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EXPERIMENTAL INVESTIGATION OF THE EFFECT OF “SECONDARY INSTABILITY” ON MIXING PROCESSES

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Abstract. Since specific volume in the ocean is a nonlinear function of salinity and internal energy, mixing results in a change in the total volume. There is a contribution to the change in potential energy due to mixing, which depends on this change in volume and on the pressure at which mixing takes place. Fofonoff has made use of this fact in defining a “secondary stability” parameter. It might be speculated that, under conditions of (secondary) instability, turbulent mixing could be generated spontaneously or that the quality of mixing would be different from that under stable conditions. An experiment to detect this phenomenon was performed, with negative results.

Introduction. The ordinary concept of stability in the oceans, as developed by Hesselberg and Sverdrup (1914), considers the change of potential energy in a layer of water resulting from vertical displacement, up or down, of a small water parcel which is not allowed to mix with the adjacent water. If this change of potential energy is positive, the water at the point under consideration is said to be stable, and vice versa. Fofonoff (1956) has widened the concept of stability by including processes in which mixing occurs and by considering the potential energy of the entire vertical water column. The term $pdv$ is thus introduced as an additional contribution to the potential energy change. This term results from the fact that specific volume is a nonlinear function of salinity and internal energy and is usually negative; thus an unstable configuration according to Hesselberg and Sverdrup’s criterion will usually be unstable according to Fofonoff’s criterion. Hesselberg and Sverdrup’s stability is referred to as “primary” stability, Fofonoff’s as “secondary.” The experiment described in this note was an attempt to observe any possible dramatic effects associated with secondary instability (such as the

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onset of turbulent motion from an initial motionless state); Fofonoff has made no such speculation.

Let us consider a configuration in which density increases with depth. Fofonoff finds that if the density gradient exceeds some critical gradient, then mixing can occur with loss of potential energy; in that case the system is said to exhibit secondary instability at this level. Therefore a system consisting of two homogeneous layers will almost always exhibit secondary instability at the interface.

To get a rough idea of the amount of mixing that would be necessary at the interface of a two-layer system in order to reach (secondary) stability in the system, let us assume that a fluid mass \( m \) immediately above the interface mixes thoroughly with an equal fluid mass \( m \) immediately below. Let \( A \) be the cross-sectional area of the water column under consideration and \( \alpha_1 \) and \( \alpha_2 \) the specific volumes of water in the upper and lower layers, respectively. For primary stability, we must have \( \alpha_1 > \alpha_2 \). After mixing there will be a homogeneous layer of water of mass \( 2m \) between the two original layers having a specific volume, say, of \( \alpha \). This layer is assumed thin enough so that inhomogeneity due to compressibility and pressure gradient may be neglected.

Before mixing, the center of gravity of the water to be mixed is at a distance \((m/4A)\left(3\alpha_2 + \alpha_1\right)\) above its lower boundary, and the potential energy relative to this level is

\[
\frac{m^2g}{2A} \left(3\alpha_2 + \alpha_1\right).
\]

After mixing, the center of gravity will be at \( \alpha m/A \) and the potential energy will be

\[
\frac{2m^2\alpha g}{A} + mp \left(2\alpha - \alpha_1 - \alpha_2\right),
\]

where the second term represents the \( p\text{d}v \) addition to the potential energy. Equating the potential energies before and after mixing to find the amount of water that must mix in order to reach (secondary) stability, we obtain

\[
\frac{m}{A} = \frac{2p}{g} \cdot \frac{\alpha_1 + \alpha_2 - 2\alpha}{4\alpha - 3\alpha_2 - \alpha_1},
\]

and it is evident that if the specific volume is governed by a linear law, no water can mix without addition of potential energy.

**The Experiments.** A two-layer system, consisting of pure water above and a 17\% NaCl solution colored with red vegetable dye
below, was put under 1000 psi pressure for 18 hours. The tank used was a rectangular aquarium 27 cm long, 17 cm wide, and 19 cm deep. In order to apply the relationship (1) to this configuration we shall neglect the change of specific volume resulting from compressibility and from conversion of potential to internal energy (which is unimportant at this pressure anyway). (Note that compressibility would play no role whatsoever if the specific volumes \( \alpha, \alpha_1 \) and \( \alpha_2 \) were all affected proportionately by the pressure.) We shall therefore use the following values of specific volume for NaCl solutions at atmospheric pressure and at 20° C:

<table>
<thead>
<tr>
<th>%NaCl</th>
<th>Specific Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0018</td>
</tr>
<tr>
<td>8.5</td>
<td>.9438</td>
</tr>
<tr>
<td>17</td>
<td>.8897</td>
</tr>
</tbody>
</table>

Taking \( p = 1000 \text{ psi} = 68 \times 10^6 \text{ dynes cm}^{-2} \) along with the above values, we obtain \( m/A = 6 \times 10^3 \text{ gm cm}^{-2} \) from relationship (1), which is equivalent to a thickness of 60 m. Even at atmospheric pressure, the thickness over which mixing should occur without need of additional potential energy is of the order of a meter. The motive in going to 1000 psi was to increase the secondary instability and the likelihood of a spectacular result.

At the end of 18 hours there was no visual evidence of turbulent mixing, but of course the sharpness at the interface had become considerably reduced as a result of molecular diffusion. A stirring apparatus consisting of \( \frac{1}{4}'' \) wire mesh mounted horizontally in the upper layer was then turned on, the mesh executing reciprocal up and down motion; no qualitative difference could be detected visually in the character of mixing at 1000 psi as compared with that at atmospheric pressure.

Even if secondary instability does not induce turbulent mixing from an initial motionless state, perhaps mixing by molecular diffusion is speeded up. The first experiment would have given no indication of this, since the dye, which was used as a visual tracer, contributes negligibly to changes in specific volume and could diffuse independently of the NaCl. Therefore, a second experiment was carried out with fresh water and a concentrated solution of CuCl\(_2\) (just under saturation at room temperature). This solution itself is bright green and no dye is needed as a tracer. As before, the two layer system was submitted to 1000 psi for 18 hours, at the end of which time there was no striking

\(^2\) The junior author has used similar apparatus in a mixing experiment at atmospheric pressure.
evidence of a qualitative difference in the diffusion rate under pressure as compared with that under atmospheric pressure. It must be emphasized that we were not set up for a quantitative experiment on diffusion rates; by the methods at our disposal we could have detected only a gross difference.

Conclusions. A simple experiment, carried out in an attempt to detect a possible effect of secondary instability, gave a negative result; this, of course, does not prove that there are no such effects. It seems probable, however, that the principal result of secondary instability, if such exists at all, is a small change in the diffusion rate in unstable configurations. In the case of primary instability, it is quite evident that the potential energy lost to the system as a result of a small perturbation tends to increase the perturbation. For secondary stability this loss of energy may go somewhere else (directly into heat, for example). In any case, energy considerations in the initial and final states alone are inadequate for predicting the outcome of such an experiment. Henry Stommel (personal communication) has suggested the possible existence of a criterion for the onset of convection in this stability problem that is similar to Rayleigh's convection criterion.

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REFERENCES

Fofonoff, N. P.
Hesselberg, Th. and H. U. Sverdrup