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A Theory of Money and Financial Institutions. Part 37. The Profit Maximizing Firm: Managers and Stockholders

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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART 37

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THE PROFIT MAXIMIZING FIRM: MANAGERS AND STOCKHOLDERS*

by

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1. *INTRODUCTION*

In general equilibrium theory consumers are presented as individual maximizers of utility whereas firms are run by shadowy automata who can best be regarded as selfless fiduciaries whose only goal is to maximize profits which are then flowed through to consumer-stockholders.

The purpose of this paper is to examine the conditions under which it can be proved that a utility maximizing manager of a firm will in fact attempt to maximize its profits; and the conditions where he will do something else.

In order to begin to investigate the necessary and sufficient conditions we must go to a level of microeconomic modelling which is usually not indulged in. Essentially all the rules of the game must be specified. In other words a complete description of individual goals and strategies must be provided.

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The basic apparatus upon which this paper builds is provided by a previous model describing trade and production as a noncooperative game [1]; the relevant aspects of that model are described below in sufficient detail that this paper is self-contained. In that model, however, managers were modelled as profit maximizing automata without goals of their own.

In a modern economy whether the corporations or large manufacturing enterprises are owned by the state or by individual stockholders, they are usually run by managers who have goals which are independent of the corporation. Most managers have some important sense of identity with the corporations they manage, but it is by no means total.

The history of capitalist and socialist economies is replete with examples of managers channeling resources under their control to serve for their own benefit. Top bureaucrats in most economies regardless of ideology tend to live well. They are in a position to remove "something off the top" [2] in the form of company cars, hunting lodges, dachas, special dining rooms, special stores, corporate yachts, planes and a whole array of other perquisites which go with occupying positions of bureaucratic power.

There are a multitude of details and reasons which account for managers serving their own ends at the expense of expected profits of the firm. In the subsequent sections of this paper we attempt to explore some of them and then to formalize models as games of strategy.

2. *CORPORATE OWNERSHIP AND VOTING SHARES*

The Arrow Debreu mathematization of an economy with exchange and production modelled the ownership of firms as consisting of the holdings of nonvoting shares entitling the owners to a share of corporate profits generated by profit maximizing firms run by automata [3]. For their purposes, their model was adequate and ingenious. However it depended upon several implicit assumptions which we make explicit here. In making them explicit we have two purposes in mind. The first is to show the nature of the added assumptions required to guarantee the existence of an efficient price system in an economy where voting shares exist and managers have their own economic goals. The second goal is to show that by changing to the more general formulation of an economy as a noncooperative game of strategy it is possible to incorporate not merely the general equilibrium results, but to produce models which are consistent with an economy where the fiduciary and control roles of top management are somewhat less than perfect.

In particular the critical assumptions concern:

(1) the thickness of factor, product and stock markets;

(2) the protection of minority stockholder rights;

(3) the limitation of managerial decision power [4];

(4) the presence or absence of accounting fudge factors.

In Section 3 we construct a model to show that utility maximizing managers of firms will profit maximize for their firms regardless of their own shareholdings provided that all markets are thick; corporate law requires equal treatment of all stockholders and a combination of accounting and managerial decision rules make it impossible for self dealing.

In actuality none of the conditions noted above are fully satisfied in any economy known to man. In the United States there is a large body of regulatory law including antitrust legislation and the laws enforced by the S.E.C.; as well as a considerable body of accounting rules and guides devoted to producing conditions as close to the ideal as possible.

Even with these safeguards the courts are filled daily with examples of managerial self serving and the price swings in corporate takeovers and mergers serve to indicate both the value of control and the gaps in value which can come about owing to difficulties in accounting. A detailed discussion of these points is given by Whitman and Shubik [2] elsewhere.

Many of the necessary limitations on managerial decisions come about by the imperfections of markets and the difficulties in accounting created by the complexities of tax laws and uncertainty and innovation in a dynamic economy. In socialist or other centralized economies the reporting systems used for control will contain aggregated data and this immediately provides those on the spot an opportunity to take advantage of biasing accounting information in their favor.

Uncertainties due to the introduction of new products and processes offer opportunities for managers to channel resources for their own purposes. These and the problems noted above are important items of detail in controlling managers in all economies. We will not pursue a study of them in further detail here, but will assume perfect accounting and no uncertainty.

The remainder of Section 2 is devoted to considering voting stock.

2.1. Prices, Voting and the Core

It is well known that virtually all voting schemes lead to games without a core. A simple example serves as an illustration. Consider an economy where n individuals jointly own a factory which can convert a valueless input into an output they all desire. Let each individual have one voting share as an initial set of resources. Let each individual i have a utility function of the form $u_i = (y_i)$ where y_i is the amount of the output he obtains. The production function of the factory is given by* $y = f(x)$ where x is the input. Suppose that the factory originally owns A units of the input.

The factory can produce any amount from 0 to $f(A)$ and it could divide its output in any manner giving $\alpha_i f(x)$ to the i^{th} individual where $\sum_{i=1}^{n} \alpha_i = 1$.

Before the economy noted above can be described as a game several rules must be specified. In particular who decides upon the level of production and who decides upon the division of the final product [5]?

The simplest model is a communal one where a simple majority vote decides everything. The model which is more in keeping with a modern economy run by managers would have a manager decide upon the production plan and the payout subject to the approval of directors voted in by stockholders. In this instance there are in general contractual limits on what each is permitted to do. In particular there may be rules which *impose* symmetric treatment of stockholders of the same class. For example the decision to pay a dividend may be made by management, but it may be required

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*Concave and increasing.

that all shares must be paid (or assessed) the same amount [6].

Reverting to our simple example where all decisions are made by a simple majority vote. If we normalize so that $f(0) = 0$ and $f(A) = 1$ then the characteristic function is given by:

$$V(S) = \{0, \ldots, 0\} \quad \text{for} \quad |S| \leq n/2$$

$$= \{\alpha_1, \ldots, \alpha_S\} \quad \text{for} \quad |S| > n/2$$

where $\sum_{i \in S} i = 1$.

It is easy to see from a simple diagram that this game has no core. Suppose we limited ourselves to symmetric payments then any coalition just

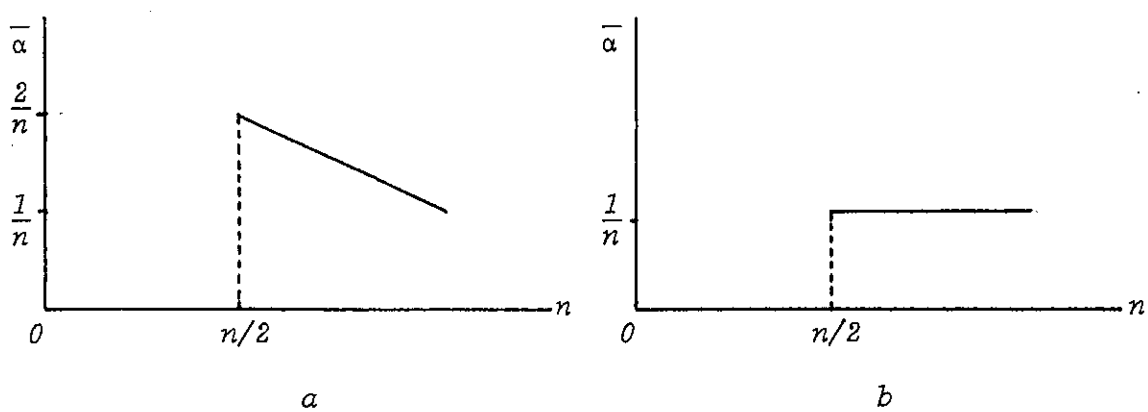

FIGURE 1

larger than $n/2$ could appropriate for itself a per capita payoff of around $2/n$. The coalition of all could only obtain $1/n$ per capita. This is shown in Figure 1a.

If there is a law which says that all profits paid out must be *pro rated* regardless of whether a shareholder is in the control group, then the maximum per capita payout will be $1/n$ regardless of the size of the control group, as is shown in Figure 1b. This minority stockholder protection clause restores the core to this game. This assumption is implicit in the Arrow Debreu treatment of shares.

In current corporate practice, given that it is required to treat all shares of the same class equally, managements must resort to other devices to reward themselves at the cost of stockholders. These devices include inflated salaries, expense accounts, pensions, private dining rooms, company jets and so forth. These can exist where the labor market for executives is less than perfect, hiring and firing costs are high and the deciphering of comparative information on productivity is low.

The other devices for enrichment include less than arms length dealing with suppliers and customers who may be relatives, friends or even business associates in other ventures. The difficulties in tax complications, accounting problems and technical evaluation offer opportunities for dealings of this variety which cannot easily be classified as breaking any law. But most of the possibilities would disappear if evaluation of economic opportunities and accounting were more accurate than they are in fact.

Although most opportunities for managerial self serving would vanish with perfect accounting and with protection of minority stockholders, even in perfect markets laws against violation of fiduciary trust are needed.

It is not accurate to state that in a perfectly competitive market an individual has no influence on price. He can sell at less than market price if he wishes. If the individual is a fiduciary and not a big enough owner; if the stakes are high enough it may pay him to sell at below market to a customer who may be a secret partner. His gain could be sufficiently large to compensate for being fired. In order to prevent this, either a rule must be introduced to forbid trades of this type at below market; or a sufficiently drastic penalty must be included to dissuade managers from such activities.

In the U.S. economy there are laws to protect stockholders against managements bent upon preempting corporate wealth. There are more than enough examples of management self-enrichment to show that these laws are needed. However it is quite possible that the U.S. economy at least has relatively little self serving, not purely because of the law, but because of noneconomic factors such as ethical standards and community pressure. These considerations take us beyond the type of analysis we can attempt here.

3. AN ECONOMY WITH UTILITY MAXIMIZING MANAGERS

For ease of presentation we make several simplifying assumptions which can each and all be dispensed with at the cost of considerably more notation and a lengthier proof. The three assumptions are:

(1) Consumers are required to sell all commodities.

(2) Managers are assumed to own all of the shares of the firms they control

and (3) We consider a game with simultaneous rather than sequential moves.

In a previous paper [1] on production and exchange none of these assumptions are made and our result on profit maximizing behavior could have been proved in that context along similar lines.

3.1. The Market $\mathcal{E}$

Let $\{I, \mathcal{C}, \mu\}$ be a measure space, where $I \equiv$ the set of agents in the market, $\mathcal{C} \equiv$ the σ-algebra of coalitions (subsets) of I, and μ is a measure* on $\{I, \mathcal{C}\}$. For our model we break up I into two disjoint sets I_1 and I_2 of positive μ-measure. I_1 will be the set of manager-consumers; I_2 the set of consumers. The initial data of the market $\mathcal{E}$ is given by the following measurable mappings [where Ω^ℓ denotes the nonnegative orthant of the Euclidean space R^ℓ of dimension ℓ]:

$$a : I \to \Omega^{m+1}$$

$$u : I \times \Omega^{m+1} \to \Omega^1$$

$$e : I_1 \to \Omega^{m+1}$$

$$Y : I_1 \to 2^{R^{m+1}}$$

For $i \in I$, $a(i)$ will be denoted by a^i etc. and $u(i,x)$ by $u^i(x)$ for $x \in \Omega^{m+1}$. Let us now explain our symbols.

$\Omega^{m+1} \equiv$ the set of commodity bundles

$a^i \equiv$ the initial endowment of consumer i

$e^j \equiv$ the initial endowment of firm j

$Y^j \equiv$ the production possibility set of firm j

$u^i(x) \equiv$ utility of consumer i for the bundle $x \in \Omega^{m+1}$.

For any $x \in \Omega^\ell$, we will denote the j^{th} component of x by x_j.

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*We do not preclude the possibility that μ has finite support.

3.2. *The Market Game* $\Gamma(\mathcal{E})$

To recast the market $\mathcal{E}$ as a game in strategic form, we single out the $(m+1)^{st}$ commodity as a money, and set up m trading-posts for the remaining m commodities. A consumer $i \in I \backslash I_1$ is now required to supply all of his first m commodities* for sale and may also bid money for their purchase. A manager-consumer $i \in I_1$ can *in addition* choose the production that firm i (under his control) will engage in. Let us use b^i to denote the bid vector of $i \in I$. [Thus b^i is in Ω^m, and b^i_j represents the bid of i in the j^{th} trading-post.] For $i \in I_1$, we will use $y^i \in \Omega^{m+1}$ to denote the output produced** by firm i.

Let

$$\tilde{S}^i = \{b^i : b^i \in \Omega^m, \sum_{j=1}^{m} b^i_j \leq a^i_{m+1}\}$$

for $i \in I$;

$$\underset{\sim}{S}^i = (e^i + Y^i) \cap \Omega^{m+1}, \quad \text{for } i \in I_1 .$$

Then the strategy sets S^i of $i \in I$ are given by:

$$S^i = \tilde{S}^i \quad \text{for } i \in I_2$$

$$S^i = \tilde{S}^i \times \underset{\sim}{S}^i \quad \text{for } i \in I_1 .$$

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*For simplicity we are using a "sell-all" model [7], rather than a bid offer model [8]. Either will do but the first is easiest.

**We could explicitly rule out the production of the $m+1^{st}$ commodity which is used as a commodity money without effecting our results. Leaving it in is as though gold producing firms can add to supply in a gold standard economy.

(Note: the mapping $i \to S^i$ is measurable.) Suppose we have a measurable selection s of strategies [$s^i \in S^i$ for $i \in I$, $s^i \equiv b^i$ for $i \in I_2$, $s^i \equiv (b^i, y^i)$ for $i \in I_1$]. Then the prices p of the first m commodities are obtained as follows:*

$$p_j(s) = \frac{\int_I b_j^i d\mu(i)}{[\int_I a_j^i d\mu(i)] + [\int_{I_1} y_j^i d\mu(i)]}$$

for $j = 1, \ldots, m$. The final money holdings of** the firms are:

$$\Pi^i(s) = y_{m+1}^i + \sum_{j=1}^{m} p_j(s) y_j^i$$

for $i \in I_2$. The final commodity bundle of consumer $i \in I$ is $x^i(s) \in \Omega^{m+1}$ where:

$$x_j^i(s) = \frac{b_j^i}{p_j(s)}, \quad j = 1, \ldots, m ; \quad \text{and} \quad i \in I$$

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*Note that a firm will be required to put up all of the first m commodities produced for sale, ruling out any self-supply by the manager. Also note that we define division by 0 to be 0 throughout this paper.

**A problem in accounting conventions and the reporting of profits for a firm in an ongoing economy could arise here if the firm has beginning and ending inventories of items other than money. For the maximization of net profit and the maximizing of final money worth for the firm to be the same we require an attribution of change in value of inventories. This is easy to do in theory in models as simple as ours, but extremely difficult to do in practice.

$$x^i_{m+1}(s) = \begin{cases} a^i_{m+1} - \sum_{j=1}^{m} b^i_j + \sum_{j=1}^{m} p_j(s)a^i_j & \text{if } i \in I_2 \\ a^i_{m+1} - \sum_{j=1}^{m} b^i_j + \sum_{j=1}^{m} p_j(s)a^i_j + \underset{\sim}{\Pi}^i(s) & \text{if } i \in I_1 \end{cases}$$

Finally the payoff to consumer $i \in I$ is $\Pi^i(s) = u^i(x^i(s))$.

This defines a game in strategic form.* A *Nash Equilibrium* (N.E.) of this game is a measurable strategy selection $\hat{s}$ such that

$$\Pi^i(\hat{s}|s^i) \leq \Pi^i(\hat{s}) \quad \text{for} \quad s^i \in S^i$$

for all $i \in I$. Here $(\hat{s}|s^i)$ is the same as $\hat{s}$, but with $\hat{s}^i$ replaced by s^i .

An *active N.E.* is an N.E. which produces positive prices in each trading-post. An *N.E. price* is a price produced at an N.E.

4. CONVERGENCE

Let us now suppose that $\mathcal{E}$ is a nonatomic market, i.e. the underlying measure μ is nonatomic. We wish to set up the description of a sequence of finite markets $\mathcal{E}_n$ which "approach" the nonatomic market $\mathcal{E}$. Consider the sequence $(I^n)_{n=1}^{\infty}$ where I^n is a finite set, $|I^n| \to \infty$. Suppose we have measurable** mappings $\{\beta_n\}_{n=1}^{\infty}$, $\beta_n : I \to I^n$ such that $\beta_n(I_1) \cap \beta_n(I_2) = \phi$. [Intuitively $i \in I^n$ can be identified now with the set $\beta_n^{-1}(i) \subset I$.]

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*We have assumed the strategy-selection s to be measurable. For a discussion of this, when μ is nonatomic, see [9].

**The measure α_n on I^n gives equal weight to all elements of I^n .

A "simple" market $\mathcal{E}_n$ with the underlying set I^n of agents is again described by the mappings:

$$_na : I^n \to \Omega^{m+1}$$

$$_nu : I^n \times \Omega^{m+1} \to \Omega^1$$

$$_ne : I_1^n \to \Omega^{m+1}$$

$$_nY : I_1^n \to 2^{R^{m+1}} .$$

Here $I_1^n \equiv \beta_n(I_1)$ and $I_2^n \equiv \beta_n(I_2) = I^n \setminus I_1^n$. The mappings $_na, \ldots, {}_nY$ have the obvious meaning.

Any mapping f from I^n (or I_1^n) to an arbitrary set X may also be viewed as a mapping from I (or I_1) to X given by $f(i) = f(\beta_n(i))$ for $i \in I$ (or $i \in I_1$). We shall often view, in this sense, the mappings $_na$, $_ne$, $_nY$ (and others as they arise in the sequel) as if they had the domains I or I_1 . Also, we will think of $_nu$ as a mapping with domain $I \times \Omega^{m+1}$, where $_nu^i(i) \equiv {}_nu^{\beta_n(i)}(x)$, for $x \in \Omega^{m+1}$.

We shall say that the sequence $\{\mathcal{E}_n\}_{n=1}^{\infty}$ of simple markets *converges* to --and write " $\mathcal{E}_n \to \mathcal{E}$ " --if

(i) $\mu(\beta_n^{-1}(S)) = |S|/|I^n|$ for every $S \subset I^n$

(ii) $_na$, $_nu$, $_ne$, $_nY$, converge* with n to a , u , e , Y , almost everywhere on I (or I_1) .

The theorem we wish to establish is:

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*The sense of convergence is as in Hildenbrand [11].

Theorem. Suppose (i) $\mathcal{E}^n \to \mathcal{E}$, (ii) ${}_np \in \Omega^m$ is an N.E. price of $\Gamma(\mathcal{E}^n)$ and ${}_np \to p > 0$. Then p is an N.E. price of $\Gamma(\mathcal{E})$.

The proof of this is along the same lines as the proof of Theorem 1 in [10], hence we shall give only its outline. We need to set up some notation to do this. Let:

$${}_n\tilde{S}^i = \{b \in \Omega^m : \sum_{j=1}^{m} b_j \le {}_na^i_{m+1}\}, \quad i \in I^n$$

$${}_n\underset{\sim}{S}^i = ({}_ne^i + {}_nY^i) \cap \Omega^{m+1}, \quad i \in I_1^n$$

$${}_nS^i = {}_n\tilde{S}^i, \quad i \in I_2^n$$

$${}_nS^i = {}_n\tilde{S}^i \times {}_n\underset{\sim}{S}^i, \quad i \in I_1^n .$$

Next, for $\hat{b} \in \Omega^m$, $\hat{y} \in \Omega^{m+1}$, $b : I^n \to \Omega^m$, $y : I_1^n \to \Omega^m$, $1 \le j \le m$, define* the following:

$${}_nQ_j^{-i}(b,y) = \sum_{k\in I^n\setminus i} {}_na_j^i + \sum_{k\in I_1^n\setminus i} y_j^i$$

$${}_nB_j^{-i}(b,y) = \sum_{k\in I^n\setminus i} b_j^i$$

If $i \in I_2^n$,

$$p_j^i[(b,y)|\hat{b}] = \frac{{}_nQ_j^{-i}(b,y)}{\hat{b}_j + {}_nB_j^{-i}(b,y)}$$

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* ${}_n\bar{a}_j \equiv \sum_{i\in I^n} {}_na_j^i$, $\bar{y}_j = \sum_{i\in I_1^n} y_j^i$.

$${}_n x^i_j[(b,y)|\hat{b}] = \hat{b}_j / p^i_j[(b,y)]\hat{b}$$

$${}_n x^i_{m+1}[(b,y)|\hat{b}] = {}_n a^i_{m+1} - \sum_{j=1}^{m} \hat{b}_j + \sum_{j=1}^{m} {}_n a^i_j p^i_j[(b,y)|\hat{b}]$$

$${}_n B^i(b,y) = \{{}_n x^i\ (b,y)|\hat{b}\ :\ \hat{b} \in {}_n S^i\}$$

$${}_n \hat{B}^i(b,y) = \{x \in {}_n B^i(b,y)\ :\ x \text{ maximizes } {}_n u^i \text{ on } {}_n B^i(b,y)\} .$$

If $i \in I^n_1$,

$$p^i_j[(b,y)|(\hat{b},\hat{y})] = \frac{{}_n Q^{-i}_j(b,y) + \hat{y}_j}{{}_n B^{-i}_j(b,y) + \hat{b}_j}$$

$${}_n x^i_j[(b,y)|(\hat{b},\hat{y})] = \hat{b}_j / p^i_j[(b,y)|(\hat{b},\hat{y})]$$

$${}_n x^i_{m+1}[(b,y)|(\hat{b},\hat{y})] = {}_n a^i_{m+1} - \sum_{j=1}^{m} \hat{b}_j + \sum_{j=1}^{m} {}_n a^i_j p^i_j[(b,y)|(\hat{b},\hat{y})]$$
$$+ \hat{y}_{m+1} + \sum_{j=1}^{m} \hat{y}_j p^i_j[(b,y)|(\hat{b},\hat{y})]$$

$${}_n B^i(b,y) = \{{}_n x^i[(b,y)|(\hat{b},\hat{y})]\ :\ (\hat{b},\hat{y}) \in {}_n S^i\}$$

$${}_n \hat{B}^i(b,y) = \{x \in {}_n B^i(b,y)\ :\ x \text{ maximizes } {}_n u^i \text{ on } {}_n B^i(b,y)\} .$$

Also let us define for $p \in \Omega^m$ $(p > 0)$, $\hat{b} \in \Omega^m$, $\hat{y} \in \Omega^{m+1}$: If $i \in I_2$,

$$x^i_j[p|\hat{b}] = \frac{\hat{b}_j}{p_j}$$

$$x^i_{m+1}[p|\hat{b}] = a^i_{m+1} - \sum_{j=1}^{m} \hat{b}_j + \sum_{j=1}^{m} a^i_j p_j$$

$$B^i(p) = \{x^i[p|\hat{b}] \; : \; \sum_{j=1}^{m} \hat{b}_j \le a^i_{m+1}\}$$

$$\hat{B}^i(p) = \{x \in B^i(p) \; : \; x \text{ maximizes } u^i \text{ on } B^i(p)\} \; .$$

If $i \in I_1$,

$$x^i_j[p|(\hat{b},\hat{y})] = \frac{\hat{b}_j}{p_j}$$

$$x^i_{m+1}[p|(\hat{b},\hat{y})] = a^i_{m+1} - \sum_{j=1}^{m} \hat{b}_j + \sum_{j=1}^{m} a^i_j p_j + \sum_{j=1}^{m} \hat{y}_j p_j + \hat{y}_{m+1}$$

$$B^i(p) = \{x^i[p|(\hat{b},\hat{y})] \; : \; \sum_{j=1}^{m} \hat{b}_j \le a^i_{m+1}, \; \hat{y} \in \underset{\sim}{S}^i\}$$

$$\hat{B}^i(p) = \{x \in B^i(p) \; : \; x \text{ maximizes } u^i \text{ on } B^i(p)\} \; .$$

Outline of Proof of Theorem. Let ${}^*_n s$ be a N.E. of $\Gamma(\mathcal{E}_n)$ which produces the price ${}_n p$, $n = 1, 2, \ldots$. Adopt the notation ${}^*_n s^i = ({}^*_n b^i)$ for $i \in I^n_1$, ${}^*_n s^i = ({}^*_n b^i, {}^*_n y^i)$ for $i \in I^n_2$. Under our assumptions, it can be shown that

(*) $\qquad \lim\sup {}_n B^i({}^*_n b, {}^*_n y) = B^i(p)$

for all $i \in I$. (See proof of Theorem 1 in [10] for details of an analogous result.) Using (*) it can then be shown that:

$$(**) \qquad \limsup_n \hat{B}^i({}^*_n b,\ {}^*_n q,\ {}^*_n y) \subset \hat{B}^i(p)\ .$$

[See [10] for a proof of (**) from (*).]

Since $({}^*_n b,\ {}^*_n y)$ is an N.E. of $\Gamma(\mathcal{E}^n)$,

$$_nA = [\int_{I^n} {}_n\hat{B}^i({}^*_n b,\ {}^*_n y)] \cap [\int_{I^n} {}_n a^i + \int_{I_1^n} {}_n \underset{\sim}{S}^i] \neq \phi\ .$$

But

$$\int_{I^n} {}_n a^i \to \int_I a^i\ , \quad \int_{I_1^n} {}_n \tilde{S}^i \to \int_{I_1} \underset{\sim}{S}^i\ ,$$

and

$$\limsup \int_{I^n} {}_n\hat{B}^i({}^*_n b,\ {}^*_n y) \subset \int_{I^n} \limsup {}_n\hat{B}^i({}^*_n b,\ {}^*_n y) \subset \int_I \hat{B}^i(p)\ .$$

[The first $\subset$ follows from Theorem 6 in Section DII in [11]; the second follows from (**).] Hence from $\limsup {}_nA \neq \phi$ we derive

$$[\int_I \hat{B}^i(p)] \cap [\int_I a^i + \int_{I_1} {}_n \underset{\sim}{S}^i] \neq \phi\ .$$

Select measurable mappings

$$x : I \to \Omega^{m+1}\ , \quad x^i \in \hat{B}^i(p)$$

$$y : I_1 \to \Omega^{m+1}\ , \quad y^i \in \underset{\sim}{S}^i$$

such that

$$\int_I x = \int_I a + \int_{I_1} y \ .$$

Define $b : I \to \Omega^m$ by $b_j^i = p_j x_j^i$, $j = 1, \ldots, m$. Then it is straightforwardly verified that (b, y) constitutes a N.E. of $\Gamma(\mathcal{E})$.

Remarks

(1) It is obvious that, if we assume that $i \in I_2$ desires money [i.e. $u^i(x + \Delta e) > u^i(x)$ for any $x \in \Omega^{m+1}$ and $\Delta > 0$ (where $e \in \Omega^{m+1}$, $e_j = 0$ for $1 \leq j \leq m$, $e_{m+1} = 1$)] then at any N.E. $({}^*b, {}^*y)$ of $\Gamma(\mathcal{E})$ the manager-consumer i chooses ${}^*y^i$ in order to maximize the revenue* $\underset{\sim}{\Pi}^i[({}^*b, {}^*y)|\hat{y}^i]$ earned by his firm. Thus our convergence theorem shows that as the finite economy $\mathcal{E}^n$ approaches the nonatomic economy $\mathcal{E}$, the choice of production made by the manager converges to that which maximizes his firm's revenues. Indeed this is the content of the relation:

$$\mathit{lim\ sup}\ {}_n\hat{B}^i({}^*b, {}^*y) \subset \hat{B}^i(p)$$

established in the proof of the theorem.

(2) If we assume that for each commodity j , $1 \leq j \leq m$, there are at least two traders (equivalently, a nonnull subset of traders in the nonatomic case) who have money and desire j then as shown in [10] active N.E. exist for the games $\Gamma(\mathcal{E})$.

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*This is tantamount to maximizing the profit made by the firm. The profit is $P^i = \Pi^i[({}^*b, {}^*y)|\hat{y}^i] - \sum_{j=1}^{m} p_j e_j^i - e_{m+1}^i$. But since the last two terms are independent of the strategy $\hat{y}^i$ picked by i , P^i and Π^i are maximized by the same $\hat{y}^i \in \underset{\sim}{S}^i$.

4. AN EXAMPLE OF MANAGERIAL OPTIMIZATION

An extremely simple example of an economy with nonatomic consumers and a single firm run by a manager serves to illustrate the conflict between profit maximization and individual maximization.

Let each individual i own $(0, o, \rho^i, M^i)$ where 0 is the initial supply of a raw material; o the initial supply of the manufactured good; ρ^i the shares of the single firm owned by i, and M^i an amount of a commodity money owned by each individual i.

There is one firm run by individual 1 as manager. It has a production function given by

(1) $$y = x$$

where y is the total output of the manufactured good where x is the amount of input. The firm owns all of the resource which is otherwise valueless. The profit of the firm is given by:

$$\Pi = px$$

where p is the price of output.

We assume the firm starts with $(A, 0, 0, 0)$ where the zeros indicate no stocks of output, no ownership of shares in itself and no money. As the example is simple we take several shortcuts in analyzing it hence it may appear that all of the details specified are not necessary; in general they are.

The utility function of an individual i is:

(2) $$u^i(x^i, y^i, z^i) = y^i - \frac{1}{2}(y^i)^2 + z^i .$$

The strategic moves of a consumer who is not a manager are to bid an amount of money b^i to buy the output.

Consumer 1 who is also the manager will decide upon not only b^1 but also upon y^f which is the amount of the output offered by the firm.

The price p is given by

$$(3) \qquad p = \int b^j / y^f \, .$$

Writing the payoffs in terms of the strategic variables a trader who is not a manager attempts to maximize

$$(4) \qquad U^i = y^i - \frac{(y^i)^2}{2} + M^i - py^i + \rho^i \Pi \quad \text{(defined for } 0 \leq y^i \leq 1 \text{)}$$

where $y^i = b^i/p$ and Π is the profit of the firm.

The consumer-manager also attempts to maximize (4) but he controls both b^1 and y^f which influences p as can be seen from (3).

We assume that all consumers are identical in resources except for the consumer manager who we differentiate in three cases. For all others the density of ownership of shares is 1 for him we consider densities of 0, 1 and 2.

First setting aside our special role for the manager we solve this market for the efficient solution or competitive equilibrium, then for the monopoly solution.

The competitive equilibrium is given by:

$$(5) \qquad y_f = 1 \, , \quad p = 0 \, , \quad \Pi = 0 \, , \quad U^i = \frac{1}{2} + M \, ;$$

and the monopoly solution is given by:

firm has any oligopolistic influence on the market then a change in price yields three influences to the consumer-manager to wit:

$$\textit{The substitution effect} + \begin{matrix}\textit{the consumer}\\ \textit{income effect}\end{matrix} + \begin{matrix}\textit{the owner}\\ \textit{income effect}\end{matrix} .$$

Depending upon the size of the manager's holdings the last effect can dominate the other two.

Perhaps it may be belaboring the obvious to demonstrate this phenomenon in the context of a closed economic model. A partial equilibrium analysis might have served to show it. We nevertheless believe that the noncooperative game model of production and exchange offers a more flexible model of the closed economy where at least a group of phenomena which exist in our economy can be identified within a context that is consistent with general equilibrium as a special case. This is consistent with the observation of Hicks [12].

There are other incentive systems such as the granting of bonuses to managers in proportion to reported accounting profits or making compensation proportional to the size of their bureaucracy which also influence their behavior in a dynamic economy. The importance of these phenomena raises several empirical questions which remain to be answered.

(6) $$y_f = \frac{1}{2}, \quad p = \frac{1}{2}, \quad \Pi = \frac{1}{4}, \quad U^i = \frac{1}{2} - \frac{1}{8} + M - \frac{1}{4} + \frac{1}{4} = \frac{3}{8} + M .$$

We now consider the manager owning a density of 0, 1 or 2 shares. He has the strategic power to chose between (5) or (6). Table 1 shows his minimum payoffs.

		C.E. Payoff	*Monopoly Payoff*
Share Density	*0*	$\frac{1}{2} + M$	$\frac{1}{8} + M$
	1	$\frac{1}{2} + M$	$\frac{3}{8} + M$
	2	$\frac{1}{2} + M$	$\frac{5}{8} + M$

TABLE 1

For the first two levels of ownership he prefers to run the firm with a price of $p = 0$. When he owns twice as many shares as any other individual he prefers the monopoly solution.

5. *CONCLUDING REMARKS*

Expressed in the simplest terms the effect on an individual consumer of a price change can be broken down into the well known substitution and income effects. This view is based upon the assumption that no individual can influence market prices to influence his income directly. This is true for a manager only if his firm is both so small that it does not influence price and if he is expressly forbidden from separate dealings with individual customers where underselling the market might favor him.

Even given restrictions against separate dealings as soon as the

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