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The General Equilibrium Model with Joint Ownership of the Corporation (Voting Stock and the Core)

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1. Introduction

This note deals with a small but important point in the modelling of a general equilibrium system. The point specifically is that the Arrow-Debreu[1] treatment of the joint ownership of industry by introducing shares which can be treated, requires further specification.

The need for further specification can be seen immediately when this model is examined not for the competitive equilibrium but for the core. It is well known that the competitive equilibrium is contained within the core.[4,6] However it will be shown in Section 3 that unless extra conditions are imposed on the control of stock the resultant game may have no core whatsoever and hence no competitive equilibrium. An example illustrating this is given.

It is my belief that the slurring over of the apparently minor fine points in microeconomic modelling such as this has been the reason why the general equilibrium theory appears to be institution free and apparently hard to reconcile with the many institutions and laws needed to enable a market economy to function.

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By paying attention specifically to economic modelling implications of phenomena such as pecuniary externalities,[7] bankruptcy laws, market structures,[8] escrow arrangements[9] the details of contract, the laws of corporate control and stock voting the gap that exists between micro and macroeconomics can be closed.

The general equilibrium models are essentially nonstrategic and hence disguise or slur over many of the subtle ways in which strategically oriented entrepreneurs (or coalitions of entrepreneurs) beat the system. By modelling the same economic structure as a cooperative or noncooperative game these strategic possibilities cannot be missed. If they are limited or controlled by society it is by means of laws and institutions which must be specified.

It is important to stress the approach suggested here does not call for ad hoc modelling of institutions. On the contrary by starting with strategic models of trade the necessity of completely specifying "the rules of game" calls for construction of laws and agencies of enforcement without any particular attempt to make them look like specific national laws or institutions.

2. The Characteristic Function, The Core and Voting

An economy or a polity can be modelled as a cooperative game and the core solution of this game can be studied by first formulating the characteristic or characterizing function* of the game.[5]

*The difference between these two, concerns the existence of a sidepayment commodity or "money" with a constant marginal utility. If we assume sidepayments then we define the characteristic function. Without sidepayments we define the characterizing function. This technical difference is of no importance here.

Without going into unnecessary technical detail the meaning of the characterizing function can be explained and illustrated by means of an economic and a political example.

The characterizing function is a listing of the degree of independent control over the outcomes exercised by every coalition. Consider a trading economy with n traders. There are <u>in toto</u> $2^n - 1$ nonempty coalitions which can be formed. Once a coalition has been formed the traders in that coalition can trade among themselves regardless of what the others do. These traders can achieve any set of outcomes obtainable within the limits of their trading group. Their optimal policy will be to limit their attention to the Pareto optimal set they can obtain.

If there are k players in a coalition where $k < n$ the optimal set they can obtain is of a lower dimension than that which the coalition of n can obtain. Figure 1 shows an example for a three trader market. The

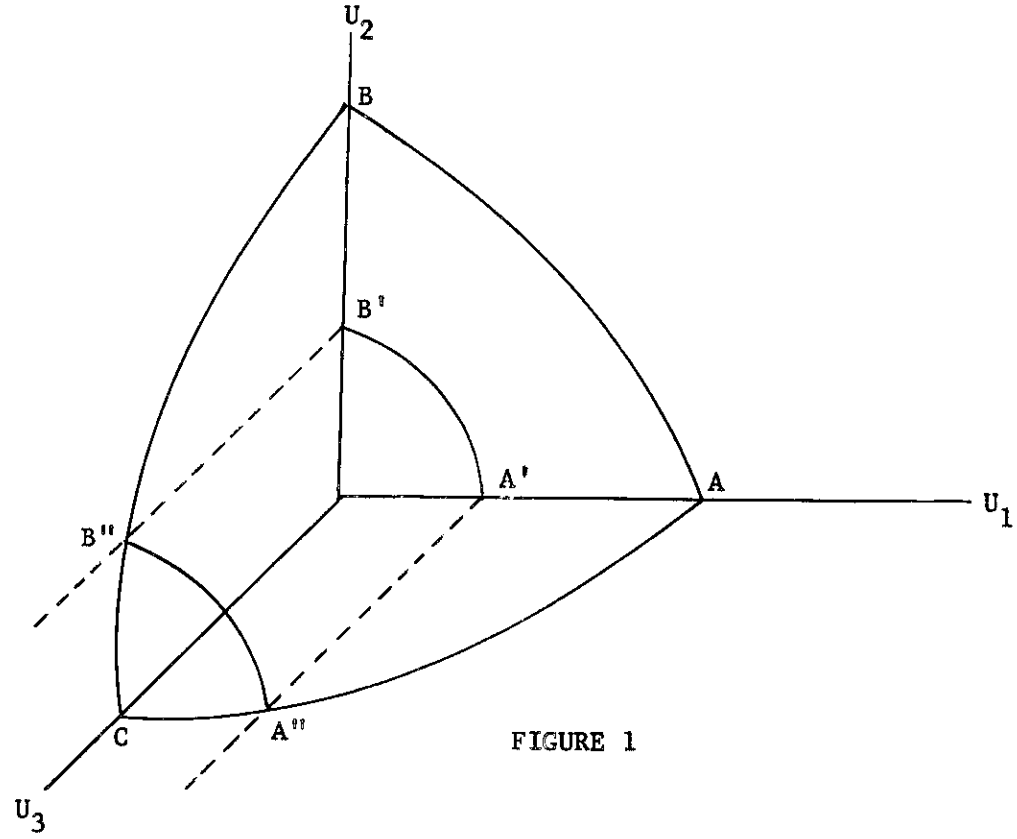

FIGURE 1

Pareto optimal surface for the three traders in coalition is ABC . If we consider 1 and 2 alone they can achieve A'B' . By the formal trick of drawing out a cylinder in the direction of U_3 this will cut the overall Pareto optimal surface at A"B" . Thus any final imputation which lies on the surface ABC in the region A"B"C can be challenged by a coalition of 1 and 2 in the sense that it offers them less than they can achieve by acting as a coalition without cooperating with 3.

The core of an n-person consists of the set of imputations* which are not dominated, i.e. cannot be challenged by any coalition of traders less than the group of all n traders.

The characterizing function is not always a good representation of the power or ability of a coalition. It is a reasonable representation when there are no externalities among groups. If there are externalities then there may be threats or other interactions which make it difficult to evaluate in a reasonable manner the amount that a coalition can achieve.

When we calculate the characterizing function for a market game involving trade among the participants once trading groups have been formed there is no difficulty in calculating the domain of effectiveness of any group because it depends only upon their own resources. A trader not included in a coalition has no influence on the payoffs of that coalition, regardless of what he does. This is not true when individuals are required to trade via formally organized markets.[7] A formally organized market

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*An imputation is a point on the Pareto optimal surface that is also individually rational, i.e. no individual obtains an outcome of less worth than he can obtain by "going it alone."

or an issue of voting shares produce externalities so that the simple evaluation of the effectiveness of a coalition cannot be made without specifying the effect of the behavior of the other traders not included in the coalition.

A game which has the property that once coalitions have been formed they are independent of each other is known as an orthogonal coalition game or a c-game.[5] The Arrow Debreu[1] model of the economy is such a game if we exclude their comments concerning the existence of shares to take care of joint ownership. The models of the economy treated by Shubik[6] and Debreu and Scarf[4] in their study of the core were also c-games.

A game which is not an orthogonal coalition game, but can be reasonably well represented by a characteristic function is the simple majority voting game. Suppose there are n individuals each with one vote (for simplicity consider that n is an odd number) who are voting on a taxation and subsidy bill or on the disposition of some other form of joint property. Suppose that the total sum at stake is 1. Furthermore suppose that a simple majority of voters are assumed to take control. Unless a specific law concerning the protection of minority rights exists a majority can enrich itself at the expense of a minority. A simple description of the power of coalition is given below:*

$$f(s) = 0 \quad \text{if} \quad s < n/2$$
$$= 1 \quad \text{if} \quad s > n/2 \ ,$$

*In this simple example as all individuals have one vote each, the game is symmetric hence any coalition of the same size has the same power thus our representation is particularly simple as only number not the identity of individuals make a difference.

where s is the number of voters in a coalition. This can be illustrated by the diagram in Figure 2a. This simple majority voting game has no core

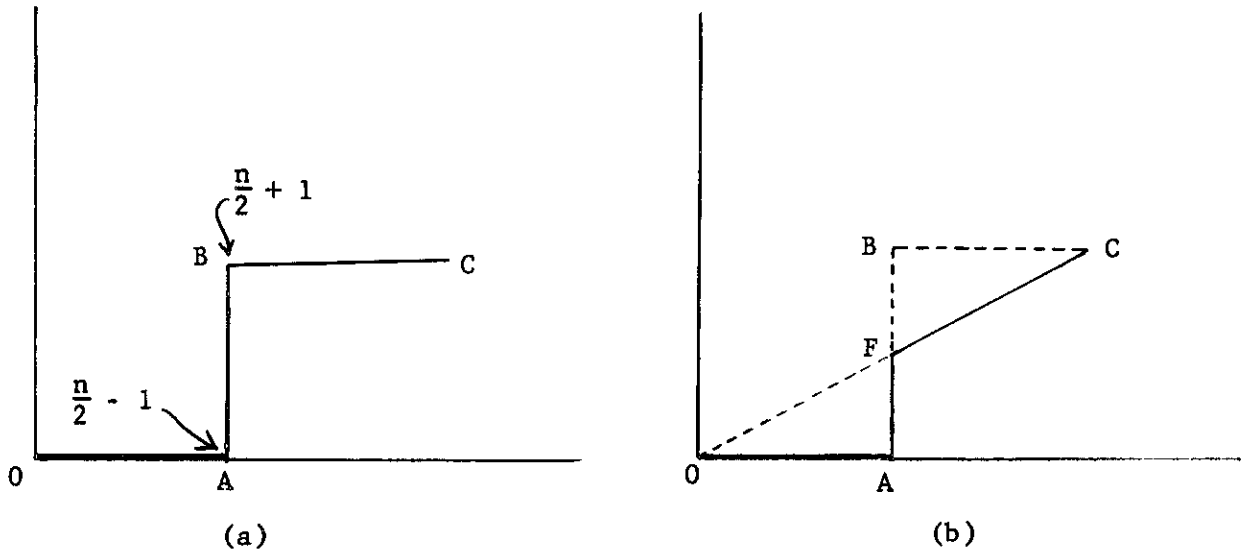

FIGURE 2

and hence unless the appropriate modification is made there cannot ever be a competitive market for votes.

There are two simple and natural modifications which can be made to this game. First we may consider a game where individuals have different numbers of votes. This corresponds to the number of votes per individual being proportional to stock ownership.

The second modification calls for the protection of minority stockholders. A rule that is in effect in U.S. corporate law is that although only a majority is required to control the corporation and thus to decide on items such as the size of dividends, any payout that is made must be made in proportion to the stockholder's equity. This is illustrated in Figure 2b. The new characteristic function is given by OAFC . This game has a core and provides the rule that is implicit in the Arrow Debreu idea of contractual participation in profits.

In general it is my belief that a key approach to the treatment of mixed voting and economic systems caused by joint ownership or other externalities is to find legal or societal modifications or rules of the game which guarantee the existence of a core in a game that otherwise fails to have one. Examples of the application of this principle are given in the recent work of Klevorick and Kramer[3] and of Don Brown.[2]

3. An Example of a Market with No Competitive Price for Shares

An extremely simple example is presented here to illustrate the need for the legal specification of payout conditions and protection of minority stockholders.

Consider a society with n individuals each with utility functions of the form

$$U_i(x,y) = U(x,y) = x + ky \ , \quad k \geq \sqrt{n} \ .$$

There is one profit maximizing enterprise with a production function given by:

$$y = f(x) \quad \text{say} \quad y = \sqrt{x} \quad \text{to be specific.}$$

It is assumed that there are n shares and that the initial holdings of each individual are $(1,0,1)$, i.e. one unit of X none of Y and 1 share.

The firm will try to maximize

$$\Pi = p_2\sqrt{x} - p_1 x$$

where p_1 is the price of X and p_2 the price of Y. We can set $p_1 = 1$. It is easy to observe that for $k \geq \sqrt{n}$ the firm will buy all n units of X and $p_2 = 2\sqrt{n}$ with profit given by:

$$\Pi = 2\sqrt{n}\sqrt{n} - n = n .$$

Hence on the assumption of equal payout to all shares, each stockholder obtains a dividend of 1. Thus his total income is 2, for which he is able to buy $1/\sqrt{n}$ units of Y with a final worth of

$$U = \frac{k}{\sqrt{n}} \geq 1 .$$

If we assume that there is no minority stockholder protection so that a simple majority controls both the corporation and its payout policy then the characteristic function is as follows

$$f(s) = s \quad \text{for} \quad s < n/2$$
$$= k\sqrt{s} \quad \text{for} \quad s > n/2$$

where $k \geq \sqrt{n}$. This game has no core.

If we assume that although the majority may control corporate policy, whenever they pay out it must be in proportion to equity then a different characteristic function will be obtained.

Assumption 1: Claims proportional to equity. Dividends will be paid to all stockholders in proportion to shares held.

Before we can define the new characteristic function, further rules of the game must be specified. They concern the limits on what the corporation

can pay suppliers who also have control of the corporation. In describing a game in coalitional form we must take care in specifying precisely what are the powers and abilities of a coalition. If a majority of the voters control a corporation but are constrained to pay out all profits in proportion to equity, if it is strategically possible it will pay them to hide profits by unduly enriching themselves in the roles of management or as suppliers to the corporation.

In the example given here (as in the Arrow Debreu model) the consumer has three roles, they are as a consumer, a supplier and a stockholder. If he is not a member of the controlling group of the corporation the greatest damage that he can do to that group is to refuse to supply the corporation. The corporation in turn if being run for the benefit of the control groups can reduce its profits to zero by paying inflated prices to the controlling shareowners in their role as preferred suppliers. If this can be done then in spite of the proportionate payout of profits condition, the characteristic function will be the same as before and the game will have no core.

The extra conditions needed are to prevent the control group from "robbing the till." These conditions also exist (but are hard to enforce and are frequently broken) in United States corporate law.

Assumption 2: Dealings with like suppliers must be on a similar basis. There can be no self-serving special treatment of "insider" suppliers.

Assumption 3: Internal remuneration of officers must not be "undue," i.e. the wage and remuneration structure of the members of the corporation must be on a similar basis as the payments made to others of like ability in like positions.

In order to enable us to be mathematically precise in formulating the above assumptions we must be explicit in our assumptions concerning the trading mechanism. Two major alternatives present themselves. (1) We can make the assumption that coalitions once formed do not have to trade through organized markets. (2) We can postulate the existence of markets and require that all trade be carried out using the markets.[7]

If we choose the second alternative then it is fairly straightforward to translate assumptions 2 and 3 into a mathematical model.

One can argue that assumptions 2 and 3 are implicit in the Arrow Debreu assumption that the firm is a straightforward profit maximizer.

The new characteristic function, in this example becomes:

$$f(s) = s \qquad \text{for} \quad s < n/2$$
$$= k\sqrt{s}(s/n) \qquad \text{for} \quad s > n/2 \ .$$

This game has a core and it is possible to obtain a market price for the shares held by any trader.

4. Conclusions

The simple example presented in Section 3 is given as an illustration to show that if a closed economic system with shares is considered from the viewpoint of the core extra rules are needed in order to well-define the model.

The laws needed to provide the extra rules are of the variety which exist in enterprise economies. Thus features which are frequently regarded as "peculiarly institutional" are necessary to fully define the game.

Not only are the laws needed, there is little doubt that in actuality they are difficult to write and to enforce. A glance at the actual operation of cooperations is sufficient to show that there is frequently ample opportunity for insider enrichment and there are many ways in which a controlling group may benefit itself at the cost of other stockholders.

Institutional economics is far too important to be left in the hands of the institutionalists. The general equilibrium theory, although mathematically elegant runs a serious danger of becoming prematurely sterile. By making the assumption of perfect competition this type of theorizing immediately reduces the individual consumer or entrepreneur to an automation or dummy. In doing so a simplification is achieved which makes it unnecessary to add all of the rules and conditions needed to control or modify the behavior of individual economic actors with strategic freedom. It is precisely at this point that the general equilibrium theory has succeeded in throwing away its connection to the underlying network of law and institutions needed for the control of any economy.

A game theoretic approach whether it uses the core, the noncooperative equilibrium or other solution concepts differs from the general equilibrium approach inasmuch as it stresses the strategic freedom of the individual. In doing so it enables us to begin to understand the need for laws and institutions at the basic level of being able to fully define the mathematical structure of the rules of the game.

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