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WIND STRESS ON AN ARTIFICIAL POND

BY

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ABSTRACT

The wind-induced slope of the surface in an 800-foot model-yacht pond has been measured to a relative accuracy of $5 \times 10^{-7}$. This slope is proportional to the sum of the surface and bottom stresses. The bottom stress was independently measured to an accuracy of 0.1 dynes cm$^{-2}$ and was found to be negligible when compared to the surface stress. The surface slope is shown empirically to be the result of two effects: first, a tangential "friction" drag, which is invariably present and which is proportional to the square of the windspeed; and second, a "form" drag, which occurs only after the wind has increased above a certain value. The second effect is related to surface waves. Application of a detergent to the water eliminates both waves and form drag. The surface current was proportional to the windspeed and independent of waves. The slope increased with heavy rain, and a theoretical model is proposed which adequately predicts the observed increase.

The present study was modeled closely after Keulegan's experiments in a 60-foot laboratory channel. The results of the two studies agree quantitatively.

INTRODUCTION

Previous attempts to determine the stress exerted by the wind on a free water surface can be roughly divided into two groups. The first consists of measuring the vertical gradient of the mean horizontal wind velocity over a convenient range of elevations, converting this gradient into a shear stress (usually by Prandtl's mixing-length theory), and extrapolating this stress to the water surface by analogy to the empirical laws describing flow over a rigid boundary or by assuming the stress to be constant over a limited range of height above the surface. The second method involves measurements of the surface slope of a confined body of water under wind action, making use of the fact that under certain assumptions the slope is nearly proportional to the surface stress. This stress is then correlated with the wind at a stated height above the surface.

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Attempts to measure stress by the second method fall into two categories: those dealing with large natural bodies of water (Corkan, 1950; Ekman, 1905; Hela, 1948; Hellstrom, 1941; Palmén and Laurila, 1938; and others) and those in laboratory channels (Francis, 1951; Hellstrom, 1941; Keulegan, 1951). This paper deals with still another series of slope measurements made on an artificial pond 220 m long. Thus the scale was roughly intermediate between the two mentioned categories.

THEORY OF SURFACE SLOPE

The theoretical treatment of the shape of the free surface of a confined body of water acted upon by a uniform and steady wind has been given by Hellstrom (1941). The special case of a long, shallow, rectangular channel was investigated by Keulegan (1951) and Haurwitz (1951), but the role of the inertial terms was not discussed explicitly. We shall give a brief account below.

If we take the x-axis along the bottom in the direction of the wind, with the z-axis upwards (Fig. 1), neglect the Coriolis force and the horizontal pressure gradient, and assume that the water is homogeneous, then the equation of horizontal motion can be integrated to the following form,

\[
\frac{\partial}{\partial t} \left( \int_0^{H+h} udz \right) + \frac{\partial}{\partial x} \left( \int_0^{H+h} u^2dz \right) = -g(H + h)\frac{\partial h}{\partial x} + \frac{\tau_s + \tau_b}{\rho},
\]

where \( u \) is the horizontal water velocity, \( H \) the undisturbed water depth, \( h \) the surface departure from the undisturbed state, \( \rho \) the water density, \( \tau_s \) and \( \tau_b \) the horizontal stresses at the surface and bottom respectively, and \( g \) the acceleration of gravity. This equation expresses the momentum budget of a column of water of unit area extending from the surface to the bottom. Assuming steady-state conditions, the first term in (1) can be neglected. The measurements of \( h \) were made at the ends of the pond, so we take the average of the remaining terms over the entire length, \( L \):

\[
\frac{1}{L} \int_0^L \frac{\partial}{\partial x} \left( \int_0^{H+h} u^2dz \right) dx = -\frac{g}{L} \int_0^L (H + h) \frac{\partial h}{\partial x} dx
\]

\[
+ \frac{1}{\rho L} \int_0^L (\tau_s + \tau_b) dx.
\]

Here \( L \) refers to the distance between the points at which \( h \) is observed, which in the present experiments was slightly less (220 m) than the actual length, \( l \), at the surface (240 m). So long as the bottom profile
is symmetrical, the only requirement is that these points be at equal distances from the ends so that (2) will apply. Under these circumstances, the advection term vanishes and, provided $H \gg h$ and $H$ does not vary with $x$, we have

$$0 = -\frac{gH}{L} (h_0 - h_L) + \frac{\tau_s + \tau_b}{\rho},$$

(3)

with the bars designating average values over the length of the pond. Henceforth the bars will be omitted. If the setup, $S$, is defined as the difference in elevation between the two ends of the pond, the equation relating the average slope to the surface stress, $\tau_s$, is simply

$$\frac{S}{L} = \frac{n\tau_s}{\rho gh},$$

(4)

where $n$ is defined as

$$n - 1 = \frac{\tau_b}{\tau_s}.$$  

(5)

It is significant that equation (4) involves no assumptions regarding the distribution of stress within the fluid, nor does it predict the velocity distribution. The surface stress is known only to the extent that the bottom stress is known.

Only in the special case of laminar flow can $n$ be determined explicitly, for then

$$\tau = \mu \cdot \frac{\partial u}{\partial z}.$$  

(6)

Inserting this value for $\tau$ in the unintegrated equation of horizontal motion, neglecting the inertial terms, and assuming steady flow, one obtains (Keulegan, 1951)

$$\frac{d^2u}{dz^2} = \frac{g}{\nu} \frac{dh}{dx},$$

(7)

where $\mu$ and $u$ are the kinematic and dynamic viscosities respectively.

If $H$ is not uniform, one must, as in the theory of seiches (Defant, 1925), regard $H$ as the mean depth, equal to the total volume of water divided by the surface area. But actually, a convenient and practical method of obtaining this depth is to observe the period, $T$, of the fundamental seiche and, using the mean length, $l$, at the surface in the direction of the wind, compute it from the formula $H = 4l\bar{u}/gT^2$. This method applies only to basins of simple shape. As an example, we consider the data obtained for the model-yacht pond on December 5, 1951, when the observed depth over 90% of the area was 240 cm. By the first method $H = 2.75 \times 10^4 m^3/1.51 \times 10^3 m^2 = 1.83 m$; and by the second method, with $l = 240 m$ and $T = 112$ sec, we obtain $H = 1.85 m$. It is apparent that shoaling at the boundaries significantly affects the average depth.
This relation can be integrated directly, in view of the requirement that the net flow through any cross section be zero. The results are:

$$\frac{dh}{dx} = \frac{6\nu u_s}{gH^2}, \quad (8)$$

and

$$\tau_s = -2\tau_b = 4\mu u_s/H, \quad (9)$$

where $u_s$ is the surface current speed. Then, because of (5),

$$n = 3/2. \quad (10)$$

The law for the velocity distribution within the fluid is given by the relation

$$u = u_s \left[3 \left(\frac{z}{H}\right)^2 - 2 \frac{z}{H}\right] \quad (11)$$

and is illustrated in Fig. 1. The flows consist of a surface current in the direction of the wind that extends down to one-third the depth and beneath it another current flowing in the opposite direction having a minimum value $u = -u_s/3$ at two-thirds the depth. The bottom stress is one-half the surface stress.
When the flow is turbulent, however, the distribution of velocity is known to differ widely from that predicted by the laminar solution, hence the stresses $\tau_b$ and $\tau_*$ cannot be explicitly determined in terms of easily measurable quantities. Hellstrom (1941), by means of the Boussinesq theory of turbulence and on the basis of analogy to the empirical laws of the turbulent flow in open channels, proposed that, in general, the limits of the parameter $n$ in (4) are $1.0 < n < 1.5$. In the present experiments a special investigation was undertaken to measure the bottom stress directly, and it was established that $\tau_b \leq 0.1 \tau_*$, which is less than the threshold sensitivity of the instrumentation. Hereafter it will be assumed that $n = 1.0$ in all cases.

EXPERIMENTAL PROCEDURE AND INSTRUMENTATION

The site for the experimental work was a model-yacht pond situated on an island near the geometric center of Mission Bay, adjacent to San Diego (Fig. 2). The pond is roughly rectangular with dimensions approximately $240 \times 60 \times 2$ m, and with the depth controllable over a total range of about one meter by a check-valve system in the 30" pipe which connects the pond to the bay. The pond, constructed
with its long axis parallel to the direction of the prevailing wind from the ocean, is separated from the Bay by about 50 m of sand with a maximum elevation of two m above the water surface.

**The Wind and Temperature Measurements.** Throughout the experiments the wind velocity and direction at 10 m elevation above the pond were continuously recorded with a three-cup Bendix anemometer mounted on a staff above the roof of a building adjacent to the pond. Thus the fetch for this anemometer was over smooth sand rather than over water. While it might be expected that the presence of the building would modify the air flow, the relative values of these windspeeds as compared to those measured at lower elevations over the pond were extremely consistent.

The wind velocity was also measured at elevations of 25 and 100 cm by two “Davis” propeller-type anemometers mounted on a staff at the downwind water’s edge; photographs were taken at five-minute intervals. All three anemometers were calibrated together against a new Bendix “Aerovane” anemometer, for which a National Bureau of Standards calibration was supplied by the manufacturer. The four instruments were mounted on a staff at 10 m elevation and were allowed to run for 10 hours; the readings were averaged over five-minute intervals. The results indicated that it would be necessary to apply a correction to the readings of the two Davis anemometers, but the ratios of the indicated windspeeds were consistent within 4%. Throughout the experiments all wind data were reported to the nearest 0.1 m/s, although this accuracy was considered to be somewhat optimistic at the higher windspeeds.

In conjunction with the wind measurements at the lower elevations, the air temperature at one and seven meters was read from shielded thermometers every 15 minutes to the nearest 0.1°C. Although transient temperature fluctuations of as much as 0.3°C often occurred at both elevations, the instantaneous difference in temperature (upper minus lower) was consistently about −0.1°C. In addition, the water temperature was taken from a bucket sample. Throughout the tests the water was invariably warmer than the air; the average value for all readings (principally daytime) was 0.9°C, the maximum difference 2.6°C.

The uniformity of the wind field over the pond as well as the temperature homogeneity of the water assumed in the basic theory were investigated separately, and it was found that the observed departure

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3 From previous studies this procedure appears to be preferable to a calibration in a wind tunnel, where the gustiness is quite different from that found under natural conditions.
from the assumed conditions was so small that the errors thus introduced in the final results could be neglected (Van Dorn, 1952).

The Measurement of Setup. In the present experiments, each reported value for the setup, \( S \), was actually the height of the column of water which corresponded to the difference in pressure at two points 220 m apart at equal elevations near the bottom of the pond and at equal distances from the ends. Because the vertical water velocity, \( w \), can be expected to be greatest near the ends of the pond, it appears desirable to estimate the error introduced into the pressure measurements by neglecting it. Assuming a steady state and noting that \( u = 0 \) at the ends, the vertical equation of motion can be written

\[
aw \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial z} - g, \tag{12}
\]

where \( p \) is the total pressure within the fluid. Integrating (12) between the bottom and the elevation, \( z' \), at which the pressure was measured, we obtain

\[
p = p_a + \rho g z' - \frac{1}{2} \rho w^2. \tag{13}
\]

Ignoring the atmospheric and hydrostatic pressures, the difference in pressure, \( \Delta p \), at the two ends of the pond due to the velocity will be

\[
\Delta p = - \rho w^2. \tag{14}
\]

Taking a conservative value of \( w = 1 \) cm/sec, the pressure error amounts to only 1 dyne/cm\(^2\), which is only one-tenth the minimum reported pressure during the experiments.

A block diagram of the instrumentation used to measure the pressure difference is shown in Fig. 3. A \( \frac{1}{4}'' \) diameter plastic tube, filled with distilled water and laid down the long axis of the pond, terminated at each end in a pair of pie tins clamped together and supported 55 cm above the bottom on a small tripod. Between the pie tins a thin, limp plastic membrane prevented the sea water from entering the tube and yet maintained adequate pressure continuity in the system. The outer tin contained a screened opening to prevent fouling of the membrane by marine organisms. In the center of the tube there was a Statham (Model 0.2-D) differential pressure gauge in an oil-filled plexiglass case; two additional membranes separated the oil from the distilled water in the tube. The Statham gauge is essentially a pressure-sensitive electric strain-gauge of the resistance-bridge type that produces a change in the direct current across the bridge which is proportional to a difference in pressure between its two pressure connections. The change in fluid capacity corresponding to full-scale
pressure differential is only about 0.16 cm³, so that almost no flow took place within the tube. Hence, the amplitude response of the system was virtually unity and the phase shift was zero for the frequencies encountered in the experiments (Iberall, 1950). In view of the low pressure levels experienced during the measurements, it was necessary to reduce the thermal response of the transducer to diurnal temperature fluctuations by burying it in the sand beneath the bottom of the pond.

The electric cable from the transducer ran to a building adjacent to the pond, where the signal was fed to a Micromax recorder through a D. C. amplifier and a long-period resistance-capacitance filter. The filter was designed for the multiple purpose of obtaining an electric impedance match between the amplifier and recorder, eliminating the surface waves from the record, and attenuating the seiche (period 112 sec).

4 The volumetric distension of the tube under the pressures encountered was separately checked and was negligible.
Figure 4. Typical records for setup and windspeed and direction at 10 m elevation. Windspeed is given by the frequency of the pips representing miles of wind, and the direction is in degrees from the axis of the pond. The sample is reproduced from an original record made on March 1, 1952.

Preliminary calibration of the instrumentation was made in a tank, where it was found that variations of the water level were reproducible within 0.01 cm of water. Throughout the actual tests the elevation difference between the two ends of the pond was read to 0.01 cm, which corresponds to a slope, $S/L$, of approximately

$$\frac{S}{L} = \frac{0.01}{2.2 \times 10^4} = 5 \times 10^{-7}.$$  (15)
Fig. 4 is a reproduction of a typical setup record and corresponding wind record at 10 m elevation. The seiche is plainly visible in the setup record even though it is attenuated by the filter to about 16% of its true amplitude. The reported value of the setup was obtained by drawing a line through the midpoints of the seiche oscillations and by integrating the area between this line and the zero-reference line over 20 minute intervals. The windspeed record was averaged over the same intervals.

The Bottom Stress Observations. The instrumentation for the bottom stress measurements was essentially identical with that used in the slope studies. However, instead of the pressure transducer and water-filled tube, the sensing element consisted of a circular plexiglass disk having an area of 1000 cm$^2$ which was suspended in the center of circular surround faired into the bottom at the approximate geometric center of the pond. The disk was supported at one edge by a pivot and was linked at the opposite edge to a Statham G-7 (Model 0.15–320) force transducer, the axis of suspension being perpendicular to the long axis of the pond. The surface of the disk was coated with a layer of pond sand embedded in cement to reproduce as nearly as possible the actual bottom roughness. Calibration of the unit was carried out in a tank by applying horizontal forces to the center of the disk with a torque arm in 0.1 g increments and by varying the transducer bias at each force level until a graph of sensitivity versus bias for each force increment was obtained. This was done so that the sensitivity could be adjusted in the field if the signals proved to be larger or smaller than anticipated. In the experiments the sensitivity was set at 10 g equal to full scale on the speedomax recorder, corresponding to a stress of 10 dynes/cm$^2$. The system had threshold accuracy of 0.01 dynes/cm$^2$, although it was found that the background noise (principally due to surface waves) was roughly ten times this amount.

SUBSIDIARY OBSERVATIONS

The Effect of Detergent. Keulegan (1951) discovered that waves in his experimental channel could be virtually eliminated by adding a detergent to the water. In this way the setup produced in the presence and absence of waves could be studied separately under nearly identical wind conditions. The same technique was successfully applied to these experiments with “Merrill’s Rich Suds,” a powder dispensed in pound packages. The detergent was scattered by hand along the upwind end of the pond so that the wind dispersed it evenly over the surface. It was necessary to apply soap$^6$ continu-

$^6$ Hereafter the terms “detergent” and “soap” are used interchangeably.
Figures 5 and 6. Two views of the downwind end of the pond taken at a wind-speed of 17 m/sec before and after the addition of detergent to the surface. The marker pole is graduated at one-foot intervals.
Figure 7. Ratios of the windspeed at elevations of 25 and 100 cm to that at 10 m as a function of the windspeed at 10 m.

Ously as the wind drove the soap slick away from the scatterers and as incipient wave formation occurred. At the highest windspeeds, circa 15 m/s, this procedure required the combined efforts of three people running at top speed. Figs. 5 and 6 show the pronounced difference in the appearance of the water surface with and without detergent.

One might expect that the addition of detergent would also affect the flow of air over the water. The effect would be towards reducing the turbulence near the surface, thus increasing the wind at lower levels with respect to that at higher levels. The observations in Fig. 7 should reveal this tendency if present, but the scatter is too large to permit any conclusions except that the change is certainly not large.

An indirect but perhaps more precise method for estimating the effect of the detergent on the wind profile can be based on the observed increase of the velocity ratios (Fig. 7) with a decrease in windspeed (hence stress). For example, according to Fig. 13, for $V = 12 \text{m/sec}$, the addition of detergent decreases the stress by an amount equal to a decrease in wind to 10.3 m/sec. The corresponding decrease in the velocity ratio in Fig. 7 is from 0.65 to 0.67, or 3%.
Figure 8. Detergent requirements as a function of windspeed for the 25 cm anemometer elevation. The data indicate the average rate at which detergent was dispensed in order to prevent the formation of waves on the surface.
Each time the soap technique was employed a record was kept of the total amount of soap consumed, and the results are shown in Fig. 8. The distribution of points is approximately linear and has an intercept at $V = 2 \text{ m/sec}$. Since the soap was used only sparingly to prevent wave formation, this suggests that no waves occur at this windspeed.

Wave Observations. Additional evidence to support the above view was provided by a series of 40 low-angle photographs of the water surface taken at one minute intervals on a morning when there was a very low intermittent breeze. The windspeed was recorded at 25 cm elevation and was averaged over the same intervals. The water retained its glassy-calm appearance up to $V = 110 \text{ cm/sec}$; incipient rippling in patches occurred from 110 to 200 cm/sec; but the surface became only uniformly covered with persistent wavelets at speeds above 200 cm/sec. Because of the large vertical gradient of the wind velocity at low elevations, no comparison is attempted between this value and those reported elsewhere in the literature; however, we can state that it falls between values reported by Jeffreys (1925) and those by Keulegan (1951) and Francis (1951).

Actually, close study of the surface indicated that it was at no time perfectly smooth under the least breath of wind; its appearance seemed to undergo a series of transitions with increasing windspeed. One point of similarity is remarkably consistent: at all winds one had the impression that, in contrast to the conventional concept of waves advancing before the wind, there were instead two sets of waves to the right and left of the wind direction. Moreover, the angle between the wind direction and the direction of wave travel appeared to depend on the windspeed, being initially about $75^\circ$ and decreasing to a minimum of about $15^\circ$ at the highest winds observed. During low winds, the intersections of corresponding wave phases traveled downwind at a much faster rate than those of the individual crests and, due to slight irregularities in wave length, they gave the surface a curious streaky appearance.

Current Measurements. By timing the drift rate of small flakes of paraffin wax, Keulegan (1951) made numerous measurements of the surface water velocities, in which the depth, $H$, and the viscosity, $\nu$, were varied separately for several windspeeds. His results are shown in dimensionless form by the curve in Fig. 9, where the windspeed was taken as the mean value in the laboratory channel. Because of dimensional considerations (see p. 271), the current measurements made throughout the present tests can be compared with those
obtained by Keulegan; these are shown in Fig. 9 by the cluster of points in the vicinity of $R = \frac{u_H}{v} = 10^6$. Since both series of measurements were made with and without soap, the close agreement between the points obtained here and the empirical curve obtained by Keulegan supports his conclusion that the magnitude of the surface current is unaffected by the presence of waves and is independent of depth at large Reynold's numbers.

THE EXPERIMENTAL RESULTS OF THE STRESS MEASUREMENTS

Setup Versus Windspeed. Measurements of setup and windspeed were made from November 1951 through March 1952 under a variety of weather conditions. From more than 300 hours of recordings, only those for which the wind direction was within 20° of the pond axis (a total of about 60 hours) were considered acceptable for analysis. All of the data for the 10 m wind elevation are shown in Fig. 10. The analogous representations for the two lower anemometer elevations are not given, but they are similar except for the scale of the windspeed. The points obtained when the surface was covered with soap tend to fall along a straight line through the origin, while those taken when waves were present initially follow the same line but diverge upwards as the wind increases.

The Scatter of the Data. In Fig. 11 we have plotted the recorded seiche amplitude (16% of actual amplitude) against windspeed, with and without soap. It will be noticed that with soap on the surface the

* The floats used in the present tests had a diameter of 1" and were cut from cork composition (0.050" thick) having almost neutral buoyancy.
Figure 10. Original data for setup as a function of windspeed at 10 m elevation. The smooth curves were drawn by inspection. Each point is a 20-minute time average for both wind and setup.

Figure 11. Recorded double-amplitude of the seiche as a function of windspeed at 10 m elevation. Each point represents the mean of two groups of 10 seiche cycles at the average windspeed over the same time interval.
seiche is reduced by a factor of three, approximately. In Fig. 10 it can be seen that the scatter of data is similarly reduced by the addition of soap. Furthermore, the seiche amplitude equals approximately the scatter of setup values for any fixed windspeed. Hence one might be inclined to ascribe the scatter to error in measuring setup as related to seiche activity. But this error was previously shown to be only of the order of 0.01 cm, and thus it cannot account for the observed scatter.

Another possibility is that the scatter may be related to the gustiness of the wind. For any given setup the scatter in windspeeds in Fig. 10 is of the order of 10% of the mean speed. This ratio is by no means excessive. From this point of view the seiche activity can be considered a result of gustiness. The reduction in seiche amplitude produced by soap implies that the addition of soap to the surface significantly reduces the gustiness of the wind in the lower layers. Further study is indicated.

**The Transient Response of the Pond.** Because the theory of surface slope assumes that a steady-state must be established, the transient response of the pond to changing wind force was examined in some detail. Haurwitz (1951), in a theoretical analysis of a rectangular basin, concluded that equilibrium will be obtained in a time of the order of one seiche period, although oscillation about the equilibrium level may persist, depending on the internal and bottom friction.

The passage of a cold front during the night of March 1, 1952 permitted an experimental examination of the transient response. Within about three minutes the wind changed direction abruptly from south to west and quadrupled in intensity. Shortly thereafter there was a sharp decrease in windspeed, followed by a subsequent oscillation of lesser magnitude. At the highest windspeed (nearly 20 m/sec) the close spacing of the mile-pips on the wind record permitted an accurate determination of the rate of change of windspeed. In Fig. 12 the observed setup is compared to the steady-state value obtained from the curve in Fig. 14 by inserting the appropriate observed windspeed. Even in this extreme case the amplitudes of the two curves correspond closely, and the phase lag is never more than two seiche periods (about 4 min.). Hence, the response-time was much more rapid than the 20 minute averages to which the data were customarily reduced.

**The Bottom Stress.** Nearly 100 hours of bottom-stress recordings were made over a range of windspeeds up to 10 m/sec (at 10 m elevation). The recorder pen always drew a nearly straight line on the chart, hence no conclusive variation of bottom stress with windspeed could be established; it can be stated only that within the accuracy of
Figure 12. The transient response of setup to changing windspeed. The observed setup was averaged over the time interval corresponding to each mile-ship of the wind record. The computed setup was taken from Fig. 14 for a wind average over the same time interval. The dotted lines indicate when the computed setup was less than the observed setup.

the instrumentation (0.1 dyne/cm²) the stress was negligible. This conclusion is supported by the observations of divers with underwater swimming equipment. They reported that sediment clouds stirred up from the bottom remained sensibly stationary for several hours and that no motion of the long sparse grasses growing upwards from the bottom could be detected. In the course of a few days, sufficient marine growth and sediment would accumulate on the surface of the sensing disk to unbalance it, and the record would gradually drift off to one side of the initial zero. With the long-period filter switched out of the recording circuit, oscillations in the record had a period roughly the same as the longer surface waves (1–2 sec.) and amplitudes of the order of 0.3 dynes/cm² together with barely perceptible modulations of seiche period (112 sec.). It is hoped that the experiments can be continued later in the year with the advent of higher winds, but meanwhile we conclude tentatively that the bulk of circulation within the pond is confined to a relatively thin layer near the surface.
INTERPRETATION OF THE RESULTS

An Empirical Law for Setup. Keulegan (1951) proposed that the total setup observed in the laboratory channel was actually the combination of two effects: first, the frictional drag on the water surface, which is present at all windspeeds; second, an additional effect attributed to wave development, which commences only after the wind has reached a certain value, $V_e$, which he defined as the "formula" velocity. He also found that the second effect could be indefinitely postponed in terms of increasing windspeed either by adding detergent to the surface or by adding sugar to the water to increase its viscosity. In addition, the ratio of surface current to windspeed, $u_s/V$, at high Reynolds numbers is observed to be constant and unaffected by the addition of soap. It appears, therefore, that such a partition of the setup has a real physical basis and is not due to some special action of the soap in "greasing" the water surface.

Keulegan's results can be represented by the following equations:

$$S_1 = aV^2, \quad S_2 = b(V - V_e)^2,$$

where $S_1$ and $S_2$ are the partial setups due to friction and waves, respectively, and

$$S = S_1 \text{ for } V \leq V_e; \quad S = S_1 + S_2 \text{ for } V > V_e. \quad (16a, b)$$

With soap on the surface, (17a) holds for all windspeeds. From dimensional considerations he proposed that

$$a = \frac{AL}{gH}, \quad b = \frac{BL}{gH} \sqrt{\frac{H}{L}},$$

where $A$ and $B$ are dimensionless constants for any given set of wind measurements. However, a question is raised here concerning the validity of the factor $\sqrt{H/L}$ in (18b). In view of equation (4), this would imply a secondary dependence of wind stress on depth. Moreover, Keulegan's experimental determination of the dependency of $S_2$ on $H$ is not conclusive, and the secondary dependence on $L$ was arbitrarily assumed for dimensional consistency. We propose, instead,

$$a = \frac{L}{gH} \alpha^2, \quad b = \frac{L}{gH} \beta^2. \quad (19a, b)$$

In order to determine whether or not the empirical results obtained in the present experiments agree with the form proposed by Keulegan, it was necessary to evaluate the quantities $\alpha$, $\beta$ and $V_e$ for each anemometer elevation. The coefficient $\alpha$ was determined from the slope of
the straight line giving the best fit to the points obtained when the surface was covered by soap, using equations (16a) and (19a). The data for total setup were then averaged over convenient class intervals of windspeed, and a mean value for \( S_2 \) for each class interval was obtained by subtracting the ordinate beneath the same straight line at the mean square windspeed within the interval. In Fig. 13, the square root of \( S_2 \) is plotted against windspeed for each anemometer elevation. The straight lines through the points were drawn to give a best fit to the points, weighting each point according to the number of data within the interval. The intercepts at \( S_2 = 0 \) were construed to be the most reasonable values of \( V_e \), and the slope of the lines gives the coefficient \( \beta \) by virtue of (16b) and (19b). The coefficients \( \alpha \) and \( \beta \) and the formula velocity \( V_e \) determined by the above procedure for each anemometer elevation are given in Table III together with the corresponding values obtained by Keulegan. Using the values of the three parameters given in Table III, the curves of Figs. 14, 15 and 16 were computed. The agreement with the observed data is satisfactory.

It will be noticed that the values obtained by Keulegan in Table III fall between those obtained for the model-yacht pond at the 25 and 100 cm anemometer elevations, being nearer the latter values. This
"equivalent height" is considerably higher than the actual height (about 10 cm) at which the wind was measured in the laboratory channel. This result is in contrast to Keulegan's conclusion that the walls and ceiling of his channel imposed no significant restriction on the air velocities.

TABLE III. COMPARISON OF THREE EMPIRICAL PARAMETERS FOR THE MODEL-YACHT POND AND LABORATORY CHANNEL

<table>
<thead>
<tr>
<th>Anemometer elevation (cm)</th>
<th>( \alpha \times 10^3 )</th>
<th>( \beta \times 10^3 )</th>
<th>( V_c ) (m/sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>2.1</td>
<td>4.6</td>
<td>3.1</td>
</tr>
<tr>
<td>100</td>
<td>1.7</td>
<td>3.2</td>
<td>4.0</td>
</tr>
<tr>
<td>1000</td>
<td>1.1</td>
<td>1.5</td>
<td>5.6</td>
</tr>
<tr>
<td>Keulegan</td>
<td>1.8</td>
<td>3.7*</td>
<td>3.9*</td>
</tr>
</tbody>
</table>

* Average for five water depths

The Relation Between Wind and Surface Current. If a Reynolds number is defined as \( R = u_s H / \nu \), then the curve in Fig. 9 can be approximated by the expressions

\[
\frac{u_s}{V} = K_1 \sqrt{\frac{u_s H}{\nu}} \quad \text{for} \quad R \ll 10^3
\]  

(20)

and

\[
\frac{u_s}{V} = K_2 \quad \text{for} \quad R \gg 10^3,
\]  

(21)

where \( K_1 \) and \( K_2 \) are constants which depend only on how the wind is measured. The influence of the viscosity in (20) led Keulegan to suppose that the entire distribution of velocity within the water might be laminar at low Reynolds numbers. Combining (20) and (8), the theoretical equation for surface slope under laminar conditions, we find that

\[
\frac{dh}{dx} = \frac{S}{L} = 6K_1^2 \cdot \frac{V^2}{gH}
\]  

(22)

is the condition to be satisfied for \( R \ll 10^3 \). But (22) has the same form as (16a); moreover, Keulegan was able to demonstrate that \( 6K_1^2 = \alpha^2 \) and thus verify his assumption. For larger bodies of water, such as that considered here, the Reynolds number will always be much greater than \( 10^3 \) for any observable surface slopes, and consequently the flow is turbulent. One would therefore expect the results of the present current measurements to obey the law given by (21). While the scatter of individual measurements has a standard deviation of 8%, there is no significant variation in the ratio \( u_s/V \) with a fourfold increase in Reynolds number.
Figure 14. Setup as a function of the square of the windspeed at 10 m elevation. The curves were computed from equations (21) and (24).

Figure 15. Setup as a function of the square of the windspeed at 100 cm elevation.

Figure 16. Setup as a function of the square of the windspeed at 25 cm elevation.
In attempting to relate the measurements in the pond to those in the laboratory channel, it was found that a systematic relationship exists between the parameters listed in Table III and the constant \( K_2 \) in (21). The four parameters \( \alpha, \beta, 1/V_c \) and \( K_2 \) as obtained under natural conditions for the two lower anemometer elevations and the corresponding values obtained under laboratory conditions have nearly the same ratios (Table IV).

**TABLE IV. The Ratios (Model-yacht Pond to Laboratory Channel) of Four Parameters**

<table>
<thead>
<tr>
<th>Anemometer elevation (cm)</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( 1/V_c )</th>
<th>( K_2 = u_s/V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>1.19</td>
<td>1.24</td>
<td>1.19</td>
<td>1.23</td>
</tr>
<tr>
<td>100</td>
<td>0.93</td>
<td>0.87</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>1000</td>
<td>0.62</td>
<td>0.40</td>
<td>0.66</td>
<td>0.62</td>
</tr>
</tbody>
</table>

The agreement for the highest anemometer is poor, which is not surprising in view of the fact that it was mounted above a building nearby rather than over the pond. It is significant that the discrepancy is greatest for \( \beta \); i.e. when \( V > V_c \). The values of \( u_s/V \) in Fig. 9 are for the 25 cm elevation and were divided by the factor 1.23.

**The Formula Velocity.** No obvious physical changes in the appearance of the water surface could be noted at the windspeed \( V_c \). Waves appeared at lower windspeeds and grew in a regular manner as the wind increased through this range. It is instructive, however, to consider the stress, \( \tau'_s \), and the surface current, \( u'_s \), which occur at the windspeed \( V_c \). Since the Reynolds number is greater than \( 10^3 \), it follows from (21) that

\[
\frac{(u'_s)_P}{(u'_s)_L} = \frac{(V_c)_P}{(V_c)_L} \cdot \frac{(K_2)_P}{(K_2)_L},
\]

where \( P \) and \( L \) refer to pond and laboratory. In view of the ratios in Table IV, this means that the surface currents in the pond and laboratory are equal at the windspeed \( V_c \). Empirically, the numerical value for \( u'_s \) is approximately 12 cm/sec. It is concluded tentatively that the stress \( \tau'_s \) is associated with a specific value of the surface current, independent of the scale of the experiment.

**The Wind Stress with a Smooth Water Surface.** Consider now the case when the water surface is smooth, defined by \( V < V_c \). By combining equations (4) and (16a), we obtain a relation between the boundary stresses and the windspeed at a specific elevation

\[
\tau_s = \rho \alpha^2 V^2.
\]
Taylor (1916) has proposed that the surface stress can be represented in terms of a resistance coefficient, $\gamma^2$, defined by

$$\tau_s = \gamma^2 \rho_a V^2,$$

(25)

where $\rho_a$ is the air density. Comparing (24) and (25), the form of the latter is verified provided that

$$\gamma^2 = \frac{\rho}{\rho_a} \alpha^2.$$

(26)

As an example we consider the two extreme anemometer elevations, 25 cm and 1000 cm. Taking $\rho_a = 1.2 \times 10^{-3}$ and $\alpha$ from Table III, the corresponding values of the resistance coefficient are $\gamma^2 = 0.0037$ and 0.0011 respectively. Although the above figures apply only to smooth water conditions, their range encompasses all values reported by 12 different investigators, as compiled by Francis (1951). It is suggested that the variability in the height at which the wind is measured may be a contributory factor in the present ambiguity regarding $\gamma^2$ (Montgomery, 1951).

**Wind Stress with a Rough Water Surface.** If, by analogy to the smooth water case, we combine equations (4) and (16), we obtain a relation for the total stress when the surface is rough; defined by $(V > V_c)$,

$$\tau_s = \rho \alpha^2 V^2 + \rho \beta^2 (V - V_c)^2.$$

(27)

The coefficients $\alpha$ and $\beta$ depend only on where the wind is measured. To a first approximation, the above expression for the wind stress appears to be independent of fetch and hence independent of the size of waves which can develop. The form of (27) is incompatible with an abrupt increase in the stress at a "critical windspeed," as proposed by Munk (1947).

If it is recalled that the addition of soap to the surface at high windspeed produces no corresponding change in the surface current, it is evident that the resulting change in setup can be due only to a change in the character of the circulation beneath the surface. Qualitative support for this view is provided by observations of drifting objects. When the surface was covered with soap, a sharp gradient of velocity with depth was apparent. Small particles in the uppermost layer (circa 1 cm) traveled at roughly twice the speed of those a centimeter or so deeper, and there was little change in speed at all greater depths within which the trajectories could be followed. The motion was essentially steady and horizontal. In contrast, when the surface was rough, bits of paper which became waterlogged and sank rather
slowly moved in sluggish vertical circles of about one foot in diameter in the direction of the wind, with a mean horizontal velocity roughly independent of depth. Since the virtual elimination of surface waves is the most obvious manifestation of the change in setup which takes place when soap is applied, it is suggested that the waves themselves, under the action of the wind, may be an effective agent in increasing the downward transport of horizontal momentum through the surface.

The Stress Due to Rainfall. If the original data in Fig. 10 are examined, it will be noticed that a group of five points giving the total setup in the vicinity of the windspeed $V = 12 \text{ m/s}$ lies consistently above all others. In attempting to resolve this anomaly in the records, it was discovered that this series of data was obtained in the early morning hours of March 12, 1952 during a period of heavy rain and that no rain had fallen throughout the remainder of the experiments. The rain gauge at Lindbergh Field, three miles away, recorded an average rate of rainfall of 0.25 inch/hour during the period in question, and it is thus possible to make an estimate of the order of magnitude of the stress due to the rain.

First it is necessary to estimate the reduction in the horizontal component of the velocity of a falling raindrop as it passes through the layer of slowly moving air near the surface. To simplify the problem we consider the coordinate axes to be moving at the windspeed, $V_0$, at an arbitrary elevation, $z_0$, with the $z$-axis positive downward. We assume that initially a raindrop has a horizontal component of velocity equal to that of the wind; therefore let $u$ and $V$ be the subsequent relative velocities of the drop and wind respectively, referred to the moving axes. We are interested in the final velocity, $u_0$, of the drop at a time, $t$, equal to that required for the drop to fall at terminal velocity, $w$, through the distance, $z_0$. The equation of motion is

$$\frac{du}{dt} = F,$$  \hfill (28)

and we take:

$$F = \delta (V - u)^2,$$  \hfill (29)
$$V = Kz,$$  \hfill (30)
$$z = wt,$$  \hfill (31)

where $K$ is a constant. The assumed linear increase in relative windspeed permits a closed solution and gives a very conservative result. Combining these four equations,

$$\frac{du}{dt} = \frac{\delta}{m} (Kwt - u)^2,$$  \hfill (32)
and, after integrating between the limits \( u = 0 \) at \( t = 0 \) and \( u = u_0 \) at \( t = z_0/w \), the solution is

\[
\frac{V_0 - u_0}{c} = \tan h \left( c \cdot \frac{\delta}{m} \cdot \frac{z_0}{w} \right),
\]

where

\[
c = \frac{mKw}{\delta}, \quad V_0 = Kz_0.
\]

Expanding (33) in Taylor’s series, we obtain the relation

\[
\frac{u_0}{V_0} = \frac{1}{3} \frac{\delta}{m} \frac{z_0}{w} - \frac{2}{5} \frac{V_0^2}{m} \left( \frac{\delta}{m} \cdot \frac{z_0}{w} \right)^2 + \ldots,
\]

which gives the ratio of the final horizontal velocity of the drop to the velocity at \( z_0 \). Assuming a spherical raindrop of diameter \( D \) and density \( \rho \), the ratio \( \delta/m \) is

\[
\frac{\delta}{m} = \frac{1}{2} \frac{\rho a C_d \left( \frac{\pi D^2}{4} \right)}{\frac{1}{\pi \rho D^3}} = \frac{3}{4} \frac{\rho a C_d}{\rho D},
\]

where \( C_d \) is the drag coefficient. According to Spilhaus (1948), reasonable values for these variables with continuous heavy rain are \( C_d = 0.2 \) and \( D = 0.6 \) cm, and the appropriate terminal velocity is \( w = 10^3 \) cm/s. Again taking \( \rho a = 1.2 \times 10^{-3} \), we find that

\[
\frac{\delta}{m} = 3 \times 10^{-4}.
\]

Combining (34) and (36) and taking only the first term of the series,

\[
\frac{u_0}{V_0} = 10^{-7} V_0 z_0 \ldots \ldots
\]

Assuming now the arbitrary values \( V_0 = 12 \) m/sec at 10 m elevation, we see that \( u_0 \) will be only 15% of \( V_0 \), or, in terms of a raindrop initially traveling with the wind and falling from a height of 10 m through a region of linearly decreasing wind, its horizontal velocity at the ground would still be about 85% of its initial value. Because the profile assumed is conservative, the change in \( u \) will be neglected in the following discussion.

We consider now the average horizontal stress, \( \tau_r \), acting at the water surface due to the horizontal momentum of the raindrops. We write
where \( u \) is now the horizontal velocity of the drops with respect to the surface. With steady rain, we neglect the term \( d(mu)/dt \) and note that \( dm/dt \) is just the rate of rainfall. Assuming, as before, that \( u = V = 12 \text{ m/s at 10 m elevation}, \) we have

\[
\tau_r = u \frac{dm}{dt} = \frac{1.2 \times 10^3 \times 0.6}{3600} = 0.23 \text{ dynes.} \tag{39}
\]

The corresponding setup as computed from equation (4) is \( S_r = 0.035 \text{ cm} \). This, when compared with the average of the five points taken from Fig. 10, which is roughly 0.04 cm, constitutes about 20% of the total effect. It thus appears as if heavy rainfall can considerably augment the stress due to the wind.

**REFERENCES**

Corkan, R. H.

Defant, A.

Ekman, V. W.

Francis, J. R. D.

Haurwitz, B.

Hela, I.

Hellstrom, B.

Iberall, A. S.

Jeffreys, H.

Keulegan, G. H.
MONTGOMERY, R. B.
1951. The present position of the study of exchange across the ocean-atmosphere interface. WHOI Contribution 576 (Unpublished).

MUNK, W. H.

Palmén, E. und E. Laurila

Spilhaus, A. F.

Taylor, G. I.

Van Dorn, W. G.