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### Product Warranties and Double Moral Hazard

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PRODUCT WARRANTIES AND DOUBLE MORAL HAZARD

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August 1984

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July 1984

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## Abstract

This paper explores a model of warranties in which moral hazard problems play a key role. The goal is to understand the important characteristics of warranties including their provision of incomplete insurance and the relationship between product quality and coverage. We analyze a model in which buyers and sellers take actions which affect a product's performance. Since these actions are not cooperatively determined, an incentives problem arises. We characterize the optimal warranty contract and undertake comparative statics to determine the predicted correlation of warranty coverage and product quality.

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## PRODUCT WARRANTIES AND DOUBLE MORAL HAZARD

### I. Introduction

Warranties are prevalent in commodity markets.<sup>1</sup> Most durables, in particular, have some type of warranty promising payment from the producer conditional on performance of the product. Three principal characteristics of these warranties interest us here:

- (i) they provide less than full insurance against unsatisfactory performance,
- (ii) they are supplied by the seller of the product rather than by independent insurance agencies, and
- (iii) the extent of warranty protection bears no general relation to the built-in quality of the good. That is, the sellers of more reliable brands of a particular product may offer more, equal or even less warranty protection than sellers of less reliable brands.

Existing warranty theories have difficulty explaining these observations. If warranties are simply insurance policies sold by the seller to the buyer, Heal's (1977) results suggest that with risk neutral sellers the insurance should be complete. This insurance theory also does not explain why insurance companies are not providing the insurance, nor does it speak to point (iii) above. Spence (1977) has argued for the signalling role of warranties. High quality producers can afford to offer close to full insurance since their products are unlikely to break down.

The signalling theory, however, also fails to explain (iii). In fact, it allows only for a positive correlation between the extent of warranty protection and a brand's reliability. While such positive correlations may indeed exist in some markets, in many others there is little or even a negative relationship between the variables.<sup>2</sup>

In this paper we explore a model of warranties in which moral hazard problems play the central role. The model is consistent with all three of the phenomena listed above. We are, of course, not the first to recognize the importance of moral hazard in shaping warranty contracts. In fact, it can be argued that moral hazard problems lie at the core of Priest's (1981) "Investment Theory" of warranties. Our approach here is both more direct and more formal, however.

There are two types of incentive problems which we see as relevant. First, buyers generally take actions which influence the performance of the product. To the extent that these actions are not monitored or detectable by the seller, a moral hazard problem will arise if warranties are present. Second, the actual qualities of many commodities are not directly observable to buyers. In such a situation, the incentives to a seller to maintain high quality are also embedded in the warranty. Hence, warranties may act as incentive mechanisms for both sides of the product market. Here we focus on the resulting problem of double moral hazard.

Recent papers by Kambhu (1982) and Mann and Wissink

(1983) have analysed models which are structurally similar to ours. In contrast to their efforts, we focus on characterizing the optimal agreement between firms and customers in the presence of double moral hazard. Kambhu is primarily interested in the role of third parties as a means of providing incentives for efficient actions. Perhaps because introducing imperfectly informed third parties would only compound the moral hazard problems, most warranty contracts involve only the buyer and the seller. For this reason, we have chosen to focus exclusively on such bilateral agreements. Mann and Wissink also consider only bilateral arrangements to overcome moral hazard problems. They investigate a special contracting mechanism which they find intuitively appealing, with particular reference to the problem of vertical integration. In this essay, we search over a broader set of contracting possibilities.

Our results indicate, as one would expect, that due to these incentive problems, warranties will offer only partial insurance. We are able to determine the effects on the levels of care exercised by buyers and quality built in by sellers, of the imperfect information and resulting double moral hazard. These distortions are shown to depend critically on whether care and quality are complements or substitutes in determining the probability that a product will work. We also study some comparative statics with respect to the costs of care and quality to the customers and firms, respectively. These results suggest that such heterogeneity

may indeed explain why the observed correlation between quality and warranty protection is sometimes positive and sometimes negative.

## II. Overview of the Model

We consider a contract between a buyer and a seller. The parties to the contract may be viewed as two firms or as one consumer and one firm. In what follows we use the latter labels. In order to concentrate our attention on the incentive problems here, we remove any concerns about risk sharing by assuming that both the buyer and the seller are risk neutral.

The contract stipulates the price ( $p$ ) to be paid by the buyer for a single unit of the commodity which will be traded. This good may or may not work after its purchase. The probability that it works is represented by  $\Pi$ , thus the probability of breakdown is  $(1-\Pi)$ .  $\Pi$  is a function of two variables:  $q$ , the quality level chosen by the seller, and  $e$  the level of care or effort expended by the buyer. Letting subscripts denote partial derivatives, we assume that  $\Pi_e > 0$ ,  $\Pi_q > 0$ ,  $\Pi_{ee} < 0$  and  $\Pi_{qq} < 0$ . This means that the inputs are productive though at a decreasing rate. The sign of  $\Pi_{eq}$  is left unspecified at this point, and it will prove crucial in the analysis that follows.

Each buyer's expected utility is represented by

$$U(e, q, p, s) = y - p + \Pi z + (1-\Pi)sz - g(e)$$

with  $g'(\cdot) \geq 0$ ,  $g'(0) = 0$  and  $g''(\cdot) > 0$ . Each consumer's initial income is  $y$  and  $p$  is spent on the commodity studied here.



If the product works (with probability  $\Pi$ ) it is worth  $z$  "dollars" to the consumer. If the product fails, the warranty provides that the buyer receive " $sz$ " as compensation from the seller. " $s$ " is then a measure of the degree of warranty protection in the contract. Product failure is always assumed to be a readily observable phenomenon. The function  $g(e)$  measures the consumer's disutility of effort.

A firm's expected profits will be

$$V(e,q,p,s) = p - C(q) - (1-\Pi)sz$$

where  $C(q)$  is an increasing, convex cost of quality function,  $C'(\cdot) \geq 0$ ,  $C'(0) = 0$  and  $C''(\cdot) > 0$ . Hence, the firm's total cost is the sum of the cost of production,  $C(q)$ , and the expected warranty payment.

The choices of  $e$  and  $q$  will affect the parties directly through the cost functions,  $g(e)$  and  $C(q)$ , and indirectly through the probability function  $\Pi(e,q)$ .

Before addressing the double moral hazard itself, we begin by stating the full-information, cooperative solution. Here, all elements of the contract  $(p,s,e,q)$  are set cooperatively and the contract is fully enforceable. Given the linearity of the problem,  $p$  and  $s$  will be indeterminate. The cooperative solution  $(e^*,q^*)$  satisfies

$$(1) \quad \Pi_e(e,q)z = g'(e)$$

$$\text{and } (2) \quad \Pi_q(e,q)z = C'(q) \quad .$$

We denote the solution to (1), for a given  $q$ , as  $e^*(q)$  and the solution to (2), for a given  $e$ , as  $q^*(e)$ . The cooperative solution, also called first-best or FB, is simply

the combination of  $(e, q)$  satisfying (1) and (2). Given the cooperative nature of the agreement,  $e^*$  and  $q^*$  are set so that marginal benefits (to the parties jointly) equal the marginal costs to each.

In general there may be multiple or no solutions to these equations. We will be more specific about the properties of the cooperative solution in the next two sections.

### III. Warranties with Unobservable Effort and Qualities: Double Moral Hazard

The cooperative agreement, characterized by (1) and (2), requires that the input levels of the two parties be set at  $(e^*, q^*)$  regardless of any incentives that may exist to alter these input levels. Hence, there must be an explicit enforcement mechanism for the implementation of the cooperative agreement. When these inputs are not costlessly observable to the parties to the contract, or the courts, an enforcement problem arises. In such a situation, the agreement must be self-enforcing so that neither party has an incentive to deviate from the agreed-upon actions.

We suppose that the price of the product and the warranty level can be cooperatively set in an enforceable manner. In contrast to the previous section of the paper, we do not allow for consumer effort or producer quality levels to be determined cooperatively. Hence, the contract, through the choice of  $p$  and  $s$ , must provide incentives for the parties to take appropriate actions.

To model this double moral hazard problem, we consider

the two-stage game played by the parties to the contract. In the first (cooperative) stage, the parties sign a binding agreement with respect to  $p$  and  $s$ . The second (noncooperative) stage takes  $(p,s)$  as given and the players choose their inputs,  $e$  and  $q$ . We focus on the Nash equilibrium of this noncooperative game.

Payoffs are made after the condition of the product is determined. We begin by analyzing the second stage of the game for arbitrary  $(p,s)$ . In fact, given the linearity of the problem, the second stage equilibrium is independent of  $p$ , though it will depend crucially on  $s$ .

When  $q$  and  $e$  are chosen noncooperatively, we can analyse the problem by looking for reaction function equilibria. For given  $s$ , buyers choose effort to maximize  $U$  with respect to  $e$ , given their conjecture about the level of  $q$ . The solution to this problem,  $\hat{e}(q;s)$ , will be independent of  $p$  and will satisfy the first order condition:

$$(3) \quad \Pi_e(e,q)(1-s)z = g'(e) \quad .$$

Similarly, the firm chooses  $q$ , given  $s$  and a conjecture about  $e$ , to maximize expected profits. The solution,  $\hat{q}(e;s)$ , satisfies

$$(4) \quad \Pi_q(e,q)sz = C'(q) \quad .$$

We assume that  $\hat{e}(0;s)$  and  $\hat{q}(0;s)$  are both positive for  $0 < s < 1$ . That is, if one party is providing no input at all, the best response of the other is to provide a strictly positive amount of input. From (3) and (4), one can easily compute the effects of changes in  $s$  on  $e$  and  $q$  (i.e.  $e$ , and

$q_s$ ):

$$e_s = \Pi_e z / [\Pi_{ee}(1-s)z - g''] < 0$$

$$q_s = -\Pi_q z / [\Pi_{qq}sz - C''] > 0 \quad .$$

We can use (3) and (4) to compare the solutions to the full-information and the noncooperative equilibria. For given  $s$ , our model reproduces the results of Kambhu and Mann-Wissink regarding the reaction curves of the two parties.<sup>3</sup>

Proposition 1: For  $0 < s < 1$ ,  $\hat{q}(e; s) < q^*(e)$  and  $\hat{e}(q; s) < e^*(q)$ .

Proof: Direct from comparing (1) and (2) with (3) and (4). ||

Simply stated, with  $0 < s < 1$ , neither party receives the full benefit of exerting more effort (for the buyer) or increasing quality (for the seller). Hence, both parties have an incentive to shirk and to reduce their inputs into the  $\Pi$  function.

To characterize the distortions in quality and effort in the second-best (SB) equilibrium, we relate the reaction functions in the FB and SB problems, i.e.,

$$(5) \quad e^*(q) = \hat{e}(q; 0)$$

and

$$(6) \quad q^*(e) = \hat{q}(e; 1).$$

Since  $s$  cannot simultaneously equal 0 and 1, it is obvious that the full information solution is not implementable as a noncooperative equilibrium.

In determining the distortions due to the double moral

hazard, we differentiate between the cases of  $\Pi_{e,q} > 0$ ,  $\Pi_{e,q} = 0$  and  $\Pi_{e,q} < 0$ . We begin with  $\Pi_{e,q} > 0$  so that quality and effort are complements. For the present discussion, we shall assume that both the FB and SB equilibria exist and that they are unique and stable. We consider the consequences of multiple equilibria in the next section.

As shown in Figure 1, (5) and (6) allow us to easily compare the two solutions when  $\Pi_{e,q} > 0$ . Point K is assumed to be a unique solution to the full information problem. Since  $\hat{e}_s(q;s) < 0$  it is clear that the noncooperative reaction function for the choice of effort lies below the cooperative function  $e^*(q)$  for  $s > 0$ . Similarly,  $\hat{q}(e;s)$  lies above  $q^*(e)$  for  $s < 1$ . So if  $s$  is interior, the curves are as shown in the Figure. Hence, A is a Nash equilibrium for a given  $s$ .

To compare quality and effort levels, it is clear from the figure that if there are unique equilibria for both the full-information and SB economies, then quality and effort will be lower when the incentive problems are present.

Proposition 2: For  $0 < s < 1$ ,  $\Pi_{e,q} > 0$ , and  $(q^*, e^*)$  solving (1) and (2), there exists a  $\hat{q} < q^*$  and  $\hat{e} < e^*$  such that  $(\hat{q}, \hat{e})$  solves (3) and (4).

Proof: Since  $\hat{q}(e;s)$  is monotonically increasing in  $e$ , we can define the inverse function  $\Phi(q;s)$  as the value of  $e$  such that  $\hat{q}(e;s) = q$ .  $\Phi(q;s)$  is increasing in  $q$ . We have assumed that  $\hat{e}(0;s) > 0$  and  $\hat{q}(0;s) > 0$  so that there exists  $\bar{q} > 0$  where  $\Phi(\bar{q}, s) = 0$ . This is shown in Figure 1 as well.

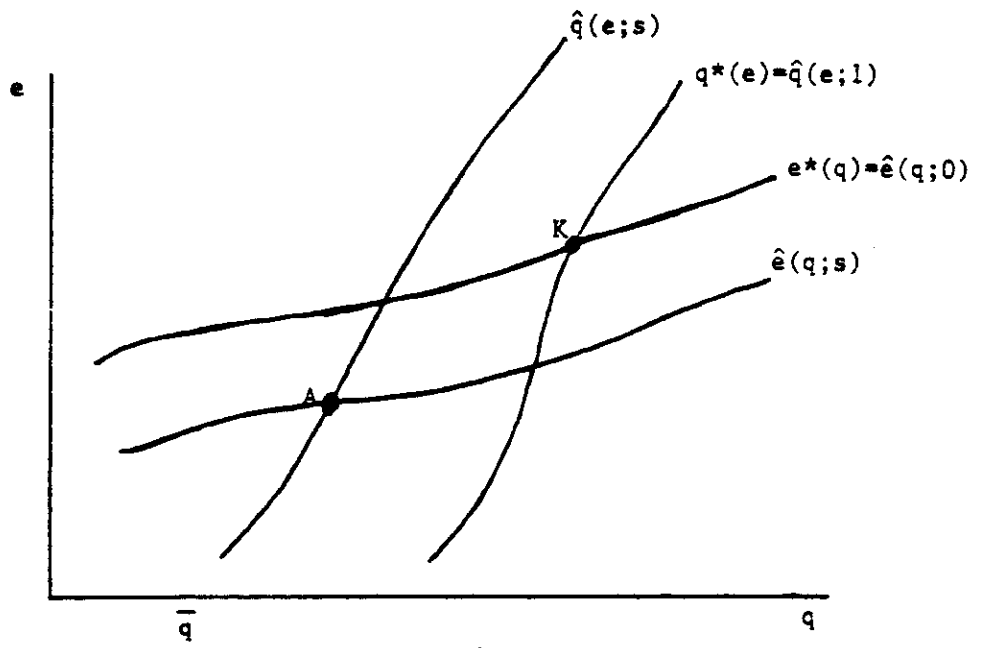


FIGURE 1

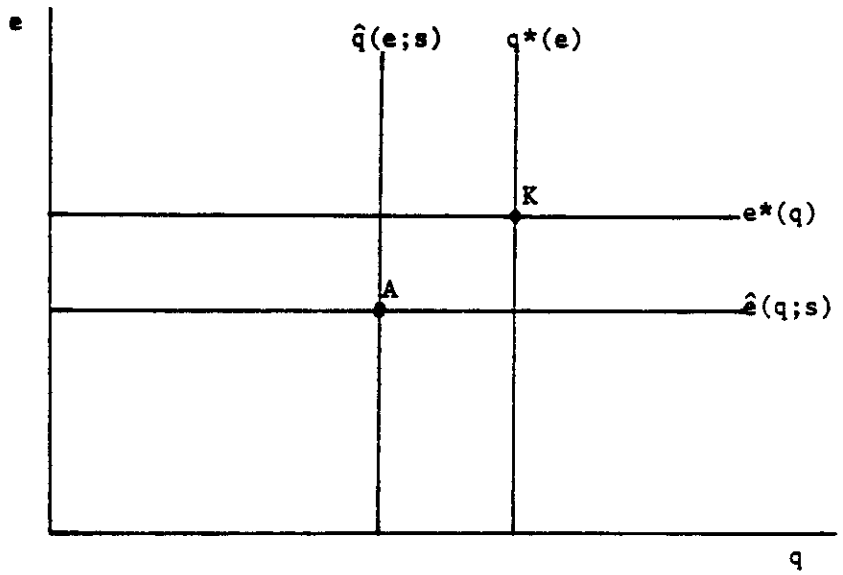


FIGURE 2

Finally, define  $D(q)$  by

$$D(q) \equiv \hat{e}(q;s) - \Phi(q;s).$$

Clearly,  $D(\hat{q}) > 0$ . At  $q^*$ ,  $\hat{e}(q^*;s) < e^*(q^*) < \Phi(q^*;s)$  from Proposition 1. Hence  $D(q^*) < 0$ . Since all of the reaction functions are continuous, there exists  $\hat{q}$  in  $(\hat{q}, q^*)$  such that  $D(\hat{q}) = 0$ . This  $\hat{q}$  is a Nash equilibrium point. Since  $\hat{e}(q;s)$  is monotonically increasing in  $q$ ,  $\hat{e} < e^*$ . ||

Figure 2 makes it clear that quality and effort will again be lower than their FB levels when  $\Pi_{e,q} = 0$ . In fact, Proposition 2 still holds when  $\Pi_{e,q} = 0$ .

When  $\Pi_{e,q} < 0$ , we can say very little in terms of comparisons. Figures 3a and 3b show the ambiguity. We do know that both  $e$  and  $q$  cannot be higher in the noncooperative solution than in the FB since, as stated before,  $e^*(q) > \hat{e}(q;s)$  and  $q^*(e) > \hat{q}(e;s)$  for  $0 < s < 1$ . However, it can be the case that either  $e$  or  $q$  will be higher in the SB, as we see from the figures.

Intuitively, the ambiguity is not surprising. Suppose we start at a cooperative solution. Then both the seller and the buyer would want to reduce their inputs taking the other party's choice as given. This was also the case for  $\Pi_{e,q} \geq 0$ . Now when  $\Pi_{e,q} < 0$ , there are some offsetting effects. When the firm reduces  $q$ , this makes the consumer's effort more productive so that  $e$  will be increased. A similar effect occurs in the firm's choice of  $q$ . As Figures 3a and 3b show, it is possible for either of these two effects to dominate for one of the parties. It is clearly impossible

for both  $e$  and  $q$  to increase.

Thus far we have concentrated on the non-cooperative game played between the seller and the buyer. Our analysis holds for arbitrary  $s \in (0,1)$  though the magnitude of  $s$  will clearly affect the actual levels chosen in equilibrium.

We can denote the set of Nash equilibria of the non-cooperative game, for given  $s$ , by  $N(s)$ . Since the choice of  $p$  does not affect the Nash equilibria, we ignore this variable. The first stage of the problem is then to choose  $s$  given  $N(s)$ . We will continue to assume that  $N(s)$  gives, for any  $s$ , a unique pair,  $(e, q)$ , that solves (3) and (4). We characterize the level of  $s$  which maximizes joint profits by solving:

$$(7) \quad \max_s \quad \bar{L} = y + \Pi z + - g(e) - C(q)$$

subject to:      $e = \hat{e}(q; s)$   
    $q = \hat{q}(e; s) \quad .$

Substituting the constraints into the problem and differentiating with respect to  $s$ , we obtain

$$(8) \quad \partial \bar{L} / \partial s = (\Pi_q z - C') q_s + (\Pi_e z - g') e_s = 0 \quad .$$

Using (3) and (4), we can rewrite (8) as

$$(9) \quad q_s \Pi_q z (1-s) + e_s \Pi_e z s = 0$$

with  $q_s > 0$  and  $e_s < 0$  as defined earlier. At  $s=0$ ,  $\partial \bar{L} / \partial s = q_s \Pi_q z > 0$ , and at  $s=1$ ,  $\partial \bar{L} / \partial s = e_s \Pi_e z < 0$ , so that the solution to (9) will be  $s^* \in (0,1)$ . Thus, the optimal SB level of



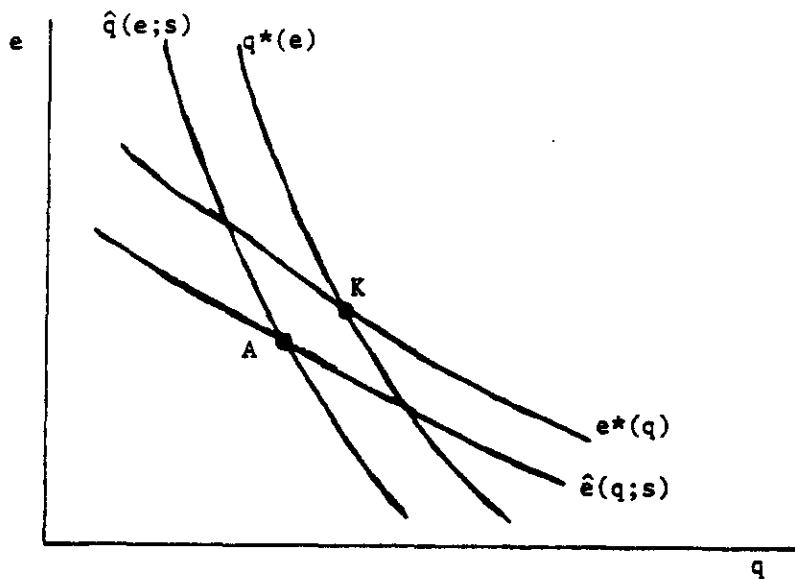


FIGURE 3a

SB has lower e and q.

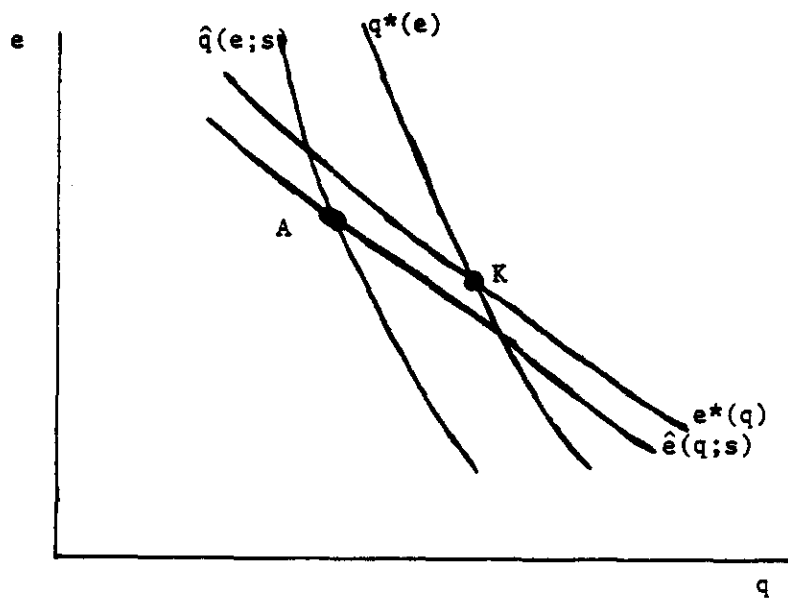


FIGURE 3b

SB has lower q and higher e

warranty protection will be interior.'

We turn now to a consideration of comparative statics on the SB equilibrium. We are particularly interested in understanding what happens to the values of the observables,  $s$  and  $q$ , when various parameters of the model change. As one would expect, the results here will depend critically on the sign of  $\Pi_{s,q}$ .

The signalling literature, as we described earlier, suggests that, within markets,  $s$  and  $q$  should be positively correlated. The story there relies on imperfect buyer information and says simply that firms with high quality goods will attempt to signal this quality by offering a more complete warranty. Since warranties are less costly to provide when the product breaks less often, high quality firms will be able to signal more cheaply, therefore a signalling equilibrium is possible.

Though positive correlations between  $s$  and  $q$  are undoubtedly observed in many markets, negative correlations are found as well. An obvious example here is the automobile market in which the Japanese manufacturers sell small cars of higher (by most accounts) quality than the domestic makers but with inferior warranty protection.

Study of the comparative statics of this model reveal conditions under which both positive and negative correlations will be observed. It would seem plausible that certain differences among buyers could generate positive correlations. For example, more risk averse customers might be

expected to demand more protection from breakdown, and this protection might, in general, involve higher levels of both  $s$  and  $q$ . With the risk neutral buyers in this model, a similar story can be told with reference to each buyer's cost of effort.

On the other hand, certain differences among firms in the market may lead them to make different choices regarding how they protect their customers. Some firms (e.g. Japanese auto makers) may have cost advantages in building quality, but suffer cost disadvantages in providing warranty protection.<sup>5</sup>

We first consider the effects of increasing the marginal disutility of effort, i.e.  $g(e)$  becomes  $\delta g(e)$  and we consider the effects of increasing  $\delta$ . By lowering  $(\Pi_{,z}-g')$  and raising  $e$ , (which is negative), this makes  $\partial \bar{L}/\partial s > 0$  and therefore pushes  $s$  upward. This makes intuitive sense. As effort becomes more expensive, we want to shift the burden of raising  $\Pi$  more toward the seller.

From (3) we see that  $\hat{e}(q;s)$  will fall with rising  $\delta$ , but we expect no effect on  $\hat{q}(e;s)$ . The rising  $s$  will further depress  $\hat{e}(q;s)$  while increasing  $\hat{q}(e;s)$ . When  $\Pi_{,q} \leq 0$  these effects all reinforce one another, resulting in a new equilibrium with lower  $e$ , higher  $q$  and higher  $s$ . Thus, we observe a positive correlation between  $s$  and  $q$  when the variance is in the buyers' costs of care.

With  $\Pi_{,q} > 0$ , the net effect is ambiguous. As  $\hat{e}(\cdot)$  shifts down it pushes  $e$  and  $q$  down, but  $\hat{q}(\cdot)$  shifting out

pushes both levels up. We can only say that as long as  $\Pi_{e,q}$  is not "too large", the net result will still be lower levels of  $e$  and higher levels of  $q$ .<sup>6</sup>

To consider differences across firms, we first add a new element to the cost of the warranty. Now the firm's expected costs are written:  $C(q) + (1 - \Pi)(sz + x)$ , where  $x$  represents some cost of the warranty that does not go to the buyer. It could simply be the cost of verifying that the product is broken and of processing the claim.<sup>7</sup>

Increasing  $x$  will not affect  $e_s$ , but it will lower  $q_s$ , making  $\partial \bar{L} / \partial s < 0$  and reducing the SB level of  $s$ . Since (4) is now

$$(4') \quad \Pi_q(e, q)(sz + x) = C'(q)$$

it is clear that  $\hat{q}(\cdot)$  will increase with increasing  $x$ . However, the falling  $s$  will push  $\hat{e}(\cdot)$  up and  $\hat{q}(\cdot)$  down. As long as the  $\hat{q}$  function rests, on net, higher, the case of  $\Pi_{e,q} \geq 0$  will give us higher levels of both  $q$  and  $e$ . When  $\Pi_{e,q} < 0$ , we cannot count on either being true.

Thus, it seems at least possible that we could observe negative correlations between  $s$  and  $q$  when the costs of providing the warranty vary across firms.

Finally, we contemplate the effect on  $q$ ,  $e$ , and  $s$  of an increase in the marginal cost of quality. This is analogous to the cost of effort case already considered. Now  $C(q)$  becomes  $\gamma C(q)$  and we consider the effects of increasing  $\gamma$ . This will lower  $q_s$ , making  $\partial \bar{L} / \partial s < 0$ . This means that the SB level of  $s$  will fall. As  $\gamma$  goes up, (4) tells us that  $\hat{q}(\cdot)$

will fall. The lower  $s$  pushes  $\hat{q}(\cdot)$  down still further and raises  $\hat{e}(\cdot)$ . For  $\Pi_{e,q} \leq 0$  we have the clear result that the SB level of  $q$  will be lower while the level of  $e$  is higher. For  $\Pi_{e,q} > 0$  the net effects on both  $e$  and  $q$  are again ambiguous.

If consumers were risk averse, so that the warranty was also serving an insurance function, there would be reason to suspect that a rising  $\gamma$  might lead to a higher  $s$ . The increase in the costs of quality would have two effects, analogous to substitution and income effects. With rising  $\gamma$  the seller would choose to switch to a lower  $q$  and higher  $s$  if he wanted to provide the same insurance protection. As we have found here, however, it will also be appropriate for the contract to encourage further effort on the part of the buyer, so that  $s$  and the amount of insurance provided may still be reduced.

Combining the results from this analysis of the model's comparative statics, we find at least the seed of an explanation for observed positive and negative correlations between  $s$  and  $q$ . Buyers with higher costs of effort will seek out sellers willing to carry more of the burden of preventing breakdowns by choosing a higher level of  $q$ . These incentives are accommodated through the selection of a higher level of  $s$ . When some sellers have a cost disadvantage in providing warranty protection, they may opt to reduce their warranty offerings and raise  $q$  (so that they have to pay off on the warranty less often).

#### IV. Multiple Equilibria

When there are multiple equilibria, it is not as obvious how to make comparisons between the FB and SB equilibria. At this level of generality, one cannot even ensure that the number of noncooperative equilibria bear any relation to the number of solutions to the full information problem. Moreover, as  $s$  varies, the number of noncooperative equilibria may vary as well. Using the insights of the literature on smooth economies (e.g. Debreu (1970) and Balasko (1978)), if we assume that  $\Pi(e,q)$  is smooth (so that the reaction functions are continuously differentiable), then we know that there will generally be an odd number of locally unique equilibria.<sup>1</sup> This local uniqueness provides a basis for the comparative statics results reported above.

One can make some statements for the multiple equilibria case along the lines of our earlier proposition. Figure 4 depicts an example with  $\Pi_{e,q} > 0$ , where there are cooperative equilibria ( $K_1$ ,  $K_2$ , and  $K_3$ ) and an equal number of noncooperative equilibria ( $A_1$ ,  $A_2$ , and  $A_3$ ). It is obvious from this figure that some noncooperative solutions will have higher  $q$  and  $e$  than some of the full information solutions (compare  $A_1$  and  $K_3$ ). It is equally apparent that Proposition 2 applies here in that  $A_1$  will have lower quality and lower  $e$  than any of the cooperative equilibria.

Suppose we were to start the noncooperative economy at any of the cooperative equilibria points. From Proposition 1, both the seller and the buyer, taking the other's choice

of input as given, will have incentives to reduce their own inputs. Without carefully specifying a dynamic story, this shirking will continue until a new Nash equilibrium is reached. So if the economy is started at either  $K_1$  or  $K_2$ , we would expect to see a noncooperative solution of  $A_1$ . Similarly, if we start at  $K_3$ , the economy would unravel to  $A_1$ . In either case, both  $q$  and  $e$  are lower in the noncooperative solutions.

The final point to notice about the figure is that for the stable full-information solutions ( $K_1$  and  $K_3$ ), there will be an associated noncooperative solution with lower  $q$  and  $e$ . Hence, if one is content to focus on stable solutions, Proposition 2 can be easily extended.

## V. Conclusions

Our general interest here lies in trying to understand the role moral hazard plays in shaping sale-warranty contracts. To this end, we have solved for the optimal second-best equilibria that obtain under the conditions of double moral hazard. We were able to describe the directions of the SB distortions, and to show the dependence of these distortions on the properties of the  $\Pi(e,q)$  function.

We also explored the sensitivity of these equilibria to changes in the values of certain parameters of the model. The results suggested conditions under which we could expect either positive or negative correlations between the extent of warranty protection offered with a particular brand of a product and the level of quality built into that brand.

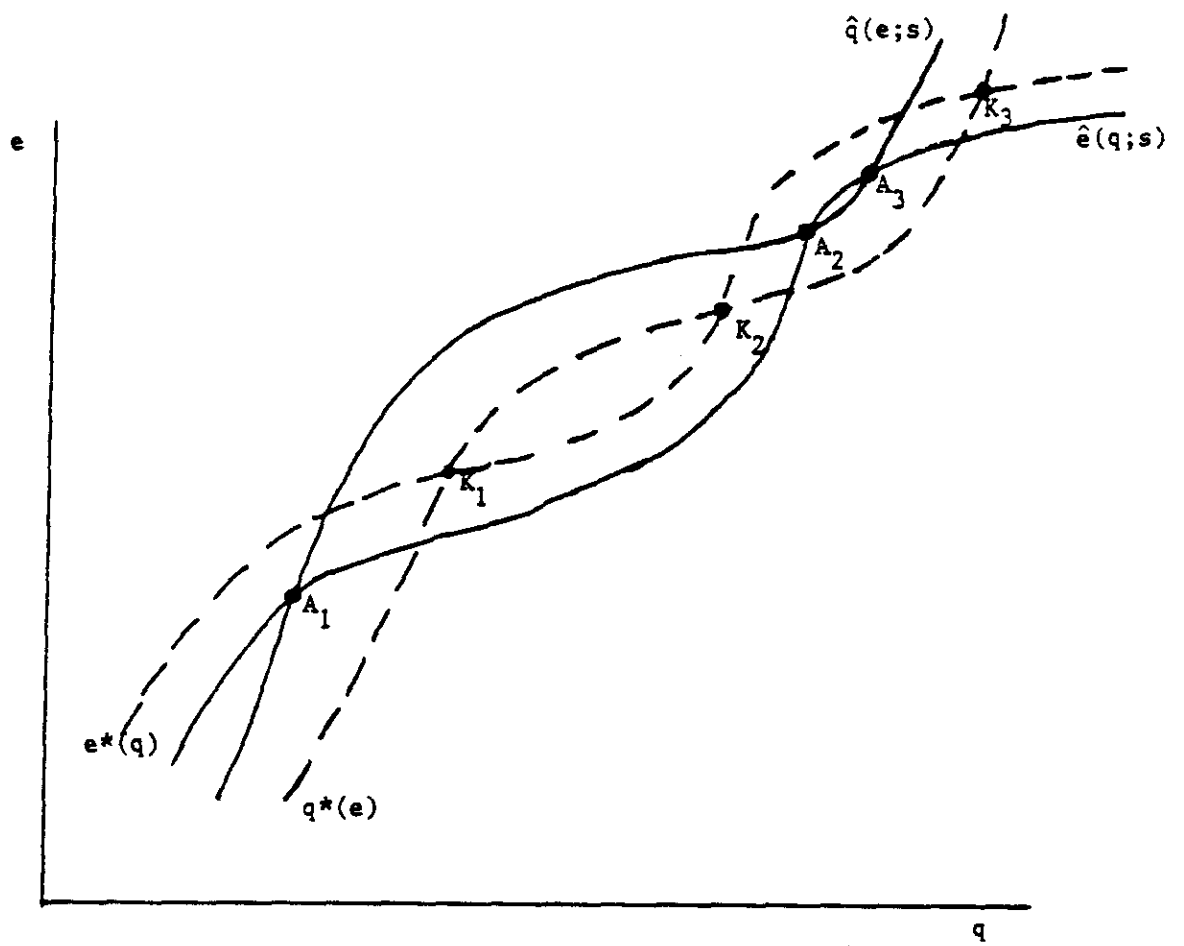


FIGURE 4



## FOOTNOTES

<sup>1</sup> Priest (1981) goes so far as to say that warranties may, in fact, be among the most common of written contracts (p. 1297).

<sup>2</sup> See Priest (1981) and Schwartz and Wilde (1983) for discussions of this empirical evidence.

<sup>3</sup> Our results do not require the additional assumptions made in Kambhu, regarding the benefit and quality functions. That is, we do not require condition (9a) in Kambhu (1982).

<sup>4</sup> The conditions  $g'(0)=C'(0)=0$  insure that (3) and (4) hold even when  $s=0$  or  $s=1$ . Hence,  $q_s > 0$  and  $e_s < 0$  for all  $s$ . Without these assumptions,  $s=0$  or  $s=1$  could be local optima, as in Kambhu (1982).

<sup>5</sup> Smaller parts inventories and dealer networks raise the costs to the Japanese car makers of providing any given level of warranty protection, at least in the short run.

<sup>6</sup> We have set aside any consideration of the additional problem of adverse selection that may arise in these markets when customers differ.

<sup>7</sup> Actually,  $x$  may even be negative. The firms may not need to spend  $sz$  to get the buyer  $sz$  dollars of benefit out of the product. They may be able to repair it perfectly and cheaply, so that the uninsured portion,  $(1-s)z$ , represents only the buyer's inconvenience cost.

<sup>8</sup> To be precise, we need to assume that:  $\lim_{q \rightarrow \infty} D(q) < 0$  for all  $s$ .

## REFERENCES

- Balasko, Y., "Economic Equilibrium and Catastrophe Theory", Econometrica, Vol. 46 No. 3, May 1978, pp. 557-569.
- Debreu, G., "Economies With a Finite Set of Equilibria", Econometrica, Vol. 38, 1970, pp. 387-392.
- Green, J., "On the Optimal Structure of Liability Laws", Bell Journal of Economics, Vol. 7, No. 2, Autumn 1976, pp. 553-574.
- Heal, G., "Guarantees and Risk Sharing," Review of Economic Studies, Vol. 44, 1977, pp. 549-560.
- Holmstrom, B., "Moral Hazard and Observability," Bell Journal of Economics, Vol. 93, 1979. pp. 541-562.
- Holmstrom, B., "Moral Hazard in Teams", Bell Journal of Economics, Vol. 13 No.2, Autumn 1982, pp. 324-341.
- Kambhu, J., "Optimal Product Quality Under Asymmetric Information and Moral Hazard," Bell Journal of Economics, Vol. 13, No. 2, Autumn 1982, pp. 483-492.
- Kambhu, J., "Product Liability Rules and Moral Hazard in Incentive Contracts", mimeo, Columbia University, June 1983.
- Mann, D. and J. Wissink, "Inside vs. Outside Production: A Contracting Approach to Vertical Integration," mimeo, University of Pennsylvania, April 1983.
- Priest, G., "A Theory of Consumer Product Warranty", Yale Law Journal, Vol. 90, No. 6, May 1981, pp. 1297-1352.
- Schwartz, A. and L. Wilde, "Imperfect Information in Markets for Contract Terms: The Examples of Warranties and Security Interests", Virginia Law Review, Vol. 69, No. 8, November 1983, pp. 1387-1485.
- Spence, A.M. "Consumer Misperceptions, Product Failure and Product Liability," Review of Economic Studies, Vol. 44, 1979, pp. 561-572.
- Wilde, L. and A. Schwartz, "Consumer Markets for Warranties", mimeo, 1982.