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A MODEL OF THE VERTICAL STRUCTURE OF MASS IN EQUATORIAL WIND-DRIVEN CURRENTS OF A BAROCLINIC OCEAN

BY

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A model for the vertical distribution of density, which approximates observed conditions in the equatorial Pacific Ocean, is presented. This consists of a homogeneous layer near the surface overlying a baroclinic medium wherein the density varies exponentially in such a manner as to approach a constant value with increasing depth. It is shown that the depth of the upper homogeneous layer can be considered as a function only of the wind stress which is exerted upon the sea surface. The relationship is developed from Sverdrup's (1947) theory of wind-driven currents in a baroclinic ocean, where it has been demonstrated that the mean horizontal distribution of mass of the upper 1,000 meters is determined by the wind stress. The mean horizontal distribution of mass is, in turn, directly related to the depth of the homogeneous layer. Consequently, it is possible to describe the threedimensional distribution of density and pressure, provided that the wind stress is known.

The distribution of density in a vertical section is constructed for the case of an east-west wind stress whose magnitude varies sinusoidally with latitude. The computed model is compared with a similar cross section of density obtained from oceanographic observations in the eastern Pacific.

THEORY

Review. In a baroclinic ocean the horizontal pressure gradient and velocity generally vanish at great depth. If \( d \) (a constant) is the selected depth of no motion and \( p = p(xyz) \) is the pressure, then the

\[ \frac{\partial p}{\partial z} = \frac{g}{\rho_0} \frac{\partial \rho}{\partial z} \]

where \( \rho \) is the density and \( \rho_0 \) is the density at the surface. The right-hand side of this equation is the term representing the gravitational acceleration and the vertical density gradient. Integrating this equation from the surface to the depth \( d \), we obtain

\[ \rho(z) = \rho_0 + \int_0^z \frac{g}{\rho_0} \frac{\partial \rho}{\partial z} \, dz \]

This expression represents the density as a function of depth. The density at the surface is given by \( \rho_0 \), and the integral represents the change in density due to the vertical pressure gradient.

\[ \rho(z) = \rho_0 + \int_0^z \frac{g}{\rho_0} \frac{\partial \rho}{\partial z} \, dz \]

The integral must be evaluated numerically, and the result can be used to construct a model of the vertical distribution of density in the ocean. The model can then be compared with observed data to verify its accuracy.

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vertical integral of pressure $P = P(xy)$, which represents the mean horizontal distribution of mass, is defined by

$$
\frac{\partial P}{\partial x} = \int_{\text{surface}}^{d} \frac{\partial P}{\partial x} \cdot dz, \quad \frac{\partial P}{\partial y} = \int_{\text{surface}}^{d} \frac{\partial P}{\partial y} \cdot dz, \quad (1)
$$

where $z$ is the vertical co-ordinate measured downward from a reference near the surface. For unaccelerated motion in a baroclinic ocean, free of lateral friction and transient changes of density, it can be shown on the basis of Sverdrup's (1947) theory that the quantity $P(xy)$ is given by the expression

$$
P = (x - x_o) \left[ \frac{\partial \bar{r}_x}{\partial y} R \cdot \tan \phi \right] + \int_{y_1}^{y} \tau_v (y') dy', \quad (2)
$$

where $\bar{r}_x = \bar{r}_x(xy)$ and $\tau_v = \tau_v(y)$ are the $x$ and $y$ components of wind stress, respectively, in a rectilinear system where $x$ and $y$ are the horizontal co-ordinates positive to the east and north, respectively, with the origin on the equator. The term $\phi$ is the latitude, positive north of the equator, and $R$ is the mean radius of the earth. The zero subscript refers to the continental boundary, and the bar superscript indicates an average value over the range $x$ to $x_o$. The limit $y_1$ represents an arbitrary limit. Expression (2) is valid provided (a) that a constant depth of no motion, $d$, does exist, (b) that a north-south vertical boundary is present, and (c) that $\tau_v$ is independent of $x$.

It can be shown that (2) is a very good approximation for the eastern equatorial Pacific, even where $x_o$—the co-ordinate of the American continental boundary—varies with latitude.

**The Model.** The selected density structure is essentially a generalization of the simple two layer system treated by Defant (1936), where the upper homogeneous layer is retained. The structure is as follows: there is [1] an upper homogeneous layer of depth $h = h(xy)$ and density $\rho_o$; beneath this is [2] a baroclinic transitional layer extending from $h$ to depth $d$, below which the water is at rest and of constant density $\rho_d$. An exponential transition will be assumed, such that the density $\rho(xyz)$ approaches $\rho_d$ asymptotically with increasing depth, in good approximation of the model. One possible form is

$$
\rho(xyz) = \rho_o = \rho_d - \Delta \rho e^{-(z-h)/h}, \quad (3)
$$

where $\Delta \rho = \rho_d - \rho_o$. The model is illustrated in Fig. 1, in which it is compared with actual distributions of $\rho$ with depth, for various values of $h$. From (3) it is possible to solve for the $P$-function in terms of $h$. 

To a very close degree of approximation the pressure is given by the hydrostatic equation,

$$p = g \int_{-h}^{z} \rho (xyz) \, dz,$$

(4)
where \( \zeta = \zeta(xy) \) is the elevation of the surface, and \( g \) is the acceleration of gravity.

It will be convenient to deal with the space derivatives of pressure rather than with the pressure itself. The pressure gradient in the \( x \)-direction is given by

\[
\left( \frac{\partial p}{\partial x} \right)_1 = g \frac{\partial}{\partial x} \int_{-\zeta}^{z} \rho_0 dz = g\rho_0 \frac{\partial \zeta}{\partial x}, \quad -\zeta \leq z \leq h, \tag{5}
\]

in the upper layer, and by

\[
\left( \frac{\partial p}{\partial x} \right)_2 = g \left[ \rho_o \left( \frac{\partial \zeta}{\partial x} + \frac{\partial h}{\partial x} \right) - \rho_0 \frac{\partial h}{\partial x} + \Delta \rho \frac{\partial}{\partial x} \left( he^{1-z/h} - h \right) \right] \tag{6}
\]

in the transition layer. Similar expressions are valid for \( \partial p/\partial y \).

At \( z > d \), the pressure gradient vanishes, and if \( d \gg h \), (6) yields the approximation

\[
\zeta = \frac{2\Delta \rho}{\rho_0} h, \tag{7}
\]

provided that the reference level is selected properly. Both \( h \) and \( \zeta \) must either be positive or vanish.

The derivative of integrated pressure is, from (1), (6) and (7)

\[
\frac{\partial P}{\partial x} = \int_{-\zeta}^{h} g\rho_0 \frac{\partial \zeta}{\partial x} \cdot dz + \int_{h}^{d} \rho_0 \frac{\partial h}{\partial x} \left( he^{1-z/h} \right) \cdot dz.
\]

This yields the approximation

\[
\frac{\partial P}{\partial x} = \frac{5g\Delta \rho}{2} \frac{\partial h^2}{\partial x}.
\]

A similar expression holds for \( \partial P/\partial y \), and it follows

\[
P = \frac{5}{2} g\Delta \rho \cdot h^2 + \text{constant}. \tag{8}
\]

The Sinusoidal Wind Stress Distribution. The east-west component of wind stress in the equatorial Pacific has been shown to vary nearly sinusoidally with latitude (Reid, 1948). It will be assumed here for simplicity that

\[
\tau_x = \tau_o + b \sin \omega (\phi - \phi_o), \quad \tau_y = 0, \tag{9}
\]

where \( \tau_o, b, \omega, \phi_o \) are constants. The function \( P \) and consequently \( h \)
Figure 2. The computed distribution of density along the Carnegie section of the central equatorial Pacific, derived from the hypothetical exponential density model and the sinusoidal wind stress distribution.

can be evaluated in terms of the position co-ordinates. Using (2), (8), and (9) it follows

\[ h^2 = \frac{2}{5g\Delta\rho} (x - x_o) \left[ \tau_o + b \sin \omega (\phi - \phi_o) - \omega b \tan \phi \cdot \cos \omega (\phi - \phi_o) \right] - K, \quad (10) \]
Figure 3. The actual distribution of density along the Carnegie section of the central equatorial Pacific, derived from oceanographic data.

and the surface elevation is

\[ h^2 = \frac{8A \rho}{5g \rho_o^2} (x - x_o) \left[ \tau_o + b \sin \omega (\phi - \phi_o) - \omega b \tan \phi \cdot \cos \omega (\phi - \phi_o) \right] \]

\[ - \left( \frac{2A \rho}{\rho_o} \right)^2 K, \]  

(11)

where \( K \) is the constant of integration, to be selected such that \( h \) is always a real quantity.
It is evident that $h^2$ and $r^2$ increase linearly with distance from the boundary for the chosen distribution of density and wind stress.

The Pressure Distribution. The form of the pressure distribution is obtained readily from the hydrostatic equation and the model (3), giving

$$p_1 = g (\rho \xi + 2\Delta \rho h), \quad -\xi \leq z \leq h,$$

and

$$p_2 = g (\rho \xi + \Delta \rho h e^{1-z/h}), \quad z \geq h.$$  \hspace{1cm} (12)

The slope of the isosteres, $i_{p2}$, in the transition layer can be shown to be

$$i_{p2} = \left(\frac{z}{h}\right) i_h, \quad z \geq h,$$  \hspace{1cm} (13)

where $i_h$ is the slope of the interface at $z = h$. It can be verified that the isobaric slopes, $i_p$, for the two layers are

$$i_{p1} = -\frac{2\Delta \rho}{\rho} i_h = i_r,$$

and

$$i_{p2} = -\frac{\Delta \rho}{\rho} \left(1 + \frac{z}{h}\right) e^{1-z/h} i_h = \frac{1}{2} \left(1 + \frac{z}{h}\right) e^{1-z/h} i_{p1}.$$  \hspace{1cm} (14)

The isobaric slope in the transition layer is opposite in sign to the slope of $h$ for all $z \geq h$. The isosteric slope, on the other hand, has the same sign as that of $h$ for $x \geq h$. This clearly demonstrates the baroclinicity of the model fluid, i.e., the isosteres intersect the isobars at all $z \geq h$.

It is evident from (14) that the isobaric slope becomes vanishingly small as $z$ approaches $d$, which is necessary to satisfy the condition of stationary water below $d$.

DISCUSSION

The observations utilized in the computations consist essentially of a line of 13 stations between Lat. 24° 57' N, Long. 137° 44' W and Lat. 10° 54' S, Long. 161° 53' W, occupied by the Carnegie between October 21 and November 4, 1929 (Fleming, et al., 1945).

A value of $K$ was selected as $3.8 \times 10^8$ so that the minimum value of $h$ for the model would correspond to that observed at about Lat. 13° N. The depth of no motion was taken as 700 m, and the mean values of $\rho_0$ and $\rho_4$ were taken for the Pacific data as 1.0230 and 1.0274 gm/cm$^3$, respectively.\footnote{The values represented here do not include the effect of compression, nor does the model. The effect is unimportant, since there is very little contribution to the gradients of $p$ and $P$ due to compression.}
Figure 4. The dependence of isobaric slope on depth, for the exponential density distribution. The ratio of the slope to that in the homogeneous layer is plotted vs. depth in per cent of the homogeneous layer depth.
The values of the wind stress constants are:
\[
\tau_\phi = -0.3 \text{ dyne/cm}^2, \omega = 13.85, \\
b = 0.3 \text{ dyne/cm}^2, \phi_\circ = 1^\circ.
\]

The hypothetical vertical density structure along the Carnegie section computed from expressions (3) and (10) and the above constants is shown in Fig. 2. The values of \((x - x_a)\) were obtained from the position of the Carnegie stations and the American continental boundary. The actual distribution of density, omitting effects of compression, is shown in Fig. 3 for comparison. A plot of the vertical variation of isobaric slope, obtained from (14), is shown in Fig. 4. There is no discontinuity in the slope at \(z = h\), as in the case of the simple two layer system [where \(\rho(z)\) is discontinuous].

**SUMMARY AND CONCLUSIONS**

The exponential transition of density below a light homogeneous layer is strikingly representative of the vertical distribution of density in the equatorial Pacific. From this model and the theory of wind-driven currents, the entire field of density and pressure is described. A cross section of the model density structure is compared with the actual distribution given by the Carnegie data. The principal characteristics of the observed structure of mass are accounted for in the model, except in the vicinity of the equator.

The conditions at the equator need further consideration, especially regarding the role of thermodynamic processes in determining the distribution of the \(P\)-function.

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