required rate of return would have occurred. In these estimates, over half the adjustment occurs in the first year.¹⁰ After four years, almost all the adjustment has occurred. The Euler equation approach generates what are possibly more plausible results because it permits more explicit consideration of the decision problem of the firm.

VI. Conclusion

This paper offers estimates of the dynamic demand for capital based on explicit consideration of the firm's decision problem. The estimates are based on the Euler equations. The q-theory and the linear-quadratic approach examine closed-form decision rules of the firm. The advantage of the Euler equation approach over the linear-quadratic approach is that it allows more flexible functional forms and a varying and uncertain rate

of discount. The advantage of the approach over the q-theory approach is that the adjustment cost in the Euler equation is not summarized with a single, reduced-form variable.

The estimated structural parameters have reasonable values and the capital stock responds at a credible rate to innovations in the factor prices. The best defense of the Euler equation approach is the plausibility of its empirical results. Specifically, the estimates presented in this paper do not imply the excessively large lags in the adjustment of the capital stock found in estimates based on q . Therefore, the investment equation of this paper may be useful to analyze the effects of changes in tax policy on the demand for capital.

The standard view is that rates of adjustment are so slow that the cost of capital will not effect investment in the short run.¹¹ To fully study the effects of changing the cost of capital on investment would require a complete model with product demand and capital and labor supply. Yet, the rapid rate of adjustment implied by the estimates in this paper provides a clear challenge to the view that factor prices do not matter for short run fluctuations of investment.

TABLE 1
 Estimates of the First-Order Conditions (7)
 and Equation for the Wage Bill (4)
 1955 QIII to 1980 QIII

Instruments	(a) Current	(b) Lagged	(c) Current	(d) Lagged
w_0	1.49 (.02) (.06)	1.50 (.02) (.08)	1.50 (.02) (.06)	1.50 (.02) (.08)
w_1	.43 (.02) (.05)	.43 (.02) (.06)	.43 (.02) (.05)	.43 (.02) (.09)
w_2	-.05 (.006) (.011)	-.05 (.006) (.020)	-.05 (.007) (.018)	-.05 (.008) (.023)
a_L	.45 (.005) (.009)	.46 (.005) (.009)	.46 (.006) (.017)	.46 (.006) (.018)
a_N	.27 (.004) (.007)	.27 (.003) (.009)	.27 (.004) (.017)	.27 (.004) (.014)
a_H	.53 (.01) (.02)	.53 (.01) (.02)	.52 (.011) (.022)	.52 (.011) (.028)
g_{KK}	.0013 (.0004) (.0011)	.0014 (.0006) (.0014)	.0011 (.0005) (.0014)	.0014 (.0005) (.0020)
g_{LL}	-.0002 (.0008) (.0018)	-.0003 (.0010) (.0030)	.0007 (.0016) (.0043)	.0006 (.0023) (.0088)
g_{NN}	.088 (.027) (.097)	.088 (.023) (.062)	.088 (.039) (.193)	.088 (.047) (.23)
g_{HH}	.00047 (.00027) (.00068)	-.0001 (.0005) (.0019)	.0007 (.0005) (.0012)	.0003 (.0006) (.0025)
g_{KL}			.0010 (.0004) (.0012)	.0010 (.0005) (.0019)

continued...

TABLE 1 (continued)

Instruments	(a) Current	(b) Lagged	(c) Current	(d) Lagged
g_{KN}			-.0005 (.0021) (.0083)	-.0006 (.0025) (.0011)
g_{KH}			-.0003 (.0003) (.0009)	-.0007 (.0004) (.0014)
g_{LN}			.0005 (.0044) (.017)	.0008 (.0074) (.038)
g_{LH}			.0004 (.0007) (.0017)	.0011 (.0011) (.0035)
g_{NH}			.0008 (.0020) (.0080)	.0006 (.0040) (.014)
J*	103.3	104.6	104.1	103.9
Significance	.017	.014	.004	.004

Standard errors in parentheses: The first standard errors are from 3SLS, the second from GMM.

* J test of overidentifying restrictions (see Hansen [1982]). J is distributed as chi-squared (85-k) where k is the number of parameters estimated. It is here calculated using the GMM rather than the least squares objective.

FOOTNOTES

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¹Hayashi [1982] shows that average q equals marginal q under certain circumstances such as constant returns to scale.

²The cost of intermediate inputs do not appear in the expression for profits so real value added minus real factor costs equals real profits.

³Sargent [1978] and Meese [1980] use a quadratic production function and do not impose or test CRS or convexity. Pindyck and Rotemberg [1983] impose and test CRS and convexity when estimating a trans-log cost function. Morrison and Berndt [1981] show how to impose CRS in a quadratic cost function. I tried a trans-log specification but failed to identify the production function parameters. In some cases the parameter of the first-order term in L_t exceeded one. It is difficult to untangle the adjustment cost parameters from the other production function parameters in the trans-log case because they both multiply second-order terms in the factors.

⁴I consider specifications where the overtime premium is asymmetric so it is paid only when hours exceed 40. The data strongly do not reject symmetry. In aggregate data some overtime is always paid; a symmetric premium appears to be a good approximation.

⁵I use the quarterly PVCCA and ITC constructed by Data Resources, Inc.

⁶The compensation data are annual. I interpolate to obtain quarterly data.

⁷I carried out the estimation in a FORTRAN program which I wrote to perform GMM. The program will handle general, non-linear problems.

⁸Rotemberg [1984] argues that if the overidentifying restrictions fail, different instrument lists could lead to vastly different estimates. That the estimates remain essentially unchanged when the timing of the instruments is changed is informal evidence that his problem does not arise with these estimates even though the J statistic is large.

⁹To calculate the root, I must make more explicit assumptions about the environment of the representative firm than in the Euler equations. With the Euler equations, I assume that the representative firm is a price taker. It would be incorrect, however, to assume that there is no feedback from factor and product markets to the representative firm. Even if the market is competitive, the representative firm will move down the market demand curve and up the labor supply curve. The competitive assumption means only that the firm does not take its effect on prices into account in its decision rule. I assume that labor is supplied inelastically, so L_t , N_t , and H_t are held constant. Wages relative to output prices are also held constant. Summers [1981] makes a similar assumption.

Using parameter values from Table 1, column b, the linearized rule for representative form is

$$K_t = 0.75 K_{t-1} - 7.1 \sum_{i=0}^{\infty} 0.93^i (E_t p_{t+i}^K - dr E_t p_{t+i+1}^K)$$

when the required rate of return is constant.

¹⁰Meese [1980, p. 151], using the technique advocated by Hansen and Sargent [1980, 1982] estimates that the root in the capital equation is 0.9563. Again, the adjustment lags are much larger than in my estimates, but not as large as in Summers. The root of 0.9563 implies that half the adjustment to the steady state takes place after four years. Moreover, it takes the economy over 25 years to get within one percent of the steady state under Meese's estimate compared to four years under the estimates presented here.

¹¹The standard view of the effect of the cost of capital on investment is well-represented by the following:

The effect of interest rates and tax changes...are likely to be felt only gradually, over long periods of time. For short-term forecasting (two years or less), the effect of moderate variations in taxes and interest rates is likely to be negligible.
(Clark [1979, p. 104])

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