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ON THE STABILITY OF GAS BUBBLES IN SEA WATER

BY

JEFFRIES WYMAN, JR., ² P. F. SCHOLANDER,³ G. A. EDWARDS⁴ AND L. IRVING⁵

ABSTRACT

Laboratory experiments have been performed to determine the rate of solution and the change in composition of air bubbles in stirred sea water as a function of depth (0–200 m) and temperature (1–27°C). At each depth the shrinking of the diameter is nearly linear. This rate of solution increases with increasing pressure up to a limiting value which is almost reached at a depth of 20 m. The rate of disappearance of a bubble is only imperceptibly influenced by temperature. When a bubble of air dissolves at a given depth it becomes richer in nitrogen until a final constant composition is reached. This final nitrogen percentage increases with increasing depth. Equations have been developed, based on the theory of diffusion, which satisfactorily account for the observed facts.

INTRODUCTION

There are many biological processes known where free gases are produced under water: gas bubble formation as a result of decomposition of organic material in lakes and swamps and in the sea, and gas bubbles from water plants as a result of photosynthesis. In many cases gases are carried by plants and animals either externally or internally or both. Gas bubbles also appear in the blood of higher organisms as a result of sudden decompression, giving rise to what is commonly called “bends.” The mechanisms of gas bubble formation have been admirably treated by E. N. Harvey and coworkers (1944). The present study deals mainly with the fate of a bubble after it has been formed.

When gas is produced in the form of free bubbles, the partial pressures of the components within the bubbles are likely to be different from the partial pressures of the same components in the water, and hence gas exchange takes place. In sea water the partial pressures of the dissolved atmospheric gases are usually close to those in the atmosphere. This holds true very exactly for nitrogen at all depths.

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(Rakestraw and Emmel, 1938; Hamm and Thompson, 1941). Any bubble in water will be at a total pressure of one atmosphere plus the prevailing hydrostatic pressure; hence it will finally dissolve. Since different gases have different solubility and diffusion characteristics, the composition of a bubble will change during the process. We have studied the rate of disappearance and the change in composition of dissolving gas bubbles at different depths by introducing bubbles into stirred sea water under pressures corresponding to depths as great as 200 m. Both disappearance rates and changes in composition have been found to follow simple physical laws that can presumably be applied also to other gases in less simple systems; the analysis we have given should be useful as a basis of calculation and interpretation to other investigators concerned with problems of gas exchange, respiration, and the like.

METHODS

Both low and high pressure systems were used to expose bubbles of gas to stirred water at different hydrostatic pressures and temperatures. The low pressure system is shown in Fig. 1B. A 300-cc glass bulb furnished with a vaccine stopper at one end and a piece of pressure tubing at the other is suspended on two bearing posts in a waterbath. The pressure tubing leads to a mercury leveling bulb which is suspended from a swivel so that the bulb can be elevated to any desired height. Through the injection port an air bubble can be introduced into the 300-cc bulb completely filled with water; the bulb is rotated by a simple drive so as to keep the bubble close to the center. The diameter of the bubble is measured through an ocular micrometer after stopping the rotation. The corresponding volume is found from an empirical calibration involving introduction of bubbles of known volume from a capillary syringe analyzer furnished with a hypodermic needle (Scholander and Roughton, 1942). The bubble is then withdrawn and analyzed with the same instrument. Pressures up to 20 m can be handled by this system.

For pressures corresponding to those at depths of 20 to 200 m, a pressure cylinder was employed which was fitted with a window and pressure gauge (see Fig. 1A). A large centrifuge tube containing about 300 cc of sea water is wedged into the cylinder. This tube is closed with a rubber stopper containing two holes, one for introduction of bubbles, the other connected with a syringe. The syringe ensures an anaerobic, sensitive pressure transmission between the nitrogen gas in the cylinder and the water containing the bubble. The desired pressure is furnished from a nitrogen cylinder. The instrument is rotated horizontally in a waterbath at a speed sufficient to keep the bubble
near the center of the water column. After stopping the rotation, the diameter of the bubble is measured through the glass window by means of an ocular micrometer, using an empirical calibration as described for the low pressure system. The sea water used in the experiments was filtered and equilibrated with air at the desired waterbath temperature.

It should be pointed out that the amount of water used in both types of apparatus is so large that the solution of the bubble does not increase the concentration of dissolved gas enough to change the diffusion gradient by as much as one per cent.

RATE OF SOLUTION OF AIR BUBBLES IN RELATION TO PRESSURE

The first experiments were made to determine the rate of solution of a single bubble of air as a function of pressure. The results obtained are summarized as follows:

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Figure 1. Methods for studying disappearance rate and changes in composition of bubbles under various hydrostatic pressures. A. The high pressure system. B. The low pressure system.
Figure 2. Rate of solution of air bubbles in sea water in relation to hydrostatic pressure (depth). Sea water was saturated with air at normal atmospheric pressure.

(a) As a bubble dissolves there is, at any given pressure, a nearly linear decrease in its diameter with time until it is too small to be seen. Actually the initial rate of shrinkage is always somewhat greater than the subsequent rate (Fig. 2).
Figure 3. Minutes required to diminish bubble diameter 1 mm at various depths. Sea water was saturated with air at normal atmospheric pressure.

(b) The rate of solution increases with pressure but approaches an upper limiting value at high pressures. In Fig. 3 the average rate of disappearance of the bubbles is represented in relation to pressure.

(c) The rate of disappearance of a bubble does not vary significantly with temperature in the range from 1 to 27° C. Table I shows the figures obtained at a total pressure of 2 atm. (depth, 10 m).

<table>
<thead>
<tr>
<th>Temp., °C.</th>
<th>Decrease of Bubble Diameter (mm/min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.065–.068 (4 observations)</td>
</tr>
<tr>
<td>15</td>
<td>.065–.069 (7 observations)</td>
</tr>
<tr>
<td>27</td>
<td>.068–.069 (2 observations)</td>
</tr>
</tbody>
</table>

These results indicate that below 10–15 m the lifetime of an air bubble of given size should not vary much with depth and should be nearly proportional to the diameter. A bubble 2 mm in diameter would last about 20 minutes.

Of course the total pressure includes the pressure of one atmosphere which prevails at the surface.
This very simple behavior of a bubble can be accounted for, as a first approximation, by a correspondingly simple treatment based on the principles of diffusion. Due to the relative motion of the water and the bubble, we assume that the composition of the water in respect to dissolved gas is everywhere uniform except in a thin shell surrounding the bubble and that the gas in the bubble is uniform right up to the gas-water interface. The inner layer of water in the shell will be in equilibrium with the gas, and the whole of the diffusion gradient which causes the gas to pass from the bubble to the water will be supported by the shell. We assume further that this gradient is uniform throughout the shell. Also, we ignore the fact that the gas actually consists of oxygen and nitrogen, and we treat the situation as if only one kind of molecule were present with a single diffusion constant.  

We introduce \( n \) to denote the number of moles of gas in the bubble; \( p \), the pressure in the bubble; \( r \), its radius; \( p_o \), the partial pressure of the gas in the water everywhere outside the thin shell; \( t \), the time; \( R \), the gas constant; and \( T \), the absolute temperature. Then, from the gas law:

\[
n = \frac{1}{RT} \frac{4}{3} \pi r^3 p .
\]

Also, from Fick's first law, which relates the amount of material which diffuses through unit area to the product of the concentration gradient and the diffusion constant:

\[
\frac{dn}{dt} = - \delta 4 \pi r^2 (p - p_o).
\]

In this expression \( \delta \) is a proportionality constant which depends on: \( d \), the thickness of the shell; on \( \Delta \), the diffusion constant of the gas in the water; and on \( \alpha \), the solubility of the gas in the water, in accordance with the relation

\[
\delta = \frac{\Delta \alpha}{d};
\]

\( \alpha \) is involved because of the necessity of converting the expression \( (p - p_o) \) on the right of (2) into a concentration difference. By

\[7\] The uniformity of the gradient is a necessary consequence of Fick's second law of diffusion in any situation involving a steady state. In the present case the pressure of the gas within the bubble, which is the same as the hydrostatic pressure, remains constant, and the concentration of gas in the surrounding water is likewise constant. Therefore there must be a steady state as regards the concentrations of gas on the two sides of the diffusion layer except for changes of composition.
differentiating (1) with respect to time and combining the result with (2) we obtain the following expression for the rate of change of the radius of the bubble with time:

\[ \frac{dr}{dt} = -RT\delta \frac{(p - p_0)}{p} . \] (3)

The underlying basis for this result is that the pressure in the bubble depends on its volume whereas the amount of gas which escapes by diffusion depends on its surface.

Equation (3) accords with the results summarized above.

(a) It shows that for any value of \( p \), corresponding to any given depth,\(^8\) the value of \( \frac{dr}{dt} \) should be constant. That is, the radius or diameter of the bubble should decrease uniformly with time.

(b) It describes the effect of pressure on the rate of disappearance of the bubble, at least as a first approximation. For example, at the surface \( p = p_o = 1 \) atm., and \( \frac{dr}{dt} = 0 \), as we know it must. At a depth of 10 m, \( p \cong 2 \) atm. At this depth

\[ \frac{dr}{dt} = -\frac{RT\delta}{2} . \]

At 200 m, where \( p = 21 \) atm.,

\[ \frac{dr}{dt} = -\frac{RT\delta}{21} , \]

or about twice this. The limiting value of \( \frac{dr}{dt} \) for very great depths is of course \(-RT\delta\), or twice that at 10 m. This is close to what is observed (see Fig. 2 and Table II).

(c) It accounts for the fact that the rate of solution of a bubble is essentially independent of temperature. It will be seen that this rate is proportional to the absolute temperature and to the product of the

\(^8\) It should be emphasized that \( p \) is the total pressure of the gas in the bubble: i.e., 1 atm. at the surface. We take no account of the slight additional pressure due to surface tension. This is equal to \( \frac{2\gamma}{r} \), in which \( \gamma \) is surface tension (for water, \( \gamma = 72.8 \) dynes/cm at 20\(^o\)). For the smallest visible drop, of diameter say 0.1 mm, this will amount to 30,000 dynes/cm\(^2\) (= 0.03 atm. = 30 mm water), which is too small to be of consequence. (Below 0.1 mm the pressure due to surface tension mounts rapidly and soon becomes the dominant factor.)
diffusion constant $\Delta$ times the solubility $\alpha$ of the gas. Of the latter quantities the first ($\Delta$) increases while the second ($\alpha$) decreases with temperature, so that their product may be expected to remain roughly constant. No data appear to be available on the diffusivities of oxygen and nitrogen as a function of temperature, but the figures given in International Critical Tables show that from 10 to $21^\circ$ the diffusivity of hydrogen in water increases from $4.3$ to $5.2$ cm$^2$sec$^{-1}$. Over the same range the solubility coefficient of nitrogen decreases from $0.0186$ to $0.0152$. If the proportional change in the diffusivity of nitrogen were the same as that of hydrogen, the product $\Delta \alpha$ would change by only about one per cent between 10 and $21^\circ$ and could be regarded as essentially independent of temperature. As a first approximation, the same conclusion may be applied to air. Since the range of absolute temperature covered by the experiments is small, we should therefore expect the rate of solution of the bubble to be nearly independent of temperature, as it is.

A more quantitative indication of the essential correctness of this interpretation is afforded by the constancy of the values of $\delta$ calculated by equation (3) from the experimental data, taking $RT$ as $2.4 \times 10^4$ cc atm./mole, which corresponds to $20^\circ$. The constancy of the values of $\delta$ so calculated is shown in Table II, where the values listed for $\frac{dr}{dt}$ are those obtained by averaging the results of the various experiments, taking the final constant value of the rate in each case. It is seen that the values of $\delta$ are indeed fairly constant but that they show a real decrease with $p$. In all probability this is to be correlated with the fact that the bubble consists of two gases, oxygen and nitrogen, which, under the conditions of the experiment, leave the bubble at different rates and that the percentage of nitrogen in the bubble during the majority of its life is higher at the higher pressures. In the next sec-

<table>
<thead>
<tr>
<th>Meters</th>
<th>$\frac{dr}{dt}$ (cm/sec)</th>
<th>$p - p_0$</th>
<th>$\delta$ $\left(\frac{\text{moles}}{\text{cm}^2 \text{sec atm}}\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$4.0 \times 10^{-5}$</td>
<td>$1/3$</td>
<td>$5.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>10</td>
<td>$5.4 \times 10^{-5}$</td>
<td>$1/2$</td>
<td>$4.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>20</td>
<td>$7.6 \times 10^{-5}$</td>
<td>$2/3$</td>
<td>$4.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>50</td>
<td>$8.3 \times 10^{-5}$</td>
<td>$5/6$</td>
<td>$4.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>100</td>
<td>$8.8 \times 10^{-5}$</td>
<td>$10/11$</td>
<td>$4.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>200</td>
<td>$9.2 \times 10^{-5}$</td>
<td>$20/21$</td>
<td>$4.0 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
tion we shall deal with the question of the composition of the bubble and its effect on the rate of solution.

Added confirmation of the simple picture given above is provided by observations on the rate of disappearance of an air bubble at 2 atm. (10 m) into degassed sea water. The observed rate is \( \frac{dr}{dt} = 0.06 \) mm/min. This is the same, within the limits of error, as the observed value for 100 or 200 m (0.055 mm/min.). This is as it should be, for in the case of degassed water, \( p_o = 0 \) and (3) reduces to

\[
\frac{dr}{dt} = -RT\delta,
\]

which is the same as the limiting form approached by the equation for great depths \((p \to \infty)\) when \( p_o = 1 \).

Before we conclude this section it is important to point out that the results on the rate of disappearance of an air bubble in water are in general agreement with much cruder results on the rate of accumulation of gas in a cavity produced by suddenly and forcibly withdrawing and briefly holding back the piston of a syringe while the nozzle is kept closed. The amount of gas which would be expected to enter the cavity produced under these conditions may be roughly estimated by means of equation (3). For this purpose we set \( p = 0 \) and \( p_o = 1 \). Then, if we denote the surface area of the cavity by \( A \), the number of moles of gas which enter it in time \( t \) should be given by

\[
\int_0^t \frac{dn}{dt} dt = A\delta p_o t. \tag{4}
\]

The volume which these molecules would occupy at a pressure of 1 atm., corresponding to the condition when the piston of the syringe is released, is

\[
RTA\delta p_o t. \tag{4.1}
\]

On the basis of Table II, \( RT\delta \) may be taken as \( 10^{-4} \) cm/sec. Several experiments were made to test this prediction under conditions for which \( A \) was estimated to be in the neighborhood of 16 cm\(^2\), though this figure was probably too low due to the irregularity of the cavity associated with the splashing of the liquid. From the figures just given, the calculated volume of gas which should have entered the cavity per second turns out to be approximately \( 1.6 \times 10^{-3} \) cm\(^3\). The observed amount is about twice this, but in view of the uncertainties involved in the calculations, the agreement may be regarded as satis-
factory and may be taken as evidence in support of the fundamental correctness of the interpretation. These results on the accumulation of gases in a cavity by diffusion from the surrounding medium are of interest in connection with various respiratory problems.

**CHANGES IN COMPOSITION ACCOMPANYING SOLUTION OF GAS BUBBLES**

When the gas obtained from freshly formed cavities is analyzed it contains significantly more oxygen than the 21% characteristic of air. In fact, analyses made as soon as possible after formation of a cavity yield values up to 35% oxygen. Correspondingly it is found that as bubbles of air introduced into water diminish in size under excess pressure they become poorer in oxygen and richer in nitrogen. These two observations really represent the same phenomenon, namely the different rates of passage of oxygen and nitrogen across the diffusion layer at the surface of the bubble (cavity). As we have already suggested, certain small discrepancies between the observations and theory encountered in the preceding section are probably attributable to this phenomenon. In order to explore this subject, experiments were carried out to determine the changes in composition accompanying the solution of gas bubbles in water, and, in particular, the effect of pressure on the phenomenon.

The bubbles, consisting either of air or of an arbitrary mixture of oxygen and nitrogen, were placed in either one of the pressure chambers previously described and were allowed to dissolve for a given time before being withdrawn for gas analysis. The initial size of the bubble used varied somewhat, but it was in the neighborhood of 3.5–4 mm in diameter. The results are shown in Fig. 4, which illustrates quantitatively the way in which the bubbles always become richer in nitrogen as they diminish in size. By the time they are disappearing they ap-

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*In accordance with considerations to be developed presently we should expect the amount of oxygen in a freshly formed cavity to be half that of nitrogen; in other words, the gas should be 33 1/3% oxygen. This follows from the same line of reasoning involved in equations (4) and (4.1). We should expect the ratio of oxygen to nitrogen in the cavity, before any gas has had a chance to escape, to be given by

\[
\frac{\delta_1}{\delta_2} \frac{p_{1o}}{p_{2o}} = 2 \times \frac{0.2}{0.8} = \frac{1}{2}.
\]

(Subscripts 1 and 2 refer to oxygen and nitrogen respectively.) The value 2 for the ratio \(\frac{\delta_1}{\delta_2}\) is based on considerations given later in this section.*
Figure 4. Change in composition of bubbles as they dissolve at different pressures (depths) in sea water saturated with air at normal atmospheric pressure. The initial diameter of the bubbles studied varied from approximately 3.4 to 4 mm.

It appears to have attained a nearly constant composition. This composition is the richer in nitrogen the higher the pressure.

The physical explanation of the phenomenon is simple. It rests on the fact that the diffusion constants of oxygen and nitrogen are nearly the same but that the solubility of oxygen is about twice that of nitrogen. Let us consider the case of an air bubble suspended in

\[ \text{SURFACE} \]

\[ \text{PERCENTAGE OXYGEN} \]

\[ \text{MINUTES} \]

\[ \text{30} \]

\[ \text{20} \]

\[ \text{10} \]

\[ \text{5 M} \]

\[ \text{10 M} \]

\[ \text{50 M} \]

\[ \text{100 M} \]

\[ \text{200 M} \]

\[ 0 \]

\[ 10 \]

\[ 20 \]

\[ 30 \]

\[ 0 \]

\[ 10 \]

\[ 20 \]

\[ 30 \]

At 19° the diffusivity of nitrogen in dilute aqueous solution is close to 1.9 and at 18° that of oxygen is also close to 1.9 cm² sec⁻¹ (International Critical Tables). The solubility coefficient of N₂ is .0155 and that of O₂ is .0310 in H₂O. These figures refer to volumes of gas (measured at 0 and 1 atm.) dissolved by 1 volume of water when the partial pressure of the gas is 1 atm.
water saturated with air at atmospheric pressure. If the pressure in
the bubble is one atmosphere, as at the surface, no gas will pass into the
liquid phase, for the system is at equilibrium (see Fig. 2). There will
be no concentration gradient anywhere to produce diffusion. Howe-
ver, if the pressure is raised, the concentration of oxygen at the air-
water interface, and consequently the concentration gradient across the
diffusion layer, will increase more in proportion to the amount of gas
in the bubble for oxygen than for nitrogen, owing to the greater solu-
bility of oxygen. Consequently, since the diffusion constants of the
two gases are the same, the percentage loss of oxygen from the bubble
will exceed that of nitrogen and the bubble will become richer in
nitrogen.

It is worth-while to formulate the situation mathematically in terms
of our simplified hypothesis. On the whole the result agrees so well
with the facts that it suggests that our picture of the physical factors
involved is correct and lends confidence to any inferences based on
that picture. In this formulation, subscript 1 refers to oxygen and 2
refers to nitrogen, and $x$ is introduced to denote the mole fraction of
oxygen in the bubble. Otherwise we use the same notation as before.
The situation is governed by the following equations:

$$n_1 = \frac{1}{RT} \frac{4\pi r^3}{3} x p, \quad n_2 = \frac{1}{RT} \frac{4\pi r^3}{3} (1 - x) p,$$

$$\frac{-dn_1}{dt} = \delta_1 4\pi r^2 (x p - p_{1o}), \quad \frac{-dn_2}{dt} = \delta_2 4\pi r^2 [(1 - x)p - p_{2o}].$$

If we differentiate (5) with respect to time we obtain:

$$\frac{dn_1}{dt} = \frac{4\pi r^2}{RT} x p \frac{dr}{dt} + \frac{4\pi r^3}{3RT} p \frac{dx}{dt},$$

$$\frac{dn_2}{dt} = \frac{4\pi r^2}{RT} (1 - x) p \frac{dr}{dt} - \frac{4\pi r^3}{3RT} p \frac{dx}{dt}.$$

Equations (6) and (7) provide four relations between the five infini-
tesimals $dn_1, dn_2, dt, dx,$ and $dr$. They may be combined to eliminate
any three of these, leaving an equation between the remaining two.
Our interest is of course limited to $t, x,$ and $r$. If the two infinities
are chosen as $dr$ and $dx$, we obtain, after taking account of the fact
that $\delta_1 = 2\delta_2$,

$$\frac{-3dr}{r} = \left[ \frac{xp + p - p_{2o} - 2p_{1o}}{x^2p - x(p + p_{2o} + 2p_{1o}) + 2p_{1o}} \right] dx.$$
Similarly, if they are chosen as $dr$ and $dt$, we obtain

$$-dr = RT\delta_2 \frac{xp + p - p_{20} - 2p_{10}}{p} dt.$$  \hfill (9)

Finally, if they are chosen as $dx$ and $dt$, we obtain

$$dx = \frac{3RT\delta_2}{rp} \left[ x^2p - x(p + p_{20} + 2p_{10}) - 2p_{10} \right] dt.$$ \hfill (10)

Of these three equations only (8) is suitable for integration, and it is apparent that the significant variable in determining $x$ is $r$, not $t$. Nevertheless (9) and (10) are of interest also. For example, (10) makes it possible to calculate the limiting steady value of the composition of a bubble as it dissolves \( \left( \frac{dx}{dt} = 0, \ r \neq 0 \right) \) and to see how this varies with $p$, $p_{20}$, and $p_{10}$. If we assign to $p_{10}$ and $p_{20}$ the values corresponding to atmospheric air, we verify that the equilibrium composition of the bubble is richer in nitrogen the greater the pressure $p$, as we find it to be.

If equation (8) is written in the form

$$\frac{3dr}{r} = \left( \frac{x + a}{b^2 - (x + c)^2} \right) dx,$$ \hfill (11)

it may be integrated to give

$$\log r = \frac{1}{6} \left[ \frac{(a - c)}{b} \log \left( \frac{b + c + x}{b - c - x} \right) \right. \left. \log \{b^2 - (x + c)^2\} \right] + \text{const.} \hfill (12)$$

Since, according to definition, the quantities $a$, $b$, and $c$ are known functions of $p$, $p_{10}$, and $p_{20}$, (12) may be used to calculate pairs of values of $x$ and $r/r_0$ for different conditions ($r_0$ being the initial radius of the bubble). Of course these must include a specification of the value of $x$ for $t = 0$ ($r/r_0 = 1$), i.e., the initial composition of the bubble. If we are interested in the case of an air bubble dissolving at pressure $p$ in water saturated with air at atmospheric pressure we may set $p_{10} = 0.2$ atm. and $p_{20} = 0.8$ atm. Theoretical curves calculated for this case are shown in Fig. 5, together with corresponding experimental points obtained by the methods already described. The agreement, on the whole, is satisfactory and there is little doubt that the interpretation we have given provides the essential explanation of the main phenomenon; i.e., the gradual increase in the nitrogen content of the
bubbles as it dissolves towards a limiting steady value is greater as the pressure (depth) increases.

By joint use of equations (9) and (12) it is a simple matter to calculate how the rate of shrinkage of the bubble should vary with size and composition and depth. Such calculations are given in Table III, and they serve to explain the fact that at all pressures the initial value of \( \frac{dr}{dt} \) is always greater than the later values, as shown by the data given in Fig. 2. They also explain why the values of \( RT\delta \) calculated from (3) and shown in Table II are not constant but decrease with pressure. Physically, of course, this is explained by the fact that initially, when the bubble is relatively rich in oxygen, it shrinks more rapidly due to the fact that the rate of loss of oxygen is greater than that of nitrogen.

From the values of \(- \frac{1}{RT\delta_2} \frac{dr}{dt}\) given in Table III and from the values of \( \frac{dr}{dt} \) given in Table II (representing final rates), \( \delta_2 \) (referring to nitrogen) may be calculated and then the thickness of the water shell...
surrounding the bubble. We first find that $RT\delta_2 = (10 \pm 0.5)^{-4}$ cm/sec. From this, taking $RT = 2.4 \times 10^4$ cc atm./mole, we obtain $\delta_2 = 4.2 \times 10^{-9}$ moles/cm²/sec./atm. ($\delta_1$ is twice this, or $8.4 \times 10^{-9}$).

However, $\delta_2 = \frac{\Delta_2\alpha_2}{d}$. At $20^\circ$, $\Delta_2 = 2 \times 10^{-5}$ cm²/sec. and $\alpha_2 = 6.9 \times 10^{-7}$ moles/cc/atm. Consequently, $d = 3.3 \times 10^{-3}$ cm. It should be realized that the picture of a sharply defined water shell supporting a uniform diffusion gradient surrounding the bubble is artificial. The bubble behaves as if this were the case, but this simply means that the mixing of the water is practically complete up to a distance from the air-water interface of the same order of magnitude as the calculated thickness of the shell.

**SUMMARY**

Observations on the rate of solution of gas bubbles in water are described and are shown to be in close accord with equations based on the theory of diffusion. As an air bubble dissolves in water saturated with the atmosphere, the rate of decrease in diameter with time is nearly constant and increases with depth up to a limiting value. However, there is a slight decrease in the rate of solution with time, and this is associated with a change in composition, the bubble becoming richer in nitrogen as it approaches a limiting composition. When a bubble is formed in water saturated with air as a result of cavitation, it is initially much richer in oxygen than air, as would be predicted. Theoretically, the percentage of oxygen to be expected in such a bubble at the moment of formation is $33\frac{1}{3}\%$. Actually the highest observed value was 35%.
The equations developed in connection with these observations are of general applicability and should be of use in the analysis of various respiratory and other physiological problems involving aquatic organisms. They may also apply in certain problems of physical oceanography.

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