A STUDY OF THE SALT BALANCE IN A COASTAL PLAIN ESTUARY[1]

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ABSTRACT

Observations of salinity and current velocity taken in the James River estuary are used in this investigation. Time means are taken over several tidal cycles and the nonadvective motions and nonadvective salt flux are related to these means. The analysis shows that in the James River estuary the horizontal advective flux and the vertical nonadvective flux of salt are the most important factors in maintaining the salt balance. The vertical advective flux is of secondary importance but is still significant. The horizontal nonadvective flux of salt is small. In addition, the vertical nonadvective flux of salt is partly related to the magnitude of the oscillatory tidal currents.

INTRODUCTION

The extensive observations of salinity and current velocity, made during the summer of 1950 in the James River estuary,[2] form the basis for the present study of the variation and relative importance of factors that maintain the salt balance. Fig. 1 shows the curves for typical mean salinity vs. depth as well as horizontal mean velocity vs. depth for the James River estuary. The salinity increases horizontally from fresh water at the head of the estuary to approximately 20 ‰ at the mouth of the James where it joins the Chesapeake Bay. The character of the vertical distribution does not change much longitudinally. Consequently the horizontal gradients do not vary appreciably with depth. This indicates that there is appreciable vertical mixing of the upper and lower layers. The lower layers are of higher salinity and flow toward the head of the estuary while the upper layers are of lower salinity and flow seaward.

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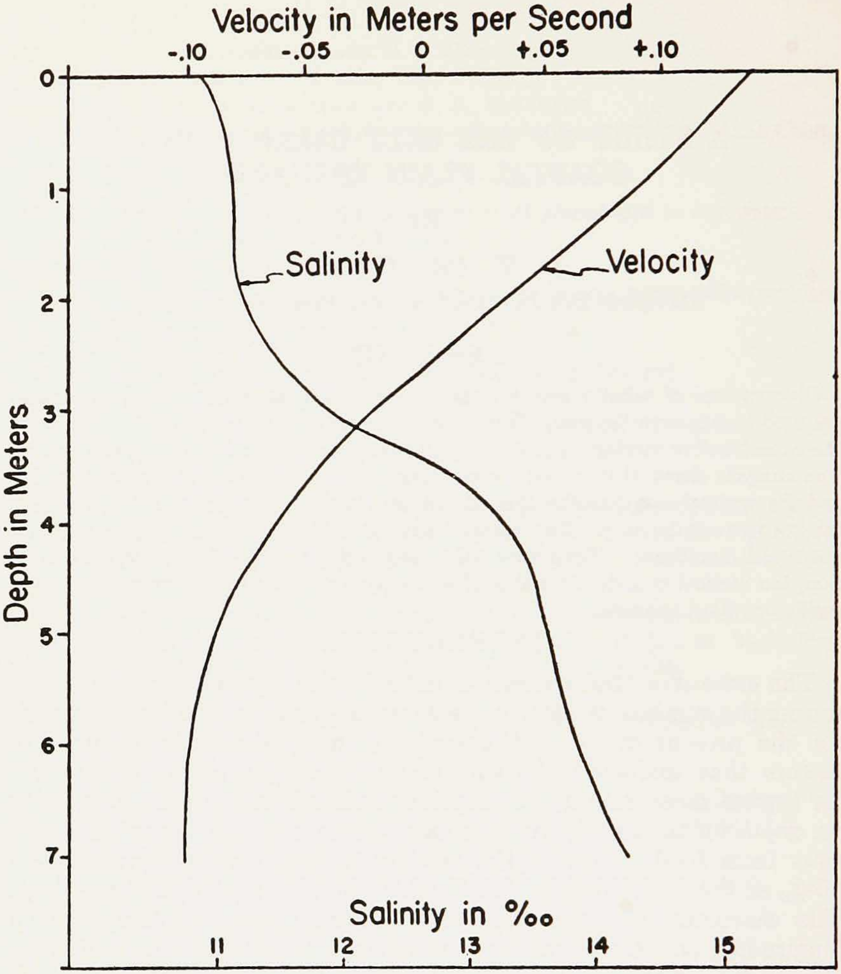
[2] Pritchard, D. W. 1952. Salinity distribution and circulation in the Chesapeake Bay estuarine system. J. Mar. Res., *11:* 106–123.



1. Typical mean salinity *versus* depth curve and horizontal mean velocity *versus* depth curve for the James River estuary.

THE BASIC SALT BALANCE EQUATION

Consider a left-handed coordinate system with the origin at the surface in the fresh water at the head of the estuary. The x_1 coordinate is directed down the estuary along the central axis, and the x_2 coordinate points vertically downward; the lateral coordinate x_3 is directed horizontally from the center of the estuary toward the right-hand shore.

Neglecting molecular diffusion, the instantaneous local rate of change of the salt concentration, s, is given by

$$\partial s/\partial t = -\frac{\partial(v_i s)}{\partial x_i} \cdot \tag{1}$$

The instantaneous velocity, v_i, may be expressed as the sum of three terms: (1) a mean velocity $\bar{v}_i$, obtained by averaging over one or more tidal cycles; (2) a tidal velocity U_i, which we assume is directed up and down the estuary and which is given simply by $U_0 \cos \varphi$, where U_0 is the amplitude of the tidal current and φ represents the sum of the angular time argument of tidal period plus a phase angle; (3) a velocity deviation relative to a time average, designated as v_i'. Thus

$$v_i = \bar{v}_i + U_i + v_i'. \tag{2}$$

In a similar manner, the instantaneous value of salinity is expressed by

$$s = \bar{s} + (s)_u + s', \tag{3}$$

where $\bar{s}$ is the time mean salt content, $(s)_u$ the periodic variation in salt content related to tidal motion [assumed to be given simply by $(s)_u = (s)_0 \sin \varphi$], and s' the salinity deviation relative to a time average. Substituting (2) and (3) into (1), and taking the time average over one or more tidal cycles, we have

$$\begin{aligned} <\partial s/\partial t> = &-\frac{\partial}{\partial x_i}(\bar{v}_i\bar{s}) - \frac{\partial}{\partial x_i}<U_i(s)_u> - \frac{\partial}{\partial x_i}<U_i s'> \\ &- \frac{\partial}{\partial x_i}<(s)_u v_i'> - \frac{\partial}{\partial x_i}<v_i' s'>. \end{aligned} \tag{4}$$

But if U_i is given by $U_0 \cos \varphi$ and $(s)_u$ by $(s)_0 \sin \varphi$, then the mean product $<U_i(s)_u>$ over one or more tidal cycles becomes zero. The terms involving $<U_i s'>$ and $<(s)_u\, v'_i>$ depend on the correlation between the velocity and salinity deviations on the one hand and the tidal period on the other. Assuming that no such correlations exist, these terms would also vanish with time averages over one or more tidal cycles. (There may well be a direct relation between the tidal period and the root mean square of the velocity deviations. However, this does not imply a correlation between the tidal period and the individual velocity deviations.) Hence (4) becomes

$$<\partial s/\partial t> = -\frac{\partial}{\partial x_i}(\bar{v}_i\bar{s}) - \frac{\partial}{\partial x_i}<v_i' s'>. \tag{5}$$

Note that the terms involving the simple harmonic tidal fluctuations have been eliminated.

Equation (5) states that the local time mean change in concentration results from two terms, one related to the mean advection, the second to nonadvective processes. Oceanographers and meteorologists have called the latter process "diffusion" and have replaced the mean product $<v_i's'>$ by the so-called diffusion terms having the general form $-A_i\,\partial\bar{s}/\partial x_i$.

Equations (1) and (5) apply to an infinitesimal element in the fluid. The nature of the observations makes it convenient to study the salt balance in an element extending across the estuary laterally. In this element the variations in the velocity and salinity in the x_3 direction are neglected and the resulting salt balance equation will differ from the basic equation (1).

Consider the continuity of salt within an element of volume extending across the estuary at level x_2, which has an infinitesimal length dx_1 and an infinitesimal height dx_2. There can be no transport of salt through the ends of the element which are the sides of the estuary. Since the width of the estuary decreases with depth, the area of the top surface of the element is greater than that of the bottom surface. Likewise, the area of the downstream surface is different from that of the upstream surface, since the width of the estuary varies in the longitudinal direction. For such an element it can be shown that

$$\partial s/\partial t = -\frac{1}{w}\frac{\partial}{\partial x_1}(wv_1s) - \frac{1}{w}\frac{\partial}{\partial x_2}(wv_2s), \tag{6}$$

where w is the width of the estuary at the depth x_2. If we now take the time mean of equation (6) in the same manner as was done with equation (1), we obtain

$$\begin{aligned} <\partial s/\partial t> = & -\frac{1}{w}\frac{\partial}{\partial x_1}(w\bar{v}_1\bar{s}) - \frac{1}{w}\frac{\partial}{\partial x_2}(w\bar{v}_2\bar{s}) \\ & -\frac{1}{w}\frac{\partial}{\partial x_1}\{w<v_1's'>\} - \frac{1}{w}\frac{\partial}{\partial x_2}\{w<v_2's'>\}. \end{aligned} \tag{7}$$

For the element under consideration, the equation of continuity is

$$\frac{\partial}{\partial x_1}(w\bar{v}_1) + \frac{\partial}{\partial x_2}(w\bar{v}_2) = 0. \tag{8}$$

Utilizing this relationship, equation (7) becomes

$$(9) \quad \langle \partial s/\partial t \rangle = -\bar{v}_1\, \partial \bar{s}/\partial x_1 - \bar{v}_2\, \partial \bar{s}/\partial x_2 - \frac{1}{w}\frac{\partial}{\partial x_1}\{w \langle v_1{}'s' \rangle\} - \frac{1}{w}\frac{\partial}{\partial x_2}\{w \langle v_2{}'s' \rangle\}.$$

Note that, as in the case of equation (5), terms involving the harmonic variations due to the tides are absent from equation (9). The variation in the terms of equation (9) and the relative importance of the processes which they represent in maintaining the salt balance are the objects of our study.

THE MEAN LOCAL TIME CHANGE

It is here assumed that the time mean of the instantaneous local time change of salinity, $\langle \partial s/\partial t \rangle$, is given approximately by the increment of mean salinity change per unit time, that is $\Delta \bar{s}/\Delta t$. The field program in the James River was undertaken during a period when conditions approximated the mean steady state in salinity distribution. During all three periods of investigation the mean local time variation $\langle \partial s/\partial t \rangle$ was less than 2×10^{-3} g/m^3/sec.

THE LONGITUDINAL TERMS IN THE SALT BALANCE EQUATION

There are two longitudinal terms in the salt balance equation, an advective term $\bar{v}_1\, \partial \bar{s}/\partial x_1$ and a nonadvective term $\frac{1}{w}\frac{\partial}{\partial x_1}\{w \langle v_1{}'s' \rangle\}$. Sufficient velocity and salinity measurements were made in the James River study to evaluate the advective term. To determine the significance of the turbulent mixing term, the following treatment of equation (1) is required.

Consider a segment of the estuary bounded at either end by a cross-section. Designating the total salt content within the segment by $S = \iiint_{Vol} s\,dv$, where V is the volume of the segment, we have from (1)

$$(10) \quad \frac{\partial S}{\partial t} = \iiint_{Vol} \frac{\partial s}{\partial t}\, dV = -\iiint_{Vol} \frac{\partial}{\partial x_i}(v_i s)\, dV = -\iint_{\sigma} v_i s\, d\sigma_i\,,$$

where $\iint_{\sigma} d\sigma_i$ represents the integral over the bounding surfaces of the segment. The time mean of this equation may be taken, following the same procedure which was utilized in obtaining equation (5) from equation (1). We then have

$$(11) \qquad \left\langle \frac{\partial S}{\partial t} \right\rangle = - \iint_{\sigma} \bar{v}_i \bar{s} d\sigma_i - \iint_{\sigma} <v_i' s'> d\sigma_i .$$

For steady state $<\partial S/\partial t> = 0$, and hence

$$(12) \qquad \iint_{\sigma} \bar{v}_i \bar{s} d\sigma_i + \iint_{\sigma} <v_i' s'> d\sigma_i = 0 .$$

Since the segment under study is bounded by the surface, bottom and sides of the estuary, only the horizontal terms in equation (12) need be considered. Equation (12) becomes

$$(13) \qquad \iint_{\sigma} \bar{v}_1 \bar{s} d\sigma_1 + \iint_{\sigma} <v_1' s'> d\sigma_1 = 0 .$$

For an evaluation of the first term of equation (13), observations of mean salinity and velocity for nine separate cases are available. It is convenient, both for the present discussion and for later analysis, to consider their first integral in two parts, one term involving the upper layer, where the mean velocity is positive, the other term involving the lower layer, where the mean velocity is negative. The value of the integral $\iint_{\sigma} <v_1' s'> d\sigma_1$, as determined from equation (13), was in all cases less than 5% and in six of the nine cases less than 1% of either of the two parts of the first term in equation (13). This means that the salinity balance within a segment bounded by the surface and the bottom is maintained primarily by horizontal advection, the horizontal diffusion being of only slight importance.

The mean value of the term $\frac{1}{w} \frac{\partial}{\partial x_1} \{w <v_1' s'>\}$, as determined from the mean value of the integral $\iint_{\sigma} <v_1' s'> d\sigma_1$, was two orders of magnitude less than the horizontal advective term $\bar{v}_1 \, \partial \bar{s}/\partial x_1$. The latter term varied from about 5×10^{-2} g/m^3/sec at the surface to -5×10^{-2}g/m^3/sec near the bottom. The horizontal nonadvective term was of the order of only 5×10^{-4}g/m^3/sec, though its relative importance increased in the upstream direction.

THE VERTICAL ADVECTIVE TERM IN THE SALT BALANCE EQUATION

The second term on the right-hand side of equation (9) involves the mean vertical velocity. This parameter may be computed from the horizontal velocities by means of the concept of volume continuity. For nondivergent flow, $\partial \bar{v}_i/\partial x_i = 0$. Taking the integral of this expression over the volume V and applying Green's formula, we have

$$\iint_{\sigma} \bar{v}_i d\sigma_i = 0\,, \tag{14}$$

a statement that the volume of water flowing into the segment bounded by the surface σ must equal the volume of water flowing out of the same segment. Equation (14) may be employed to evaluate the mean vertical velocity as a function of depth from measurements of the horizontal velocity at two sections.

A second method of computation of the mean vertical velocity involves the vertical integration of the equation of continuity in the form of equation (8). The horizontal gradient of the product $(w\bar{v}_1)$ can be found graphically for each depth and the integration can be carried out numerically, by using the boundary condition that the mean vertical velocity must be zero at the surface.

Computations by both methods were carried out for three periods by using three sections in the James River estuary. The calculations were made for depth increments of one-half meter. Though both methods are based on the same continuity concept, the mechanics of application lead to slightly different results. The average of the two computations of the mean vertical velocity for a typical section for the three periods is shown in Fig. 2.

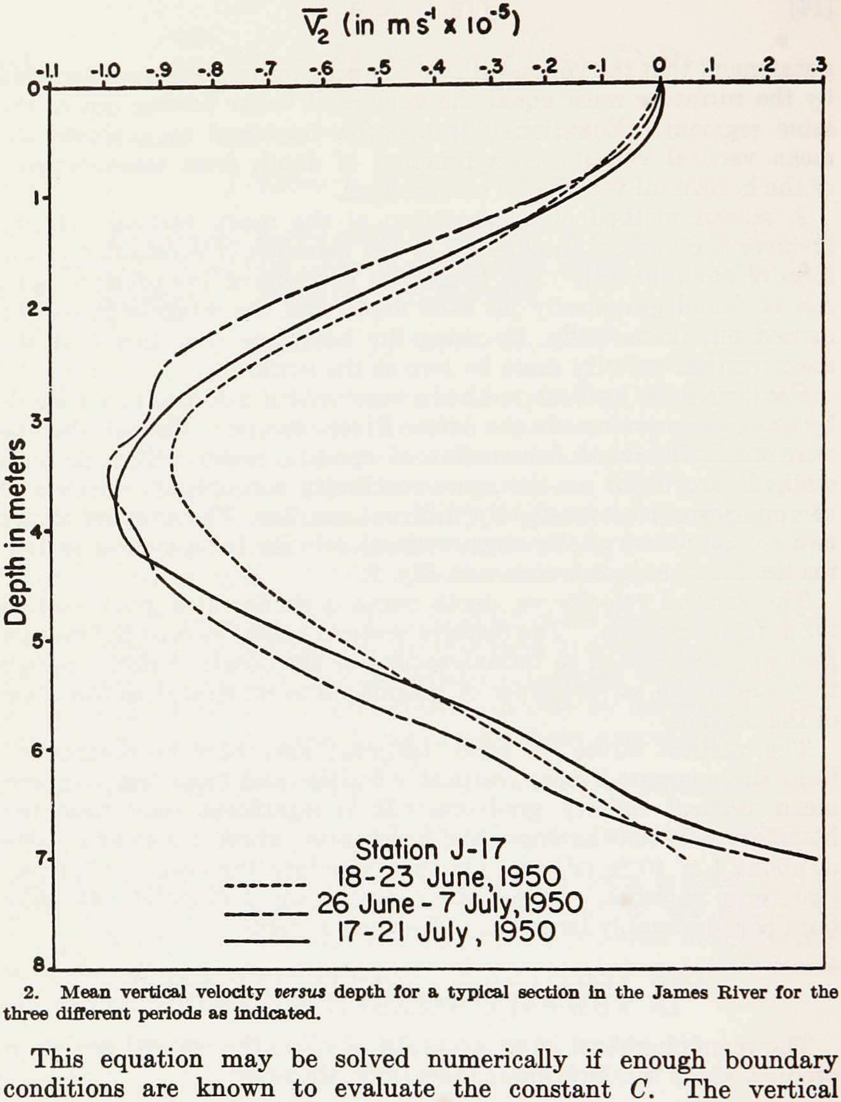
The vertical velocity vs. depth curve is similar at a given station for different periods. The positive vertical velocities near the bottom probably are related to inconsistencies in the observed data, though they are of the correct order of magnitude to be related to the slope of the bottom.

The vertical advective term, $\bar{v}_2\partial\bar{s}/\partial x_2$, may now be determined from the computed mean vertical velocities and from the observed mean vertical salinity gradients. It is significant only near the boundary between the upper and lower layer, where it attains a value of about 1×10^{-2}g/m^3/sec. On this boundary the horizontal advective term vanishes. At all other depths the horizontal advective term is considerably larger than the vertical term.

THE VERTICAL NONADVECTIVE TERM IN THE SALT BALANCE EQUATION

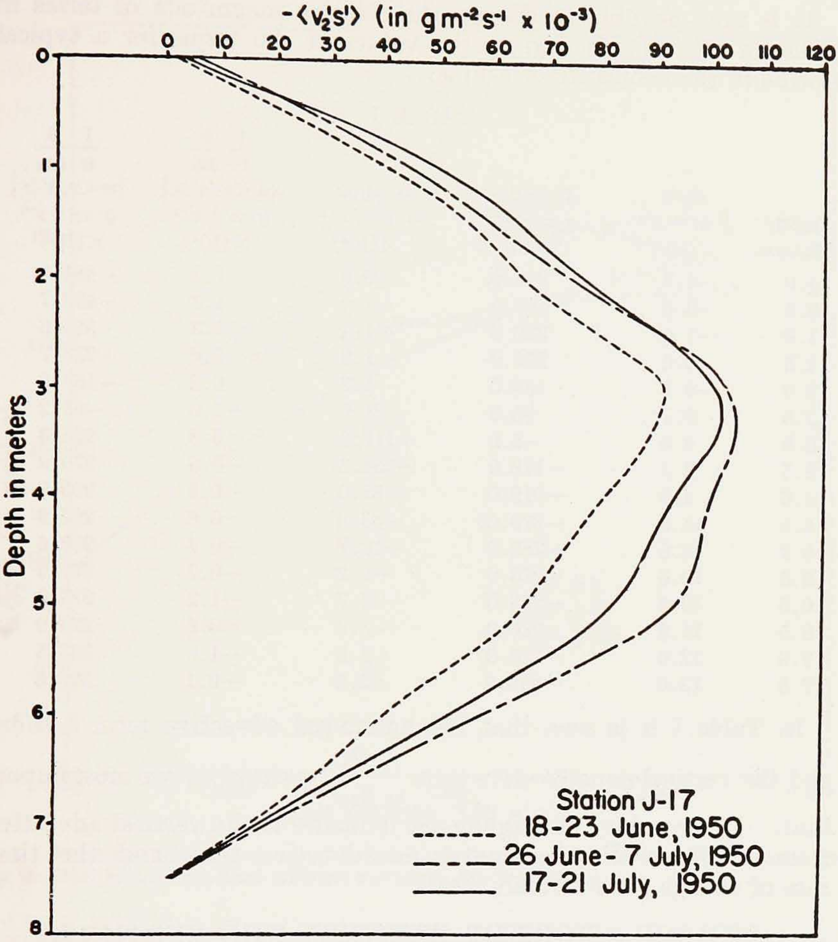
The remaining term in equation (9) involves the vertical deviation product $\langle v_2's'\rangle$. Solving for this term, we have

$$\langle v_2's'\rangle = -\frac{1}{w}\left\{\int w\left[\partial\bar{s}/\partial t + \bar{v}_1\,\partial\bar{s}/\partial x_1 + \bar{v}_2\,\partial\bar{s}/\partial x_2 + \frac{1}{w}\frac{\partial}{\partial x_2}\{w\langle v_1's'\rangle\}\right]dx_2 + C\right\}. \tag{15}$$



2. **Mean vertical velocity *versus* depth for a typical section in the James River for the three different periods as indicated.**

This equation may be solved numerically if enough boundary conditions are known to evaluate the constant C. The vertical deviation product should be zero both at the bottom and at the surface (neglecting the effect of rainfall and evaporation at the surface). Only one of these boundary conditions is needed to evaluate C. The consistency of the basic equation (9) and of the analysis of the data may be checked by the degree to which the second condition is met.



3. The vertical nonadvective flux of salt *versus* depth for a typical section in the James River for the three periods as indicated.

The value of $<v_2's'>$ was taken as zero at the bottom boundary, and equation (15) was evaluated for $<v_2's'>$ at each half meter interval of depth. The results of this evaluation for a typical section in the James River estuary for three different periods are shown in Fig. 3. The boundary condition at the surface is nearly satisfied in all cases, a result which serves to check the consistency of the form of the salt balance equation used here with the observed data.

It is now possible to discuss the relative magnitude of terms in equation (9). Table I gives the values of the terms for a typical station in the James River estuary.

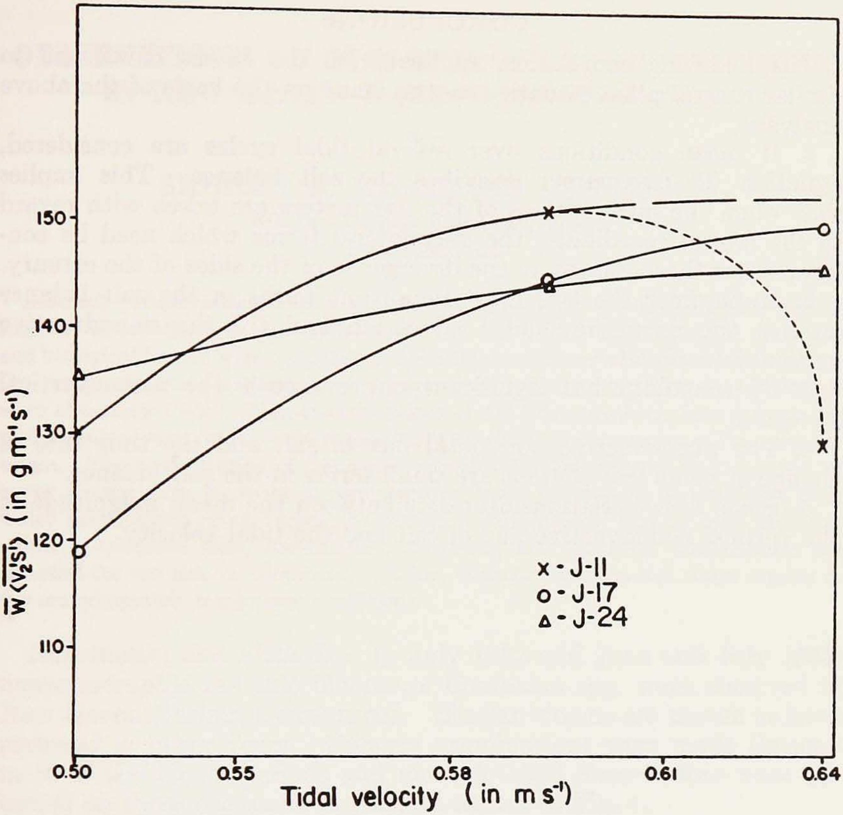
TABLE I

Depth Meters	$\partial\bar{s}/\partial t$ $g\cdot m^{-3}\cdot s^{-1}$ $\times(10^4)$	$\bar{v}_1\,\partial\bar{s}/\partial x_1$ $g\cdot m^{-3}\cdot s^{-1}$ $\times(10^4)$	$\bar{v}_2\,\partial\bar{s}/\partial x_2$ $g\cdot m^{-3}\cdot s^{-1}$ $\times(10^4)$	$\frac{1}{w}\frac{\partial}{\partial x_1}\{w<v_1's'>\}$ $g\cdot m^{-3}\cdot s^{-1}$ $\times(10^4)$	$\frac{1}{w}\frac{\partial}{\partial x_2}\{w<v_2's'>\}$ $g\cdot m^{-3}\cdot s^{-1}$ $\times(10^4)$
0.0	−4.2	484.0	0.0	1.3	−481.1
0.5	−3.6	407.0	−1.0	1.3	−403.7
1.0	−1.0	326.0	−1.4	1.3	−324.6
1.5	0.0	238.0	−1.9	0.6	−236.7
2.0	−0.1	159.0	−4.2	0.2	−154.9
2.5	0.1	88.0	−21.9	−1.0	−65.2
3.0	0.8	−8.0	−117.0	−0.8	125.0
3.5	2.5	−118.0	−154.8	−0.6	270.9
4.0	4.8	−219.0	−81.0	−0.6	295.8
4.5	15.5	−279.0	−33.8	−0.6	297.9
5.0	14.5	−288.0	−13.7	−0.4	287.6
5.5	10.0	−278.0	−9.2	−0.7	277.9
6.0	10.5	−278.0	−6.7	−1.2	275.4
6.5	11.9	−286.0	−5.2	−0.6	279.9
7.0	12.0	−318.0	+3.5	−1.0	303.5
7.5	12.0	−345.0	+3.5	−0.1	329.6

In Table I it is seen that the horizontal advective term $\bar{v}_1\partial\bar{s}/\partial x_1$ and the vertical nonadvective term $-\frac{1}{w}\frac{\partial}{\partial x_2}\{w<v_2's'>\}$ are most important. Of secondary but significant influence is the vertical advective term, $\bar{v}_2\partial\bar{s}/\partial x_2$. The horizontal nonadvective term and the time rate of change are relatively small.

THE RELATIONSHIP BETWEEN THE VERTICAL NONADVECTIVE TERM AND THE TIDE

The computed values of $<v_2's'>$ differ somewhat from place to place and from time to time, with the difference due to location being more pronounced than the difference due to time (Fig. 3). The change in $<v_2's'>$ with time should be related to a corresponding time change in the mechanism which leads to eddy mixing. Since, as stated earlier, the tide is considered to be the most important factor governing the mixing processes, there should be a relationship between the tidal velocities and the magnitude of the vertical deviation product.



4. The mean value of the vertical nonadvective flux of salt as a function of the amplitude of the tidal velocity for three different sections in the James River.

Predicted magnitudes of the maximum flood and ebb tidal velocities, taken at a reference station near the mouth of the James River estuary during each of the periods under consideration, were obtained from the C&GS tables and were plotted against the mean value of $<v_2's'>$. The relation for each station is shown in Fig. 4. In all but one case there appears to be a definite relation between the tidal velocities and the vertical nonadvective term. This result is significant for two reasons. First, it confirms the hypothesis that the mixing processes are related primarily to tidal action. Second, it suggests the possibility of predicting the eddy diffusion terms from the tidal velocities.

CONCLUSIONS

The following conclusions applicable to the James River and to similar coastal plain estuaries can be made on the basis of the above analysis.

1. If mean conditions over several tidal cycles are considered, equation (9) adequately describes the salt balance. This implies that when the mean values of the parameters are taken with regard to the lateral coordinate, the only lateral terms which need be considered are those related to the divergence of the sides of the estuary.

2. In summer the two most important terms in the salt balance are (a) the mean horizontal advection and (b) the nonadvective vertical flux of salt.

3. Of secondary but significant importance is the mean vertical advection.

4. The nonadvective horizontal flux of salt and the time rate of change of mean salt content are small terms in the salt balance.

5. Some direct relationship exists between the mean magnitude of the vertical nonadvective flux of salt and the tidal velocity.