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FISCAL INCIDENCE IN A DYNAMIC LIFE-CYCLE MODEL WITH LAND

Christophe Chamley and Brian D. Wright

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FISCAL INCIDENCE IN A DYNAMIC LIFE-CYCLE MODEL WITH LAND

by

Christophe Chamley and Brian D. Wright*

I. Introduction

The life-cycle model has become popular in studies of fiscal incidence in general equilibrium framework.¹ Although it is only a very stylized description of a real economy, it provides a useful tool for analysis of the incidence of fiscal policies on the welfare of different generations, and on endogenous economic variables.

A standard assumption is the perfect substitutability between consumption goods and investment goods as the outputs of the production technology. The price of capital in terms of consumption is thus fixed and independent of any fiscal policy. Accordingly all incidence effects occur through changes in the size of the stock of non-labor assets, rather than through changes in asset prices. Since a significant fraction of savings is invested in assets which have a very long life time and are not easily substitutable with consumption, one may well ask whether the inclusion of "capitalization" effects may importantly affect the results on the incidence of fiscal policy.

In an important extension of the life-cycle approach, Feldstein (1977) introduced a third, fixed factor, "land," and considered the effects of an "uncompensated" land rent tax on the price of land. He showed that such a tax induces a reallocation of demand between the fixed asset, land, and the variable asset, capital, and that this effect

radically modifies the traditional result that the land price falls by the amount of the tax. In fact he obtained the surprising result that the price of land may even rise in the long run in his model. Though his life-cycle analysis concentrated solely on comparative statics effects on land prices, he further claimed (p. 353) that the initial older generation might gain from imposition of his "uncompensated" land rent tax.

But the incidence of taxes on the members of the initial asset-holding generation is directly affected by the gains or losses in land value which accrue to them. These inter-generational welfare effects which occur outside the steady state cannot be assessed by the conventional comparative static analysis performed in previous studies.

In order to go beyond the comparative statics approach we construct a new dynamic life-cycle analysis of the effects of fiscal interventions. We use the standard stylized framework of a two period overlapping generations model, modified to include both capital and a fixed factor, land. In Section II we analyze in detail the stability properties of the model for two reasons. First, the stability question is not trivial, for as Calvo (1978) has shown in a simpler but similar model, the price of land may not always be uniquely determined. Also some incidence results are directly related to the stability properties of the model; the "unstable" eigenvalue of the dynamic system is an important parameter in the incidence results which we present.

The short-run impact of an uncompensated land tax on the land price is analyzed in Section III. We find that if the consumption levels in the two periods of an individual's life are sufficiently complementary, the price of land may rise immediately when the land rent tax is

introduced. However we show that this price rise, if it occurs, is always smaller than one-half of the tax revenues collected per unit of land if the tax is "uncompensated," that is, if the revenues are disposed of in a way which does not affect individual utilities. Therefore, the land owners bear at least one-half of the tax burden.

In the fourth section, we consider the incidence of the tax on the life-time utilities of all generations. Except for the old generation which owns the land at the time of tax reform, the price of land is not an argument in the indirect life-time utilities of individuals, which depend only on the net factor prices. In particular, the welfare incidence of an uncompensated land rent tax across generations depends only on the variation of the capital stock which is induced by a reallocation of land and capital in private portfolios. This effect is similar to the Tobin affect (1965) of inflation in a monetary economy.² One should add that the effect of the land tax may be quantitatively more important since the value of land dwarfs the stock of the monetary base. In general, the impact of the land tax on capital accumulation is ambiguous, but if we eliminate the parametric values for which "crowding in" (the reverse of crowding out) occurs, the tax stimulates capital accumulation. Note that the existence of operative bequests neutralizes this effect in both cases (for a monetary economy, see Sidrauski, 1967, and for a model with land, see Calvo, Kotlikoff and Rodriguez, 1979).

Finally, although we concentrate mainly on the land tax in order to simplify the argument, we extend our framework in Section V to analyze the incidence of the taxes on the other factors. The presence of a fixed factor like land may significantly alter the traditional results. For example, the introduction of a system of intergenerational transfers

from the young to the old, as in an unfunded social security scheme, has a negative impact on the land price which reduces the magnitude of the net transfer received by the first generation.

II. The Model

Individuals live for two periods. They supply a fixed quantity (one unit) of labor and earn a wage w_t in the first period, when output is produced. Since we assume that the population size is constant and that any individual is identical to all others of his generation, we can normalize the population size at unity.

The production technology is represented by a twice-differentiable neoclassical production function $F(K,L,T)$ with positive decreasing first derivatives and constant returns to scale. The three inputs: capital, K , labor, L , and land, T , are awarded their marginal returns, r , w , and m respectively. The supply of land is fixed and normalized at unity. The produced good is the numeraire, and P_t is the price of land.

Savings are invested partly as capital K which does not depreciate and is a perfect substitute for the consumption good, and partly in land purchased from the older generation who are in the second period of life, and consume all of their accumulated wealth, leaving no bequests. The level of savings of the young generation at time t , S_t , depends on the wage rate w_t , the perfectly anticipated rate of return on capital, r_{t+1} , and the structure of tax rates and transfers. For simplicity, we first consider only a tax on land which is uncompensated in the sense defined above.

The savings equation is then given by:

$$(1) \quad S(w_t, r_{t+1}) = K_{t+1} + P_t,$$

where P_t is the price of land. Consumption in both periods is a normal good.

Since capital and land are perfect portfolio substitutes, they have the same rate of return:

$$(2) \quad 1 + r_{t+1} = \frac{\left((1-\theta)m_{t+1} + P_{t+1} \right)}{P_t}$$

where m_t is the marginal product of land in period t , and θ is the land rent tax rate. Note that the return on land includes both rent and any change in value relative to consumption. (Since capital is always a perfect substitute for consumption, its return includes no price appreciation component.) The inclusion of the intra-period price appreciation in the return to holding land is important because, unlike Feldstein (1977), we do not restrict ourselves to steady state analysis.

Since w_t and r_{t+1} are functions of K_t and K_{t+1} , respectively, equation (1) implicitly defines K_{t+1} as a function of K_t and P_t :

$$(3) \quad K_{t+1} = A(K_t, P_t).$$

From equation (2), P_{t+1} is a function of K_{t+1} , P_t , and θ :

$$(4) \quad P_{t+1} = C(K_{t+1}, P_t, \theta).$$

By substitution of the function A for K_{t+1} , we find that P_{t+1} can be expressed as a function of K_t , P_t , and θ :

$$(5) \quad P_{t+1} = B(K_t, P_t, \theta) .$$

The dynamic properties of the model are entirely determined by equations (3) and (5). In order to analyze them, we consider the case with no taxes or transfers, that is $\theta = 0$. We assume that there is a unique steady state, with the values K^* and P^* . By linearization of A and B around these values, we find that

$$(6) \quad \begin{pmatrix} \tilde{K}_{t+1} \\ \tilde{P}_{t+1} \end{pmatrix} = \begin{pmatrix} A_1 & A_2 \\ B_1 & B_2 \end{pmatrix} \begin{pmatrix} \tilde{K}_t \\ \tilde{P}_t \end{pmatrix} \equiv M \begin{bmatrix} \tilde{K}_t \\ \tilde{P}_t \end{bmatrix}$$

where a tilde (\sim) denotes the difference between a variable and its steady state value, and the elements of the matrix M are the partial derivatives of A and B with respect to their first two arguments. The dynamic properties of the model can be described by the following nontechnical remarks:

In the beginning of an arbitrary period taken as zero, the deviation of the capital stock \tilde{K}_0 from the steady state value is predetermined by the past savings. The deviation of the land price \tilde{P}_0 is determined in the land market between the young and the old by the expectations about the next period, as expressed in equation (4).

Assume that such an equilibrium price is given in period t . By iteration, the system (6) determines the entire path of the values $(\tilde{K}_t, \tilde{P}_t)$. These values tend to zero as t tends to infinity (i.e., the dynamic path converges to the steady state), if and only if the initial values of \tilde{K}_0 and \tilde{P}_0 are such that the initial vector $(\tilde{K}_0, \tilde{P}_0)$ is in the space spanned by the eigenvectors of M which are associated with eigenvalues of modulus smaller than one.

Since the value of \tilde{K}_0 is predetermined, an admissible value for \tilde{P}_0 exists and is unique if and only if there is only one eigenvalue with a modulus smaller than one. We will call λ_1 this eigenvalue, and its associated eigenvectors are defined by the ratio between their two components v . The value of \tilde{P}_0 is then determined by the relation:

$$(7) \quad \tilde{P}_0 = v\tilde{K}_0 .$$

In general, near the steady state the dynamic system (6) reduces to

$$(8) \quad \begin{cases} \tilde{K}_t = \lambda_1^t \tilde{K}_0 \\ \tilde{P}_t = v\tilde{K}_t \end{cases}$$

Calling λ_2 the other eigenvalue of M , we deduce from the above discussion that there exists a unique dynamic path near the steady state if and only if $|\lambda_1| < 1$ and $|\lambda_2| > 1$. We will make this assumption throughout the paper.

III. The Short-Run Impact of a Land Rent Tax on the Land Price

We assume first that the economy is in the steady state with no tax. It is defined by the equations (3) and (5), (with $\theta = 0$). This state corresponds to the point A in Figure 1. In period zero, before the members of the old generation sell their land, a tax on the income of land is announced and implemented. Its rate is assumed to be small and constant for all future periods. After the announcement, the market for land opens (the initial holders trading land with the next generation in exchange for retirement consumption), and its price jumps to a new level P_0 (see the figure). In the subsequent periods,

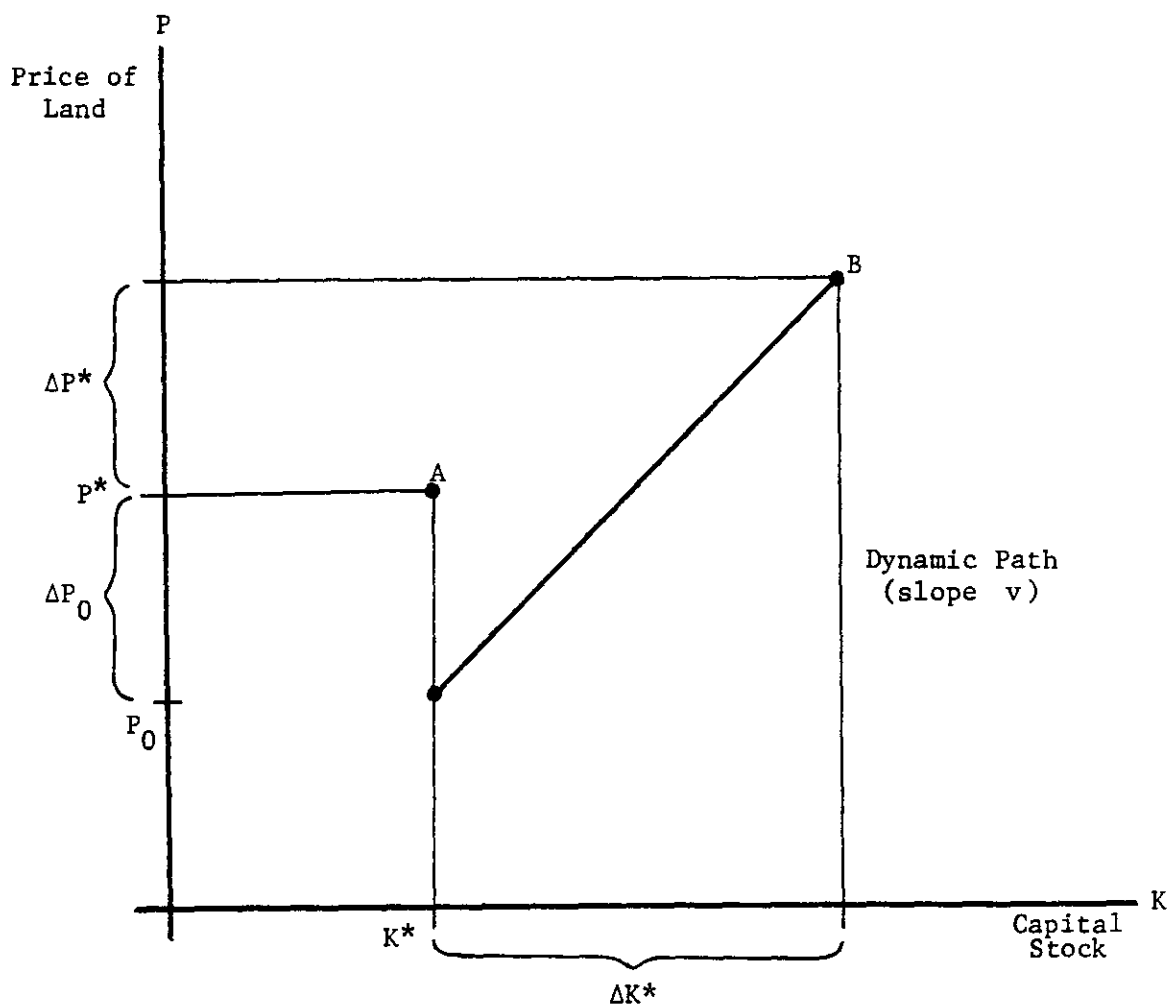


FIGURE 1

Point A: Steady state with no tax.

Point B: Steady state with a land rent tax.

the values of K_t and P_t are represented by points on the dynamic path which converge to the new steady state B .

As discussed in Section II above, the slope of the dynamic path close to the new steady state is v . From Figure 1, it is clear that the price jump ΔP_0 is equal to

$$(9) \quad P_0 = \Delta P^* - v\Delta K^* .$$

We now proceed to determine the value of ΔP_0 in the following two steps.

III.A. The Long-Term Incidence

To a first order approximation, the comparative statics responses ΔK^* and ΔP^* to the tax are determined by differentiation of the equations (3) and (5) in the steady state:

$$(10) \quad \begin{pmatrix} \Delta K^* \\ \Delta P^* \end{pmatrix} = (I-M)^{-1} \begin{pmatrix} 0 \\ m\theta \end{pmatrix} .$$

We normalize the tax rate θ such that the amount of tax revenue is equal to one.³

Define $\psi(\cdot)$ as the characteristic polynomial of M . Then $\psi(1)$ is the determinant of $(I-M)$, and

$$(11) \quad (I-M)^{-1} = \frac{1}{\psi(1)} \begin{pmatrix} 1 - B_2 & A_2 \\ B_1 & 1 - A_1 \end{pmatrix} .$$

Substituting in (10) where $m\theta$ is equal to one,

$$(12) \quad \Delta K^* = \frac{A_2}{\psi(1)} , \quad \Delta P^* = \frac{1 - A_1}{\psi(1)} .$$

These expressions measure the long-run impact of taxation. As implied by the comparative static analysis of Feldstein (1977), their signs are in general ambiguous. Below we will compare these results to the short-term impact of the land tax, which is our main concern here.

III.B. The Immediate Impact on the Land Price

We have seen in the previous section that the slope of the dynamic path v is such that $\begin{pmatrix} 1 \\ v \end{pmatrix}$ is an eigenvector of the matrix M associated with the eigenvalue λ_1 . By definition, we have:

$$(13) \quad A_1 + A_2 v = \lambda_1 ,$$

or

$$v = \frac{\lambda_1 - A_1}{A_2} .$$

By substituting in (9) the values of ΔK^* , ΔP^* , v found in (12) and (13), the value of the price jump is equal to

$$(14) \quad \Delta P_0 = \frac{1 - \lambda_1}{\psi(1)} .$$

Since $\psi(1) = (1 - \lambda_1)(1 - \lambda_2)$, we have the following result:

Proposition 1. The immediate impact of the unanticipated introduction of a permanent land rent tax on the price of land, per unit of tax revenues, is equal to $1/(1 - \lambda_2)$, where λ_2 is the eigenvalue of the dynamic system with a modulus greater than one.

The sign of the price variation is in general ambiguous. It is positive when λ_2 is smaller than one. But under the assumption of a

unique stable path, if λ_2 is smaller than one, it must be smaller than -1 (since λ_1 is the unique eigenvalue between -1 and 1). Therefore, if the price variation is positive, it is smaller than $1/2$. Further, the total effect of the introduction of the tax on the landowners is the sum of the tax payment and the capital gain on land (positive or negative). This implies the following result.

Proposition 2. The immediate response of the land price to an uncompensated land rent tax is always smaller than $1/2$ per unit of tax revenues. The consumption loss of the landowners is at least equal to $1/2$ per unit of revenues.

Note that this proposition rules out a conjecture of Feldstein (1977, p. 353) that the old generation can gain from the introduction of the land tax. This result depends only on the existence and uniqueness of a convergent path. We should emphasize at this point that it is possible to find parameters of the model which will generate any choice of eigenvalues λ_1 and λ_2 . Additional results therefore require further assumptions. To this effect, we consider the property of crowding-out in the long-run. It is verified if the introduction of the government debt decreases the level of capital in the steady state (the debt is perfectly substitutable with capital and its return is financed by a labor income tax).

Lemma. The introduction of a land tax increases the level of the capital stock in the long-run if and only if the parameters of the model are such that the property of long-run crowding-out is satisfied.

The lemma is proven in the appendix. We will maintain the

assumption of crowding-out in the rest of the paper.

We now discuss the sign of the term A_2 . It represents the change of the savings invested in capital by the young generation, in response to a marginal increase in the price of land which is bought by the young (equation (1)), taking into account the effect of this capital accumulation on the rate of return in the next period. We would expect that such a price change crowds out capital accumulation in the short run (in the same period), and therefore, $A_2 < 0$. The sign of A_2 is also negative if an exogenous increase in the income of the young has a positive impact on capital accumulation, ignoring the change of the land price.

We prove in the Appendix the following results:

Proposition 3. When the term A_2 (discussed in the previous paragraph) is negative, the immediate impact of the uncompensated land tax is to lower the value of land.

The amount of this loss is not greater than the full capitalization of the tax if and only if the production function has the following property: An increase in the capital stock, keeping constant the inputs of land and labor, lowers the ratio between the factor prices of capital and land.⁴

A sufficient condition for the term A_2 to be negative is that the interest elasticity of savings η is not strictly negative. As an illustration, let us consider the case of a Cobb-Douglas form for the production technology and the utility function ($\eta = 0$), with the shares of capital, labor and land equal to .15, .7 and .15, respectively, and a rate of return over a generation equal to unity. The short-run impact on the land price is equal to $-1/2$, i.e., half the capitalization of the tax revenues.

Proposition 4. The immediate impact of a land rent tax is to raise the price of land if and only if the assumption of long-run crowding-out is verified and the term A_2 is positive.

A necessary condition for A_2 to be negative is that the consumption levels in the two periods are strongly complementary, and that η is strictly negative. Also, one can show after a straightforward manipulation, that there is a negative value η_0 , such that if $\eta < \eta_0$, crowding-in occurs.⁵ In this unlikely case, the value of land falls by an amount greater than the discounted value of all future tax revenues.

Our results can be compared with those obtained in other studies. Consider first the case where the interest elasticity of savings η is positive and tends to infinity. The effect of policy on the rate of return becomes smaller because it is dampened by the larger response of private savings to variations of the rate of return. At the limit, the rate of return is fixed, and the land tax has no incidence on the level of the capital stock. One can also verify this analytically by noting that in this case $A_2 = 0$ in (12). To maintain the portfolio equilibrium (equation (2)), the price of land adjusts immediately by the full capitalization of the tax: $\Delta P_0 = -1/r$. This result is analogous to that of Calvo, Kotlikoff and Rodriguez (1979) in a model with intergenerational transfers, though the interpretation of the intergenerational incidence is quite different in our framework. If the asset demand by the young generation is infinitely elastic, the landowners (the old generation) bear the full capitalization of the land tax revenues. The welfare of the future generations is unaffected.

Finally, we can compare the short-run and the long-run impacts of the tax on the land price. From (12), the long-run impact ΔP^* is equal to:

$$(15) \quad \Delta P^* = \frac{1 - A_1}{1 - \lambda_1} \Delta P_0 ,$$

where ΔP_0 is the short-run impact. By the assumption of stability, the term $1 - \lambda_1$ is positive, but the sign of $1 - A_1$ is in general ambiguous and does not relate to the stability properties of the model (which depend only on λ_1 and λ_2). It is possible that the land price decreases in the short-run and increases in the long-run. However, if capital and labor are complements in the technology, one can show that if the land price increases in the short-run, it also increases in the long-run.

IV. Welfare Incidence

The incidence of tax reform on the land price has a direct effect on the utilities only of the individuals who own the land when the tax is introduced. All other individuals have life-time indirect utilities which depend on the rate of return and the wage rate, and are determined by the level of capital. Therefore, the introduction of the land tax has an impact on their utilities only through the incidence on capital accumulation. For example, we have seen above that in the extreme case where life-time savings are infinitely elastic with respect to the interest rate, there is no incidence on the capital stock, and the initial generation of landowners bears the full burden of the present discounted value of all tax revenues.

In this section, we will assume the property of long-run crowding-out. According to the previous lemma, the introduction of the tax induces an increase of the capital stock. The difference between the utility of individuals born in period t and the utility of individuals in the

steady state with no tax is measured by the equivalent income loss, keeping prices unchanged. Using the properties of the indirect utility function, a first order approximation of this income variation is equal to:⁶

$$(16) \quad \Delta I_t = \Delta w_t + \frac{K+P}{1+r} \Delta r_{t+1} ,$$

or

$$(17) \quad \Delta I_t = w' \Delta K_t + \frac{K+P}{1+r} r' \Delta K_{t+1} ,$$

where w' and r' represent the partial derivatives of w and r with respect to the capital stock, and $\Delta K_t = K_t - K_0$.

The discussion below will be simplified if we rewrite this expression as follows:

$$(18) \quad \Delta I_t = G \cdot \Delta H_t ,$$

with
$$\Delta H_t = E \cdot \Delta K_t - \Delta K_{t+1} ,$$

and
$$G = \frac{(K+P)(-r')}{1+r} ,$$

$$E = \frac{w'(1+r)}{(K+P)(-r')} .$$

The term G is positive. For simplicity, we assume that labor and capital are complementary, so that E is positive. From (8), $\Delta K_t = (1 - \lambda_1^t) \Delta K^*$. We deduce the equivalent income variation of the first generation:

$$(19) \quad \Delta I_0 = -(1 - \lambda_1) G \Delta K^* < 0 .$$

The individuals born at the time of tax reform suffer a welfare loss.

The equivalent income variation of the individuals born in period t ($t \geq 1$) is equal to:

$$(20) \quad \begin{aligned} \Delta I_t &= \Delta I_0 + (1 - \lambda_1^t)(E - \lambda_1)G\Delta K^* \\ &= \left(E - 1 - \lambda_1^t(E - \lambda_1) \right) G\Delta K^* . \end{aligned}$$

In the steady state with taxation, the equivalent income variation is equal to

$$(21) \quad \Delta I^* = (E-1)G\Delta K^*$$

The land tax induces an increase of individual savings in productive capital and therefore a shift of resources towards future generations. If the initial level of capital is relatively low (with respect to its Golden Rule value), the long-term gain of a higher capital stock overshadows the costs of tax payments by individuals; in expression (21), the term E is greater than one.⁷

For the uncompensated land tax the two possible situations are represented in Figure 2. Notice also that the utility of individuals born at time t increases monotonically with t ($t \geq 0$).⁸ From Proposition 2 and relation (19), we know that the introduction of the uncompensated land tax has a negative impact at least on the utilities of all individuals living in period zero, old and young (provided that the interest elasticity of saving is finite). The relative magnitudes of the effects on these two generations is in general indeterminate, and depends on the parameters of the model.

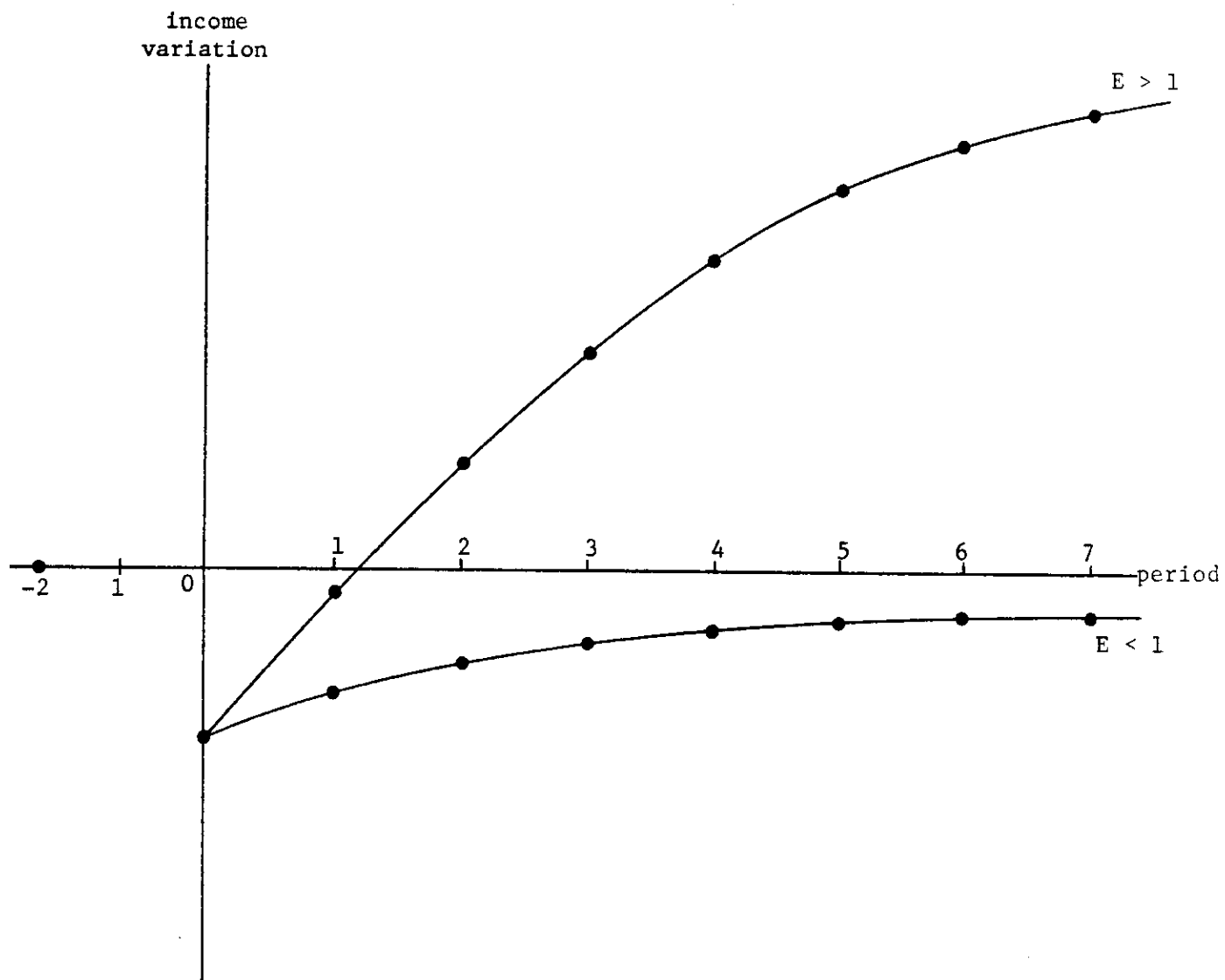


FIGURE 2

Equivalent Income Variation of Individuals Born at Time t ($t \neq -1$)

V. Other Fiscal Policies

The framework developed in Section III is easily extended for the analysis of a wage tax, and of a tax on the incomes of all assets (capital and land) at a uniform rate. The results for the immediate impact of tax reform on the price of land are represented in Table 1. (A derivation of the results in the first row is given in the Appendix.)

The terms in the table satisfy the inequalities in parentheses when the assumption of crowding-out is verified (see the above lemma). For the inequalities with an asterisk, we have the stronger sufficient condition that $A_2 < 0$.

We consider also the effect of a transfer of the tax revenues to the young. Since the labor supply is fixed, this transfer is equivalent to a subsidy on labor. By addition of the two effects, we obtain the results in the second row of the table. The transfer to the young has a positive impact on the savings of the young, the demand for land, and therefore on the price of land.

When the tax revenues are received in the second period of the life cycle (a transfer to the old), the impact on savings is equal to $\frac{\sigma-1}{1+r}$, per unit of revenues, where σ is the marginal propensity to save in period 1. Replacing σ by this term in the second row, we deduce the results in the last row of the table.

A tax on capital income which generates one unit of revenue is a combination of the tax on the incomes of all assets with revenues equal to $1+\gamma/\alpha$, and of a subsidy on the land rent equal to γ/α , where α and γ are the shares of capital and land, respectively. The incidence results are then derived immediately by taking a linear combination of the first and the third column in the table.

The total impact on the consumption of the old individuals in

TABLE 1
Initial Land Price Response, P_0 , per Unit of Revenue

Refund of Revenues	Land Rent	Tax on Wages	Income of All Assets (Land and Capital)
None	Q (< 0) [*] ($< 1/2$)	$-\sigma J$ (< 0)	H (< 1)
To the Young	$Q + \sigma J$ (< 0) [*]	0	$H + \sigma J$ (< 1)
To the Old	$Q - \frac{1-\sigma}{1+r} J$ (< 0) [*]	$-\frac{1+r\sigma}{1+r} J$ (< 0)	$-\frac{\epsilon^c J}{1+r}$ (< 0)

Notation: $Q = 1/(1-\lambda_2)$, $J = 1+rQ$, $H = -(\eta J/1+r)$, σ is the marginal propensity to save ($\sigma = \partial s/\partial w$), η is the interest elasticity of savings ($\eta = \frac{(1+r)}{s} \frac{\partial s}{\partial w}$), and ϵ^c is the compensated elasticity of the second period consumption with respect to $1+r$ ($\epsilon^c > 0$). The signs in parentheses apply under the assumption of crowding-out (or under the assumption that $A_2 < 0$ when there is an asterisk)

period zero is found by subtracting the tax revenues, normalized at unity, from the terms in the first two rows and in the first and third columns of the table, and by adding the unit revenue to the middle entry in the last row. This impact is in most cases negative. We could have also presented other sufficient conditions for a negative impact.⁹

It is important to note that the effects described in the table depend not only on the taxes and the transfers which occur in the first period, but also on all future taxes and transfers which are anticipated with perfect foresight. For example, the transfer of tax revenues to the old has a negative impact on the land price because young individuals reduce their savings when they anticipate that transfer benefits will accrue to them in their retirement period.

The case of the labor tax with transfer to the old is especially interesting. It is equivalent to Social Security system on a pay-as-you-go basis. The initial impact on the price of land which is owned by the old is negative, if A_2 is negative, and reduces the net benefits which accrue to them. From Proposition 3, we know that the value of J is between 0 and 1; therefore, the capital loss is smaller than one and is not greater than the transfer received by the old. Its value may be significant however. For the numerical example described in the previous section,¹⁰ it is equal to about one-third of the transfer from young to old.

Note that when A_2 is positive the value of capital loss may exceed the amount of the transfer. In this case, the unanticipated introduction of a permanent transfer system from the young to the old has a negative impact on welfare of the first generation of old individuals who receive the transfers--"Social Security" could harm the first recipients!

VI. Conclusion

In this paper we have studied incidence from both a comparative static and a dynamic viewpoint. We have found that in most cases, the comparative statics results obtained in a model with land are in line with those which might be inferred by analogy to other two-factor models and models which include money. In particular, land rent taxes generally increase the capital stock, the effects of taxes involving capital depend on the elasticity of supply of savings, and an unfunded social security scheme generally lowers the steady state capital stock.

But our dynamic analysis shows that the presence of land significantly modifies the incidence of fiscal packages on the generation which owns assets at the time of the fiscal change. With the reasonable assumption that the term A_2 is negative, a fiscal package including a tax on land rent always reduces consumption of the initial old (land-owning) generation. The reduction in consumption is always greater than the current tax revenue, unless the latter is returned to them. However full Ricardian capitalization is, for these specifications, a limiting case. On the other hand, taxes on income from capital may increase the consumption of the generation which currently holds assets, and a uniform tax on income from all their assets may be at least partially shifted. Finally, a pay-as-you-go social security scheme increases the consumption of the old (when $A_2 < 0$), but by less than the value of the revenues they receive. And because of its depressing effect on fixed asset values, the effect on capital formation will be smaller than the reduction in the saving of the young.

These findings, and others reported above, show the importance of specifying the use to which tax revenues are put, of separating asset

price effects from incidence effects, and of clearly distinguishing the dynamic incidence on the initial older (asset-holding) generation (which has not previously been analyzed in this context) from comparative statics results. Further conditions necessary to uniqueness and stability of the dynamic solution place bounds on both types of incidence results, even in a model with a quite general specification of production technology and consumption behavior.

Although we have concentrated on land price changes in this model, capitalization of fiscal effects would be much more pervasive in a more realistic model which recognized that most capital investments are of the putty-clay type. Fiscal changes would be reflected in the prices of all such finitely-lived assets, and proportional price effects would vary inversely with rates of depreciation. Though we trust that our model throws some light on incidence in such a model, its structure is sufficiently different to deter rash generalizations from the propositions derived here.

APPENDIX

1. Proof of the Lemma

When the level of the public debt is equal to B , and its service is financed by a labor tax, equation (1) takes the following form in the steady state:

$$(A-1) \quad S(w - rB, r) = K + P + B$$

The portfolio equation is unchanged:

$$(A-2) \quad 1+r = \frac{m+P}{P} .$$

This system determines the value of the steady state variables K^* and P^* as functions of B . Differentiating with respect to B , at $B = 0$, we find that:

$$\begin{pmatrix} \Delta K^* \\ \Delta P^* \end{pmatrix} = -(I-M)^{-1} \begin{pmatrix} A_2 \\ C_1 A_2 \end{pmatrix} (1+\sigma r) B .$$

Using the equation (11),

$$\Delta K^* = -r \frac{A_2}{\psi(1)} (1+\sigma r) B$$

ΔK^* is negative if and only if $A_2/\psi(1)$ is positive, which proves the lemma (compare with (12)).

2. Proof of Proposition 3

Using the relations $B_1 = C_1 A_1$ and $B_2 = C_1 A_2 + C_2$, the characteristic polynomial of the matrix M is equal to

$$Q(\lambda) = \lambda^2 - (A_1 + C_1 A_2 + C_2)\lambda + A_1 C_2,$$

with $C_1 = \text{Pr}\left(\frac{r'}{r} - \frac{m'}{m}\right)$, $C_2 = 1+r$, and $A_1 = -\sigma w' A_2$. All derivatives of factor prices are taken with respect to the capital stock, and $\sigma = \partial s / \partial w$.

We place the root λ_2 with respect to $1+r$ by determining the sign of $Q(1+r)$:

$$Q(1+r) = -C_1 A_2 (1+r).$$

If $A_2 < 0$, and $C_1 < 0$, $Q(1+r)$ is negative, which implies that $\lambda_2 > 1+r$, and since $\Delta P_0 = 1/(1-\lambda_2)$, $-1/r \leq P_0 < 0$.

When C_1 is positive, λ_2 is smaller than $1+r$. We now prove that $\lambda_2 \geq 1$:

$$Q(0) = A_1 C_2 = -\sigma w' A_2 (1+r).$$

If $C_1 \geq 0$, n' is negative, and w' is strictly positive (since by the homogeneity of F , $kr' + w' + u' = 0$). Therefore $Q(0)$ is strictly positive. When C_1 varies, the value of λ_2 which is continuous in C_1 , cannot "cross" the value zero. For all values of C_1 , λ_2 is positive. Since the dynamic path is unique, λ_2 is greater than one, and $\Delta P_0 > -1/r$.

3. Proof of Proposition 4

If the immediate impact on the land price is positive, Proposition 3 implies that $A_2 > 0$. Since $1 - \lambda_2$ is positive, the right hand side is positive in the expression for K^* in (12), and by the lemma the property of crowding-out is satisfied. The sufficiency part of the proposition is trivial.

4. Proof of the Results in Table 1

When a tax policy is introduced, equations (3) and (4) in the text, take the following form:

$$(A-3) \quad K_{t+1} = A(K_t, P_t, z)$$

$$(A-4) \quad P_{t+1} = C(K_{t+1}, P_t, z) = B(K_t, P_t, z)$$

where the functional forms and the variable z depend on the policy under consideration.

Differentiating this system in the steady state with respect to z (for $z = 0$), we have:

$$(A-5) \quad \begin{pmatrix} \Delta K^* \\ \Delta P^* \end{pmatrix} = (I-M)^{-1} \begin{pmatrix} A_3 \\ C_1 A_3 + C_3 \end{pmatrix} \Delta z .$$

The impact on the land price in period zero is equal to $\Delta P_0 = \Delta P^* - v \Delta K^*$, with $v = (\lambda_1 - A_1)/A_2$. Multiplying both sides of (A-5) by the vector $(-v)$, and using (11), we find:

$$(A-6) \quad \Delta P_0 = \left[\left(\frac{\lambda_1 - A_1}{A_2} \Gamma + C_1 \right) A_3 + (1 - \lambda_1) C_3 \right] \Delta z$$

The results in the first row of Table 1 are obtained by substituting in (A-6) the following values for A_3 and C_3 (Δz is normalized to one):

$$\text{labor tax: } A_3 = \sigma A_2 \text{ with } \sigma = \frac{\partial S}{\partial w}; \quad C_3 = 0$$

$$\text{capital income tax: } A_3 = \frac{A_2}{K} \frac{\partial S}{\partial r}; \quad C_3 = -\frac{P}{K}.$$

The impact of a tax on all assets at a uniform rate, which generates one unit of revenues, is obtained by adding the impacts of a capital income tax and a land tax which generate revenues equal to α and γ respectively (the shares of the incomes of capital and labor).

FOOTNOTES

1. There is a vast literature on the subject. Representative studies have been presented by Hall (1969), Diamond (1970), Pestieau (1974), Atkinson and Sandmo (1980), and Summers (1981).
2. For an analysis of this effect in a life-cycle model of optimization, see Drazen (1981).
3. All changes should then be interpreted per unit of tax revenues, where the total amount of revenue is small. Also, changes in the marginal productivity of land m , are negligible because they introduce only terms of order higher than one.
4. This property may be violated when marginal increase of the labor input lowers the ratio r/m : because $F(K, L, T)$ is homothetic of degree one, a marginal increase of K (with L and T constant), has the same effect on the ratio r/n as a marginal proportional reduction of L and T (keeping K constant). If the level of land is small, the effect of the variation of T can be neglected (it is nil if $T = 0$), and the ratio r/m falls. Also, note that the property of the proposition is satisfied when the production function is separable in labor, and has the form $F(K, L, T) = G(H(K, t), L)$.
5. There may also be a range of negative values of η such that the dynamic path is not unique and the price of land is indeterminate.
6. We use the property that if $v(w_t, r_{t+1})$ is the indirect utility,

$$\frac{\partial v}{\partial r_{t+1}} = - \frac{C_2}{(1+r)^2} \frac{\partial v}{\partial w_t}, \text{ and } C_2 = (K+P)(1+r).$$
7. For the case of a Cobb-Douglas technology, E is greater than one if $r > \gamma/(\alpha\beta)$, where α , β and γ are the shares of capital, labor and land, respectively ($\alpha+\beta+\gamma = 1$).
8. One can prove that $E < \lambda_1$ by analyzing the sign of the characteristic polynomial $P(\lambda)$ for $\lambda = E$.
9. For example, using $Q < 1/2$, one can show that a compensated land tax (with the compensation to the old), has a negative impact on the consumption of the old in the first period, if the consumption level in the first period of the life is greater than that in the second.
10. In this example, production and utility functions are of the Cobb-Douglas type (see Section III above).

REFERENCES

- Atkinson, A. and A. Sandmo, "Welfare Implications of the Taxation of Savings," Economic Journal, 90 (1970), pp. 529-547.
- Barro, Robert J., "Are Government Bonds Net Wealth?" Journal of Political Economy, 82, No. 6 (November/December 1974), pp. 1085-1117.
- Becker, Gary S., "A Theory of Social Interactions," Journal of Political Economy, 82, No. 6 (November/December 1974), pp. 1063-1083.
- Calvo, Guillermo A., "On the Indeterminacy of Interest Rates and Wages with Perfect Foresight," Journal of Economic Theory, Vol. 19 (1978), pp. 326-337.
- _____, Lawrence Kotlikoff and Carlos Alfredo Rodriguez, "The Incidence of a Tax on Pure Rent: A New (?) Reason for an Old Answer," Journal of Political Economy, Vol. 87, No. 4 (1979), pp. 869-874.
- Diamond, Peter A., "National Debt in a Neoclassical Growth Model," American Economic Review, 55 (1965), pp. 1126-50.
- _____, "Incidence of an Interest Income Tax," Journal of Economic Theory, 2 (1970), pp. 211-224.
- Drazen, Allan, "Inflation and Capital Accumulation under a Finite Horizon," Journal of Monetary Economics, 8 (1981), pp. 247-260.
- Feldstein, Martin, "The Surprising Incidence of a Tax on Pure Rent: A New Answer to an Old Question," Journal of Political Economy, Vol. 85, No. 2 (1977), pp. 349-360.
- _____, and S. C. Tsiang, "The Interest Rate, Taxation, and the Personal Savings Incentive," Quarterly Journal of Economics, 82 (August 1968), pp. 419-434.
- Hall, Robert E., "Consumption Taxes versus Income Taxes: Implications for Economic Growth," Proceedings of the Sixty-First National Tax Association Conference, 1968.
- Pestieau, Pierre, "Optimal Taxation and Discount Rate for Public Investment in a Growth Setting," Journal of Public Economics, 75 (1973), pp. 67-82.
- Samuelson, Paul A., "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," Journal of Political Economy, Vol. 66, No. 6 (December 1958), pp. 467-482.
- Summers, Laurence H., "Capital Taxation and Accumulation in a Life Cycle Growth Model," American Economic Review, 71, No. 4 (1981), pp. 533-544.
- Tobin, James, "Money and Economic Growth," Econometrica, 33 (1965), pp. 671-684.