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Properties of Manipulative Government Forecasts

John J. Beggs

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

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Introduction

If the educated lay-man has trouble dealing with economic forecasts, in particular, those emanating for government sources, it is that they are apparently too variable and appear to chop-and-change too often. While the author does not necessarily accept either of these propositions, they are so pervasively well known and discussed that an attempt should be made to establish plausible circumstances under which these two phenomena arise. In the model offered below the government forecaster takes the role of a manipulator, trying to take account of sluggish reactions on the part of other economic agents.

By overstating the forecast of the relevant variable, in one direction or the other, the intent is to elicit extra response from those who use the forecast. Devil's circle problems are present, but to start the process in motion explicit assumptions are made about the way in which economic agents use the government forecast.

Model

Consider a time series on an economic variable, $\{y_t\}$. It is assumed that the variable follows a stationary ARMA process and that its realizations are not dependent upon actions of the agents in the forecasting model. Let the private economic agents (those using the government forecast) have adaptive expectations of the following form:

$$\hat{\tilde{y}}_t = \beta\hat{\tilde{y}}_{t-1} + (1-\beta)\hat{y}_t \tag{1}$$

where $\hat{\tilde{y}}_t$ is the forecast of the private agents in period t,

$\hat{y}_t$ is the forecast made by the government for period t (this is the forecast to be manipulated),

β is a weighting coefficient, $0 < \beta < 1$.

The private agents in this model are then taking a weighted average of the decision taken last year, and this year's government forecast, to arrive at current period forecast.

The government forecasters are assumed to know the adaptive expectations rule of the private agents, and are also assumed to be capable of making minimum mean squared error forecasts $\{y_t^*\}$ of the true series $\{y_t\}$. These forecasts are obtained as

$$y_t^* = Q(L)y_{t-1} \tag{2}$$

where $Q(L)$ is a polynomial function in the lag operator, L . The intent of the government's manipulative forecast is to cause private agents to employ the best forecast, i.e. the minimum mean squared error forecast. The $\hat{y}_t$ then is set so that:

$$y_t^* = \hat{\tilde{y}}_t = \beta\hat{\tilde{y}}_{t-1} + (1-\beta)\hat{y}_t \ . \tag{3}$$

Solving for $\hat{y}_t$,

$$\hat{y} = \frac{(1-\beta L)}{(1-\beta)} y_t^* \ . \tag{4}$$

Denoting power spectra by $f(\cdot)$, then

$$(5) \qquad f_y(\lambda) = \frac{(1 + \beta^2 - 2\beta \text{ Cos } \beta)}{(1-\beta)^2} \cdot f_{y*}(\lambda)$$

$$= \kappa(\lambda) \cdot f_{y*}(\lambda) \ .$$

Interpretation

An examination of the function $\kappa(\lambda)$ reveals that $\kappa(\lambda) > 1$ for all $0 \leq \lambda \leq \pi/2$. Hence the variance of the manipulative series, $\{\hat{y}_t\}$, is greater than that of the minimum mean squared error forecasts for all frequencies. This is the phenomenon alluded to in the opening paragraph, that manipulated series will have high variance. It is clear that the manipulated series has larger overall variance than the minimum mean square error forecasts. A familiar property of "good" forecasts is that they have smaller variance than the se ies they are forecasting, but it can also be seen that the manipulative forecasts may have larger variance than the actual series being forecast, the $\{y_t\}$. This can be illustrated with a simple example. Suppose the process being forecast is of the form,

$$(6) \qquad Y_t = \varepsilon_t + \alpha\varepsilon_{t-1}$$

where the $\{\varepsilon_t\}$ are independent and identically distributed $(0, \sigma_\varepsilon^2)$. The minimum mean squared error forecast is

$$(7) \qquad y_t^* = \alpha\varepsilon_{t-1}$$

and the manipulative forecast is

$$\hat{y}_t = \frac{(1 - \beta L)\alpha\varepsilon_t}{(1-\beta)} . \quad (8)$$

The variance of the actual process and the manipulative forecast are given by σ_y^2 and $\sigma_{\hat{y}}^2$ respectively, as

$$\sigma_y^2 = (1 + \alpha^2)\sigma_\varepsilon^2 \quad (9)$$

$$\sigma_{\hat{y}}^2 = \frac{\alpha^2(1 + \beta^2)}{(1-\beta)^2} \cdot \sigma_\varepsilon^2 . \quad (10)$$

With straightforward algebra it can be seen that

$$\sigma_{\hat{y}}^2 > \sigma_y^2 \quad (11)$$

when α satisfies the condition,

$$\alpha^2 > \frac{(1-\beta)^2}{2\beta} , \quad (12)$$

and the proposition is established by example.

The second feature of the model warrenting interpretation is the feature that the function $\kappa(\lambda)$ is monotone increasing in λ over the range zero to $\pi/2$. Herein lies the answer to the second puzzler posed in the introductory paragraph. The manipulation of the forecast in the manner described causes the increase in the variance of the series to be greater at the higher frequencies, hence giving the strong impression of chopping-and-changing of the forecast from one period to the next.

Finally it is worthwhile to draw attention to the intrinsic (devil's) circularity in this model (and all similar models with manipulative agents). Suppose the private agents, those using the government forecast, were themselves to become versed in the ways of minimum mean squared error forecasting, and were to forsake their crude adaptive expectations decision rule for the current mean squared error forecast, they would learn that the mean squared error forecast was exactly the same as that derived from the old decision rule. There is then an incentive for the private agents to simply continue to adjust for the obviously poor government forecasts, and for the government to continue to produce "manipulated" forecasts because they clearly elicit the desired result from the private agents.