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A Short-Run Two-Commodity Macroeconomic Model

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by

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Standard IS-LM analysis assumes a single homogeneous commodity which can be purchased at a unique price and either consumed or added to the capital stock. This is a very powerful assumption which greatly simplifies the mathematical and graphical analysis of the model.

Although the realism of this assumption is seldom discussed, it can be motivated by the argument that the relative prices of commodities are fixed in the shortrun. Perhaps the most natural reason for this would be some group of consumers, producers or hoarders which view the different commodities as perfect substitutes. Alternatively, if both labor and capital are immobile and wages and prices are flexible, then the equilibria of the separate labor markets can determine relative commodity prices independently of financial markets and the demand for commodities. Another possibility is that wages and prices are fixed in the short run.

A few published papers have distinguished between the prices of consumption and investment goods. Probably the most widely known is Foley and Sidrauski's classic Monetary and Fiscal Policy in a Growing Economy; its relevance as a shortrun model is however considerably weakened by the assumption that capital is perfectly mobile between sectors. More

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recent contributions by Park and by Sargent and Henderson are given some shortrun flavor by the assumption that the nominal wage is fixed; but they continue to assume mobile capital.

The only two-sector immobile capital model that we have found is in an article by Benavie which was published as this paper was being completed. Although our papers were written independently, they are strikingly similar in spirit. The scope of the papers does however differ in two dimensions. We maintain the usual assumption that bonds and equities are perfect substitutes while Benavie also examines the more general case in which they are imperfect substitutes. On the other hand, Benavie assumes that nominal wages are fixed while we additionally analyze the more general case of flexible wages. There are also a few interesting differences in detail, one of which is responsible for a qualitatively different conclusion in the fixed wage and perfect substitutes case in which our papers overlap.

Section I lays out the basic model and analyzes monetary and fiscal policy when there are no real balance or interest rate effects on consumption. The key endogenous variables are sequentially determined in this model. Employment, output and relative commodity prices are determined by consumption goods equilibrium. Given these variables, the interest rate is determined by investment goods equilibrium; and given these equilibria the money or bond-equity equilibria determine the level of prices.

Section II considers the extended model in which consumption demand is influenced by interest rates and the level of prices. These factors make the markets interrelated so that exogenous shocks generally influence all of the endogenous variables. Government fiscal policies are found to change the composition of output, stimulating production in one sector while discouraging it in the other. Government open market purchases increase investment and reduce consumption, while open market sales have the opposite effects. This section also considers a variety of special cases.

In Section III we then consider the case of rigid nominal wages. Here open market purchases stimulate both sectors while fiscal policies stimulate production of the commodity purchased by the government and have ambiguous effects on the other sector. Since this section overlaps Benavie, we compare our models and conclusions.

Overall, we find that working with two commodities instead of one substantially increases the complexity of the analysis and the ambiguity of the results. In particular, many propositions which are now entrenched in the public as well as academic discussion of monetary and fiscal policies need not hold in a two-sector model. If the profession judges the relevant parameter values to be such that these familiar propositions are justified, then the one sector model has a clear expository advantage. If, however, their validity rests only upon their correctness in a one-sector model, then serious attention should be given to more complicated models.

I. The Basic Model

There are two commodities (consumption and investment goods), three financial assets (money, bonds, and equity), and three sectors (households, corporations, and government). There are no taxes since they would complicate the model without adding anything of substance. Expectations are static (i.e. inelastic) since more substantive assumptions would create too many ambiguities. It is a discrete model in which end of period stock demands are constrained by beginning of period stock holdings and flows during the period. These flow purchases and sales determine the values of the market clearing variables. We are examining only the current period (in which beginning of period stocks are fixed) and asking the comparative statics question: how would the flow decisions and market clearing prices be different if the government pursued different monetary and fiscal policies?

Corporations

There are two competitive sectors which produce consumption and investment goods with two-factor constant returns to scale production functions

$$Q_C = F(K_C, N_C)$$

$$Q_I = G(K_I, N_I) .$$

Labor is mobile but capital is not. Profit maximizing output is at the level of employment such that the marginal revenue product is equal to the marginal cost

$$P_C F_2 = w = P_I G_2 .$$

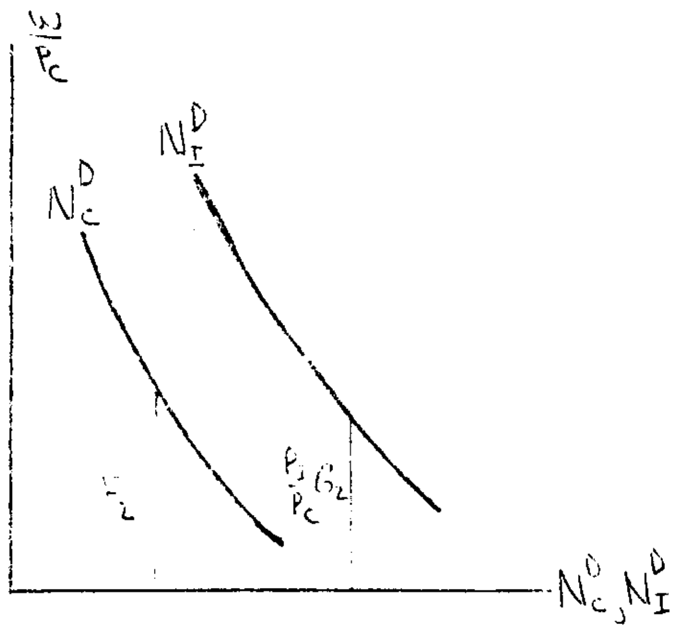
The sectoral labor demands are consequently given by the levels of employment that solve

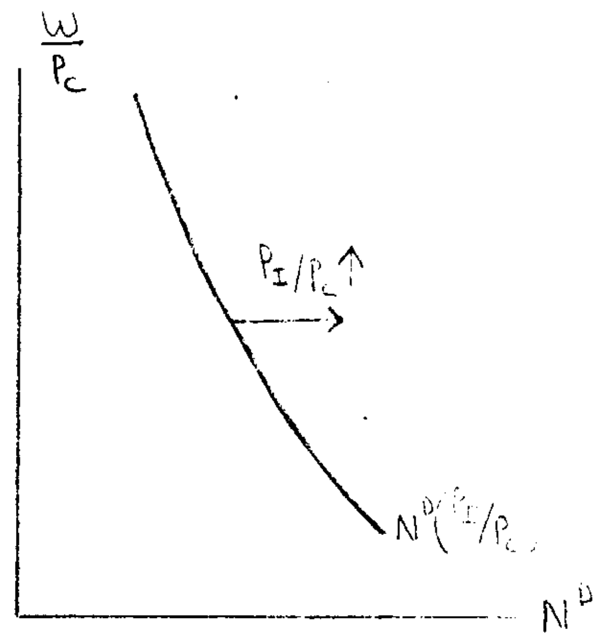
$$F_2 = \frac{w}{P_C} = \frac{P_I}{P_C} G_2 .$$

Thus,

$$N^D = N^D\left(\frac{w}{P_C}, \frac{P_I}{P_C}\right) \quad , \quad \frac{\partial N^D}{\partial \frac{w}{P_C}} > 0 \ , \quad \frac{\partial N^D}{\partial \frac{P_I}{P_C}} > 0 \ .$$

Graphically,





Labor is paid its marginal revenue product and the remainder of revenue (the marginal revenue product of capital) is distributed to shareholders. Letting E be the number of shares outstanding and P_E the per share price of equity, the rates of return on equity are

$$r_{E_C} = \frac{P_C F_1 K_C}{P_{E_I} E_C} \quad \text{and} \quad r_{E_I} = \frac{P_I G_1 K_I}{P_{E_C} E_I} \ .$$

These required rates of return will be less than the implicit rates of return on capital

$$r_{K_C} = \frac{P_C F_1}{P_I} \qquad\qquad r_{K_I} = G_1$$

if and only if the market value of the firm is greater than the replacement cost of its capital stock, i.e. if Tobin's q is greater than one:

$$q^C = \frac{P_{E_C} E_C}{P_I K_C} = \frac{r_{K_C}}{r_{E_C}}$$

$$q^I = \frac{P_{E_I} E_I}{P_I K_I} = \frac{r_{K_I}}{r_{E_I}} .$$

Thus, physical investment is given by

$$\Delta K^D = I^D = I_C^D(q^C - 1) + I_I^D(q^I - 1)$$

Such investment is financed entirely by the sale of new equity

$$E_C^S - E_C = \frac{P_I I_C^D}{P_{E_C}} , \quad E_I^S - E_I = \frac{P_I I_I^D}{P_{E_I}} .$$

<u>Government</u>

The government prints money and sells single period bonds (paying a rate of interest r) to finance its purchase of consumption and investment goods

$$M^S - M + B^S - B = P_C G_C + P_I G_I .$$

The physical units of money, consumption goods, and investment goods are treated as exogenous policy instruments with the bond supply residually determined by the government's budget constraint. Thus, an increase in G_C represents a bond financed increase in government purchases of consumption goods; an increase in M^S is similarly accomplished by the purchase of bonds.

Households

The supply of labor depends only upon the real wage, $N^S = N^S(w/P_C)$. While this is a powerful assumption which dichotomizes the demand and supply sides of the economy, it is a strong tradition in neoclassical macroeconomics. The introduction of money illusion or interest rate and Pigou effects to the labor supply would add a considerable amount of ambiguity to our results.

Household purchases of consumption goods, government bonds, equity, and money are constrained by wealth and factor income

$$Y + W = C + \frac{M^D}{P_C} + \frac{B^D}{P_C} + \frac{P_{E_C} E_C^D}{P_C} + \frac{P_{E_I} E_I^D}{P_C}$$

where

$$Y = Q_C + P_I Q_I / P_C$$

$$W = (M + B + P_{E_C} E_C + P_{E_I} E_I)/P_C \ .$$

We will assume that bonds and equities are perfect substitutes, so that $r = r_E^C = r_E^I$. The real demands will then be written as

$$C = C(r,Y,W)$$

$$\frac{M^D}{P_C} = L(r,Y,W)$$

$$\frac{B^D + P_E^C E_C^D + P_E^I E_I^D}{P_C} = H(r,Y,W) \ .$$

The adding up restrictions are that

$$\frac{\partial C}{\partial r} + \frac{\partial L}{\partial r} + \frac{\partial H}{\partial r} = 0$$

$$\frac{\partial C}{\partial Y} + \frac{\partial L}{\partial Y} + \frac{\partial H}{\partial Y} = 1$$

$$\frac{\partial C}{\partial W} + \frac{\partial L}{\partial W} + \frac{\partial H}{\partial W} = 1 .$$

We will initially assume that

$$\frac{\partial C}{\partial r} = 0 , \quad \frac{\partial L}{\partial r} < 0 , \quad \frac{\partial H}{\partial r} > 0$$

$$\frac{\partial C}{\partial Y} > 0 , \quad \frac{\partial L}{\partial Y} > 0 , \quad \frac{\partial H}{\partial Y} > 0$$

$$\frac{\partial C}{\partial W} = 0 \quad \frac{\partial L}{\partial W} = 0 , \quad \frac{\partial H}{\partial W} = 1 .$$

The restrictions $\partial C/\partial r = \partial C/\partial W = \partial L/\partial W = 0$ are frequently used in IS-LM analysis and are of some simplifying value. In Section II we will examine the effects of relaxing these assumptions.

The complete model can now be written as follows:

$$N_C\left(\frac{w}{P_C}\right) + N_I\left(\frac{w}{P_I}\right) = N^S\left(\frac{w}{P_C}\right)$$

$$Q_C = F\left[K_C, N_C\left(\frac{w}{P_C}\right)\right]$$

$$Q_I = G\left[K_I, N_I\left(\frac{w}{P_I}\right)\right]$$

$$Y = Q_C + \frac{P_I}{P_C}Q_I$$

$$C(Y) + G_C = Q_C$$

$$I^D\left(\frac{P_C}{P_I}\frac{F_1}{r}, \frac{G_1}{r}\right) + G_I = Q_I$$

$$L(r,Y) = \frac{M^S}{P_C}$$

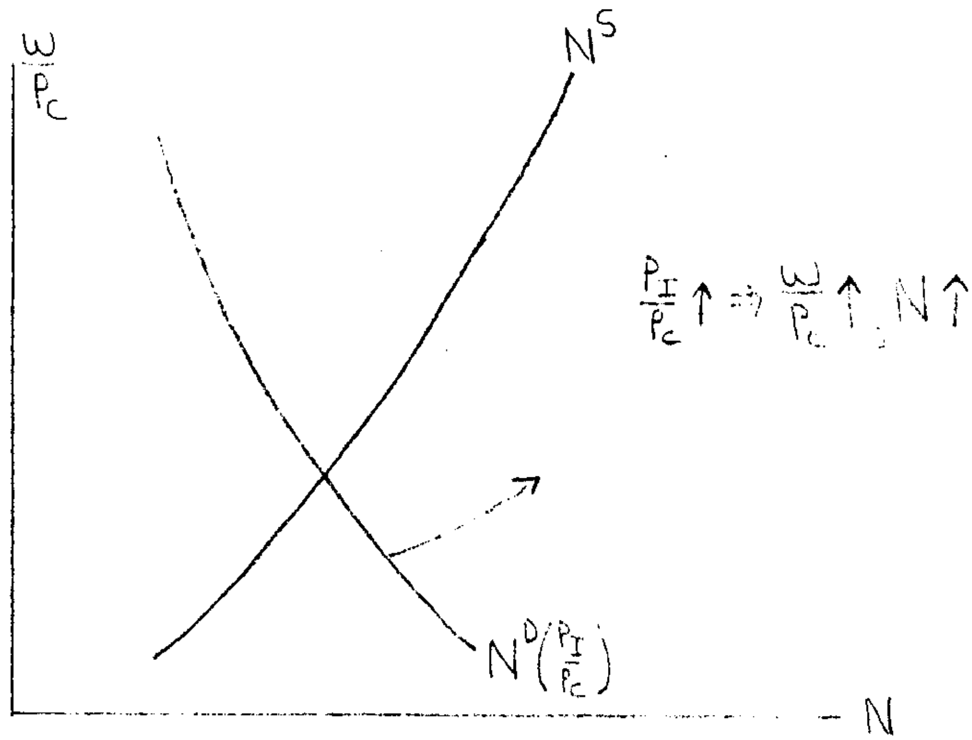
$$H(r,Y,W) = \left(G_C + \frac{P_I G_I}{P_C} + \frac{B+M-M^S}{P_C}\right) + \left(\frac{F_1 K_C}{r} + \frac{P_I}{P_C}\frac{G_1 K_I}{r} + \frac{P_I I^D}{P_C}\right)$$

$$W = \frac{M+B}{P_C} + \frac{F_1 K_C}{r} + \frac{P_I}{P_C}\frac{G_1 K_I}{r} .$$

Written in this way there are 7 exogenous variables (K_C, K_I, M, B, G_C, G_I, M^S) and 8 endogenous variables (w, P_C, P_I, r, Q_C, Q_I, Y, W). One of the commodity or financial market equilibrium conditions is redundant (Walras' Law) and we will follow tradition by deleting the bond-equity market.

The labor market and production function can be separately solved for the real wage, employment and output as functions of the relative price of investment and consumption goods.* Graphically,

*Notice that if labor were immobile, production and employment would be insulated from commodity demands and financial markets. In this case, w/P_C and consumer goods employment and output would be determined by equilibrium in the consumer goods labor market. Similarly, equilibrium in the investment goods labor market would determine w/P_I and investment employment and output. The allocation of income would only affect the level of prices and the relative prices of financial assets.



At a given level of w/P_C an increase in P_I relative to P_C increases the demand for workers by the investment goods industry. To hire more workers w/P_C is bid up; this increases the supply of labor and reduces the consumption goods sector's demand for labor. The new equilibrium is at a higher real wage with higher overall employment and lower employment in the consumption goods sector; employment in the investment goods industry has consequently risen. Mathematically,

$$0 < \frac{d\left(\frac{w}{P_C}\right) \Big/ \frac{w}{P_C}}{d\left(\frac{P_I}{P_C}\right) \Big/ \frac{P_I}{P_C}} = \frac{-\frac{dN_I}{d\frac{w}{P_I}} \frac{P_C}{P_I}}{\frac{dN^S}{d\left(\frac{w}{P_C}\right)} - \frac{\partial N_C}{\partial \frac{w}{P_C}} - \frac{\partial N_I}{\partial \frac{w}{P_I}} \frac{P_C}{P_I}} < 1$$

formally shows that a rise in P_I/P_C increases w/P_C and reduces w/P_I. We can consequently write

$$Q_C = Q_C\left(\frac{P_I}{P_C}\right) \qquad \frac{dQ_C}{d\frac{P_I}{P_C}} < 0$$

$$Q_I = Q_I\left(\frac{P_I}{P_C}\right) \qquad \frac{dQ_I}{d\frac{P_I}{P_C}} > 0 \; .$$

The net effect on income $Q_C + Q_I(P_I/P_C)$ is ambiguous, depending upon the capital intensities and size of the two sectors and the relative price ratio. In any case, it would be difficult to make welfare statements about changes in Y since we are only considering voluntary full employment situations and an intergenerational social welfare function need not weight consumption and investment goods by current prices.

The commodity and financial market equilibrium conditions can now be written as

$$C\left[Q_C\left(\frac{P_I}{P_C}\right) + \frac{P_I}{P_C}Q_I\left(\frac{P_I}{P_C}\right)\right] + G_C = Q_C\left(\frac{P_I}{P_C}\right)$$

$$I^D\left[\frac{P_I}{P_C}, r\right] + G_I = Q_I\left(\frac{P_I}{P_C}\right)$$

$$L\left[Q_C\left(\frac{P_I}{P_C}\right) + \frac{P_I}{P_C}Q_I\left(\frac{P_I}{P_C}\right), r\right] = \frac{M^S}{P_C} \; .$$

The only ambiguous sign is $\partial I^D/\partial(P_I/P_C)$. An increase in P_I/P_C increases employment, the marginal productivity of capital, and q in the investment goods industry. On the other hand, employment and the marginal productivity of capital fall in the consumer goods industry; this and the falling price ratio P_C/P_I both lower q for this industry.

We will here make the reasonable assumption that the effects on marginal productivity are minor, so that an increase in P_I/P_C unambiguously reduces the demand for investment goods relative to their supply. If an increase in P_I/P_C were to instead increase the demand for investment goods faster than the supply, this would suggest a potentially unstable system; for example, with this section's model the new equilibrium for a bond financed increase in government consumption spending would require a fall in the bond rate.

In this model there is a sharp distinction between the implications of commodity and financial decisions. Interest rates, relative commodity prices, employment, and output are completely determined in the commodity markets and unaffected by the demand and supply of money relative to bonds and equities. These financial decisions only determine the level of prices. The comparative statics reduced form equations are displayed below with all terms within parentheses being positive:

$$\Delta\frac{P_I}{P_C} = \frac{-\Delta G_C}{(A)}$$

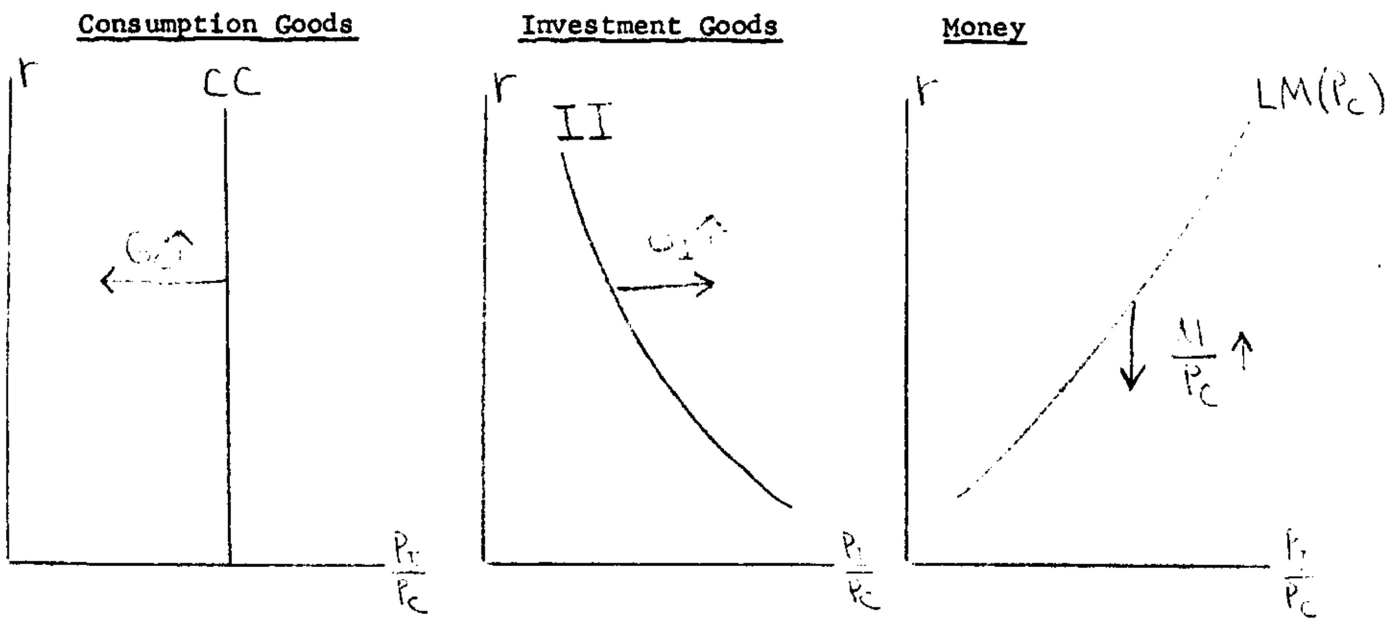
$$\Delta r = \frac{\partial G_I}{\left(-\frac{\partial I^D}{\partial r}\right)} + \frac{\left(\frac{\partial Q_I}{\partial \frac{P_I}{P_C}} - \frac{\partial I^D}{\partial \frac{P_I}{P_C}}\right)\Delta G_C}{\left(-\frac{\partial I^D}{\partial r}\right)(A)}$$

$$\Delta P_C = \frac{P_C \Delta M^S}{M^S} + \frac{\left(-\frac{\partial L}{\partial r}\right)\Delta G_I}{\left(-\frac{\partial I^D}{\partial r}\right)\left(\frac{M^S}{P_C^2}\right)} + \frac{\left[\left(\frac{\partial Q_I}{\partial \frac{P_I}{P_C}} - \frac{\partial I^D}{\partial \frac{P_I}{P_C}}\right)\left(-\frac{\partial L}{\partial r}\right) + \left(-\frac{\partial I^D}{\partial r}\right)\left(\frac{\partial L}{\partial r}\right)\left[\frac{\partial Y}{\partial \frac{P_I}{P_C}}\right]\right]\Delta G_C}{(A)\left(-\frac{\partial I^D}{\partial r}\right)\left(\frac{M^S}{P_C^2}\right)}$$

where

$$A = \left(1 - \frac{\partial C}{\partial Y}\right)\left(- \frac{\partial Q_C}{\partial \frac{P_I}{P_C}}\right) + Q_I + \frac{P_I}{P_C}\left(\frac{\partial Q_I}{\partial \frac{P_I}{P_C}}\right) .$$

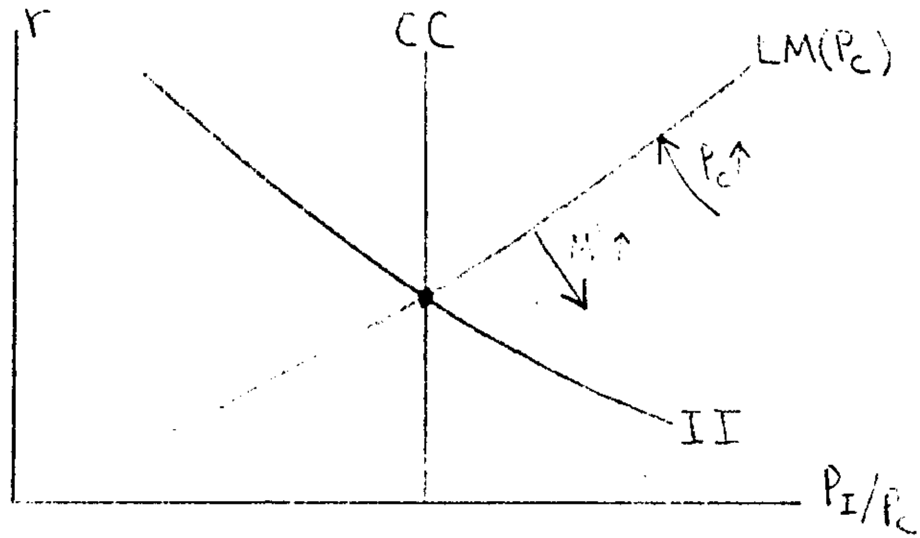
One graphical depiction of the three equilibria conditions is as follows:



The LM schedule has been drawn assuming $\partial Y/\partial(P_I/P_C) > 0$. The schedule would be horizontal if this term were zero and negatively sloped if the term were negative; in each case an increase in M^S/P_C would shift the LM curve downward.

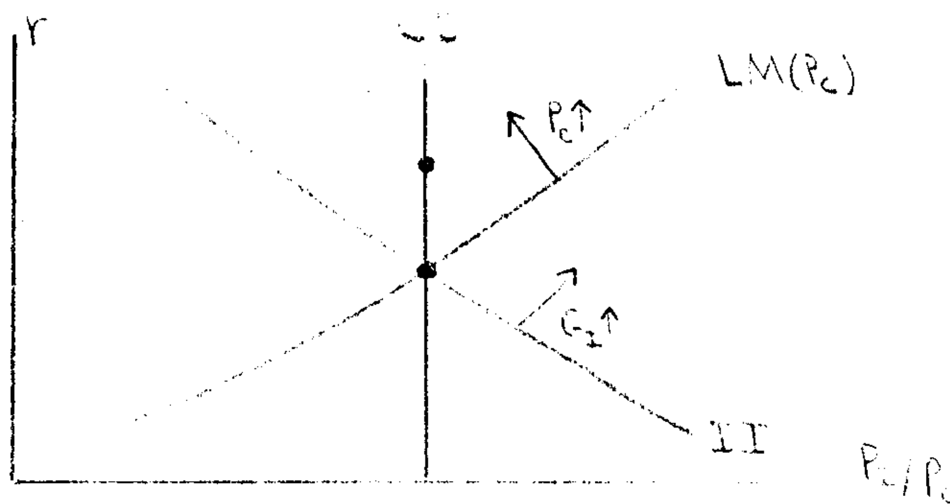
Notice that there is a complete sequential determination of the endogenous variables. Since consumption goods equilibrium depends only upon the relative price ratio P_I/P_C , this ratio is determined by this market. Investment goods equilibrium depends upon the interest rate and P_I/P_C . With the latter already determined, this market determines the interest rate. With P_I/P_C and r determined, the demand and supply of money fixes the level of prices.

A bond financed increase in the money supply consequently has no



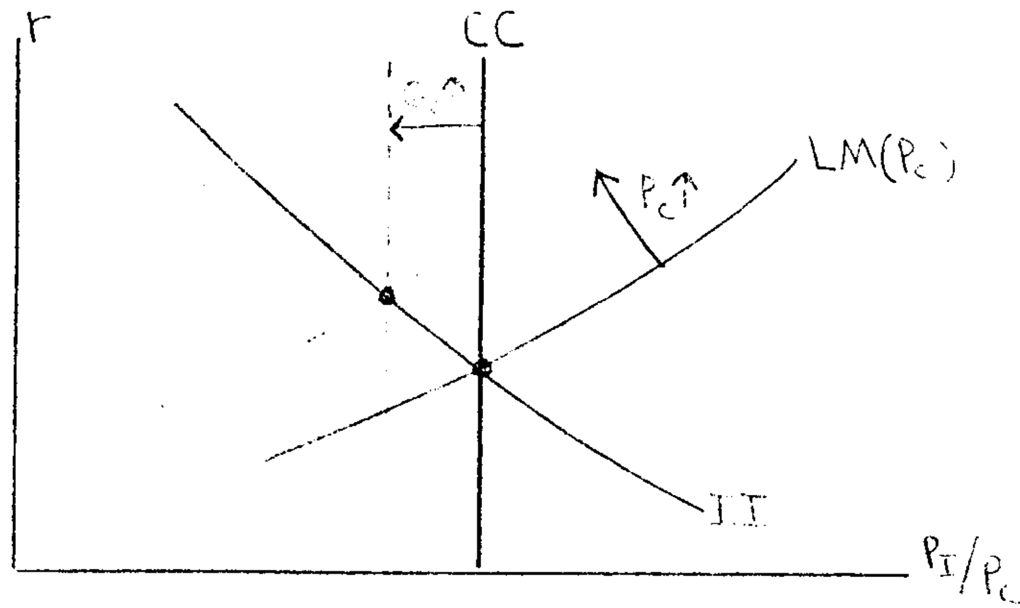
real effects, but only increases the level of prices proportionately.

A bond financed increase in government purchases of investment goods cannot affect P_I/P_C (and hence output) since there is only one



price ratio consistent with consumption goods equilibrium. Instead, the level of prices rises (reducing the real money supply) and interest rates rise, which reduces real money demand and discourages private investment. In the new equilibrium, the increased government purchases have fully crowded out an equal amount of private purchases of investment goods. If the government investment goods are unproductive (tanks and monuments) then there could be adverse future consequences from the private sector's reduced capital stock.

A bond financed increase in government purchases of consumption



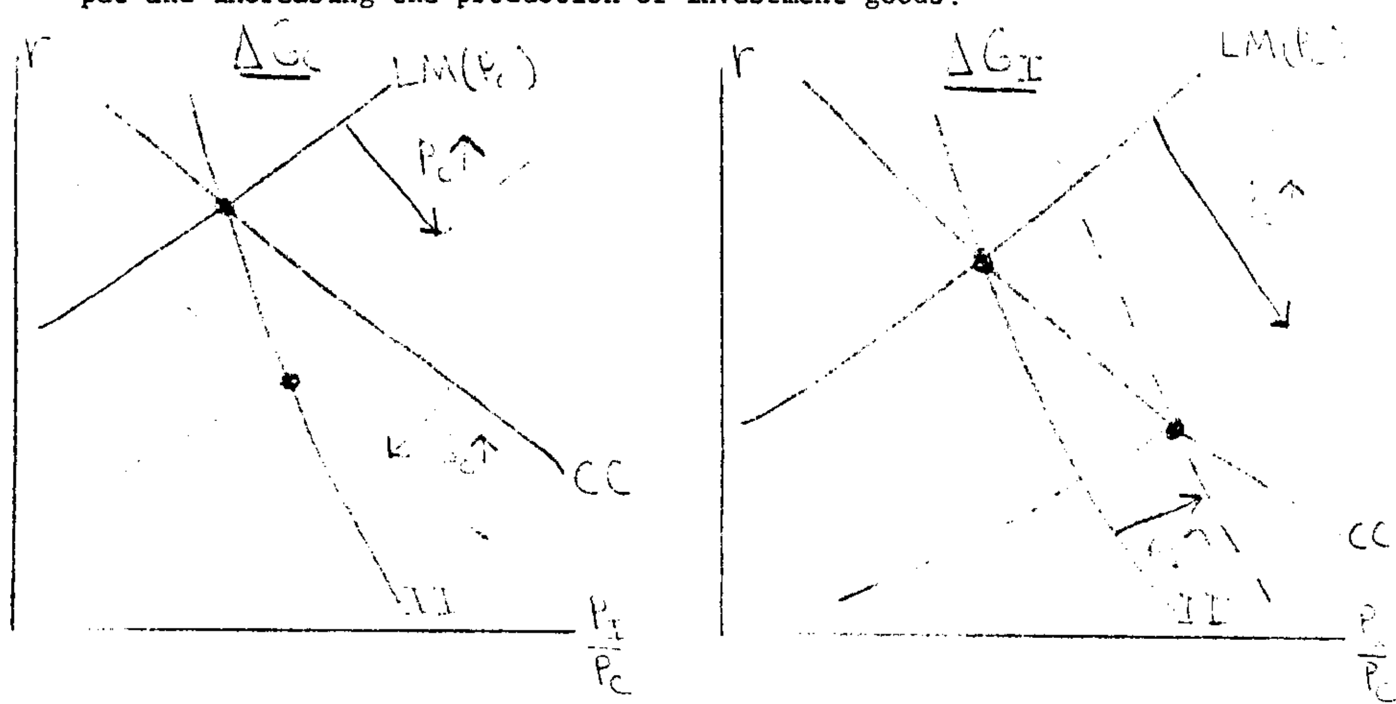
goods pushes up the price of consumption goods relative to investment goods. This stimulates the production of consumption goods and reduces the supply of investment goods relative to demand; a higher level of interest rates maintains investment goods equilibrium. On balance, employment falls and real income probably declines too. Higher interest rates and lower income reduce the real demand for money, and a higher commodity price level is necessary to reestablish equilibrium. If real income were to instead increase, equilibrium could require a lower level of commodity prices. (Formally, the LM curve would have to be negative sloped and steeper than the II curve; none of our other results depend upon the slope of the LM curve.)

II. The Extended Model

In this section, we will relax the assumption that consumption demand is unaffected by the interest rate and the level of prices. We will first consider separately interest elastic consumption demand since it is often argued that real balance effects are small. We will then analyze the complete case in which both prices and interest rates are important. Finally, we will examine the significance of the extreme assumptions regarding the interest elasticity of the demand for money and investment goods which are usually analyzed in one-sector IS-LM models.

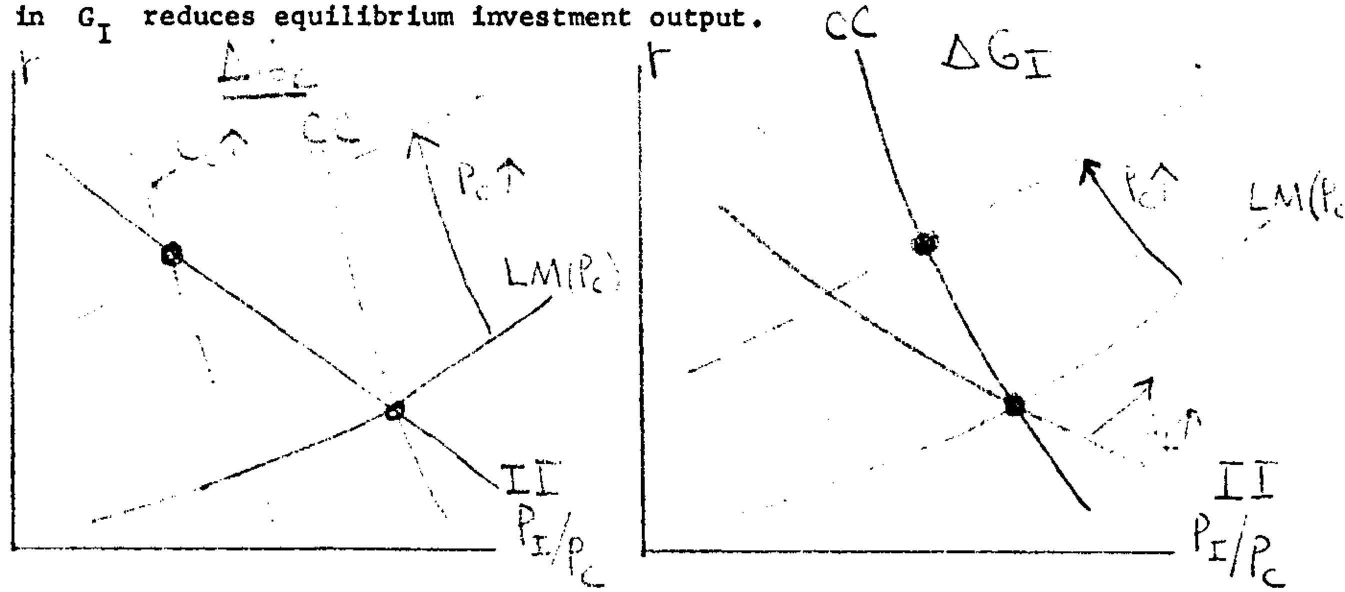
Interest Elastic Demand for Consumption Goods

If we allow interest rates to affect consumption demand, then the consumption and investment goods markets will simultaneously determine P_I/P_C and r. However, a variety of "perverse" cases can arise. If $\partial C/\partial r$ is sufficiently positive so that the CC curve is negatively sloped and flatter than the II curve, then any bond financed increase in government spending will lower interest rates and an increase in government consumption spending will raise P_I/P_C, lowering consumption output and increasing the production of investment goods:



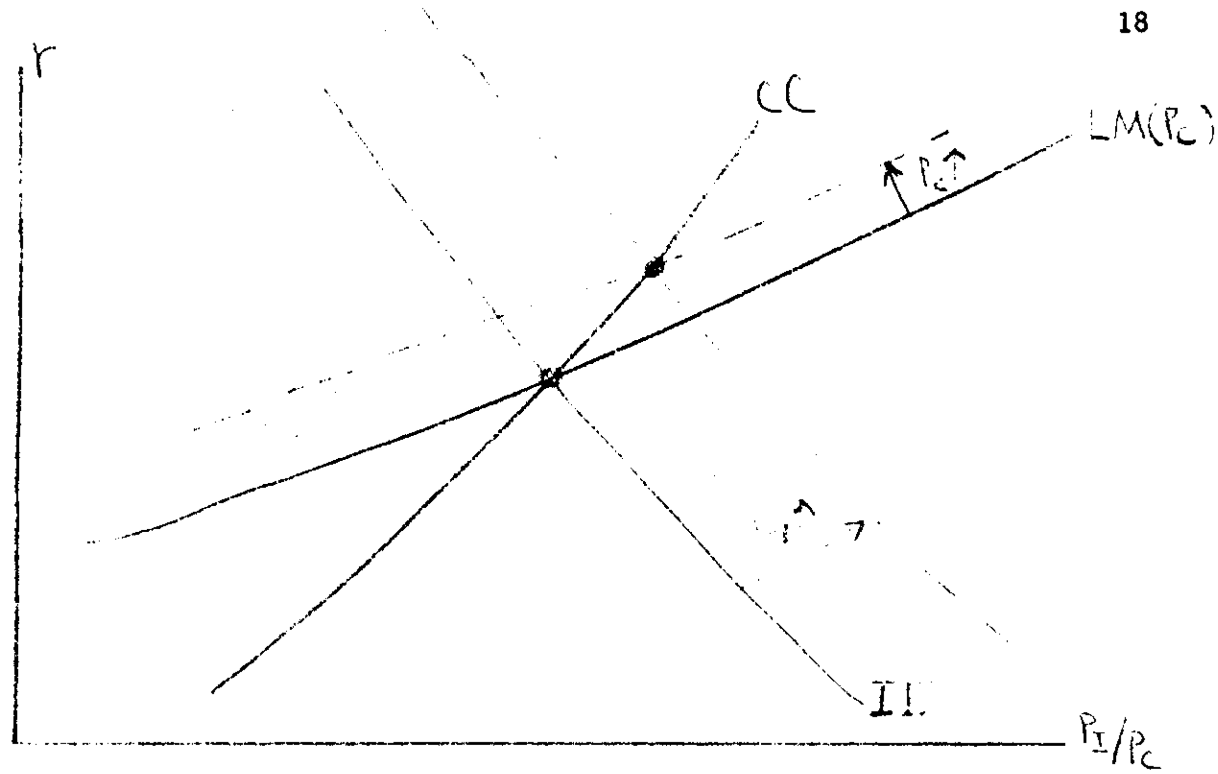
In addition, prices will fall except in the unlikely case where $\partial Y/\partial(P_I/P_C)$ is strongly negative; specifically, the LM curve would need to be negatively sloped and steeper than both the II and CC curves. All of these factors strongly suggest a potentially unstable model and the irrelevance of comparative statics.

A negatively sloped CC curve which is steeper than the II curve avoids these problems but introduces the perverse result that an increase in G_I reduces equilibrium investment output.

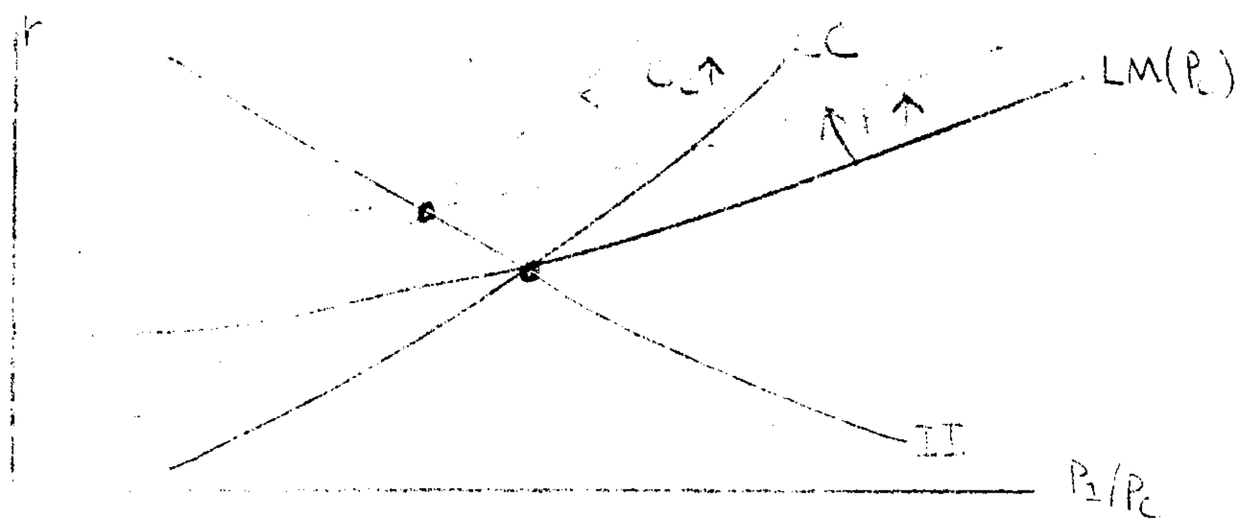


The most plausible comparative statics results are obtained when $\partial C/\partial r < 0$ so that the CC curve is positively sloped. Fortunately, this is also a very reasonable assumption if we take into account the fact that an increase in r reduces the real value of household wealth; this factor is discussed more fully below in connection with Pigou effects.

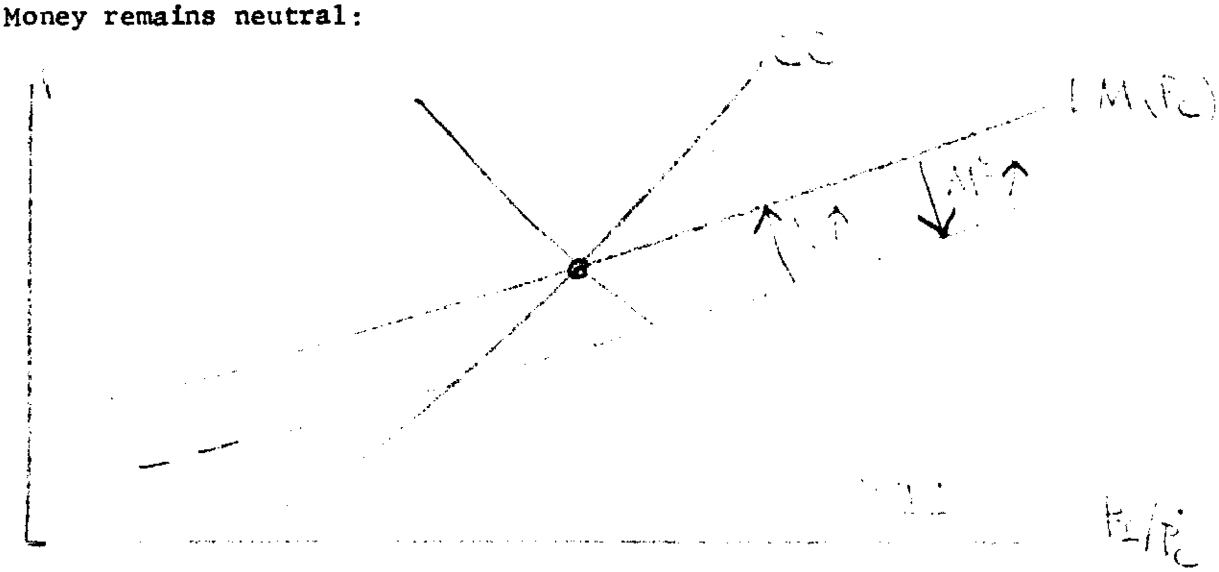
A bond financed increase in government investment spending now increases the relative price ratio P_I/P_C, encouraging the production of investment goods and depressing consumer goods output. Interest rates increase and the level of prices rise unless the LM curve is steeper than the CC curve.



A bond financed increase in consumer goods spending has qualitatively the same effects as when $\partial C/\partial r = 0$:



Money remains neutral:



Wealth Effects

Up until now, we have assumed that revaluations of real wealth

$$W = \frac{M+B}{P_C} + \frac{F_1 K_C}{r} + \frac{P_I}{P_C}\frac{G_1 K_I}{r}$$

affect only the demand for bonds and equities and do not alter the real demands for money and consumer goods. We shall see in this section that the major consequence of relaxing this assumption is to allow the level of prices and hence monetary policy to influence the commodity markets.

The partial effects on real wealth of an increase in the interest rate or the price of consumer goods are both clearly negative. The partial effect of an increase in relative prices

$$\frac{\partial W}{\partial \frac{P_I}{P_C}} = \left(-\frac{\partial F_1}{\partial N_C}\right)\left(-\frac{\partial N_C}{\partial \frac{P_I}{P_C}}\right)\frac{K_C}{r} + \left(\frac{\partial G_1}{\partial N_I}\right)\left(\frac{\partial N_I}{\partial \frac{P_I}{P_C}}\right)\frac{P_I}{P_C}\frac{G_1 K_I}{r} + \frac{G_1 K_I}{r}$$

is formally ambiguous. We will again assume that the effects on marginal productivity are minor, so that $\partial W/\partial(P_I/P_C) > 0$.

The incorporation of wealth into the demands for money and commodities

$$C\left[Q_C\left(\frac{P_I}{P_C}\right) + \frac{P_I}{P_C}Q_I\left(\frac{P_I}{P_C}\right),\ W\right] + G_C = Q_C\left(\frac{P_I}{P_C}\right)$$

$$I\left[r,\ \frac{P_I}{P_C}\right] + G_I = Q_I\left(\frac{P_I}{P_C}\right)$$

$$L\left[Q_C\left(\frac{P_I}{P_C}\right) + \frac{P_I}{P_C}Q_I\left(\frac{P_I}{P_C}\right),\ r,\ W\right] = \frac{M^S}{P_C}$$

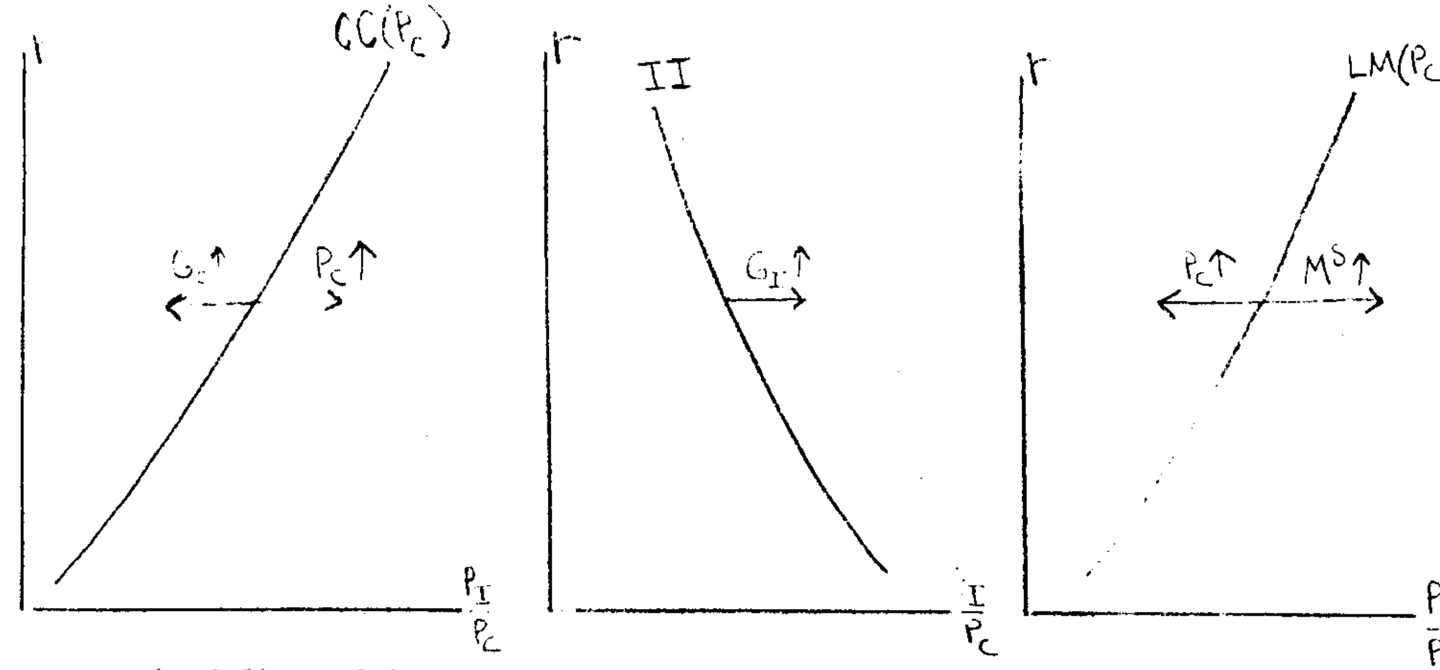
$$W = \frac{M+B}{P_C} + \frac{F_1 K_C}{r} + \frac{P_I}{P_C}\frac{G_1 K_I}{r}$$

introduces the level of consumer goods prices into the consumer goods equilibrium condition and reinforces our previously assumed sign restrictions with one exception. An increase in P_C lowers the real supply of money but now also lowers the real demand by reducing real wealth. The net effect

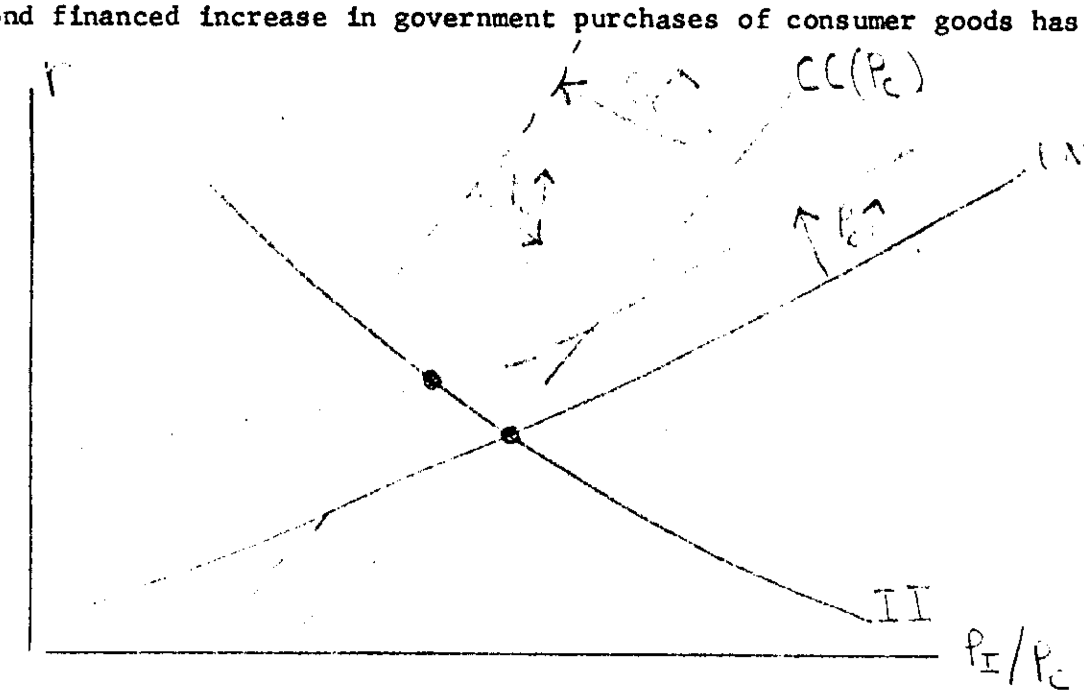
$$\frac{\partial(L - M^S/P_C)}{\partial P_C} = \frac{M^S - \frac{\partial L}{\partial W}(M+B)}{P_C^2}$$

is formally ambiguous. In the usual single commodity IS-LM analysis with Pigou effects this term is assumed positive on the grounds that $\partial L/\partial W$ is small--in particular that it is smaller than $M^S/(M+B)$. The contrary case results in a variety of perverse comparative statics results. If the term is slightly negative, then in our model a bond financed increase in G_C reduces equilibrium interest rates and consumption output; if the term is strongly negative, then these peculiarities vanish but a bond financed increase in the money supply raises equilibrium interest rates, reduces income and reduces both commodity prices. We will only give serious attention to the comparative statics results for the conventional case where $\partial(L - M^S/P_C)/\partial P_C > 0$.

The equilibrium conditions can be graphically represented as follows:

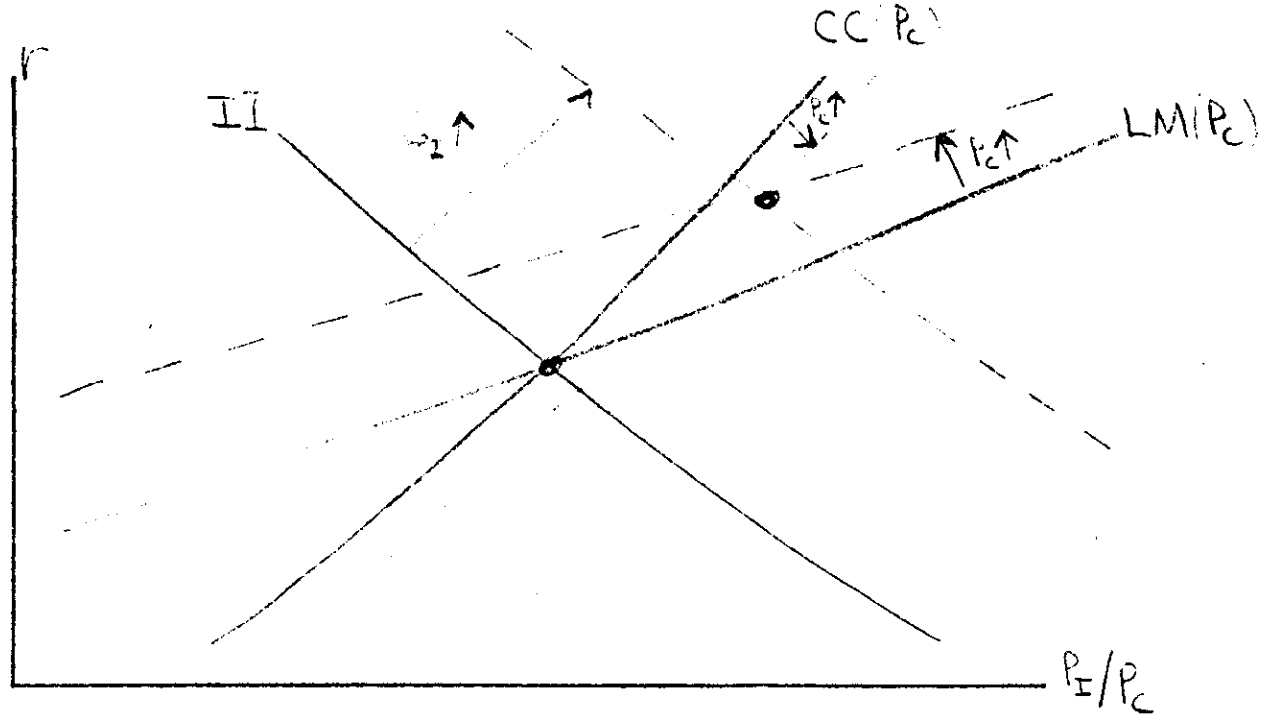


A bond financed increase in government purchases of consumer goods has



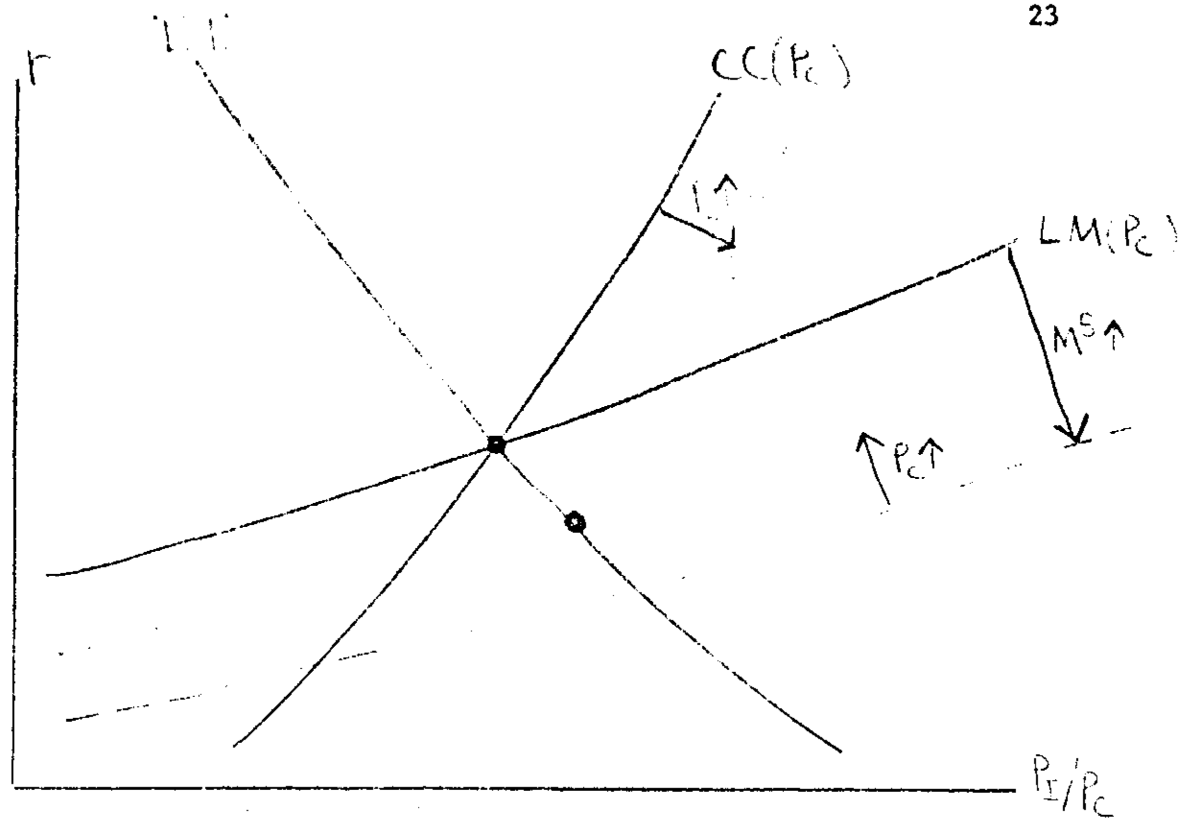
the same qualitative effects as in the case of an interest elastic demand for consumption goods with no wealth effects. The increase in G_C raises P_C relative to P_I, stimulating consumption goods output and depressing the production of investment goods. The demand for investment goods is reduced by higher interest rates. An increase in the level of consumer goods prices reduces the real money supply to a level consistent with the reduced demand. The allowance for wealth effects does reduce the magnitude of the changes in the endogenous variables since the increase in G_C is partly offset by the reduced private consumption resulting from the depressing effects of higher P_C, higher r, and lower P_I/P_C on real wealth.

A bond financed increase in government purchases of investment goods raises the relative price ratio, increasing the production of



investment goods and reducing that of consumption goods. The higher level of interest rates crowds out some private investment, but not enough to fully offset the increased government purchases. The price of consumer goods rises when the CC curve is steeper than the LM curve and falls when it is flatter. The only qualitative importance of the wealth effects is to more vigorously motivate the interest elasticity of consumer goods spending, which provides for less than complete crowding out.

A bond financed increase in the money supply now lowers interest rates (stimulating consumption and investment spending) and raises the level of prices (depressing consumer goods spending); on balance the

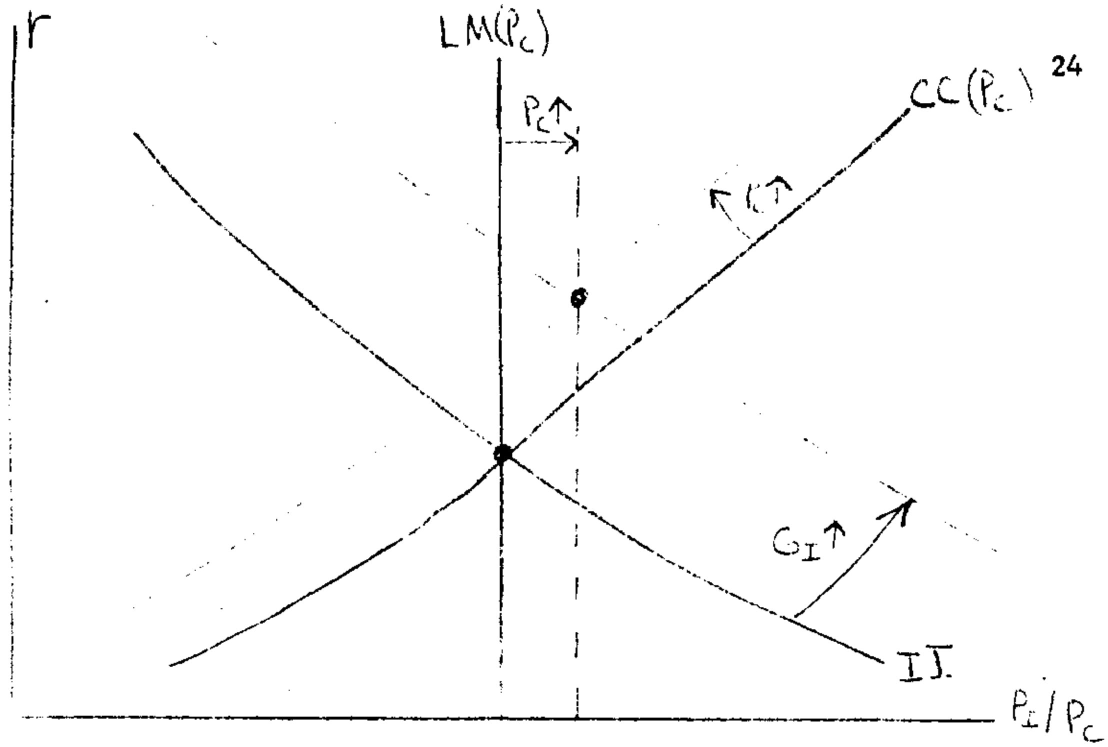


relative price ratio P_I/P_C increases, which reduces consumer goods output and increases investment production. Total employment expands. The Pigou effect directly eliminates the nonneutrality of money by allowing the level of prices as well as relative prices to influence commodity markets.

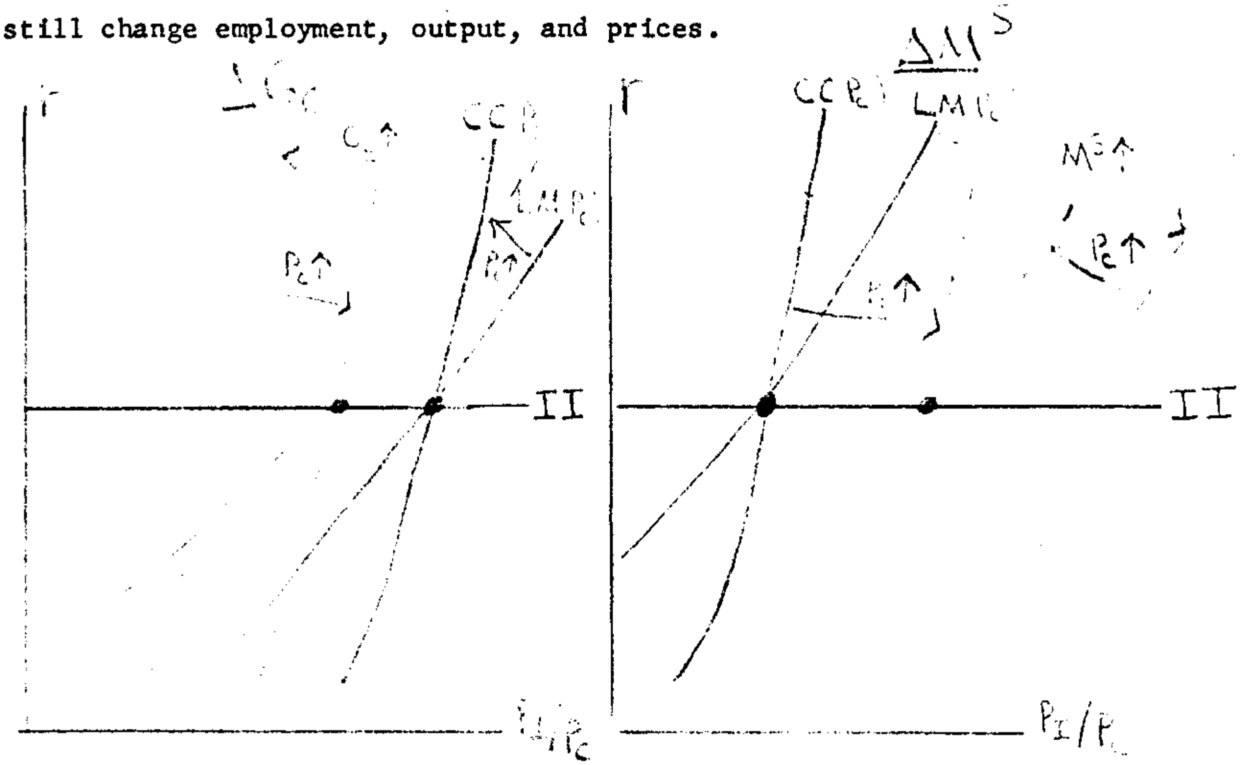
Special Cases

In the single commodity IS-LM model, frequently mention is made of two extreme cases ($\partial L/\partial r = 0$ and $\partial I/\partial r = -\infty$) which emasculate fiscal policy and two opposite extremes ($\partial L/\partial r = -\infty$ and $\partial I/\partial r = 0$) in which monetary policy is impotent. In the two commodity model, this contrast is considerably more clouded.

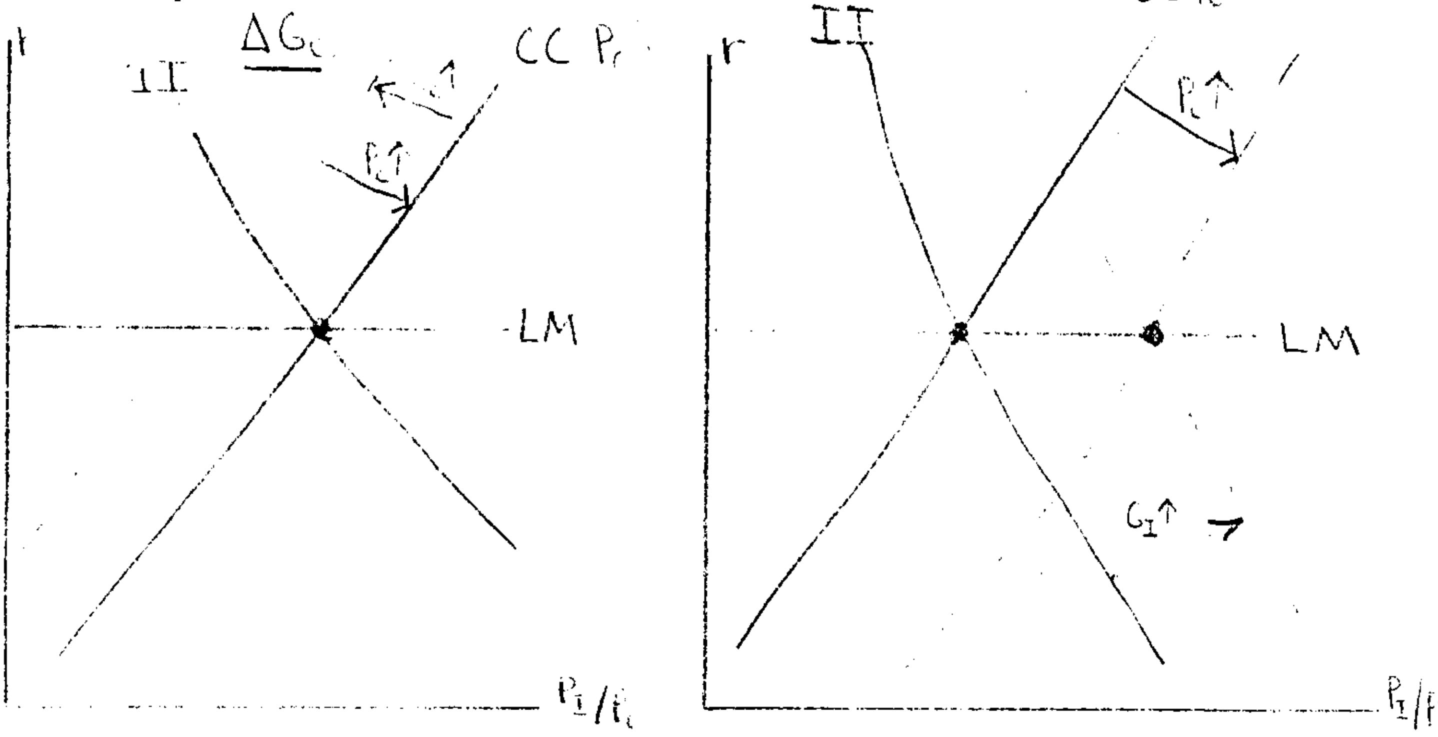
The only qualitative effect of an interest insensitive demand for money ($\partial L/\partial r = 0$) is to insure that the LM curve is steeper than the CC curve, which means that a bond financed increase in government investment spending unambiguously lowers the price of consumer goods:



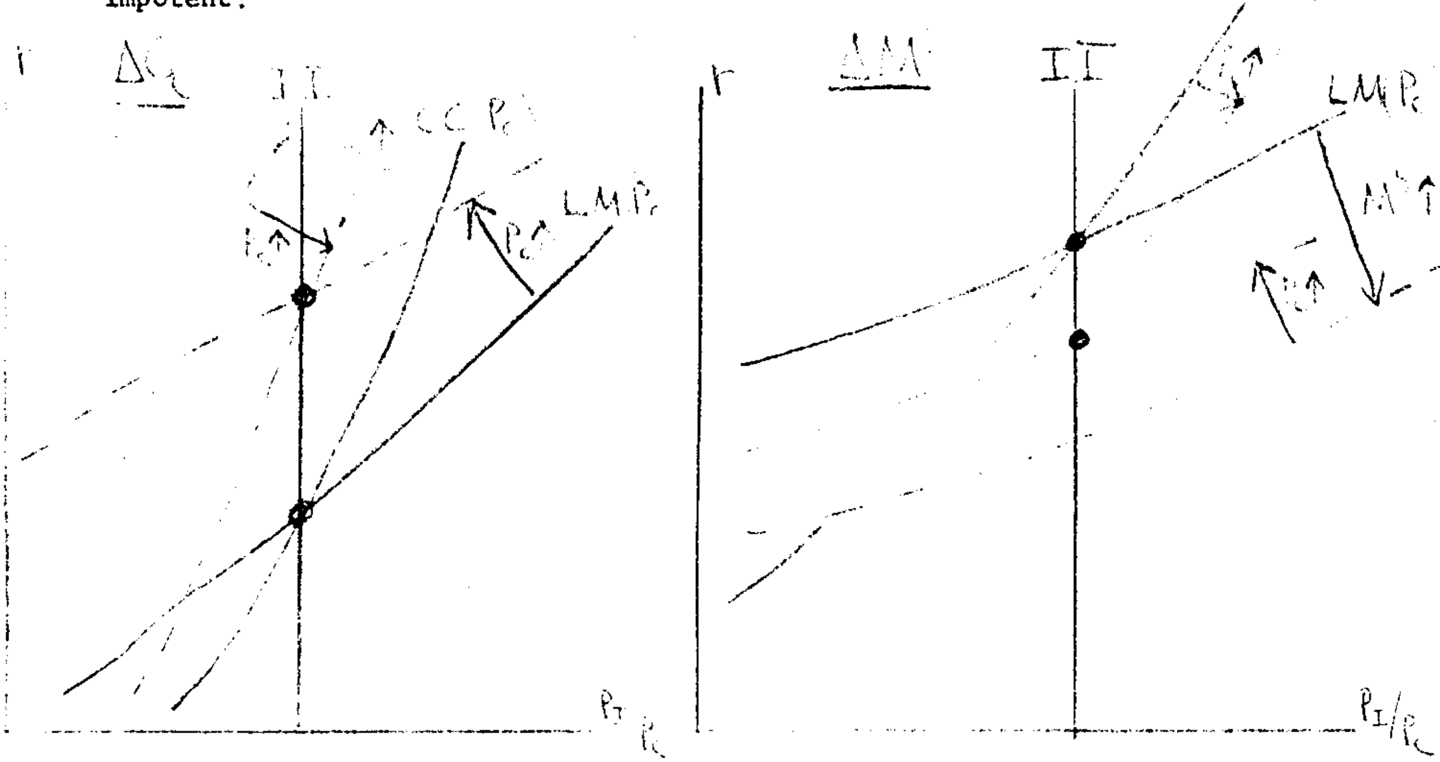
An infinitely interest elastic investment demand ($\partial I/\partial r = -\infty$) does fix the interest rate and emasculate government investment purchases. However, government consumer goods purchases as well as monetary policy can still change employment, output, and prices.



A liquidity trap ($\partial L/\partial r = -\infty$) would also fix the interest rate. Here monetary policy would have no effect and government consumption purchases would only raise the level of prices, while investment purchases are powerful:



Finally, interest insensitive investment $(I_r = 0)$ makes investment purchases powerful and monetary policy and consumer goods purchases impotent:



III. Fixed Nominal Wages

A common device for introducing unemployment into macro models is to assume that the nominal wage is exogenously set at some level w_0 at which the supply of labor is greater than the demand. Actual employment is then assumed to be equal to demand

$$N_C = N_C^D\left(\frac{w_0}{P_C}\right) \qquad N_I = N_I^D\left(\frac{w_0}{P_I}\right)$$

so that the output of each commodity is positively related to the price of the commodity

$$Q_C = Q_C(P_C) \qquad \frac{dQ_C}{dP_C} > 0$$

$$Q_I = Q_I(P_I) \qquad \frac{dQ_I}{dP_I} > 0 \ .$$

We will consequently now work in terms of the three market clearing variables r , P_C and P_I . The real market value of wealth

$$W = \frac{M+B}{P_C} + \frac{F_1(P_C)K_C}{r} + \frac{P_I}{P_C}\frac{G_1(P_I)K_I}{r}$$

is negatively related to r and positively related to P_I . An increase in P_C will raise the marginal productivity of capital in the consumer goods industry (by increasing employment in this industry) but lower the purchasing power of money, bonds, and equity in the investment goods industry. We will again assume that the changes in marginal product are minor and that $\partial W/\partial P_C < 0$. There is also an ambiguity in investment demand

$$I^D = I^D\left[\frac{P_C}{P_I}\frac{F_1(P_C)}{r}, \frac{G_1(P_I)}{r}\right] , \quad \frac{\partial I^D}{\partial r} < 0 , \quad \frac{\partial I^D}{\partial P_C} > 0 .$$

We will once again assume that G_1 is sufficiently inelastic so that $\partial I^D/\partial P_I$ is negative or, if positive, less than $\partial Q_I/\partial P_I$. Finally, we will as before make the usual assumption that an increase in P_C raises the demand for money relative to the supply.

With these assumptions, the total differentiation of the model

$$C(Y,r,W) + G_C = Q_C(P_C)$$

$$I(P_C, P_I, r) + G_I = Q_I(P_I)$$

$$L(Y,r,W) = M^S/P_C$$

where $Y = Q_C(P_C) + P_I Q_I(P_I)/P_C$ and W is as previously defined, yields

$$\begin{bmatrix} -a_{11} & a_{12} & -a_{13} \\ a_{21} & -a_{22} & -a_{23} \\ a_{31} & a_{32} & -a_{33} \end{bmatrix} \begin{bmatrix} \Delta P_C \\ \Delta P_I \\ r \end{bmatrix} = \begin{bmatrix} -\Delta G_C \\ -\Delta G_I \\ \frac{1}{P_C}\Delta M^S \end{bmatrix}$$

where each of the elements a_{ij} is positive. The determinant

$$|A| = -a_{13}(a_{21}a_{32} + a_{22}a_{31}) - a_{23}(a_{11}a_{32} + a_{12}a_{31})$$
$$- a_{33}(a_{11}a_{22} - a_{12}a_{21})$$

will be assumed negative since if it were positive an increase in G_C would lower equilibrium consumer goods output, an increase in G_I would lower investment goods output, and an increase in M^S would reduce the prices and production of both commodities. The comparative statics are displayed below:

$$\begin{bmatrix} \Delta P_C \\ \Delta P_I \\ \Delta r \end{bmatrix} = \frac{-1}{|A|} \begin{bmatrix} a_{22}a_{33}+a_{23}a_{32} & a_{13}a_{33}-a_{13}a_{32} & a_{12}a_{23}+a_{13}a_{22} \\ a_{21}a_{33}-a_{23}a_{31} & a_{11}a_{33}+a_{13}a_{31} & a_{11}a_{23}+a_{13}a_{21} \\ a_{21}a_{32}+a_{22}a_{31} & a_{11}a_{32}+a_{12}a_{31} & a_{12}a_{21}-a_{11}a_{22} \end{bmatrix} \begin{bmatrix} \Delta G_C \\ \Delta G_I \\ \Delta M^S/P_C \end{bmatrix}$$

A bond financed increase in government spending increases interest rates and production of the commodity purchased, and has ambiguous effects on the output of the other commodity. For an intuitive explanation of these results, consider an increase in government consumption purchases. The excess demand for consumption goods puts upward pressure on their price, driving down real wages and increasing production in this sector. Higher consumer goods prices and employment increase this sector's demand for capital. At the same time, the bond financing creates an excess supply of bonds, raising the interest rate on bonds. Since equity is a perfect substitute with bonds, equity prices fall and this lower valuation (and higher required return) discourages investment. The net effect on investment demand (and hence price and production) is ambiguous. There are of course mixed fiscal policies involving both consumption and investment purchases which stimulate both sectors.

When the government prints money in order to purchase bonds, the new equilibrium is with higher prices and production of both commodities. Interest rates may be either higher or lower; if they are sufficiently higher, however, the determinant $|A|$ will become positive and we will have the many implausible comparative statics results noted earlier. Such open market purchases will directly lower the bond rate, reducing the required rate and increasing the price of equity. This will stimulate both consumption and investment, driving up the price of each commodity. The increased price and employment in the consumer goods sector will further increase the demand for investment goods; similarly, the increased price and employment in the investment sector will raise real income and wealth, leading to more consumption spending. If these cross effects do not dominate the direct effects,* then a higher level of P_C and P_I will restore equilibrium in both commodity markets. Since these price increases and the lower interest rate raise the demand for money relative to supply, they can give a full four market equilibrium.

If, however, the indirect effects dominate then some qualitatively different equilibria are possible. One possibility is that r, P_I, and P_C will all fall; this corresponds to $|A| > 0$ and since the fiscal comparative statics are also implausible we have dismissed it as probably unstable. The other possibility is that r, P_I, and P_C will all rise; while it is not as difficult to imagine how such an equilibria might be reached, there is still reason to be concerned about stability.

Finally, we will briefly note the usual special cases. $\partial L/\partial r = 0$ insures that government consumption spending does reduce investment production and that government investment reduces consumption production. At the other extreme, $\partial L/\partial r = -\infty$ insures that government spending stimulates both sectors and that monetary policy is impotent. $\partial I/\partial r = -\infty$

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*Specifically, holding r constant an increase in P_C has the net effect of raising the supply of consumption goods and the demand for investment while an increase in P_I raises consumption demand and investment supply. Thus in both markets, a higher level of P_I requires a higher level of P_C to maintain equilibrium. The case of "dominant direct effects" is where for a given increase in P_I it takes less of an increase in P_C to restore equilibrium in the consumer goods market than is necessary in the investment goods market.

completely immobilizes government investment and also implies that government consumption reduces investment, $\partial I/\partial r = 0$ reverses these results.

The version of Benavie's recent model in which bonds and equities are perfect substitutes is very similar to the one we have analyzed in this section, and the only qualitative difference in our presented multipliers is that he finds $\partial P_C/\partial G_I$ to be unambiguously positive. There are however a few differences in our approaches which merit discussion.

He works with a continuous model in which asset demands are constrained by current asset stocks. As a consequence, when he assumes that the demand for money is positively related to income, he is forced to assume that the demand for bonds plus equity is negatively related to income. However, the motives for holding money in a continuous model are not at all clearcut, and reversing his sign assumption would modify some of his results. In our discrete model, asset demands are constrained by wealth plus saving, and each can be positively related to income.

Similarly, Benavie does not allow corporate and government saving to affect asset markets. Thus investment has no effect on the supply of equity and government spending has no effect on the supply of money or bonds. One of the consequences of this is that it makes no difference to the equilibria whether government spending is financed by printing money or selling bonds. In contrast, stock and flow equilibria in our model are identical and all sectoral budget constraints are respected.

Benavie also differs from us in assuming that consumption is interest inelastic despite the dependence of real wealth on interest rates. It is because of our assumption that $\partial C/\partial r < 0$, that we find a bond financed increase in government investment purchases to have an ambiguous effect on the production of consumption goods. Benavie argues that his restriction "is justified by the failure of [the interest rate] to appear as a significant predictor in studies of consumption spending." On the contrary, the market value of wealth is widely used in consumption functions and is on both theoretical and empirical grounds strongly dependent upon interest rates.

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