The Journal of Marine Research, one of the oldest journals in American marine science, published important peer-reviewed original research on a broad array of topics in physical, biological, and chemical oceanography vital to the academic oceanographic community in the long and rich tradition of the Sears Foundation for Marine Research at Yale University.

An archive of all issues from 1937 to 2021 (Volume 1–79) are available through EliScholar, a digital platform for scholarly publishing provided by Yale University Library at https://elischolar.library.yale.edu/.

Requests for permission to clear rights for use of this content should be directed to the authors, their estates, or other representatives. The Journal of Marine Research has no contact information beyond the affiliations listed in the published articles. We ask that you provide attribution to the Journal of Marine Research.

Yale University provides access to these materials for educational and research purposes only. Copyright or other proprietary rights to content contained in this document may be held by individuals or entities other than, or in addition to, Yale University. You are solely responsible for determining the ownership of the copyright, and for obtaining permission for your intended use. Yale University makes no warranty that your distribution, reproduction, or other use of these materials will not infringe the rights of third parties.

This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License. https://creativecommons.org/licenses/by-nc-sa/4.0/
PROFILES OF WIND-CREATED WATER WAVES IN THE CAPILLARY-GRAVITY TRANSITION REGION

By

ALLEN H. SCHOOLEY
U. S. Naval Research Laboratory
Washington 25, D. C.

ABSTRACT

Crapper (1957) has theoretically predicted that pure capillary waves have profiles that peak downward, which is the reverse of the gravity wave case. This paper confirms Crapper's theory and shows enlarged profiles of capillary waves that were photographed with a high-speed motion picture camera in a small water-wind tunnel. Under the conditions of measurements, the maximum vertical distance between trough and crest for the capillary waves observed was about 0.5 wavelength. This is roughly 70% of Crapper's theoretical figure of 0.73 and is about 3.5 times the maximum of 0.14 for gravity waves. Wave profile pictures in the transition region from capillary to gravity waves are also shown. Examples are given where capillary waves having the same velocity as the gravity waves tend to ride just in front of the start of the crests of the gravity waves.

INTRODUCTION

Large water waves having gravity as a restoring force have long been studied because of their effects on ships, breakwaters, docks, and beaches. Capillary water waves having surface tension as a restoring force have become of considerable interest in recent years because of their effect on the reflection and scattering of light and microwave radar waves from the wind-swept surface of the sea. The trochoidal shape of single wavelength gravity waves in deep water is well known. However, only recently has an exact solution been found for the shape of pure capillary waves. Crapper (1957) has calculated the wave shapes shown graphically in Fig. 1. Each wave shape has an associated amplitude divided by wavelength (a/λ) number. If any particular wave shape is taken as the water surface, the lines below it are streamlines. The wave form of greatest height occurs when the crest-to-trough distance or amplitude is 0.73 wavelength. Higher waves are not expected to occur because the sides of adjacent waves would touch. The maximum a/λ number for gravity waves is 0.14.
Crapper's theoretical capillary wave profiles for various amplitude-to-wavelength ($a/\lambda$) ratios.

Figure 1. Crapper's theoretical capillary wave profiles for various amplitude-to-wavelength ($a/\lambda$) ratios.

Crapper's capillary wave shapes are peaked downward whereas the trochoidal shape of gravity waves are peaked upward. On the basis of pictures of wind-generated waves in a transparent channel, it is the purpose of this paper to present evidence that confirms Crapper's prediction of the shape of capillary waves and to show that under some circumstances gravity and capillary waves of the same velocity tend to propagate together.

EXPERIMENTAL PROCEDURE

The small transparent water channel shown in Fig. 2 has a provision for drawing air at various measured velocities over the water surface; this apparatus has been described previously (Schooley, 1955). Close-up pictures were taken for various wind velocities by means of a 16-mm motion picture camera operating at 72 frames per second and having a shutter speed of about 1/900 second per frame. The camera was focused on the central portion of the channel wall so as to show the water-air boundary. The view subtended slightly over 4 cm in width. A 2 cm scale was included for reference in wave length and wave height measurements. The velocity of the waves was measured by the distance they traveled between one frame and the next on the motion
picture film. To provide accurate timing, the frame rate of the camera was calibrated to better than 3% accuracy under the conditions used in taking the pictures.

RESULTS

Figs. 3 through 9 are enlargements of selected frames taken with the 16-mm motion picture camera. The light area below is the illuminated water and the dark area above is the air. The independent variable in the pictures is wind speed. The range of wind speeds was from 10 to 20 knots and the direction of air flow was from left to right.

Fig. 3 shows the starting of wave formation. Here the amplitude is about 0.07 cm and the wavelength about 1.4 cm, giving an $a/\lambda$ ratio of 0.05. This wave was traveling at about the minimum velocity, corresponding to the transition region between gravity and capillary waves. The wind velocity for Fig. 3 is approximately 10 knots, which indicates that wave formation starts at a wind velocity slightly below this value. This is considerably higher than that experienced for the initiation of wave formation on open bodies of water (Schooley, 1954). The high threshold velocity, observed previously (Schooley, 1955), was attributed to possible inaccurate wind velocity measurements and to the possibility that water contamination lowered the surface tension. In the present experiments the wind velocity measurements were carefully checked and appear to be consistent with the earlier results. As will be evident later in this paper, the surface tension of the water channel appears not to be appreciably depressed by contaminants. Thus, it is believed that low air turbulence due to the short fetch may have an effect on the threshold wind velocity at which waves form.

Figs. 4–6 are presented for a 12 knot wind because Crapper's wave shapes are most clearly evident at this velocity. The waves are not of a single frequency and amplitude because the driving force of the wind tends to generate a spectrum of waves. However, the peaked downward shape is shown in portions of these figures. The maximum value of $a/\lambda$ for Figs. 4–6 is about 0.23, 0.30, and 0.29, respectively. These values of $a/\lambda$ would place the wave profiles in the central region of Fig. 1. However, the sharp troughs shown in Figs. 5 and 6 appear to match more nearly the region between $a/\lambda = 0.53$ and $a/\lambda = 0.73$. Actually it is difficult to measure precisely $a/\lambda$ in the pictures because of the thin water film that tends to cling to the transparent wall of the channel along the average water-level mark. Due to this film, the profile of waves that are below the average water mark are emphasized by a dark band while the profile of waves above the average water line are suppressed, except for large gravity waves at high wind velocities.
Figure 2 (upper left): water channel with provision for controlled wind over the water surface. Figure 3 (upper right): water channel wave profile for 10 knot wind; (scale, 2 cm long). Figure 4 (lower left): 12 knot wind shows Crapper's wave profiles best. Figure 5 (lower right): another 12 knot wind sample.
Figure 6 (upper left): still an other 12 knot sample. Figure 7 (upper right): 14 knot wind. Figure 8 (lower left): 16 knot wind. Figure 9 (lower right): 20 knot wind.
Fig. 7 is representative of a transition region where the wavelength spectrum excited by the wind starts to show the characteristics of both long gravity waves and short capillary waves. In this case the capillary waves are about 0.7 cm long, the gravity wave about 4.6 cm. From a study of the motion picture record it is not apparent that there is a consistent phase relationship between the capillary and gravity waves.

The left side of Figs. 8 and 9 show the start of crests of gravity waves that have wavelengths of about 8 cm. It is evident that the crests are above the water film that clings to the channel wall; the troughs in the center and on the right side of the pictures are below. Preceding the start of the crests in both Figs. 8 and 9 are capillary waves having wavelengths in the general region of 0.3 cm. There is evidence that gravity waves and capillary waves travel at the same velocity because the type of phenomena shown in these figures was more or less typical of many frames of the motion picture film that was examined. In other words, there was a distinct tendency for capillary waves to ride at the start of the crest of the gravity waves, with the wavelength of each adjusted so that the velocities are identical.

It is interesting to note the train of capillary waves in front of the gravity wave in Fig. 9. The maximum $a/\lambda$ in this train is slightly over 0.5, which is the largest observed in all measurements that were made. It is quite evident, however, that the film of water clinging to the side of the channel may affect the apparent shape of the waves and the accuracy to which amplitude and wavelength measurements can be made. The apparent tipping back of the crests of the capillary waves may be due to the presence of the film of water, and it also may be due to forward movement of the water in this region.

The solid curves in Fig. 10 show the relationship between wavelength and wave velocity for water waves as defined by the approximate equation

$$v = \left(\frac{2\pi T}{\lambda \rho} + \frac{g\lambda}{2\pi}\right)^{1/2}$$

where $v$ is the water wave velocity in cm per sec, $T$ surface tension in dynes per cm, $\rho$ water density in gm per cm$^2$, and $\lambda$ wavelength in cm. In plotting the curves in Fig. 10, it was assumed that $\rho$ equaled unity, $g$ equaled 980, and $T$ equaled 70 and 35. The handbook value of $T$ for pure water at room temperature is listed between 70 and 75 by various references. Since the presence of organic impurities at a water surface markedly decreases the surface tension, a curve for $T$ equal 35 is also shown. It was originally thought that fresh Washington tap water, when placed in the water channel, would have appreciable contamina-
tion and would thus depress the surface tension. This proved not to be the case, as will be explained below.

The experimental points in Fig. 10 were derived from pictures such as those shown in Figs. 3 through 9. Wave velocity was measured by noting the wave movement on successive frames of the motion picture film and multiplying this distance by the frame-rate of taking the pictures. Wavelength was measured directly from the pictures for the capillary waves. Since the gravity waves were longer than the picture width, it was necessary to note the number of frames for a given wave to pass a given point and multiply this by the distance traveled per frame in order to obtain the wavelength.

The experimental points in Fig. 10 appear on the average to be about $30\%$ higher in velocity than would be expected from the upper theoretical curve. Except for this displacement they appear to follow the general shape of the $T = 70$ curve. This indicates that the surface tension of the water in the channel was not appreciably depressed by organic contaminants.

The $30\%$ higher velocity at all wavelengths is believed to be significantly greater than the measuring errors. It may be expected that the phase velocity of the waves in the four-inch wide channel may be greater than that for an infinite surface. This would be particularly significant if the channel width was on the order of one half-wavelength or less. Actually, the channel width was about twice the longest half-wavelength measured, which would indicate a phase velocity down the
channel of about 10\% above the wave velocity on a large surface; for shorter wavelengths the phase velocities should approach nearer and nearer the large surface velocity. Thus, some other interpretation is needed to explain why the experimental points in Fig. 10 are significantly above the theoretical curve.

The distribution of points with wind velocity in Fig. 10 is particularly interesting. At the low wind velocities, waves start to form at the minimum wave velocity transition region between gravity and capillary waves. As the wind velocity increases, both gravity and capillary waves having substantially the same velocities are formed. The measurements covered a range of wavelengths between about 0.2 and 11 cm. Assuming that Crapper's $a/\lambda$ maximum of 0.73 for capillary waves is correct, it can be expected that the amplitude of capillary waves directly associated with long gravity waves will become small. For example, for a gravity wave of 100 cm wavelength, the maximum amplitude of an equal velocity capillary wave would be about 0.02 cm. Similarly, a 1000 cm gravity wave could only have associated with it a capillary wave of about 0.002 cm amplitude.

**CONCLUSIONS**

On the basis of this investigation, it can be stated:

(a) Crapper's theoretical description of capillary wave profiles with troughs peaked downward can be observed on capillary waves created by the wind.

(b) For a 13 inch fetch in a water channel with controlled velocity air drawn across the surface, the clearest evidence of Crapper's waves is experienced with a 12 knot wind.

(c) Under the circumstances of the experiments, waves started to form when the wind velocity was just under 10 knots. This is high compared to open water measurements and may indicate that low air turbulence due to short fetch is a factor in the minimum air velocity for wave formation.

(d) The wavelength of wind generated waves that first form is in the 1 to 2 cm region corresponding to the minimum velocity in the gravity-capillary transition region.

(e) As the wind velocity is increased, both gravity and capillary waves having substantially the same velocity are formed simultaneously.

(f) At wind velocities between 16 and 20 knots it is common for capillary waves having the same velocity as the gravity waves to ride just at the beginning of the crest of the gravity waves.
(g) Crapper's prediction for the maximum ratio of amplitude to wavelength of 0.73 for capillary waves was approached closely enough to give confidence in its correctness.

(h) Assuming the 0.73 figure to be correct, the maximum amplitude of capillary waves that can be associated with the starting of the crest of gravity waves becomes very small as the gravity wave wavelength is increased beyond 100 cm.

ACKNOWLEDGMENT

The author wishes to thank Prof. Willard Pierson, Jr., for bringing the work of Dr. Crapper to his attention.

REFERENCES

Crapper, G. D.

Schooley, A. H.