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### A Theory of Money and Financial Institutions. Part 26. On the Number of Types of Markets with Trade in Money

Martin Shubik

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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART 26

ON THE NUMBER OF TYPES OF MARKETS WITH TRADE IN MONEY

Martin Shubik

January 14, 1976

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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART 26

ON THE NUMBER OF TYPES OF MARKETS WITH TRADE IN MONEY

by

Martin Shubik\*\*

1. INTRODUCTION

In several previous papers<sup>6, 3, 4, 5</sup> models of closed exchange economies with trading using a money<sup>\*\*\*</sup> have been built. This paper explores the number of different market structures that are to be expected under certain reasonable restrictions on the nature of trade. In particular we limit ourselves to markets without bargaining, haggling or recontracting mechanisms. A move by a trader will be construed as a bid or offer which is accepted or rejected.

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\*\*The author wishes to thank Professors Schmeidler, Postlethwaite and Okuno for several valuable conversations.

\*\*\*These can also include credit, but for simplicity credit is not discussed further here.

## 2. BIDS, OFFERS AND MARKETS

Consider an economy with  $m$  commodities and a money (which may or may not be a commodity). We consider a market to consist of a set of buyers, a set of sellers and a mechanism which matches their bids and offers.

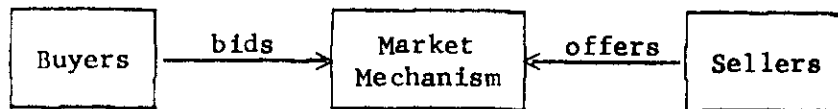


FIGURE 1

We assume that all buyers and sellers bid simultaneously and independently. Furthermore no complex contingent bids or offers are considered.\*

It is assumed that markets are for single commodities and that the method and mechanism of exchange in each market is the same.

A buyer must use money to bid. He may specify a price or accept the market price and he may specify a specific quantity or buy what he can. Limiting ourselves to money payments and to specifications concerning price and quantity to be purchased we have:

### 2.1. Bids

#### (1) A Money Bid

A trader  $i$  bids an amount of money  $b_j^i$  for the  $j^{\text{th}}$  commodity. He has no reserve price and takes what the market gives him. This provides an extremely simple quantity bid that enables us to construct a mechanism similar to that of Cournot.

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\*The meaning of this will be made clear in 2.5.

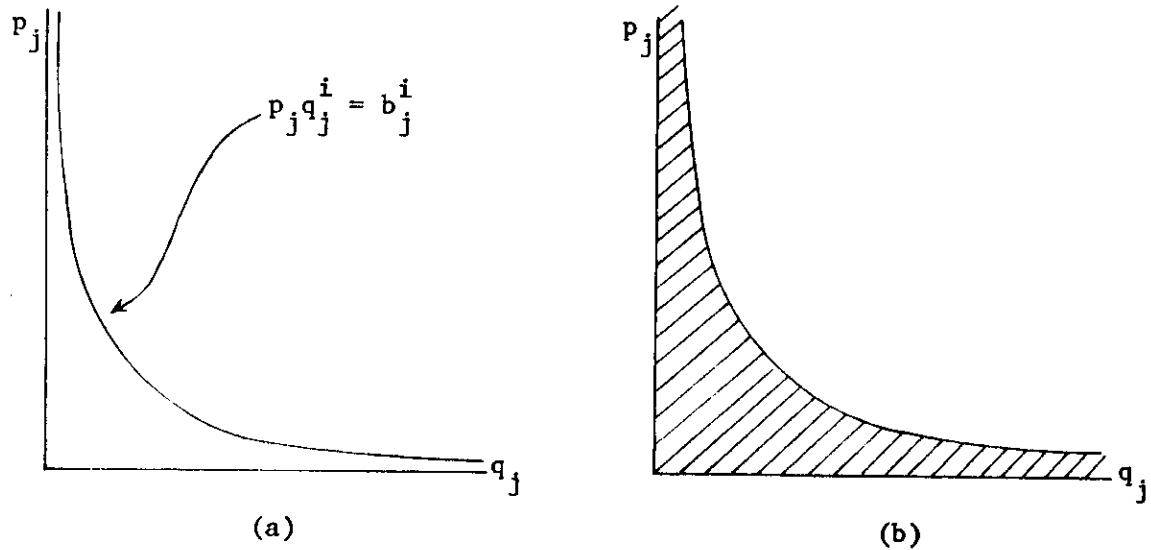


FIGURE 2

The market clearing mechanism will give the trader an amount  $q_j^i$  which lies on the hyperbola  $p_j q_j^i = b_j^i$  as is shown in Figure 2a (Model 1.1).

Suppose that the trader bids  $b_j^i$ , but agrees to trade for less than all of the money bid. The feasible set of trades includes the shaded area in Figure 2b (Model 1.2).

There are nine minor variations of these two models to take into account whether or not upper or lower bounds are imposed on prices or quantities which will be accepted by the bidder.

$p_j \backslash q_j^i$	u.b.	l.b.	none
u.b.			
l.b.			
none			

TABLE 1

These variations truncate the range of the hyperbola.

(2) The Price-Quantity Bid

Suppose that a trader offers a price  $p_j^i$  for an amount  $q_j^i$ . It is reasonable to expect that he is willing to buy  $q_j^i$  for a price less than  $p_j^i$  hence the outcomes acceptable to him are indicated by the line AB in Figure 3a (Model 2.1).

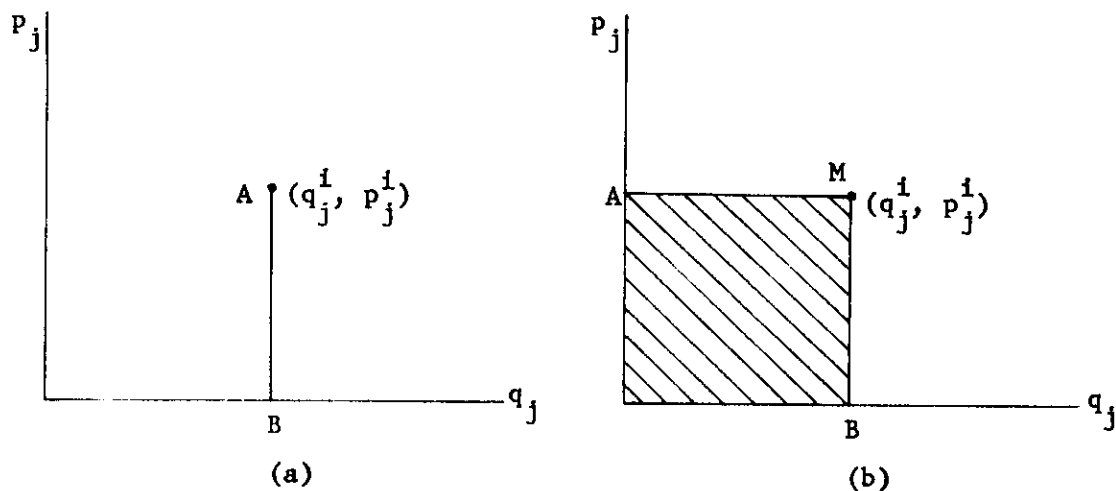


FIGURE 3

There is an implicit limit in this bid inasmuch as  $q_j^i, p_j^i$  must be less than or equal to the credit line and cash of the individual.

It is possible that the trader will accept an amount less than  $q_j^i$ , if he is unable to fill his total order. If he is willing to take a partial fulfillment then any outcome in the shaded rectangle indicated in Figure 3b is feasible (Model 2.2).

If the bidder has to expose his bid to the seller (as, for example in an advertisement for purchase) the bid may be at a single price  $p_j^i$  for a specific quantity  $q_j^i$  (Model 2.3) this is shown by the point M in Figure 3b. A single price bid may be for any amount up to some limit. This is shown by the line AM in Figure 3b (Model 2.4).

(3) The Price Bid

The trader states a price  $p_j^i$  and stands willing to buy any amount he can obtain at  $p_j^i$  or less. This has an implicit bound in it as his expenditures cannot exceed his cash and credit. Any point in Figure 4 is feasible. The point M is where the credit constraint becomes relevant (Model 3.1).

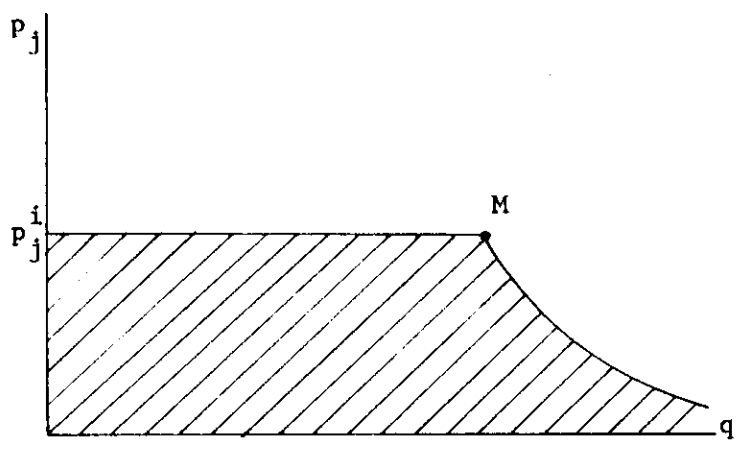


FIGURE 4

We can modify this bid by introducing an upper or lower bound on the quantity to be accepted (or both a lower and upper bound).

(4) The Quantity Bid

The individual bids for a quantity  $q_j^i$  at any price. If credit constraints are introduced this gives the same model as Model 2.1.

The trader could bid for  $q_j^i$  or less. This gives as a feasible set of outcomes those shown in Figure 5 (Model 4.1).



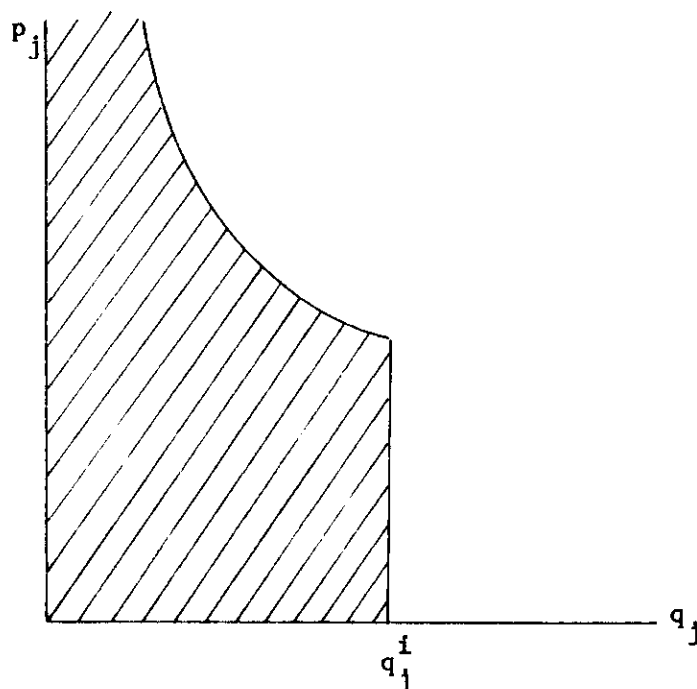


FIGURE 5

## 2.2. Offers

The seller or offerer does not have quite the same possibilities as a buyer. It is theoretically possible that he could announce a willingness to sell any amount  $q_j^i$  at any price  $p_j^i$  so long as  $q_j^i p_j^i \geq M^i$  where  $M^i$  were his monetary requirement. However it appears more reasonable to restrict the seller to price and/or quantity offers of the commodity being sold, whereas the buyer more naturally has bids involving not only price and the quantity of the commodity but also or alternatively the quantity of money.

### (1) The Price Quantity Offer

The seller may offer  $q_j^i$  at  $p_j^i$  or more. This is shown in Figure 6a (Model 5.1). Alternatively his offer could be for the sale of  $q_j^i$  or less at  $p_j^i$  or more as is shown in Figure 6b (Model 5.2).

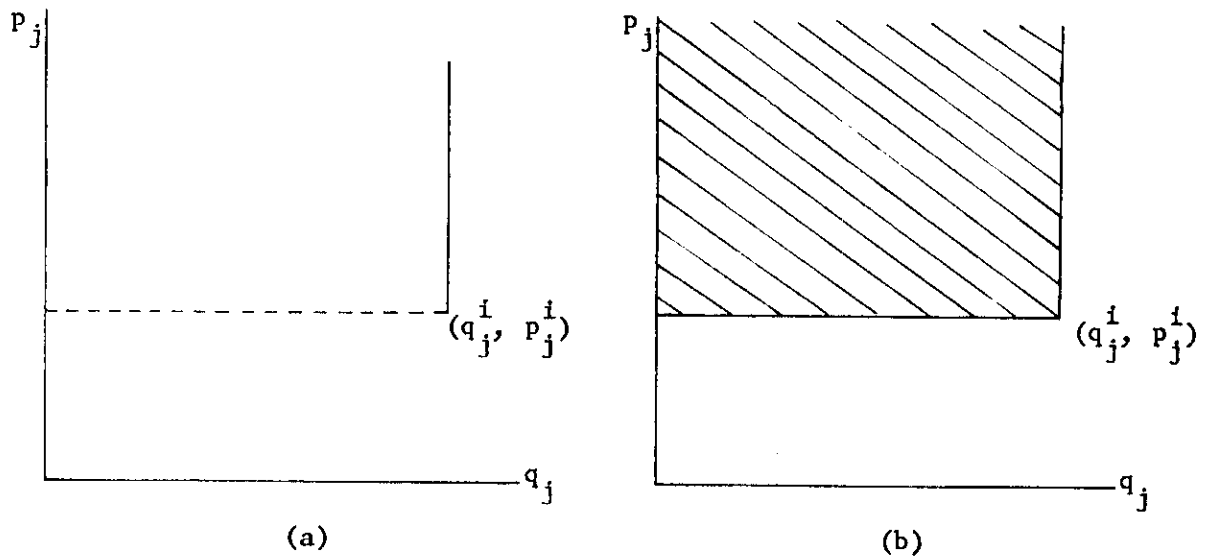


FIGURE 6

(2) The Price Offer

A seller offers any amount at  $p_j^i$  or above. This offer must be modified by the limit imposed by the amount of the commodity he has available. When this limit is imposed we have essentially Model 5.2.

If the seller's offer is disclosed ahead of time to the buyer then we may wish to consider a simple statement of price  $p_j^i$ . This is shown in Figure 7. The outcomes are restricted to the line  $AM$ . Most retail

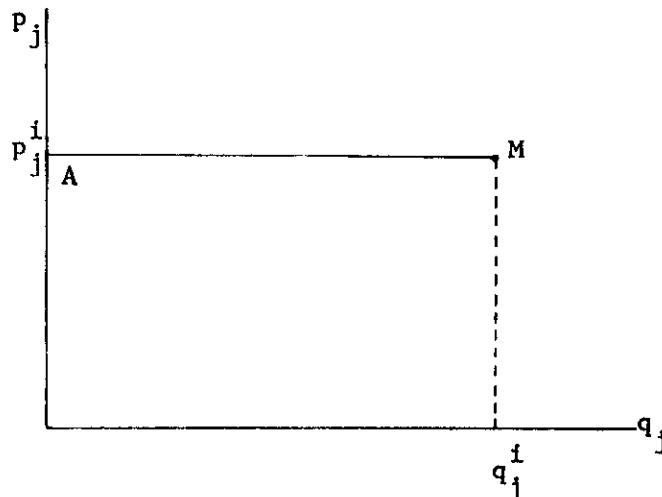


FIGURE 7

sales involve offers of this variety\* (Model 6.1).

### (3) The Quantity Offer

A seller may offer  $q_j^i$  at any price. This case can be included in Model 5.1 by setting  $p_j^i = 0$ ; similarly the sale of  $q_j^i$  or less is covered by Model 5.2.

In a single market with simultaneous moves it appears as if at most the 6 types of bids and 2 types of offers noted can reasonably be considered as inputs to a market clearing mechanism. These are considered further in 2.4.

### 2.3. Credit Constraints and Transactions Costs

Although 6 types of bids and 2 types of offers have been suggested there are a host of minor variations which depend upon credit conditions and market technology. These conditions appear as ad hoc requirements in any attempt to completely specify "the rules of the game" for any market mechanism.

Several examples of the types of conditions and market details to be covered are given.

(a) Credit: An individual may wish to bid beyond his immediate cash resources. He may have to arrange bank credit and bid by certified cheque before his bid is considered valid. The seller in some instances might be willing to extend direct credit, hence no third party will be required.

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\*The point to be stressed here is that the information conditions make the more flexible bid (of  $p_j^i$  or above) essentially of no advantage over the bid  $p_j^i$ . Thus even minute communication or transmission costs rule out the more complicated bid in favor of the simple. Stores do not advertise a radio for sale at "\$100 or more."

(b) Transactions Costs: The transportation, storage, display, packing and unpacking of many commodities may be extremely expensive. Ripe tomatoes sent to market which are not sold are essentially confiscated, in the sense that the costs of retrieval may easily exceed worth.

Even a money bid which is not accepted involves costs. The money has been tied up during the market and there may be broker or specialist fees to pay.

#### 2.4. Market Mechanisms and Clearing Devices

From one point of view a market is a function, a mechanism or a transformation which takes in bids and offers and transforms them into purchases and sales at a market price.

Different institutional conditions may be reflected in the sequencing of bids, information and communication. Here however we restrict our enquiry to the three simplest of single move games. They are (1) where buyers and sellers bid and offer simultaneously and a market mechanism resolves trade; (2) sellers move simultaneously but their offers are disclosed to the buyers before they move, and (3) buyers bid simultaneously, but their bids are disclosed to the sellers before they move.

Let  $B^i$  be the set of bids available to buyer  $i$  and  $S^j$  be the set of offers available to seller  $j$ . A market mechanism is a mapping  $T$  which transforms the vector of bids and offers  $(b_1^1, b_2^1, \dots, b_m^1; s_1^1, s_2^1, \dots, s_m^1)$ , where  $n_1$  is the number of bidders and  $n_2$  the number of sellers, into a vector of trades and prices.

More precisely let us consider  $n$  individuals where individual  $i$  has an endowment  $A_j^i$  of good  $j$  where  $j = 1, \dots, m$  and an endowment of money of  $M^i$ . Let there be  $m$  individual markets, one for each commodity  $j$  where that commodity is exchanged for money.

It might be a reasonable limitation to assume that individuals are not simultaneously buyers and sellers of the same commodity at the same time. However this rules out the oligopolistic stock market phenomenon of "wash sales" where an individual buys and sells in order to transmit a false signal of market activity. Although this assumption may be convenient for some analyses, in general, it would be preferable to deduce the conditions under which an individual is not simultaneously a buyer and seller of the same commodity.

If there were no overall credit constraints each market could be treated independently so that in each market  $j$  an individual  $i$  would have a set of moves in market  $j$ ,  $B_j^i \times S_j^i$  where a move could be described by a pair  $(b_j^i, s_j^i)$ . If we wished to assume that an individual is at most a buyer or seller we could add the condition  $b_j^i s_j^i = 0$ .

In each market  $j$  there is a mechanism such that:

$$(1) \quad (b_j^1, s_j^1, b_j^2, s_j^2, \dots, b_j^n, s_j^n; A_j^1, A_j^2, \dots, A_j^n) \xrightarrow{T_j} \\ (p_j^1, p_j^2, \dots, p_j^n; A_j^1 + x_j^1 - z_j^1, \dots, A_j^n + x_j^n - z_j^n) .$$

This transformation takes all bids and offers and initial holdings in market  $j$  into prices, amounts bought  $(x_j^i)$  and amounts sold  $(z_j^i)$  of all commodities. This also enables us to calculate net changes in money holdings.

A reasonable restriction on the mechanism is that all trade is required to take place at the same price. This requires that  $p_j^i = p_j$  for  $i = 1, \dots, n$ .

In general we cannot assume that bids in one market are independent of bids in another. There is at least a credit interlinkage which may

or may not be independent of the nature of the bids (for example different "margin" requirements may make an individual's credit line a function of his bids). Without credit constraints the set of all moves in all markets for  $i$  is:

$$(2) \quad \prod_{j=1}^m B_j^i \times S_j^i .$$

With a credit constraint, the size of the set of feasible moves must be appropriately restricted.

## 2.5. Moves, Strategies and Contingencies

Markets were characterized in 2.4 as devices which map market moves (bids and offers) and initial holdings of goods into prices, a new array of resources and a new set of net balances of money.

A move is not in general, the same as a strategy. A strategy is a plan which an individual uses to select his moves as a function of the information available when he is called upon to move.

Limiting ourselves to markets with simultaneous moves by the buyers and sellers and with symmetric knowledge by all about the states of nature, then even though the market clearing mechanism may be complicated and the bids complex, they cannot reflect knowledge of the moves of others. There is no recontracting.

If one set of individuals move first and these moves are announced before the others move then the strategies of the latter group will call for moves to be selected contingent on the behavior of the former.

## 2.6. Information and Search

There have been a considerable number of articles recently on search models of trade where individuals check on different employment opportunities sequentially, or go from gas station to gas station or check sequences of car dealers or house brokers. Rothschild has a survey of some of this literature<sup>2</sup>.\*

When these models are viewed from the perspective of markets with strategic traders they are clearly not ones with symmetric information conditions. The search processes are complicated strategies dependent upon what was found out by the potential buyer after having examined offers by some sellers.

For example consider 1 buyer, wanting to buy a unit of a single commodity facing 2 sellers. We may imagine that all sellers announce a price simultaneously. The buyer does not know these prices. At a cost of  $c$  per inspection he can check on a single price. He then has the choice to buy or to sample again. Suppose the payoff to a seller  $j$  is  $p_j$  if he sells a unit at price  $p_j$  otherwise it is 0. The payoff to the buyer is as follows

$$\begin{aligned}
 P_B &= 0, -c, -2c \text{ if no purchase after } 0, 1 \text{ or } 2 \\
 &\text{ searches} \\
 &= K - p_j - c \text{ or } K - p_j - 2c \text{ if purchase from } j \text{ after} \\
 &1 \text{ or } 2 \text{ searches.}
 \end{aligned}$$

We may wish to assume that if both sellers have been inspected, then there either is or is not an extra cost to return from the second

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\*With little reference to marketing or bidding literature.

to buy at the first. In this example assume no extra cost. The game tree below is drawn as if the price bids were discrete.

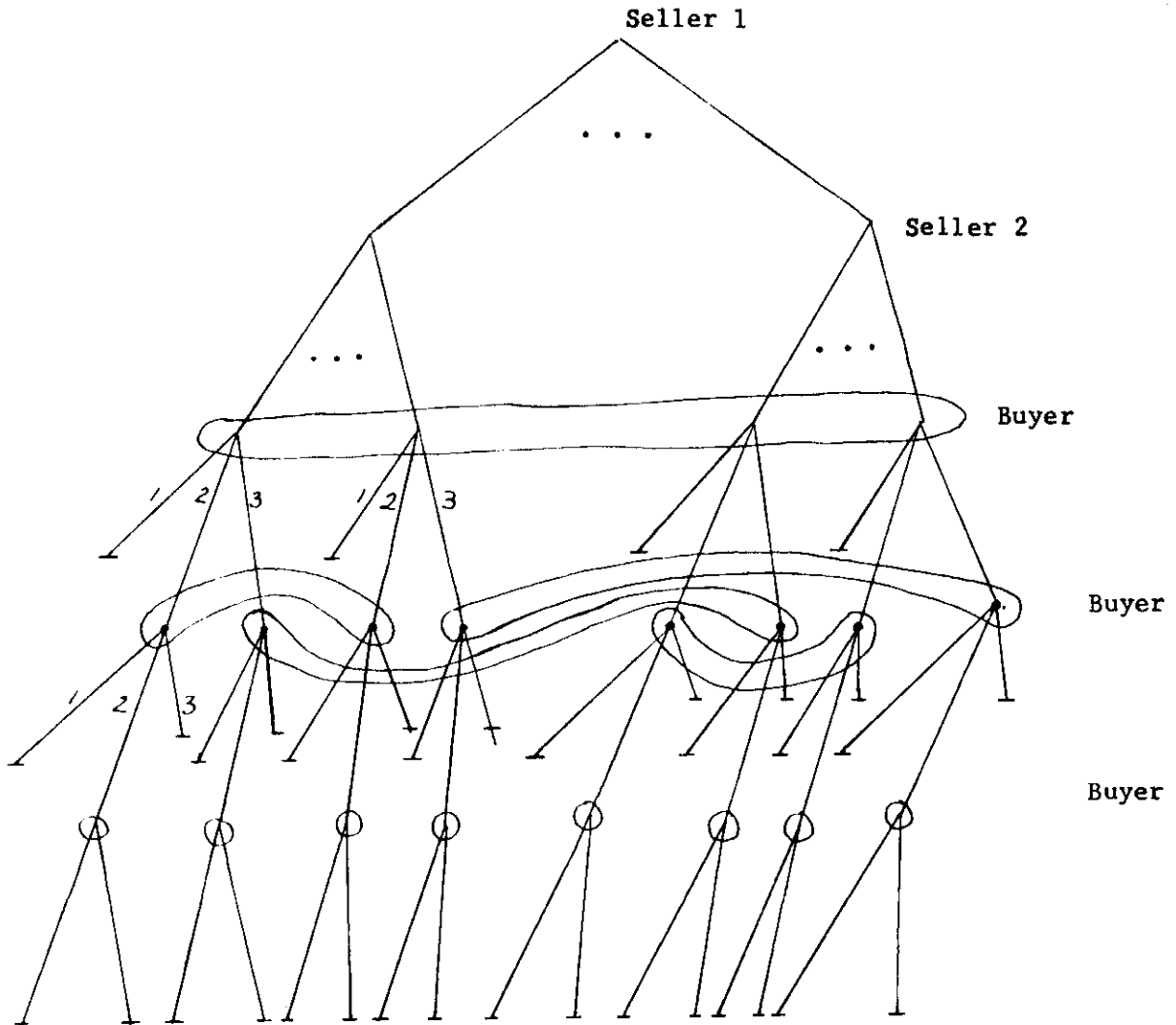


FIGURE 8

The sellers simultaneously each set a price. The buyer can opt out with a score of 0 or pay  $c$  and decide to find out the price of seller 1, or the price of seller 2. These are represented by the branches 1, 2, 3 at his first move. He then moves again, he can opt out with a score of  $-c$  or he can buy obtaining a score of  $K - p_1 - c$  or  $K - p_2 - c$  or he can obtain more information. These are indicated by branches 1,



3 and 2 in his second move. At his third move he has complete information and must abstain from buying which gives him a score of  $-2c$  or he can buy and obtain a score of  $K - \min[p_1, p_2] - 2c$ , (where if  $p_1 = p_2$  the buyer randomizes between the sellers).

Assume  $K \geq 2c$ . Let  $\phi(p_j)$  be the probability that seller  $j$  charges  $p_j$ . The expected payoff to the buyer can be expressed as follows

$$P_B = \begin{cases} K - c - \int_0^{\infty} p_j d\phi(p_j) & \text{if he buys after 1 sample} \\ -c & \text{if he quits after 1 sample} \end{cases}$$

$$P_B = \begin{cases} K - 2c - (1 - \phi(p_j))p_j - \int_0^{p_j} p_1 d\phi(p_1) & \text{if he buys after 2 samples} \\ -2c & \text{if he quits after 2 samples} \end{cases}$$

where  $\phi$  is the cumulative distribution of  $\phi$ .

These type of games provide representations of many interesting problems in marketing. The stress in this paper is not on the actual solution of specific problems of this variety, but to show that in contrast with the simultaneous strategy markets, the basic feature of markets with search is the nonsymmetry of information conditions, the possibility of the purchase of additional information and the recognition by all parties of the nonsymmetry in the information structure.

In an example such as the one above, when looked at as a game of strategy for all, or even given subjective probabilities for the price policies of the sellers, the buyer has a potentially complex sampling strategy.

In the context of a closed economic system there are some difficult modelling problems to be faced in accounting for the search costs. As the major concern of this paper is with bidding, offers and market mechanisms in general without search costs, these problems are not analyzed any further here.

### 3. ON DIFFERENT MARKETS

#### 3.1. Stock and Commodity Double Auction Markets

In 2.1 and 2.2 it was suggested that when there is symmetry in strategic information (i.e. bidders and sellers must make their bids on offers in ignorance of each others' moves) there are 6 types of bids and 2 types of offers\* to consider. Leaving aside minor variants this would give us 12 possibilities in any market combining each bid type with an offer type.

The market mechanism must then take these bids and offers into prices and final goods and money distributions. Shubik<sup>6, 7</sup>, Shapley<sup>3</sup>, Shapley and Shubik<sup>5</sup> and Dubey and Shubik<sup>1</sup> have already considered several bidding offering combinations where the mechanism produces one market price in each market.

We can observe that of the 12 combinations many will be extremely similar. In particular for the reason given below we could pick 1.2 over 1.1, 2.2 over 2.1 and 5.2 over 5.1. This cuts down the combinations to  $4 \times 1 = 4$ .

The alternatives eliminated are the ones in which "all or none"

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\*2 bids and 1 offer naming a specific price, which were more natural to nonsymmetric information were also noted.

bids or offers are involved. When numbers of traders are few, such bids are difficult to handle by market clearing houses. The similar bids which also accept partial fulfillment of an order are easier to match. However when numbers of traders are large the difference between the two becomes vanishingly small if the size of the bidders and sellers relative to the market becomes small.

The simplest way to generate a price is to sum the bids and offers and name the price at the intersection. The summation can always be done as the bids and offers are in commensurate units.

Case (1.2-5.2)

The bidders bid a sum of money (or less) and each seller  $i$  offers to sell an amount  $q_j^i$  or less for a price  $p_j^i$  or more. The market clearing house sums bids and offers together and obtains an aggregate supply and demand picture as is shown in Figure 9. Price is set at  $p_j^*$ , the

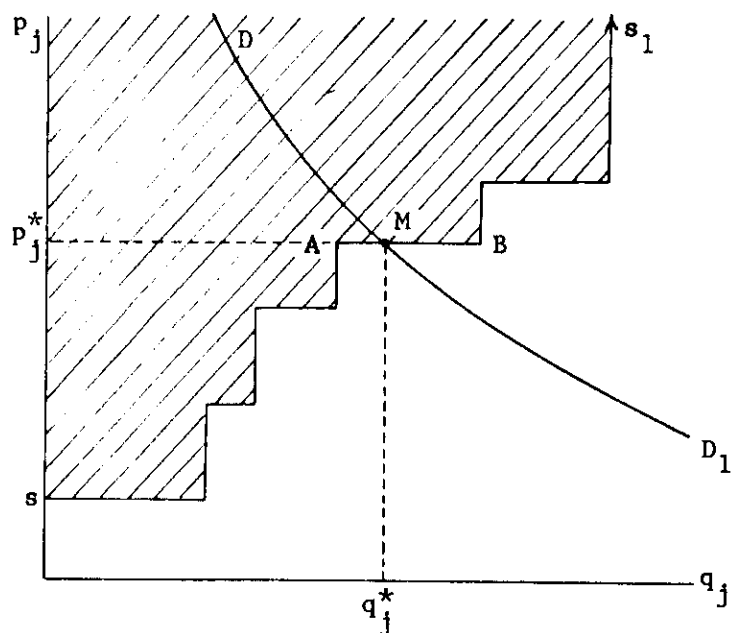


FIGURE 9

volume of trade at  $q_j^*$  and a device for rationing along the supply segment AB must be specified. There are several ways in which rationing can be done. For example, those suppliers whose offers are represented by the segment AB could be selected randomly until AM is obtained, or their sales could be prorated.\*

Cases (1.1-5.1), (1.1-5.2) and (1.2-5.1) are essentially the same.

The model originally suggested by Shubik<sup>6</sup> is a simplified version of (1.1-5.1) where a buyer  $i$  offers a sum of money  $b_j^i$  and all sellers offer all of their goods with a reserve price of  $p_j = 0$ . This gives clearance as shown in Figure 10.

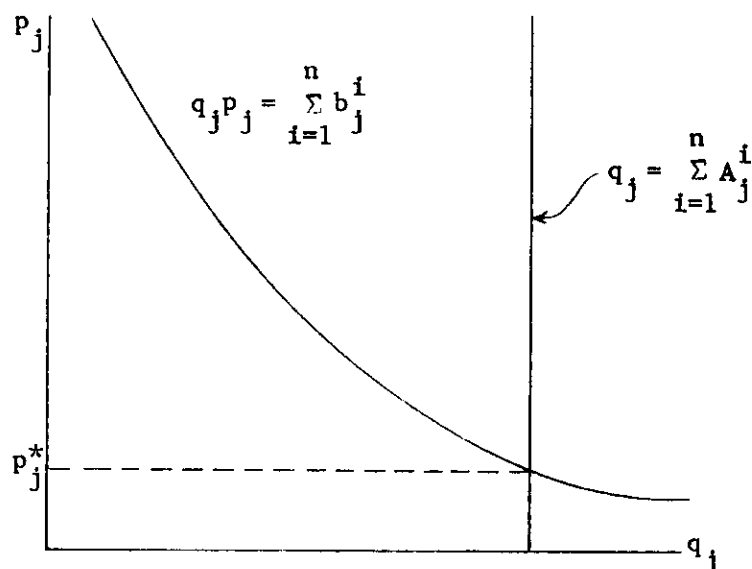


FIGURE 10

This market mechanism helps to define a noncooperative bidding game which has a noncooperative equilibrium point arbitrarily close to the competitive equilibrium for a large enough group of traders<sup>6, 3, 5</sup>.

\*For a full definition of the market mechanism what happens when bids are 0 and/or offers are 0 must be defined.

Case (2.2-5.2)

The bidder  $i$  demands an amount  $q_j^i$  or less for a price  $p_j^i$  or less. A seller  $k$  offers an amount  $q_j^k$  or less for a price  $p_j^k$  or more. The market lines up bids in descending order of price and offers in ascending order as is shown in Figure 11. This model has been discussed in detail by Shubik<sup>7</sup> who showed that in a two commodity trade there is a noncooperative equilibrium arbitrarily close to a competitive equilibrium for a large enough group of traders. It is conjectured that this result is true more generally.

In this model it is possible that the aggregate supply and bid curves do not intersect. A reasonable convention is to assume no trade during this period. The details of tie-breaking and the possible rationing of marginal bidders or offerers are discussed elsewhere<sup>7</sup>. The three further cases (2.1-5.1), (2.2-5.1) and (2.1-5.2) are substantially the same.

Case (3.1-5.2)

The bidder  $i$  offers to take all he can get at a price  $p_j^i$  or less. This bid must be modified by a total amount of credit he has available. A seller  $k$  offers  $q_j^k$  or less for a price of  $p_j^k$  or more. The difference between this and (2.2-5.2) is that the aggregate bid curve will have some hyperbolic scallops in it. These are shown in Figure 11.

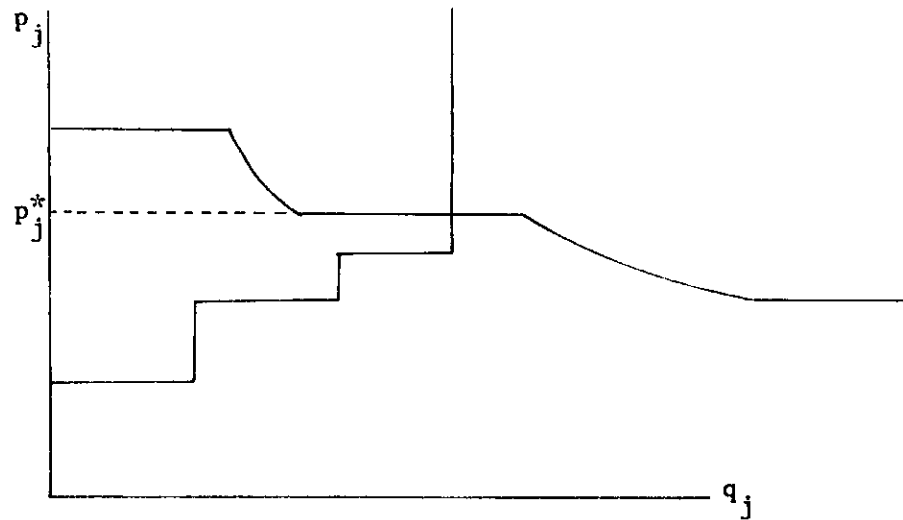


FIGURE 11

Case (3.1-5.1) is close to (3.1-5.2). No analysis of these cases has been made. It is conjectured that they have limit noncooperative equilibria which are close to competitive equilibria.

Case (4.1-5.2)

The bidder specifies the quantity  $q_j^i$  (or less) that he is willing to buy at any price. After  $q_j^i p_j$  exceeds the money and credit of the bidder the quantity demanded must be scaled back. This is shown in Figure 12 where the location of  $D_1$  is given by summing all of the  $q_j^i$ . As

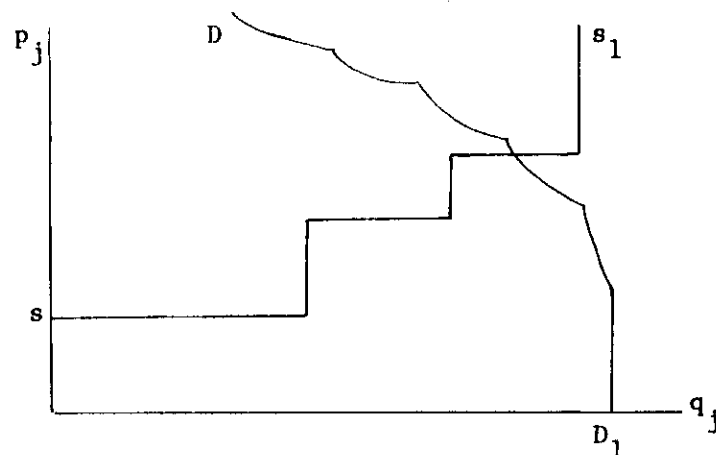


FIGURE 12

the price  $p_j$  increases gradually various bidders cannot pay for a full  $q_j^i$  demanded thus the hyperbolic scallops appear as demand falls off. Case (4.1-5.1) is close to (4.1-5.2).

In order to examine with complete rigor the existence of a non-cooperative equilibrium and its behavior when the number of traders is large it is necessary to be precise in the definition of the rationing and credit mechanisms. Examples of the level of detail required are given elsewhere.<sup>5, 7</sup> It is my belief that with many traders (in the sense of replication of a market) that all of these market models have noncooperative equilibria arbitrarily close to the competitive equilibria; i.e., there are many detailed marketing arrangements which when used as the basis for a noncooperative game with many traders of all types have non-cooperative equilibria close to the competitive equilibria of the barter exchange or Walrasian system.

### 3.2. Retail Trade

For the most part the symmetry of information condition suggested for the market mechanisms in 3.1 does not hold for retail markets. For many items such as goods in department stores and supermarkets a price is displayed. The customer faces the type of problem noted in 2.6.

The purchase of a house or automobile or even other major consumer durables tend to involve both nonsymmetric information and search and face-to-face communication and bargaining. Offers and counteroffers are made, and negotiations frequently break off while the buyer looks elsewhere and the seller deals with other buyers.

The simplest market model where information about the moves of players is not symmetric is that shown in Figure 8 in 2.6 where the buyer

can see the prices named by the sellers before he has to act.

Although in Section 2 and 3.1 above, much discussion has been given to simultaneous move markets with the double auction market being a prime example, it is by no means clear how well even the New York stock market approximates this idealized trading.

### 3.3. Auctions, Dutch Auctions and Sealed Bids

There is now an extensive literature on bidding and a comprehensive bibliography has been prepared by Stark and Rothkopf<sup>10</sup>. This literature is highly related to the problems discussed in Section 2. The major difference being that for the most part the markets being examined are essentially "open" or onesided. The seller or buyer is assumed as given. The competition is all on one side of the market as among bidders at an auction or contractors making sealed bids in order to gain a contract.

The condition calling for one price for each commodity is not even necessarily true for a simultaneous bid market. For example in bidding for Treasury issues once a market clearing price has been calculated, those whose bids were higher have their orders filled at the higher prices. In open auctions it is not uncommon to have a sequence of identical items auctioned sequentially with different sales prices being obtained for them.

## 4. MONEY GAMES

### 4.1. Market Models and Disequilibrium

The Walrasian auctioneer with his mysterious tatonnement process was not a completely useful simplification in general equilibrium theorizing. By avoiding the specification of exactly how price is to be formed the key element needed for the incorporation of markets, money and financial



institutions was thrown away in general equilibrium theory. The possibility for fully appreciating this limitation imposed by the modelling was minimized by using the competitive equilibrium solution concept rather than the noncooperative equilibrium solution concept. The former is non-strategic and in the search for an equilibrium which is essentially a static concept it is easy to avoid defining the state of the economic system for all positions of disequilibrium. Aggregate excess supply and demand conditions are deemed to be enough.

Even though the noncooperative equilibrium solution concept is essentially static; the rigid requirements of game theoretic modelling force us to define a completely explicit model for all positions of disequilibrium. This entails being explicit about price formation in the markets; credit conditions; the sequencing of moves and about the information conditions. Thus although the noncooperative solution is static, the model of the economy is basically dynamic in the sense that it describes a mechanism for the explicit calculation of all feasible states of the system.

The mathematical economist approaching the problem of modelling an explicit market mechanism for a closed economic system might at first be horrified in contemplation of a morass of alternatives, each with different institutional overtones. The argument here is that if we require the conditions that bids are simultaneous and that the mechanism must generate only one price for each commodity, then there are only 4 significantly different reasonable mechanisms\* with 8 more, closely related ones and a host of minor differences.

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\*When production is modelled a new set of difficulties are encountered owing to the sequencing of moves and the information conditions required by the nature of the production cycle.

Regardless of which mechanism appears to be the best, the act of modelling with any one of them leads to the construction of closed economic models where the mathematical necessity of well defining the system creates the need for conditions which amount to markets, money, credit and a variety of financial institutions<sup>8, 9</sup>.

#### 4.2. Dynamics, Adjustment and Institutions

When an economy is in equilibrium, the role of markets, financial institutions and money tends to disappear. The institutions such as organized markets, firms and banks are the carriers of process and a major part of the information and communication flow of an economy. In disequilibrium they appear clearly.

The first understanding of the role of money and financial institutions and where they fit into a general closed economic model, can be obtained with a static solution concept, but a dynamic model. A deeper understanding requires the development of dynamics. Thus, for example, it is likely that the disequilibrium paths of the 4 market models in 3.1 are different even though they might have the noncooperative equilibria which approach the same limit.

The design of financial and economic institutions is intimately related to the way in which we select adjustment processes in any economy. Many different institutions may have the same static efficiency properties, but it is possible that they manifest considerably different dynamic properties. The questions concerning the selection of optimal financial institutions in a fully dynamic context have hardly been asked in a precise form, let alone answered.

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