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Essay in Social Coordination

Jacob Derechin
Yale University Graduate School of Arts and Sciences, jacob.derechin@gmail.com

Abstract
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Social Coordination is an essential feature of any social system. Coordination is a precondition for certain kinds of cooperation, as cooperation implies agents working together for a common cause, while coordination only implies agents synchronizing their behavior. For example, social contagion suggests a mechanism for groups to socially coordinate without necessarily cooperating. When behaviors spread across social ties the people may not be actively intending to spread the behavior, so they are not always cooperating. The necessity of coordination does not imply that it is easy to achieve. In Chapter 1, I survey two theoretical frameworks for operationalizing the challenges to coordination: The Prisoner's Dilemma Game and The Generals Problem. In the Prisoner's Dilemma game, the agents will both be better off if they cooperate, but have strong incentives to betray each other. In this classic case coordination is expected but it is not Pareto-optimal, highlighting the differences between agents simply coordinating (both playing the same strategy) and cooperation for each other's benefit. Here, the impediment to cooperation is that of misaligned incentives. The General's Problem presents a different barrier to coordination: unreliable communication channels. The Generals all have aligned incentives to coordinate and work together, but due to the chance that messages may fail to be transmitted, they are unable to effectively do so. The Byzantine Generals Problem in effect combines the challenges of coordinating with both misaligned incentives and faulty communication. In this setting, some of the agents are trying to prevent the rest of the group from

reach consensus.

Chapter 2 is a global consensus experiment in a "Byzantine" setting. The players have 10 rounds of communication to reach consensus among a set of arbitrary identifiers. The players are able to send full text messages to each other. This setting is Byzantine because some of the players disconnect from the game either through technical problems on their end or through failing to send their messages in enough time. Additionally, we found that a subset of the players did not perfectly understand the instructions of the game and made errors, thus demonstrating that players can engage in Byzantine behavior. The players were arranged into a Watts-Strogatz network and one of the interventions was altering the fraction of odd vertices in these graphs. The other intervention was to alter the instructions so as to change the story the players were told about why they were trying to reach consensus. We found that groups were more likely to reach consensus in groups with a lower fraction of odd vertices, but did not find the changing story about why players were coordinating had much impact. We found that the human consensus does exhibit some byzantine fault tolerance. For example, the effect of players dropping out of the game, a classic type of byzantine fault, had a negligible effect on the outcome. However, we also found that the fraction of players with misunderstandings or errors negatively impacted the consensus process. The players did demonstrate the ability to correct misunderstandings in others, but sometimes misunderstandings were contagious. Notably the effect of misunderstandings is comparable to effect of the fraction of players trying to vote for the first identifier in alphabetical order. This suggests that in this setting using a bad protocol is comparable in effect to a byzantine fault.

Chapter 3 is a methodological exploration of creating preference-indifferent identifiers. Throughout the testing of identifiers to use in the consensus experiment in Chapter 1, we found that the players expressed preferences over random strings of letters and numbers. To remedy this, we generated nonsense words with an alternating vowel and consonant pattern to make them easily pronounceable to English speakers. We developed a software platform for users to evaluate these nonce words as forced-choice paired comparisons. We then used the Elo algorithm to generate scores for each of these words. We also developed techniques to find unobserved heterogeneity in ratings for this setting. We found that human raters do indeed have significant preferences even over these nonsense words, implying that even if the identifiers are randomly generated, they are not necessarily preference-equivalent. We also compared the preferences we observed with the predictions of Phonological Cue Theory and found that our results were not entirely consistent. While this was not initially devised as a Phonology experiment, the platform we develop may have benefits for conducting Phonology experiments.

Chapter 4 is an agent-based model assessing the impact of capacity constraints in a threshold contagion model. Many sorts of contagious phenomenon, such as music, do not exist in isolation but as part of a competitive marketplace. In these settings there are often superstars with out-sized popularity along with a large number of flops with little popularity. I suggest that capacity constraints may be a structural factor that influences these disparities. In this model, there are multiple potentially cascading states that the agent can potentially occupy. The agents have a certain capacity of states that they can occupy at once. For example, suppose someone has a workout playlist that lasts 1 hour. As they discover new music to add to the playlist, they have to remove songs currently in the playlist to keep the playlist 1 hour. Thus, in this setting, the states indirectly trade off with each other by virtue of the capacity constraint. Increasing the number of states in excess of capacity increased the unpredictability of which states become popular as well as increased the disparities between popular and unpopular states. This suggests that capacity constraints may play a role in explaining market concentration and superstar phenomenon.

Essays in Social Coordination

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Contents

Chapter 1

Introduction

Cooperation is a fundamental building block of any social system. The ability of multiple agents to work together toward a common cause is a key aspect of social systems. However, just because the capacity to cooperate is necessary for social functioning, that does not mean it is sufficient. Agents may still face misaligned incentives as well as uncertainty about each other, thus presenting challenges to the effectiveness of the group working together. Given the different scenarios under which cooperation can take place, it is often operationalized differently. First, I will survey operationalizing cooperation as the Prisoner's Dilemma and then as the Generals Problem to showcase the differences between these two approaches. Then, I present examples of byzantine faults in real system. Next, I survey operationalizing coordination without cooperation as social contagion. Then, I present an overview of the following chapters.

1.1 The Prisoner's Dilemma

In the classic Prisoner's Dilemma game, agents have two choices: cooperation and defection. Cooperation can be interpreted as selfless or prosocial behavior while defection can be interpreted as selfish or antisocial behavior. Table 1.1 shows the payoff matrix for a Prisoner's Dilemma game, where $T > R > P > S$ and $a > \frac{T+S}{2}$ (Rapoport et al., 1965). Thus, it is clear in this game Defection is the dominant strategy. Conditional on the other player cooperating, it is preferable to Defect since $T > R$, and conditional on the other player defecting it is still optimal to Defect as $P > S$. The condition of $R > \frac{T+S}{2}$ ensures that sustained cooperation is more beneficial than alternating cooperation and defection, which, while not relevant in the one-shot version of this game, is relevant in the repeated version. This means that while both players Cooperating would be pareto-optimal, they both have incentives not to do so. While the prospects are quite grim in a one-shot game, extending the Prisoner's Dilemma game to a repeated game context or a population games context can provide avenues for cooperation to emerge.

1.1.1 Repeated Games

While in a single interaction there may be no incentive to cooperate, in a repeated game context things can turn out differently. The classic iterated prisoner's dilemma tournaments by Axlerod showed the success of the Tit-For-Tat strategy (Axelrod, 1980, 1981). The Tit-For-Tat strategy begins by cooperating initially and then will copy the strategy it receives from its partner in the next round. Since two Tit-For-Tat players start out cooperating, they will cooperate forever achieving the pareto-optimal outcome. Tit-For-Tat also has the ability to punish defectors, as it will defect when defected upon. Unlike the Grim Trigger strategy, where the player will cooperate until its partner defects and then defects forever, Tit-For-Tat has the capacity to forgive defection (Axelrod, 1980). If its partner returns to cooperation Tit-For-Tat will as well. This can be problematic in a setting with errors, as Tit-For-Tat can be caught in a defection spiral. Since Tit-For-Tat has a memory of one round if the initial round has a cooperator and a defector, the Tit-For-Tat players will alternate between cooperation and defection forever. To break out of this a new strategy Generous Tit-For-Tat has a chance to cooperate when it's neighbor defects, allowing the defection spiral to break (Molander, 1985). This demonstrates the ability of deterrence via credible threats as well as the forgiveness of errors to promote cooperation.

1.1.2 Population Games

In a population game setting, instead of two players playing against each other, there are many players who all have different strategies playing some subset of the players over time. Here, the overall composition of the population depends on payoffs the players using each given strategy attain. Population games can showcase the emergence of cooperation at a population level as well as showcase how it might be vulnerable to exploitation from defection strategies.

Evolutionary stability is a state where, when the population is seeded with one agent of a different strategy, it cannot grow and invade the population (Smith, 1974). While unconditionally playing the dominant strategy Defect is evolutionary stable, there are some conditions where other strategies can be evolutionary stable as well (Axelrod and Hamilton, 1981). For example, if the players are likely to play each other again, Tit-For-Tat can be evolutionary stable (Axelrod and Hamilton, 1981). Nowak and Sigmund (1992) show that Tit-For-Tat can act as the stepping stone for even more cooperative strategies to dominate the population, as once Tit-For-Tat has taken hold it is vulnerable to invasion from strategies like Generous Tit-For-Tat. Nowak and Sigmund (1993) introduce a strategy PAVLOV which cooperated if both player played the same action in the previous round and defects otherwise, which can also out preform Tit-For-Tat under some conditions. Thus, the population composition can matter considerably for determining which strategies are optimal. Spatial structure can also influence strategic choice. For example, on a wide array of network structures cooperation can emerge as long as the benefit to cost ratio of cooperating is greater than the average degree (Ohtsuki et al., 2006). Increased cooperation when the benefit cost ratio is greater than the average degree has been replicated with human players (Rand et al., 2014). Network structure also influences evolutionary dynamics as some graphs have been found to promote selection while others promote random drift (Lieberman et al., 2005). Non-network spatial structure can also matter, as in lattice prisoners dilemma games chaotic patterns can emerge (Nowak and May, 1992).

	C	D
C	R,R	S,T
D	T,S	P,P

Table 1.1: Prisoner's Dilemma Game

1.2 The Generals Problem

The Generals Problem represents a different way to understand (and a different kind of) cooperation. Instead of players resisting the temptation to defect, in the Generals Problem the player's incentives are aligned, and the difficulty comes from coordination. Table 1.2 shows an example of the payoff Matrix for a Generals Problem. Here, the players get a nonzero payoff as long as they either both play 1 or both play 2, but if they play different actions, they get a payoff of zero. This is an example of a pure coordination game as described by (Schelling, 1960). Thus, the players need to coordinate or fail. In theory, the two actions would be identical; in practice, this is not always the case, as people can have expectations about each other's behavior from outside of the game allowing one of the actions to serve as a focal point thus making coordination easier (Schelling, 1960; Mehta et al., 1994).

Now suppose that there is a small probability that one of the messages between the players is not transmitted. It has been shown that, in this setting, it would require infinite messages for the players to achieve common knowledge about each other's actions and fully confirm their coordination (Akkoyunlu et al., 1975; Halpern and Moses, 1990). Common knowledge is also related to consensus through Aumann's classic result about the failure to agree to disagree, which states that if two agents start with the same priors, if their posteriors the event in question are common knowledge, they will be equivalent (Aumann, 1976). Thus, under these circumstances common knowledge guarantees consensus in a certain sense, as even agents using different information will reach the same conclusion in Aumann's setting. Samet (1990) has found that impossibility of Agreeing to Disagree can be extended to the setting where agents does not know what they do not know, showing that this works in many reasonable epistemic situations. This demonstrates that even among agents that are trying their best to cooperate, coordination can still fail due to communication failures. These failures are not necessarily absolute, as with slight variations to the game form and sufficiently small probability of the communication being lost, coordination can occur most of the time (Rubinstein, 1989; Morris and Shin, 1997). The key difference is that generals are no longer coordinating on an arbitrary identifier, now the enemy is either prepared or unprepared. It is advantageous to attack when the enemy is prepared but disadvantageous to attack when the enemy is prepared. Table's 1.3 and 1.4 show the payoff matrix for the coordinated attack game as presented by Morris and Shin (Morris and Shin, 1997). They found in this setting that if the probability of communication error is less than $\frac{1}{M+1}$, the first general will send the second general a message to attack, the first general will attack without receiving confirmation from the second general, and the second general will attack upon receiving the message to attack (Morris and Shin, 1997). They found that this results in the generals attacking most of the time that the enemy is unprepared (Morris and Shin, 1997). They also find that when the payoffs are symmetric across players, as shown in Tables 1.5 and 1.6, and the chance that the enemy is prepared is very small, coordinated attack almost always occurs (Morris and Shin, 1997). This suggests that in practice coordination is easier to reach than the pure two generals problems suggests. Many real settings have built in structures to generate common knowledge. For example, when advertising at the super bowl people who see the add know that others have seen it since the event is so popular (Chwe, 2013).

	1	2
1	a,a	0,0
2	0,0	a,a

Table 1.2: The Generals Game

	Attack	No Action
Attack	-M,-M	-M,0
No Action	0,-M	0,0

Table 1.3: Coordinated Attack Game: Enemy Prepared

	Attack	No Action
Attack	1,1	-M,0
No Action	0,-M	0,0

Table 1.4: Coordinated Attack Game: Enemy Unprepared

	Attack	No Action
Attack	-M,-M	-M,-M
No Action	-M,-M	0,0

Table 1.5: Coordinated Attack Game: Enemy Prepared Symmetric

	Attack	No Action
Attack	1,1	-M,-M
No Action	-M,-M	0,0

Table 1.6: Coordinated Attack Game: Enemy Unprepared Symmetric

1.3 The Byzantine Generals Problem

The standard Generals Problem assumes that the players all have aligned incentives and the players do not make errors. The Byzantine Generals Problem relaxes that assumption. Now the information received in messages from players might be incorrect because the sender is actively trying to mislead the group, is simply mistaken, or stops sending messages altogether for whatever reason (Lamport et al., 1982). This adds in uncertainly into all of the messages as the players now need to determine if the message is authentic or "byzantine".

While it is not generically possible to solve the byzantine generals problem, there are communication protocols that are robust to certain levels of byzantine action and are still able to coordinate on a value (Lamport et al., 1982). These protocols are called Byzantine Fault Tolerant. There are many real world systems that utilize Byzantine Fault Tolerance, like Aircraft Flight Control (Yeh, 2001), Cryptocurrency Protocols (Nakamoto, 2008; Buterin, 2014), and computer file systems (Castro and Liskov, 2002).

Sometimes, people respond counter-intuitively to their environment, as the introduction of noise can improve the ability of human groups to coordinate (Shirado and Christakis, 2017). Thus, it is not a given that all types of Byzantine faults would necessarily be detrimental to the consensus process, even if that is the expected outcome. Additionally adding the ability for players to communicate in Prisoners Dilemma games (Dawes, 1980) and Battle of the Sexes games (Cooper et al., 1989) has been shown the increase the ability of the group to cooperate.

1.4 Faults and Fault Tolerance in Human Systems

The Byzantine Generals framework can be an interesting way to analyze consensus problems in human systems. It combines the misaligned incentives problem that is present in the prisoners dilemma game with the communication problem from the Standard Generals problem. Unfortunately, these dynamics are present in many important human systems, which can manifest as errors. A notable example of an error prone human system is medicine. Medical errors are widespread and impact hundreds of thousands of people each year in the United States alone (Leape, 2000; Weingart et al., 2000) and there is likely significant under reporting (Classen et al., 2011). Medicine is a complex system with many agents interacting, transferring information and thus many failure points (Leape, 1997). Thus instead of focusing on individual errors it is appropriate to look to structural factors as generating errors (Leape, 1997; Reason, 2000).

Interventions based on structural factors have proven effective, for example it has been shown that implementing computerized physician order entry, to standardize the drug ordering process and ensure legibility, can reduce the frequency of medication errors (Kaushal et al., 2003). One can think about this setting as a consensus problem where both the doctor and the pharmacist are working together to get the patient the proper medication. Suppose the doctor writes the prescription, but has poor handwriting, so the name of the intended drug (A) appears to be a totally different drug (B). This could be analogous to a byzantine fault, as if the pharmacist fills the prescription for B, then in a sense the doctor and the pharmacist did not reach consensus. Thus, computerized physician order entry could be seen as making the system more fault tolerant by eliminating the possibility of handwriting faults.

These structural problems are not limited to medicine but are pervasive. For example, in military applications, energy production, aviation, and shipping, it has been found that for organizational reasons, systems are often not optimized for the ease of use by those who operate them, leading to errors (Perrow, 1983). Thus, the normal accident theory proposes that faults are in inherent part of complex systems due to propagation of errors (Perrow, 2011, 1999, 1994). Byzantine faults represent some of the errors that Perrow studied. For example, Perrow reviews multiple false alarms in nuclear early warning systems, some were caused by hardware malfunctions, and one was caused by a bear climbing a fence at a military base causing the nuclear attack alarm to sound instead of the intruder alarm (Perrow, 2011, 1994). The fact that nuclear war did not occur, shows that the nuclear early warning system was able to tolerate these types of byzantine faults, but this demonstrates that byzantine faults can occur in even the most high stakes systems (Perrow, 2011).

Byzantine faults can occur in communication systems and present as misinformation. When dealing with human actors you cannot always guarantee that they will interpret accurate information correctly. For example, a study of Covid-19 risk perceptions found that showing a picture of a beach (a relatively low risk location) before articles that accurately present the risks of locations for Covid-19 caused some participants to rate beaches as riskier and restaurant as less risky, which is the opposite of what the articles presented (Derechin et al., 2021). This can break down the ability of public actions to build towards common knowledge, since once could presume there is a chance the action will be misinterpreted. Additionally, false information commonly appears on social media sites (Lazer et al., 2018). Accuracy in discriminating "fake news" from real news has been shown to depend on a number of psychological factors. For example, repeated exposure to plausible false news headlines increases the fraction of people who will rate it as true (Pennycook et al., 2018). The ability to discriminate between real news headlines and false headlines has been shown to depend on analytical ability (Pennycook and Rand, 2019), and increasing deliberation time also increases accuracy in this kind of task (Bago et al., 2020). Finally, it was found that having people rate the accuracy of headlines beforehand made people more likely to report wanting to share true headlines (Pennycook et al., 2020).

Altogether, this suggests that byzantine faults occur across a wide variety of domains and that human error can be a source of byzantine faults. While structural intervention can often improve the robustness of the system, these changes can be difficult to implement due to organizational factors (Cohen et al., 1972; Perrow, 1994).

1.5 Social Contagion

Social contagion represents an alternate pathway to coordination that does not necessarily require active cooperation. Under this model, people are influenced by the behavior of others and this influence decays as it propagates outward through social networks (until about a geodesic distance of 3) (Christakis and Fowler, 2013). This phenomenon has been observed on a wide range of behaviors including: obesity (Christakis and Fowler, 2007), loneliness (Cacioppo et al., 2009), smoking (Christakis and Fowler, 2008), happiness (Fowler and Christakis, 2008), sleep loss (Mednick et al., 2010), drug use (Mednick et al., 2010), depression (Rosenquist et al., 2011), and general emotional states (Hill et al., 2010). Social influence models have also been effective for describing the behavior of crowds (Helbing and Molnar, 1995; Helbing et al., 2005).

Unfortunately, these effects can be difficult to empirically identify. Manski presents 3 key challenges to identifying peer effects: shared environment, properties of the group itself, and reflection (Manski, 1993). Shared environment is pretty self-explanatory, people are similar because they are exposed to similar environments. The group property refers to some property of the group determining the similarity and not the interactions within the group, for example seeing a higher incidence of breast cancer in a group of predominantly women than a group of predominantly men. Reflection refers to the problem of reverse causality in social influence: is it the individual influencing the group or the group influencing the individual (Manski, 1993)? The effects can also be problematic to identify, as accounting for social influence can introduce many weak instruments biasing the results (Angrist, 2014). When social influence is restricted to an agent's neighborhood it is possible to identify spillover effects using matching (Forastiere et al., 2021). An experiment at the US Air Force Academy tried to use a reduced form peer effects model to produce optimal squadron compositions for improving academic performance, but their intervention ending up making things worse for the lowest preforming students (Carrell et al., 2013). Both experiments and agent-based models can be useful for overcoming these methodological challenges. Experiments can do this by randomizing participants over different social settings. In agent-based models, the modeler has control of the the exact mechanism of social contagion in question. This way they can directly simulate what would happen under those conditions. Both agent-based models and experiments sacrifice external validity for internal validity. I find this trade off worthwhile and utilize both experiments and agent-based models.

1.6 Overview

The following chapters proceed as follows:

Chapter 2 is an online experiment conducted in a Byzantine setting. The players are working to achieve global consensus and have 10 rounds of synchronous messaging before they vote. The players can send full text messages to each other allowing for a rich array of messages that can be sent. This allows us to observe how human players behave and attempt to solve the Byzantine Consensus problem abstracted from the complexity of real systems.

Chapter 3 is a methodological exploration on the creation of preference-indifferent identifiers. During the piloting of the experiment in Chapter 2, we noticed that the players had preferences over random strings of numbers and letters we were using as the objects of consensus. This presented a problem for the experiment as the choices between alternatives were no longer arbitrary. We developed a forced choice platform in combination with the Elo algorithm to generate rankings over nonce words designed to be pronounceable to English speakers. We also developed techniques to assess ranking heterogeneity in this setting.

Chapter 4 goes in another direction; it describes an agent based model studying the capacity constraint in cascade processes with multiple cascading states. In this model, agents have a limited number states which they can occupy at once which is less than the total number of possible states. Thus, the states can trade off with each other by virtue of the agents being over capacity, requiring them to drop previously adopted states.

Chapter 2

BFT

2.1 Motivation

The Byzantine Generals Problem is a variation of the classic Two Generals Problem where global consensus could be inhibited by both lack of message confirmation and incorrect message information (Lamport et al., 1982). There is now a new class of agents in the Byzantine Generals problem, the "Byzantine agents", who are not trying to reach consensus. They could feed malicious or error-riddled information into the system or simply stop responding.

While this problem was initially formulated with computer systems in mind, human systems face these kinds of challenges as well. Even people who are trying their best to cooperate with others might sometimes make accidental mistakes, and people regularly have to deal with people of varying levels of competence at navigating institutions. A novice player at a game might still be trying their best to win, but may blunder because they do not fully understand the rules. In addition to mistakes, people also have to cope with others with incentives that are not always aligned. The door-to-door salesman selling a miracle cure claiming treat to all of your ails, may not have one's best interest at heart.

People particularly in social systems do not always have the resources to independently verify all of the information they encounter and must rely on the testimony of others to navigate their epistemic landscape, exposing them to the problem of "bullshit" (Wakeham, 2017). For example, in one study some people rate sentences composed of random buzzwords as profound (Pennycook et al., 2015). In this sense, discriminating between truly meaningful information and information that simply appears meaningful is likely to be an important aspect of the kinds of Byzantine challenges people can face.

Given the wide range of possible problems it's clear that not all Byzantine actors are created equal. For example, imagine a group of friends trying to decide what restaurant to eat at, and in one scenario the Byzantine actor provides information about pineapple cultivation while another, the Byzantine actor is telling different agents different real restaurants. People would expect that the agents being exposed to random pineapple facts would probably be able to safely ignore this, while the agents getting real restaurants might have some more difficulty. Understanding the ways in which people are able to reach a shared understanding of the social world and coordinate their actions is thus both complicated and crucial.

2.2 Methods

2.2.1 Experimental Design

We utilized the Breadboard platform to design the game and interface with Amazon Mechanical Turk (McKnight and Christakis, 2016). 4343 participants passed the comprehension quiz to start a run of experiment, but only 3419 completed their run. In this experiment participants, are arranged in a Watts-Strogatz graph (Watts and Strogatz, 1998) and they were given the objective of all agreeing to the same arbitrary identifier. Each participant was assigned an arbitrary 5-letter identifier which represents the object of consensus as well as an arbitrary 4-letter identifier that represents their name as the player. These were not English words but were designed by us to have similar patterns to English words and experimentally tested to be preference-indifferent (Sankaran et al., 2021).

we used a 10-by-3 factorial design with 10 vignette arms and 3 structural intervention arms. The vignette arms involve modifying the instructions to the game to manipulate the context in which the participants are coordinating. We refer to these different vignettes as "skins". They are: Managers firing a Worker, Managers hiring a Candidate, Members of an orchestra selecting a Venue, Members of a city council selecting a Mascot, Electors choosing a Leader for a nation, Generals choosing a Fort to attack, Delegates selecting a Host City for the Olympics, Astronauts selecting a Planet to colonize, Members of the International Horticultural Society selecting a Plant Name, and Friends choosing a Restaurant to eat at. The full instructions for the Generals skin is listed in Appendix A.1 while tables of the text that is swapped between skins is in Appendix A.2.
The structural intervention manipulates the fraction of vertices in the graph that are even or odd. The 3 levels for fraction of odd vertices are 0.2,0.5, and 0.8. This structural intervention was chosen because it would alter the kinds of votes players could receive from their neighbors to be gridlocked. For example, if a player has a even number of neighbors and half of their neighbors vote for MUXIL and half vote for QUPEP, the player is the deciding vote. If on the other hand the player has an odd number of vertices then it is not possible for half of their neighbors to vote for one identifier and half to vote for the other, but when you include the players own vote it is. This can help determine how players count their own vote in this process.

In order to quantify the differences between the different Vignettes, we also used the Elo algorithm (Elo, 1978) to assess each of the scenarios on four dimensions: Complexity, Familiarity, Stakes, and Emotionally Charge. We recruited an additional

1693 participants from Amazon Mechanical Turk. The participants were ineligible if they participated in a run of the main experiment and were subsequently not eligible to participate in the main experiment. The participants were instructed to read through the two sets of instructions for two different skins. After the participants finished reading each of the vignettes, they were asked what the group in each that scenario was trying to accomplish as an open-ended attention check. Only 1507 participants successfully passed this attention check. Afterwards the participants were asked the following four questions forced choice questions:

Which scenario do you think is more complex?

Which scenario do you think is higher stakes?

Which scenario do you think is more emotionally charged?

Which scenario do you think is more familiar?

After these questions, the participants were also optionally allowed to submit open-ended comments. Figure 2.1 shows the Elo values of each Vignette on each of the four dimensions. These results largely mapped onto our expectations. For example, Generals leading an army and Electors picking the leader for a nation were rated as much more complex and higher stakes than friends choosing a Restaurant to eat at, and friends choosing a restaurant was much more familiar. These results suggest that the different Vignettes do capture a wide range of potential mechanisms for how ability to reach consensus might be effected by the context of the challenge. Furthermore, these results suggest that our skins reasonably express the kinds of scenarios present in real life. Figure 2.2 shows the Elo values normalized to (0,1) within each dimension overlaid with the fraction of groups that reach consensus. There is not a clear relationship between the normalized Elo values and the fraction of groups that reach consensus.

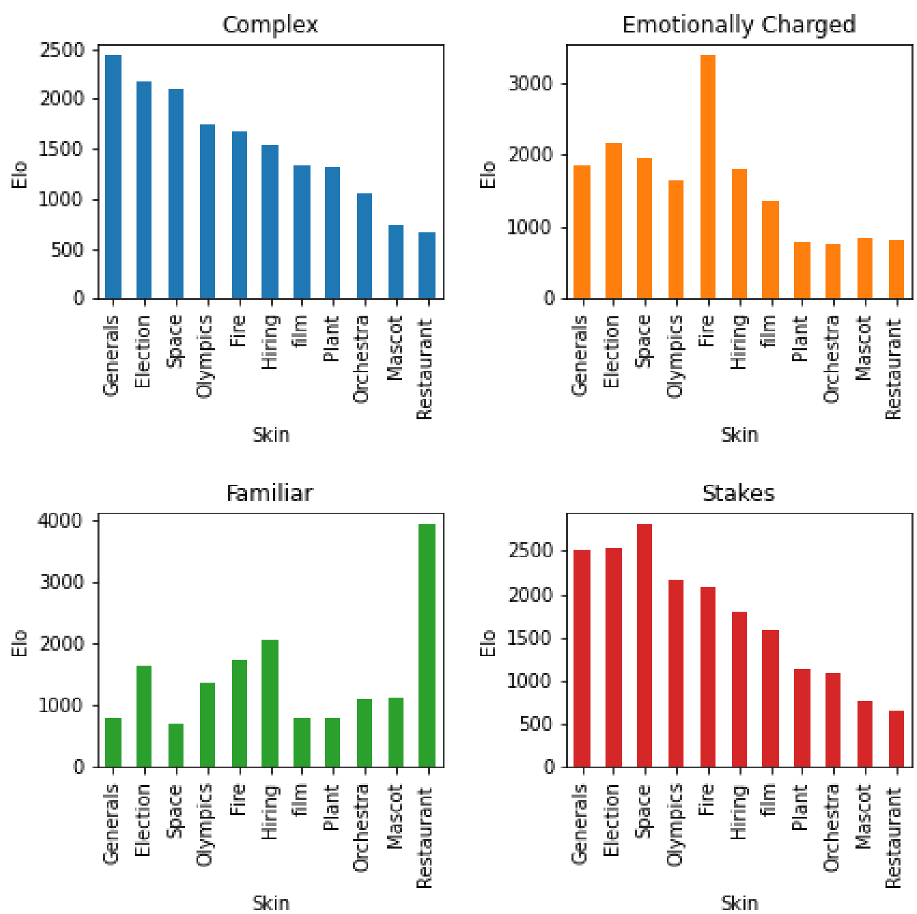
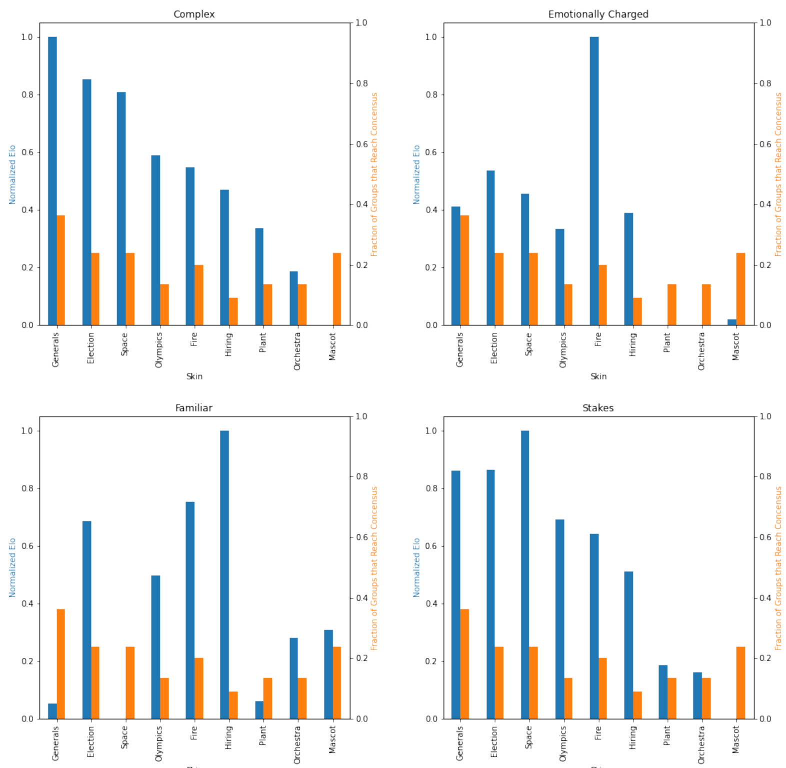

Complex
Emotionally Charged
Familiar
Stakes
Elo
Skin
Generals
Election
Space
Olympics
Fire
Hiring
film
Plant
Orchestra
Mascot
Restaurant

Figure 2.1



Complex
Emotionally Charged
Familiar
Stakes
Normalized Elo
Fraction of Groups that Reach Concensus
Skin
Generals
Election
Space
Olympics
Fire
Hiring
Plant
Orchestra
Mascot

Figure 2.2

2.2.2 Gameplay

When the participants first enter the game upon accepting the Amazon Mechanical Turk HIT, they are presented with a set of slides showing the instructions for that game. The instructions for each skin are different, but only the framing information of the scenario differs; not the core gameplay mechanics. The participants are told that the character they are playing and what they are trying to accomplish. It is explained that they will be a four-letter identifier representing their name as a player, like AHIQ, as well as a five-letter identifier representing the target of cooperation assigned to them, for example BUMAF.

The objective of the game is global consensus, so it does not matter if any given player's target is chosen so long as the whole group agrees. We also explain the networked nature of the game to the player: each player has a certain number of players they can message directly, but there are other players in the game they cannot directly message. They players see a graph on their screen in the UI showing the names of the players they can message as well as their own name.

The players have 10 rounds of message passing to reach consensus. There is an initial messaging step where players have a limited number of time to send messages to other players. There is a text box labeled with the name of each player they are connected to so players can send different neighbors different messages. Next, there is a second part of the round where the players receive the messages sent by their neighbors. Each message is marked with the name of the player who sent it. There is also a scratch area where players can type notes to persist across rounds. Players are instructed that if they do not complete each part of the game within the time limit, they may be dropped from the game for being idle. After the 10 rounds are complete, the player will be presented with a final dropdown list to make their choice. The participants are informed of the payment structure, and then may take a comprehension quiz to determine whether they understand the rules well enough to be eligible to play the game. We limited the number of players to between 15 and 25.

We did our best to prevent repeat play, so once players have been paid either via being dropped from the game or completing the game, they are not eligible to play in the future. Players who failed the comprehension quiz were allowed to play again, as well as players who were dropped due to over-recruitment or under-recruitment. The questions used for the comprehension quiz are presented in Appendix A.3. Ideally, players will be playing the actual game for the first time even if they have been exposed to the instructions more than once. We were only able to filter players at the Amazon Mechanical Turk account level, so if players have multiple accounts or are working on tasks in a group, we were not able stop them. These are things that Amazon tries to prevent on the platform but they have not been completely successful. It is likely the impact of this is small.

2.2.3 Coding

Messages were coded in 3 ways: "state", "protocol", and "confusion". "State" refers to the 5-letter identifiers used in the message. As a first pass, exact matches to our set of identifiers were extracted using string manipulation with common misspellings corrected by hand. We did not consider whether or not players were actively encouraging other players to choose the states in question, so the messages "choose PISOF or DOMIF" and "choose PISOF not DOMIF" would be coded the same. This is because the player is presenting information about both states in their message and indicates that they are considering those identifiers that round. For "protocol", it was common for players to suggest and alphabetical strategy. There were multiple variants like: "Select the first name in the final drop-down list alphabetically" or "pass around every identifier you see and pick the first alphabetically". Since adopting the alphabetical protocol did not necessarily require passing an identifier, we coded this separately. In our analysis, we considered players who adopted the alphabetical strategy without passing any identifiers as occupying a separate state from any of the identifiers. We also marked players as confused if did not play correctly like trying to coordinate on player names instead of identifiers (using 4-letter words vs 5-letter words), or behaving suspiciously, like typing gibberish over and over or repeating greetings every round. This category was out best effort to identify low-effort players (as well as any bots who made it through our attention checks and qualifications).

2.2.4 Outcomes of Interest

The two primary outcomes of interest are: (1) whether or not the group reaches global consensus (wins the game), and (2) the discrete metric standard deviation in the final set of votes. The discrete metric is defined as:

$$m_d(a,b) = \begin{cases} 1, & \text{if } a \neq b. \\ 0, & \text{if } a = b. \end{cases}$$

Each of the identifiers in the final votes are words and thus non numeric, so there is no defined mean over any of the words themselves. The discrete metric allows us to quantify equivalences among the identifiers, to measure on a pairwise level if there is consensus. For example, the discrete distance between the identifier HIDEP and HIDEP is 0, while the discrete distance between HIDEP and CERIY is 1. Variance and thus standard deviation, can only be calculated via applying the discrete metric to the identifiers. Zhang et al. (2012) developed a way to calculate variance via only paired comparisons within the population:

$$s^2 = \frac{1}{2N^2}\sum_i^N\sum_j^N (x_i - x_j)^2$$

This formulation is equivalent to calculating variance under the euclidean metric for scalars since the euclidean metric is:

$$m_e(a, b) = \sqrt{(a-b)^2}$$

Inserting this into the variance formula yields:

$$m_e(x_i, x_j)^2 = \left(\sqrt{(x_i - x_j)^2}\right)^2 = (x_i - x_j)^2$$

To calculate variance in this way using the discrete metric we can simply swap it for the euclidean metric:

$$s_d^2 = \frac{1}{2N^2}\sum_i^N\sum_j^N m_d(x_i, x_j)^2$$

Since standard deviation is the square root of variance it can be calculated as:

$$s_d = \sqrt{\frac{1}{2N^2}\sum_i^N\sum_j^N m_d(x_i, x_j)^2}$$

This allows us to have a continuous measure of the degree which the group has reached consensus. This way, we are able to see if the interventions bring groups closer to consensus even when they do not reach it. This technique of variance calculation could be potentially useful for other types of string comparison like Levenshtein distance (Levenshtein, 1966) or Hamming distance (Hamming, 1950) at the population level.

2.3 Results

2.3.1 Basic Results

The following plots show the average levels of the two outcomes of interest by each experimental condition (altering the fraction of odd vertices, or the vignettes used to frame the challenge). Figure 2.3 shows the average win rate and discrete standard deviation by fraction of odd vertices, and Figure 2.4 shows these results by Skin. Figure 2.5 is a heat map which shows the average win rate and the discrete standard deviation by each pair of experimental conditions. Figure 2.3 suggests a linear relationship between the fraction of odd vertices and consensus, with fewer odd vertices being associated with better consensus prospects. This is the opposite of what we initially expected when designing this experiment. The results by, Skin as shown in Figure 2.4, suggest that, with respect to achieving global consensus there is substantial variation across skins, but the differences are much more modest when considering final discrete variance. Figure 2.5 shows that within some skins there is considerable variation across the fraction of odd vertices, like the election skin, while other like the hiring skin do not show drastic changes. There are only about 7 replicates for each skin - fraction of odd vertices pair, so it is difficult to interpret the results at this level. While we test the statistical significance and robustness of these results later using regressions, the following subsections explore possible mechanisms that could generate this kind of difference.

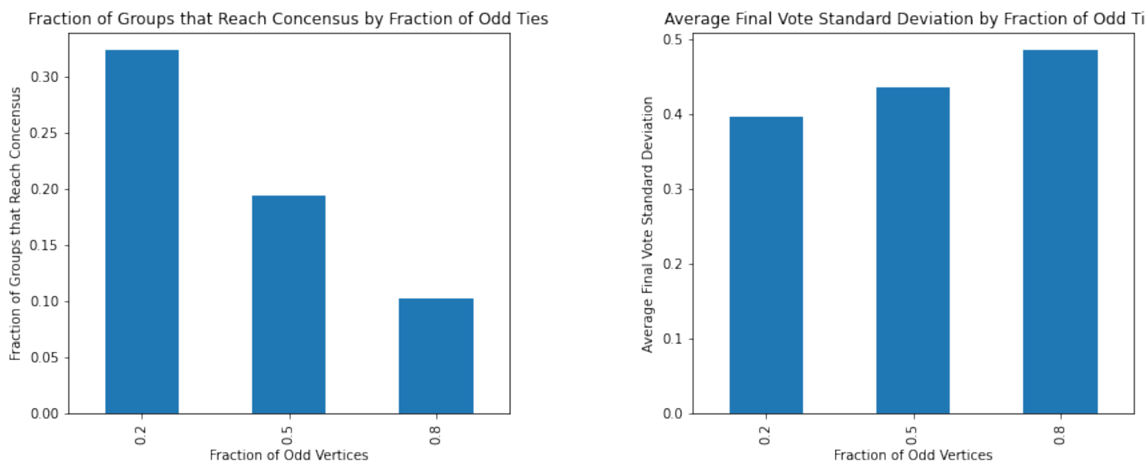


Figure 2.3

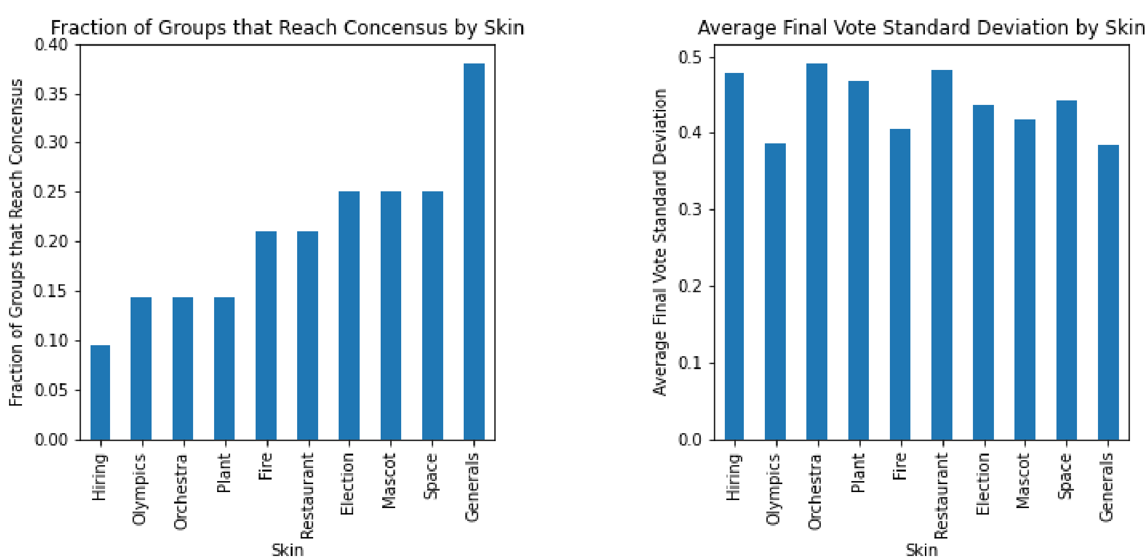


Figure 2.4

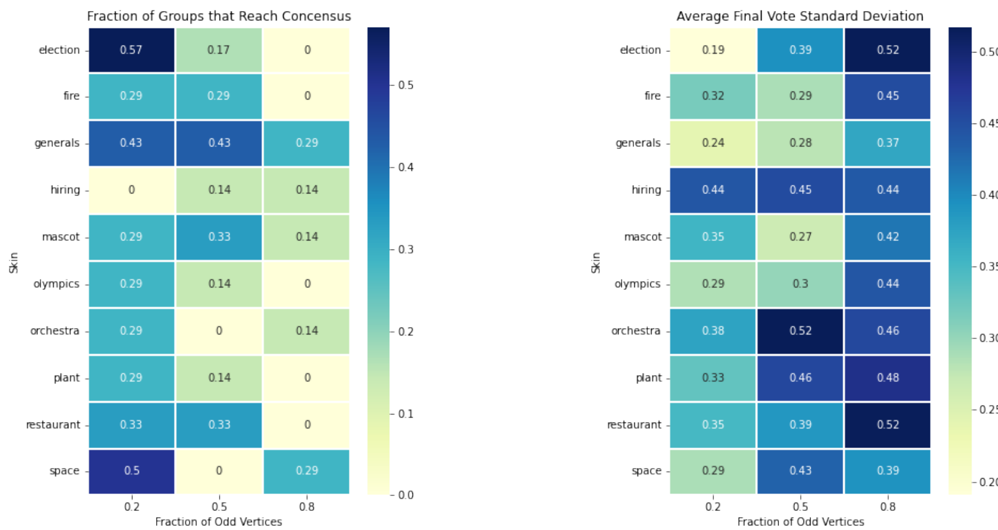

Fraction of Groups that Reach Concensus
Average Final Vote Standard Deviation
Skin
election
fire
generals
hiring
mascot
olympics
orchestra
plant
restaurant
space
Fraction of Odd Vertices
0.2
0.5
0.8

Figure 2.5

2.3.2 Gridlocks

One of the mechanisms we hypothesized for how the fraction of even and odd vertices might influence game play is through gridlock. We define an agent as gridlocked if the set of identifiers they receive that round all have equal votes. For example, if the agent receives 1 vote for FARIW, 1 vote for LANOQ, and 1 vote for JUTUR they are gridlocked. We define an agent as weakly gridlocked if the maximal set of identifiers proposed that round all have equal votes. For example if the agent receives 2 votes for FARIW, 2 votes for LANOQ and 1 vote for JUTUR, that agent is weakly gridlocked. Every agent who is strongly gridlocked is also weakly gridlocked, as the total set of identifiers is the set of identifiers with maximal votes.
For each type of gridlock, we can define the set of identifiers at each round as either only being from messages from the agent's neighbors or including their own messages from that round. This can help distinguish between which types of gridlocks really matter, and to what degree agents include their own state when making decisions.
Figure 2.6 shows the average gridlocks by round by fraction odd when agent's own states are considered, while Figure 2.7 shows the situation where only the neighbor's states are considered. Figure 2.8 shows this type of graph for weak gridlocks with the agent's own state included, while in Figure 2.9 there are only states from neighbors. Across both strong and weak gridlocks, the core trend is similar; with a gradual decrease in gridlocks as time goes on, which is consistent with the consensus process progressing. Sometimes, the average number of gridlocks increases from round 1 to round 2. There also tends to be a difference in average gridlocks across the game by fraction odd, with runs where the fraction of odd vertices is 0.8 tending to have higher gridlock that the runs where the fraction of odd vertices is 0.2.

Gridlocks near the end of the game can serve as an indicator that the consensus process is not going well, because if it was those gridlocks would not exist. To test for end game differences, we pooled together the data for rounds 9 and 10 and tested for differences in means using Welch's T-test (Welch, 1947). We test whether the means in the 0.2 fraction odd condition is less than the mean in the 0.5 fraction odd condition, whether the means in the 0.5 fraction odd condition is less than the mean in the 0.8 fraction odd condition, and whether the means in the 0.2 fraction odd condition is less than the mean in the 0.8 fraction odd condition across the different types of gridlocks. The results of these tests are presented in Table 2.1. In all types of gridlocks, the mean in the 0.2 condition is lower than that of the 0.8 condition, and the mean in the 0.5 condition is less than the mean in the 0.8 condition. When comparing the means of the 0.2 condition to the 0.5 condition the results are more ambiguous; the difference in means is statistically significant when looking at neighbors only gridlocks, but not when the self state is included. This suggests that however gridlocks are defined, having a higher fraction of odd vertices is worse, and suggests that whatever mechanism is inhibiting consensus leads to an increase in gridlocks at the end of the game.

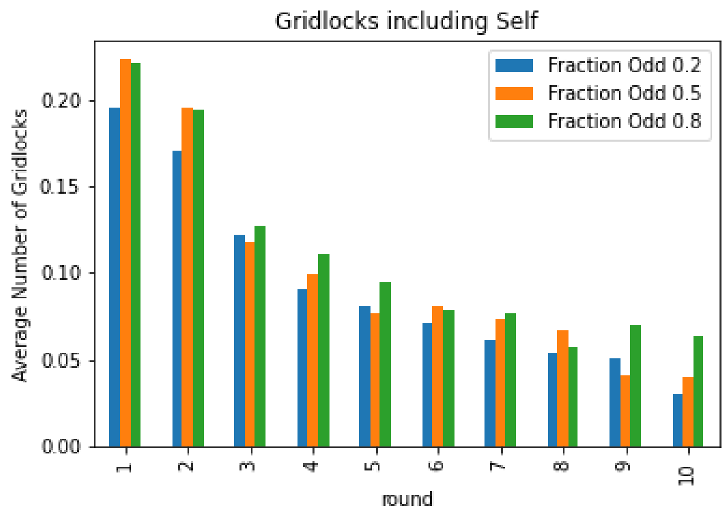


Figure 2.6

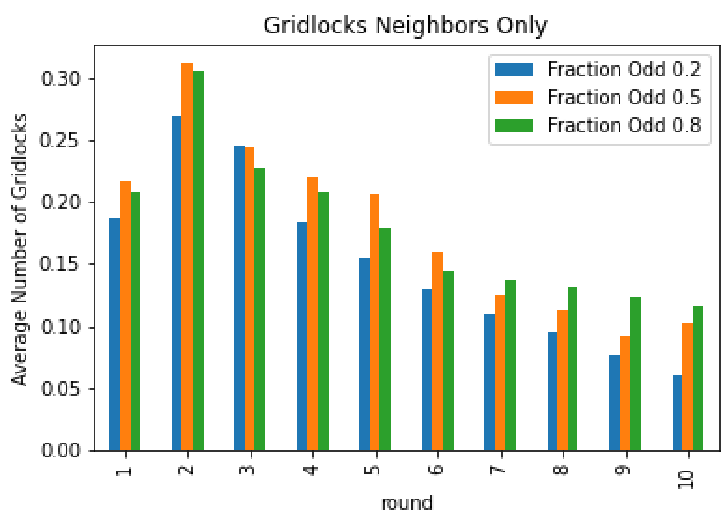


Figure 2.7

Welch's t-test rounds 9 and 10, t (p)	$0.2 < 0.5$	$0.5 < 0.8$	$0.2 < 0.8$
Gridlocks Including Self	-0.027 (0.488)	-3.091 (0.001)	-3.323 (0.000)
Gridlocks Neighbors Only	-2.508 (0.006)	-1.768 (0.0390)	-4.473 (0.000)
Weak Gridlocks Including Self	-0.238 (0.406)	-3.026 (0.001)	-3.346 (0.000)
Weak Gridlocks Neighbors Only	-2.263 (0.012)	-2.452 (0.007)	-4.915 (0.000)

Table 2.1

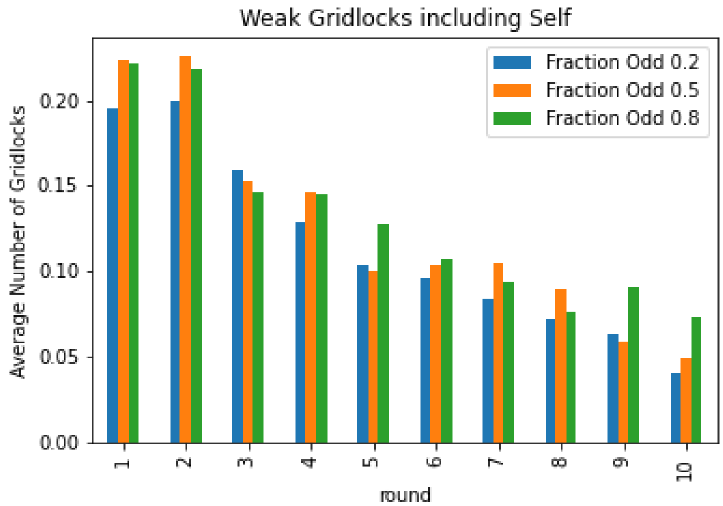


Figure 2.8

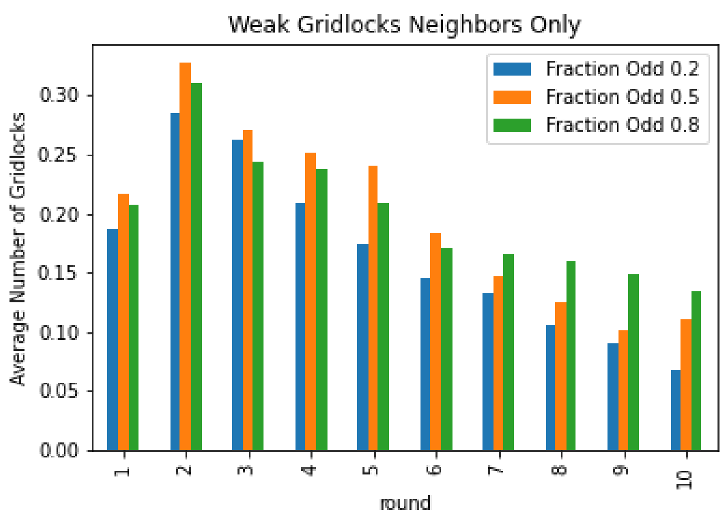


Figure 2.9

2.3.3 State Changes

We measure State Changes among the players if they change their set of proposed identifiers from time t to time t+1. This could mean include adding or subtracting an identifier, as well as keeping to total number of identifiers proposed the same but changing the specific identifiers proposed. One of the ways variations in gridlocks could impact the outcomes is through changing the frequency that players change states. Gridlocked players changed states at a higher rate than non-gridlocked players. Figures 2.10 and 2.11 show the average number of state changes per round by gridlock status when calculating strong gridlocks when using self states or neighbors' states only respectively. Generally, gridlocked players change state at a higher rate than non-gridlocked players (the only exception is in round 2 when considering only neighbors states). Figures 2.12 and 2.13 replicate these same graphs using weak gridlocks and the results are similar.

This suggests that being gridlocked is an unstable state and the players do work the resolve them. Another thing to consider is whether or not changing the fraction of odd vertices impacts the rate at which players change states more generally. Figure 2.14 shows the average number of state changes at each round by fraction of odd vertices and the differences are small. Similarly to the analysis for gridlocks, we preformed Welch's t-tests across the experimental conditions for end game values of state changes. In this case, since state changes is not defined for round 10, we pool round 8 and 9 together instead of 9 and 10. Table 2.2 shows the results of these tests and only the difference between the 0.2 condition and the 0.8 condition are statistically significant. This suggests that players in the 0.8 condition do change their state more frequently at the end of the game than player in the 0.2 condition do. This could also be an indicator of the difficulty in the consensus process, as closer the players are to consensus the fewer times one would expect them to change their state.

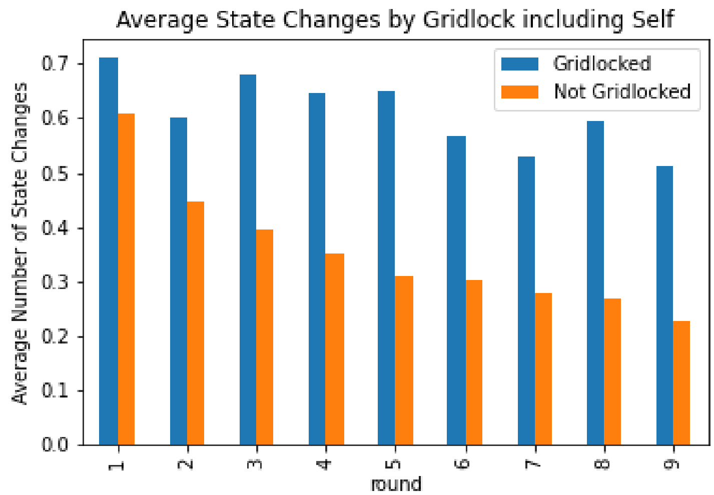


Figure 2.10

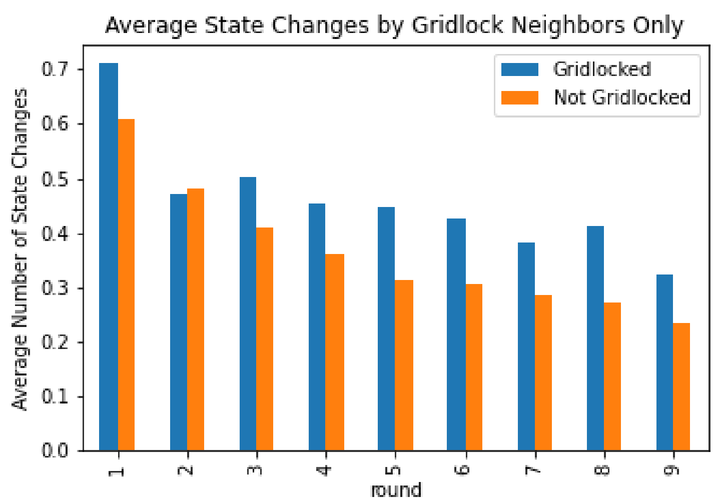


Figure 2.11

Welch's t-test rounds 8 and 9, t (p)	$0.2 < 0.5$	$0.5 < 0.8$	$0.2 < 0.8$
State Changes	-0.875 (0.190)	-1.082 (0.139)	-1.994 (0.023)

Table 2.2

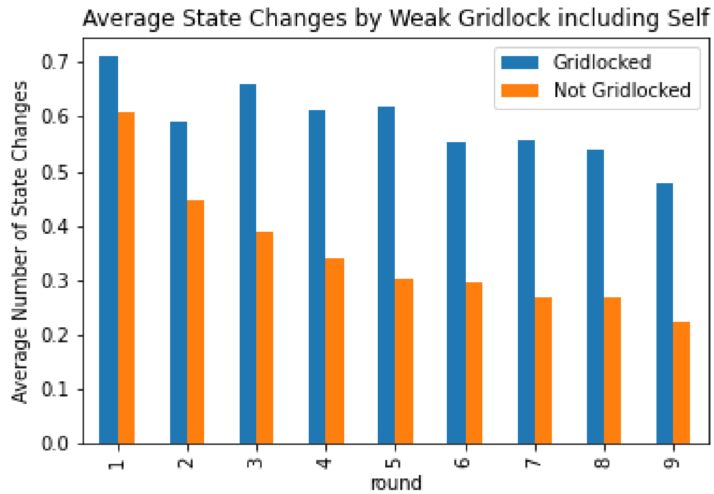


Figure 2.12

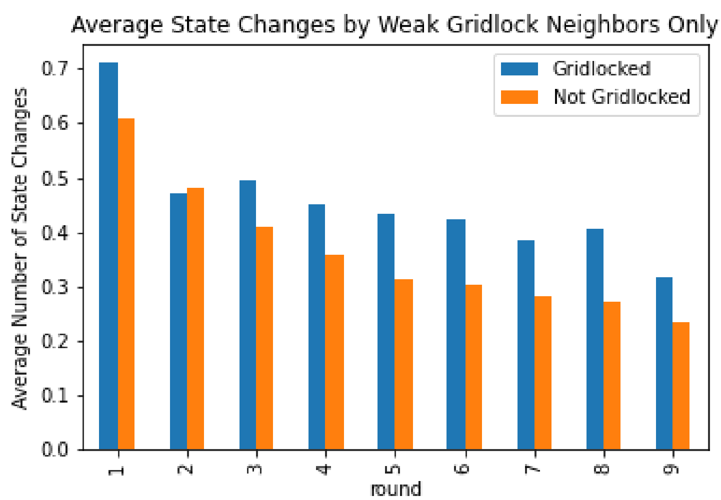


Figure 2.13

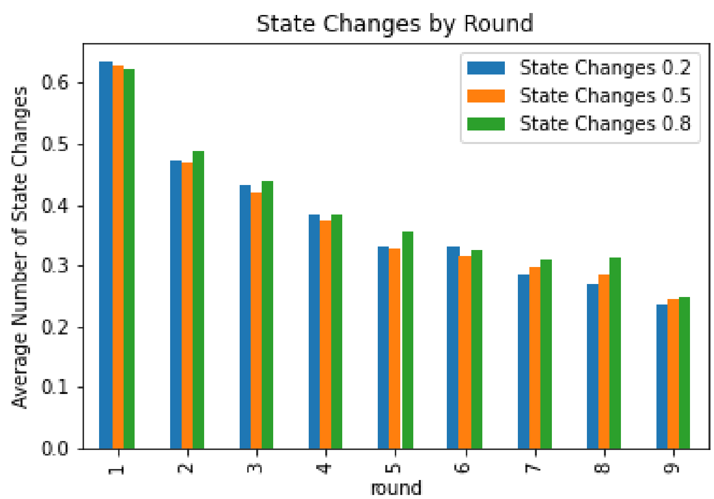

State Changes by Round
State Changes 0.2
State Changes 0.5
State Changes 0.8
Average Number of State Changes
0.0
0.1
0.2
0.3
0.4
0.5
0.6
1
2
3
4
5
6
7
8
9
round

Figure 2.14

2.3.4 Resolved Gridlocks

We define a gridlock as resolved if the player who is gridlocked at time t is no longer gridlocked at time t+1. Gridlock resolution can be applied whichever way gridlocks are calculated. Figure 2.15 shows the average number of resolved strong gridlocks when the self state is included by round and fraction of odd vertices, and Figure 2.16 shows the average number of resolved gridlocks with only neighbors' states by round and fraction of odd vertices. Figures 2.17 and 2.18 show resolved weak gridlocks by round and fraction of odd vertices when self state is included and neighbors only, respectively.

The resolution of gridlocks can show how the consensus process is progressing. If a high fraction of gridlocks are resolved that suggests that the group is moving closer to consensus. Table 2.3 shows the Welch's t-tests across comparing the different conditions for rounds 8 and 9. Only one test was statistically significant, which was resolved weak gridlocks when self is concluded for the hypothesis that the 0.2 condition has a smaller mean than the 0.5 condition. This suggests that gridlocks are resolved comparably across the different experimental conditions, suggesting that whatever process negatively impacts consensus, does not impact the ability of the group to resolve gridlocks.

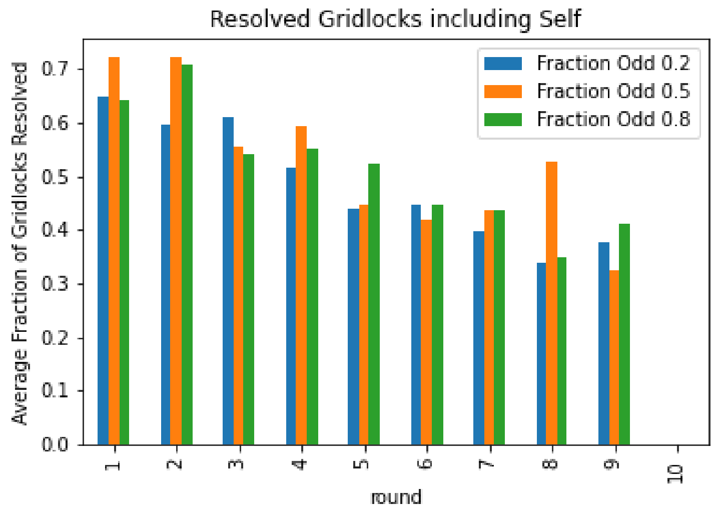


Figure 2.15

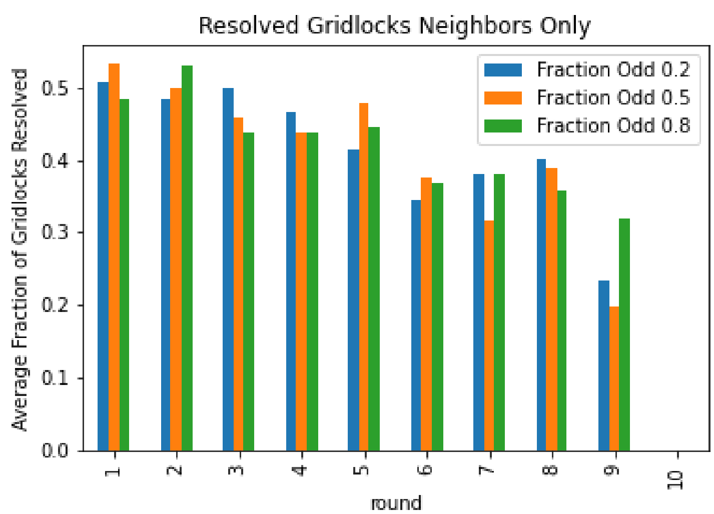


Figure 2.16

Welch's t-test rounds 8 and 9, t (p)	0.2 $<$ 0.5	0.5 $<$ 0.8	0.2 $<$ 0.8
Resolved Gridlocks Including Self	-1.192 (0.117)	0.830 (0.796)	-0.382 (0.351)
Resolved Gridlocks Neighbors Only	0.519 (0.698)	-1.013 (0.155)	-0.472 (0.318)
Resolved Weak Gridlocks Including Self	-1.674 (0.047)	1.807 (0.964)	0.086 (0.534)
Resolved Weak Gridlocks Neighbors Only	0.253 (0.600)	-1.496 (0.067)	-1.234 (0.109)

Table 2.3

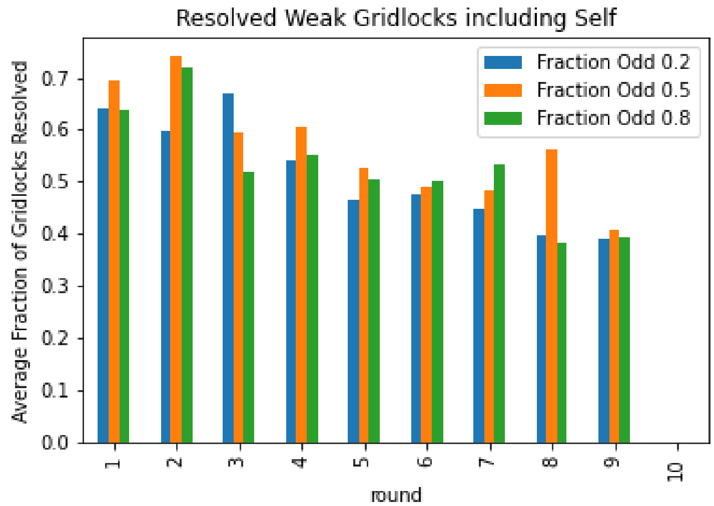


Figure 2.17

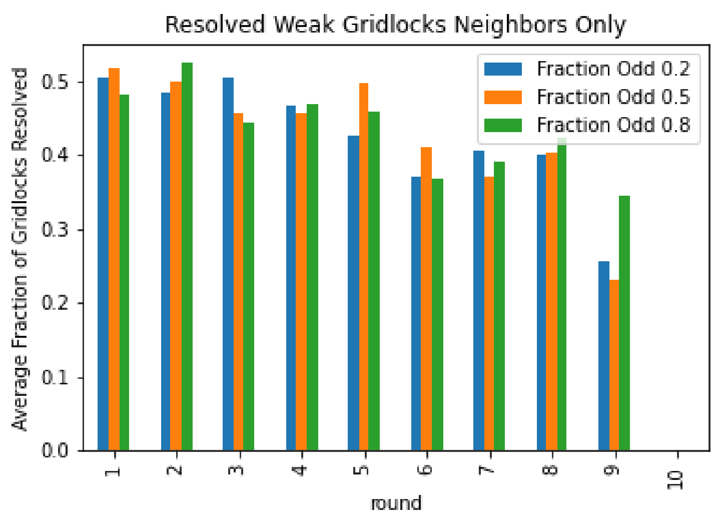


Figure 2.18

2.3.5 Player Strategies

The players came up with a rich array of strategies to address this problem. Some players came up with strategies to induce preferences over the identifiers, like the alphabetical protocol, while others just simply told other players to vote for a specific option, and still others tried to hold the rest of the group hostage claiming that they would vote for their preferred choice regardless of what the other players did. Some players attempted to deceive their neighbors about how many other players were truly supporting their preferred choice, and some of those who claimed to unconditionally vote for a specific identifier ended up voting with the group. In this sense, deception was a valid method for trying to induce cooperation as many of the agents using such strategies were still trying to win.

We also observed some malicious actions from the players, for example sometimes players would get tired of playing the game or demoralized and just give up trying to cooperate. There were also some players that stated that they were simply trying to get the game over with as soon as possible and weren't really trying to win. We set up our payment structure to try and prevent this from happening, and luckily this was not a common occurrence. Still, the fact that this happened suggests that even in settings where people have incentives to cooperate, not everyone will do so. A few players expected to see bots in this context and would some try to Turing test the other players to determine whether or not they were human. Some players would also banter with each other throughout the game talking about a wide variety of topics such as the weather or current events; this also was not common as most of the players were focused on trying to achieve consensus, especially since the amount of time they had to type messages was constrained. While most players were cordial some resorted to insults when they believed other players weren't cooperative. It is unclear what the ultimate impact of this was on the consensus process.

One of the reasons we gave the players unique names was so that they could refer to each other using their names and sign their messages. This is not the same as signed messages in the original formulation of the Byzantine Generals problem (Lamport et al., 1982), since the players could forge each other's signatures. The signatures here are not definitive confirmation that the player did in fact send the message claimed, but would allow players to group players and their votes into ledgers to pass around. While players did not always refer to each other by their names or pass ledgers, they sometimes did. An important example of this was querying their neighbors to try to convince a specific player to go along with the vote. This kind of strategy shows understanding of the networked structure of the game which in early testing we had difficulty conveying to the players. Overall, the full text messages allowed us to observe a much richer array of behavior then simply having the players click buttons for votes. Table 2.4 shows examples of the types of messages mentioned here.

Strategy	Player Message
Alphabetical	Lets vote for the name thats higher in the alphabet, starting with A. Pass it on!
Specific Identifier	We all want the money tell everyone to pick COKUD!!!
Unconditional / Hostage Taking	Alright to avoid confusion any further I will vote on Bexoh no matter what it will be my final vote and we should work to get that to be the worker we all decide on
Deception	We need to be united. All other members are going for FAHIJ. The bonus is the same...dont let us drag this please. (This message was sent in round 2 and the group was not close to consensus and the player was sent messages that were not FAHIJ in the previous round.)
Giving Up	I guess we will have to be happy with what we get, since no one will agree on anything.
Low Effort	Let's pick one and get this over with.
Ledger	Suggest we share all the candidate names and pick the first name in alphabetic order. Don't know how many names there are total, so propagate the message out, add to the list if you get new names. Names: BESAM BEXOH BUMAF GIGOX HITEJ KOSOB PADEB PESAM PIQEZ QOMAP
Mentioning a specific player	Ok great! User OSIB is now trying to back out, I think? What the heck! I'll try to convince them otherwise.
Banter	you crack me up, IWUQ XD
Turing Test	GIGOX for the love of god. I'd swear some of you were bots but even bots could use context clues and switch.
Insults	You are an idiot. you will mess this up for everyone.

Table 2.4

2.4 Analysis

2.4.1 Main Regressions

To account for the possibility that different structural factors in addition to the fraction of odd vertices influences the consensus process we adjust for the graph's diameter, average path length, transitivity, and the fraction edges that are shortcuts. We define an edge to be a shortcut if when the edge is dropped the shortest path between the head and tail vertex is greater than two. Watts and Strogatz (1998) found that shortcuts were responsible for some of the interesting properties of Watts-Strogatz graphs. The diameter shows the worst scenario for perfect message passing (assuming all agents pass all of the information they receive, which real players do not do). However, the average path speaks to the average distance between players. Given that that there are only 10 rounds, message passing on a graph with lower diameter and lower average path length is likely easier than one with relatively higher diameter or average path length. Transitivity could suggest the potential for feedback in the information from messages. Since all of the ties are undirected it is possible for there to be feedback on a cycle size of two, but feedback in larger size cycles has more potential to obfuscated to the players. Although the graphs for each level of odd vertices was drawn from the same distribution, there is still the possibility for random variation on all of these factors. Unfortunately for various reasons such as random disconnections, or player idleness the structure of the graph is not guaranteed to be the same across the duration of the game. To account for this, we run regressions considering structural variables at round 1, round 10 as well as the average of those structural variables across the 10 rounds. This can help determine whether initial conditions, final conditions or average conditions are more relevant for consensus. We also parameterize the treatment of the initial fraction of odd vertices in two ways, as dummies and as a continuous variable. Table 2.5 shows the regression on final standard deviation for average values of the structural variables, when Dummies are used for Fraction of Odd Vertices. In this regression only the coefficient for Fraction of Odd Vertices = 0.2 is statistically significant at the 5% level. None of the dummies for skins are statistically significant. In Table 2.6, instead of dummies for the skins we use the four dimensions of each of the skins: Complexity, Familiarity, Stakes and Emotionally Charged, but Fraction Odd is still a dummy and the structural variables are averages. None of the skin dimensions are statistically significant. As before, only Fraction of Odd Vertices = 0.2 is statistically significant tat the 5% level. Table 2.7 is the same type of model as Table 2.5, except that this time the outcome of interest is win rate not discrete standard deviation. In this case, only Fraction Odd = 0.2 is statistically significant at the 5% level. Table 2.8 is the same type of model as Table 2.6 when looking at the win rate and in this model once again, Fraction Odd = 0.2 is the only coefficient significant at the 5% level. In Tables 2.9 through 2.12 show the same type of models for average structural controls but in these models Fraction of Odd Vertices is a continuous variable instead of dummies. The results are broadly similar to treating Fraction of Odd Vertices as a dummy variable. Tables 2.13 through 2.20 show the series of regressions but with structural controls taken from the first round, while Tables 2.21 through 2.29 show the regressions with structural controls from round 10. Across both round 1 models and round 10 models the coefficient for fraction of odd vertices was always statistically significant. In the round 1 models the fraction of odd coefficient was the only coefficient that was statistically significant at the 5% level. In the round 10 models average path length was also statistically significant at the 5% level. These results suggest that higher average path length is associated with higher final standard deviation and lower probability of winning. The signs of the coefficients for these variables were consistent across models, suggesting that these effects are robust to changes over time.

Dep. Variable:	final_std	**R-squared:**	0.246
Model:	OLS	**Adj. R-squared:**	0.181
Method:	Least Squares	**F-statistic:**	3.844
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	3.50e-06
Time:	22:57:37	**Log-Likelihood:**	47.812
No. Observations:	202	**AIC:**	-61.62
Df Residuals:	185	**BIC:**	-5.383
Df Model:	16		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.1996	0.323	-0.618	0.537	-0.833	0.434
skin: fire	-0.0145	0.077	-0.189	0.850	-0.165	0.136
skin: generals	-0.0522	0.076	-0.690	0.490	-0.200	0.096
skin: hiring	0.0688	0.071	0.969	0.332	-0.070	0.208
skin: mascot	-0.0160	0.076	-0.210	0.834	-0.165	0.133
skin: olympics	-0.0187	0.067	-0.279	0.780	-0.151	0.113
skin: orchestra	0.0648	0.071	0.908	0.364	-0.075	0.205
skin: plant	0.0517	0.069	0.750	0.453	-0.083	0.187
skin: restaurant	0.0522	0.077	0.679	0.497	-0.098	0.203
skin: space	0.0065	0.073	0.089	0.929	-0.136	0.150
Fraction Odd 0.2	-0.1102	0.036	-3.076	0.002	-0.180	-0.040
Fraction Odd 0.5	-0.0426	0.035	-1.216	0.224	-0.111	0.026
frac_shortcut_avg	0.0184	0.313	0.059	0.953	-0.595	0.632
diameter_avg	-0.0027	0.040	-0.066	0.947	-0.082	0.076
avg_path_length_avg	0.2567	0.176	1.460	0.144	-0.088	0.601
transitivity_avg	-0.0875	0.740	-0.118	0.906	-1.538	1.363
count_avg	0.0048	0.007	0.644	0.520	-0.010	0.019

Omnibus:	22.116	**Durbin-Watson:**	2.072
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	18.346
Skew:	-0.647	**Prob(JB):**	0.000104
Kurtosis:	2.288	**Cond. No.**	560.

Table 2.5

Dep. Variable:	final_std	**R-squared:**	0.226
Model:	OLS	**Adj. R-squared:**	0.181
Method:	Least Squares	**F-statistic:**	5.159
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	4.49e-07
Time:	22:57:37	**Log-Likelihood:**	45.082
No. Observations:	202	**AIC:**	-66.16
Df Residuals:	190	**BIC:**	-26.47
Df Model:	11		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.1909	0.326	-0.586	0.558	-0.829	0.448
Fraction Odd 0.2	-0.1091	0.035	-3.116	0.002	-0.178	-0.040
Fraction Odd 0.5	-0.0399	0.035	-1.143	0.253	-0.108	0.029
frac_shortcut_avg	0.0682	0.287	0.238	0.812	-0.494	0.631
diameter_avg	-0.0043	0.039	-0.111	0.911	-0.081	0.072
avg_path_length_avg	0.2619	0.174	1.509	0.131	-0.078	0.602
transitivity_avg	-0.0360	0.669	-0.054	0.957	-1.346	1.274
count_avg	0.0051	0.007	0.712	0.477	-0.009	0.019
Complex	-3.638e-05	0.000	-0.356	0.722	-0.000	0.000
Familiar	1.032e-05	2.18e-05	0.473	0.636	-3.25e-05	5.31e-05
Stakes	1.667e-05	8.16e-05	0.204	0.838	-0.000	0.000
Emotionally_Charged	-1.762e-05	3.14e-05	-0.561	0.575	-7.92e-05	4.39e-05

Omnibus:	25.071	**Durbin-Watson:**	2.005
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	19.116
Skew:	-0.648	**Prob(JB):**	7.06e-05
Kurtosis:	2.230	**Cond. No.**	9.63e+04

Table 2.6

Dep. Variable:	win	**R-squared:**	0.243
Model:	OLS	**Adj. R-squared:**	0.177
Method:	Least Squares	**F-statistic:**	2.819
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	0.000415
Time:	22:57:37	**Log-Likelihood:**	-76.352
No. Observations:	202	**AIC:**	186.7
Df Residuals:	185	**BIC:**	242.9
Df Model:	16		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.3860	0.714	1.943	0.052	-0.012	2.784
skin: fire	-0.0350	0.139	-0.251	0.802	-0.308	0.238
skin: generals	0.0988	0.148	0.669	0.503	-0.191	0.388
skin: hiring	-0.1271	0.129	-0.985	0.325	-0.380	0.126
skin: mascot	-0.0054	0.144	-0.038	0.970	-0.287	0.276
skin: olympics	-0.1043	0.118	-0.884	0.377	-0.336	0.127
skin: orchestra	-0.0699	0.126	-0.554	0.579	-0.317	0.177
skin: plant	-0.0930	0.126	-0.737	0.461	-0.341	0.154
skin: restaurant	-0.0195	0.136	-0.144	0.886	-0.285	0.246
skin: space	0.0045	0.132	0.034	0.973	-0.255	0.264
Fraction Odd 0.2	0.1802	0.064	2.795	0.005	0.054	0.307
Fraction Odd 0.5	0.0442	0.063	0.701	0.483	-0.079	0.168
frac_shortcut_avg	-0.0838	0.759	-0.110	0.912	-1.572	1.405
diameter_avg	-5.928e-05	0.074	-0.001	0.999	-0.146	0.146
avg_path_length_avg	-0.5738	0.349	-1.643	0.100	-1.258	0.111
transitivity_avg	0.3152	1.870	0.169	0.866	-3.350	3.980
count_avg	6.418e-06	0.015	0.000	1.000	-0.030	0.030

Omnibus:	26.800	**Durbin-Watson:**	2.036
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	34.510
Skew:	1.012	**Prob(JB):**	3.21e-08
Kurtosis:	3.011	**Cond. No.**	560.

Table 2.7

Dep. Variable:	win	**R-squared:**	0.224
Model:	OLS	**Adj. R-squared:**	0.180
Method:	Least Squares	**F-statistic:**	4.077
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	2.23e-05
Time:	22:57:37	**Log-Likelihood:**	-78.776
No. Observations:	202	**AIC:**	181.6
Df Residuals:	190	**BIC:**	221.3
Df Model:	11		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.3858	0.705	1.965	0.049	0.004	2.768
Fraction Odd 0.2	0.1765	0.064	2.762	0.006	0.051	0.302
Fraction Odd 0.5	0.0408	0.063	0.652	0.514	-0.082	0.163
frac_shortcut_avg	-0.1738	0.673	-0.258	0.796	-1.494	1.146
diameter_avg	-0.0007	0.070	-0.010	0.992	-0.138	0.137
avg_path_length_avg	-0.6276	0.336	-1.868	0.062	-1.286	0.031
transitivity_avg	0.1897	1.645	0.115	0.908	-3.035	3.415
count_avg	0.0026	0.014	0.182	0.856	-0.026	0.031
Complex	0.0002	0.000	0.756	0.450	-0.000	0.001
Familiar	8.568e-06	4e-05	0.214	0.830	-6.98e-05	8.69e-05
Stakes	-8.015e-05	0.000	-0.511	0.610	-0.000	0.000
Emotionally_Charged	-5.944e-07	5.83e-05	-0.010	0.992	-0.000	0.000

Omnibus:	29.383	**Durbin-Watson:**	1.977
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	38.860
Skew:	1.074	**Prob(JB):**	3.64e-09
Kurtosis:	3.059	**Cond. No.**	9.63e+04

Table 2.8

Dep. Variable:	final_std	**R-squared:**	0.246
Model:	OLS	**Adj. R-squared:**	0.185
Method:	Least Squares	**F-statistic:**	4.079
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	2.02e-06
Time:	22:57:37	**Log-Likelihood:**	47.719
No. Observations:	202	**AIC:**	-63.44
Df Residuals:	186	**BIC:**	-10.51
Df Model:	15		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.3326	0.325	-1.022	0.307	-0.970	0.305
skin: fire	-0.0139	0.076	-0.182	0.855	-0.163	0.136
skin: generals	-0.0522	0.075	-0.694	0.488	-0.200	0.095
skin: hiring	0.0690	0.071	0.976	0.329	-0.070	0.208
skin: mascot	-0.0162	0.076	-0.213	0.831	-0.165	0.132
skin: olympics	-0.0189	0.067	-0.282	0.778	-0.150	0.113
skin: orchestra	0.0652	0.071	0.913	0.361	-0.075	0.205
skin: plant	0.0519	0.069	0.754	0.451	-0.083	0.187
skin: restaurant	0.0517	0.076	0.676	0.499	-0.098	0.202
skin: space	0.0069	0.073	0.095	0.924	-0.136	0.150
frac_shortcut_avg	0.0132	0.315	0.042	0.967	-0.605	0.631
diameter_avg	-0.0023	0.040	-0.057	0.954	-0.081	0.076
avg_path_length_avg	0.2565	0.175	1.462	0.144	-0.087	0.600
transitivity_avg	-0.1027	0.748	-0.137	0.891	-1.568	1.363
count_avg	0.0044	0.007	0.603	0.547	-0.010	0.019
Fraction Odd	0.1835	0.059	3.085	0.002	0.067	0.300

Omnibus:	22.616	**Durbin-Watson:**	2.071
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	18.438
Skew:	-0.646	**Prob(JB):**	9.91e-05
Kurtosis:	2.276	**Cond. No.**	556.

Table 2.9

Dep. Variable:	final_std	**R-squared:**	0.225
Model:	OLS	**Adj. R-squared:**	0.184
Method:	Least Squares	**F-statistic:**	5.623
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	2.36e-07
Time:	22:57:37	**Log-Likelihood:**	44.959
No. Observations:	202	**AIC:**	-67.92
Df Residuals:	191	**BIC:**	-31.53
Df Model:	10		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.3196	0.326	-0.981	0.327	-0.958	0.319
frac_shortcut_avg	0.0623	0.290	0.215	0.830	-0.506	0.631
diameter_avg	-0.0039	0.039	-0.100	0.920	-0.080	0.072
avg_path_length_avg	0.2616	0.173	1.510	0.131	-0.078	0.601
transitivity_avg	-0.0537	0.679	-0.079	0.937	-1.384	1.277
count_avg	0.0047	0.007	0.660	0.509	-0.009	0.019
Complex	-3.68e-05	0.000	-0.361	0.718	-0.000	0.000
Familiar	1.007e-05	2.16e-05	0.465	0.642	-3.24e-05	5.25e-05
Stakes	1.675e-05	8.15e-05	0.206	0.837	-0.000	0.000
Emotionally_Charged	-1.726e-05	3.11e-05	-0.555	0.579	-7.82e-05	4.37e-05
Fraction Odd	0.1816	0.058	3.126	0.002	0.068	0.295

Omnibus:	25.985	**Durbin-Watson:**	2.003
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	19.161
Skew:	-0.643	**Prob(JB):**	6.91e-05
Kurtosis:	2.211	**Cond. No.**	9.56e+04

Table 2.10

Dep. Variable:	win	**R-squared:**	0.240
Model:	OLS	**Adj. R-squared:**	0.179
Method:	Least Squares	**F-statistic:**	2.955
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	0.000307
Time:	22:57:37	**Log-Likelihood:**	-76.716
No. Observations:	202	**AIC:**	185.4
Df Residuals:	186	**BIC:**	238.4
Df Model:	15		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.5752	0.723	2.177	0.029	0.157	2.993
skin: fire	-0.0372	0.139	-0.268	0.789	-0.310	0.235
skin: generals	0.0990	0.147	0.672	0.501	-0.190	0.388
skin: hiring	-0.1280	0.129	-0.995	0.320	-0.380	0.124
skin: mascot	-0.0048	0.143	-0.033	0.973	-0.285	0.276
skin: olympics	-0.1036	0.119	-0.871	0.384	-0.337	0.130
skin: orchestra	-0.0712	0.128	-0.558	0.577	-0.321	0.179
skin: plant	-0.0936	0.126	-0.743	0.457	-0.341	0.153
skin: restaurant	-0.0178	0.135	-0.131	0.895	-0.283	0.247
skin: space	0.0031	0.134	0.023	0.982	-0.260	0.266
frac_shortcut_avg	-0.0645	0.772	-0.084	0.933	-1.578	1.449
diameter_avg	-0.0014	0.074	-0.019	0.985	-0.147	0.145
avg_path_length_avg	-0.5733	0.348	-1.646	0.100	-1.256	0.109
transitivity_avg	0.3710	1.908	0.194	0.846	-3.368	4.110
count_avg	0.0011	0.015	0.075	0.940	-0.029	0.031
Fraction Odd	-0.2997	0.107	-2.804	0.005	-0.509	-0.090

Omnibus:	26.359	**Durbin-Watson:**	2.027
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	33.893
Skew:	1.003	**Prob(JB):**	4.37e-08
Kurtosis:	2.953	**Cond. No.**	556.

Table 2.11

Dep. Variable:	win	**R-squared:**	0.222
Model:	OLS	**Adj. R-squared:**	0.181
Method:	Least Squares	**F-statistic:**	4.417
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	1.36e-05
Time:	22:57:38	**Log-Likelihood:**	-79.157
No. Observations:	202	**AIC:**	180.3
Df Residuals:	191	**BIC:**	216.7
Df Model:	10		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.5670	0.708	2.215	0.027	0.180	2.954
frac_shortcut_avg	-0.1547	0.687	-0.225	0.822	-1.501	1.192
diameter_avg	-0.0021	0.070	-0.030	0.976	-0.140	0.136
avg_path_length_avg	-0.6267	0.335	-1.872	0.061	-1.283	0.030
transitivity_avg	0.2471	1.685	0.147	0.883	-3.055	3.549
count_avg	0.0038	0.014	0.263	0.793	-0.024	0.032
Complex	0.0002	0.000	0.763	0.445	-0.000	0.001
Familiar	9.404e-06	3.95e-05	0.238	0.812	-6.81e-05	8.69e-05
Stakes	-8.041e-05	0.000	-0.509	0.610	-0.000	0.000
Emotionally_Charged	-1.786e-06	5.78e-05	-0.031	0.975	-0.000	0.000
Fraction Odd	-0.2935	0.106	-2.771	0.006	-0.501	-0.086

Omnibus:	28.924	**Durbin-Watson:**	1.968
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	38.229
Skew:	1.066	**Prob(JB):**	5.00e-09
Kurtosis:	2.996	**Cond. No.**	9.56e+04

Table 2.12

Dep. Variable:	final_std	**R-squared:**	0.239
Model:	OLS	**Adj. R-squared:**	0.173
Method:	Least Squares	**F-statistic:**	3.560
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	1.33e-05
Time:	23:00:21	**Log-Likelihood:**	46.865
No. Observations:	202	**AIC:**	-59.73
Df Residuals:	185	**BIC:**	-3.490
Df Model:	16		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.2695	0.487	-0.554	0.580	-1.224	0.685
skin: fire	-0.0231	0.077	-0.298	0.766	-0.175	0.129
skin: generals	-0.0645	0.075	-0.854	0.393	-0.212	0.083
skin: hiring	0.0764	0.077	0.992	0.321	-0.075	0.227
skin: mascot	-0.0289	0.076	-0.380	0.704	-0.178	0.120
skin: olympics	-0.0126	0.069	-0.181	0.856	-0.149	0.123
skin: orchestra	0.0564	0.071	0.792	0.428	-0.083	0.196
skin: plant	0.0514	0.071	0.721	0.471	-0.088	0.191
skin: restaurant	0.0580	0.079	0.732	0.464	-0.097	0.213
skin: space	0.0014	0.074	0.019	0.985	-0.143	0.146
Fraction Odd 0.2	-0.1139	0.038	-2.997	0.003	-0.188	-0.039
Fraction Odd 0.5	-0.0486	0.036	-1.354	0.176	-0.119	0.022
frac_shortcut_1	-0.0422	0.465	-0.091	0.928	-0.953	0.869
diameter_1	-0.0172	0.042	-0.411	0.681	-0.099	0.065
avg_path_length_1	0.3770	0.389	0.968	0.333	-0.386	1.140
transitivity_1	-0.1660	1.224	-0.136	0.892	-2.566	2.234
count_1	0.0012	0.019	0.064	0.949	-0.036	0.038

Omnibus:	21.009	**Durbin-Watson:**	2.099
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	19.728
Skew:	-0.696	**Prob(JB):**	5.20e-05
Kurtosis:	2.363	**Cond. No.**	839.

Table 2.13

Dep. Variable:	final_std	**R-squared:**	0.215
Model:	OLS	**Adj. R-squared:**	0.169
Method:	Least Squares	**F-statistic:**	4.666
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	2.65e-06
Time:	23:00:21	**Log-Likelihood:**	43.670
No. Observations:	202	**AIC:**	-63.34
Df Residuals:	190	**BIC:**	-23.64
Df Model:	11		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.1276	0.447	-0.285	0.775	-1.003	0.748
Fraction Odd 0.2	-0.1140	0.037	-3.111	0.002	-0.186	-0.042
Fraction Odd 0.5	-0.0465	0.036	-1.304	0.192	-0.116	0.023
frac_shortcut_1	-0.0130	0.422	-0.031	0.975	-0.840	0.814
diameter_1	-0.0093	0.040	-0.231	0.817	-0.088	0.070
avg_path_length_1	0.2769	0.345	0.802	0.422	-0.400	0.953
transitivity_1	-0.1723	1.094	-0.158	0.875	-2.316	1.971
count_1	0.0047	0.017	0.279	0.780	-0.028	0.038
Complex	-4.769e-05	0.000	-0.469	0.639	-0.000	0.000
Familiar	1.635e-05	2.15e-05	0.760	0.447	-2.58e-05	5.85e-05
Stakes	3.093e-05	8.03e-05	0.385	0.700	-0.000	0.000
Emotionally_Charged	-2.238e-05	3.02e-05	-0.741	0.459	-8.16e-05	3.68e-05

Omnibus:	23.205	**Durbin-Watson:**	2.034
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	20.493
Skew:	-0.699	**Prob(JB):**	3.55e-05
Kurtosis:	2.306	**Cond. No.**	1.39e+05

Table 2.14

Dep. Variable:	win	**R-squared:**	0.230
Model:	OLS	**Adj. R-squared:**	0.163
Method:	Least Squares	**F-statistic:**	2.458
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	0.00211
Time:	23:00:21	**Log-Likelihood:**	-78.080
No. Observations:	202	**AIC:**	190.2
Df Residuals:	185	**BIC:**	246.4
Df Model:	16		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.4977	0.957	1.564	0.118	-0.379	3.374
skin: fire	-0.0182	0.141	-0.129	0.897	-0.295	0.258
skin: generals	0.1337	0.146	0.917	0.359	-0.152	0.419
skin: hiring	-0.1443	0.143	-1.006	0.314	-0.425	0.137
skin: mascot	0.0266	0.146	0.182	0.855	-0.259	0.313
skin: olympics	-0.1099	0.126	-0.869	0.385	-0.358	0.138
skin: orchestra	-0.0438	0.125	-0.349	0.727	-0.289	0.202
skin: plant	-0.0855	0.132	-0.646	0.518	-0.345	0.174
skin: restaurant	-0.0273	0.142	-0.192	0.848	-0.307	0.252
skin: space	0.0192	0.137	0.140	0.888	-0.249	0.287
Fraction Odd 0.2	0.1958	0.070	2.803	0.005	0.059	0.333
Fraction Odd 0.5	0.0580	0.065	0.896	0.370	-0.069	0.185
frac_shortcut_1	0.1090	1.051	0.104	0.917	-1.950	2.168
diameter_1	0.0004	0.075	0.006	0.996	-0.146	0.147
avg_path_length_1	-0.7625	0.808	-0.944	0.345	-2.345	0.820
transitivity_1	0.4693	2.834	0.166	0.868	-5.085	6.023
count_1	0.0056	0.041	0.137	0.891	-0.074	0.085

Omnibus:	29.747	**Durbin-Watson:**	2.066
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	39.432
Skew:	1.081	**Prob(JB):**	2.74e-09
Kurtosis:	3.083	**Cond. No.**	839.

Table 2.15

Dep. Variable:	win	**R-squared:**	0.203
Model:	OLS	**Adj. R-squared:**	0.157
Method:	Least Squares	**F-statistic:**	3.391
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	0.000263
Time:	23:00:21	**Log-Likelihood:**	-81.521
No. Observations:	202	**AIC:**	187.0
Df Residuals:	190	**BIC:**	226.7
Df Model:	11		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.2657	0.902	1.403	0.161	-0.503	3.034
Fraction Odd 0.2	0.1968	0.068	2.874	0.004	0.063	0.331
Fraction Odd 0.5	0.0556	0.064	0.872	0.383	-0.069	0.181
frac_shortcut_1	0.0420	0.916	0.046	0.963	-1.754	1.838
diameter_1	-0.0253	0.070	-0.362	0.717	-0.162	0.112
avg_path_length_1	-0.6007	0.701	-0.857	0.391	-1.975	0.773
transitivity_1	0.4256	2.435	0.175	0.861	-4.347	5.198
count_1	0.0017	0.035	0.049	0.961	-0.068	0.071
Complex	0.0002	0.000	0.900	0.368	-0.000	0.001
Familiar	-6.069e-06	3.97e-05	-0.153	0.879	-8.39e-05	7.18e-05
Stakes	-0.0001	0.000	-0.733	0.464	-0.000	0.000
Emotionally_Charged	5.818e-06	5.48e-05	0.106	0.916	-0.000	0.000

Omnibus:	32.489	**Durbin-Watson:**	2.000
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	44.435
Skew:	1.148	**Prob(JB):**	2.24e-10
Kurtosis:	3.095	**Cond. No.**	1.39e+05

Table 2.16

Dep. Variable:	final_std	**R-squared:**	0.239
Model:	OLS	**Adj. R-squared:**	0.178
Method:	Least Squares	**F-statistic:**	3.816
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	6.59e-06
Time:	23:00:22	**Log-Likelihood:**	46.824
No. Observations:	202	**AIC:**	-61.65
Df Residuals:	186	**BIC:**	-8.716
Df Model:	15		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.4169	0.497	-0.839	0.401	-1.390	0.556
skin: fire	-0.0227	0.077	-0.295	0.768	-0.173	0.128
skin: generals	-0.0647	0.075	-0.861	0.389	-0.212	0.082
skin: hiring	0.0764	0.077	0.996	0.319	-0.074	0.227
skin: mascot	-0.0293	0.076	-0.387	0.699	-0.178	0.119
skin: olympics	-0.0127	0.069	-0.183	0.855	-0.148	0.123
skin: orchestra	0.0565	0.071	0.794	0.427	-0.083	0.196
skin: plant	0.0515	0.071	0.725	0.469	-0.088	0.191
skin: restaurant	0.0577	0.079	0.733	0.464	-0.097	0.212
skin: space	0.0015	0.074	0.020	0.984	-0.143	0.146
frac_shortcut_1	-0.0437	0.464	-0.094	0.925	-0.954	0.866
diameter_1	-0.0175	0.042	-0.420	0.674	-0.099	0.064
avg_path_length_1	0.3807	0.385	0.988	0.323	-0.375	1.136
transitivity_1	-0.1747	1.220	-0.143	0.886	-2.566	2.217
count_1	0.0009	0.019	0.048	0.961	-0.036	0.038
Fraction Odd	0.1897	0.063	3.006	0.003	0.066	0.313

Omnibus:	21.165	**Durbin-Watson:**	2.099
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	19.754
Skew:	-0.695	**Prob(JB):**	5.13e-05
Kurtosis:	2.358	**Cond. No.**	841.

Table 2.17

Dep. Variable:	final_std	**R-squared:**	0.214
Model:	OLS	**Adj. R-squared:**	0.173
Method:	Least Squares	**F-statistic:**	5.137
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	1.20e-06
Time:	23:00:21	**Log-Likelihood:**	43.606
No. Observations:	202	**AIC:**	-65.21
Df Residuals:	191	**BIC:**	-28.82
Df Model:	10		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.2738	0.452	-0.606	0.545	-1.160	0.612
frac_shortcut_1	-0.0148	0.423	-0.035	0.972	-0.845	0.815
diameter_1	-0.0096	0.040	-0.240	0.811	-0.088	0.069
avg_path_length_1	0.2812	0.342	0.821	0.412	-0.390	0.952
transitivity_1	-0.1833	1.095	-0.167	0.867	-2.330	1.963
count_1	0.0043	0.017	0.259	0.795	-0.028	0.037
Complex	-4.785e-05	0.000	-0.472	0.637	-0.000	0.000
Familiar	1.627e-05	2.14e-05	0.762	0.446	-2.56e-05	5.81e-05
Stakes	3.092e-05	8.01e-05	0.386	0.699	-0.000	0.000
Emotionally_Charged	-2.21e-05	2.99e-05	-0.738	0.460	-8.08e-05	3.66e-05
Fraction Odd	0.1898	0.061	3.119	0.002	0.071	0.309

Omnibus:	23.505	**Durbin-Watson:**	2.033
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	20.462
Skew:	-0.696	**Prob(JB):**	3.60e-05
Kurtosis:	2.296	**Cond. No.**	1.39e+05

Table 2.18

Dep. Variable:	win	**R-squared:**	0.228
Model:	OLS	**Adj. R-squared:**	0.165
Method:	Least Squares	**F-statistic:**	2.599
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	0.00144
Time:	23:00:22	**Log-Likelihood:**	-78.351
No. Observations:	202	**AIC:**	188.7
Df Residuals:	186	**BIC:**	241.6
Df Model:	15		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.7374	0.979	1.774	0.076	-0.182	3.657
skin: fire	-0.0201	0.140	-0.143	0.886	-0.295	0.255
skin: generals	0.1346	0.145	0.927	0.354	-0.150	0.419
skin: hiring	-0.1444	0.143	-1.007	0.314	-0.425	0.137
skin: mascot	0.0283	0.145	0.195	0.845	-0.257	0.313
skin: olympics	-0.1095	0.127	-0.861	0.390	-0.359	0.140
skin: orchestra	-0.0440	0.126	-0.348	0.728	-0.291	0.203
skin: plant	-0.0858	0.132	-0.649	0.516	-0.345	0.173
skin: restaurant	-0.0262	0.142	-0.184	0.854	-0.304	0.252
skin: space	0.0188	0.138	0.136	0.892	-0.252	0.289
frac_shortcut_1	0.1161	1.062	0.109	0.913	-1.965	2.197
diameter_1	0.0017	0.074	0.023	0.982	-0.143	0.146
avg_path_length_1	-0.7802	0.804	-0.971	0.332	-2.356	0.795
transitivity_1	0.5109	2.860	0.179	0.858	-5.095	6.117
count_1	0.0071	0.040	0.175	0.861	-0.072	0.086
Fraction Odd	-0.3257	0.116	-2.806	0.005	-0.553	-0.098

Omnibus:	29.366	**Durbin-Watson:**	2.059
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	38.884
Skew:	1.074	**Prob(JB):**	3.60e-09
Kurtosis:	3.042	**Cond. No.**	841.

Table 2.19

Dep. Variable:	win	**R-squared:**	0.201
Model:	OLS	**Adj. R-squared:**	0.159
Method:	Least Squares	**F-statistic:**	3.703
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	0.000149
Time:	23:00:22	**Log-Likelihood:**	-81.822
No. Observations:	202	**AIC:**	185.6
Df Residuals:	191	**BIC:**	222.0
Df Model:	10		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.5045	0.913	1.647	0.100	-0.286	3.295
frac_shortcut_1	0.0493	0.930	0.053	0.958	-1.773	1.872
diameter_1	-0.0241	0.069	-0.349	0.727	-0.159	0.111
avg_path_length_1	-0.6184	0.699	-0.885	0.376	-1.989	0.752
transitivity_1	0.4702	2.471	0.190	0.849	-4.373	5.313
count_1	0.0033	0.035	0.094	0.925	-0.066	0.072
Complex	0.0002	0.000	0.903	0.367	-0.000	0.001
Familiar	-5.733e-06	3.93e-05	-0.146	0.884	-8.27e-05	7.12e-05
Stakes	-0.0001	0.000	-0.728	0.466	-0.000	0.000
Emotionally_Charged	4.686e-06	5.44e-05	0.086	0.931	-0.000	0.000
Fraction Odd	-0.3273	0.114	-2.877	0.004	-0.550	-0.104

Omnibus:	32.153	**Durbin-Watson:**	1.992
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	43.966
Skew:	1.143	**Prob(JB):**	2.84e-10
Kurtosis:	3.049	**Cond. No.**	1.39e+05

Table 2.20

Dep. Variable:	final_std	**R-squared:**	0.255
Model:	OLS	**Adj. R-squared:**	0.191
Method:	Least Squares	**F-statistic:**	4.350
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	3.24e-07
Time:	23:00:45	**Log-Likelihood:**	49.027
No. Observations:	202	**AIC:**	-64.05
Df Residuals:	185	**BIC:**	-7.814
Df Model:	16		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.2953	0.238	-1.241	0.214	-0.762	0.171
skin: fire	-0.0025	0.076	-0.033	0.974	-0.152	0.147
skin: generals	-0.0386	0.075	-0.518	0.604	-0.185	0.107
skin: hiring	0.0670	0.068	0.985	0.325	-0.066	0.200
skin: mascot	-0.0099	0.074	-0.133	0.894	-0.156	0.136
skin: olympics	-0.0232	0.065	-0.357	0.721	-0.151	0.104
skin: orchestra	0.0737	0.069	1.063	0.288	-0.062	0.210
skin: plant	0.0609	0.068	0.891	0.373	-0.073	0.195
skin: restaurant	0.0569	0.074	0.768	0.443	-0.088	0.202
skin: space	0.0090	0.071	0.126	0.899	-0.130	0.148
Fraction Odd 0.2	-0.1108	0.036	-3.089	0.002	-0.181	-0.040
Fraction Odd 0.5	-0.0376	0.035	-1.075	0.282	-0.106	0.031
frac_shortcut_10	0.0400	0.194	0.206	0.837	-0.340	0.420
diameter_10	-0.0154	0.033	-0.470	0.638	-0.079	0.049
avg_path_length_10	0.3175	0.134	2.371	0.018	0.055	0.580
transitivity_10	0.0051	0.411	0.012	0.990	-0.801	0.811
count_10	0.0034	0.005	0.634	0.526	-0.007	0.014

Omnibus:	22.402	**Durbin-Watson:**	2.040
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	17.352
Skew:	-0.615	**Prob(JB):**	0.000171
Kurtosis:	2.259	**Cond. No.**	501.

Table 2.21

Dep. Variable:	final_std	**R-squared:**	0.235
Model:	OLS	**Adj. R-squared:**	0.191
Method:	Least Squares	**F-statistic:**	5.713
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	6.17e-08
Time:	23:00:45	**Log-Likelihood:**	46.333
No. Observations:	202	**AIC:**	-68.67
Df Residuals:	190	**BIC:**	-28.97
Df Model:	11		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.2672	0.234	-1.142	0.254	-0.726	0.191
Fraction Odd 0.2	-0.1103	0.035	-3.153	0.002	-0.179	-0.042
Fraction Odd 0.5	-0.0352	0.035	-1.013	0.311	-0.103	0.033
frac_shortcut_10	0.0817	0.179	0.457	0.648	-0.269	0.433
diameter_10	-0.0148	0.031	-0.471	0.638	-0.076	0.047
avg_path_length_10	0.3132	0.130	2.402	0.016	0.058	0.569
transitivity_10	0.0470	0.366	0.129	0.898	-0.670	0.764
count_10	0.0040	0.005	0.778	0.436	-0.006	0.014
Complex	-1.604e-05	0.000	-0.156	0.876	-0.000	0.000
Familiar	7.838e-06	2.12e-05	0.370	0.711	-3.36e-05	4.93e-05
Stakes	-5.161e-06	8.25e-05	-0.063	0.950	-0.000	0.000
Emotionally_Charged	-1.225e-05	3.19e-05	-0.384	0.701	-7.47e-05	5.02e-05

Omnibus:	25.634	**Durbin-Watson:**	1.976
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	18.099
Skew:	-0.615	**Prob(JB):**	0.000117
Kurtosis:	2.201	**Cond. No.**	8.95e+04

Table 2.22

Dep. Variable:	win	**R-squared:**	0.252
Model:	OLS	**Adj. R-squared:**	0.187
Method:	Least Squares	**F-statistic:**	3.174
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	8.06e-05
Time:	23:00:45	**Log-Likelihood:**	-75.127
No. Observations:	202	**AIC:**	184.3
Df Residuals:	185	**BIC:**	240.5
Df Model:	16		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.4733	0.515	2.861	0.004	0.464	2.482
skin: fire	-0.0571	0.137	-0.417	0.677	-0.326	0.212
skin: generals	0.0724	0.146	0.496	0.620	-0.214	0.359
skin: hiring	-0.1223	0.123	-0.994	0.320	-0.363	0.119
skin: mascot	-0.0143	0.139	-0.103	0.918	-0.287	0.258
skin: olympics	-0.1004	0.113	-0.892	0.373	-0.321	0.120
skin: orchestra	-0.0860	0.121	-0.709	0.478	-0.324	0.152
skin: plant	-0.1097	0.123	-0.892	0.372	-0.351	0.131
skin: restaurant	-0.0287	0.133	-0.215	0.829	-0.290	0.232
skin: space	0.0004	0.127	0.003	0.997	-0.248	0.249
Fraction Odd 0.2	0.1833	0.065	2.840	0.005	0.057	0.310
Fraction Odd 0.5	0.0333	0.063	0.529	0.597	-0.090	0.157
frac_shortcut_10	-0.1134	0.460	-0.246	0.805	-1.016	0.789
diameter_10	0.0281	0.058	0.483	0.629	-0.086	0.142
avg_path_length_10	-0.6306	0.253	-2.494	0.013	-1.126	-0.135
transitivity_10	0.1745	1.064	0.164	0.870	-1.911	2.260
count_10	-0.0005	0.010	-0.054	0.957	-0.020	0.019

Omnibus:	24.882	**Durbin-Watson:**	2.013
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	31.424
Skew:	0.966	**Prob(JB):**	1.50e-07
Kurtosis:	2.964	**Cond. No.**	501.

Table 2.23

Dep. Variable:	win	**R-squared:**	0.236
Model:	OLS	**Adj. R-squared:**	0.192
Method:	Least Squares	**F-statistic:**	4.564
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	3.84e-06
Time:	23:00:45	**Log-Likelihood:**	-77.234
No. Observations:	202	**AIC:**	178.5
Df Residuals:	190	**BIC:**	218.2
Df Model:	11		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.4112	0.500	2.823	0.005	0.432	2.391
Fraction Odd 0.2	0.1803	0.064	2.830	0.005	0.055	0.305
Fraction Odd 0.5	0.0294	0.062	0.472	0.637	-0.093	0.151
frac_shortcut_10	-0.1755	0.415	-0.423	0.672	-0.989	0.638
diameter_10	0.0253	0.054	0.466	0.641	-0.081	0.132
avg_path_length_10	-0.6552	0.243	-2.699	0.007	-1.131	-0.179
transitivity_10	0.0936	0.939	0.100	0.921	-1.747	1.934
count_10	0.0008	0.010	0.085	0.932	-0.018	0.020
Complex	0.0001	0.000	0.567	0.571	-0.000	0.001
Familiar	1.234e-05	3.9e-05	0.317	0.752	-6.4e-05	8.87e-05
Stakes	-4.009e-05	0.000	-0.257	0.797	-0.000	0.000
Emotionally_Charged	-1.015e-05	5.81e-05	-0.175	0.861	-0.000	0.000

Omnibus:	27.120	**Durbin-Watson:**	1.964
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	35.050
Skew:	1.020	**Prob(JB):**	2.45e-08
Kurtosis:	3.012	**Cond. No.**	8.95e+04

Table 2.24

Dep. Variable:	final_std	**R-squared:**	0.254
Model:	OLS	**Adj. R-squared:**	0.194
Method:	Least Squares	**F-statistic:**	4.598
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	1.96e-07
Time:	23:00:45	**Log-Likelihood:**	48.836
No. Observations:	202	**AIC:**	-65.67
Df Residuals:	186	**BIC:**	-12.74
Df Model:	15		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.4196	0.241	-1.739	0.082	-0.893	0.053
skin: fire	-0.0017	0.075	-0.022	0.982	-0.150	0.146
skin: generals	-0.0389	0.074	-0.524	0.600	-0.184	0.107
skin: hiring	0.0674	0.068	0.994	0.320	-0.065	0.200
skin: mascot	-0.0103	0.074	-0.139	0.889	-0.155	0.135
skin: olympics	-0.0232	0.065	-0.358	0.720	-0.150	0.104
skin: orchestra	0.0742	0.069	1.067	0.286	-0.062	0.210
skin: plant	0.0611	0.068	0.896	0.370	-0.073	0.195
skin: restaurant	0.0564	0.074	0.763	0.445	-0.088	0.201
skin: space	0.0095	0.071	0.135	0.893	-0.130	0.149
frac_shortcut_10	0.0332	0.193	0.172	0.863	-0.345	0.412
diameter_10	-0.0147	0.033	-0.452	0.652	-0.078	0.049
avg_path_length_10	0.3133	0.133	2.358	0.018	0.053	0.574
transitivity_10	-0.0136	0.411	-0.033	0.974	-0.819	0.792
count_10	0.0031	0.005	0.586	0.558	-0.007	0.014
Fraction Odd	0.1845	0.060	3.096	0.002	0.068	0.301

Omnibus:	23.317	**Durbin-Watson:**	2.040
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	17.571
Skew:	-0.615	**Prob(JB):**	0.000153
Kurtosis:	2.241	**Cond. No.**	498.

Table 2.25

Dep. Variable:	final_std	**R-squared:**	0.233
Model:	OLS	**Adj. R-squared:**	0.193
Method:	Least Squares	**F-statistic:**	6.204
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	3.42e-08
Time:	23:00:45	**Log-Likelihood:**	46.098
No. Observations:	202	**AIC:**	-70.20
Df Residuals:	191	**BIC:**	-33.81
Df Model:	10		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.3877	0.234	-1.654	0.098	-0.847	0.072
frac_shortcut_10	0.0744	0.178	0.418	0.676	-0.275	0.424
diameter_10	-0.0140	0.031	-0.447	0.655	-0.075	0.047
avg_path_length_10	0.3084	0.129	2.386	0.017	0.055	0.562
transitivity_10	0.0263	0.365	0.072	0.943	-0.689	0.742
count_10	0.0037	0.005	0.723	0.470	-0.006	0.014
Complex	-1.697e-05	0.000	-0.166	0.868	-0.000	0.000
Familiar	7.562e-06	2.1e-05	0.360	0.719	-3.36e-05	4.87e-05
Stakes	-4.669e-06	8.24e-05	-0.057	0.955	-0.000	0.000
Emotionally_Charged	-1.185e-05	3.15e-05	-0.376	0.707	-7.35e-05	4.98e-05
Fraction Odd	0.1838	0.058	3.160	0.002	0.070	0.298

Omnibus:	27.185	**Durbin-Watson:**	1.975
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	18.257
Skew:	-0.610	**Prob(JB):**	0.000109
Kurtosis:	2.176	**Cond. No.**	8.89e+04

Table 2.26

Dep. Variable:	win	**R-squared:**	0.248
Model:	OLS	**Adj. R-squared:**	0.187
Method:	Least Squares	**F-statistic:**	3.324
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	5.99e-05
Time:	23:00:45	**Log-Likelihood:**	-75.725
No. Observations:	202	**AIC:**	183.4
Df Residuals:	186	**BIC:**	236.4
Df Model:	15		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.6410	0.524	3.132	0.002	0.614	2.668
skin: fire	-0.0599	0.136	-0.439	0.661	-0.327	0.208
skin: generals	0.0731	0.146	0.503	0.615	-0.212	0.358
skin: hiring	-0.1236	0.122	-1.010	0.312	-0.363	0.116
skin: mascot	-0.0131	0.139	-0.095	0.925	-0.285	0.258
skin: olympics	-0.1005	0.114	-0.885	0.376	-0.323	0.122
skin: orchestra	-0.0875	0.123	-0.712	0.477	-0.328	0.153
skin: plant	-0.1104	0.123	-0.899	0.368	-0.351	0.130
skin: restaurant	-0.0269	0.133	-0.203	0.839	-0.287	0.233
skin: space	-0.0014	0.129	-0.011	0.991	-0.255	0.252
frac_shortcut_10	-0.0911	0.459	-0.199	0.843	-0.990	0.808
diameter_10	0.0258	0.058	0.443	0.658	-0.088	0.140
avg_path_length_10	-0.6169	0.249	-2.474	0.013	-1.106	-0.128
transitivity_10	0.2357	1.065	0.221	0.825	-1.852	2.324
count_10	0.0003	0.010	0.029	0.977	-0.019	0.020
Fraction Odd	-0.3050	0.107	-2.846	0.004	-0.515	-0.095

Omnibus:	24.568	**Durbin-Watson:**	2.004
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	31.013
Skew:	0.958	**Prob(JB):**	1.84e-07
Kurtosis:	2.886	**Cond. No.**	498.

Table 2.27

Dep. Variable:	win	**R-squared:**	0.231
Model:	OLS	**Adj. R-squared:**	0.191
Method:	Least Squares	**F-statistic:**	4.935
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	2.37e-06
Time:	23:00:45	**Log-Likelihood:**	-77.868
No. Observations:	202	**AIC:**	177.7
Df Residuals:	191	**BIC:**	214.1
Df Model:	10		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.5707	0.498	3.157	0.002	0.595	2.546
frac_shortcut_10	-0.1533	0.413	-0.371	0.711	-0.963	0.656
diameter_10	0.0229	0.055	0.420	0.675	-0.084	0.130
avg_path_length_10	-0.6406	0.239	-2.679	0.007	-1.109	-0.172
transitivity_10	0.1567	0.939	0.167	0.867	-1.684	1.997
count_10	0.0017	0.010	0.175	0.861	-0.017	0.021
Complex	0.0001	0.000	0.581	0.561	-0.000	0.001
Familiar	1.317e-05	3.85e-05	0.342	0.732	-6.23e-05	8.87e-05
Stakes	-4.158e-05	0.000	-0.264	0.792	-0.000	0.000
Emotionally_Charged	-1.137e-05	5.74e-05	-0.198	0.843	-0.000	0.000
Fraction Odd	-0.2999	0.106	-2.837	0.005	-0.507	-0.093

Omnibus:	26.740	**Durbin-Watson:**	1.954
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	34.579
Skew:	1.013	**Prob(JB):**	3.10e-08
Kurtosis:	2.926	**Cond. No.**	8.89e+04

Table 2.28

2.4.2 The Effect of Skins

The effects of the skins either by the 4 dimensions or skin dummies were not statistically significant. In order to test whether or not the fraction of odd vertices was overpowering any sort of effect from skins, Tables 2.29 through Table 2.32 show regressions where the fraction of odd vertices is not included. This did not end up making a difference as none of the skin variables were statistically significant in these models. Average path length was statistically significant at the 10% level in all of these models and statistically significant at the 5% level in Table 2.32. This suggests that average path length and fraction of odd vertices may impact consensus through similar mechanisms.

Another possibility is that the differences between the skins are small and thus do not show up when all of the skin dummies are included in the regression. To test for this, we sub sampled some skin pairs where we expected to find large differences. Tables 2.33 through 2.36 show the regressions for the Hiring - Generals sub sample, Tables 2.37 through 2.40 show the regressions for the Olympics - Orchestra, sub sample and Tables 2.41 through 2.44 show the regressions for the Space Exploration - Firing sub sample. The effect of the vignette in the hiring-generals sub sample was statistically significant at the 5% level. None of the vignette effects across these regressions in the other sub samples were statistically significant at the 5% level. This suggests that across some pairs vignettes there may be some effect, but since these sub samples have a greatly reduced sample size than the main regressions. This means it may not be sensitive enough to detect an effect, and is more susceptible to noise.

Dep. Variable:	final_std	**R-squared:**	0.205
Model:	OLS	**Adj. R-squared:**	0.145
Method:	Least Squares	**F-statistic:**	3.672
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	2.04e-05
Time:	23:00:02	**Log-Likelihood:**	42.415
No. Observations:	202	**AIC:**	-54.83
Df Residuals:	187	**BIC:**	-5.207
Df Model:	14		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.2911	0.327	-0.890	0.373	-0.932	0.350
skin: fire	-0.0172	0.079	-0.217	0.828	-0.172	0.138
skin: generals	-0.0502	0.078	-0.642	0.521	-0.204	0.103
skin: hiring	0.0668	0.074	0.907	0.364	-0.077	0.211
skin: mascot	-0.0152	0.077	-0.197	0.844	-0.167	0.136
skin: olympics	-0.0201	0.071	-0.283	0.777	-0.159	0.119
skin: orchestra	0.0644	0.075	0.864	0.387	-0.082	0.210
skin: plant	0.0518	0.072	0.718	0.473	-0.090	0.193
skin: restaurant	0.0529	0.081	0.655	0.512	-0.105	0.211
skin: space	0.0103	0.076	0.136	0.892	-0.138	0.158
frac_shortcut_avg	0.0342	0.312	0.110	0.913	-0.578	0.646
diameter_avg	-0.0050	0.041	-0.124	0.901	-0.085	0.075
avg_path_length_avg	0.2990	0.177	1.691	0.091	-0.048	0.646
transitivity_avg	-0.1346	0.733	-0.184	0.854	-1.571	1.301
count_avg	0.0025	0.007	0.340	0.734	-0.012	0.017

Omnibus:	24.332	**Durbin-Watson:**	1.993
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	19.955
Skew:	-0.676	**Prob(JB):**	4.64e-05
Kurtosis:	2.262	**Cond. No.**	556.

Table 2.29

Dep. Variable:	final_std	**R-squared:**	0.185
Model:	OLS	**Adj. R-squared:**	0.147
Method:	Least Squares	**F-statistic:**	5.140
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	3.04e-06
Time:	23:00:02	**Log-Likelihood:**	39.895
No. Observations:	202	**AIC:**	-59.79
Df Residuals:	192	**BIC:**	-26.71
Df Model:	9		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.2753	0.329	-0.837	0.402	-0.920	0.369
frac_shortcut_avg	0.0809	0.288	0.281	0.779	-0.484	0.646
diameter_avg	-0.0066	0.039	-0.168	0.866	-0.084	0.070
avg_path_length_avg	0.3023	0.175	1.725	0.085	-0.041	0.646
transitivity_avg	-0.0888	0.667	-0.133	0.894	-1.396	1.218
count_avg	0.0029	0.007	0.400	0.689	-0.011	0.017
Complex	-3.732e-05	0.000	-0.361	0.718	-0.000	0.000
Familiar	1.031e-05	2.23e-05	0.463	0.643	-3.33e-05	5.39e-05
Stakes	1.948e-05	8.35e-05	0.233	0.816	-0.000	0.000
Emotionally_Charged	-1.971e-05	3.13e-05	-0.629	0.529	-8.11e-05	4.17e-05

Omnibus:	27.499	**Durbin-Watson:**	1.939
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	20.111
Skew:	-0.660	**Prob(JB):**	4.29e-05
Kurtosis:	2.197	**Cond. No.**	9.55e+04

Table 2.30

Dep. Variable:	win	**R-squared:**	0.208
Model:	OLS	**Adj. R-squared:**	0.149
Method:	Least Squares	**F-statistic:**	2.975
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	0.000395
Time:	23:00:02	**Log-Likelihood:**	-80.866
No. Observations:	202	**AIC:**	191.7
Df Residuals:	187	**BIC:**	241.4
Df Model:	14		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.5074	0.724	2.081	0.037	0.088	2.927
skin: fire	-0.0319	0.145	-0.220	0.826	-0.315	0.251
skin: generals	0.0958	0.151	0.635	0.526	-0.200	0.391
skin: hiring	-0.1243	0.132	-0.944	0.345	-0.383	0.134
skin: mascot	-0.0063	0.146	-0.043	0.965	-0.292	0.279
skin: olympics	-0.1017	0.125	-0.814	0.416	-0.347	0.143
skin: orchestra	-0.0699	0.132	-0.528	0.597	-0.330	0.190
skin: plant	-0.0935	0.132	-0.709	0.478	-0.352	0.165
skin: restaurant	-0.0197	0.143	-0.138	0.891	-0.300	0.261
skin: space	-0.0024	0.139	-0.017	0.986	-0.275	0.270
frac_shortcut_avg	-0.0989	0.768	-0.129	0.898	-1.605	1.407
diameter_avg	0.0031	0.075	0.041	0.968	-0.144	0.151
avg_path_length_avg	-0.6427	0.348	-1.844	0.065	-1.326	0.040
transitivity_avg	0.4231	1.881	0.225	0.822	-3.264	4.110
count_avg	0.0043	0.015	0.282	0.778	-0.026	0.034

Omnibus:	28.823	**Durbin-Watson:**	1.945
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	38.128
Skew:	1.064	**Prob(JB):**	5.26e-09
Kurtosis:	2.961	**Cond. No.**	556.

Table 2.31

Dep. Variable:	win	**R-squared:**	0.191
Model:	OLS	**Adj. R-squared:**	0.153
Method:	Least Squares	**F-statistic:**	4.360
Date:	Mon, 07 Mar 2022	**Prob (F-statistic):**	3.45e-05
Time:	23:00:02	**Log-Likelihood:**	-83.051
No. Observations:	202	**AIC:**	186.1
Df Residuals:	192	**BIC:**	219.2
Df Model:	9		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.4953	0.708	2.112	0.035	0.108	2.883
frac_shortcut_avg	-0.1847	0.684	-0.270	0.787	-1.525	1.156
diameter_avg	0.0023	0.070	0.032	0.974	-0.135	0.140
avg_path_length_avg	-0.6923	0.334	-2.076	0.038	-1.346	-0.039
transitivity_avg	0.3039	1.664	0.183	0.855	-2.957	3.565
count_avg	0.0068	0.014	0.470	0.638	-0.021	0.035
Complex	0.0002	0.000	0.764	0.445	-0.000	0.001
Familiar	9.01e-06	4.07e-05	0.221	0.825	-7.08e-05	8.88e-05
Stakes	-8.483e-05	0.000	-0.529	0.597	-0.000	0.000
Emotionally_Charged	2.178e-06	5.91e-05	0.037	0.971	-0.000	0.000

Omnibus:	30.711	**Durbin-Watson:**	1.898
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	41.394
Skew:	1.109	**Prob(JB):**	1.03e-09
Kurtosis:	3.015	**Cond. No.**	9.55e+04

Table 2.32

Dep. Variable:	final_std	**R-squared:**	0.395
Model:	OLS	**Adj. R-squared:**	0.248
Method:	Least Squares	**F-statistic:**	3.985
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.00214
Time:	00:29:02	**Log-Likelihood:**	14.084
No. Observations:	42	**AIC:**	-10.17
Df Residuals:	33	**BIC:**	5.472
Df Model:	8		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.7070	1.514	0.467	0.641	-2.261	3.675
skin: hiring	0.1656	0.073	2.274	0.023	0.023	0.308
Fraction Odd 0.2	-0.0386	0.081	-0.478	0.632	-0.197	0.120
Fraction Odd 0.5	-0.0438	0.108	-0.405	0.685	-0.255	0.168
frac_shortcut_avg	-1.5208	0.927	-1.640	0.101	-3.338	0.296
diameter_avg	-0.0832	0.101	-0.825	0.409	-0.281	0.115
avg_path_length_avg	0.6912	0.692	0.999	0.318	-0.665	2.047
transitivity_avg	-3.0143	2.559	-1.178	0.239	-8.029	2.001
count_avg	-0.0113	0.026	-0.427	0.670	-0.063	0.040

Omnibus:	2.630	**Durbin-Watson:**	2.247
Prob(Omnibus):	0.269	**Jarque-Bera (JB):**	2.126
Skew:	-0.414	**Prob(JB):**	0.345
Kurtosis:	2.272	**Cond. No.**	926.

Table 2.33

Dep. Variable:	win	**R-squared:**	0.433
Model:	OLS	**Adj. R-squared:**	0.296
Method:	Least Squares	**F-statistic:**	2.918
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.0141
Time:	00:29:02	**Log-Likelihood:**	-11.816
No. Observations:	42	**AIC:**	41.63
Df Residuals:	33	**BIC:**	57.27
Df Model:	8		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.6786	2.375	-0.286	0.775	-5.333	3.976
skin: hiring	-0.3446	0.136	-2.525	0.012	-0.612	-0.077
Fraction Odd 0.2	-0.0442	0.138	-0.319	0.750	-0.315	0.227
Fraction Odd 0.5	0.0723	0.191	0.378	0.705	-0.302	0.447
frac_shortcut_avg	3.2067	1.508	2.126	0.033	0.251	6.162
diameter_avg	0.0880	0.184	0.478	0.632	-0.273	0.448
avg_path_length_avg	-1.1126	1.158	-0.961	0.337	-3.382	1.157
transitivity_avg	6.1757	4.090	1.510	0.131	-1.840	14.192
count_avg	0.0186	0.044	0.419	0.676	-0.068	0.106

Omnibus:	1.037	**Durbin-Watson:**	2.089
Prob(Omnibus):	0.595	**Jarque-Bera (JB):**	1.078
Skew:	0.307	**Prob(JB):**	0.583
Kurtosis:	2.511	**Cond. No.**	926.

Table 2.34

Dep. Variable:	final_std	**R-squared:**	0.393
Model:	OLS	**Adj. R-squared:**	0.268
Method:	Least Squares	**F-statistic:**	4.737
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.000853
Time:	00:29:02	**Log-Likelihood:**	14.005
No. Observations:	42	**AIC:**	-12.01
Df Residuals:	34	**BIC:**	1.892
Df Model:	7		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.6190	1.107	0.559	0.576	-1.551	2.789
skin: hiring	0.1669	0.071	2.367	0.018	0.029	0.305
frac_shortcut_avg	-1.4863	0.684	-2.174	0.030	-2.826	-0.146
diameter_avg	-0.0789	0.099	-0.793	0.427	-0.274	0.116
avg_path_length_avg	0.6595	0.616	1.071	0.284	-0.548	1.867
transitivity_avg	-2.8752	1.750	-1.643	0.100	-6.304	0.554
count_avg	-0.0094	0.025	-0.381	0.703	-0.058	0.039
Fraction Odd	0.0677	0.126	0.539	0.590	-0.179	0.314

Omnibus:	2.862	**Durbin-Watson:**	2.282
Prob(Omnibus):	0.239	**Jarque-Bera (JB):**	2.335
Skew:	-0.449	**Prob(JB):**	0.311
Kurtosis:	2.274	**Cond. No.**	892.

Table 2.35

Dep. Variable:	win	**R-squared:**	0.424
Model:	OLS	**Adj. R-squared:**	0.306
Method:	Least Squares	**F-statistic:**	3.630
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.00500
Time:	00:29:02	**Log-Likelihood:**	-12.155
No. Observations:	42	**AIC:**	40.31
Df Residuals:	34	**BIC:**	54.21
Df Model:	7		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.5965	1.620	-0.368	0.713	-3.772	2.579
skin: hiring	-0.3495	0.134	-2.618	0.009	-0.611	-0.088
frac_shortcut_avg	3.0736	1.113	2.761	0.006	0.892	5.255
diameter_avg	0.0714	0.180	0.396	0.692	-0.282	0.425
avg_path_length_avg	-0.9906	0.980	-1.011	0.312	-2.912	0.930
transitivity_avg	5.6393	2.802	2.013	0.044	0.147	11.131
count_avg	0.0114	0.039	0.292	0.771	-0.065	0.088
Fraction Odd	0.0607	0.217	0.279	0.780	-0.365	0.487

Omnibus:	1.231	**Durbin-Watson:**	2.180
Prob(Omnibus):	0.540	**Jarque-Bera (JB):**	1.223
Skew:	0.366	**Prob(JB):**	0.542
Kurtosis:	2.596	**Cond. No.**	892.

Table 2.36

Dep. Variable:	final_std	**R-squared:**	0.354
Model:	OLS	**Adj. R-squared:**	0.197
Method:	Least Squares	**F-statistic:**	3.721
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.00337
Time:	00:29:15	**Log-Likelihood:**	17.905
No. Observations:	42	**AIC:**	-17.81
Df Residuals:	33	**BIC:**	-2.172
Df Model:	8		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.6432	0.943	-0.682	0.495	-2.492	1.205
skin: orchestra	0.0822	0.072	1.144	0.253	-0.059	0.223
Fraction Odd 0.2	-0.1233	0.084	-1.472	0.141	-0.287	0.041
Fraction Odd 0.5	0.0023	0.080	0.029	0.977	-0.154	0.158
frac_shortcut_avg	0.5125	0.666	0.770	0.441	-0.793	1.818
diameter_avg	0.0475	0.077	0.620	0.535	-0.103	0.198
avg_path_length_avg	0.1570	0.299	0.525	0.599	-0.429	0.743
transitivity_avg	0.6994	1.661	0.421	0.674	-2.556	3.955
count_avg	0.0062	0.014	0.443	0.658	-0.021	0.034

Omnibus:	2.620	**Durbin-Watson:**	1.869
Prob(Omnibus):	0.270	**Jarque-Bera (JB):**	1.812
Skew:	-0.499	**Prob(JB):**	0.404
Kurtosis:	3.197	**Cond. No.**	1.06e+03

Table 2.37

Dep. Variable:	win	**R-squared:**	0.582
Model:	OLS	**Adj. R-squared:**	0.480
Method:	Least Squares	**F-statistic:**	5.726
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.000136
Time:	00:29:15	**Log-Likelihood:**	2.8149
No. Observations:	42	**AIC:**	12.37
Df Residuals:	33	**BIC:**	28.01
Df Model:	8		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	2.3221	1.222	1.901	0.057	-0.072	4.717
skin: orchestra	0.0497	0.097	0.512	0.609	-0.141	0.240
Fraction Odd 0.2	0.1892	0.122	1.549	0.121	-0.050	0.429
Fraction Odd 0.5	-0.1251	0.114	-1.094	0.274	-0.349	0.099
frac_shortcut_avg	-1.2919	0.972	-1.329	0.184	-3.197	0.613
diameter_avg	-0.1146	0.110	-1.043	0.297	-0.330	0.101
avg_path_length_avg	-0.5160	0.445	-1.159	0.247	-1.389	0.357
transitivity_avg	-0.7403	2.387	-0.310	0.756	-5.418	3.937
count_avg	0.0090	0.020	0.447	0.655	-0.030	0.048

Omnibus:	16.302	**Durbin-Watson:**	1.645
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	20.983
Skew:	1.223	**Prob(JB):**	2.78e-05
Kurtosis:	5.451	**Cond. No.**	1.06e+03

Table 2.38

Dep. Variable:	final_std	**R-squared:**	0.332
Model:	OLS	**Adj. R-squared:**	0.195
Method:	Least Squares	**F-statistic:**	3.199
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.0103
Time:	00:29:15	**Log-Likelihood:**	17.202
No. Observations:	42	**AIC:**	-18.40
Df Residuals:	34	**BIC:**	-4.503
Df Model:	7		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
Intercept	-0.7121	0.946	-0.753	0.452	-2.566	1.142
skin: orchestra	0.0838	0.072	1.157	0.247	-0.058	0.226
frac_shortcut_avg	0.5133	0.710	0.722	0.470	-0.879	1.906
diameter_avg	0.0486	0.076	0.636	0.525	-0.101	0.199
avg_path_length_avg	0.1228	0.297	0.413	0.680	-0.460	0.706
transitivity_avg	0.7007	1.738	0.403	0.687	-2.705	4.107
count_avg	0.0059	0.015	0.403	0.687	-0.023	0.035
Fraction Odd	0.2042	0.139	1.467	0.142	-0.069	0.477

Omnibus:	4.582	**Durbin-Watson:**	1.815
Prob(Omnibus):	0.101	**Jarque-Bera (JB):**	3.459
Skew:	-0.674	**Prob(JB):**	0.177
Kurtosis:	3.398	**Cond. No.**	1.06e+03

Table 2.39

Dep. Variable:	win	**R-squared:**	0.500
Model:	OLS	**Adj. R-squared:**	0.397
Method:	Least Squares	**F-statistic:**	3.602
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.00524
Time:	00:29:15	**Log-Likelihood:**	-0.94642
No. Observations:	42	**AIC:**	17.89
Df Residuals:	34	**BIC:**	31.79
Df Model:	7		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	2.2461	1.463	1.536	0.125	-0.621	5.113
skin: orchestra	0.0439	0.108	0.406	0.685	-0.168	0.256
frac_shortcut_avg	-1.2946	1.183	-1.094	0.274	-3.614	1.025
diameter_avg	-0.1186	0.115	-1.027	0.304	-0.345	0.108
avg_path_length_avg	-0.3985	0.473	-0.842	0.400	-1.326	0.529
transitivity_avg	-0.7449	2.855	-0.261	0.794	-6.341	4.851
count_avg	0.0100	0.022	0.451	0.652	-0.034	0.054
Fraction Odd	-0.3109	0.210	-1.478	0.139	-0.723	0.101

Omnibus:	19.545	**Durbin-Watson:**	1.574
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	27.380
Skew:	1.430	**Prob(JB):**	1.13e-06
Kurtosis:	5.733	**Cond. No.**	1.06e+03

Table 2.40

Dep. Variable:	final_std	**R-squared:**	0.477
Model:	OLS	**Adj. R-squared:**	0.333
Method:	Least Squares	**F-statistic:**	9.322
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	2.89e-06
Time:	00:30:21	**Log-Likelihood:**	14.849
No. Observations:	38	**AIC:**	-11.70
Df Residuals:	29	**BIC:**	3.041
Df Model:	8		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.9266	0.650	1.425	0.154	-0.348	2.201
skin: space	0.0403	0.076	0.527	0.598	-0.109	0.190
Fraction Odd 0.2	-0.0997	0.094	-1.066	0.286	-0.283	0.084
Fraction Odd 0.5	-0.0481	0.086	-0.561	0.575	-0.216	0.120
frac_shortcut_avg	-0.4672	0.535	-0.872	0.383	-1.517	0.582
diameter_avg	0.0649	0.102	0.634	0.526	-0.136	0.266
avg_path_length_avg	-0.2340	0.430	-0.544	0.586	-1.077	0.609
transitivity_avg	-2.1733	1.036	-2.098	0.036	-4.203	-0.143
count_avg	0.0197	0.018	1.112	0.266	-0.015	0.054

Omnibus:	5.372	**Durbin-Watson:**	2.169
Prob(Omnibus):	0.068	**Jarque-Bera (JB):**	4.395
Skew:	-0.825	**Prob(JB):**	0.111
Kurtosis:	3.233	**Cond. No.**	878.

Table 2.41

Dep. Variable:	win	**R-squared:**	0.533
Model:	OLS	**Adj. R-squared:**	0.404
Method:	Least Squares	**F-statistic:**	14.96
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	2.07e-08
Time:	00:30:21	**Log-Likelihood:**	-6.9652
No. Observations:	38	**AIC:**	31.93
Df Residuals:	29	**BIC:**	46.67
Df Model:	8		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.4718	1.273	-0.371	0.711	-2.967	2.023
skin: space	0.0100	0.121	0.082	0.935	-0.228	0.248
Fraction Odd 0.2	0.2233	0.151	1.475	0.140	-0.073	0.520
Fraction Odd 0.5	-0.0377	0.157	-0.240	0.810	-0.345	0.270
frac_shortcut_avg	0.6103	1.061	0.575	0.565	-1.470	2.691
diameter_avg	-0.0272	0.158	-0.173	0.863	-0.336	0.282
avg_path_length_avg	0.0274	0.995	0.027	0.978	-1.924	1.979
transitivity_avg	4.0777	2.040	1.999	0.046	0.079	8.076
count_avg	-0.0223	0.044	-0.505	0.614	-0.109	0.064

Omnibus:	6.127	**Durbin-Watson:**	2.133
Prob(Omnibus):	0.047	**Jarque-Bera (JB):**	5.020
Skew:	0.871	**Prob(JB):**	0.0813
Kurtosis:	3.371	**Cond. No.**	878.

Table 2.42

Dep. Variable:	final_std	**R-squared:**	0.477
Model:	OLS	**Adj. R-squared:**	0.355
Method:	Least Squares	**F-statistic:**	9.128
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	5.24e-06
Time:	00:30:21	**Log-Likelihood:**	14.848
No. Observations:	38	**AIC:**	-13.70
Df Residuals:	30	**BIC:**	-0.5956
Df Model:	7		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.7959	0.599	1.329	0.184	-0.378	1.970
skin: space	0.0403	0.074	0.544	0.587	-0.105	0.186
frac_shortcut_avg	-0.4700	0.480	-0.978	0.328	-1.412	0.472
diameter_avg	0.0653	0.100	0.652	0.514	-0.131	0.261
avg_path_length_avg	-0.2329	0.427	-0.546	0.585	-1.069	0.603
transitivity_avg	-2.1815	0.854	-2.555	0.011	-3.855	-0.508
count_avg	0.0196	0.017	1.145	0.252	-0.014	0.053
Fraction Odd	0.1660	0.154	1.081	0.280	-0.135	0.467

Omnibus:	5.433	**Durbin-Watson:**	2.169
Prob(Omnibus):	0.066	**Jarque-Bera (JB):**	4.448
Skew:	-0.829	**Prob(JB):**	0.108
Kurtosis:	3.241	**Cond. No.**	852.

Table 2.43

Dep. Variable:	win	**R-squared:**	0.510
Model:	OLS	**Adj. R-squared:**	0.395
Method:	Least Squares	**F-statistic:**	12.86
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	1.64e-07
Time:	00:30:21	**Log-Likelihood:**	-7.8778
No. Observations:	38	**AIC:**	31.76
Df Residuals:	30	**BIC:**	44.86
Df Model:	7		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.3686	1.169	-0.315	0.753	-2.660	1.923
skin: space	0.0053	0.123	0.044	0.965	-0.235	0.246
frac_shortcut_avg	0.8507	0.930	0.915	0.360	-0.971	2.673
diameter_avg	-0.0575	0.153	-0.375	0.708	-0.358	0.243
avg_path_length_avg	-0.0667	1.007	-0.066	0.947	-2.041	1.907
transitivity_avg	4.7731	1.638	2.914	0.004	1.563	7.983
count_avg	-0.0104	0.042	-0.245	0.807	-0.093	0.073
Fraction Odd	-0.3555	0.248	-1.432	0.152	-0.842	0.131

Omnibus:	7.056	**Durbin-Watson:**	2.064
Prob(Omnibus):	0.029	**Jarque-Bera (JB):**	5.709
Skew:	0.881	**Prob(JB):**	0.0576
Kurtosis:	3.707	**Cond. No.**	852.

Table 2.44

2.4.3 The Alphabetical Protocol

By design, the difference between each of the identifiers was arbitrary, and in this environment, it is reasonable to expect the players to come up their own methods of discrimination. The Elo technique we describe in Chapter 3 ensures that each of the identifiers are approximately equally attractive to the players, but some of the identifiers start with different letters of the alphabet. The strategy of players proposing to pick the first identifier in alphabetical order was incredibly common. In about 61.8% of the games at least one player proposed a variant of the alphabetical protocol at some point. As we mentioned when describing the coding scheme, there were a couple of different variations of this protocol; some involved players sharing the identifiers that they have seen to try and determine the first identifier alphabetically, while others involved players instructing other players to select the first in alphabetical order from the drop-down list at the end.

The pure alphabetical strategy, where no identifiers are passed, has some downsides compared other versions where identifiers are passed: player error in the final choice and difficulty pivoting. When not coordinating on a specific identifier, the players could accidentally pick different identifiers in the drop down causing them to not reach consensus. It also more difficult to pivot away from as a strategy, as they are not passing around identifiers, so it may be less clear to the players what the space of identifiers is for that game. When identifiers are passed, the alphabetical strategy can act as a dimensionality reduction, since identifiers with first letters not near the beginning of the alphabet are not considered. This way, even if the players are not able to commit to the alphabetical protocol throughout the entire game, its presence early on can serve to winnow the field. Figure 2.20 shows the frequency of the alphabetical protocol by round, and the alphabetical protocol peaks in frequency early and decays as the game progresses.

Figure 2.19 shows the distribution of first letters in final votes for runs where the alphabetical protocol was present, where it was not present, and the baseline distribution of our identifiers. In games where the alphabetical protocol is present, identifiers starting with the letter b are much more common than when it is not present. Similarly, identifiers starting with the letter q are much more common in runs where the alphabetical protocol is not present, than when it is.

The alphabetical protocol had a negative impact on the consensus process. In order to compare across runs we calculated an alphabetical score for each run: the sum of the average fraction of alphabetical messages at each round. Tables 2.45 and 2.46 show the regressions final vote standard deviation and win rate respectively, without any structural adjustments. In these regressions the coefficients for alphabetical score were not statistically significant. The signs on the coefficients here did suggest that they negatively impact consensus. On the other hand, Tables 2.47 and 2.48 do have structural adjustments and for both final standard deviation and win rate the coefficients on alphabetical score are statistically significant. This suggests that the ability of the alphabetical protocol may be impacted by graphical structure. These results show the alphabetical protocol having a small negative effect on the consensus process.

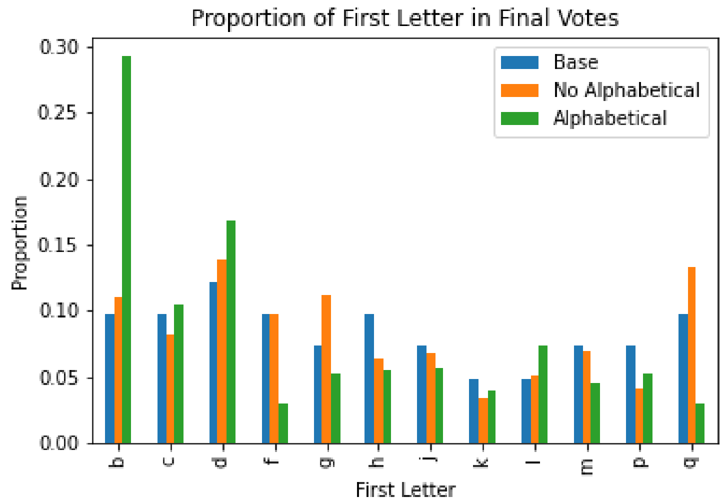


Figure 2.19

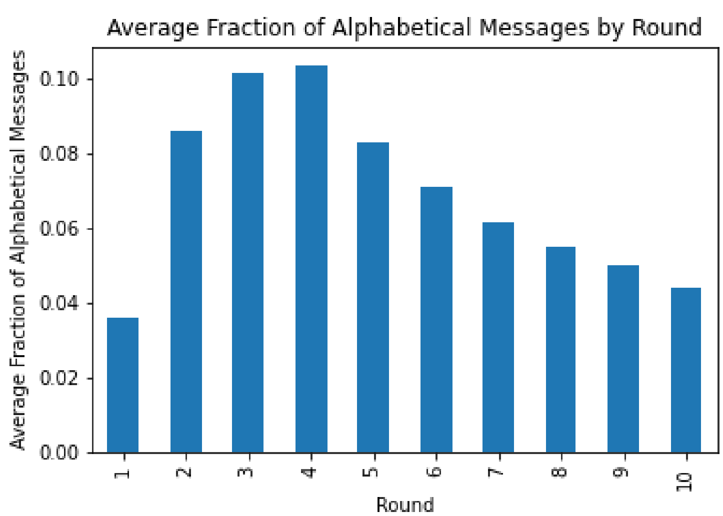


Figure 2.20

Dep. Variable:	final_std	**R-squared:**	0.009
Model:	OLS	**Adj. R-squared:**	0.004
Method:	Least Squares	**F-statistic:**	2.783
Date:	Wed, 09 Mar 2022	**Prob (F-statistic):**	0.0968
Time:	02:35:14	**Log-Likelihood:**	20.190
No. Observations:	202	**AIC:**	-36.38
Df Residuals:	200	**BIC:**	-29.76
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.3687	0.018	20.211	0.000	0.333	0.404
alphabetical_avg	0.0198	0.012	1.668	0.095	-0.003	0.043

Omnibus:	33.016	**Durbin-Watson:**	1.890
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	27.123
Skew:	-0.801	**Prob(JB):**	1.29e-06
Kurtosis:	2.191	**Cond. No.**	1.93

Table 2.45

Dep. Variable:	win	**R-squared:**	0.006
Model:	OLS	**Adj. R-squared:**	0.001
Method:	Least Squares	**F-statistic:**	1.818
Date:	Wed, 09 Mar 2022	**Prob (F-statistic):**	0.179
Time:	02:35:15	**Log-Likelihood:**	-103.89
No. Observations:	202	**AIC:**	211.8
Df Residuals:	200	**BIC:**	218.4
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.2274	0.034	6.673	0.000	0.161	0.294
alphabetical_avg	-0.0283	0.021	-1.348	0.178	-0.069	0.013

Omnibus:	43.758	**Durbin-Watson:**	1.889
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	68.354
Skew:	1.425	**Prob(JB):**	1.44e-15
Kurtosis:	3.059	**Cond. No.**	1.93

Table 2.46

Dep. Variable:	final_std	**R-squared:**	0.231
Model:	OLS	**Adj. R-squared:**	0.203
Method:	Least Squares	**F-statistic:**	8.364
Date:	Wed, 09 Mar 2022	**Prob (F-statistic):**	6.37e-09
Time:	02:35:16	**Log-Likelihood:**	45.783
No. Observations:	202	**AIC:**	-75.57
Df Residuals:	194	**BIC:**	-49.10
Df Model:	7		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.4870	0.307	-1.588	0.112	-1.088	0.114
alphabetical_avg	0.0282	0.012	2.425	0.015	0.005	0.051
frac_shortcut_avg	0.0769	0.286	0.269	0.788	-0.483	0.637
diameter_avg	-0.0080	0.036	-0.218	0.827	-0.079	0.063
avg_path_length_avg	0.3231	0.166	1.944	0.052	-0.003	0.649
transitivity_avg	0.0025	0.675	0.004	0.997	-1.320	1.325
count_avg	0.0031	0.007	0.458	0.647	-0.010	0.016
Fraction Odd	0.1868	0.057	3.268	0.001	0.075	0.299

Omnibus:	22.189	**Durbin-Watson:**	1.935
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	18.384
Skew:	-0.647	**Prob(JB):**	0.000102
Kurtosis:	2.287	**Cond. No.**	541.

Table 2.47

Dep. Variable:	win	**R-squared:**	0.229
Model:	OLS	**Adj. R-squared:**	0.201
Method:	Least Squares	**F-statistic:**	7.236
Date:	Wed, 09 Mar 2022	**Prob (F-statistic):**	1.06e-07
Time:	02:35:17	**Log-Likelihood:**	-78.174
No. Observations:	202	**AIC:**	172.3
Df Residuals:	194	**BIC:**	198.8
Df Model:	7		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.8754	0.650	2.887	0.004	0.602	3.149
alphabetical_avg	-0.0446	0.022	-2.021	0.043	-0.088	-0.001
frac_shortcut_avg	-0.1800	0.648	-0.278	0.781	-1.450	1.090
diameter_avg	0.0066	0.066	0.099	0.921	-0.124	0.137
avg_path_length_avg	-0.7214	0.316	-2.279	0.023	-1.342	-0.101
transitivity_avg	0.1464	1.577	0.093	0.926	-2.945	3.237
count_avg	0.0058	0.013	0.437	0.662	-0.020	0.032
Fraction Odd	-0.3004	0.104	-2.878	0.004	-0.505	-0.096

Omnibus:	30.029	**Durbin-Watson:**	1.939
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	40.014
Skew:	1.090	**Prob(JB):**	2.05e-09
Kurtosis:	3.060	**Cond. No.**	541.

Table 2.48

2.4.4 Byzantine Faults

There are two primary sources of byzantine faults in our experiment: dropouts and player actions. Dropouts can occur for a variety of reasons such as server-client clock desynchronization, poor internet connection, power outages, and player inattention. It is also a concern that the population of the players could have changed significantly over the time the experiment ran for. The longer an experiment is out in the wild, the more likely it is that new players will have exposure to the experiment, if for example, the instructions were posted to an Amazon Mechanical Turk workers forum. It is also possible that the players could be lower quality over time as not allowing repeat play can exhaust the pool of potential players. These exogenous factors could influence the likelihood of byzantine player action. We try to account for this by adjusting for the rate at which players pass the comprehension quiz, as well as accounting for the date the experiment took place (expressed as julian day). Additionally, we also account for dropouts, since if players are more/less likely to dropout over time, that could be problematic. Tables 2.49 and 2.50 address the impact of these variables on the final game standard deviation. None of the variables were statistically significant in these regressions other than fraction of odd vertices, suggesting that these factors do not significantly impact the consensus process. Table 2.53 shows the results for the regression of quiz pass rate versus julian day. The coefficient for julian day is statistically significant, but the coefficient is also very small suggesting that the variation in quiz pass rate over time is not practically significant. These results suggest that there were not major shifts in the population over the run time of this experiment and any changes that did happen did not meaningfully impact the results. The fact that the fraction of players who drop out did not statistically significantly impact the final vote standard deviation suggests that the human consensus process can tolerate these kinds of faults.

The second type of byzantine faults present in our experiment is due to player action. We coded this as confusion represented players who were playing improperly like sending gibberish, repeated greetings, and trying to coordinate on invalid identifiers. An interesting example of this occurred in a run where the group achieved consensus on the identifier: PESAM. PESAM was one of the words we generated in other work (Sankaran et al., 2021), but we used this word in the tutorial as an example, and, due to concerns about priming, we never assigned it to a player. This is a great example of byzantine action, in the form of bullshit, or information that appears like true information but is meaningless. From the perspective of a player, PESAM appears to be a perfectly valid identifier because it has 5 letters and follows the consonant - vowel - consonant - vowel - consonant pattern. Thus, other than the initial player who proposed PESAM, none of the players were doing anything unreasonable from their perspective. Unfortunately, since the players had no reason to expect that PESAM would not appear in the drop-down list, they did not plan for contingencies and ultimately lost. Thus, even if the mechanism the players use to achieve consensus are sound, if the data they are operating on is wrong they will reach the wrong conclusion. In the case of 4-letter identifiers, the players were often able to recover from this type of fault. Usually, another player would remind them that the proper identifiers use were 5-letter identifiers, such as in this message a player sent: "Four letter words are names of delegates, 5 letter are cities. Choose KOSOB." This demonstrates some degree of tolerance to these kinds of faults.

Unfortunately, the players were not completely tolerant of confusion. Confusion via 4-letter identifiers sometimes spreads to other players as seen by this message: "both of you had chosen EPUB. so, i will choose EPUB too". They adopted a confused state from their neighbors, even though they should know better. We aggregated the level of confusion across games using the same method we used to aggregate the alphabetical protocol. Tables 2.54 and 2.55 show the regressions final vote standard deviation and win rate respectively, without any structural adjustments. In these regressions the coefficients for confused score were statistically significant at the 5% level for both outcome. The signs on the coefficients here did suggest that they negatively impact consensus. On the other hand, Tables 2.56 and 2.57 do have structural adjustments and for both final standard deviation and win rate the coefficients on confused score are not statistically significant. Similar to alphabetical protocol, confusion seems to have a small but negative effect on the consensus process. Notably the coefficients for confused score and alphabetical score are similar, so their impact on gameplay is similar.

Dep. Variable:	final_std	**R-squared:**	0.049
Model:	OLS	**Adj. R-squared:**	0.028
Method:	Least Squares	**F-statistic:**	2.618
Date:	Wed, 09 Mar 2022	**Prob (F-statistic):**	0.0366
Time:	03:30:23	**Log-Likelihood:**	26.050
No. Observations:	187	**AIC:**	-42.10
Df Residuals:	182	**BIC:**	-25.94
Df Model:	4		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	767.0080	677.330	1.132	0.257	-560.535	2094.551
Frac Odd	0.1750	0.067	2.614	0.009	0.044	0.306
Quiz Pass Rate	0.0805	0.188	0.428	0.669	-0.289	0.450
Fraction of Dropouts	-0.1691	0.200	-0.845	0.398	-0.562	0.223
Julian Day	-0.0003	0.000	-1.132	0.258	-0.001	0.000

Omnibus:	25.326	**Durbin-Watson:**	1.881
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	22.610
Skew:	-0.769	**Prob(JB):**	1.23e-05
Kurtosis:	2.269	**Cond. No.**	9.79e+10

Table 2.49

Dep. Variable:	final_std	**R-squared:**	0.185
Model:	OLS	**Adj. R-squared:**	0.143
Method:	Least Squares	**F-statistic:**	4.490
Date:	Wed, 09 Mar 2022	**Prob (F-statistic):**	2.53e-05
Time:	03:30:24	**Log-Likelihood:**	40.452
No. Observations:	187	**AIC:**	-60.90
Df Residuals:	177	**BIC:**	-28.59
Df Model:	9		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	-139.7280	658.970	-0.212	0.832	-1431.285	1151.829
Frac Odd	0.1550	0.064	2.429	0.015	0.030	0.280
Quiz Pass Rate	-0.0878	0.195	-0.450	0.653	-0.470	0.295
Fraction of Dropouts	0.0114	0.220	0.052	0.959	-0.419	0.442
Julian Day	5.678e-05	0.000	0.212	0.832	-0.000	0.001
frac_shortcut_avg	-0.2228	0.332	-0.671	0.502	-0.874	0.428
diameter_avg	-0.0291	0.040	-0.723	0.470	-0.108	0.050
avg_path_length_avg	0.3653	0.186	1.961	0.050	0.000	0.730
transitivity_avg	-0.8308	0.794	-1.046	0.295	-2.387	0.726
count_avg	0.0022	0.009	0.243	0.808	-0.015	0.020

Omnibus:	23.051	**Durbin-Watson:**	1.884
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	18.421
Skew:	-0.670	**Prob(JB):**	0.000100
Kurtosis:	2.246	**Cond. No.**	1.07e+11

Table 2.50

Dep. Variable:	Number of Dropouts	**R-squared:**	0.000
Model:	OLS	**Adj. R-squared:**	-0.005
Method:	Least Squares	**F-statistic:**	0.002208
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.963
Time:	01:13:08	**Log-Likelihood:**	-361.77
No. Observations:	187	**AIC:**	727.5
Df Residuals:	185	**BIC:**	734.0
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	-190.5435	4122.062	-0.046	0.963	-8269.636	7888.549
Julian Day	7.877e-05	0.002	0.047	0.963	-0.003	0.003

Omnibus:	7.549	**Durbin-Watson:**	2.040
Prob(Omnibus):	0.023	**Jarque-Bera (JB):**	7.311
Skew:	0.465	**Prob(JB):**	0.0258
Kurtosis:	3.269	**Cond. No.**	8.49e+10

Table 2.51

Dep. Variable:	Fraction of Dropouts	**R-squared:**	0.024
Model:	OLS	**Adj. R-squared:**	0.019
Method:	Least Squares	**F-statistic:**	4.019
Date:	Fri, 04 Feb 2022	**Prob (F-statistic):**	0.0465
Time:	03:43:51	**Log-Likelihood:**	202.62
No. Observations:	187	**AIC:**	-401.2
Df Residuals:	185	**BIC:**	-394.8
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	-446.3521	222.728	-2.004	0.045	-882.890	-9.814
Julian Day	0.0002	9.06e-05	2.005	0.045	4.05e-06	0.000

Omnibus:	4.239	**Durbin-Watson:**	1.989
Prob(Omnibus):	0.120	**Jarque-Bera (JB):**	4.280
Skew:	0.342	**Prob(JB):**	0.118
Kurtosis:	2.713	**Cond. No.**	8.49e+10

Table 2.52

Dep. Variable:	Quiz Pass Rate	**R-squared:**	0.231
Model:	OLS	**Adj. R-squared:**	0.227
Method:	Least Squares	**F-statistic:**	59.21
Date:	Fri, 04 Feb 2022	**Prob (F-statistic):**	8.19e-13
Time:	03:43:07	**Log-Likelihood:**	185.15
No. Observations:	187	**AIC:**	-366.3
Df Residuals:	185	**BIC:**	-359.8
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	-1701.8081	221.224	-7.693	0.000	-2135.399	-1268.217
Julian Day	0.0007	9e-05	7.695	0.000	0.001	0.001

Omnibus:	0.999	**Durbin-Watson:**	1.747
Prob(Omnibus):	0.607	**Jarque-Bera (JB):**	1.025
Skew:	0.063	**Prob(JB):**	0.599
Kurtosis:	2.660	**Cond. No.**	8.49e+10

Table 2.53

Dep. Variable:	final_std	**R-squared:**	0.015
Model:	OLS	**Adj. R-squared:**	0.010
Method:	Least Squares	**F-statistic:**	3.859
Date:	Wed, 09 Mar 2022	**Prob (F-statistic):**	0.0509
Time:	02:35:15	**Log-Likelihood:**	20.754
No. Observations:	202	**AIC:**	-37.51
Df Residuals:	200	**BIC:**	-30.89
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.3671	0.018	20.547	0.000	0.332	0.402
confused_avg	0.0328	0.017	1.964	0.049	7.49e-05	0.065

Omnibus:	34.343	**Durbin-Watson:**	1.934
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	27.390
Skew:	-0.801	**Prob(JB):**	1.13e-06
Kurtosis:	2.171	**Cond. No.**	1.73

Table 2.54

Dep. Variable:	win	**R-squared:**	0.011
Model:	OLS	**Adj. R-squared:**	0.006
Method:	Least Squares	**F-statistic:**	3.957
Date:	Wed, 09 Mar 2022	**Prob (F-statistic):**	0.0480
Time:	02:35:15	**Log-Likelihood:**	-103.32
No. Observations:	202	**AIC:**	210.6
Df Residuals:	200	**BIC:**	217.3
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.2323	0.033	6.960	0.000	0.167	0.298
confused_avg	-0.0525	0.026	-1.989	0.047	-0.104	-0.001

Omnibus:	43.119	**Durbin-Watson:**	1.929
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	66.964
Skew:	1.410	**Prob(JB):**	2.88e-15
Kurtosis:	3.037	**Cond. No.**	1.73

Table 2.55

Dep. Variable:	final_std	**R-squared:**	0.219
Model:	OLS	**Adj. R-squared:**	0.191
Method:	Least Squares	**F-statistic:**	7.760
Date:	Wed, 09 Mar 2022	**Prob (F-statistic):**	2.85e-08
Time:	02:35:16	**Log-Likelihood:**	44.268
No. Observations:	202	**AIC:**	-72.54
Df Residuals:	194	**BIC:**	-46.07
Df Model:	7		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.3558	0.307	-1.158	0.247	-0.958	0.246
confused_avg	0.0222	0.014	1.570	0.117	-0.006	0.050
frac_shortcut_avg	0.0437	0.290	0.150	0.880	-0.525	0.613
diameter_avg	-0.0018	0.038	-0.048	0.961	-0.077	0.073
avg_path_length_avg	0.2616	0.169	1.552	0.121	-0.069	0.592
transitivity_avg	-0.0874	0.685	-0.128	0.898	-1.430	1.255
count_avg	0.0046	0.007	0.672	0.502	-0.009	0.018
Fraction Odd	0.1715	0.058	2.939	0.003	0.057	0.286

Omnibus:	27.099	**Durbin-Watson:**	1.976
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	18.888
Skew:	-0.629	**Prob(JB):**	7.92e-05
Kurtosis:	2.186	**Cond. No.**	534.

Table 2.56

Dep. Variable:	win	**R-squared:**	0.220
Model:	OLS	**Adj. R-squared:**	0.192
Method:	Least Squares	**F-statistic:**	6.670
Date:	Wed, 09 Mar 2022	**Prob (F-statistic):**	4.43e-07
Time:	02:35:19	**Log-Likelihood:**	-79.377
No. Observations:	202	**AIC:**	174.8
Df Residuals:	194	**BIC:**	201.2
Df Model:	7		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.6695	0.643	2.595	0.009	0.409	2.931
confused_avg	-0.0324	0.023	-1.398	0.162	-0.078	0.013
frac_shortcut_avg	-0.1288	0.654	-0.197	0.844	-1.410	1.152
diameter_avg	-0.0026	0.068	-0.038	0.969	-0.136	0.131
avg_path_length_avg	-0.6263	0.316	-1.984	0.047	-1.245	-0.008
transitivity_avg	0.2877	1.592	0.181	0.857	-2.832	3.408
count_avg	0.0034	0.013	0.258	0.797	-0.023	0.030
Fraction Odd	-0.2775	0.106	-2.616	0.009	-0.485	-0.070

Omnibus:	29.294	**Durbin-Watson:**	1.965
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	38.921
Skew:	1.075	**Prob(JB):**	3.53e-09
Kurtosis:	2.981	**Cond. No.**	534.

Table 2.57

2.5 Discussion

One way to assess the difficulty of this problem is to consider the probability that a solution would occur by chance with actors playing randomly. If there are N players with N distinct states, a player voting uniformly randomly has a $\frac{1}{N}$ chance to choose any particular state. Assuming each player chooses independently, the probability of consensus is $\frac{1}{N^N}$. With 20 players and 20 states, which is a reasonable size for this experiment, the probability of random consensus is $\frac{1}{20^{20}}$, which is incredibly small. Now consider a dimensionality reduction where the 20 players have managed to reduce the 20 states to 5 to choose randomly over; the probability of random consensus is now $\frac{1}{5^{20}}$ which, while being much better, is still very small. Comparing this to the win probabilities observed in this game, which are about $\frac{1}{10}$ with 0.8 fraction of odd vertices, about $\frac{1}{5}$ with fraction of 0.5 odd vertices, and about $\frac{1}{3}$ when the fraction of odd vertices is 0.2. Thus, even in the worst experimental condition the human players are many orders of magnitude better than random play. While random play might seem to be a low bar, this explains how problem difficulty can scale with the number of players, and explains why the number of players alone was so strongly associated the likelihood of consensus. This also suggests that, when referenced against random play, reducing the number of states probably matters less than reducing the number of players. Notably, we did not find a significant effect for the number of players but average path length did sometimes have a significant effect. This suggests that the structure of the network matters more than the size of the network over the range of network sizes we considered.

We did not find strong evidence that the context of the vignettes mattered much to the consensus process. We expected that would behave differently based on context, and to some extent that was true. Some of the players would get into character; for example, in the orchestra scenario some players would say the venue they were voting for was made up positive qualities. Here is an example of a player doing this: "Like IWUQ, I support HEDUL as the venue. You know it's convenient and beautiful and has many amenities.". This seems to suggest that players did internalize the setting, but that it did not impact their strategy much. While we might expect the consensus process to work differently in real life when you have generals planning an attack versus friends going out to eat, it is likely that this game is too far removed from that to really show the difference. Even if the players can rate the scenarios as being different across the four dimensions, they do not seem to be acting like this mattered.

We found that the structural manipulation of altering the fraction of even and odd vertices in the initial graph did matter for the group to reach consensus. Unfortunately, the mechanism by which it does so is unclear. Our initial hypothesis was that more odd vertices would be better, as it would help players break gridlocks. There actually were slightly more gridlocks when the fraction of odd vertices was

0.8 versus 0.2 and gridlocked players were more likely to change their state not less. Additionally, the aggregate number of state changes across the different fraction of odd vertices was quite small. This suggests that altering the fraction of even versus odd vertices may be a fruitful intervention in different contexts.

2.6 Conclusion

We found that manipulating the Fraction of Odd vertices had a significant impact on the ability of a group to reach consensus in a Byzantine setting. The mechanism does not seem to be decision paralysis induced by gridlocks related to conflicting information. We found that gridlocked players were more likely to change their state than non gridlocked players. Further research is needed to determine what exactly about the fraction of odd vertices impacts the consensus process. Though, our results do suggest that this process leads to more gridlocks near the end of the game. Notably gridlocks were higher in the 0.8 fraction odd condition regardless of the type, so whether or not the players were considering their own state there were still more gridlocks. If the mechanism is structural, then the fraction of odd vertices may be relevant in other games on graphs. Alternatively, if the mechanism is psychological, then this could have implications for optimal ways to communicate information to foster agreement.

The human players preformed incredibly well when compared against the expected results of random actors, the win rate was still low suggesting that this is a difficult problem. However, we did find evidence of byzantine fault tolerance in this setting. Player dropout did not have a significant effect on the outcomes of interest. Player confusion did have a modest and negative effect on consensus, but this was overpowered by the fraction of odd vertices. The size of this effect was similar to that of the alphabetical protocol, suggesting that, in this setting choosing, a bad protocol is comparable in effect to byzantine action.

An important aspect of this experiment is that we did not experimentally introduce any byzantine faults into this experiment, they all occurred on their own. The methodological implications of this are that online experiments are a byzantine setting and that needs to be accounted for in experimental designs. The richness of the messages also allowed us deeper insights into the motivations, strategies and understandings of the players. This allowed us to detect when players did not understand the rule of the game, but in more limited settings these errors could have gone unnoticed. Thus, we recommend incorporating some sort of full text input into online experiments.

One advantage of this design is that the full text messages allow the players to both communicate in a rich way but also one that is more similar to real communication given the prevalence of email and other online messaging. Thus, it is likely that these results generalize better to online interactions better than in person interactions. In many real scenarios the information is not as stratified as it is in our setting, with each player being assigned a unique initial state with only the ability to communicate across network ties. It is reasonable for people to encounter those with different information sets than themselves. For example, people who consume liberal versus conservative media might have different knowledge about a certain event, or scientific collaborators who come from different fields could know different things about the same topic. Even so, effect of the skins was not significant suggesting, that this experiment was far enough removed from the stakes and complications of real consensus problems to have a substantial effect. Nevertheless, these results are promising and suggest avenues for further research in ecologically relevant settings.

2.7 Robustness Checks

2.7.1 Final Voting Bug

In the post-experimental analysis, we detected a bug in the experimental software that impacted some of the votes in the game. In an earlier version of this experiment, we assigned each player a 4 digit code to serve as their player identifier composed from their AMT ids. These include both letters and numbers. While these codes were not supposed to be used, under certain conditions, these codes could make it into the final drop-down list for voting, but were not assigned to any players. Since these options were the wrong number of letters and sometimes contained numbers, they were different enough from what they players were told to expect that we would expect players who passed the comprehension quiz to not vote for these options. While these incorrect choices appeared in the final drop-down lists in 139 out of our 202 runs, 63 runs were completely unaffected while players only voted for one of these options in 9 runs. Since these identifiers didn't appear until the final voting drop down, most players were already going into this with an idea of what they wanted to vote for already. This suggest that the vast majority of players were not impacted by this bug. Interestingly, players only voted for this in games where the alphabetical protocol was present suggesting that this and other strategies that coordinate on information in the drop down may have been the motivation to vote for these options. Even though the impact of this bug is minimal, Tables 2.58 through 2.85 show replications of some of the main regressions done on the sub sample of unaffected runs. We did not do this for skin paired comparisons because the sample size is already small. The coefficients on the variables representing the Fraction of Odd vertices were the same sign in these regressions even if they were not always statistically significant at the 5% level. Given that these regressions were done on slightly less than one third of the sample, the loss in power is not surprising this still suggests that these results are robust and were broadly not impacted by this bug.

Dep. Variable:	final_std	**R-squared:**	0.284
Model:	OLS	**Adj. R-squared:**	0.035
Method:	Least Squares	**F-statistic:**	1.100
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.383
Time:	00:28:09	**Log-Likelihood:**	22.275
No. Observations:	63	**AIC:**	-10.55
Df Residuals:	46	**BIC:**	25.88
Df Model:	16		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.2642	0.796	0.332	0.740	-1.297	1.825
skin: fire	0.0911	0.298	0.306	0.760	-0.493	0.675
skin: generals	0.2740	0.358	0.765	0.444	-0.428	0.976
skin: hiring	0.3037	0.309	0.983	0.326	-0.302	0.909
skin: mascot	0.3200	0.320	1.000	0.317	-0.307	0.947
skin: olympics	0.1153	0.293	0.393	0.694	-0.459	0.690
skin: orchestra	0.2133	0.294	0.725	0.468	-0.363	0.790
skin: plant	0.1992	0.298	0.668	0.504	-0.385	0.784
skin: restaurant	0.1842	0.307	0.600	0.549	-0.418	0.786
skin: space	0.1880	0.304	0.618	0.536	-0.408	0.784
Fraction Odd 0.2	-0.1683	0.087	-1.925	0.054	-0.340	0.003
Fraction Odd 0.5	-0.0675	0.076	-0.886	0.376	-0.217	0.082
frac_shortcut_avg	-0.0221	0.716	-0.031	0.975	-1.426	1.382
diameter_avg	0.0093	0.083	0.112	0.911	-0.153	0.172
avg_path_length_avg	0.0460	0.400	0.115	0.909	-0.738	0.830
transitivity_avg	-0.9074	1.928	-0.471	0.638	-4.686	2.871
count_avg	0.0042	0.018	0.232	0.816	-0.031	0.040

Omnibus:	4.448	**Durbin-Watson:**	2.429
Prob(Omnibus):	0.108	**Jarque-Bera (JB):**	4.385
Skew:	-0.629	**Prob(JB):**	0.112
Kurtosis:	2.703	**Cond. No.**	1.28e+03

Table 2.58

Dep. Variable:	final_std	**R-squared:**	0.211
Model:	OLS	**Adj. R-squared:**	0.041
Method:	Least Squares	**F-statistic:**	1.118
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.367
Time:	00:28:09	**Log-Likelihood:**	19.219
No. Observations:	63	**AIC:**	-14.44
Df Residuals:	51	**BIC:**	11.28
Df Model:	11		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.2841	0.792	0.359	0.720	-1.268	1.837
Fraction Odd 0.2	-0.1459	0.077	-1.905	0.057	-0.296	0.004
Fraction Odd 0.5	-0.0650	0.074	-0.884	0.376	-0.209	0.079
frac_shortcut_avg	0.0875	0.685	0.128	0.898	-1.254	1.429
diameter_avg	-0.0092	0.077	-0.119	0.905	-0.160	0.142
avg_path_length_avg	-0.0213	0.376	-0.057	0.955	-0.759	0.716
transitivity_avg	-0.2091	1.876	-0.111	0.911	-3.885	3.467
count_avg	0.0192	0.015	1.288	0.198	-0.010	0.048
Complex	9.568e-06	0.000	0.035	0.972	-0.001	0.001
Familiar	-3.418e-06	4.79e-05	-0.071	0.943	-9.73e-05	9.05e-05
Stakes	-2.004e-05	0.000	-0.110	0.913	-0.000	0.000
Emotionally_Charged	-2.815e-05	5.23e-05	-0.539	0.590	-0.000	7.43e-05

Omnibus:	5.187	**Durbin-Watson:**	2.245
Prob(Omnibus):	0.075	**Jarque-Bera (JB):**	5.083
Skew:	-0.690	**Prob(JB):**	0.0787
Kurtosis:	2.827	**Cond. No.**	2.09e+05

Table 2.59

Dep. Variable:	win	**R-squared:**	0.263
Model:	OLS	**Adj. R-squared:**	0.007
Method:	Least Squares	**F-statistic:**	0.5271
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.919
Time:	00:28:09	**Log-Likelihood:**	-13.607
No. Observations:	63	**AIC:**	61.21
Df Residuals:	46	**BIC:**	97.65
Df Model:	16		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.1636	1.419	0.820	0.412	-1.617	3.944
skin: fire	-0.3487	0.688	-0.507	0.612	-1.696	0.999
skin: generals	-0.5480	0.833	-0.658	0.511	-2.181	1.085
skin: hiring	-0.5380	0.692	-0.777	0.437	-1.895	0.819
skin: mascot	-0.5573	0.725	-0.768	0.442	-1.979	0.864
skin: olympics	-0.4925	0.673	-0.732	0.464	-1.811	0.826
skin: orchestra	-0.3643	0.683	-0.533	0.594	-1.704	0.975
skin: plant	-0.3953	0.687	-0.576	0.565	-1.741	0.950
skin: restaurant	-0.3427	0.697	-0.491	0.623	-1.710	1.024
skin: space	-0.3674	0.696	-0.528	0.598	-1.732	0.997
Fraction Odd 0.2	0.2460	0.161	1.529	0.126	-0.069	0.561
Fraction Odd 0.5	0.1375	0.131	1.046	0.295	-0.120	0.395
frac_shortcut_avg	-0.1258	1.324	-0.095	0.924	-2.721	2.470
diameter_avg	-0.0531	0.148	-0.360	0.719	-0.342	0.236
avg_path_length_avg	-0.3354	0.761	-0.441	0.659	-1.826	1.155
transitivity_avg	0.9526	3.383	0.282	0.778	-5.678	7.584
count_avg	0.0048	0.033	0.145	0.885	-0.059	0.069

Omnibus:	15.463	**Durbin-Watson:**	2.203
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	17.574
Skew:	1.236	**Prob(JB):**	0.000153
Kurtosis:	3.766	**Cond. No.**	1.28e+03

Table 2.60

Dep. Variable:	win	**R-squared:**	0.187
Model:	OLS	**Adj. R-squared:**	0.012
Method:	Least Squares	**F-statistic:**	0.6696
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.760
Time:	00:28:09	**Log-Likelihood:**	-16.721
No. Observations:	63	**AIC:**	57.44
Df Residuals:	51	**BIC:**	83.16
Df Model:	11		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.0057	1.357	0.741	0.459	-1.654	3.666
Fraction Odd 0.2	0.2088	0.138	1.513	0.130	-0.062	0.479
Fraction Odd 0.5	0.1132	0.129	0.876	0.381	-0.140	0.366
frac_shortcut_avg	-0.2731	1.283	-0.213	0.831	-2.788	2.242
diameter_avg	-0.0489	0.140	-0.350	0.726	-0.322	0.225
avg_path_length_avg	-0.2406	0.714	-0.337	0.736	-1.640	1.159
transitivity_avg	0.1042	3.270	0.032	0.975	-6.305	6.513
count_avg	-0.0112	0.026	-0.430	0.667	-0.062	0.040
Complex	8.055e-05	0.001	0.149	0.881	-0.001	0.001
Familiar	1.299e-05	8.56e-05	0.152	0.879	-0.000	0.000
Stakes	-5.086e-05	0.000	-0.146	0.884	-0.001	0.001
Emotionally_Charged	8.494e-06	9.66e-05	0.088	0.930	-0.000	0.000

Omnibus:	19.511	**Durbin-Watson:**	2.088
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	24.281
Skew:	1.402	**Prob(JB):**	5.34e-06
Kurtosis:	4.178	**Cond. No.**	2.09e+05

Table 2.61

Dep. Variable:	final_std	**R-squared:**	0.283
Model:	OLS	**Adj. R-squared:**	0.054
Method:	Least Squares	**F-statistic:**	1.202
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.304
Time:	00:28:09	**Log-Likelihood:**	22.223
No. Observations:	63	**AIC:**	-12.45
Df Residuals:	47	**BIC:**	21.84
Df Model:	15		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.0044	0.794	0.006	0.996	-1.551	1.560
skin: fire	0.1023	0.295	0.347	0.729	-0.476	0.680
skin: generals	0.2814	0.340	0.827	0.408	-0.386	0.948
skin: hiring	0.3054	0.309	0.989	0.323	-0.300	0.911
skin: mascot	0.3179	0.319	0.995	0.320	-0.308	0.944
skin: olympics	0.1204	0.293	0.411	0.681	-0.454	0.695
skin: orchestra	0.2145	0.295	0.728	0.467	-0.363	0.792
skin: plant	0.2049	0.298	0.688	0.492	-0.379	0.789
skin: restaurant	0.1886	0.308	0.612	0.540	-0.415	0.792
skin: space	0.1942	0.303	0.641	0.522	-0.400	0.788
frac_shortcut_avg	0.0134	0.716	0.019	0.985	-1.390	1.417
diameter_avg	0.0092	0.082	0.112	0.911	-0.151	0.169
avg_path_length_avg	0.0458	0.388	0.118	0.906	-0.714	0.806
transitivity_avg	-0.8135	1.891	-0.430	0.667	-4.519	2.892
count_avg	0.0044	0.018	0.249	0.804	-0.030	0.039
Fraction Odd	0.2767	0.141	1.963	0.050	0.000	0.553

Omnibus:	4.495	**Durbin-Watson:**	2.420
Prob(Omnibus):	0.106	**Jarque-Bera (JB):**	4.447
Skew:	-0.629	**Prob(JB):**	0.108
Kurtosis:	2.668	**Cond. No.**	1.24e+03

Table 2.62

Dep. Variable:	final_std	**R-squared:**	0.211
Model:	OLS	**Adj. R-squared:**	0.059
Method:	Least Squares	**F-statistic:**	1.256
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.279
Time:	00:28:09	**Log-Likelihood:**	19.207
No. Observations:	63	**AIC:**	-16.41
Df Residuals:	52	**BIC:**	7.161
Df Model:	10		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.0704	0.734	0.096	0.924	-1.368	1.509
frac_shortcut_avg	0.1030	0.672	0.153	0.878	-1.214	1.420
diameter_avg	-0.0089	0.076	-0.117	0.907	-0.158	0.140
avg_path_length_avg	-0.0202	0.363	-0.055	0.956	-0.732	0.692
transitivity_avg	-0.1676	1.795	-0.093	0.926	-3.685	3.350
count_avg	0.0192	0.015	1.317	0.188	-0.009	0.048
Complex	1.331e-05	0.000	0.051	0.960	-0.001	0.001
Familiar	-3.318e-06	4.63e-05	-0.072	0.943	-9.4e-05	8.74e-05
Stakes	-2.273e-05	0.000	-0.129	0.897	-0.000	0.000
Emotionally_Charged	-2.667e-05	4.9e-05	-0.544	0.586	-0.000	6.94e-05
Fraction Odd	0.2424	0.124	1.950	0.051	-0.001	0.486

Omnibus:	5.225	**Durbin-Watson:**	2.244
Prob(Omnibus):	0.073	**Jarque-Bera (JB):**	5.142
Skew:	-0.693	**Prob(JB):**	0.0765
Kurtosis:	2.813	**Cond. No.**	2.04e+05

Table 2.63

Dep. Variable:	win	**R-squared:**	0.263
Model:	OLS	**Adj. R-squared:**	0.028
Method:	Least Squares	**F-statistic:**	0.5541
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.894
Time:	00:28:09	**Log-Likelihood:**	-13.620
No. Observations:	63	**AIC:**	59.24
Df Residuals:	47	**BIC:**	93.53
Df Model:	15		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.4610	1.370	1.067	0.286	-1.223	4.145
skin: fire	-0.3390	0.685	-0.495	0.621	-1.682	1.004
skin: generals	-0.5415	0.818	-0.662	0.508	-2.145	1.062
skin: hiring	-0.5365	0.689	-0.778	0.436	-1.887	0.814
skin: mascot	-0.5591	0.721	-0.776	0.438	-1.972	0.854
skin: olympics	-0.4881	0.670	-0.728	0.467	-1.802	0.826
skin: orchestra	-0.3633	0.681	-0.534	0.594	-1.698	0.971
skin: plant	-0.3903	0.684	-0.570	0.568	-1.731	0.951
skin: restaurant	-0.3390	0.697	-0.486	0.627	-1.706	1.028
skin: space	-0.3620	0.690	-0.525	0.600	-1.715	0.991
frac_shortcut_avg	-0.0949	1.305	-0.073	0.942	-2.653	2.463
diameter_avg	-0.0532	0.143	-0.372	0.710	-0.333	0.227
avg_path_length_avg	-0.3355	0.740	-0.454	0.650	-1.785	1.114
transitivity_avg	1.0341	3.235	0.320	0.749	-5.307	7.375
count_avg	0.0049	0.032	0.154	0.878	-0.058	0.068
Fraction Odd	-0.4134	0.259	-1.598	0.110	-0.921	0.094

Omnibus:	15.586	**Durbin-Watson:**	2.211
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	17.745
Skew:	1.239	**Prob(JB):**	0.000140
Kurtosis:	3.790	**Cond. No.**	1.24e+03

Table 2.64

Dep. Variable:	win	**R-squared:**	0.187
Model:	OLS	**Adj. R-squared:**	0.030
Method:	Least Squares	**F-statistic:**	0.7606
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.665
Time:	00:28:09	**Log-Likelihood:**	-16.725
No. Observations:	63	**AIC:**	55.45
Df Residuals:	52	**BIC:**	79.03
Df Model:	10		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.2629	1.265	0.998	0.318	-1.217	3.743
frac_shortcut_avg	-0.2560	1.261	-0.203	0.839	-2.727	2.215
diameter_avg	-0.0486	0.138	-0.353	0.724	-0.319	0.221
avg_path_length_avg	-0.2393	0.686	-0.349	0.727	-1.583	1.104
transitivity_avg	0.1503	3.148	0.048	0.962	-6.019	6.320
count_avg	-0.0112	0.025	-0.441	0.659	-0.061	0.039
Complex	8.471e-05	0.001	0.164	0.870	-0.001	0.001
Familiar	1.31e-05	8.3e-05	0.158	0.875	-0.000	0.000
Stakes	-5.385e-05	0.000	-0.160	0.873	-0.001	0.001
Emotionally_Charged	1.013e-05	9.55e-05	0.106	0.915	-0.000	0.000
Fraction Odd	-0.3490	0.224	-1.558	0.119	-0.788	0.090

Omnibus:	19.509	**Durbin-Watson:**	2.092
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	24.276
Skew:	1.401	**Prob(JB):**	5.35e-06
Kurtosis:	4.184	**Cond. No.**	2.04e+05

Table 2.65

Dep. Variable:	final_std	**R-squared:**	0.198
Model:	OLS	**Adj. R-squared:**	-0.036
Method:	Least Squares	**F-statistic:**	0.8399
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.624
Time:	00:28:21	**Log-Likelihood:**	18.679
No. Observations:	63	**AIC:**	-7.358
Df Residuals:	48	**BIC:**	24.79
Df Model:	14		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.0344	0.808	0.043	0.966	-1.550	1.619
skin: fire	0.1382	0.385	0.359	0.719	-0.615	0.892
skin: generals	0.2533	0.390	0.650	0.516	-0.511	1.017
skin: hiring	0.3089	0.391	0.790	0.429	-0.457	1.075
skin: mascot	0.2405	0.398	0.604	0.546	-0.540	1.021
skin: olympics	0.1377	0.382	0.361	0.718	-0.611	0.886
skin: orchestra	0.2174	0.382	0.568	0.570	-0.532	0.967
skin: plant	0.1801	0.384	0.469	0.639	-0.572	0.933
skin: restaurant	0.2330	0.391	0.596	0.551	-0.533	0.999
skin: space	0.2271	0.394	0.576	0.565	-0.546	1.000
frac_shortcut_avg	0.2295	0.757	0.303	0.762	-1.255	1.714
diameter_avg	-0.0062	0.087	-0.071	0.944	-0.177	0.165
avg_path_length_avg	0.0373	0.405	0.092	0.927	-0.757	0.831
transitivity_avg	-0.3487	1.841	-0.189	0.850	-3.957	3.260
count_avg	0.0039	0.019	0.204	0.838	-0.033	0.041

Omnibus:	6.277	**Durbin-Watson:**	2.410
Prob(Omnibus):	0.043	**Jarque-Bera (JB):**	6.399
Skew:	-0.753	**Prob(JB):**	0.0408
Kurtosis:	2.590	**Cond. No.**	1.23e+03

Table 2.66

Dep. Variable:	final_std	**R-squared:**	0.138
Model:	OLS	**Adj. R-squared:**	-0.009
Method:	Least Squares	**F-statistic:**	0.9839
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.464
Time:	00:28:21	**Log-Likelihood:**	16.416
No. Observations:	63	**AIC:**	-12.83
Df Residuals:	53	**BIC:**	8.600
Df Model:	9		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	0.0461	0.708	0.065	0.948	-1.342	1.434
frac_shortcut_avg	0.3265	0.691	0.472	0.637	-1.028	1.681
diameter_avg	-0.0147	0.080	-0.183	0.855	-0.172	0.143
avg_path_length_avg	-0.0354	0.371	-0.095	0.924	-0.763	0.692
transitivity_avg	0.2490	1.698	0.147	0.883	-3.079	3.577
count_avg	0.0172	0.015	1.143	0.253	-0.012	0.047
Complex	-3.326e-05	0.000	-0.128	0.898	-0.001	0.000
Familiar	1.474e-05	4.07e-05	0.362	0.717	-6.5e-05	9.45e-05
Stakes	3.119e-05	0.000	0.167	0.867	-0.000	0.000
Emotionally_Charged	-2.738e-05	5.58e-05	-0.491	0.623	-0.000	8.19e-05

Omnibus:	7.211	**Durbin-Watson:**	2.314
Prob(Omnibus):	0.027	**Jarque-Bera (JB):**	7.539
Skew:	-0.829	**Prob(JB):**	0.0231
Kurtosis:	2.653	**Cond. No.**	2.03e+05

Table 2.67

Dep. Variable:	win	**R-squared:**	0.200
Model:	OLS	**Adj. R-squared:**	-0.033
Method:	Least Squares	**F-statistic:**	0.4909
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.927
Time:	00:28:21	**Log-Likelihood:**	-16.195
No. Observations:	63	**AIC:**	62.39
Df Residuals:	48	**BIC:**	94.54
Df Model:	14		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.4163	1.402	1.010	0.312	-1.331	4.163
skin: fire	-0.3926	0.805	-0.488	0.626	-1.971	1.186
skin: generals	-0.4995	0.863	-0.579	0.563	-2.190	1.191
skin: hiring	-0.5418	0.803	-0.675	0.500	-2.115	1.032
skin: mascot	-0.4436	0.828	-0.536	0.592	-2.067	1.180
skin: olympics	-0.5140	0.792	-0.649	0.517	-2.067	1.039
skin: orchestra	-0.3676	0.802	-0.458	0.647	-1.940	1.205
skin: plant	-0.3532	0.802	-0.440	0.660	-1.926	1.219
skin: restaurant	-0.4054	0.816	-0.497	0.619	-2.004	1.193
skin: space	-0.4113	0.815	-0.505	0.614	-2.008	1.186
frac_shortcut_avg	-0.4178	1.401	-0.298	0.766	-3.164	2.328
diameter_avg	-0.0303	0.147	-0.206	0.837	-0.319	0.258
avg_path_length_avg	-0.3228	0.750	-0.430	0.667	-1.793	1.147
transitivity_avg	0.3396	3.229	0.105	0.916	-5.989	6.668
count_avg	0.0058	0.034	0.169	0.866	-0.061	0.072

Omnibus:	17.111	**Durbin-Watson:**	2.260
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	20.209
Skew:	1.317	**Prob(JB):**	4.09e-05
Kurtosis:	3.874	**Cond. No.**	1.23e+03

Table 2.68

Dep. Variable:	win	**R-squared:**	0.137
Model:	OLS	**Adj. R-squared:**	-0.010
Method:	Least Squares	**F-statistic:**	0.8094
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.610
Time:	00:28:21	**Log-Likelihood:**	-18.602
No. Observations:	63	**AIC:**	57.20
Df Residuals:	53	**BIC:**	78.64
Df Model:	9		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.2980	1.208	1.075	0.283	-1.069	3.665
frac_shortcut_avg	-0.5778	1.310	-0.441	0.659	-3.146	1.990
diameter_avg	-0.0402	0.138	-0.291	0.771	-0.311	0.230
avg_path_length_avg	-0.2173	0.672	-0.323	0.746	-1.534	1.100
transitivity_avg	-0.4496	3.066	-0.147	0.883	-6.459	5.560
count_avg	-0.0084	0.025	-0.340	0.734	-0.057	0.040
Complex	0.0002	0.001	0.303	0.762	-0.001	0.001
Familiar	-1.289e-05	7.58e-05	-0.170	0.865	-0.000	0.000
Stakes	-0.0001	0.000	-0.390	0.697	-0.001	0.001
Emotionally_Charged	1.115e-05	0.000	0.109	0.913	-0.000	0.000

Omnibus:	23.047	**Durbin-Watson:**	2.203
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	31.191
Skew:	1.584	**Prob(JB):**	1.69e-07
Kurtosis:	4.357	**Cond. No.**	2.03e+05

Table 2.69

Dep. Variable:	final_std	**R-squared:**	0.352
Model:	OLS	**Adj. R-squared:**	0.126
Method:	Least Squares	**F-statistic:**	1.724
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.0758
Time:	00:28:42	**Log-Likelihood:**	25.392
No. Observations:	63	**AIC:**	-16.78
Df Residuals:	46	**BIC:**	19.65
Df Model:	16		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	2.0979	1.235	1.699	0.089	-0.323	4.519
skin: fire	0.0587	0.214	0.274	0.784	-0.362	0.479
skin: generals	0.2494	0.229	1.090	0.276	-0.199	0.698
skin: hiring	0.2596	0.225	1.156	0.248	-0.180	0.700
skin: mascot	0.2891	0.231	1.254	0.210	-0.163	0.741
skin: olympics	0.1039	0.210	0.494	0.621	-0.308	0.516
skin: orchestra	0.2032	0.203	1.001	0.317	-0.195	0.601
skin: plant	0.1982	0.217	0.911	0.362	-0.228	0.624
skin: restaurant	0.1991	0.217	0.917	0.359	-0.227	0.625
skin: space	0.1748	0.210	0.832	0.405	-0.237	0.587
Fraction Odd 0.2	-0.1936	0.086	-2.243	0.025	-0.363	-0.024
Fraction Odd 0.5	-0.0573	0.071	-0.806	0.420	-0.197	0.082
frac_shortcut_1	-0.6306	0.873	-0.722	0.470	-2.342	1.080
diameter_1	0.0411	0.082	0.502	0.616	-0.119	0.202
avg_path_length_1	-1.0388	0.835	-1.244	0.214	-2.676	0.598
transitivity_1	-2.2767	2.201	-1.035	0.301	-6.590	2.036
count_1	0.0518	0.038	1.350	0.177	-0.023	0.127

Omnibus:	2.127	**Durbin-Watson:**	2.341
Prob(Omnibus):	0.345	**Jarque-Bera (JB):**	1.967
Skew:	-0.423	**Prob(JB):**	0.374
Kurtosis:	2.821	**Cond. No.**	1.63e+03

Table 2.70

Dep. Variable:	final_std	**R-squared:**	0.292
Model:	OLS	**Adj. R-squared:**	0.139
Method:	Least Squares	**F-statistic:**	1.586
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.131
Time:	00:28:42	**Log-Likelihood:**	22.610
No. Observations:	63	**AIC:**	-21.22
Df Residuals:	51	**BIC:**	4.498
Df Model:	11		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	2.3814	1.144	2.082	0.037	0.140	4.623
0.2 Fraction Odd	-0.1805	0.076	-2.374	0.018	-0.329	-0.031
0.5 Fraction Odd	-0.0549	0.068	-0.808	0.419	-0.188	0.078
frac_shortcut_1	-0.6362	0.799	-0.796	0.426	-2.203	0.931
diameter_1	0.0329	0.069	0.478	0.633	-0.102	0.168
avg_path_length_1	-1.2021	0.748	-1.608	0.108	-2.667	0.263
transitivity_1	-1.8742	2.090	-0.897	0.370	-5.971	2.223
count_1	0.0669	0.036	1.876	0.061	-0.003	0.137
Complex	-1.068e-05	0.000	-0.046	0.963	-0.000	0.000
Familiar	-3.66e-06	3.93e-05	-0.093	0.926	-8.07e-05	7.34e-05
Stakes	-6.896e-06	0.000	-0.042	0.966	-0.000	0.000
Emotionally_Charged	-4.422e-05	5.6e-05	-0.790	0.429	-0.000	6.55e-05

Omnibus:	1.286	**Durbin-Watson:**	2.142
Prob(Omnibus):	0.526	**Jarque-Bera (JB):**	1.320
Skew:	-0.312	**Prob(JB):**	0.517
Kurtosis:	2.665	**Cond. No.**	2.54e+05

Table 2.71

Dep. Variable:	win	**R-squared:**	0.283
Model:	OLS	**Adj. R-squared:**	0.034
Method:	Least Squares	**F-statistic:**	0.5681
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.891
Time:	00:28:42	**Log-Likelihood:**	-12.751
No. Observations:	63	**AIC:**	59.50
Df Residuals:	46	**BIC:**	95.94
Df Model:	16		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-1.9174	2.416	-0.794	0.427	-6.652	2.818
skin: fire	-0.2516	0.605	-0.416	0.678	-1.438	0.935
skin: generals	-0.4310	0.642	-0.671	0.502	-1.690	0.828
skin: hiring	-0.3784	0.603	-0.628	0.530	-1.560	0.803
skin: mascot	-0.3988	0.629	-0.634	0.526	-1.632	0.835
skin: olympics	-0.4298	0.586	-0.733	0.463	-1.579	0.719
skin: orchestra	-0.2804	0.588	-0.477	0.633	-1.432	0.872
skin: plant	-0.3427	0.599	-0.572	0.567	-1.517	0.831
skin: restaurant	-0.3416	0.599	-0.570	0.569	-1.516	0.833
skin: space	-0.2922	0.594	-0.492	0.622	-1.456	0.871
Fraction Odd 0.2	0.2910	0.168	1.730	0.084	-0.039	0.621
Fraction Odd 0.5	0.1159	0.125	0.928	0.353	-0.129	0.361
frac_shortcut_1	0.9535	1.680	0.568	0.570	-2.338	4.245
diameter_1	-0.1139	0.163	-0.699	0.485	-0.433	0.206
avg_path_length_1	1.7051	1.754	0.972	0.331	-1.732	5.142
transitivity_1	2.5391	4.044	0.628	0.530	-5.386	10.464
count_1	-0.0971	0.078	-1.249	0.212	-0.250	0.055

Omnibus:	13.807	**Durbin-Watson:**	2.031
Prob(Omnibus):	0.001	**Jarque-Bera (JB):**	15.044
Skew:	1.145	**Prob(JB):**	0.000541
Kurtosis:	3.696	**Cond. No.**	1.63e+03

Table 2.72

Dep. Variable:	win	**R-squared:**	0.238
Model:	OLS	**Adj. R-squared:**	0.074
Method:	Least Squares	**F-statistic:**	0.8225
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.618
Time:	00:28:42	**Log-Likelihood:**	-14.669
No. Observations:	63	**AIC:**	53.34
Df Residuals:	51	**BIC:**	79.06
Df Model:	11		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-2.4624	2.143	-1.149	0.251	-6.663	1.738
Fraction Odd 0.2	0.2832	0.145	1.949	0.051	-0.002	0.568
Fraction Odd 0.5	0.0906	0.126	0.718	0.473	-0.157	0.338
frac_shortcut_1	0.8343	1.541	0.541	0.588	-2.186	3.854
diameter_1	-0.1561	0.129	-1.211	0.226	-0.409	0.097
avg_path_length_1	2.2019	1.489	1.479	0.139	-0.717	5.121
transitivity_1	1.7569	3.765	0.467	0.641	-5.623	9.137
count_1	-0.1223	0.071	-1.729	0.084	-0.261	0.016
Complex	5.09e-05	0.000	0.119	0.905	-0.001	0.001
Familiar	-7.6e-06	7.37e-05	-0.103	0.918	-0.000	0.000
Stakes	-5.169e-05	0.000	-0.176	0.860	-0.001	0.001
Emotionally_Charged	3.585e-05	0.000	0.329	0.742	-0.000	0.000

Omnibus:	13.594	**Durbin-Watson:**	1.950
Prob(Omnibus):	0.001	**Jarque-Bera (JB):**	14.810
Skew:	1.145	**Prob(JB):**	0.000608
Kurtosis:	3.631	**Cond. No.**	2.54e+05

Table 2.73

Dep. Variable:	final_std	**R-squared:**	0.345
Model:	OLS	**Adj. R-squared:**	0.136
Method:	Least Squares	**F-statistic:**	1.743
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.0745
Time:	00:28:42	**Log-Likelihood:**	25.078
No. Observations:	63	**AIC:**	-18.16
Df Residuals:	47	**BIC:**	16.13
Df Model:	15		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.6863	1.172	1.438	0.150	-0.611	3.984
skin: fire	0.0843	0.216	0.390	0.696	-0.339	0.508
skin: generals	0.2686	0.220	1.223	0.221	-0.162	0.699
skin: hiring	0.2651	0.231	1.148	0.251	-0.187	0.718
skin: mascot	0.2842	0.238	1.192	0.233	-0.183	0.751
skin: olympics	0.1159	0.218	0.532	0.595	-0.311	0.543
skin: orchestra	0.2047	0.213	0.963	0.335	-0.212	0.621
skin: plant	0.2105	0.223	0.944	0.345	-0.227	0.648
skin: restaurant	0.2070	0.225	0.920	0.358	-0.234	0.648
skin: space	0.1890	0.216	0.875	0.382	-0.235	0.613
frac_shortcut_1	-0.5256	0.868	-0.606	0.545	-2.227	1.176
diameter_1	0.0360	0.080	0.452	0.651	-0.120	0.192
avg_path_length_1	-0.9798	0.796	-1.230	0.219	-2.541	0.581
transitivity_1	-2.0258	2.207	-0.918	0.359	-6.351	2.299
count_1	0.0497	0.037	1.356	0.175	-0.022	0.121
Fraction Odd	0.3139	0.139	2.256	0.024	0.041	0.586

Omnibus:	2.379	**Durbin-Watson:**	2.342
Prob(Omnibus):	0.304	**Jarque-Bera (JB):**	2.321
Skew:	-0.441	**Prob(JB):**	0.313
Kurtosis:	2.675	**Cond. No.**	1.59e+03

Table 2.74

Dep. Variable:	final_std	**R-squared:**	0.286
Model:	OLS	**Adj. R-squared:**	0.149
Method:	Least Squares	**F-statistic:**	1.676
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.112
Time:	00:28:42	**Log-Likelihood:**	22.358
No. Observations:	63	**AIC:**	-22.72
Df Residuals:	52	**BIC:**	0.8585
Df Model:	10		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.9668	1.023	1.923	0.055	-0.038	3.972
frac_shortcut_1	-0.5377	0.776	-0.693	0.488	-2.058	0.982
diameter_1	0.0281	0.068	0.415	0.678	-0.105	0.161
avg_path_length_1	-1.1328	0.699	-1.621	0.105	-2.502	0.237
transitivity_1	-1.6338	2.028	-0.806	0.420	-5.608	2.341
count_1	0.0643	0.034	1.898	0.058	-0.002	0.131
Complex	8.105e-06	0.000	0.036	0.971	-0.000	0.000
Familiar	-2.791e-06	3.85e-05	-0.072	0.942	-7.83e-05	7.27e-05
Stakes	-1.907e-05	0.000	-0.119	0.905	-0.000	0.000
Emotionally_Charged	-3.767e-05	5.21e-05	-0.723	0.469	-0.000	6.44e-05
Fraction Odd	0.2957	0.123	2.409	0.016	0.055	0.536

Omnibus:	1.906	**Durbin-Watson:**	2.148
Prob(Omnibus):	0.386	**Jarque-Bera (JB):**	1.859
Skew:	-0.349	**Prob(JB):**	0.395
Kurtosis:	2.530	**Cond. No.**	2.47e+05

Table 2.75

Dep. Variable:	win	**R-squared:**	0.282
Model:	OLS	**Adj. R-squared:**	0.053
Method:	Least Squares	**F-statistic:**	0.6073
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.854
Time:	00:28:42	**Log-Likelihood:**	-12.804
No. Observations:	63	**AIC:**	57.61
Df Residuals:	47	**BIC:**	91.90
Df Model:	15		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-1.4143	2.269	-0.623	0.533	-5.861	3.033
skin: fire	-0.2709	0.608	-0.446	0.656	-1.462	0.920
skin: generals	-0.4454	0.629	-0.709	0.479	-1.678	0.787
skin: hiring	-0.3824	0.605	-0.632	0.527	-1.568	0.803
skin: mascot	-0.3952	0.632	-0.625	0.532	-1.634	0.844
skin: olympics	-0.4388	0.588	-0.747	0.455	-1.591	0.713
skin: orchestra	-0.2816	0.592	-0.476	0.634	-1.441	0.878
skin: plant	-0.3520	0.600	-0.587	0.557	-1.527	0.823
skin: restaurant	-0.3475	0.603	-0.576	0.564	-1.529	0.834
skin: space	-0.3029	0.594	-0.510	0.610	-1.467	0.861
frac_shortcut_1	0.8748	1.659	0.527	0.598	-2.378	4.127
diameter_1	-0.1101	0.155	-0.711	0.477	-0.413	0.193
avg_path_length_1	1.6608	1.675	0.991	0.321	-1.622	4.944
transitivity_1	2.3509	3.987	0.590	0.555	-5.464	10.166
count_1	-0.0955	0.075	-1.274	0.203	-0.243	0.051
Fraction Odd	-0.4784	0.270	-1.773	0.076	-1.007	0.051

Omnibus:	13.512	**Durbin-Watson:**	2.026
Prob(Omnibus):	0.001	**Jarque-Bera (JB):**	14.681
Skew:	1.139	**Prob(JB):**	0.000649
Kurtosis:	3.633	**Cond. No.**	1.59e+03

Table 2.76

Dep. Variable:	win	**R-squared:**	0.234
Model:	OLS	**Adj. R-squared:**	0.087
Method:	Least Squares	**F-statistic:**	0.9179
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.524
Time:	00:28:42	**Log-Likelihood:**	-14.830
No. Observations:	63	**AIC:**	51.66
Df Residuals:	52	**BIC:**	75.23
Df Model:	10		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-1.8339	1.908	-0.961	0.336	-5.573	1.906
frac_shortcut_1	0.6922	1.502	0.461	0.645	-2.252	3.637
diameter_1	-0.1492	0.124	-1.199	0.230	-0.393	0.095
avg_path_length_1	2.1019	1.380	1.523	0.128	-0.603	4.806
transitivity_1	1.4101	3.693	0.382	0.703	-5.828	8.648
count_1	-0.1185	0.067	-1.775	0.076	-0.249	0.012
Complex	2.38e-05	0.000	0.058	0.954	-0.001	0.001
Familiar	-8.853e-06	7.17e-05	-0.123	0.902	-0.000	0.000
Stakes	-3.412e-05	0.000	-0.118	0.906	-0.001	0.001
Emotionally_Charged	2.64e-05	0.000	0.243	0.808	-0.000	0.000
condition_int	-0.4646	0.236	-1.966	0.049	-0.928	-0.002

Omnibus:	13.401	**Durbin-Watson:**	1.937
Prob(Omnibus):	0.001	**Jarque-Bera (JB):**	14.661
Skew:	1.150	**Prob(JB):**	0.000655
Kurtosis:	3.544	**Cond. No.**	2.47e+05

Table 2.77

Dep. Variable:	final_std	**R-squared:**	0.277
Model:	OLS	**Adj. R-squared:**	0.026
Method:	Least Squares	**F-statistic:**	1.106
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.377
Time:	02:22:21	**Log-Likelihood:**	21.975
No. Observations:	63	**AIC:**	-9.950
Df Residuals:	46	**BIC:**	26.48
Df Model:	16		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.1004	0.748	-0.134	0.893	-1.567	1.366
skin: fire	0.0970	0.295	0.328	0.743	-0.482	0.676
skin: generals	0.2876	0.349	0.823	0.410	-0.397	0.972
skin: hiring	0.2908	0.305	0.954	0.340	-0.307	0.888
skin: mascot	0.2991	0.320	0.936	0.349	-0.327	0.926
skin: olympics	0.1051	0.288	0.365	0.715	-0.460	0.670
skin: orchestra	0.2087	0.291	0.718	0.473	-0.361	0.778
skin: plant	0.1733	0.295	0.587	0.557	-0.406	0.752
skin: restaurant	0.1701	0.307	0.554	0.580	-0.432	0.772
skin: space	0.1784	0.303	0.588	0.556	-0.416	0.773
Fraction Odd 0.2	-0.1677	0.087	-1.938	0.053	-0.337	0.002
Fraction Odd 0.5	-0.0637	0.078	-0.817	0.414	-0.216	0.089
frac_shortcut_10	0.1021	0.614	0.166	0.868	-1.100	1.305
diameter_10	-0.0352	0.074	-0.478	0.633	-0.180	0.109
avg_path_length_10	0.2443	0.329	0.742	0.458	-0.401	0.889
transitivity_10	-0.3151	1.567	-0.201	0.841	-3.387	2.757
count_10	0.0015	0.014	0.107	0.915	-0.026	0.029

Omnibus:	4.714	**Durbin-Watson:**	2.442
Prob(Omnibus):	0.095	**Jarque-Bera (JB):**	4.659
Skew:	-0.632	**Prob(JB):**	0.0973
Kurtosis:	2.580	**Cond. No.**	1.04e+03

Table 2.78

Dep. Variable:	final_std	**R-squared:**	0.205
Model:	OLS	**Adj. R-squared:**	0.033
Method:	Least Squares	**F-statistic:**	1.006
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.454
Time:	02:22:21	**Log-Likelihood:**	18.959
No. Observations:	63	**AIC:**	-13.92
Df Residuals:	51	**BIC:**	11.80
Df Model:	11		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.0823	0.716	-0.115	0.908	-1.486	1.321
Fraction Odd 0.2	-0.1449	0.076	-1.911	0.056	-0.294	0.004
Fraction Odd 0.5	-0.0661	0.075	-0.880	0.379	-0.213	0.081
frac_shortcut_10	0.1801	0.571	0.315	0.752	-0.939	1.299
diameter_10	-0.0490	0.071	-0.690	0.490	-0.188	0.090
avg_path_length_10	0.2167	0.321	0.675	0.500	-0.413	0.846
transitivity_10	0.1839	1.490	0.123	0.902	-2.736	3.103
count_10	0.0131	0.012	1.128	0.259	-0.010	0.036
Complex	1.365e-05	0.000	0.048	0.961	-0.001	0.001
Familiar	-3.388e-06	4.66e-05	-0.073	0.942	-9.47e-05	8.79e-05
Stakes	-2.266e-05	0.000	-0.119	0.905	-0.000	0.000
Emotionally_Charged	-2.23e-05	5.55e-05	-0.402	0.688	-0.000	8.65e-05

Omnibus:	5.442	**Durbin-Watson:**	2.257
Prob(Omnibus):	0.066	**Jarque-Bera (JB):**	5.474
Skew:	-0.705	**Prob(JB):**	0.0648
Kurtosis:	2.687	**Cond. No.**	1.78e+05

Table 2.79

Dep. Variable:	win	**R-squared:**	0.272
Model:	OLS	**Adj. R-squared:**	0.019
Method:	Least Squares	**F-statistic:**	0.5525
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.902
Time:	02:22:21	**Log-Likelihood:**	-13.230
No. Observations:	63	**AIC:**	60.46
Df Residuals:	46	**BIC:**	96.89
Df Model:	16		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.6980	1.314	1.292	0.196	-0.878	4.274
skin: fire	-0.4030	0.673	-0.599	0.549	-1.721	0.915
skin: generals	-0.6018	0.810	-0.743	0.458	-2.190	0.986
skin: hiring	-0.5493	0.683	-0.805	0.421	-1.887	0.788
skin: mascot	-0.5600	0.720	-0.778	0.437	-1.972	0.852
skin: olympics	-0.5084	0.662	-0.768	0.442	-1.805	0.788
skin: orchestra	-0.3860	0.673	-0.574	0.566	-1.705	0.933
skin: plant	-0.3922	0.676	-0.580	0.562	-1.717	0.933
skin: restaurant	-0.3559	0.696	-0.512	0.609	-1.719	1.007
skin: space	-0.3884	0.687	-0.566	0.572	-1.734	0.958
Fraction Odd 0.2	0.2424	0.156	1.551	0.121	-0.064	0.549
Fraction Odd 0.5	0.1310	0.134	0.977	0.329	-0.132	0.394
frac_shortcut_10	-0.4347	1.081	-0.402	0.688	-2.554	1.685
diameter_10	0.0154	0.122	0.126	0.899	-0.224	0.255
avg_path_length_10	-0.5287	0.595	-0.889	0.374	-1.694	0.637
transitivity_10	-0.1624	2.657	-0.061	0.951	-5.370	5.045
count_10	0.0047	0.024	0.191	0.848	-0.043	0.053

Omnibus:	15.292	**Durbin-Watson:**	2.188
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	17.280
Skew:	1.222	**Prob(JB):**	0.000177
Kurtosis:	3.779	**Cond. No.**	1.04e+03

Table 2.80

Dep. Variable:	win	**R-squared:**	0.194
Model:	OLS	**Adj. R-squared:**	0.020
Method:	Least Squares	**F-statistic:**	0.6246
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.799
Time:	02:22:21	**Log-Likelihood:**	-16.453
No. Observations:	63	**AIC:**	56.91
Df Residuals:	51	**BIC:**	82.62
Df Model:	11		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.5075	1.206	1.250	0.211	-0.856	3.871
Fraction Odd 0.2	0.2100	0.138	1.521	0.128	-0.061	0.481
Fraction Odd 0.5	0.1070	0.129	0.832	0.405	-0.145	0.359
frac_shortcut_10	-0.4374	1.029	-0.425	0.671	-2.454	1.579
diameter_10	0.0456	0.125	0.366	0.715	-0.199	0.290
avg_path_length_10	-0.5914	0.567	-1.043	0.297	-1.702	0.520
transitivity_10	-0.5992	2.554	-0.235	0.814	-5.605	4.406
count_10	-0.0053	0.019	-0.277	0.781	-0.043	0.032
Complex	7.974e-05	0.001	0.142	0.887	-0.001	0.001
Familiar	1.468e-05	8.56e-05	0.171	0.864	-0.000	0.000
Stakes	-4.707e-05	0.000	-0.129	0.898	-0.001	0.001
Emotionally_Charged	-7.578e-06	9.19e-05	-0.082	0.934	-0.000	0.000

Omnibus:	18.690	**Durbin-Watson:**	2.068
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	22.809
Skew:	1.367	**Prob(JB):**	1.11e-05
Kurtosis:	4.102	**Cond. No.**	1.78e+05

Table 2.81

Dep. Variable:	final_std	**R-squared:**	0.276
Model:	OLS	**Adj. R-squared:**	0.044
Method:	Least Squares	**F-statistic:**	1.209
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.299
Time:	02:22:21	**Log-Likelihood:**	21.903
No. Observations:	63	**AIC:**	-11.81
Df Residuals:	47	**BIC:**	22.48
Df Model:	15		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.3605	0.760	-0.474	0.635	-1.851	1.130
skin: fire	0.1097	0.292	0.376	0.707	-0.462	0.682
skin: generals	0.2960	0.328	0.904	0.366	-0.346	0.938
skin: hiring	0.2926	0.304	0.962	0.336	-0.304	0.889
skin: mascot	0.2970	0.318	0.933	0.351	-0.327	0.921
skin: olympics	0.1117	0.288	0.388	0.698	-0.453	0.676
skin: orchestra	0.2099	0.291	0.722	0.470	-0.360	0.780
skin: plant	0.1809	0.294	0.615	0.539	-0.396	0.758
skin: restaurant	0.1762	0.307	0.574	0.566	-0.425	0.778
skin: space	0.1853	0.302	0.614	0.539	-0.406	0.777
frac_shortcut_10	0.1480	0.606	0.244	0.807	-1.040	1.336
diameter_10	-0.0327	0.074	-0.442	0.658	-0.178	0.112
avg_path_length_10	0.2321	0.325	0.715	0.475	-0.404	0.869
transitivity_10	-0.1982	1.515	-0.131	0.896	-3.168	2.772
count_10	0.0022	0.014	0.157	0.876	-0.025	0.029
Fraction Odd	0.2745	0.140	1.965	0.049	0.001	0.548

Omnibus:	4.879	**Durbin-Watson:**	2.430
Prob(Omnibus):	0.087	**Jarque-Bera (JB):**	4.814
Skew:	-0.640	**Prob(JB):**	0.0901
Kurtosis:	2.557	**Cond. No.**	997.

Table 2.82

Dep. Variable:	final_std	**R-squared:**	0.205
Model:	OLS	**Adj. R-squared:**	0.052
Method:	Least Squares	**F-statistic:**	1.139
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.352
Time:	02:22:21	**Log-Likelihood:**	18.952
No. Observations:	63	**AIC:**	-15.90
Df Residuals:	52	**BIC:**	7.671
Df Model:	10		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	-0.2895	0.672	-0.431	0.667	-1.607	1.028
frac_shortcut_10	0.1938	0.556	0.348	0.728	-0.896	1.284
diameter_10	-0.0483	0.070	-0.689	0.491	-0.186	0.089
avg_path_length_10	0.2144	0.315	0.681	0.496	-0.402	0.831
transitivity_10	0.2203	1.396	0.158	0.875	-2.516	2.956
count_10	0.0132	0.011	1.171	0.242	-0.009	0.035
Complex	1.693e-05	0.000	0.064	0.949	-0.001	0.001
Familiar	-3.204e-06	4.52e-05	-0.071	0.943	-9.17e-05	8.53e-05
Stakes	-2.492e-05	0.000	-0.138	0.890	-0.000	0.000
Emotionally_Charged	-2.124e-05	5.25e-05	-0.404	0.686	-0.000	8.17e-05
Fraction Odd	0.2407	0.123	1.956	0.050	-0.000	0.482

Omnibus:	5.495	**Durbin-Watson:**	2.256
Prob(Omnibus):	0.064	**Jarque-Bera (JB):**	5.534
Skew:	-0.708	**Prob(JB):**	0.0628
Kurtosis:	2.680	**Cond. No.**	1.71e+05

Table 2.83

Dep. Variable:	win	**R-squared:**	0.272
Model:	OLS	**Adj. R-squared:**	0.040
Method:	Least Squares	**F-statistic:**	0.5752
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.879
Time:	02:22:21	**Log-Likelihood:**	-13.236
No. Observations:	63	**AIC:**	58.47
Df Residuals:	47	**BIC:**	92.76
Df Model:	15		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	2.0033	1.316	1.522	0.128	-0.576	4.583
skin: fire	-0.3968	0.673	-0.590	0.555	-1.716	0.922
skin: generals	-0.5977	0.793	-0.754	0.451	-2.151	0.956
skin: hiring	-0.5484	0.681	-0.805	0.421	-1.883	0.786
skin: mascot	-0.5610	0.717	-0.783	0.434	-1.966	0.844
skin: olympics	-0.5052	0.661	-0.764	0.445	-1.801	0.791
skin: orchestra	-0.3854	0.672	-0.574	0.566	-1.702	0.931
skin: plant	-0.3885	0.676	-0.575	0.565	-1.713	0.936
skin: restaurant	-0.3529	0.697	-0.506	0.613	-1.719	1.014
skin: space	-0.3849	0.683	-0.563	0.573	-1.724	0.955
frac_shortcut_10	-0.4123	1.054	-0.391	0.696	-2.478	1.653
diameter_10	0.0167	0.122	0.136	0.892	-0.223	0.256
avg_path_length_10	-0.5347	0.589	-0.908	0.364	-1.689	0.620
transitivity_10	-0.1053	2.469	-0.043	0.966	-4.944	4.733
count_10	0.0050	0.024	0.210	0.834	-0.042	0.052
Fraction Odd	-0.4064	0.253	-1.604	0.109	-0.903	0.090

Omnibus:	15.308	**Durbin-Watson:**	2.194
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	17.293
Skew:	1.221	**Prob(JB):**	0.000176
Kurtosis:	3.789	**Cond. No.**	997.

Table 2.84

Dep. Variable:	win	**R-squared:**	0.194
Model:	OLS	**Adj. R-squared:**	0.039
Method:	Least Squares	**F-statistic:**	0.7131
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	0.708
Time:	02:22:21	**Log-Likelihood:**	-16.453
No. Observations:	63	**AIC:**	54.91
Df Residuals:	52	**BIC:**	78.48
Df Model:	10		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	1.7830	1.171	1.522	0.128	-0.513	4.079
frac_shortcut_10	-0.4331	1.014	-0.427	0.669	-2.420	1.553
diameter_10	0.0459	0.124	0.371	0.710	-0.196	0.288
avg_path_length_10	-0.5921	0.556	-1.064	0.287	-1.683	0.498
transitivity_10	-0.5878	2.423	-0.243	0.808	-5.336	4.161
count_10	-0.0053	0.019	-0.280	0.779	-0.042	0.032
Complex	8.077e-05	0.001	0.153	0.879	-0.001	0.001
Familiar	1.474e-05	8.31e-05	0.177	0.859	-0.000	0.000
Stakes	-4.778e-05	0.000	-0.137	0.891	-0.001	0.001
Emotionally_Charged	-7.246e-06	9.05e-05	-0.080	0.936	-0.000	0.000
Fraction Odd	-0.3502	0.224	-1.565	0.117	-0.789	0.088

Omnibus:	18.676	**Durbin-Watson:**	2.069
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	22.785
Skew:	1.366	**Prob(JB):**	1.13e-05
Kurtosis:	4.102	**Cond. No.**	1.71e+05

Table 2.85

Chapter 3

Curmelo

3.1 Introduction

This research has already been published in PLOS ONE here: (Sankaran et al., 2021).

In this paper, we detail the theory and practice of CurmElo, a forced-choice based approach to producing a preference ranking of an arbitrary set of objects. CurmElo was originally designed for the purpose of producing sets of approximately preference-indifferent identifiers, which we define as identifiers that are relatively equally preferred across a population of subjects. In our original use case, those identifiers were sets of nonsense words of four and five letters.

This work has three motivations. The first motivation is that when eliciting preference, forced-choice based questions are preferable to Likert-style scales in a number of circumstances. The second motivation is that confounding preference for identifiers of various kinds rears its head in numerous unexpected places in social science research, and that it is essential to use some explicit form of preference elicitation, ideally using the population targeted by the research as raters, to control for these effects. The third is that preference heterogeneity induced polarization in preferences among raters and also intransitivity in preference rankings can render naive attempts to control for identifier preference inadequate, and that some method for dealing with these issues is necessary before the preference rankings can be used.

In the section below, we outline the three topics and detail our initial motivating use case for CurmElo, the production of approximately preference-indifferent four-letter and five-letter nonsense identifiers. In the rest of the paper, we use this motivating use case to demonstrate how CurmElo incorporates these insights into a comprehensive method for preference elicitation.

3.1.1 Motivations

Our motivating use case: four and five letter nonsense identifiers

In early pilot versions of the BFT experiment in Chapter 2, the vignette was only Generals attacking a fort. We used sequences from the player's Amazon Mechanical Turk HIT ID as their player identifiers and the numbers 1-12 as our objects of consensus. In this version of the vignette the players were told that these numbers represented the time of day when the attack would be conducted instead of the location of a fort out of many. We found strong preferences for the numbers 9, 1, and 5, which roughly correspond to a morning evening and lunchtime attack. It has been found that when people are asked to pick a "random" number from an interval, there is clumping around specific parts of interval, so this result is consistent with what is already known Heywood (1972); Kubovy and Psotka (1976). It is also possible that the players were bringing in their prior ideas about when a good time would be to conduct a military operation, so it was possible that these choices were not truly arbitrary. It was around this time that we realized that if the number of players exceeded the number of identifiers, whichever one that was seeded to more than one player via the pigeonhole principle would be artificially more likely to succeed, also breaking the parity between choices. Thus, it was necessary for us to choose an equal number of arbitrary identifiers to the number of players. We next tried using sequences from the AMT HIT IDs as objects of consensus. These identifiers were convenient to use because they were unique across players and reasonably unlikely to reoccur from run to run. One problem was because they were a random string of numbers and letters like, 341AXM, they could be difficult to pronounce or remember versus a word like '30EJON' (not a real example). Reading through the messages that the players were sending, we observed players using similarity with real words as a mnemonic to help remember the words as well as a justification for voting for it. Thus, even though the identifiers were arbitrary we observed that the players had heterogeneous preferences over them which is problematic for studying consensus in this context. To ensure the least level of bias in the experiment we required identifiers that were easy for the players to understand and that they were preference indifferent over.

To produce our preference-indifferent identifiers, we first generated two very large sets of four and five-letter identifiers using formats based on the general rules of English phonology to ensure that they were reasonably pronounceable and memorable. For each set, we removed any identifier that had (as of January 2018) been used previously across the large and representative English-language Google Ngrams corpus Michel et al. (2011). Then, we used a version of the Elo rating system, initially formulated by Arpad Elo to rank chess players Elo (1978), to derive ratings for each identifier from individual pairwise comparisons to form a population-level ranking. After this, we applied a novel technique based on monotonicity breaks to remove identifiers that might be polarizing but achieve middling values in Elo ratings. Finally, we extracted a set of similarly-rated identifiers from the middle of the ranking distribution. These are our preference-indifferent identifiers.

Our approach allows us both to make claims about which identifiers are equally preferred by raters and also to make claims about which identifiers, overall, are more or less preferred by the raters. For instance, the following pairs of identifiers are equally preferred: (1) camaz and bumak; (2) lujaf and piqez; and (3) cixuq and quhuq. But the first pair is much liked by the raters, the third pair much disliked, and the middle pair has middling ratings (the subjects are neutral about neutral choices, as it were). Finally, we also provide (in the appendix) a list of 1,000 4-letter and 5-letter identifiers and their ratings, which might be useful to others, from social scientists to fiction writers, facing similar objectives.

Why forced choice?

In many applications, preferences are elicited from raters using Likert-type questions or scales Likert (1932). Whether numerical (e.g., "rate your preference from 1-5") or descriptive (e.g., 'strongly disagree'-'strongly agree'), the variance inherent in individual perception of points in the scale and ordering effects with regard to samples presented to each subject make it challenging to extract an accurate group-level preference ranking from the data in aggregate. Normalization procedures used to compensate for these issues run the risk of imposing arbitrary ordering based on the specifics of the algorithm used – for example, it has been found that the effect of question wording (positive vs. negative wording) does not generalize across different scales Kam (2018). As a result, there is some controversy around the use of such scales, especially single Likert questions as opposed to comparisons across multiple questions, to measure preference and sentiment Jamieson et al. (2004); Gliem and Gliem (2003); Carifio and Perla (2008). Additionally, within the Likert scale literature, there are significant inconsistencies about what the optimal size of the scale is. Some empirical results suggest that, consistent with the predictions of information theory, scales with greater numbers of points (1-7 vs 1-11 for example) are better Alwin (1997), while other empirical results suggest precisely the opposite, that scales with more points tended to be less reliable Revilla et al. (2014); Alwin and Krosnick (1991). Moreover, the optimal parity – even or odd – of the scale used is also contentious: while a sizeable number of deployments of Likert-type surveys appear to use odd-parity scales, research into these instruments suggests that survey participants will often times use a middle option, only available on an odd-parity scale, to express that they don't know or don't have an opinion about the question instead of an actual opinion corresponding with the middle value, even when an "I don't know" option is available, in many cases materially changing the final results Sturgis et al. (2014).

A diverse body of research Ray (1980); Jackson et al. (2000); Bartlett et al. (1960); Bartram (2007) including publicly described but unpublished work within the technology industry Roettgers (2017) suggests that a series of forced-choice binary questions – yes or no, this one or that one – extracts more accurate information about preferences than Likert scales when samples of questions and raters are suitably large.

In our particular use case, a Likert-based preference elicitation method would likely be even more unreliable due to the unfamiliarity of the raters with the identifiers they are being asked to compare – unlike familiar objects like actual English words or, say, human faces, they may have no solid internal baseline for preference for these nonsense words, whereas comparing two identifiers requires no such preexisting knowledge or baseline. CurmElo presents forced-choice questions to raters to avoid these issues.

Why preference-rank identifiers?

CurmElo was originally designed for the purpose of producing sets of approximately preference-indifferent identifiers, which we define as identifiers that are relatively equally preferred across a population of subjects. In our original use case, those identifiers were sets of nonsense words of four and five letters. While at first glance it may seem reasonable to expect that preference across a set of nonsense words generated randomly will not differ significantly, it is well established that people have innate preferences for particular numbers, letters, and strings of numbers and letters – examples of this include the name-letter effect, where people prefer letters in their own name over others Nuttin Jr (1985), and the people's documented preference for the number seven over other single-digit numbers Heywood (1972); Kubovy and Psotka (1976). Research from cognitive science suggests that the map between the form of a word and its meaning is not entirely arbitrary Dingemanse et al. (2015), and that human raters impute category information to nonsense words in systematically different ways Lupyan and Casasanto (2015). The existence of these preferences is also illustrated by work on the passwords people choose for online services Riddle et al. (1989); Bonneau (2012).

It seems likely that this sort of identifier preference extends not just to nonsense words, but potentially to any class of object that might be used as an identifier: images, sounds, physical objects, colours, etc. There is work in psychology that suggests that novel and nonsense stimuli of many kinds can prime people just as much as sensical and familiar stimuli Duckworth et al. (2002). This has serious implications for the use of identifiers in experimental social science.

Here is non-exhaustive set of examples of experimental social science work where we believe that identifier preference may be a confounder: work employing the Minimal Group Paradigm Tajfel (1970); Diehl (1990); Billig and Tajfel (1973), and more generally any work where groups need identifiers; work involving inter-subject interaction where subjects have identifiers; work involving goals or target that need identifiers (our motivating use case fits in this and the previous category); work involving participants reading or listening to narratives where identifiers are used for specific characters. In work of these kinds, we believe that identifier preference, if left unaccounted for, might significantly skew results by heterogeneously changing effect size on a per-identifier basis, as well as make replication difficult due to cross-population preference heterogeneity. We suspect that identifier preference may be unacknowledged confounder for a large number of experiments in these areas.

As such, we believe that preference needs to be explicitly dealt with in some fashion in any social science work where preference for identifiers can be a confounder. This may take several forms. In certain experiments, such as in our motivating use case, one might control for identifier preference by using approximately preference-indifferent identifiers. In other settings, it might be useful to produce identifiers that are quantifiably different, up to a specified tolerance, on some dimension of preference for the raters, for example to measure interaction effects between identifier preference and some other variable.

Preference-conscious identifier generation may also be of value in other empirical or applied circumstances where the objective is to name people, objects or places in such way as to accord them neutral or specific preference of some kind, such as in game design, fiction writing, and bias training.

In this paper, we detail the motivation for the development of CurmElo for our specific use case, that is, issues with identifier preference we observed in our network science experiments, as well demonstrate that randomness in identifier generation and selection do not sufficiently mitigate these effects. We then propose a workable solution.

Why consider preference heterogeneity induced polarization and intransitivity in preference rankings?

CurmElo uses a version of the Elo algorithm to convert a set of forced-choice binary comparisons within a set of objects into ratings for each of those objects to form a totally-ordered ranking of the set. Consider the case where we want to find preference-indifferent objects of some kind. If we were to interpret these rankings naively, we would extract a subset of objects from the middle of the ranking distribution that are sufficiently similar in rating and call those objects preference-indifferent. It may be the case, however, that some of these objects are not so much preference-indifferent as 'polarizing', that is, strongly preferred by one subset of the population and strongly dispreferred by another. This sort of heterogeneity in preference may be the result of some hard-to-detect form of population heterogeneity, and could be a significant confounder if the objects are being used as identifiers in experiments, for example.

CurmElo uses a novel technique based on counting breaks in win-percentage monotonicity in Elo rankings to detect latent heterogeneity and identify polarizing objects. Crucially, this method is distinct from other formulations of the latent population heterogeneity problem since we need to measure no identifying characteristics of the populations other than their choices Pearl (2017), and as such this could be a valuable method of measuring population-level heterogeneity via preference.

Transitivity is the property that given, some objects a, b, and c, where a is preferred to and ranked above b, and b is ranked above and preferred to c, then a will be preferred to c. To see why breaks in transitivity matter, consider a case where we want to run an experiment to investigate the interaction between identifier preference and some other variable or test condition. Now imagine that our set of identifiers is objects a, b, and c, except that now there is a transitivity break manifesting as a preference cycle such that c is preferred to a. This would completely disrupt any attempt to use preference as an independent variable in the experiment since the ranking is no longer coherent – one cannot say, for example, that a is always most preferred since in this case this is dependent on what it is being compared to – and thus analysis of data collected using these identifiers can produce problematic results. CurmElo uses a technique based on counting breaks in preference transitivity in Elo rankings to identify sets of objects that break transitivity.

3.2 Materials and methods

This research was approved by the Yale Human Research Protection Program Institutional Review Boards IRB Protocol ID: 2000023887 This research contains no consent form as we did not collect personally identifying information.

Preference Data Collection

We first formulated two sets of candidate identifiers using phonological formats that mimic name identifiers the English language Chomsky and Halle (1968).

For one set of identifiers, we generated all 4-character identifiers of the type:

vowel-consonant-vowel-consonant (VCVC).

For example: ayiz, erik.

There were 11025 of these four-letter identifiers in total.

For the second set of identifiers, we generated all 5-character identifiers of the type:

consonant-vowel-consonant-vowel-consonant (CVCVC).

For example: yezak, roman.

There were 231525 of these five-letter identifiers in total.

We then implemented the following procedure: (1) we removed any identifier that occurred once or more (as of January 2018) in the Google Ngram corpus of published work in English Michel et al. (2011) (this left 118061 of the five-letter and 5969 of the four-letter identifiers), then (2) we randomly selected 1000 of the remaining identifiers from each of the sets.

These sets of 1000 identifiers were then randomly matched up against each other (within and not across each set), in pairs on our custom-built CurmElo software platform. For each of the two sets, we used 400 unique US-based raters on Amazon Mechanical Turk to perform head-to-head preference comparisons of pairs of identifiers within the set. Note that these AMT raters were recruited from the same population from which we recruit participants in the experiments in which we would subsequently use the identifiers produced using this process, so we have high internal validity for the preference rankings. Each rater was shown 50 random pairs of non-identical identifiers, one pair at a time, and asked the following each time: "Which of the two names below do you prefer? Please do not answer randomly."

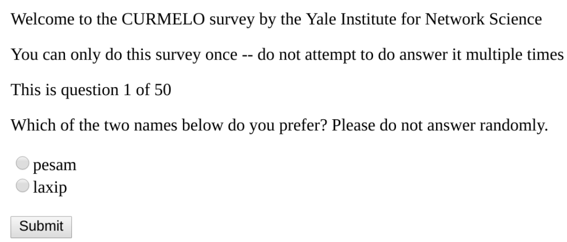


Figure 3.1: A screenshot of the CurmElo interface with two candidate CVCVC identifiers

There were 400 workers used for each set, and each worker was shown 50 pairs of identifiers. Given that there are 1000 identifiers in total, each identifier ended up with an average of 40 comparison data points. There is some variation in this number, but no identifier ended up with significantly fewer than 30 comparisons.

Querying other features and dimensions of preference using CurmElo

While in this use case, we asked raters which name identifier they preferred in general, one could use CurmElo to query any other specific feature or dimension of preference. For example, one might as "Which name sounds better?" or "Which name makes you happier?" or even "Which name seems reddest?". If your objects are pictures of faces, one might ask "Which face appears angriest?" or "Which face is sharpest?". The rankings created from the data thus collected would then correspond to the ranking of the objects relative to that specific feature (redness, anger, sharpness) or dimension of preference (sounds, happiness-of-feeling).

Using non-textual objects

CurmElo can be deployed for any set of objects that can practicably be exposed to raters. In an online-only setting such as Amazon Mechanical Turk, anything displayable on a webpage, including but not limited to audio, images, video, and interactive animations may be used. In a lab setting, physical objects may be used given that they can be uniquely identified and randomized systematically.

3.2.1 Theory

The Elo Algorithm

The Elo algorithm produces a relative rating across a set of objects. The algorithm is initialized by setting all objects to a some common initial rating, R_0. Then, objects are matched against each other, with some external input determining an outcome where one objects ‘wins’ and the other ‘loses’. In CurmElo, a match is simply a comparison of two objects by a human participant (the external input) being asked to choose a winner and loser among them. Different applications may use different matching systems; for example, if Elo ratings are used for some sort of competitive activity, it may make sense to match objects – in this case players – with similar ratings. In our setting, we use random matching, as it allows the Elo ratings to quickly converge to their stationary distributionJabin and Junca (2015). Consider objects a and b along with their corresponding Elo ratings $\mathrm{R_a}$ and $\mathrm{R_b}$. If a and b are matched, and object a wins, the ratings are updated as followsElo (1978); Aldous (2017):

$$R'_a = R_a + k\frac{1}{1 + e^{\frac{R_a - R_b}{R_D}}}$$

$$R'_b = R_b - k\frac{1}{1 + e^{\frac{R_a - R_b}{R_D}}}$$

If a and b are matched, and object b wins, the ratings are updated as followsElo (1978); Aldous (2017):

$$R'_a = R_a - k\frac{1}{1 + e^{\frac{R_b - R_a}{R_D}}}$$

$$R'_b = R_b + k\frac{1}{1 + e^{\frac{R_b - R_a}{R_D}}}$$

In this setting, k and $\mathrm{R_D}$ are free parameters, used to tune how sensitive that rating is to the results of new matches. It is possible to use a broader class of update functions other than $k\frac{1}{1+e^{\frac{R_a - R_b}{R_D}}}$ as long as it satisfies the conditions for a strong utility distribution, which will be discussed in the next sectionBlock et al. (1959); Aldous (2017). We use the logistic update function because it is commonly used for Elo applications.

This process continues until all matches – in our case, comparisons – are complete, and we refer to the Elo ratings after all matches have occurred to be the “final Elo rating.” In contrast to applications in sports or gaming, where the number of matches is exogenously built in to the structure of a tournament, in social scientific applications the number of matches can be chosen by the researcher depending on how big a sample of comparisons is needed. Jabin and Junca show that in settings with a large number of objects and intrinsic win probabilities that are not time dependent (such as our motivating example), the distribution of Elo ratings converges to a stationary distribution that represents the underlying preferenceJabin and Junca (2015).

Stochastic Preferences

Preference is a primitive that underlies many important social phenomena. In this sections, we discuss the basic formalism of deterministic and stochastic preferences.

A preference $\succeq$ must be complete and transitive in order to admit a utility representation . Let A be the finite set of objects. Completeness requires that $\forall a, b \in A$ either $a \succeq b$ or $b \succeq a$. Transitivity requires that $\forall a, b, c \in A$ if $a \succeq b$ and $b \succeq c$ then $a \succeq c$ Mas-Colell et al. (1995).

In many real systems, choices are stochastic and not deterministic, so the definitions of preferences and transitivity must be extended to accommodate the fact that, in a choice between a and b, where $a \succeq b$, b will still sometimes be chosen. Block and Marschak extend the notion of preferences by stipulating that when choosing between a and b, $a \succeq b$ if and only if a is chosen with probability greater than or equal to 50% Block et al. (1959).

Cattelan shows three different ways to apply the definition of transitivity to stochastic choice: Weak Stochastic Transitivity; Moderate Stochastic Transitivity; Strong Stochastic Transitivity Cattelan (2012). Let π_{ab} be the probability that a is chosen when the agent is presented with a choice between a and b. Consider $\forall a, b, c \in A$ when $\pi_{ab} \geq .5$ and $\pi_{bc} \geq .5$ if $\pi_{ac} \geq .5$; then $\succeq$ satisfies Weak Stochastic Transitivity; if $\pi_{ac} \geq min(\pi_{ab}, \pi_{bc})$, then $\succeq$ satisfies Moderate Stochastic Transitivity; or if $\pi_{ac} \geq max(\pi_{ab}, \pi_{bc})$, then $\succeq$ satisfies Strong Stochastic Transitivity Cattelan (2012). Let $\mathrm{u_a}$ and $\mathrm{u_b}$ be the utility representations for objects a and b respectively. The stochastic definition of preferences also imposes requirements on the probabilities a given object is chosen. Let $\pi_{ab} = W(u_a - u_b)$, where W is the win probability function Aldous (2017). W corresponds to the Block and Marschak strong utility distribution and has the following properties: $W : \mathbb{R} \to (0, 1)$, W is continuous, W is strictly increasing, $\lim_{u\to\infty} W(u) = 1$, and $W(-u) + W(u) = 1 \forall u \in \mathbb{R}$ Block et al. (1959); Aldous (2017).

Heterogeneous Preference, Polarization, and Transitivity Breaks

As discussed in the motivation section, the rankings produced using the Elo algorithm may be subject to the problem of 'polarizing' objects resulting from heterogeneous preference. This is the situation where, for some given object, one subset of the population has a strong preference for it and another subset has a strong dispreference for it, and this is not accounted for in the Elo rating. This would manifest in the object being chosen more or less often than its rating would suggest against certain objects, and signals some unobserved heterogeneity within the population. We call this the "polarization in ratings problem" and provide a method to detect when an object is polarized, as well as latent heterogeneity in preference more generally. This method is distinct from other formulations of the latent population heterogeneity problem since we measure no identifying characteristics of the populations other than their choices Pearl (2017).

We also provide a method to detect whether an object induces intransitivity in a preference ranking via calculating a normalized 'transitivity breaks score' of the number of transitivity breaks in the ranking the object is involved in.

Our methods work on the basis that while, in theory, the Win Probability function must be monotonically increasing and the ratings must satisfy stochastic transitivity for stochastic preferences to be well defined, in practice this is not always the case. Heterogeneity in preference can induce breaks in the monotonicity in the win rate among objects and, intuitively speaking, we 'count' the number of these breaks to estimate a normalized 'polarization score' (min of 0.0, max of 1.0) for a given object. In addition, real preferences rankings of various kinds may well be truly intransitive to some degree, and we similarly 'count' the number of transitivity breaks an object is involved in to estimate a normalized 'transitivity breaks score' (min of 0.0, max of 1.0) for it.

For applications where a well-behaved preference ranking is essential, in particular in order to rely on the predictions of much of the work referenced in the theory section, it is necessary to remove polarizing and transitivity breaking objects.

We first present a Pairwise Polarization Estimator based on monotonicity breaks.

Pairwise Polarization Estimator

The monotonicity assumption of the Elo algorithm is that, for a given object, it should have a higher win rate when compared against lower Elo objects than higher Elo objects. Thus, for a given object, we assess its win rate when compared against all other objects in the set. Next, we look at all pairs of these win rates to see if they match up to the expectations of higher Elo win rates being smaller than lower Elo win rates. We count all violations of this assumption normalized by the number of possible ways this rule could be broken. The process is formalized below.

Assume that there are N total objects for agents to choose from and they are presented in menus of size two. Thus, for each menu, the agent has a choice between two objects, i and j. Let $\mathrm{W_{ij}}$ represent the rate a which object i is chosen compared against object j. $\mathrm{W_{ijk}}$ refers to the kth sample of a Bernoulli random variable which is 1 if object i is chosen and 0 otherwise, when compared against object j. $\bar{W}_{ij}$ represents the sample estimator of $\mathrm{W_{ij}}$. Let $\mathrm{R_i}$ represent the final Elo rating of object i. N is the total number of objects. $\mathcal{N}$ represents the normalization factor and represents the total number of possible breaks in monotonicity implied by the Elo rating. P is the estimator for pairwise polarization.

$$\bar{W}_{ij} = \frac{1}{n}\sum_{k=0}^{n} W_{ijk}$$

$$\mathcal{N} = \sum_{l=0}^{N-1}(l-1)$$

$$P_i = \frac{1}{\mathcal{N}} \sum_{j \neq i} \sum_{\substack{k \\ R_k < R_j}} \mathbb{1}_{(\bar{W}_{ij} - \bar{W}_{ik} > 0)}$$

Use of Quantiles

In our use case, and in many practical applications where there are a reasonably large number of objects, it would be prohibitively expensive to get enough data points comparing any specific pair to use pairwise estimators with any degree of reliability. Instead, we rely on dividing the objects in the ranking distribution into quantiles, and perform comparisons between a single object and quantiles to estimate the polarization score of the object.

Quantile Polarization Estimator

Let $\{ \mathrm{Q_1,...,Q_q}\}$ be the q-quantiles of the final Elo distribution R. By convention, quantiles with higher integer values contain lower rated objects. So in a setting with 5 Quantiles, $\mathrm{Q_5}$ refers to the bottom 5th of the Elo distribution and $\mathrm{Q_1}$ refers to the top 5th of the Elo distribution. Let $\mathrm{W_{iQ_j}}$ represent the rate at which object i is chosen compared against objects in $\mathrm{Q_j}$. $\mathrm{W_{iQ_jk}}$ refers to the kth sample of a Bernoulli random variable which is 1 if object i is chosen and 0 otherwise, when compared against an object in $\mathrm{Q_j}$. By convention, we assume that there were a total of n comparisons of object i against objects in $\mathrm{Q_j}$.

$$\bar{W}_{iQ_j} = \frac{1}{n} \sum_{k=0}^{n} W_{iQ_jk}$$

$$\mathcal{N} = \sum_{l=0}^{q} (l-1)$$

$$P_i = \frac{1}{\mathcal{N}} \sum_{j \leq q} \sum_{\substack{k \\ j < k < q}} \mathbb{1}_{(\bar{W}_{iQ_j} - \bar{W}_{iQ_k} > 0)}$$

Quantile Transitivity Breaks Estimator

To count transitivity breaks, for all pairs of quantiles we count the number of times the object is stochastically preferred to a given quantile, while simultaneously not stochastically preferred to a lower quantile than the given quantile. We normalize this count by the number of ways this is possible to produce a 'transitivity breaks score' (min of 0.0, max of 1.0). We formalize this process below.

Let $W_{Q_iQ_j}$ represent the rate a which objects in Q_i is chosen compared against objects in Q_j. $W_{Q_iQ_jk}$ refers to the kth sample of a Bernoulli random variable which is 1 if the object in Q_i is chosen and 0 otherwise when compared against an object in Q_j. We call $W_{Q_iQ_j}$ the Inter Quantile Win Rate. If the Elo rating is behaving as expected, one would expect that $W_{Q_iQ_j} > .5$ if $i < j$. This would imply that quantiles with higher rated words tend to be preferred to quantiles with lower rated words. We assume that $W_{Q_iQ_j} > .5$ if $i < j$ as otherwise that implies that the ratings do not represent the preference. We use the definition of weak stochastic transitivity for this estimator.

$$\bar{W}_{Q_iQ_j} = \frac{1}{n}\sum_{k=0}^{n} W_{Q_iQ_jk}$$

$$\mathcal{N} = \sum_{l=0}^{q}(l-1)$$

$$T_i = \frac{1}{\mathcal{N}}\sum_{j\le q}\sum_{\substack{k\\ j<k<q}} \mathbb{1}_{[(\bar{W}_{iQ_j}<.5)\bigwedge(\bar{W}_{iQ_k}\ge .5)]}$$

3.3 Results

We analyzed the data using the following parameters: k= 20, $R_0 = 1000$, $R_D = 400$. Table 3.1 shows the summary statistics for the Elo ratings. The mean Elo rating for both identifiers are both close to R_0. Additionally, this table shows that there is significant variation in the final Elo ratings of both identifiers. Thus, the fact that the identifiers are arbitrary and nonsensical by construction, then subsequently randomly sampled to produce sets of 1000, does not imply that the identifiers are equally preferred. Table 3.2 shows summary statistics for the Polarization of each identifier. The summary statistics for polarization are quite similar for both the 4-Letter and 5-Letter identifiers. Table 3.3 shows the summary statistics for Transitivity breaks for the identifiers. It appears that there are about twice as many breaks in win rate monotonicity as there are in transitivity.

Figures 3.2 and 3.3 show histograms of the number of identifiers for each Polarization Score for the 4 and 5 character identifiers respectively. Figures 3.4 and 3.5 show histograms of the number of identifiers for each Transitivity Breaks Score for the 4 and 5 character identifiers respectively.

For our final sets of approximately preference-indifferent identifiers of 4 and 5 characters, we looked for identifiers with Elo values between the range of about 990-1010 and filtered out all identifiers with polarization values greater than 0.2. We

	Mean	Standard Deviation	Min	Max
4-Letter Identifiers	1010.183524	89.31063206	777.725104	1396.567372
5-Letter Identifiers	1022.774282	118.8151442	767.036162	1614.585634

Table 3.1: Summary Statistics for Elo Ratings

	Mean	Standard Deviation	Min	Max
4-Letter Identifiers	0.2756	0.164047406	0	0.8
5-Letter Identifiers	0.2453	0.155634611	0	1

Table 3.2: Summary Statistics for Polarization

	Mean	Standard Deviation	Min	Max
4-Letter Identifiers	0.1016	0.127254209	0	0.6
5-Letter Identifiers	0.0658	0.104696329	0	0.5

Table 3.3: Summary Statistics for Transitivity Breaks

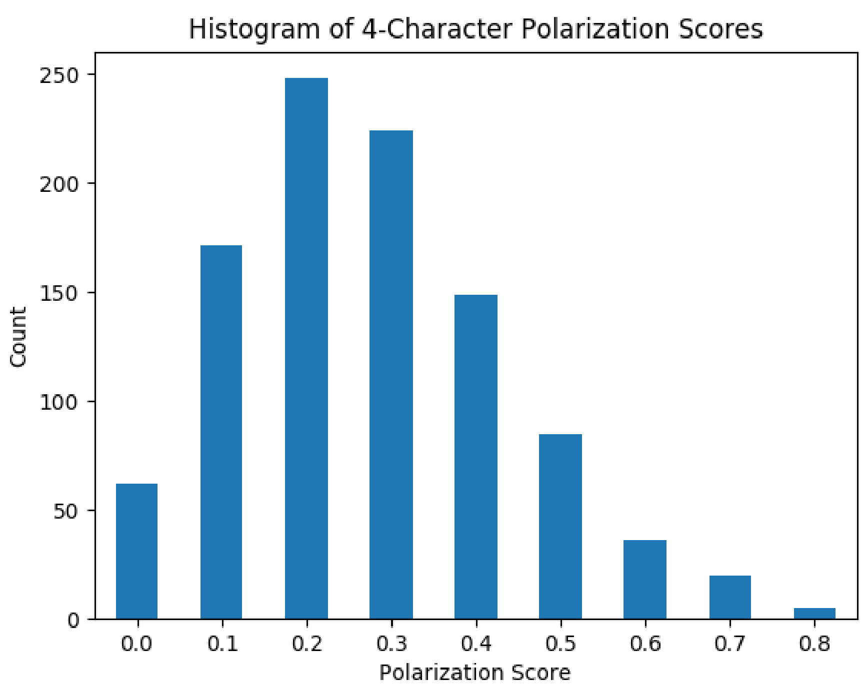


Figure 3.2: Histogram of 4-character Polarization Scores

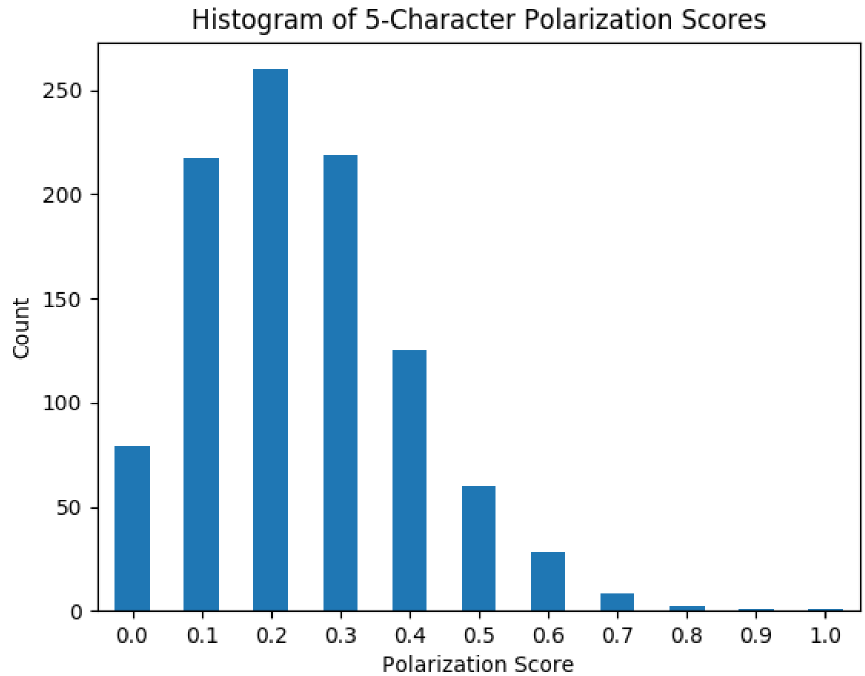


Figure 3.3: Histogram of 5-character Polarization Scores

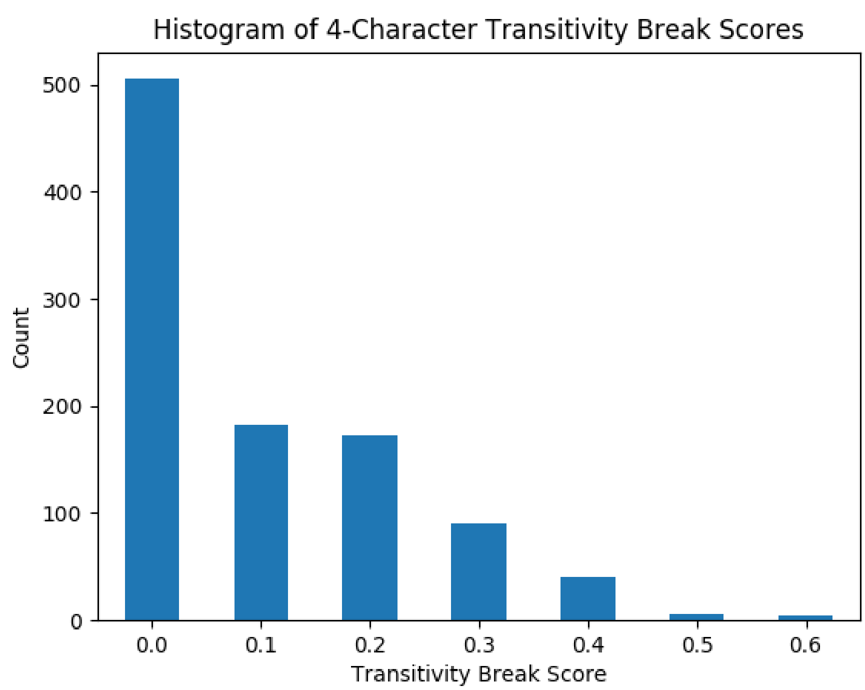


Figure 3.4: Histogram of 4-character Transitivity Breaks Scores

chose this band because the Elo algorithm was initialized at a value of 1000, so these identifiers are very close to the center of the Elo distribution.

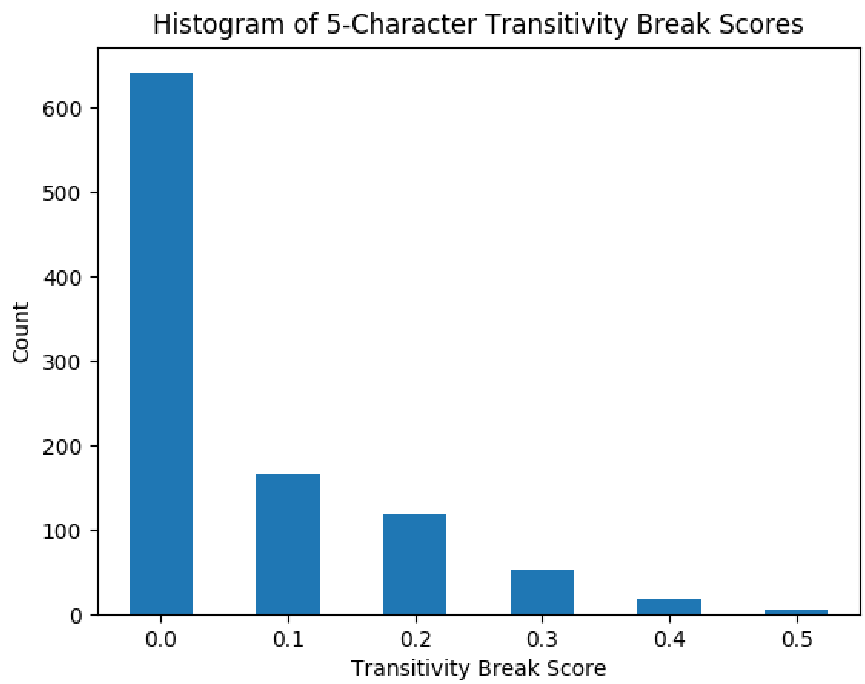


Figure 3.5: Histogram of 5-character Transitivity Breaks Scores

3.4 Discussion

3.4.1 Inter-Quantile Win Rates

One of the assumptions of the Quantile Polarization Estimator is that the Average Win rates for the quantiles against each other satisfies monotonicity. If there are monotonicity breaks at the quantile level, this indicates departure from the stationary distribution. That could indicate either that the one is using too many quantiles, so there are an insufficient number of samples per quantile, or that the overall number of samples is too low. The Inter Quantile Win Rate Matrix is calculating the average win rate of objects in the quantile represented by the rows against objects in the quantile represented by the columns. For our own application, we used quintiles, so the Inter Quantile Win Rate Matrix is a 5×5. Table 3.4 shows the Inter Quantile Win Matrix for the Target 5-character identifiers and Table 5 shows the Inter Quantile Win Matrix for the 4-character identifiers . These have the properties as expected: values close to .5 along the diagonal and monotonicity in win rates.

3.4.2 Analysis of Polarization and Transitivity Breaks

For the 5-character identifiers we found that 92.1% of them had a nonzero polarization score and for the 4-character identifiers 93.7% had a nonzero polarization score. This suggests that some level of polarization is not uncommon in this kind of preference data. This serves to underscore the importance of testing for polarization in preference data. Breaks in Transitivity were less common with 36% of the 5-character identifiers having a nonzero number of Transitivity breaks and 49.5% of the 4-character identifiers having a nonzero number of Transitivity breaks. This suggests that even in preferences over nonsense words, intransitivity in preference must be accounted for.

We tested the distribution of polarization against the distribution of ratings. If there are objects that are very likely to win against highly rated objects and lose against low rated objects, we would expect their final rating to be in the middle of the distribution. If this is the case, we would expect to find a statistically significant and negative coefficient in a regression where centered Elo ratings are the explanatory variable. Alternatively, if polarization is higher at the tails of the Elo distribution, we would expect the coefficient in the quadratic model to be positive. If Elo rating is not predictive of polarization, we would expect either non-statistically-significant or precisely identified zeros in both linear and quadratic models.

Table 3.6 shows the results of the linear regression model for the 5-Letter Identifiers. The coefficient on centered Elo ratings is small and not statistically significant. Table 3.7 shows the results of the quadratic model for the 5-letter Target Identifiers. The coefficient on centered Elo ratings squared is small, positive and not statistically significant. Based on these results, there is no clear relationship between the Elo ratings and the polarization scores. Table 3.8 shows the results of the linear model for the 4-letter Subject Identifiers. The coefficient on the centered Elo ratings is small and not statistically significant. Table 3.9 shows the results of the quadratic model for the Subject Identifiers. The coefficient on the centered Elo rating squared is negative, small and not statistically significant. These results are also consistent with the hypothesis that whatever is causing the polarization is uniformly distributed across rating. Based on our setting, it is likely that this is due to unobserved heterogeneity in the population of raters used here. This finding may be relevant to other work using populations of US-based Amazon Mechanical Turk workers, especially work involving preference.

	5	4	3	2	1
5	0.495723	0.368918	0.300589	0.231730	0.190450
4	0.623822	0.499651	0.418548	0.350965	0.266467
3	0.697948	0.583093	0.503599	0.428252	0.309600
2	0.762648	0.654072	0.551350	0.494170	0.386135
1	0.814080	0.744856	0.679105	0.619651	0.498692

Table 3.4: Inter Quantile Win Rates for 5-Letter Identifiers

	5	4	3	2	1
5	0.499399	0.401221	0.323861	0.282018	0.227306
4	0.599208	0.503693	0.435614	0.377869	0.292094
3	0.674439	0.549963	0.506442	0.428634	0.328867
2	0.720698	0.627391	0.555608	0.509095	0.388713
1	0.768008	0.706379	0.662835	0.610826	0.501591

Table 3.5: Inter Quantile Win Rates for 4-Letter Identifiers

Dep. Variable:	rank_breaks	**R-squared:**	0.000
Model:	OLS	**Adj. R-squared:**	-0.001
Method:	Least Squares	**F-statistic:**	0.002043
Date:	Fri, 11 May 2018	**Prob (F-statistic):**	0.964
Time:	16:51:59	**Log-Likelihood:**	-1860.8
No. Observations:	1000	**AIC:**	3726.
Df Residuals:	998	**BIC:**	3735.
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	2.4526	0.051	48.497	0.000	2.353	2.552
centered	1.826e-05	0.000	0.045	0.964	-0.001	0.001

Omnibus:	84.487	**Durbin-Watson:**	2.036
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	108.155
Skew:	0.714	**Prob(JB):**	3.27e-24
Kurtosis:	3.745	**Cond. No.**	123.

Table 3.6: 5-Letter Identifiers Linear Regression

Dep. Variable:	rank_breaks	**R-squared:**	0.000
Model:	OLS	**Adj. R-squared:**	-0.001
Method:	Least Squares	**F-statistic:**	0.1152
Date:	Fri, 11 May 2018	**Prob (F-statistic):**	0.734
Time:	16:51:41	**Log-Likelihood:**	-1860.7
No. Observations:	1000	**AIC:**	3725.
Df Residuals:	998	**BIC:**	3735.
Df Model:	1		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
Intercept	2.4444	0.056	43.648	0.000	2.335	2.554
np.power(centered, 2)	5.908e-07	1.74e-06	0.339	0.734	-2.82e-06	4e-06

Omnibus:	84.774	**Durbin-Watson:**	2.036
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	108.655
Skew:	0.715	**Prob(JB):**	2.55e-24
Kurtosis:	3.749	**Cond. No.**	3.59e+04

Table 3.7: 5-Letter Identifiers Quadratic Regression

Dep. Variable:	rank_breaks	**R-squared:**	0.000
Model:	OLS	**Adj. R-squared:**	-0.001
Method:	Least Squares	**F-statistic:**	0.02374
Date:	Fri, 11 May 2018	**Prob (F-statistic):**	0.878
Time:	16:52:06	**Log-Likelihood:**	-1913.4
No. Observations:	1000	**AIC:**	3831.
Df Residuals:	998	**BIC:**	3841.
Df Model:	1		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
Intercept	2.7570	0.052	53.254	0.000	2.656	2.858
centered	-9.626e-05	0.001	-0.154	0.878	-0.001	0.001

Omnibus:	44.566	**Durbin-Watson:**	1.921
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	49.735
Skew:	0.546	**Prob(JB):**	1.59e-11
Kurtosis:	3.053	**Cond. No.**	90.4

Table 3.8: 4-Letter Identifiers Linear Regression

Dep. Variable:	rank_breaks	**R-squared:**	0.006
Model:	OLS	**Adj. R-squared:**	0.005
Method:	Least Squares	**F-statistic:**	4.120
Date:	Fri, 11 May 2018	**Prob (F-statistic):**	0.0426
Time:	16:51:49	**Log-Likelihood:**	-1910.5
No. Observations:	1000	**AIC:**	3825.
Df Residuals:	998	**BIC:**	3835.
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
Intercept	2.6811	0.062	43.107	0.000	2.559	2.803
np.power(centered, 2)	9.28e-06	4.57e-06	2.030	0.042	3.19e-07	1.82e-05

Omnibus:	42.909	**Durbin-Watson:**	1.932
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	47.692
Skew:	0.535	**Prob(JB):**	4.40e-11
Kurtosis:	3.040	**Cond. No.**	1.84e+04

Table 3.9: 4-Letter Identifiers Quadratic Regression

3.5 Phonological Preference and Polarization: A Further Illustrative Application

Preferences over the identifiers in our corpus could be due to phonological aspects of the identifiers. For example, raters may prefer identifiers that more like a well formed English word than not. Given these types preferences would operate at the linguistic level, one would not expect them the contribute to polarization given that all raters are expected to agree on the phonological conventions of English. Within phonology, other experiments have been conducted using human raters evaluating nonsense words, and have found that how the word is constructed influences how acceptable raters find the word Ohala and Ohala (1986); Coleman and Pierrehumbert (1997); Frisch et al. (2000); Bailey and Hahn (2001); Hammond (2004); Albright (2008). Importantly these studies were assessing how much like a real word the raters thought the nonsense words were and presented the words aurally (Bailey and Hahn also ran an experiment with only visual stimulus) Ohala and Ohala (1986); Coleman and Pierrehumbert (1997); Frisch et al. (2000); Bailey and Hahn (2001); Albright (2008). These results may not necessarily map onto preferences in affinity over nonsense words. For example, one may recognize that "moist" is a proper English word but that does not necessary imply that they like it.

Given our particular application, and just for completeness, we tested the impact of the five following phonological constructions on both Elo Rating and Polarization: The first consonant in the word is a nasal (Initial Nasal); the last consonant the word is a voiced obstruent (Terminal Voiced Obstruent); the last consonant the word is voiceless (Terminal Voiceless); the last consonant the word is a fricative (Terminal Fictive); and the last consonant the word is a stop (Terminal Stop). We only use single letter vowels and consonants in our data set, so, for our purposes, the nasals are: ('m','n'), the fricatives are: ('f','s','v', 'z'), the stops are: ('p','t','k','b','d','g'), the voiced obstruents are: ('b','d','g','v','z'), and the voiceless consonants are: ('p','t','k','f','s','h','c','x') Hammond (1999).
Phonological Cue Theory predicts that word terminal fricatives should be preferred, word terminal stops should be dispreferred, nasals early in the word should be preferred, and voiced obstruent in the word terminal position should dispreferred Wright (2004). Table 3.10 summarizes the results for our regressions of the phonological constructions on Elo, and the individual models are detailed in S1 Appendix: Elo Regression Models . Table 3.11 summarizes the results for our regressions of the phonological constructions on Polarization, and the individual models are detailed in S2 Appendix: Polarization Regression Models. The construction Initial Nasal has the most robust effect on Elo, with a statistically significant and positive coefficient in all models. This is consistent with what Phonological Cue Theory predicts. For the rest of the constructions, the results were mixed and *not* entirely consistent with Phonological Cue Theory. With respect to polarization, we find that only the constructions

Initial Nasal and Terminal Fricative have a statistically significant relationship. The construction Initial Nasal was found to reduce polarization, which is in line with our predictions, but Terminal Fricative was found to increase polarization, which was surprising. The coefficients for Terminal Fricative in the Elo regression were consistent with the predictions of Phonological Cue Theory, so we expected the presence of this construction to reduce polarization. This suggests that the polarization process is more complex than we expected, and that work in phonology may be unknowingly affected by polarization problems.

It is important not to over-interpret our results given that this was not initially designed as a phonology experiment. For example, we planned to test whether sibilant fricatives in the word initial position impacted the elo ratings, but it turns out none of the potential words in both the 4-letter and 5-letter survived our Google Ngram filter. Since the Ngram filter involves a comparison to real English words, it is possible that the corpora suffer from significant selection bias. In addition, phonological experiments are typically conducted with aural stimuli, and here we have raters visually reading the words. Nonetheless, we still see that our design and the CurmElo system can be of use to experimental phonologists. Of the experiments we surveyed, only Ohala and Ohala use forced choice paired comparison for ratings Ohala and Ohala (1986), additionally Frisch, Large and Pisoni had a trial that used a binary rating for words Frisch et al. (2000); and the rest of the studies use Likert Scales Coleman and Pierrehumbert (1997); Bailey and Hahn (2001); Albright (2008). We believe that, in this setting, forced choice will perform better than Likert scales for rating applications. It is also worth noting that our number of raters is much larger than those of the experiments we surveyed: Ohala and Ohala had 16 raters in one experiment and 21 raters in a second experiments Ohala and Ohala (1986); Coleman and Pierrehumbert had 6 raters Coleman and Pierrehumbert (1997); Frisch, Large and Pisoni had two experiments with 24 raters in each arm; and Bailey and Hahn had one experiment with 24 raters and a second experiment with 12 raters Bailey and Hahn (2001). While some of these results have been shown to replicate, Hammond (2004); Albright (2008) the number of raters per experiment is still quite low and there may still be reproducibility and generalizability issues that have not been uncovered. The CurmElo system can straightforwardly be adapted to accommodate audio stimuli, so we believe it would be possible to design phonology experiments using CurmElo with a large number of raters relatively easily.

		4-Letter	5-Letter
Initial Nasal			
	Statistically Significant	All Models	All Models
	Sign	+	+
	Consistent	Yes	Yes
Terminal Voiced Obstruent			
	Statistically Significant	All Models	No Models
	Sign	+	Mixed
	Consistent	No	No
Terminal Voiceless			
	Statistically Significant	Some Models	No Models
	Sign	+	+
	Consistent	Yes	Yes
Terminal Fricative			
	Statistically Significant	All Models	No Models
	Sign	+	+
	Consistent	Yes	Yes
Terminal Stop			
	Statistically Significant	All Models	No Models
	Sign	+	+
	Consistent	No	No

Table 3.10: Elo Linguistics Results Summary

		4-Letter	5-Letter
Initial Nasal			
	Statistically Significant	Yes	No
	Sign	-	-
Terminal Voiced Obstruent			
	Statistically Significant	No	No
	Sign	+	-
Terminal Voiceless			
	Statistically Significant	No	No
	Sign	+	-
Terminal Fricative			
	Statistically Significant	No	Yes
	Sign	+	+
Terminal Stop			
	Statistically Significant	No	No
	Sign	+	-

Table 3.11: Polarization Linguistics Results Summary

3.6 Sociocultural Preference

It is well known that preferences over identifiers can be socially mediated. For example heterogeneous response has been documented in audit studies attempting to evaluate ethnoracial bias based on randomly assigning names to putative applicants for jobs Bertrand and Mullainathan (2004); Booth et al. (2012). Audit studies have also shown that the name of the applicant can affect responses to rental applications Carpusor and Loges (2006); Edelman et al. (2017). These naming preferences go both ways as there is significant evidence that patterns of naming children vary based on education and race Lieberson and Bell (1992); Lieberson and Mikelson (1995); Fryer Jr and Levitt (2004). Thus, one might expect there to be heterogeneity in the preferences in the identifiers in corpora of words such as ours based on these sociocultural factors. It is not entirely straightforward to test this, but we believe a potentially informative approach would be to use CurmElo to produce a ranking of words relative to the features of 'blackness' or 'whiteness' (in the racial sense) or other axes. Such efforts might be useful in future audit studies.

3.7 Conclusion

In this paper, we introduced CurmElo, a forced-choice approach to producing a preference ranking of an arbitrary set of object that combines the Elo algorithm with a novel technique for detecting and correcting for heterogeneity and polarization in preferences among raters.

We detailed the application of CurmElo to the problem of generating approximately preference-neutral identifiers, in this case four and five letter nonsense words that are patterned on the phonological conventions of the English language. We provided evidence that human raters have significant preferences over even a randomly selected set of identifiers that were arbitrary and nonsensical by construction, indicating that some method of preference-ranking is necessary to control for preference. We also demonstrate the existence of significant polarization in identifier preference in our population of US-based Amazon Mechanical Turk raters, indicating both that this heterogeneous preference could have been a significant and tricky confounder if left unaddressed.

We further demonstrated that the preference ranking produced is only somewhat consistent with the predictions of existing work in phonological preference, in particular that polarization appears to affect phonological features of words that are predicted to increase preference by Phonological Cue Theory, suggesting that experiments in phonology based on preference would benefit from using CurmElo to detect and control for such polarization. While our CurmElo phonology experiments have much larger subject populations and numbers of data points than the phonology work we reference, our experiments were not originally designed for phonological analysis and as such suffer from selection (real words removed) and presentation (visual versus aural) issues, so they are limited.

We believe that the polarization-corrected Elo framework we detail is a theoretically strong method for generating preference rankings. In particular, we see it as superior to Likert scales for the purposes of extracting a population's preference ranking of a large number of objects. We believe that CurmElo could be deployed confidently across a wide range of settings where there may be unobserved heterogeneity in the target population, and that it is a robust method for preference elicitation generally, and identifier generation specifically, across a variety of domains.

We also believe that approximately preference-indifferent identifiers should be used in any social science work where preference for identifiers can be a confounder, for example for subject and group identifiers in work employing the Minimal Group Paradigm or Vignette Studies involving arbitrary names. We believe that identifier preference is an unacknowledged confounder for many experiments of this nature, in particular in experiments in using Amazon Mechanical Turk populations, for which we have already demonstrated significantly non-uniform identifier preference and preference polarization. CurmElo can be used to produce rankings of arbitrary features or dimensions of preference of a set of objects relative to a population of raters.

3.8 Supporting information

S1 Appendix: Elo Regression Models **Regressions for Phonological Constructions on Elo.**

Dep. Variable:	elo	**R-squared:**	0.015
Model:	OLS	**Adj. R-squared:**	0.014
Method:	Least Squares	**F-statistic:**	15.07
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.000110
Time:	19:47:17	**Log-Likelihood:**	-6188.3
No. Observations:	1000	**AIC:**	1.238e+04
Df Residuals:	998	**BIC:**	1.239e+04
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1016.3492	4.059	250.385	0.000	1008.393	1024.305
initial_nasal	40.1567	10.343	3.883	0.000	19.885	60.428

Omnibus:	127.444	**Durbin-Watson:**	0.033
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	200.368
Skew:	0.864	**Prob(JB):**	3.09e-44
Kurtosis:	4.351	**Cond. No.**	2.81

Table 3.12: 5-Letter Identifiers Elo vs Initial Nasal

Dep. Variable:	elo	**R-squared:**	0.001
Model:	OLS	**Adj. R-squared:**	0.000
Method:	Least Squares	**F-statistic:**	1.116
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.291
Time:	19:47:17	**Log-Likelihood:**	-6195.5
No. Observations:	1000	**AIC:**	1.239e+04
Df Residuals:	998	**BIC:**	1.240e+04
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1019.6074	5.003	203.814	0.000	1009.802	1029.412
terminal_voiceless	7.9770	7.552	1.056	0.291	-6.824	22.778

Omnibus:	124.464	**Durbin-Watson:**	0.003
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	189.968
Skew:	0.862	**Prob(JB):**	5.61e-42
Kurtosis:	4.261	**Cond. No.**	2.45

Table 3.13: 5-Letter Identifiers Elo vs Terminal Voiceless Consonant

Dep. Variable:	elo	**R-squared:**	0.000
Model:	OLS	**Adj. R-squared:**	-0.001
Method:	Least Squares	**F-statistic:**	0.009826
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.921
Time:	19:47:17	**Log-Likelihood:**	-6196.0
No. Observations:	1000	**AIC:**	1.240e+04
Df Residuals:	998	**BIC:**	1.241e+04
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1022.9633	4.342	235.596	0.000	1014.453	1031.474
terminal_obstruents	-0.8554	8.629	-0.099	0.921	-17.768	16.057

Omnibus:	121.775	**Durbin-Watson:**	0.001
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	183.882
Skew:	0.851	**Prob(JB):**	1.18e-40
Kurtosis:	4.230	**Cond. No.**	2.55

Table 3.14: 5-Letter Identifiers Elo vs Terminal Voiced Obstruent

Dep. Variable:	elo	**R-squared:**	0.016
Model:	OLS	**Adj. R-squared:**	0.014
Method:	Least Squares	**F-statistic:**	8.151
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.000308
Time:	19:47:17	**Log-Likelihood:**	-6187.9
No. Observations:	1000	**AIC:**	1.238e+04
Df Residuals:	997	**BIC:**	1.240e+04
Df Model:	2		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1013.6635	5.129	197.632	0.000	1003.611	1023.716
initial_nasal	39.8241	10.388	3.834	0.000	19.464	60.184
terminal_voiceless	6.8990	7.528	0.916	0.359	-7.856	21.654

Omnibus:	129.467	**Durbin-Watson:**	0.036
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	205.081
Skew:	0.872	**Prob(JB):**	2.93e-45
Kurtosis:	4.372	**Cond. No.**	3.07

Table 3.15: 5-Letter Identifiers Elo vs Initial Nasal and Terminal Voiceless Consonant

Dep. Variable:	elo	**R-squared:**	0.016
Model:	OLS	**Adj. R-squared:**	0.013
Method:	Least Squares	**F-statistic:**	5.454
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.00101
Time:	19:47:17	**Log-Likelihood:**	-6187.8
No. Observations:	1000	**AIC:**	1.238e+04
Df Residuals:	996	**BIC:**	1.240e+04
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1012.2185	6.751	149.930	0.000	998.986	1025.451
initial_nasal	39.8237	10.384	3.835	0.000	19.472	60.175
terminal_voiceless	8.3442	8.710	0.958	0.338	-8.726	25.415
terminal_obstruents	3.9430	9.837	0.401	0.689	-15.337	23.223

Omnibus:	130.985	**Durbin-Watson:**	0.036
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	208.912
Skew:	0.877	**Prob(JB):**	4.32e-46
Kurtosis:	4.392	**Cond. No.**	3.67

Table 3.16: 5-Letter Identifiers Elo vs Initial Nasal,Terminal Voiceless Consonant, and Terminal Voiced Obstruent

Dep. Variable:	elo	**R-squared:**	0.001
Model:	OLS	**Adj. R-squared:**	-0.000
Method:	Least Squares	**F-statistic:**	1.033
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.310
Time:	19:47:17	**Log-Likelihood:**	-6195.5
No. Observations:	1000	**AIC:**	1.240e+04
Df Residuals:	998	**BIC:**	1.240e+04
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1021.0435	4.198	243.217	0.000	1012.815	1029.272
terminal_fricative	9.5621	9.408	1.016	0.309	-8.878	28.002

Omnibus:	123.381	**Durbin-Watson:**	0.003
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	187.486
Skew:	0.858	**Prob(JB):**	1.94e-41
Kurtosis:	4.248	**Cond. No.**	2.70

Table 3.17: 5-Letter Identifiers Elo vs Terminal Fricative

Dep. Variable:	elo	**R-squared:**	0.001
Model:	OLS	**Adj. R-squared:**	-0.000
Method:	Least Squares	**F-statistic:**	0.5403
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.462
Time:	19:47:17	**Log-Likelihood:**	-6195.8
No. Observations:	1000	**AIC:**	1.240e+04
Df Residuals:	998	**BIC:**	1.241e+04
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1021.1161	4.498	227.036	0.000	1012.301	1029.931
terminal_stop	6.0078	8.173	0.735	0.462	-10.011	22.026

Omnibus:	123.301	**Durbin-Watson:**	0.002
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	187.515
Skew:	0.857	**Prob(JB):**	1.91e-41
Kurtosis:	4.251	**Cond. No.**	2.45

Table 3.18: 5-Letter Identifiers Elo vs Terminal Stop

Dep. Variable:	elo	**R-squared:**	0.002
Model:	OLS	**Adj. R-squared:**	0.000
Method:	Least Squares	**F-statistic:**	1.042
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.353
Time:	19:47:17	**Log-Likelihood:**	-6195.0
No. Observations:	1000	**AIC:**	1.240e+04
Df Residuals:	997	**BIC:**	1.241e+04
Df Model:	2		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1017.9530	5.301	192.025	0.000	1007.563	1028.343
terminal_fricative	12.6527	9.949	1.272	0.203	-6.848	32.153
terminal_stop	9.1710	8.641	1.061	0.289	-7.766	26.107

Omnibus:	125.877	**Durbin-Watson:**	0.005
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	193.449
Skew:	0.866	**Prob(JB):**	9.84e-43
Kurtosis:	4.281	**Cond. No.**	3.24

Table 3.19: 5-Letter Identifiers Elo vs Terminal Fricative and Terminal Stop

Dep. Variable:	elo	**R-squared:**	0.017
Model:	OLS	**Adj. R-squared:**	0.014
Method:	Least Squares	**F-statistic:**	5.872
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.000565
Time:	19:47:17	**Log-Likelihood:**	-6187.3
No. Observations:	1000	**AIC:**	1.238e+04
Df Residuals:	996	**BIC:**	1.240e+04
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1011.6812	5.443	185.867	0.000	1001.013	1022.349
terminal_fricative	12.7265	9.837	1.294	0.196	-6.554	32.007
terminal_stop	8.6200	8.615	1.001	0.317	-8.265	25.505
initial_nasal	40.0655	10.360	3.867	0.000	19.760	60.371

Omnibus:	131.246	**Durbin-Watson:**	0.037
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	209.601
Skew:	0.878	**Prob(JB):**	3.06e-46
Kurtosis:	4.396	**Cond. No.**	3.30

Table 3.20: 5-Letter Identifiers Elo vs Terminal Fricative,Terminal Stop, and Initial Nasal

Dep. Variable:	elo	**R-squared:**	0.014
Model:	OLS	**Adj. R-squared:**	0.013
Method:	Least Squares	**F-statistic:**	13.69
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.000228
Time:	19:47:17	**Log-Likelihood:**	-5903.3
No. Observations:	1000	**AIC:**	1.181e+04
Df Residuals:	998	**BIC:**	1.182e+04
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1007.3665	2.898	347.604	0.000	1001.686	1013.047
initial_nasal	43.3391	11.715	3.700	0.000	20.379	66.300

Omnibus:	61.410	**Durbin-Watson:**	0.030
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	74.083
Skew:	0.588	**Prob(JB):**	8.19e-17
Kurtosis:	3.627	**Cond. No.**	4.07

Table 3.21: 4-Letter Identifiers Elo vs Initial Nasal

Dep. Variable:	elo	**R-squared:**	0.003
Model:	OLS	**Adj. R-squared:**	0.002
Method:	Least Squares	**F-statistic:**	2.694
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.101
Time:	19:47:17	**Log-Likelihood:**	-5909.2
No. Observations:	1000	**AIC:**	1.182e+04
Df Residuals:	998	**BIC:**	1.183e+04
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1006.6304	3.532	285.034	0.000	999.709	1013.552
terminal_voiceless	9.6553	5.882	1.641	0.101	-1.874	21.185

Omnibus:	59.339	**Durbin-Watson:**	0.006
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	70.363
Skew:	0.585	**Prob(JB):**	5.26e-16
Kurtosis:	3.565	**Cond. No.**	2.42

Table 3.22: 4-Letter Identifiers Elo vs Terminal Voiceless Consonant

Dep. Variable:	elo	**R-squared:**	0.011
Model:	OLS	**Adj. R-squared:**	0.010
Method:	Least Squares	**F-statistic:**	12.26
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.000484
Time:	19:47:17	**Log-Likelihood:**	-5905.0
No. Observations:	1000	**AIC:**	1.181e+04
Df Residuals:	998	**BIC:**	1.182e+04
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1004.3789	3.394	295.911	0.000	997.726	1011.031
terminal_obstruents	21.0314	6.007	3.501	0.000	9.257	32.806

Omnibus:	68.273	**Durbin-Watson:**	0.023
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	84.435
Skew:	0.622	**Prob(JB):**	4.63e-19
Kurtosis:	3.691	**Cond. No.**	2.45

Table 3.23: 4-Letter Identifiers Elo vs Terminal Voiced Obstruent

Dep. Variable:	elo	**R-squared:**	0.018
Model:	OLS	**Adj. R-squared:**	0.016
Method:	Least Squares	**F-statistic:**	8.767
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.000168
Time:	19:47:17	**Log-Likelihood:**	-5901.5
No. Observations:	1000	**AIC:**	1.181e+04
Df Residuals:	997	**BIC:**	1.182e+04
Df Model:	2		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1003.1573	3.593	279.201	0.000	996.115	1010.199
initial_nasal	44.7962	11.742	3.815	0.000	21.781	67.811
terminal_voiceless	11.1807	5.858	1.909	0.056	-0.301	22.663

Omnibus:	60.388	**Durbin-Watson:**	0.037
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	72.137
Skew:	0.588	**Prob(JB):**	2.17e-16
Kurtosis:	3.592	**Cond. No.**	4.42

Table 3.24: 4-Letter Identifiers Elo vs Initial Nasal and Terminal Voiceless Consonant

Dep. Variable:	elo	**R-squared:**	0.038
Model:	OLS	**Adj. R-squared:**	0.035
Method:	Least Squares	**F-statistic:**	12.44
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	5.44e-08
Time:	19:47:17	**Log-Likelihood:**	-5891.0
No. Observations:	1000	**AIC:**	1.179e+04
Df Residuals:	996	**BIC:**	1.181e+04
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	989.1805	4.890	202.280	0.000	979.596	998.765
initial_nasal	42.8709	11.457	3.742	0.000	20.416	65.326
terminal_voiceless	25.2412	6.727	3.752	0.000	12.057	38.425
terminal_obstruents	32.3465	6.839	4.730	0.000	18.943	45.750

Omnibus:	72.454	**Durbin-Watson:**	0.081
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	90.873
Skew:	0.643	**Prob(JB):**	1.85e-20
Kurtosis:	3.726	**Cond. No.**	4.55

Table 3.25: 4-Letter Identifiers Elo vs Initial Nasal,Terminal Voiceless Consonant, and Terminal Voiced Obstruent

Dep. Variable:	elo	**R-squared:**	0.003
Model:	OLS	**Adj. R-squared:**	0.002
Method:	Least Squares	**F-statistic:**	2.770
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.0964
Time:	19:47:17	**Log-Likelihood:**	-5909.3
No. Observations:	1000	**AIC:**	1.182e+04
Df Residuals:	998	**BIC:**	1.183e+04
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1007.9550	3.195	315.490	0.000	1001.693	1014.217
terminal_fricative	11.3121	6.797	1.664	0.096	-2.010	24.635

Omnibus:	62.066	**Durbin-Watson:**	0.006
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	74.614
Skew:	0.596	**Prob(JB):**	6.28e-17
Kurtosis:	3.609	**Cond. No.**	2.63

Table 3.26: 4-Letter Identifiers Elo vs Terminal Fricative

Dep. Variable:	elo	**R-squared:**	0.013
Model:	OLS	**Adj. R-squared:**	0.012
Method:	Least Squares	**F-statistic:**	13.31
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.000278
Time:	19:47:17	**Log-Likelihood:**	-5904.1
No. Observations:	1000	**AIC:**	1.181e+04
Df Residuals:	998	**BIC:**	1.182e+04
Df Model:	1		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1003.6893	3.363	298.432	0.000	997.097	1010.281
terminal_stop	22.3171	6.117	3.648	0.000	10.328	34.307

Omnibus:	63.532	**Durbin-Watson:**	0.028
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	76.680
Skew:	0.604	**Prob(JB):**	2.23e-17
Kurtosis:	3.617	**Cond. No.**	2.43

Table 3.27: 4-Letter Identifiers Elo vs Terminal Stop

Dep. Variable:	elo	**R-squared:**	0.021
Model:	OLS	**Adj. R-squared:**	0.019
Method:	Least Squares	**F-statistic:**	10.74
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	2.44e-05
Time:	19:47:17	**Log-Likelihood:**	-5899.8
No. Observations:	1000	**AIC:**	1.181e+04
Df Residuals:	997	**BIC:**	1.182e+04
Df Model:	2		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	997.6954	4.021	248.151	0.000	989.815	1005.575
terminal_fricative	21.5718	7.222	2.987	0.003	7.417	35.727
terminal_stop	28.3109	6.502	4.354	0.000	15.568	41.054

Omnibus:	68.137	**Durbin-Watson:**	0.046
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	83.539
Skew:	0.627	**Prob(JB):**	7.24e-19
Kurtosis:	3.657	**Cond. No.**	3.26

Table 3.28: 4-Letter Identifiers Elo vs Terminal Fricative and Terminal Stop

Dep. Variable:	elo	**R-squared:**	0.036
Model:	OLS	**Adj. R-squared:**	0.033
Method:	Least Squares	**F-statistic:**	11.80
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	1.35e-07
Time:	19:47:17	**Log-Likelihood:**	-5892.2
No. Observations:	1000	**AIC:**	1.179e+04
Df Residuals:	996	**BIC:**	1.181e+04
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	994.6756	4.073	244.231	0.000	986.693	1002.658
terminal_fricative	21.6764	7.167	3.025	0.002	7.630	35.723
terminal_stop	28.7500	6.468	4.445	0.000	16.073	41.427
initial_nasal	44.1755	11.624	3.800	0.000	21.393	66.958

Omnibus:	69.990	**Durbin-Watson:**	0.077
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	86.930
Skew:	0.632	**Prob(JB):**	1.33e-19
Kurtosis:	3.700	**Cond. No.**	4.35

Table 3.29: 4-Letter Identifiers Elo vs Terminal Fricative,Terminal Stop, and Initial Nasal

S2 Appendix: Polarization Regression Models Regressions for Phonological Constructions on Polarization.

Dep. Variable:	rank_breaks	**R-squared:**	0.002
Model:	OLS	**Adj. R-squared:**	0.000
Method:	Least Squares	**F-statistic:**	1.283
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.278
Time:	22:26:53	**Log-Likelihood:**	-1859.6
No. Observations:	1000	**AIC:**	3725.
Df Residuals:	997	**BIC:**	3740.
Df Model:	2		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	2.4947	0.068	36.550	0.000	2.361	2.628
initial_nasal	-0.2034	0.129	-1.578	0.115	-0.456	0.049
terminal_voiceless	-0.0231	0.100	-0.232	0.817	-0.218	0.172

Omnibus:	83.175	**Durbin-Watson:**	2.034
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	105.922
Skew:	0.709	**Prob(JB):**	9.99e-24
Kurtosis:	3.730	**Cond. No.**	3.07

Table 3.30: 5-Letter Identifiers Polarization vs Initial Nasal and Terminal Voiceless Consonant

Dep. Variable:	rank_breaks	**R-squared:**	0.002
Model:	OLS	**Adj. R-squared:**	-0.001
Method:	Least Squares	**F-statistic:**	0.8567
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.463
Time:	22:26:53	**Log-Likelihood:**	-1859.6
No. Observations:	1000	**AIC:**	3727.
Df Residuals:	996	**BIC:**	3747.
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	2.4963	0.084	29.599	0.000	2.331	2.662
initial_nasal	-0.2034	0.129	-1.576	0.115	-0.456	0.049
terminal_voiceless	-0.0247	0.112	-0.221	0.825	-0.244	0.194
terminal_obstruents	-0.0044	0.134	-0.033	0.974	-0.267	0.258

Omnibus:	83.181	**Durbin-Watson:**	2.034
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	105.940
Skew:	0.709	**Prob(JB):**	9.89e-24
Kurtosis:	3.730	**Cond. No.**	3.67

Table 3.31: 5-Letter Identifiers Polarization vs Initial Nasal,Terminal Voiceless Consonant, and Terminal Voiced Obstruent

Dep. Variable:	rank_breaks	**R-squared:**	0.007
Model:	OLS	**Adj. R-squared:**	0.004
Method:	Least Squares	**F-statistic:**	2.034
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.107
Time:	22:26:53	**Log-Likelihood:**	-1857.5
No. Observations:	1000	**AIC:**	3723.
Df Residuals:	996	**BIC:**	3743.
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	2.4497	0.071	34.569	0.000	2.311	2.589
terminal_fricative	0.2501	0.140	1.784	0.074	-0.025	0.525
terminal_stop	-0.0348	0.111	-0.314	0.753	-0.252	0.183
initial_nasal	-0.2019	0.129	-1.566	0.117	-0.455	0.051

Omnibus:	81.069	**Durbin-Watson:**	2.040
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	102.251
Skew:	0.701	**Prob(JB):**	6.26e-23
Kurtosis:	3.699	**Cond. No.**	3.30

Table 3.32: 5-Letter Identifiers Polarization vs Terminal Fricative, Terminal Stop, and Initial Nasal

Dep. Variable:	rank_breaks	**R-squared:**	0.005
Model:	OLS	**Adj. R-squared:**	0.003
Method:	Least Squares	**F-statistic:**	2.715
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.0667
Time:	22:26:53	**Log-Likelihood:**	-1910.7
No. Observations:	1000	**AIC:**	3827.
Df Residuals:	997	**BIC:**	3842.
Df Model:	2		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	2.7255	0.065	42.232	0.000	2.599	2.852
initial_nasal	-0.3777	0.201	-1.881	0.060	-0.771	0.016
terminal_voiceless	0.1496	0.110	1.360	0.174	-0.066	0.365

Omnibus:	42.601	**Durbin-Watson:**	1.915
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	47.276
Skew:	0.532	**Prob(JB):**	5.42e-11
Kurtosis:	3.053	**Cond. No.**	4.42

Table 3.33: 4-Letter Identifiers Polarization vs Initial Nasal and Terminal Voiceless Consonant

Dep. Variable:	rank_breaks	**R-squared:**	0.006
Model:	OLS	**Adj. R-squared:**	0.003
Method:	Least Squares	**F-statistic:**	2.053
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.105
Time:	22:26:53	**Log-Likelihood:**	-1910.4
No. Observations:	1000	**AIC:**	3829.
Df Residuals:	996	**BIC:**	3848.
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	2.6859	0.086	31.338	0.000	2.518	2.854
initial_nasal	-0.3832	0.201	-1.907	0.056	-0.777	0.011
terminal_voiceless	0.1894	0.124	1.533	0.125	-0.053	0.432
terminal_obstruents	0.0915	0.127	0.723	0.470	-0.157	0.340

Omnibus:	42.704	**Durbin-Watson:**	1.914
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	47.399
Skew:	0.533	**Prob(JB):**	5.10e-11
Kurtosis:	3.055	**Cond. No.**	4.55

Table 3.34: 4-Letter Identifiers Polarization vs Initial Nasal,Terminal Voiceless Consonant, and Terminal Voiced Obstruent

Dep. Variable:	rank_breaks	**R-squared:**	0.005
Model:	OLS	**Adj. R-squared:**	0.002
Method:	Least Squares	**F-statistic:**	1.955
Date:	Wed, 29 Aug 2018	**Prob (F-statistic):**	0.119
Time:	22:26:53	**Log-Likelihood:**	-1910.7
No. Observations:	1000	**AIC:**	3829.
Df Residuals:	996	**BIC:**	3849.
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	2.7241	0.074	36.706	0.000	2.579	2.870
terminal_fricative	0.0480	0.136	0.354	0.724	-0.218	0.314
terminal_stop	0.1648	0.122	1.351	0.177	-0.074	0.404
initial_nasal	-0.3922	0.202	-1.939	0.053	-0.789	0.004

Omnibus:	43.935	**Durbin-Watson:**	1.911
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	48.905
Skew:	0.541	**Prob(JB):**	2.40e-11
Kurtosis:	3.067	**Cond. No.**	4.35

Table 3.35: 4-Letter Identifiers Polarization vs Terminal Fricative, Terminal Stop, and Initial Nasal

S3 Appendix: List of Identifiers **This table contains the complete list of identifiers used.**

4-Letter	5-Letter
ivek	pesam
asiq	jayor
avox	kewos
ejon	mavey
omog	coyam
otif	fexen
ojav	pebel
ebud	jupar
ufix	cosop
anix	lozir
awek	koduk
epek	kovaq
izus	mazob
owek	camaz
ozis	bumak
azop	kujol
ixel	falom
emev	poqit
opaw	gosaw
uzus	hosop
uzok	madik
ovay	jutah
axoy	naroc
ozir	lenim
imeg	lelip
atux	gomos
otiz	nubul
uvap	paqel
awax	nuvis
izop	kozec
okoy	dajav
uboz	hamok
ebak	luwog
icoy	korav
enud	puzat
omaj	coxar
oyek	kehab
ebek	gupac
ibuy	hugum

ezip	nalaj
ovox	jaxam
eviv	nejix
ewop	kayus
equm	helad
ezub	nawam
opaz	neroy
enux	maviz
ikab	nipiq
ewoq	cebis
axah	gixan
emuv	mibob
osaz	banom
okap	lilap
ubob	nadog
evuz	pizaz
uvaq	pukap
oxef	deneh
eluz	melox
ufam	nowan
uweg	pefay
ovaq	hazud
enub	finoy
oliq	qavan
oyix	cazup
uzab	mezed
emoc	carir
ewiz	moruz
ecox	kisop
oxoh	mideh
umeg	bavez
etug	peber
oxim	jomic
abux	kojeb
ageg	dapup
ojay	midux
ayom	koyev
ewut	laral
ipox	luxos
ehoc	cupun
ugot	lobud
ezej	neliz

umaj	luzaf
agud	hawoh
udur	kogas
azuc	jusol
omeb	mefer
ecup	mavaz
eqiv	maqet
uwan	livud
izip	mumek
ezot	pixed
azod	pagol
usiv	jatet
awup	dacoy
uzim	jalav
ovax	fozer
opeg	leyah
upad	casuk
oxaz	hidar
obeq	mikaw
ezac	demox
ukiz	hinaj
oniq	ceyos
esoy	mofex
ebaj	gozik
ayug	maqis
ihub	bajoc
umaw	kimoz
uqiz	jezed
afav	jufah
owat	jolot
odiz	mifun
udog	humeg
uxal	miroj
uraj	panuf
olab	cunim
uwos	naroy
odaj	fazos
edav	kivam
oqal	lezul
emix	meqac
atez	pedax
adug	gupil

aqan	lohaw
ogix	jareh
ataj	povip
afab	gudem
adez	deyal
exaz	mosiv
urib	micax
esug	luqem
ojep	culah
owok	panoc
ufom	jawok
inaz	jaluk
obuk	gidem
adoy	pimac
umix	gemey
ovix	fazov
onoq	kuzac
eyab	koriv
uvir	defos
icaw	bokac
iwid	cufay
evev	luciy
odip	bageg
oqim	fazip
ozec	dafir
iwal	levid
ezif	nizaz
ucaw	kehid
ehum	diriv
ejav	namuc
esok	mogip
axog	mekeb
omeq	mepoh
imeq	nayim
olaq	kusit
oteq	jecid
eraw	bomav
umez	nirab
upez	hurod
oqos	mutiz
oket	hunaq
aqic	meyir

osuk	cudip
aqom	qador
ozit	korit
azij	cinok
ohep	kuyem
aqed	maheg
azub	pogok
avaw	hodev
ojeb	koyem
izax	kenux
odab	muvoc
idoz	famoy
ofaj	pemeg
aqac	duzin
izig	mivon
ojaw	muvit
aduw	gulac
ucog	hikop
esuz	miloc
urix	legom
axeb	davuk
atix	dedow
ezeb	pujom
uzid	pocor
orob	fabox
opaq	jigas
uzif	lulaq
ojad	pizah
avux	gawus
ubaz	ganur
uqil	gumad
atev	pufun
owuk	foraj
owol	kadul
ozaw	muluq
emoz	lifix
uzev	cofic
ajun	hukar
ocuz	jahaw
apaj	mojaw
usay	bejon
odid	hisaf

ojex	necuz
azuy	payom
esiz	gumon
ameq	decek
ejox	kebad
ebop	kulem
okif	nuxos
ijun	geroz
oyok	jixil
uleq	gemud
uxir	diroc
ewun	mojic
icaj	kuwis
axez	neyus
ocaz	jurib
atug	jusev
ezog	mafum
ubix	jajoy
oyak	bulem
arux	bapas
oqey	jexic
areq	mafur
opup	laxip
esog	lamoz
oxux	natap
onip	kefoz
aqaq	gunek
ozat	kugos
oqon	qetas
udiz	pudew
ukul	kegit
agug	namot
exoc	juzil
axuz	bemaf
aqij	finij
ovus	negaz
uxer	niwal
egev	kedew
enex	canof
ihol	fanap
ayaf	hopaf
uxix	lugof

ohiv	lejag
osot	cadej
acax	melaq
ozej	golug
aleq	bacoy
izik	hixit
ehuz	bodof
uruq	nidac
igak	jukug
utiq	kopip
eqob	gaqon
azup	cuzoy
ujok	hadof
ataw	cecig
ijex	maded
ezoh	hikas
uxim	laqiz
ozov	mumaf
iqex	jufat
ejul	jahev
ezup	foxuy
ebaf	butaz
eqon	locun
ajag	muhaz
ujen	nofen
efob	pamaq
eqot	lipob
uqad	mopot
acag	baziq
osih	lecuh
ozaj	jedap
usex	nisev
uqal	doqor
ejom	fawun
ulot	busix
abej	bizej
unuv	bamuc
okof	mufur
ulob	nofiz
izot	bozit
ogaf	bobeh
ogaw	cusud

obuf	buseq
apey	boliy
ohuz	hagac
esif	dihet
osoy	lajak
isuc	cepoz
opef	miseh
ateg	luyak
aqad	janoz
ibud	fabix
afep	jelag
apew	pakih
oxap	pevid
emiz	nanic
idug	babap
uboy	hocun
omob	forek
ubop	nugeb
ogem	bonij
obeg	fehaz
esul	husig
ujaj	mexoh
izec	galuv
imaw	fajul
anuz	payex
oxat	fanuv
evaw	jupel
imom	camux
izep	mabub
enib	dodeg
odoh	qehad
oxex	firup
ipeq	cusem
ejim	japaf
ojug	bewej
ekit	lokip
azeh	bizuq
ilox	nejux
oxeb	gufas
oyey	luveh
efux	fahah
ajav	bafen

oyoj	butaf
ezex	lidag
ozob	dimal
egif	qovel
ekux	domav
iduh	bajuw
ejiv	bexod
eqoc	gilox
opuf	copox
epof	lisiy
ucaz	dutos
usuk	loqen
emuj	jewem
egur	pomew
isuv	hiwer
iwek	lapaq
imif	kisid
ewof	dawos
akiy	dowir
owef	harok
ejem	qesex
eneg	ciriv
ajof	kenuy
obiz	jiwin
avew	pibab
atey	lofow
ikox	lufim
osob	faroq
oqaz	dawom
ivip	hikip
eruz	jekoq
exeb	qapor
uvub	nobuq
akeb	hayup
efay	pucax
iwiz	lijat
icev	bamam
uqox	gigig
izuj	bavim
ubot	godoh
ureh	lofic
ekaf	neqet

oqet	kujih
owik	fekey
edup	dizus
ojil	dimuz
epum	cetef
upuq	juwac
aseg	cimoz
uqaw	jahuh
uzub	mibom
ixoy	kagat
izoc	bijek
upaj	gakaz
efaq	ducoz
iwof	kuroc
avuk	bocut
umuy	maxih
utiv	fapod
ecuf	manup
ivuz	kuzaj
igab	focom
efib	hehum
ozuz	jiroy
edih	gupeq
idix	neyat
obef	jucin
iwoj	husaw
ipob	nalew
asiv	qinik
ekat	qenon
ihak	buzaz
axak	bibil
owet	gemeh
ohax	jizof
izex	jeqon
amup	fuxum
exuh	qukum
ehaf	kajip
uhip	qimix
ekik	biwuk
uzuf	hokuj
ubox	muhaf
usuq	fijot

awur	dayip
owug	hahek
ojon	jirem
ofok	mituh
ivaw	hucaw
ukoz	gitab
ubup	nigis
ibek	hizid
uziz	moruv
ezuv	boveq
opiw	qayuk
ajif	kafux
ogab	qotey
udub	micok
ixeb	lepey
exoz	nibat
ucah	bilaq
umav	dijeh
uwab	bisup
ekew	furub
uyox	japoz
ufub	deqeb
ehax	dukup
ixak	heyoz
oyis	gipum
uvim	gavuc
odod	bucom
onuj	capeb
owud	kazuj
ozix	nupub
otaj	jekox
ayuf	kawir
ujur	noyat
ajeb	niqin
aniw	cuxur
asuh	dijun
ibup	bumuy
ucib	bopad
ezuw	nuyak
uteb	kofup
evaj	mifep
uwec	ponum

ehix	gewom
iboq	lovuj
ojix	qaric
efif	kakox
ocav	cotoj
utaj	jowog
okut	dobot
ihoz	kipoh
axaq	bakux
ocox	bumec
omoq	nelit
eqin	kuqoc
otof	mibad
uxas	febaq
oxop	qacam
orud	fojub
uhim	gapib
aqux	giqar
uyac	pelub
uwap	jinop
asiw	lafec
ogoc	laxub
oqeb	cicum
efok	dosoz
ojel	bihir
ekok	mugeh
unih	fizeb
exuk	qamuh
uloj	bohud
okuc	kexuw
ufuy	lulew
awaf	foyar
ikak	bipam
ehof	hihug
oqan	maxaq
ipuz	nifum
iwak	fugum
icud	notak
ebup	mehor
avep	gujad
uboc	loxab
ifeg	biyom

ufug	maxoj
eteb	fojix
eguk	qupep
ofov	fihar
obeb	bofef
owev	dodas
uvut	padeb
avuv	gigox
izaj	bumaf
ohiz	fipev
ejij	jehas
eqis	gokuv
eyix	jujuc
iwuq	mudut
uluv	deguv
upuv	hidep
ubag	loxoz
ecow	cudol
odij	qozus
osib	dibed
eqec	domif
etew	ceriy
oqur	muxil
awut	kosob
osah	pelif
odiq	fariw
ohet	qaxol
ituk	lujaf
uhoc	piqez
upip	diwab
uxaq	koxus
ewit	lanoq
uwur	dabaf
ocig	buduf
ocix	jutur
okog	bexoh
uvaj	gafux
ejap	jiyag
epab	fazuf
emob	keleq
ufez	qameh
uvow	japum

ahiq	hexaq
ujes	cibam
ewav	kesuv
ukuy	cokud
omeh	dekip
udej	perap
ekak	hitej
onaf	pisof
otih	hedul
ogep	nuvok
afoq	biwar
ewad	hivoy
osuz	liper
ihib	nunif
iqem	qirab
uhod	mabod
ofeb	govez
utav	kekap
omih	giniq
utif	haceh
eqol	jafaf
esud	nolur
ayuq	buqov
oyur	burug
uwit	fahij
owep	gonuc
oyic	qomap
onuv	nagiq
ekuh	fapif
aziy	koguf
aheh	kixan
ujip	qobeh
iqom	cukep
uqat	gafig
uluh	qujet
ahuc	hidiz
utib	kamip
aliw	pewom
acuh	dafum
uveq	cuzul
ezaw	dahow
upuj	pexih

ipej	joyuk
ogaj	diqom
ofib	niniq
ewik	meyiy
enag	jaroj
idux	pecoq
inuj	cucuk
iqew	gerev
ezuc	becef
utof	boruy
owav	nefag
apuq	nicoq
unub	facuz
ewuf	jidit
ayuy	jekiy
isiz	fasij
ofuk	fukoh
iyit	kunag
igoc	kacex
ajuy	qemor
ifaf	pivun
ivoh	nomep
okiv	dilux
ecej	cecay
ihaq	biseq
iyeh	bowal
uvud	dedik
ukec	luhid
atuy	qodeb
utog	kimof
akof	nogex
imuh	jataj
ovug	fisok
ehoq	qawep
ucux	badod
ucax	kujod
ikum	fahuk
ekoz	jojuy
akeg	gudik
ovib	foxih
uzuq	gewus
uhux	hetaq

apuv	nakix
ihig	leqop
uyov	juyox
eviz	lihih
abuq	lojut
azuw	mohiw
ojoz	cebuq
uvek	dejiw
ifiz	qoroc
egeb	nipoz
ojuh	baxud
ixid	cukip
ucuf	lagoc
iqed	kigog
ekid	joxiy
equx	jucev
ugos	qemak
ujoz	lezax
aduv	besow
uxuy	guyep
ikuz	hozej
ecur	qecim
uyip	decix
ucod	gosac
eyep	juwem
ewuh	gegiv
owid	bufav
upuz	dihep
ajep	ciqog
iloq	qiruc
ajiv	nuzot
inup	didiv
owig	dofuc
ufit	lowop
atuj	hifan
ocof	fiteb
uboq	dolih
ucuw	bedif
edeg	ciziy
ogeb	baqey
ibef	naqup
iwum	linuw

uyut	dokek
oqek	fujed
enaw	cukol
ogog	fizim
edaq	fopiv
eziy	qehab
uzeb	jecir
icag	bijag
edaj	lojed
ahoz	dogoc
ogoq	febum
igah	fafuf
afuz	pikaq
uweq	nojiy
eqez	bodid
iwef	fizuz
iqef	midut
egux	bequv
ovof	nibod
axuj	cunoj
owom	guluw
ulim	dadef
ucay	mumop
ujir	pehem
uwal	kecic
uzuw	qucir
aqov	coyul
ugic	lohuh
icuv	heqij
izux	gapif
uvoq	kiyir
atuh	leqem
oxud	jukig
ixow	fovif
ozuy	qegoy
ekoc	deruq
avaj	huhuk
acew	pahif
eduw	nuxok
umew	bomip
ivez	fuyoh
ecuh	nonoj

osij	jaqew
ixaf	qegem
ufeb	hoyil
asoz	mugiy
ohuj	fijiv
ikof	gisif
efiq	nuhuz
uvej	bihib
ugeg	jagij
eboj	mufuf
ixal	pojik
oqib	lacuc
onoj	hodud
obup	bobod
ewip	humiy
ujus	hodog
ihel	nanuw
ukuk	papiq
usux	mipef
uyux	pezuh
ijoq	qayiv
eqes	dudoq
ufoh	mukiy
ayow	kogiy
ulej	qepar
ecaf	jajix
uvab	jojaq
equh	cagiq
ihuw	kugux
ayiw	hefom
iwac	qamom
ejeb	qonip
iduy	jemej
epuk	giviq
ujim	jociy
uhiz	qibog
ekoq	navep
ehog	pojuv
ohub	gupok
ukaq	joviw
abup	geluq
ofec	fevut

iwip	mijiz
iwiq	mapej
uhuv	pifil
ehuq	lawuv
eyir	luwub
opih	gedij
ugor	duyit
ofuq	husit
idej	newew
uxop	juwos
ugiz	nijef
iwuc	heham
ipib	gimaw
ixac	hufuh
itaw	cikoz
ehoh	dewil
uxam	neriy
uvav	qeguz
uyeh	kugog
oguw	qugac
iqoz	goqaj
uzof	kukij
uxac	moqac
evuq	qexix
axuc	geciv
uqef	qapem
oqut	juzud
uyif	qimec
omuy	jofoc
unaj	jebut
uniz	pituf
aguf	pobuz
ikuk	ququl
oqez	pomiy
oqul	pewep
uyem	nuguk
uwil	fisof
aqiy	jufad
iwuy	dufic
ibip	dajuf
uwaw	neqeq
ahek	qucic

uyis	jufej
utej	muxuk
uqiq	bigaj
uzug	mikox
opoq	kazij
icak	baziy
ipul	femof
uqiw	keqif
ejev	jekeh
ujux	qebux
epuy	gibep
umub	fapih
ufob	peqom
ebip	bewef
ituw	qecut
uhih	kixex
ugiv	fuqeh
ojip	gasuq
otoj	gosod
uneb	qewaj
uyey	gicuk
iduw	jaqej
ewiv	lejij
ekiy	codej
uvux	kocuj
efoh	cunuh
udud	baqoc
egay	cigup
udoj	kuhix
unuy	huqiv
icuh	pewuk
uroj	gahoz
upuf	duzug
ejep	fimup
ageq	qeyaw
ixod	qofis
iyut	jaxug
oxak	gitif
ayeq	bapef
icih	futuj
ofih	juyed
ezeg	gileq

ubij	miguw
igex	boqon
uyaw	capeh
ewiq	gixub
obuj	meyiq
uneh	geqor
ovuq	pucoc
ozoq	pahih
oqak	qiruq
iviq	cecec
uriy	hepoj
ojux	qoneb
imuw	lunij
igaq	jepiv
ofoc	qawaj
oyiv	qepov
iyej	qimuz
iwom	kigay
upiw	biqix
esuw	qupov
ijol	huvib
ujud	cemif
awih	nagip
ugeh	dedav
uyeb	quhec
ugaq	nuhux
aqey	bapup
uzaj	fuqug
ojig	nacih
ulij	qidew
afeg	jucuj
ukib	difiv
uqex	boguw
uxut	goqeg
uziy	hociy
oqiw	ciyip
uxaw	bowiv
oxuj	fufob
ipuj	gevip
iwet	hajoj
otuj	hofoc
ihif	nekuc

equq	qicot
awus	qexay
upej	furep
iqen	ceqem
adih	fihoq
aqaj	qihux
owiv	muyoy
oqok	foviz
ugej	qaqac
iruq	jujif
eyoj	qekah
ixud	qamoh
egew	jixek
aqeg	noduq
ahef	cicaj
obik	kuxaj
uvep	qoyin
ucuv	qoqaw
iyag	mihid
afoj	guyaj
uraw	ligeg
ufiv	norur
uqeg	lereg
ucoh	qadug
ohix	nosif
iyev	fugiq
ujiv	giheb
uyul	gowoc
oqic	guqur
uwuz	qediy
ujif	jucew
ixub	gugej
oxoq	cewoh
ugez	guxun
ugih	qevoj
ugeq	desaq
iquw	qafun
eqoj	jiyuy
uqoj	cefuc
ocij	guheh
iwev	gexud
ubuq	jijob

ugop	ligib
ugaj	giqir
ihog	nujuj
awoj	fukik
ifeh	gosux
iyiq	fideh
iyif	hiziq
igub	hehil
uyez	gusuz
uxev	fodeh
ihiy	nuzoq
iqug	kogiw
ukiw	qejev
uyeq	jilih
ukij	hihuh
ujaq	qexag
ejiw	nutuj
uguf	fakag
uwiq	qanej
ipuw	qezuv
igew	peqef
ijaj	pigef
ubih	noxoj
oqof	qagiw
idiy	qoqiq
ikux	liliw
ifof	nafug
iqet	feget
uvih	guhiz
uwux	cusij
oquv	peqeg
ucuj	qavew
uhew	hejic
ihiq	qafij
afuj	fixej
oxok	cacij
egej	jufeh
iqaj	fowiv
ukuh	joqiw
uduj	qosid
uyuw	qupof
okiw	gijek

uluy	gabiy
egub	biquc
omuq	qijaq
oqig	qoyih
ixij	ditij
iceh	jigej
ojiy	guqip
ujoq	gufek
oqug	jikeq
uriw	qeyaf
uyiw	guhuy
udik	fucuj
iquq	gewih
ufof	qupef
iriw	qaviw
iwih	fepoj
uxej	geqog
uruw	qefib
uzup	gogib
iyaj	leheq
iyuf	cahiy
uxuj	cijej
aqej	hisoq
esih	giqic
uxuk	cuwuq
ujih	cixuq
oqiq	quhuq
eyik	qixew
iruw	gihin
uyoq	qaqep
igay	gayij
ogiw	gikif
owoq	bibiw
exuw	hegug
usiy	fihuw
uxij	qiyag
ihih	nufiw

Chapter 4

Cascades in Capacity Constrained Agents

4.1 Introduction

Diffusion processes are an important social phenomenon that has been studied across multiple domains such as consumer goods (Bass, 1969), adoption of hybrid corn (Griliches, 1957), the spread of disease (Kermack and McKendrick, 1927), information on social media (Goel et al., 2015), revolutions (Kuran, 1991), as well as many theoretical explorations (Boorman, 1974; Granovetter, 1978; Granovetter and Soong, 1983; Macy, 1991; Centola et al., 2007; Watts, 2011)

Cascades often do not occur in isolation, but exist in an environment with multiple potential cascades that could occur simultaneously. One way to model multiple simultaneous cascades or contagions is through direct interaction between the contagions. These models are typically used in an epidemiological setting where there is some sort of infection that is spreading and a social behavior like vaccination or distancing also spreads through the population, which modulates the spread of the infection but is also driven by the infection (Epstein et al., 2008; Perra et al., 2011; Bauch and Galvani, 2013; Fu et al., 2017). The interaction between multiple contagions can become computationally challenging but in well mixed populations it has been shown that these models are equivalent to complex contagion models (Hébert-Dufresne et al., 2020).

Another way to model simultaneous cascades is to think of them as substitutes for each other. For example, no matter how many washing machine brands are on the market, I as a consumer only need one washing machine and would be unlikely to buy two washing machines of different brands. Thus once my capacity for washing machines is met, I have no need to buy any more washing machines. While products that someone only needs one of can easily demonstrate the capacity constraint, the capacity for other products could be greater than one. For example, one could regularly listen to a playlist of songs that lasts 1 hour, changing out the different songs over time as their tastes change instead of adding new songs to the end. It is also reasonable that different people could have different capacities for the same product, for example a family of four probably wants more spoons than an individual who lives alone does. Given the similarities between this process and the types of congestion White observed in human communication networks, it is reasonable to expect these kinds of dynamics to influence a wide range of products (White, 1973).

Given that the music market regularly experiences multiple overlapping cascades, the stylized characteristics of the market can be useful for understanding how multicascade processes empirically function. The music industry historically has experienced high levels of firm concentration and this concentration is associated with a lower level musical diversity (Peterson and Berger, 1975, 1996). At the level of artists the music industry also shows signs of concentration where a small number of popular artists have out sized influence at any given time (Rosen, 1981; Hamlen Jr, 1991; Krueger, 2005). There are multiple mechanisms that may explain the features of this market. Experimental evidence suggests that social influence and more typical cascade dynamics are at play (Salganik et al., 2006; Salganik and Watts, 2008), but more recent analysis suggests that these effects may only be temporary perturbations from songs' fundamental value (Van de Rijt, 2019). In their experiment (Salganik and Watts, 2008) presented their participants with 48 different songs to potentially listen to and download for free and found that they listened to only 7 songs on average only downloaded 1 on average. This suggests that the their participants may be capacity constrained in both their interest in listening to songs and downloading them.

4.2 Model

This model is a variation of the Threshold Cascade model as described in (Macy and Evtushenko, 2020). There are N agents in the population who each have a cascade capacity of C. This means that if that agent would adopt a cascading state that would bring the number of states adopted greater than C, one of the currently adopted states is randomly dropped. There are S total states that can cascade and in order for the capacity constraint to be binding $S > C$. Agents can adopt a new state through either Threshold Cascading or Random adoption. Threshold cascading follows the process described in (Granovetter, 1978), each agent as a threshold T_i and adopts the state if the number of agents who have already adopted the state is greater than or equal to T_i. In this model the agents have the same threshold for each state. As in (Granovetter, 1978), the thresholds drawn from a normal distribution with mean μ and variance σ. Since the number of states adopted is discrete, the values of output by the normal distribution were rounded. For Random adoption, each agent is challenged each time step to adopt a random state. If the agent has already adopted the state, nothing changes. If the agent has not already adopted the state it adopts the state with probability p as defined by $p = \frac{1}{1+e^{mT_i}}$. A similar formulation for random adoption was used by (Macy and Evtushenko, 2020). At each time step each agent first checks for threshold adoption, then random adoption and finally checks for capacity. Since states are randomly dropped at the end of the round, it is possible for a state to be adopted and dropped within the same round.

The threshold contagion model is actually a form of complex contagion model. What differentiates this model from the standard formulation that Centola and Macy present is that in this setting all adoption and dropping actions are public, so the graph is complete (Centola and Macy, 2007). Instead of focusing on spread between individuals this model is better suited for describing population level phenomenon. While it is true than individual and network level dynamics can matter, the presence of small global signals can overpower local diffusion (Rossman and Fisher, 2021).

There have been some attempts to quantify thresholds at the individual and network level (Valente, 1996; Romero et al., 2011) , but attempts to merge threshold models with more classical diffusion models have also been successful Hedström (1994); Braun (1995). This suggests that even while thresholds may be difficult to empirically measure the results of these models can be compared to other classes of diffusion models.

4.2.1 Parameter Space

This model has 7 parameters. N represents the total number of agents in the population. T represents the total number of time steps. m represents the slope parameter in the random adoption function. C represents the capacity of each agent. S represents the total number of states. μ represents the mean of the threshold distribution and σ represents the variance of the threshold distribution. Table 4.1 shows the ranges of each of the parameters contained in this experiment. There are 80000 different combinations in this parameter space and each unique parameter combination was replicated 100 times so there were 8000000 total runs in this experiment. This experiment was constructed using the python package AgentPy. (Foramitti, 2021)

	Range
N	100
T	100
m	0.2
C	3,4
S	5,10,15,20
μ	0-100
σ	0-100

Table 4.1: Parameter Space

4.3 Analysis

4.3.1 Outcomes of Interest

The outcomes I tracked in this experiment can be split into those which track attributes within a given state over time and those that measure properties across states. These outcomes depend on the number of agents which are currently adopting the given state at a given time step. I will refer to this as counts going forward. For the within state outcomes I track the average number of counts over time (average counts), variance in counts over time (variance in counts), the maximum count reached (max counts), the time to reach the maximum (time to max), the sum of counts over time (final counts), the minimum count after the maximum count was reached (min after max), the time between the maximum and the minimum after maximum (decay time), as well as the Shannon entropy (Shannon, 1948) in counts (entropy counts). In order to aggregate these within state values to a per run level, I look at the mean, median, variance, mean absolute deviation, maximum and range (maximum - minimum) across states per run. The across state measures all depend on the total sum of counts for each state over time, the final count. For the across state outcomes I track, kurtosis in final count, number of states with final count of zero and the Herfindahl-Hirschman Index (Herfindahl, 1950; Hirschman, 1945) for final count. Table 4.2 shows the summary statistics for these outcomes.

Figure 4.1 shows the average counts averaged across states for each μ and σ value grouped by number of excess states, while Figures 4.2 and 4.3 shows the same type of plot for final counts and decay time respectively. These plots have similar fan like shape across both excess states as well as the range of μ and σ combinations for which there are interesting count values. Figures 4.4 and 4.5 show the Herfindahl index and kurtosis respectively averaged by each μ and σ pair grouped by number of excess states. Given the similarity between these groups of graphs, it suggests that the Herfindahl index and kurtosis are both effectively functioning as measures of concentration across the states. Notably both the Herfindahl index and kurtosis reach high values in a band across the bottom right corner going from around $\mu = 40$ to $\sigma = 30$.

Figure 4.6 shows the counts over time from four different runs with different configurations of parameters. The lower left graph has $\mu = 41$,$\sigma = 5$, and 2 excess states putting it within the high concentration band. There are not many total counts in this run, and since the high concentration bad overlaps with areas of low final and average counts generally this suggest that in this high concentration there is minimal adoption of any states. Thus, this region of the parameter space somewhat resembles a natural monopoly, where it is difficult to successfully enter and those that do dominate.

	Mean	Min	Max	Standard Deviation
Final Counts	338.0956	0	9489	1067.4220
Max Counts	9.3467	0	100	14.7336
Time to Max	25.6855	0	99	29.9113
Average Counts	6.8460	0	94.89	10.7935
Variance in Counts	22.4925	0	2311.4848	117.7511
Min after Max	6.7575	0	100	11.6436
Decay Time	7.5308	0	98	14.6412
Entropy in Counts	4.5658	0	19	4.6644
Herfindahl Index	0.1544	0	1	0.1418
Kurtosis in Counts	0.7549	-3.3333	20.0000	3.0163
Number of Zero Counts	1.7832	0	20	4.6321

Table 4.2: Summary Statistics

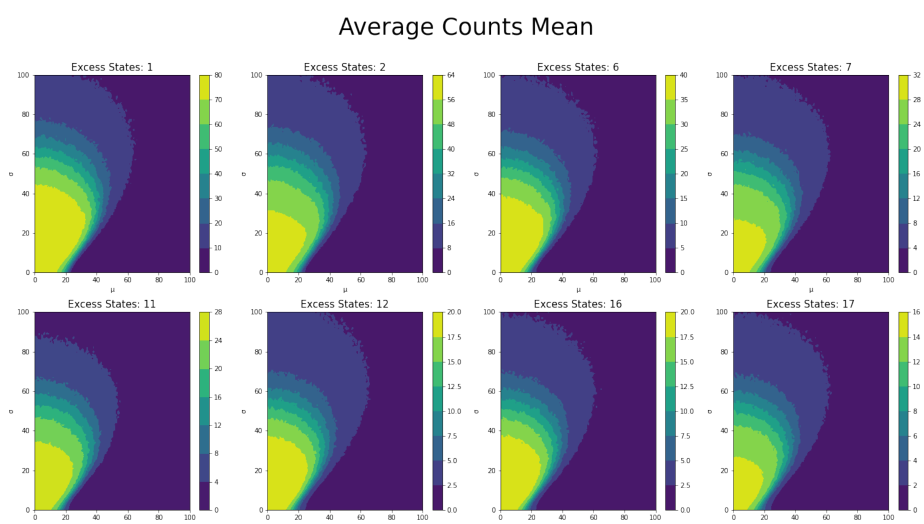


Figure 4.1

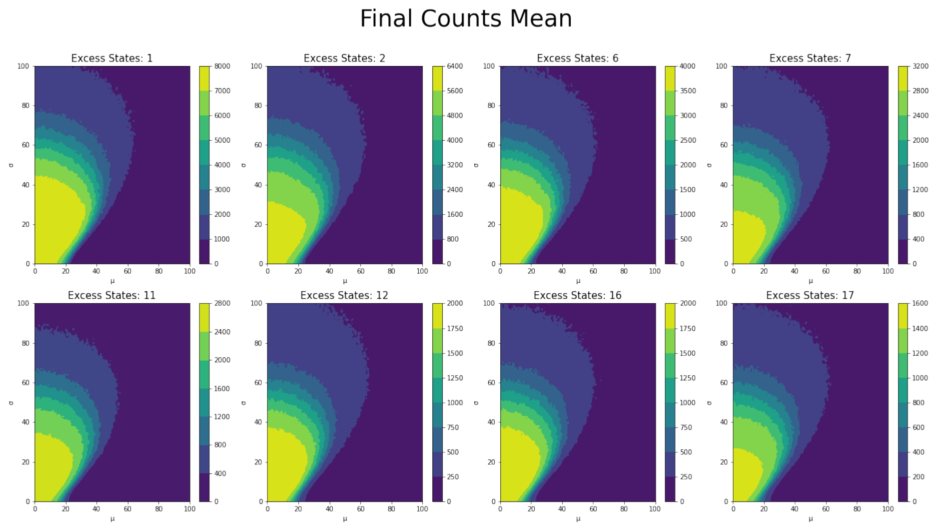


Figure 4.2

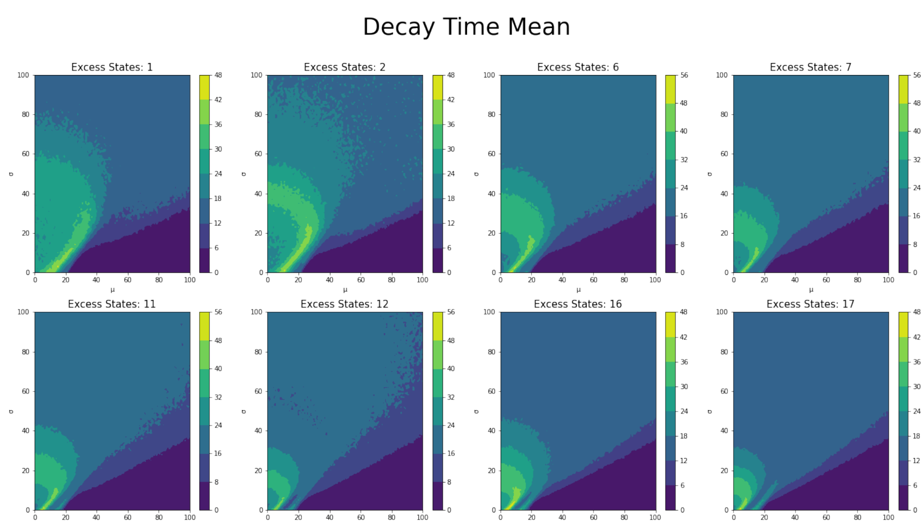


Figure 4.3

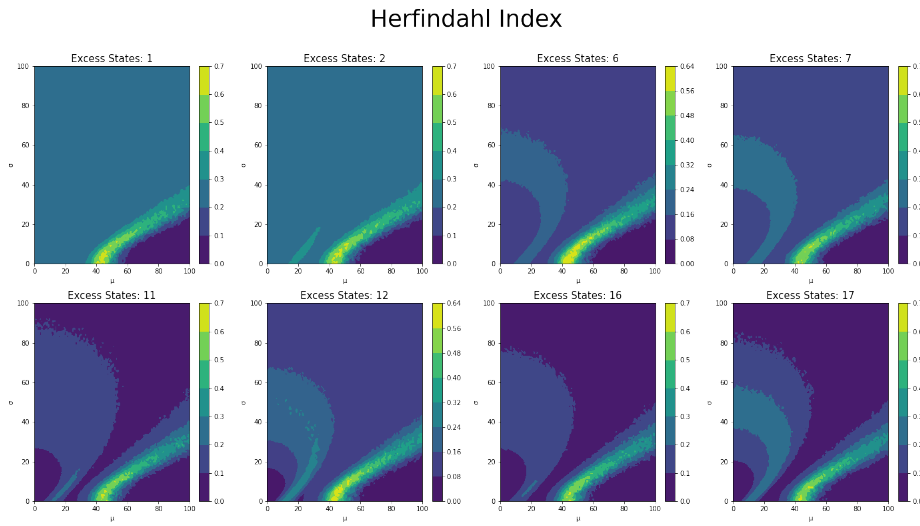


Figure 4.4

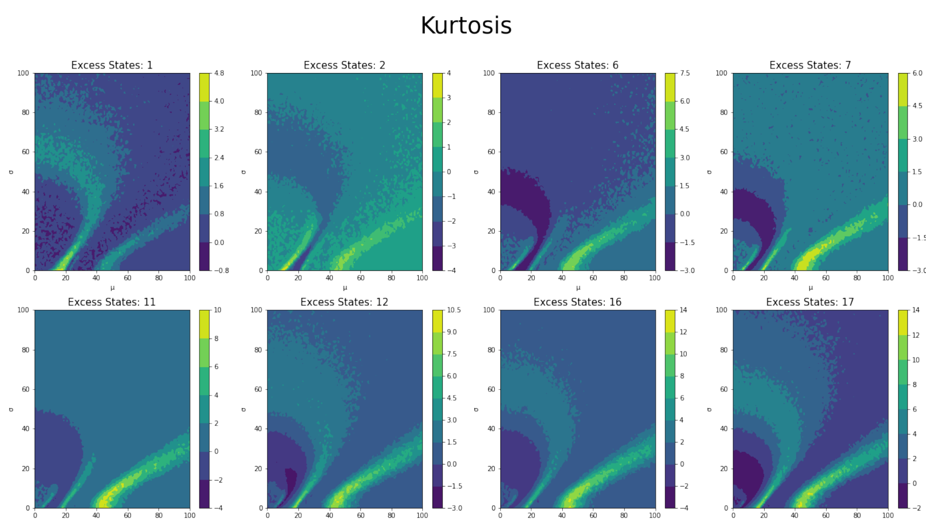


Figure 4.5

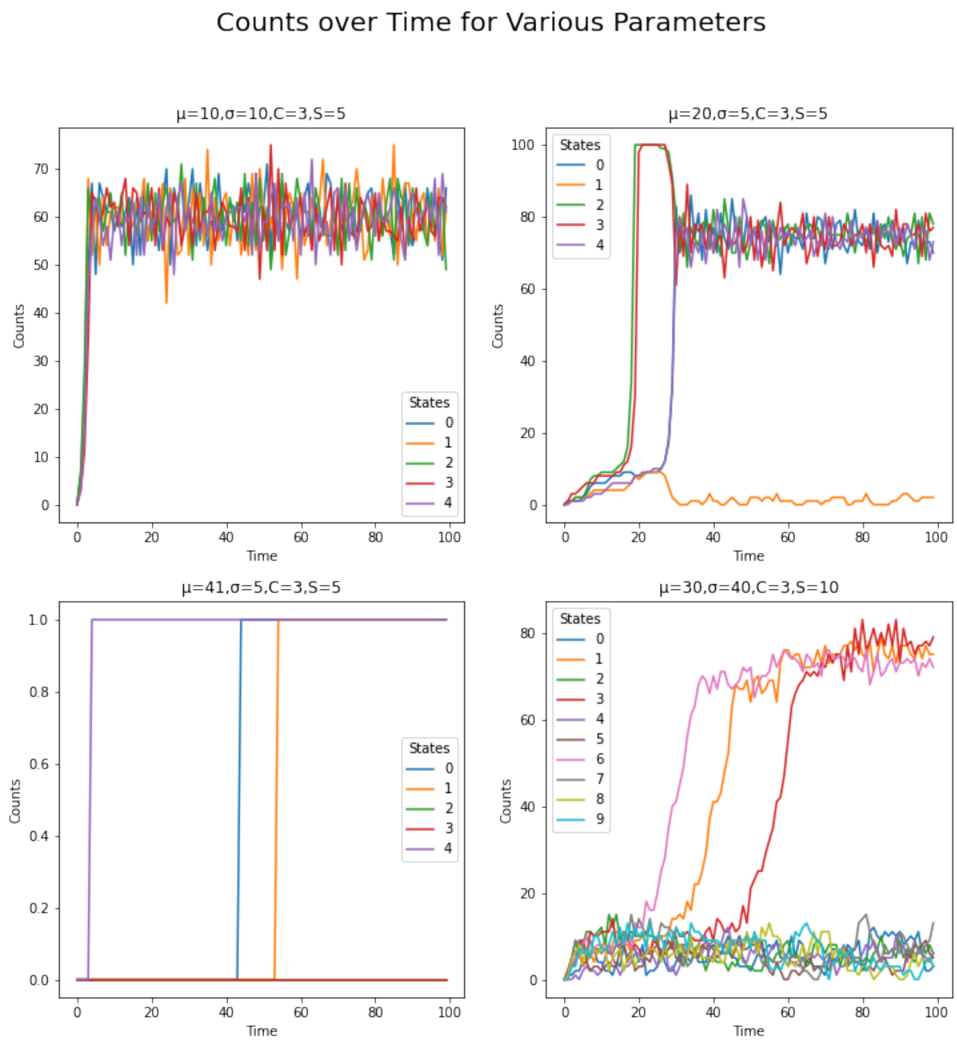

Counts over Time for Various Parameters
μ=10,σ=10,C=3,S=5
μ=20,σ=5,C=3,S=5
μ=41,σ=5,C=3,S=5
μ=30,σ=40,C=3,S=10
Counts
Time
States

Figure 4.6

4.3.2 Regressions

The primary independent variable of interest is excess states, which is S-C. The OLS regressions follow the form: $Y = \beta_1 ExcessStates + \beta_2\mu + \beta_3\sigma + c$. Tables 4.3 and 4.4 show the regression coefficients for the variable excess states for all outcomes of interest. All coefficients listed in this table are statistically significant at the 1% level. In Table 4.3 the columns Mean, Median, Standard Deviation,Mean Absolute Deviation, Max, and Range refer to the method of aggregation across states within a run. Given that the distribution of outcomes across states may be fat tailed differentiating between the Mean and Median as well as the Standard Deviation and Mean Absolute Deviation can signal whether the effects are driven by outliers across states. Aggregating via the maximum demonstrates how the maximum is directly effected, since it is expected to be an outlier, and the gap between the maximum and minimum shows the full range of the extremes. As the results in Table 4.3 show there were no differences in sign between the regressions using the Mean vs Median as the aggregation function and the signs are all negative. This suggests that increasing the number of excess stats is associated with less overall adoption of states, smaller maximum counts, shorter time to reach the maximum counts, smaller time averaged counts, smaller time variance in counts, smaller mimima after the maximum is reached and lower entropy.

Alternatively, the coefficient for the range aggregation has a positive coefficient for all of the outcomes of interest. With the exception of Time to Maximum and Variance in Counts, and Decay Time, the coefficients for the maximum aggregation are negative. In these cases, given the coefficients for the means, medians, and the range this implies that while the increase in excess states reduces the average and maximum level, the impact disproportional effects the minimum since the range increases. When looking at the number of zeros, the regression coefficient for excess states is also positive, providing additional evidence for disproportionate effect of increasing excess states on the minimum counts. For Time to Maximum and Variance in Counts, it is possible that the positive effect of excess states on the range is due to the increases in the maximum. The broadly suggests that increasing the number of states can widen the disparities between the popularity of the states.

Similar to the range, the regression coefficients for the standard deviation aggregation are positive for all outcomes. Macy and Evtushenko (2020) use standard deviation as a measure of unpredictability, but while they were measuring unpredictability across runs this measures unpredictability across states. This suggests that increasing the excess states makes all of the outcomes of interest less predictable. Mean absolute deviation was used as an alternative measure of dispersion and with the exception of variance in counts, minimum after maximum, and decay time the signs of the regression coefficients for excess states are the same as they are for standard deviation. It is possible that outliers play a role in this difference, since mean absolute deviation is less outlier sensitive.

The full regression tables are listed in Appendix B

	Mean	Median	Standard Deviation	Mean Absolute Deviation	Max	Range
Final Counts	-92.7826	-113.2765	5.7807	1.8463	-45.3861	31.9112
Max Counts	-1.1052	-1.3967	0.1345	0.0734	-0.3916	0.6076
Time to Max	-0.6973	-0.8792	0.3482	0.2892	0.4659	1.7882
Average Counts	-0.9278	-1.1328	0.0578	0.0185	-0.4539	0.3191
Variance in Counts	-4.1909	-5.8243	0.2933	-0.2283	1.1350	3.1665
Min after Max	-0.9768	-1.2578	0.0210	-0.0281	-0.4292	0.2506
Decay Time	-0.1589	-0.2224	0.0115	-0.0518	0.7465	0.8525
Entropy in Counts	-0.0537	-0.0593	0.0194	0.0155	-0.0120	0.0759

Table 4.3: Excess States Regression Coefficient

	Coefficient
Herfindahl Index	-0.0064
Kurtosis in Counts	0.1312
Number of Zero Counts	0.1599

Table 4.4: Excess States Regression Coefficient For Global Variables

4.4 Discussion

The parameter configurations shown in Figure 4.6 demonstrated four different competitive regimes. For example while both $\mu = 10, \sigma = 5, C = 3, S = 5$ and $\mu = 20, \sigma = 5, C = 3, S = 5$ are both highly competitive, there is a clear losing state in $\mu = 20, \sigma = 5, C = 3, S = 5$. On the other hand $\mu = 30, \sigma = 40, C = 3, S = 10$ is characterized by the emergence of a few clear winners with the rest of the states struggling to compete. As I mentioned earlier, in $\mu = 41, \sigma = 5, C = 3, S = 5$ adoption is very difficult, so the three states that end up getting adopted are only adopted once. This suggests that the threshold parameters do capture a rich enough space of possible outcomes.

One way to think of these results is in terms of observed and unobserved cascades, in the sense that we are more likely to observe cascades that succeed, but not those that fizzle. In the context of music, consider that for every artist that becomes an "overnight success" there may be many others toiling away in obscurity. In this sense the given states that do not achieve popularity could be thought of counter factually as ones that could have. Each of the cascading states was facing the same distribution of thresholds as each other, with the only differences in their outcomes being due to chance. This shows that large disparities in popularity can occur even without and underlying differences in the "quality" of the cascading state, and purely arise from structure. Due to the tradeoff between the different states imposed by the capacity constraint, single cascade models and multicascade models without the capacity constraint will miss the effect unpopular states have on successful states. This also suggests that just because something is popular, that does not imply it is high quality.

Since the capacity constraint provides a mechanism for agents to regularly remove states, this can help determine their behavior as they fade away. The decay time results suggest that while on average increases excess states reduces the decay time, it actually increases the maximum decay time. This suggests that the in more competitive environments the most stable states are even more stable. A similar pattern is shown with time to maximum, suggesting that in more competitive environments even though the average peak is earlier, the maximum peak is delayed. It is possible that this could be due to a sort of lock in effect, where once a state is adopted by a certain threshold of agents there is a minimum level it can no longer dip below. The analysis of minimum post maximum, suggest that this may not be the case as adding excess states decreases the maximum minimum post maximum. If there is lock in, this suggests that increasing excess states reduces the floor that states are locked in above.

4.5 Conclusion

These results suggest that the capacity constraint may play an important role in the diffusion dynamics of environments with multiple states that could potentially cascade. Increasing the number of total states in excess of capacity is associated with increased concentration of popularity, larger disparities between popular and unpopular states as well as greater unpredictability in which states will become popular, even while the popularity of a given state over time tended to become more predictable. Unsurprisingly increased competition from greater excess states tended to reduce average popularity overall, the heterogeneous impact suggests that capacity constraints may play a role in driving the superstar phenomena that Rosen (1981) describes. Since each of the states begin equally preferable, this suggests a mechanism for how random chance and structure can drive popularity as opposed to underlying value. Thus, more empirical work is needed to measure people's capacities as well as determine the influence of the capacity constraint on real systems.

Bibliography

Akkoyunlu, E. A., Ekanadham, K., and Huber, R. V. (1975). Some constraints and tradeoffs in the design of network communications. In *Proceedings of the fifth ACM symposium on Operating systems principles*, pages 67–74.

Albright, A. (2008). From clusters to words: grammatical models of nonce word acceptability. *Handout of talk presented at 82nd LSA, Chicago*.

Aldous, D. (2017). Elo ratings and the sports model: A neglected topic in applied probability? *Statistical Science*, 32(4):616–629.

Alwin, D. F. (1997). Feeling thermometers versus 7-point scales: Which are better? *Sociological Methods & Research*, 25(3):318–340.

Alwin, D. F. and Krosnick, J. A. (1991). The reliability of survey attitude measurement: The influence of question and respondent attributes. *Sociological Methods & Research*, 20(1):139–181.

Angrist, J. D. (2014). The perils of peer effects. *Labour Economics*, 30:98–108.

Aumann, R. J. (1976). Agreeing to disagree. *The annals of statistics*, 4(6):1236–1239.

Axelrod, R. (1980). Effective choice in the prisoner's dilemma. *Journal of conflict resolution*, 24(1):3–25.

Axelrod, R. (1981). The emergence of cooperation among egoists. *American political science review*, 75(2):306–318.

Axelrod, R. and Hamilton, W. D. (1981). The evolution of cooperation. *science*, 211(4489):1390–1396.

Bago, B., Rand, D. G., and Pennycook, G. (2020). Fake news, fast and slow: Deliberation reduces belief in false (but not true) news headlines. *Journal of experimental psychology: general*, 149(8):1608.

Bailey, T. M. and Hahn, U. (2001). Determinants of wordlikeness: Phonotactics or lexical neighborhoods? *Journal of Memory and Language*, 44(4):568–591.

Bartlett, C. J., Quay, L. C., and Wrightsman Jr, L. S. (1960). A comparison of two methods of attitude measurement: Likert-type and forced choice. *Educational and Psychological Measurement*, 20(4):699–704.

Bartram, D. (2007). Increasing validity with forced-choice criterion measurement formats. *International Journal of Selection and Assessment*, 15(3):263–272.

Bass, F. M. (1969). A new product growth for model consumer durables. *Management science*, 15(5):215–227.

Bauch, C. T. and Galvani, A. P. (2013). Social factors in epidemiology. *Science*, 342(6154):47–49.

Bertrand, M. and Mullainathan, S. (2004). Are emily and greg more employable than lakisha and jamal? a field experiment on labor market discrimination. *American economic review*, 94(4):991–1013.

Billig, M. and Tajfel, H. (1973). Social categorization and similarity in intergroup behaviour. *European Journal of Social Psychology*, 3(1):27–52.

Block, H. D., Marschak, J., et al. (1959). Random orderings and stochastic theories of response. Technical report, Cowles Foundation for Research in Economics, Yale University.

Bonneau, J. (2012). The science of guessing: analyzing an anonymized corpus of 70 million passwords. In *Security and Privacy (SP), 2012 IEEE Symposium on*, pages 538–552. IEEE.

Boorman, S. A. (1974). Island models for takeover by a social trait facing a frequency-dependent selection barrier in a mendelian population. *Proceedings of the National Academy of Sciences*, 71(5):2103–2107.

Booth, A. L., Leigh, A., and Varganova, E. (2012). Does ethnic discrimination vary across minority groups? evidence from a field experiment. *Oxford Bulletin of Economics and Statistics*, 74(4):547–573.

Braun, N. (1995). Individual thresholds and social diffusion. *Rationality and Society*, 7(2):167–182.

Buterin, V. (2014). A next-generation smart contract and decentralized application platform.

Cacioppo, J. T., Fowler, J. H., and Christakis, N. A. (2009). Alone in the crowd: the structure and spread of loneliness in a large social network. *Journal of personality and social psychology*, 97(6):977.

Carifio, J. and Perla, R. (2008). Resolving the 50-year debate around using and misusing likert scales. *Medical education*, 42(12):1150–1152.

Carpusor, A. G. and Loges, W. E. (2006). Rental discrimination and ethnicity in names. *Journal of Applied Social Psychology*, 36(4):934–952.

Carrell, S. E., Sacerdote, B. I., and West, J. E. (2013). From natural variation to optimal policy? the importance of endogenous peer group formation. *Econometrica*, 81(3):855–882.

Castro, M. and Liskov, B. (2002). Practical byzantine fault tolerance and proactive recovery. *ACM Transactions on Computer Systems (TOCS)*, 20(4):398–461.

Cattelan, M. (2012). Models for paired comparison data: A review with emphasis on dependent data. *Statistical Science*, pages 412–433.

Centola, D., Eguíluz, V. M., and Macy, M. W. (2007). Cascade dynamics of complex propagation. *Physica A: Statistical Mechanics and its Applications*, 374(1):449–456.

Centola, D. and Macy, M. (2007). Complex contagions and the weakness of long ties. *American journal of Sociology*, 113(3):702–734.

Chomsky, N. and Halle, M. (1968). The sound pattern of english.

Christakis, N. A. and Fowler, J. H. (2007). The spread of obesity in a large social network over 32 years. *n engl j med*, 2007(357):370–379.

Christakis, N. A. and Fowler, J. H. (2008). The collective dynamics of smoking in a large social network. *New England journal of medicine*, 358(21):2249–2258.

Christakis, N. A. and Fowler, J. H. (2013). Social contagion theory: examining dynamic social networks and human behavior. *Statistics in medicine*, 32(4):556–577.

Chwe, M. S.-Y. (2013). *Rational ritual*. Princeton University Press.

Classen, D. C., Resar, R., Griffin, F., Federico, F., Frankel, T., Kimmel, N., Whittington, J. C., Frankel, A., Seger, A., and James, B. C. (2011). 'global trigger tool'shows that adverse events in hospitals may be ten times greater than previously measured. *Health affairs*, 30(4):581–589.

Cohen, M. D., March, J. G., and Olsen, J. P. (1972). A garbage can model of organizational choice. *Administrative science quarterly*, pages 1–25.

Coleman, J. and Pierrehumbert, J. (1997). Stochastic phonological grammars and acceptability. *arXiv preprint cmp-lg/9707017*.

Cooper, R., DeJong, D. V., Forsythe, R., and Ross, T. W. (1989). Communication in the battle of the sexes game: some experimental results. *The RAND Journal of Economics*, pages 568–587.

Dawes, R. M. (1980). Social dilemmas. *Annual review of psychology*, 31(1):169–193.

Derechin, J., Sankaran, S., and Christakis, N. A. (2021). Accurate text generating inaccurate beliefs: a study on the effect of publishing beach pictures alongside articles about covid-19.

Diehl, M. (1990). The minimal group paradigm: Theoretical explanations and empirical findings. *European review of social psychology*, 1(1):263–292.

Dingemanse, M., Blasi, D. E., Lupyan, G., Christiansen, M. H., and Monaghan, P. (2015). Arbitrariness, iconicity, and systematicity in language. *Trends in cognitive sciences*, 19(10):603–615.

Duckworth, K. L., Bargh, J. A., Garcia, M., and Chaiken, S. (2002). The automatic evaluation of novel stimuli. *Psychological science*, 13(6):513–519.

Edelman, B., Luca, M., and Svirsky, D. (2017). Racial discrimination in the sharing economy: Evidence from a field experiment. *American Economic Journal: Applied Economics*, 9(2):1–22.

Elo, A. E. (1978). *The rating of chessplayers, past and present.* Arco Pub.

Epstein, J. M., Parker, J., Cummings, D., and Hammond, R. A. (2008). Coupled contagion dynamics of fear and disease: mathematical and computational explorations. *PLoS One*, 3(12):e3955.

Foramitti, J. (2021). Agentpy: A package for agent-based modeling in python. *Journal of Open Source Software*, 6(62):3065.

Forastiere, L., Airoldi, E. M., and Mealli, F. (2021). Identification and estimation of treatment and interference effects in observational studies on networks. *Journal of the American Statistical Association*, 116(534):901–918.

Fowler, J. H. and Christakis, N. A. (2008). Dynamic spread of happiness in a large social network: longitudinal analysis over 20 years in the framingham heart study. *Bmj*, 337:a2338.

Frisch, S. A., Large, N. R., and Pisoni, D. B. (2000). Perception of wordlikeness: Effects of segment probability and length on the processing of nonwords. *Journal of memory and language*, 42(4):481.

Fryer Jr, R. G. and Levitt, S. D. (2004). The causes and consequences of distinctively black names. *The Quarterly Journal of Economics*, 119(3):767–805.

Fu, F., Christakis, N. A., and Fowler, J. H. (2017). Dueling biological and social contagions. *Scientific Reports*, 7.

Gliem, J. A. and Gliem, R. R. (2003). Calculating, interpreting, and reporting cronbach's alpha reliability coefficient for likert-type scales. Midwest Research-to-Practice Conference in Adult, Continuing, and Community Education.

Goel, S., Anderson, A., Hofman, J., and Watts, D. J. (2015). The structural virality of online diffusion. *Management Science*, 62(1):180–196.

Granovetter, M. (1978). Threshold models of collective behavior. *American journal of sociology*, 83(6):1420–1443.

Granovetter, M. and Soong, R. (1983). Threshold models of diffusion and collective behavior. *Journal of Mathematical sociology*, 9(3):165–179.

Griliches, Z. (1957). Hybrid corn: An exploration in the economics of technological change. *Econometrica, Journal of the Econometric Society*, pages 501–522.

Halpern, J. Y. and Moses, Y. (1990). Knowledge and common knowledge in a distributed environment. *Journal of the ACM (JACM)*, 37(3):549–587.

Hamlen Jr, W. A. (1991). Superstardom in popular music: Empirical evidence. *The Review of Economics and Statistics*, pages 729–733.

Hamming, R. W. (1950). Error detecting and error correcting codes. *The Bell system technical journal*, 29(2):147–160.

Hammond, M. (1999). *The Phonology of English: A Prosodic Optimality-Theoretic Approach: A Prosodic Optimality-Theoretic Approach*. Oxford University Press, UK.

Hammond, M. (2004). Gradience, phonotactics, and the lexicon in english phonology. *International Journal of English Studies*, 4(2):1–24.

Hedström, P. (1994). Contagious collectivities: On the spatial diffusion of swedish trade unions, 1890-1940. *American journal of sociology*, 99(5):1157–1179.

Helbing, D., Buzna, L., Johansson, A., and Werner, T. (2005). Self-organized pedestrian crowd dynamics: Experiments, simulations, and design solutions. *Transportation science*, 39(1):1–24.

Helbing, D. and Molnar, P. (1995). Social force model for pedestrian dynamics. *Physical review E*, 51(5):4282.

Herfindahl, O. C. (1950). *Concentration in the steel industry*. Thesis.

Heywood, S. (1972). The popular number seven or number preference. *Perceptual and Motor Skills*, 34(2):357–358.

Hill, A. L., Rand, D. G., Nowak, M. A., and Christakis, N. A. (2010). Emotions as infectious diseases in a large social network: the sisa model. *Proceedings of the Royal Society of London B: Biological Sciences*, 277(1701):3827–3835.

Hirschman, A. O. (1945). *National power and the structure of foreign trade*, volume 105. Univ of California Press.

Hébert-Dufresne, L., Scarpino, S. V., and Young, J.-G. (2020). Macroscopic patterns of interacting contagions are indistinguishable from social reinforcement. *Nature Physics*, pages 1–6.

Jabin, P.-E. and Junca, S. (2015). A continuous model for ratings. *SIAM Journal on Applied Mathematics*, 75(2):420–442.

Jackson, D. N., Wroblewski, V. R., and Ashton, M. C. (2000). The impact of faking on employment tests: Does forced choice offer a solution? *Human Performance*, 13(4):371–388.

Jamieson, S. et al. (2004). Likert scales: how to (ab) use them. *Medical education*, 38(12):1217–1218.

Kam, C. C. S. (2018). Why do we still have an impoverished understanding of the item wording effect? an empirical examination. *Sociological Methods & Research*, 47(3):574–597.

Kaushal, R., Shojania, K. G., and Bates, D. W. (2003). Effects of computerized physician order entry and clinical decision support systems on medication safety: a systematic review. *Archives of internal medicine*, 163(12):1409–1416.

Kermack, W. O. and McKendrick, A. G. (1927). A contribution to the mathematical theory of epidemics. In *Proceedings of the Royal Society of London A: mathematical, physical and engineering sciences*, volume 115, pages 700–721. The Royal Society.

Krueger, A. B. (2005). The economics of real superstars: The market for rock concerts in the material world. *Journal of Labor Economics*, 23(1):1–30.

Kubovy, M. and Psotka, J. (1976). The predominance of seven and the apparent spontaneity of numerical choices. *Journal of Experimental Psychology: Human Perception and Performance*, 2(2):291.

Kuran, T. (1991). Now out of never: The element of surprise in the east european revolution of 1989. *World politics*, 44(1):7–48.

Lamport, L., Shostak, R., and Pease, M. (1982). The byzantine generals problem. *ACM Transactions on Programming Languages and Systems*, 4(3):382–401.

Lazer, D. M., Baum, M. A., Benkler, Y., Berinsky, A. J., Greenhill, K. M., Menczer, F., Metzger, M. J., Nyhan, B., Pennycook, G., and Rothschild, D. (2018). The science of fake news. *Science*, 359(6380):1094–1096.

Leape, L. L. (1997). A systems analysis approach to medical error. *Journal of evaluation in clinical practice*, 3(3):213–222.

Leape, L. L. (2000). Institute of medicine medical error figures are not exaggerated. *Jama*, 284(1):95–97.

Levenshtein, V. I. (1966). Binary codes capable of correcting deletions, insertions, and reversals. In *Soviet physics doklady*, volume 10, pages 707–710. Soviet Union.

Lieberman, E., Hauert, C., and Nowak, M. A. (2005). Evolutionary dynamics on graphs. *Nature*, 433(7023):312–316.

Lieberson, S. and Bell, E. O. (1992). Children's first names: An empirical study of social taste. *American Journal of Sociology*, 98(3):511–554.

Lieberson, S. and Mikelson, K. S. (1995). Distinctive african american names: An experimental, historical, and linguistic analysis of innovation. *American Sociological Review*, pages 928–946.

Likert, R. (1932). A technique for the measurement of attitudes. *Archives of psychology*.

Lupyan, G. and Casasanto, D. (2015). Meaningless words promote meaningful categorization. *Language and Cognition*, 7(2):167–193.

Macy, M. and Evtushenko, A. (2020). Threshold models of collective behavior ii: The predictability paradox and spontaneous instigation. *Sociological Science*, 7:628–648.

Macy, M. W. (1991). Chains of cooperation: Threshold effects in collective action. *American Sociological Review*, pages 730–747.

Manski, C. F. (1993). Identification of endogenous social effects: The reflection problem. *The review of economic studies*, 60(3):531–542.

Mas-Colell, A., Whinston, M. D., Green, J. R., et al. (1995). *Microeconomic theory*, volume 1. Oxford university press New York.

McKnight, M. E. and Christakis, N. A. (2016). Breadboard: Software for online social experiments.

Mednick, S. C., Christakis, N. A., and Fowler, J. H. (2010). The spread of sleep loss influences drug use in adolescent social networks. *PloS one*, 5(3):e9775.

Mehta, J., Starmer, C., and Sugden, R. (1994). The nature of salience: An experimental investigation of pure coordination games. *The American Economic Review*, 84(3):658–673.

Michel, J.-B., Shen, Y. K., Aiden, A. P., Veres, A., Gray, M. K., Pickett, J. P., Hoiberg, D., Clancy, D., Norvig, P., Orwant, J., et al. (2011). Quantitative analysis of culture using millions of digitized books. *science*, 331(6014):176–182.

Molander, P. (1985). The optimal level of generosity in a selfish, uncertain environment. *Journal of Conflict Resolution*, 29(4):611–618.

Morris, S. and Shin, H. S. (1997). Approximate common knowledge and co-ordination: Recent lessons from game theory. *Journal of Logic, Language and Information*, 6(2):171–190.

Nakamoto, S. (2008). A peer-to-peer electronic cash system.

Nowak, M. and Sigmund, K. (1993). A strategy of win-stay, lose-shift that outperforms tit-for-tat in the prisoner's dilemma game. *Nature*, 364(6432):56–58.

Nowak, M. A. and May, R. M. (1992). Evolutionary games and spatial chaos. *Nature*, 359(6398):826–829.

Nowak, M. A. and Sigmund, K. (1992). Tit for tat in heterogeneous populations. *Nature*, 355(6357):250–253.

Nuttin Jr, J. M. (1985). Narcissism beyond gestalt and awareness: The name letter effect. *European Journal of Social Psychology*, 15(3):353–361.

Ohala, J. J. and Ohala, M. (1986). Testing hypotheses regarding the psychological manifestation of morpheme structure constraints. *Experimental phonology*, pages 239–252.

Ohtsuki, H., Hauert, C., Lieberman, E., and Nowak, M. A. (2006). A simple rule for the evolution of cooperation on graphs and social networks. *Nature*, 441(7092):502–505.

Pearl, J. (2017). Detecting latent heterogeneity. *Sociological Methods & Research*, 46(3):370–389.

Pennycook, G., Cannon, T. D., and Rand, D. G. (2018). Prior exposure increases perceived accuracy of fake news. *Journal of experimental psychology: general*, 147(12):1865.

Pennycook, G., Cheyne, J. A., Barr, N., Koehler, D. J., and Fugelsang, J. A. (2015). On the reception and detection of pseudo-profound bullshit. *Judgment and Decision making*, 10(6):549–563.

Pennycook, G., McPhetres, J., Zhang, Y., Lu, J. G., and Rand, D. G. (2020). Fighting covid-19 misinformation on social media: Experimental evidence for a scalable accuracy-nudge intervention. *Psychological science*, 31(7):770–780.

Pennycook, G. and Rand, D. G. (2019). Lazy, not biased: Susceptibility to partisan fake news is better explained by lack of reasoning than by motivated reasoning. *Cognition*, 188:39–50.

Perra, N., Balcan, D., Gonçalves, B., and Vespignani, A. (2011). Towards a characterization of behavior-disease models. *PloS one*, 6(8):e23084.

Perrow, C. (1983). The organizational context of human factors engineering. *Administrative science quarterly*, pages 521–541.

Perrow, C. (1994). The limits of safety: the enhancement of a theory of accidents. *Journal of contingencies and crisis management*, 2(4):212–220.

Perrow, C. (1999). Organizing to reduce the vulnerabilities of complexity. *Journal of contingencies and crisis management*, 7(3):150–155.

Perrow, C. (2011). *Normal accidents.* Princeton university press.

Peterson, R. A. and Berger, D. G. (1975). Cycles in symbol production: The case of popular music. *American sociological review*, pages 158–173.

Peterson, R. A. and Berger, D. G. (1996). Measuring industry concentration, diversity, and innovation in popular music. *American Sociological Review*, 61(1):175–178.

Rand, D. G., Nowak, M. A., Fowler, J. H., and Christakis, N. A. (2014). Static network structure can stabilize human cooperation. *Proceedings of the National Academy of Sciences*, 111(48):17093–17098.

Rapoport, A., Chammah, A. M., and Orwant, C. J. (1965). *Prisoner's dilemma: A study in conflict and cooperation*, volume 165. University of Michigan press.

Ray, J. J. (1980). The comparative validity of likert, projective, and forced-choice indices of achievement motivation. *The Journal of Social Psychology*, 111(1):63–72.

Reason, J. (2000). Human error: models and management. *Bmj*, 320(7237):768–770.

Revilla, M. A., Saris, W. E., and Krosnick, J. A. (2014). Choosing the number of categories in agree–disagree scales. *Sociological Methods & Research*, 43(1):73–97.

Riddle, B. L., Miron, M. S., and Semo, J. A. (1989). Passwords in use in a university timesharing environment. *Computers & Security*, 8(7):569–579.

Roettgers, J. (2017). Netflix replacing star ratings with thumbs ups and thumbs downs. *Variety*. `http://variety.com/2017/digital/news/netflix-thumbs-vs-stars-1202010492/`.

Romero, D. M., Meeder, B., and Kleinberg, J. (2011). Differences in the mechanics of information diffusion across topics: idioms, political hashtags, and complex contagion on twitter. In *Proceedings of the 20th international conference on World wide web*, pages 695–704.

Rosen, S. (1981). The economics of superstars. *The American economic review*, 71(5):845–858.

Rosenquist, J. N., Fowler, J. H., and Christakis, N. A. (2011). Social network determinants of depression. *Molecular psychiatry*, 16(3):273–281.

Rossman, G. and Fisher, J. C. (2021). Network hubs cease to be influential in the presence of low levels of advertising. *Proceedings of the National Academy of Sciences*, 118(7).

Rubinstein, A. (1989). The electronic mail game: Strategic behavior under" almost common knowledge". *The American Economic Review*, pages 385–391.

Salganik, M. J., Dodds, P. S., and Watts, D. J. (2006). Experimental study of inequality and unpredictability in an artificial cultural market. *science*, 311(5762):854–856.

Salganik, M. J. and Watts, D. J. (2008). Leading the herd astray: An experimental study of self-fulfilling prophecies in an artificial cultural market. *Social psychology quarterly*, 71(4):338–355.

Samet, D. (1990). Ignoring ignorance and agreeing to disagree. *Journal of Economic Theory*, 52(1):190–207.

Sankaran, S., Derechin, J., and Christakis, N. A. (2021). Curmelo: The theory and practice of a forced-choice approach to producing preference rankings. *PloS one*, 16(5):e0252145.

Schelling, T. C. (1960). *The strategy of conflict*. Harvard university press.

Shannon, C. E. (1948). A mathematical theory of communication. *The Bell system technical journal*, 27(3):379–423.

Shirado, H. and Christakis, N. A. (2017). Locally noisy autonomous agents improve global human coordination in network experiments. *Nature*, 545(7654):370–374.

Smith, J. M. (1974). The theory of games and the evolution of animal conflicts. *Journal of theoretical biology*, 47(1):209–221.

Sturgis, P., Roberts, C., and Smith, P. (2014). Middle alternatives revisited: how the neither/nor response acts as a way of saying "i don't know"? *Sociological Methods & Research*, 43(1):15–38.

Tajfel, H. (1970). Experiments in intergroup discrimination. *Scientific American*, 223(5):96–103.

Valente, T. W. (1996). Social network thresholds in the diffusion of innovations. *Social networks*, 18(1):69–89.

Van de Rijt, A. (2019). Self-correcting dynamics in social influence processes. *American journal of sociology*, 124(5):1468–1495.

Wakeham, J. (2017). Bullshit as a problem of social epistemology. *Sociological Theory*, 35(1):15–38.

Watts, D. J. (2011). *A simple model of global cascades on random networks*, pages 497–502. Princeton University Press.

Watts, D. J. and Strogatz, S. H. (1998). Collective dynamics of 'small-world'networks. *nature*, 393(6684):440–442.

Weingart, N. S., Wilson, R. M., Gibberd, R. W., and Harrison, B. (2000). Epidemiology of medical error. *Bmj*, 320(7237):774–777.

Welch, B. L. (1947). The generalization of 'student's'problem when several different population varlances are involved. *Biometrika*, 34(1-2):28–35.

White, H. C. (1973). Everyday life in stochastic networks. *Sociological Inquiry*, 43(1):43–49.

Wright, R. (2004). A review of perceptual cues and cue robustness. *Phonetically based phonology*, pages 34–57.

Yeh, Y. (2001). Safety critical avionics for the 777 primary flight controls system. In *20th DASC. 20th Digital Avionics Systems Conference (Cat. No. 01CH37219)*, volume 1, pages 1C2/1–1C2/11 vol. 1. IEEE.

Zhang, Y., Wu, H., and Cheng, L. (2012). Some new deformation formulas about variance and covariance. In *2012 Proceedings of International Conference on Modelling, Identification and Control*, pages 987–992. IEEE.

Contents

Appendix A

BFT Instructions and Quiz

A.1 Slides for the Generals Instructions

Below are images of the slides used for the generals skin. In the actual game this is displayed to the player as an HTML slideshow and players can freely move back and forth throughout the slide show. We programmaticly generated these slideshows and Section A.2 shows the tables of the text that was swapped between skins.

In this game, you are a General in an army.

The point of the game is to choose an enemy Fort for all of the Generals to attack together.

Each General has been sent to scout out a different enemy Fort.

Each Fort has been judged to be equally strategically useful, but only one is to be attacked.

Each General has a different four-letter name, and each Fort has a different five-letter name.

At the start of the game, each General only knows the name of the Fort they have initially been sent to.

You are General **IVEK**

The code of the place you have been sent to is: **PESAM**

The current round is: 1

These names are examples.

You will see your name and the name of the Fort you have been given to on the right side of your screen when the game begins.

Your objective is to get all of the Generals to agree on a single enemy Fort to attack.

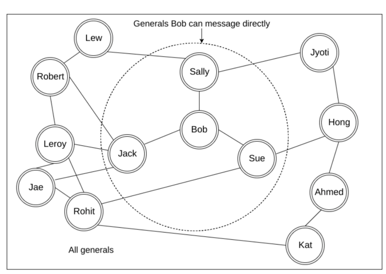
The enemy has large numbers of soldiers at each Fort, so all Generals must choose the same Fort to attack in order for the attack to succeed.

It does not matter which Fort is chosen as long as all Generals agree on the same Fort. All Forts are equally strategically important.

You do not get an additional bonus if the Fort whose name you were initially given is chosen. You only get the full bonus if all Generals agree on the same Fort.

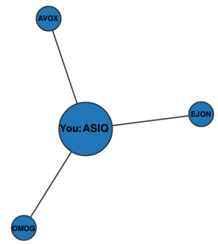
You will coordinate with the other Generals by sending and receiving messages.

You will only be able to directly message some of the Generals.



You are indirectly connected to all the Generals through the Generals you can directly message.

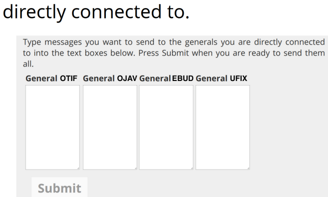
On the left side of your screen, you will see an image containing the names of the Generals you can directly message.



This diagram is an example of what you will see. Note that there may be other Generals in the game.

The game has 10 rounds.

During each round, you will first send messages to all Generals you are directly connected to.



This is a screenshot of the message interface.

You can send any text message you want to each of the Generals you are directly connected to.

Next, you will see all of the messages you have received during this round.

You are General IVEK

The code of the place you have been sent to is: PESAM

The current round is: 1

Below are the messages the generals you are directly connected to have sent you this round. Press next when you are ready to continue.

General ANIX has sent you the message: **Example**

General AWEK has sent you the message: **Example**

General EPEK has sent you the message: **Example**

General IZUS has sent you the message: **Example**

You must send all your messages for each round within one minute and thirty seconds.

There is a timer at the top of your screen showing how much time you have left.

In this example, there are 42 seconds left.

If you do not send your messages in the allotted time, you will be dropped from the game.

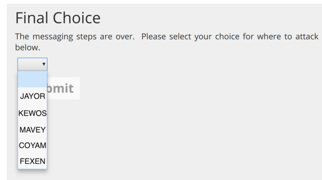
You have been dropped for being idle.

Please return this HIT.

If you have any questions or feedback please contact: yins.amt@gmail.com

This is a screenshot of the message you will receive if you are dropped for being idle.

After the 10 rounds, each General will be asked to choose which Fort they want to attack.



These Fort names are examples and will be different in your game.

You will receive the full bonus of $8.00 if all Generals choose the same Fort, and thus that Fort is successfully attacked,

If you finish the game, but all Generals do not choose the same Fort, and thus no Fort is successfully attacked. your bonus will only be $4.00.

If you are dropped from the game, you will not receive a bonus.

You must pass the comprehension quiz to be eligible for any payment.

Click the 'Start Quiz' button below to begin the quiz.

A.2 Skin Difference Tables

Tables A.1 through A.10 show the textual difference between the instructions for each skin. Each row represents a point where the text is swapped. So in the restaurant skin the title page is: Restaurant Game, while in the Olympics skin the title page is: Olympic Host City Selection Game. The images in each set of instructions are also modified for each skin to represent the players and the target of consensus. Players are Generals in the generals skin, Friends in the restaurant skin, Delegates in the olympics skin, etc. While in the generals skin the target is a Fort, in the restaurant skin the target is a Restaurant, in the olympics skin the target is a Host City, etc.

General's Coordination Game
In this game, you are a General in an army.
The point of the game is to choose an enemy Fort for all of the Generals to attack together.
Each General has been sent to scout out a different enemy Fort.
Each Fort has been judged to be equally strategically useful, but only one is to be attacked.
At the start of the game, each General only knows the name of the Fort they have initially been sent to.
You will see your General name and the name of the enemy Fort you have been initially assigned on the right side of your screen when the game begins.
Your objective is to get all of the Generals to agree on a single enemy Fort to attack.
The enemy has large numbers of soldiers at each Fort, so all Generals must choose the same Fort to attack in order for the attack to succeed.
, and thus that Fort is successfully attacked,
, and thus no Fort is successfully attacked.
is chosen
equally strategically important
General
General's Coordination game
Forts
Fort
to attack

Table A.1: Instruction Changes for Generals Skin

Restaurant Game
In this game, you are a Friend in a friend group trying to plan a group dinner.
The point of the game is to choose a Restaurant to eat at together.
Each Friend knows about one Restaurant.
Each Restaurant has been judged to be equally good, but only one may be chosen to attend all together.
At the start of the game each Friend knows the name of only one Restaurant they have been initially sent to.
You will see your Friend name and the name of the Restaurant you have been initially assigned on the right side of your screen when the game begins.
Your objective is to get all of your Friends to choose a single Restaurant to eat at.
Your Friend group decided long ago that all decisons must be unanimous, so all Friends must choose the same Restaurant, otherwise the dinner will be cancelled.
, and thus that dinner happens.
, and thus the dinner does not happen,
is chosen
equally good
Friend
Restaurant game
Restaurants
Restaurant
to choose

Table A.2: Instruction Changes for Restaurant Skin

Olympic Host City Selection Game
In this game, you are a Delegate on the Olympic Host City Selection Committee.
The point of the game is to choose a Host City for the next Olympics.
Each Delegate has been given the name of one qualified potential Host City.
Each Host City has been judged to be equally good, but only one is to be chosen.
At the start of the game, each Delegate is given the name of one Host City they have initially considered.
You will see your Delegate name and the name of the Host City you have been initially given on the right side of your screen when the game begins.
Your objective is to get all of the Delegates to choose a single Host City for the next Olympics.
Due to allegations of corruption in the Olympic Host Selection Committee, all Delegates must choose the same Host City, otherwise the Olympics will be cancelled.
and thus the Olympics will proceed as cheduled.
, and thus the Olympics do not happen,
is selected
equally good
Delegate
Host Selection game
Host Cities
Host City
to select

Table A.3: Instruction Changes for Olympics Skin

Layoff Game
In this game, you are a Manager at a workplace.
The point of the game is to choose a Worker to fire during a round of layoffs.
Each Manager has nominated one low-performing Worker to be fired.
Each low-performing Worker has been judged to be equally bad, but only one is to be fired.
At the start of the game, each Manager only knows the name of the Worker they have initially nominated.
You will see your Manager name and the name of the low-performing Worker you have been initially assigned on the right side of your screen when the game begins.
Your objective is to get all of the Managers to agree on a single low-performing Worker to fire.
For liability reasons, all firing decisions must be unanimous, so all Managers must choose to fire the same low-performing Worker, otherwise they themselves will be fired by their bosses.
, and thus that Worker is fired.
, and thus all the Managers (including you) are fired,
is chosen
equally bad
Manager
Layoff Game
Workers
Worker
to choose

Table A.4: Instruction Changes for Firing Skin

Hiring Game
In this game, you are a Manager at a workplace.
The point of the game is to choose a Candidate to hire to replace a critically important employee who has left.
Each Manager has nominated one Candidate to be hired.
Each Candidate has been judged to be equally good, but only one is to be hired.
At the start of the game, each Manager only knows the name of the Candidate they have initially nominated.
You will see your Manager name and the name of the Candidate you have been initially assigned on the right side of your screen when the game begins.
Your objective is to get all of the Managers to agree on a single Candidate to hire.
For liability reasons, all hiring decisions must be unanimous, so all Managers must choose to hire the same Candidate, otherwise no hire will be made and the company will shut down.
, and thus that Candidate is hired.
, and thus the company shuts down,
is chosen
equally good
Manager
Hiring game
Candidates
Candidate
to choose

Table A.5: Instruction Changes for Hiring Skin

Mascot Selection Game
In this game, you are a Member of a city's Council.
The point of the game is to choose a Mascot that all Members agree on.
Each Member has nominated one Mascot.
Each Mascot has been judged to be equally good, but only one may be selected.
At the start of the game, each Member only knows the name of one Mascot they have been initally assigned.
You will see your Member name and the name of the Mascot you have been initially assigned on the right side of your screen when the game begins.
Your objective is to get all of the Members to agree on a single Mascot to select.
Votes in the City Council must be unanimous, so all Members must choose the same Mascot in order for it to be selected.
, and thus that Mascot is selected.
, and thus no Mascot is selected,
is selected
equally good
Member
Mascot Selection Game
Mascots
Mascot
to select

Table A.6: Instruction Changes for Mascot Skin

Venue Choice Game
In this game, you are a Member of an Orchestra.
The point of the game is to choose a Venue for the Orchestra to perform at during their scheduled tour stop in an unfamiliar country.
Each Member knows about one potential Venue in the country.
Each Venue has been judged to be equally good, but only one may be chosen.
At the start of the game, each Member only knows the name of one Venue, they have been initially assigned.
You will see your Member name and the name of the Venue you have been initially assigned on the right side of your screen when the game begins.
Your objective is to get all of the Members to agree on a single Venue to choose for the performance.
Your Orchestra decided long ago that all decisions must be unanimous, so all Members must choose the same Venue, otherwise the performance will be canceled.
, and thus that performance happens.
, and thus the performance is canceled,
is selected
equally good
Member
Venue Choice Game
Venues
Venue
to select

Table A.7: Instruction Changes for Orchestra Skin

Plant Naming Game
In this game, you are a Member of the International Horticultural Society, which is a global organization of plant experts.
The point of the game is to select a Plant Name for an exciting new species of plant that has just been discovered.
Each Member has shortlisted one potential Plant Name from a randomly assigned region of the world they have been given responsibility for.
Each Plant Name has been judged to be equally good, but only one may be chosen.
Each Member starts the game knowing only the potential Plant Name they shortlisted.
You will see your Member name and the Plant Name you have been initially assigned on the right side of your screen when the game begins.
Your objective is to get all of the Members to agree on a single Plant Name.
By the code of the International Horticultural Society, all naming decisions must be unanimous, so all Members must select the same Plant Name or the plant will not be named at all, leaving it doomed to be ignored in future research.
, and thus the plant is named.
, and thus the plant is not named,
is selected
equally good
Member
Plant Naming Game
Plant Names
Plant Name
to select

Table A.8: Instruction Changes for Plant Skin

Space Exploration Game
In this game, you are an Astronaut on a large spaceship with a mission to establish a base on a habitable Planet.
The point of this game is to choose a Planet to land on.
Each Astronaut has been sent out on an small space dinghy to explore a different potential Planet to land on.
Each Planet explored by the surviving Astronauts have been judged to be equally habitable, but only one may be chosen to establish a base on.
At the start of the game each Astronaut only knows the name of the Planet they have been initially assigned.
You will see your Astronaut name and the Planet you have been initially assigned on the right side of your screen when the game begins.
Your objective is to get all of the Astronauts to agree on one Planet to land on.
Under the Universal Code of Space Exploration, all landings must be agreed upon unanimously. The large spaceship is running out of food and fuel, so if a unanimous decision is not reached, the ship will just drift in space and all passengers onboard will die from hunger.
, and thus the spaceship lands.
, and thus all the passengers on the spaceship die,
is chosen
equally habitable
Astronaut
Space Exploration Game
Planets
Planet
to land on

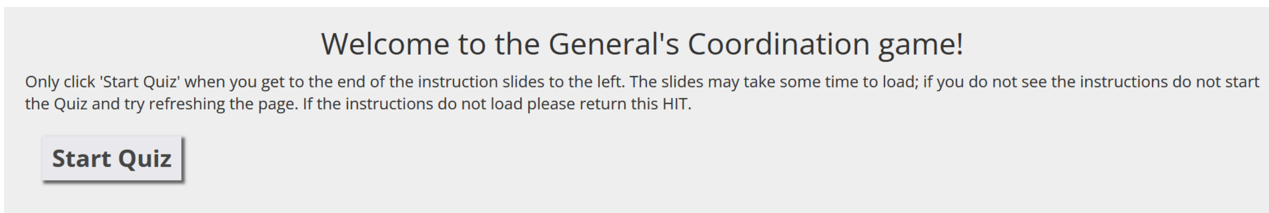
Table A.9: Instruction Changes for Space Skin

Leader Election Game
In this game, you are a member of the Electoral Council of the country Maximus Democraticus. Members of the Electoral Council are known as Electors.
The point of the game is to elect a new Leader for the nation.
Each Elector has been given the name of one potential Leader.
Each Leader remaining at this stage has been judged to be equally qualified, but only one may be elected.
At the start of the game, each Elector knows the name of one Leader.
You will see your Elector name and the name of the Leader you have been initially assigned on the right side of your screen when the game begins.
Your objective is to get all of the Electors to agree to elect a single Leader.
Elections in Maximus Democraticus must be unanimous, so all Electors must choose the same Leader in order for them to be elected.
, and thus that Leader is elected.
, and thus no Leader is elected,
is elected
equally qualified
Elector
Leader Election game
Leaders
Leader
to elect

Table A.10: Instruction Changes for Election Skin

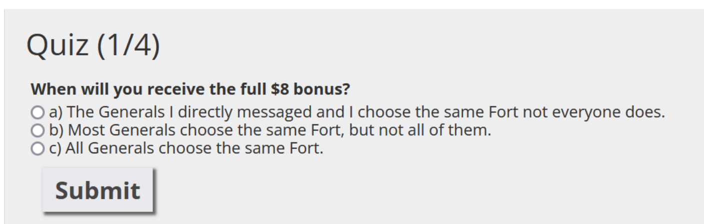
A.3 Comprehension Quiz

This the comprehension quiz for the generals skin. The order of the questions is randomized but the order of the multiple choice answers within the questions is not. These questions assess core concepts of the game such that anyone who read an understood the instructions would be able to get all of these questions correct. Question 1 assessed whether or not the players understand that this game is a global consensus game not a local consensus game. Question 2 ensures that the players understand the timed nature of the game. Question 3 shows that the players understand that they can send full text messages. Question 4 was the question that the players found the trickiest, which demonstrates the difference between the 4-letter and 5-letter identifiers as well as the networked structure of the game. Key words such as Generals and Fort varied across skins to the appropriate word for that skin.


Welcome to the General's Coordination game!

Only click 'Start Quiz' when you get to the end of the instruction slides to the left. The slides may take some time to load; if you do not see the instructions do not start the Quiz and try refreshing the page. If the instructions do not load please return this HIT.

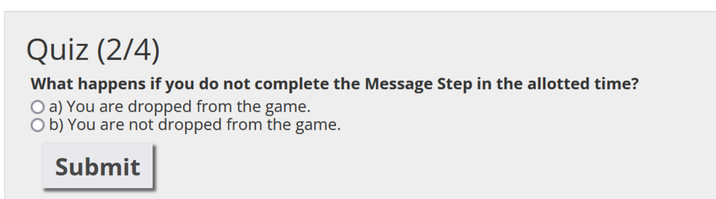
Start Quiz


Quiz (1/4)

When will you receive the full $8 bonus?

a) The Generals I directly messaged and I choose the same Fort not everyone does.
b) Most Generals choose the same Fort, but not all of them.
c) All Generals choose the same Fort.

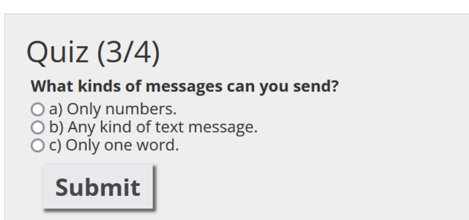
Submit


Quiz (2/4)

What happens if you do not complete the Message Step in the allotted time?

a) You are dropped from the game.
b) You are not dropped from the game.

Submit

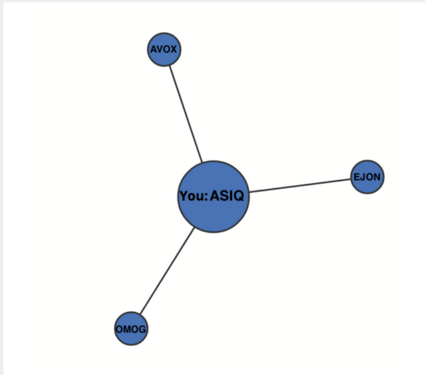

Quiz (3/4)

What kinds of messages can you send?

a) Only numbers.
b) Any kind of text message.
c) Only one word.

Submit

Quiz (4/4)



In the above image what does ASIQ represent?

- a) The name of the Fort given to you.
- b) Your name.

Submit

Appendix B

Regressions for Capacity Constrained Agent Models

B.1 Excess States Regressions

B.1.1 Average Counts

Dep. Variable:	avg_count_gap	**R-squared:**	0.288
Model:	OLS	**Adj. R-squared:**	0.288
Method:	Least Squares	**F-statistic:**	9.447e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:58:02	**Log-Likelihood:**	-3.4781e+07
No. Observations:	8160800	**AIC:**	6.956e+07
Df Residuals:	8160796	**BIC:**	6.956e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	32.6709	0.023	1400.572	0.000	32.625	32.717
mu	-0.3600	0.000	-1662.594	0.000	-0.360	-0.360
sigma	-0.0826	0.000	-427.090	0.000	-0.083	-0.082
excess_states	0.3159	0.001	302.717	0.000	0.314	0.318

Omnibus:	1631115.655	**Durbin-Watson:**	0.594
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	3071101.915
Skew:	1.243	**Prob(JB):**	0.00
Kurtosis:	4.689	**Cond. No.**	237.

Table B.1: Average Counts Range

Dep. Variable:	avg_count_mad	**R-squared:**	0.257
Model:	OLS	**Adj. R-squared:**	0.257
Method:	Least Squares	**F-statistic:**	7.881e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:58:00	**Log-Likelihood:**	-2.6300e+07
No. Observations:	8160800	**AIC:**	5.260e+07
Df Residuals:	8160796	**BIC:**	5.260e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	11.5330	0.009	1295.548	0.000	11.516	11.550
mu	-0.1178	7.77e-05	-1515.654	0.000	-0.118	-0.118
sigma	-0.0335	6.81e-05	-491.900	0.000	-0.034	-0.033
excess_states	0.0168	0.000	49.073	0.000	0.016	0.017

Omnibus:	2517993.677	**Durbin-Watson:**	0.596
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	7577134.524
Skew:	1.616	**Prob(JB):**	0.00
Kurtosis:	6.441	**Cond. No.**	237.

Table B.2: Average Counts Mean Absolute Deviation

Dep. Variable:	avg_count_max	**R-squared:**	0.429
Model:	OLS	**Adj. R-squared:**	0.429
Method:	Least Squares	**F-statistic:**	1.883e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:58:01	**Log-Likelihood:**	-3.4933e+07
No. Observations:	8160800	**AIC:**	6.987e+07
Df Residuals:	8160796	**BIC:**	6.987e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	53.3927	0.023	2275.741	0.000	53.347	53.439
mu	-0.4936	0.000	-2334.758	0.000	-0.494	-0.493
sigma	-0.1396	0.000	-669.382	0.000	-0.140	-0.139
excess_states	-0.4531	0.001	-406.037	0.000	-0.455	-0.451

Omnibus:	620131.537	**Durbin-Watson:**	0.504
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	770501.902
Skew:	0.736	**Prob(JB):**	0.00
Kurtosis:	3.316	**Cond. No.**	237.

Table B.3: Average Counts Max

Dep. Variable:	avg_count_mean	**R-squared:**	0.431
Model:	OLS	**Adj. R-squared:**	0.431
Method:	Least Squares	**F-statistic:**	1.057e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:57:56	**Log-Likelihood:**	-3.1013e+07
No. Observations:	8160800	**AIC:**	6.203e+07
Df Residuals:	8160796	**BIC:**	6.203e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	34.2824	0.021	1642.613	0.000	34.242	34.323
mu	-0.2554	0.000	-1768.630	0.000	-0.256	-0.255
sigma	-0.0845	0.000	-635.295	0.000	-0.085	-0.084
excess_states	-0.9259	0.001	-1116.350	0.000	-0.928	-0.924

Omnibus:	3145797.326	**Durbin-Watson:**	0.271
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	15543140.469
Skew:	1.825	**Prob(JB):**	0.00
Kurtosis:	8.691	**Cond. No.**	237.

Table B.4: Average Counts Mean

Dep. Variable:	avg_count_median	**R-squared:**	0.358
Model:	OLS	**Adj. R-squared:**	0.358
Method:	Least Squares	**F-statistic:**	6.461e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:57:57	**Log-Likelihood:**	-3.1914e+07
No. Observations:	8160800	**AIC:**	6.383e+07
Df Residuals:	8160796	**BIC:**	6.383e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	31.6108	0.024	1343.327	0.000	31.565	31.657
mu	-0.2092	0.000	-1290.013	0.000	-0.210	-0.209
sigma	-0.0686	0.000	-483.950	0.000	-0.069	-0.068
excess_states	-1.1305	0.001	-1219.789	0.000	-1.132	-1.129

Omnibus:	3558389.800	**Durbin-Watson:**	0.344
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	18037445.920
Skew:	2.102	**Prob(JB):**	0.00
Kurtosis:	8.948	**Cond. No.**	237.

Table B.5: Average Counts Median

Dep. Variable:	avg_count_std	**R-squared:**	0.269
Model:	OLS	**Adj. R-squared:**	0.269
Method:	Least Squares	**F-statistic:**	8.427e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:57:58	**Log-Likelihood:**	-2.7203e+07
No. Observations:	8160800	**AIC:**	5.441e+07
Df Residuals:	8160796	**BIC:**	5.441e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	12.9439	0.010	1339.384	0.000	12.925	12.963
mu	-0.1358	8.64e-05	-1571.864	0.000	-0.136	-0.136
sigma	-0.0369	7.64e-05	-482.704	0.000	-0.037	-0.037
excess_states	0.0562	0.000	141.946	0.000	0.055	0.057

Omnibus:	2062722.073	**Durbin-Watson:**	0.594
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	4835154.987
Skew:	1.426	**Prob(JB):**	0.00
Kurtosis:	5.466	**Cond. No.**	237.

Table B.6: Average Counts Standard Deviation

B.1.2 Herfindahl Index

Dep. Variable:	counts_herfindahl_index	**R-squared:**	0.084
Model:	OLS	**Adj. R-squared:**	0.084
Method:	Least Squares	**F-statistic:**	3.092e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:52:50	**Log-Likelihood:**	4.7133e+06
No. Observations:	8160800	**AIC:**	-9.427e+06
Df Residuals:	8160796	**BIC:**	-9.427e+06
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	0.2602	0.000	1865.669	0.000	0.260	0.260
mu	-0.0004	1.52e-06	-288.290	0.000	-0.000	-0.000
sigma	-0.0005	1.98e-06	-262.212	0.000	-0.001	-0.001
excess_states	-0.0064	8.49e-06	-754.010	0.000	-0.006	-0.006

Omnibus:	6869240.495	**Durbin-Watson:**	1.343
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	189368993.012
Skew:	4.040	**Prob(JB):**	0.00
Kurtosis:	25.172	**Cond. No.**	237.

Table B.7: Herfindahl Index

B.1.3 Kurtosis in Counts

Dep. Variable:	counts_kurtosis	**R-squared:**	0.065
Model:	OLS	**Adj. R-squared:**	0.065
Method:	Least Squares	**F-statistic:**	1.489e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:52:24	**Log-Likelihood:**	-2.0326e+07
No. Observations:	8160800	**AIC:**	4.065e+07
Df Residuals:	8160796	**BIC:**	4.065e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	-0.3005	0.003	-106.306	0.000	-0.306	-0.295
mu	0.0037	3.29e-05	112.506	0.000	0.004	0.004
sigma	-0.0062	3.68e-05	-169.933	0.000	-0.006	-0.006
excess_states	0.1319	0.000	666.169	0.000	0.131	0.132

Omnibus:	4600457.530	**Durbin-Watson:**	1.582
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	42503178.768
Skew:	2.604	**Prob(JB):**	0.00
Kurtosis:	12.893	**Cond. No.**	237.

Table B.8: Kurtosis in Counts

B.1.4 Number of Zero Counts

Dep. Variable:	counts_zeros	**R-squared:**	0.383
Model:	OLS	**Adj. R-squared:**	0.383
Method:	Least Squares	**F-statistic:**	9.160e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:52:02	**Log-Likelihood:**	-2.2079e+07
No. Observations:	8160800	**AIC:**	4.416e+07
Df Residuals:	8160796	**BIC:**	4.416e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1.4880	0.003	523.774	0.000	1.482	1.494
mu	0.0533	4.51e-05	1181.231	0.000	0.053	0.053
sigma	-0.0763	5.18e-05	-1471.237	0.000	-0.076	-0.076
excess_states	0.1584	0.000	646.781	0.000	0.158	0.159

Omnibus:	2185557.836	**Durbin-Watson:**	0.225
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	5437351.853
Skew:	1.481	**Prob(JB):**	0.00
Kurtosis:	5.686	**Cond. No.**	237.

Table B.9: Number of Zero Counts

B.1.5 Decay Time

Dep. Variable:	decay_time_gap	**R-squared:**	0.272
Model:	OLS	**Adj. R-squared:**	0.272
Method:	Least Squares	**F-statistic:**	1.051e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	06:01:47	**Log-Likelihood:**	-3.6483e+07
No. Observations:	8160800	**AIC:**	7.297e+07
Df Residuals:	8160796	**BIC:**	7.297e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	36.2969	0.024	1499.280	0.000	36.249	36.344
mu	-0.3008	0.000	-1137.571	0.000	-0.301	-0.300
sigma	0.2819	0.000	1046.671	0.000	0.281	0.282
excess_states	0.8525	0.001	645.696	0.000	0.850	0.855

Omnibus:	436487.112	**Durbin-Watson:**	1.083
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	198735.052
Skew:	0.172	**Prob(JB):**	0.00
Kurtosis:	2.318	**Cond. No.**	237.

Table B.10: Decay Time Range

Dep. Variable:	decay_time_mad	**R-squared:**	0.230
Model:	OLS	**Adj. R-squared:**	0.230
Method:	Least Squares	**F-statistic:**	7.933e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	06:01:45	**Log-Likelihood:**	-2.5904e+07
No. Observations:	8160800	**AIC:**	5.181e+07
Df Residuals:	8160796	**BIC:**	5.181e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	12.1728	0.007	1670.044	0.000	12.159	12.187
mu	-0.0824	7.35e-05	-1120.644	0.000	-0.083	-0.082
sigma	0.0696	7.33e-05	949.526	0.000	0.069	0.070
excess_states	-0.0518	0.000	-139.938	0.000	-0.053	-0.051

Omnibus:	264340.764	**Durbin-Watson:**	1.128
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	290966.492
Skew:	0.462	**Prob(JB):**	0.00
Kurtosis:	3.038	**Cond. No.**	237.

Table B.11: Decay Time Mean Absolute Deviation

Dep. Variable:	decay_time_max	**R-squared:**	0.268
Model:	OLS	**Adj. R-squared:**	0.268
Method:	Least Squares	**F-statistic:**	1.006e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	06:01:46	**Log-Likelihood:**	-3.6569e+07
No. Observations:	8160800	**AIC:**	7.314e+07
Df Residuals:	8160796	**BIC:**	7.314e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	39.7249	0.025	1595.967	0.000	39.676	39.774
mu	-0.3149	0.000	-1177.655	0.000	-0.315	-0.314
sigma	0.2771	0.000	1014.156	0.000	0.277	0.278
excess_states	0.7465	0.001	557.379	0.000	0.744	0.749

Omnibus:	474993.712	**Durbin-Watson:**	1.060
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	203284.835
Skew:	0.159	**Prob(JB):**	0.00
Kurtosis:	2.296	**Cond. No.**	237.

Table B.12: Decay Time Max

Dep. Variable:	decay_time_mean	**R-squared:**	0.253
Model:	OLS	**Adj. R-squared:**	0.253
Method:	Least Squares	**F-statistic:**	9.605e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	06:01:43	**Log-Likelihood:**	-2.8655e+07
No. Observations:	8160800	**AIC:**	5.731e+07
Df Residuals:	8160796	**BIC:**	5.731e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	19.9590	0.012	1728.357	0.000	19.936	19.982
mu	-0.1389	0.000	-1324.900	0.000	-0.139	-0.139
sigma	0.0774	0.000	743.095	0.000	0.077	0.078
excess_states	-0.1589	0.001	-302.590	0.000	-0.160	-0.158

Omnibus:	831597.514	**Durbin-Watson:**	0.979
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1314284.788
Skew:	0.750	**Prob(JB):**	0.00
Kurtosis:	4.270	**Cond. No.**	237.

Table B.13: Decay Time Mean

Dep. Variable:	decay_time_median	**R-squared:**	0.179
Model:	OLS	**Adj. R-squared:**	0.179
Method:	Least Squares	**F-statistic:**	6.471e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	06:01:44	**Log-Likelihood:**	-2.9103e+07
No. Observations:	8160800	**AIC:**	5.821e+07
Df Residuals:	8160796	**BIC:**	5.821e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	17.1357	0.013	1333.955	0.000	17.110	17.161
mu	-0.1213	0.000	-1093.740	0.000	-0.122	-0.121
sigma	0.0478	0.000	456.545	0.000	0.048	0.048
excess_states	-0.2224	0.001	-390.717	0.000	-0.223	-0.221

Omnibus:	2661773.611	**Durbin-Watson:**	1.296
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	11243982.866
Skew:	1.566	**Prob(JB):**	0.00
Kurtosis:	7.822	**Cond. No.**	237.

Table B.14: Decay Time Median

Dep. Variable:	decay_time_std	**R-squared:**	0.235
Model:	OLS	**Adj. R-squared:**	0.235
Method:	Least Squares	**F-statistic:**	7.907e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	06:01:44	**Log-Likelihood:**	-2.7231e+07
No. Observations:	8160800	**AIC:**	5.446e+07
Df Residuals:	8160796	**BIC:**	5.446e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	13.9582	0.008	1677.240	0.000	13.942	13.975
mu	-0.0968	8.58e-05	-1128.497	0.000	-0.097	-0.097
sigma	0.0858	8.65e-05	992.582	0.000	0.086	0.086
excess_states	0.0115	0.000	26.390	0.000	0.011	0.012

Omnibus:	199176.875	**Durbin-Watson:**	1.110
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	178649.798
Skew:	0.310	**Prob(JB):**	0.00
Kurtosis:	2.623	**Cond. No.**	237.

Table B.15: Decay Time Standard Deviation

B.1.6 Entropy in Counts

Dep. Variable:	entropy_gap	**R-squared:**	0.476
Model:	OLS	**Adj. R-squared:**	0.476
Method:	Least Squares	**F-statistic:**	2.145e+06
Date:	Sat, 26 Feb 2022	**Prob (F-statistic):**	0.00
Time:	00:15:25	**Log-Likelihood:**	-4.3191e+06
No. Observations:	6707759	**AIC:**	8.638e+06
Df Residuals:	6707755	**BIC:**	8.638e+06
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	0.6471	0.001	1039.602	0.000	0.646	0.648
mu	-0.0033	6.2e-06	-538.870	0.000	-0.003	-0.003
sigma	-0.0001	7.4e-06	-15.385	0.000	-0.000	-9.93e-05
excess_states	0.0759	3.06e-05	2479.107	0.000	0.076	0.076

Omnibus:	72119.733	**Durbin-Watson:**	1.278
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	93015.772
Skew:	0.169	**Prob(JB):**	0.00
Kurtosis:	3.468	**Cond. No.**	241.

Table B.16: Entropy in Counts Range

Dep. Variable:	entropy_mad	**R-squared:**	0.277
Model:	OLS	**Adj. R-squared:**	0.277
Method:	Least Squares	**F-statistic:**	9.200e+05
Date:	Sat, 26 Feb 2022	**Prob (F-statistic):**	0.00
Time:	00:15:21	**Log-Likelihood:**	2.9093e+06
No. Observations:	6707759	**AIC:**	-5.819e+06
Df Residuals:	6707755	**BIC:**	-5.819e+06
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	0.2600	0.000	1173.052	0.000	0.260	0.260
mu	-0.0013	2.04e-06	-645.169	0.000	-0.001	-0.001
sigma	-0.0005	2.49e-06	-193.504	0.000	-0.000	-0.000
excess_states	0.0154	9.97e-06	1548.365	0.000	0.015	0.015

Omnibus:	675794.693	**Durbin-Watson:**	1.170
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1057522.336
Skew:	0.748	**Prob(JB):**	0.00
Kurtosis:	4.244	**Cond. No.**	241.

Table B.17: Entropy in Counts Mean Absolute Deviation

Dep. Variable:	entropy_max	**R-squared:**	0.517
Model:	OLS	**Adj. R-squared:**	0.517
Method:	Least Squares	**F-statistic:**	3.040e+06
Date:	Sat, 26 Feb 2022	**Prob (F-statistic):**	0.00
Time:	00:15:23	**Log-Likelihood:**	-4.2294e+06
No. Observations:	6707759	**AIC:**	8.459e+06
Df Residuals:	6707755	**BIC:**	8.459e+06
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	3.4132	0.001	6225.408	0.000	3.412	3.414
mu	-0.0160	6.77e-06	-2369.912	0.000	-0.016	-0.016
sigma	-0.0014	8.31e-06	-168.035	0.000	-0.001	-0.001
excess_states	-0.0121	3.12e-05	-387.380	0.000	-0.012	-0.012

Omnibus:	396425.870	**Durbin-Watson:**	0.837
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	495429.858
Skew:	-0.583	**Prob(JB):**	0.00
Kurtosis:	3.641	**Cond. No.**	241.

Table B.18: Entropy in Counts Max

Dep. Variable:	entropy_mean	**R-squared:**	0.587
Model:	OLS	**Adj. R-squared:**	0.587
Method:	Least Squares	**F-statistic:**	3.538e+06
Date:	Sat, 26 Feb 2022	**Prob (F-statistic):**	0.00
Time:	00:15:15	**Log-Likelihood:**	-3.5899e+06
No. Observations:	6707759	**AIC:**	7.180e+06
Df Residuals:	6707755	**BIC:**	7.180e+06
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	3.0888	0.001	5230.658	0.000	3.088	3.090
mu	-0.0137	6.47e-06	-2118.855	0.000	-0.014	-0.014
sigma	-0.0010	7.95e-06	-131.158	0.000	-0.001	-0.001
excess_states	-0.0538	2.85e-05	-1891.351	0.000	-0.054	-0.054

Omnibus:	917679.346	**Durbin-Watson:**	0.644
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1828895.580
Skew:	-0.857	**Prob(JB):**	0.00
Kurtosis:	4.899	**Cond. No.**	241.

Table B.19: Entropy in Counts Mean

Dep. Variable:	entropy_median	**R-squared:**	0.547
Model:	OLS	**Adj. R-squared:**	0.547
Method:	Least Squares	**F-statistic:**	3.069e+06
Date:	Sat, 26 Feb 2022	**Prob (F-statistic):**	0.00
Time:	00:15:17	**Log-Likelihood:**	-4.1582e+06
No. Observations:	6707759	**AIC:**	8.316e+06
Df Residuals:	6707755	**BIC:**	8.316e+06
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	3.0734	0.001	4780.826	0.000	3.072	3.075
mu	-0.0130	6.95e-06	-1864.723	0.000	-0.013	-0.013
sigma	-0.0007	8.35e-06	-82.612	0.000	-0.001	-0.001
excess_states	-0.0594	3.06e-05	-1940.723	0.000	-0.059	-0.059

Omnibus:	587417.206	**Durbin-Watson:**	0.757
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1150353.180
Skew:	-0.595	**Prob(JB):**	0.00
Kurtosis:	4.643	**Cond. No.**	241.

Table B.20: Entropy in Counts Median

Dep. Variable:	entropy_std	**R-squared:**	0.328
Model:	OLS	**Adj. R-squared:**	0.328
Method:	Least Squares	**F-statistic:**	1.146e+06
Date:	Sat, 26 Feb 2022	**Prob (F-statistic):**	0.00
Time:	00:15:19	**Log-Likelihood:**	2.4188e+06
No. Observations:	6707759	**AIC:**	-4.838e+06
Df Residuals:	6707755	**BIC:**	-4.837e+06
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	0.2839	0.000	1192.108	0.000	0.283	0.284
mu	-0.0014	2.21e-06	-625.981	0.000	-0.001	-0.001
sigma	-0.0004	2.69e-06	-162.652	0.000	-0.000	-0.000
excess_states	0.0194	1.1e-05	1764.293	0.000	0.019	0.019

Omnibus:	342185.716	**Durbin-Watson:**	1.170
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	449870.753
Skew:	0.503	**Prob(JB):**	0.00
Kurtosis:	3.774	**Cond. No.**	241.

Table B.21: Entropy in Counts Standard Deviation

B.1.7 Final Counts

Dep. Variable:	final_counts_sums_gap	**R-squared:**	0.288
Model:	OLS	**Adj. R-squared:**	0.288
Method:	Least Squares	**F-statistic:**	9.447e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:51:37	**Log-Likelihood:**	-7.2362e+07
No. Observations:	8160800	**AIC:**	1.447e+08
Df Residuals:	8160796	**BIC:**	1.447e+08
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	3267.0891	2.333	1400.572	0.000	3262.517	3271.661
mu	-35.9975	0.022	-1662.594	0.000	-36.040	-35.955
sigma	-8.2631	0.019	-427.090	0.000	-8.301	-8.225
excess_states	31.5854	0.104	302.717	0.000	31.381	31.790

Omnibus:	1631115.655	**Durbin-Watson:**	0.594
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	3071101.915
Skew:	1.243	**Prob(JB):**	0.00
Kurtosis:	4.689	**Cond. No.**	237.

Table B.22: Final Counts Range

Dep. Variable:	final_counts_sums_mad	**R-squared:**	0.257
Model:	OLS	**Adj. R-squared:**	0.257
Method:	Least Squares	**F-statistic:**	7.881e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:51:34	**Log-Likelihood:**	-6.3881e+07
No. Observations:	8160800	**AIC:**	1.278e+08
Df Residuals:	8160796	**BIC:**	1.278e+08
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1153.2991	0.890	1295.548	0.000	1151.554	1155.044
mu	-11.7792	0.008	-1515.654	0.000	-11.794	-11.764
sigma	-3.3510	0.007	-491.900	0.000	-3.364	-3.338
excess_states	1.6823	0.034	49.073	0.000	1.615	1.750

Omnibus:	2517993.677	**Durbin-Watson:**	0.596
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	7577134.524
Skew:	1.616	**Prob(JB):**	0.00
Kurtosis:	6.441	**Cond. No.**	237.

Table B.23: Final Counts Mean Absolute Deviation

Dep. Variable:	final_counts_sums_max	**R-squared:**	0.429
Model:	OLS	**Adj. R-squared:**	0.429
Method:	Least Squares	**F-statistic:**	1.883e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:51:36	**Log-Likelihood:**	-7.2515e+07
No. Observations:	8160800	**AIC:**	1.450e+08
Df Residuals:	8160796	**BIC:**	1.450e+08
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	5339.2654	2.346	2275.741	0.000	5334.667	5343.864
mu	-49.3608	0.021	-2334.758	0.000	-49.402	-49.319
sigma	-13.9586	0.021	-669.382	0.000	-13.999	-13.918
excess_states	-45.3142	0.112	-406.037	0.000	-45.533	-45.095

Omnibus:	620131.537	**Durbin-Watson:**	0.504
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	770501.902
Skew:	0.736	**Prob(JB):**	0.00
Kurtosis:	3.316	**Cond. No.**	237.

Table B.24: Final Counts Max

Dep. Variable:	final_counts_sums_mean	**R-squared:**	0.431
Model:	OLS	**Adj. R-squared:**	0.431
Method:	Least Squares	**F-statistic:**	1.057e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:51:30	**Log-Likelihood:**	-6.8595e+07
No. Observations:	8160800	**AIC:**	1.372e+08
Df Residuals:	8160796	**BIC:**	1.372e+08
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	3428.2430	2.087	1642.613	0.000	3424.152	3432.334
mu	-25.5423	0.014	-1768.630	0.000	-25.571	-25.514
sigma	-8.4502	0.013	-635.295	0.000	-8.476	-8.424
excess_states	-92.5901	0.083	-1116.350	0.000	-92.753	-92.428

Omnibus:	3145797.326	**Durbin-Watson:**	0.271
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	15543140.469
Skew:	1.825	**Prob(JB):**	0.00
Kurtosis:	8.691	**Cond. No.**	237.

Table B.25: Final Counts Mean

Dep. Variable:	final_counts_sums_median	**R-squared:**	0.358
Model:	OLS	**Adj. R-squared:**	0.358
Method:	Least Squares	**F-statistic:**	6.461e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:51:32	**Log-Likelihood:**	-6.9496e+07
No. Observations:	8160800	**AIC:**	1.390e+08
Df Residuals:	8160796	**BIC:**	1.390e+08
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	3161.0797	2.353	1343.327	0.000	3156.468	3165.692
mu	-20.9205	0.016	-1290.013	0.000	-20.952	-20.889
sigma	-6.8616	0.014	-483.950	0.000	-6.889	-6.834
excess_states	-113.0523	0.093	-1219.789	0.000	-113.234	-112.871

Omnibus:	3558389.800	**Durbin-Watson:**	0.344
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	18037445.920
Skew:	2.102	**Prob(JB):**	0.00
Kurtosis:	8.948	**Cond. No.**	237.

Table B.26: Final Counts Median

Dep. Variable:	final_counts_sums_std	**R-squared:**	0.269
Model:	OLS	**Adj. R-squared:**	0.269
Method:	Least Squares	**F-statistic:**	8.427e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:51:33	**Log-Likelihood:**	-6.4785e+07
No. Observations:	8160800	**AIC:**	1.296e+08
Df Residuals:	8160796	**BIC:**	1.296e+08
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1294.3946	0.966	1339.384	0.000	1292.500	1296.289
mu	-13.5788	0.009	-1571.864	0.000	-13.596	-13.562
sigma	-3.6877	0.008	-482.704	0.000	-3.703	-3.673
excess_states	5.6180	0.040	141.946	0.000	5.540	5.696

Omnibus:	2062722.073	**Durbin-Watson:**	0.594
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	4835154.987
Skew:	1.426	**Prob(JB):**	0.00
Kurtosis:	5.466	**Cond. No.**	237.

Table B.27: Final Counts Standard Deviation

B.1.8 Max Counts

Dep. Variable:	max_count_gap	**R-squared:**	0.330
Model:	OLS	**Adj. R-squared:**	0.330
Method:	Least Squares	**F-statistic:**	1.159e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:54:21	**Log-Likelihood:**	-3.6894e+07
No. Observations:	8160800	**AIC:**	7.379e+07
Df Residuals:	8160796	**BIC:**	7.379e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	45.4376	0.030	1523.461	0.000	45.379	45.496
mu	-0.5034	0.000	-1837.724	0.000	-0.504	-0.503
sigma	-0.1389	0.000	-530.258	0.000	-0.139	-0.138
excess_states	0.6036	0.001	444.031	0.000	0.601	0.606

Omnibus:	1060741.770	**Durbin-Watson:**	0.595
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1531725.409
Skew:	0.999	**Prob(JB):**	0.00
Kurtosis:	3.715	**Cond. No.**	237.

Table B.28: Max Counts Range

Dep. Variable:	max_count_mad	**R-squared:**	0.279
Model:	OLS	**Adj. R-squared:**	0.279
Method:	Least Squares	**F-statistic:**	9.162e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:54:18	**Log-Likelihood:**	-2.8726e+07
No. Observations:	8160800	**AIC:**	5.745e+07
Df Residuals:	8160796	**BIC:**	5.745e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	16.1317	0.012	1363.978	0.000	16.109	16.155
mu	-0.1658	0.000	-1628.004	0.000	-0.166	-0.166
sigma	-0.0522	9.41e-05	-554.483	0.000	-0.052	-0.052
excess_states	0.0716	0.000	153.835	0.000	0.071	0.073

Omnibus:	2172230.256	**Durbin-Watson:**	0.606
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	5312313.363
Skew:	1.481	**Prob(JB):**	0.00
Kurtosis:	5.617	**Cond. No.**	237.

Table B.29: Mac Counts Mean Absolute Deviation

Dep. Variable:	max_count_max	**R-squared:**	0.463
Model:	OLS	**Adj. R-squared:**	0.463
Method:	Least Squares	**F-statistic:**	2.335e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:54:19	**Log-Likelihood:**	-3.7099e+07
No. Observations:	8160800	**AIC:**	7.420e+07
Df Residuals:	8160796	**BIC:**	7.420e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	74.0974	0.029	2518.700	0.000	74.040	74.155
mu	-0.6955	0.000	-2600.316	0.000	-0.696	-0.695
sigma	-0.1942	0.000	-679.942	0.000	-0.195	-0.194
excess_states	-0.3916	0.001	-273.990	0.000	-0.394	-0.389

Omnibus:	286063.586	**Durbin-Watson:**	0.492
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	302636.579
Skew:	0.453	**Prob(JB):**	0.00
Kurtosis:	2.735	**Cond. No.**	237.

Table B.30: Max Counts Max

Dep. Variable:	max_count_mean	**R-squared:**	0.467
Model:	OLS	**Adj. R-squared:**	0.467
Method:	Least Squares	**F-statistic:**	1.519e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:54:14	**Log-Likelihood:**	-3.2945e+07
No. Observations:	8160800	**AIC:**	6.589e+07
Df Residuals:	8160796	**BIC:**	6.589e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	47.9440	0.025	1921.333	0.000	47.895	47.993
mu	-0.3726	0.000	-2118.329	0.000	-0.373	-0.372
sigma	-0.0998	0.000	-573.077	0.000	-0.100	-0.099
excess_states	-1.1035	0.001	-1091.627	0.000	-1.106	-1.102

Omnibus:	1891976.014	**Durbin-Watson:**	0.286
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	5223094.631
Skew:	1.235	**Prob(JB):**	0.00
Kurtosis:	6.042	**Cond. No.**	237.

Table B.31: Max Counts Mean

Dep. Variable:	max_count_median	**R-squared:**	0.384
Model:	OLS	**Adj. R-squared:**	0.384
Method:	Least Squares	**F-statistic:**	8.783e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:54:15	**Log-Likelihood:**	-3.3940e+07
No. Observations:	8160800	**AIC:**	6.788e+07
Df Residuals:	8160796	**BIC:**	6.788e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	44.6282	0.029	1542.931	0.000	44.572	44.685
mu	-0.3131	0.000	-1564.284	0.000	-0.313	-0.313
sigma	-0.0761	0.000	-408.042	0.000	-0.076	-0.076
excess_states	-1.3947	0.001	-1205.997	0.000	-1.397	-1.392

Omnibus:	2914052.157	**Durbin-Watson:**	0.397
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	10925182.194
Skew:	1.786	**Prob(JB):**	0.00
Kurtosis:	7.401	**Cond. No.**	237.

Table B.32: Max Counts Median

Dep. Variable:	max_count_std	**R-squared:**	0.295
Model:	OLS	**Adj. R-squared:**	0.295
Method:	Least Squares	**F-statistic:**	9.936e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:54:17	**Log-Likelihood:**	-2.9580e+07
No. Observations:	8160800	**AIC:**	5.916e+07
Df Residuals:	8160796	**BIC:**	5.916e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	18.1492	0.013	1423.465	0.000	18.124	18.174
mu	-0.1911	0.000	-1702.789	0.000	-0.191	-0.191
sigma	-0.0588	0.000	-558.386	0.000	-0.059	-0.059
excess_states	0.1327	0.001	248.591	0.000	0.132	0.134

Omnibus:	1651457.151	**Durbin-Watson:**	0.603
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	3084298.882
Skew:	1.265	**Prob(JB):**	0.00
Kurtosis:	4.635	**Cond. No.**	237.

Table B.33: Max Counts Standard Deviation

B.1.9 Time of Max

Dep. Variable:	max_time_gap	**R-squared:**	0.343
Model:	OLS	**Adj. R-squared:**	0.343
Method:	Least Squares	**F-statistic:**	1.342e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:56:53	**Log-Likelihood:**	-3.7189e+07
No. Observations:	8160800	**AIC:**	7.438e+07
Df Residuals:	8160796	**BIC:**	7.438e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	49.0413	0.023	2098.208	0.000	48.996	49.087
mu	-0.2766	0.000	-955.829	0.000	-0.277	-0.276
sigma	0.3614	0.000	1117.858	0.000	0.361	0.362
excess_states	1.7915	0.001	1227.237	0.000	1.789	1.794

Omnibus:	547411.149	**Durbin-Watson:**	0.763
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	663689.163
Skew:	-0.685	**Prob(JB):**	0.00
Kurtosis:	3.277	**Cond. No.**	237.

Table B.34: Time of Max Range

Dep. Variable:	max_time_mad	**R-squared:**	0.259
Model:	OLS	**Adj. R-squared:**	0.259
Method:	Least Squares	**F-statistic:**	7.456e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:56:50	**Log-Likelihood:**	-2.7872e+07
No. Observations:	8160800	**AIC:**	5.574e+07
Df Residuals:	8160796	**BIC:**	5.574e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	16.3502	0.008	2023.008	0.000	16.334	16.366
mu	-0.0887	9.12e-05	-972.564	0.000	-0.089	-0.088
sigma	0.1066	9.96e-05	1069.460	0.000	0.106	0.107
excess_states	0.2898	0.000	614.288	0.000	0.289	0.291

Omnibus:	21842.995	**Durbin-Watson:**	0.943
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	21934.665
Skew:	-0.124	**Prob(JB):**	0.00
Kurtosis:	2.943	**Cond. No.**	237.

Table B.35: Time of Max Mean Absolute Deviation

Dep. Variable:	max_time_max	**R-squared:**	0.358
Model:	OLS	**Adj. R-squared:**	0.358
Method:	Least Squares	**F-statistic:**	8.452e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:56:51	**Log-Likelihood:**	-3.7308e+07
No. Observations:	8160800	**AIC:**	7.462e+07
Df Residuals:	8160796	**BIC:**	7.462e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	68.3824	0.024	2825.602	0.000	68.335	68.430
mu	-0.3068	0.000	-1042.190	0.000	-0.307	-0.306
sigma	0.5075	0.000	1513.458	0.000	0.507	0.508
excess_states	0.4646	0.001	314.834	0.000	0.462	0.467

Omnibus:	973106.179	**Durbin-Watson:**	0.520
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1357568.967
Skew:	-0.974	**Prob(JB):**	0.00
Kurtosis:	3.442	**Cond. No.**	237.

Table B.36: Time of Max Max

Dep. Variable:	max_time_mean	**R-squared:**	0.398
Model:	OLS	**Adj. R-squared:**	0.398
Method:	Least Squares	**F-statistic:**	1.518e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:56:46	**Log-Likelihood:**	-3.4478e+07
No. Observations:	8160800	**AIC:**	6.896e+07
Df Residuals:	8160796	**BIC:**	6.896e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	38.7235	0.020	1946.830	0.000	38.685	38.762
mu	-0.1420	0.000	-749.685	0.000	-0.142	-0.142
sigma	0.4177	0.000	1899.202	0.000	0.417	0.418
excess_states	-0.7001	0.001	-651.268	0.000	-0.702	-0.698

Omnibus:	55935.690	**Durbin-Watson:**	0.664
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	57097.878
Skew:	0.204	**Prob(JB):**	0.00
Kurtosis:	3.045	**Cond. No.**	237.

Table B.37: Time of Max Mean

Dep. Variable:	max_time_median	**R-squared:**	0.358
Model:	OLS	**Adj. R-squared:**	0.358
Method:	Least Squares	**F-statistic:**	1.467e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:56:47	**Log-Likelihood:**	-3.6053e+07
No. Observations:	8160800	**AIC:**	7.211e+07
Df Residuals:	8160796	**BIC:**	7.211e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	35.3592	0.025	1424.551	0.000	35.311	35.408
mu	-0.1167	0.000	-512.103	0.000	-0.117	-0.116
sigma	0.4704	0.000	1850.346	0.000	0.470	0.471
excess_states	-0.8824	0.001	-680.563	0.000	-0.885	-0.880

Omnibus:	153774.387	**Durbin-Watson:**	0.918
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	157750.010
Skew:	0.326	**Prob(JB):**	0.00
Kurtosis:	2.803	**Cond. No.**	237.

Table B.38: Time of Max Median

Dep. Variable:	max_time_std	**R-squared:**	0.266
Model:	OLS	**Adj. R-squared:**	0.266
Method:	Least Squares	**F-statistic:**	7.406e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:56:48	**Log-Likelihood:**	-2.8650e+07
No. Observations:	8160800	**AIC:**	5.730e+07
Df Residuals:	8160796	**BIC:**	5.730e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	19.0580	0.009	2169.535	0.000	19.041	19.075
mu	-0.0994	0.000	-984.158	0.000	-0.100	-0.099
sigma	0.1163	0.000	1039.725	0.000	0.116	0.117
excess_states	0.3489	0.001	671.351	0.000	0.348	0.350

Omnibus:	160390.258	**Durbin-Watson:**	0.878
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	170070.247
Skew:	-0.354	**Prob(JB):**	0.00
Kurtosis:	3.008	**Cond. No.**	237.

Table B.39: Time of Max Standard Deviation

B.1.10 Min after Max

Dep. Variable:	min_post_max_gap	**R-squared:**	0.281
Model:	OLS	**Adj. R-squared:**	0.281
Method:	Least Squares	**F-statistic:**	1.071e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	06:00:38	**Log-Likelihood:**	-3.6014e+07
No. Observations:	8160800	**AIC:**	7.203e+07
Df Residuals:	8160796	**BIC:**	7.203e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	39.4390	0.027	1459.286	0.000	39.386	39.492
mu	-0.4185	0.000	-1768.852	0.000	-0.419	-0.418
sigma	-0.0748	0.000	-325.779	0.000	-0.075	-0.074
excess_states	0.2486	0.001	201.727	0.000	0.246	0.251

Omnibus:	2059957.091	**Durbin-Watson:**	0.709
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	4674290.947
Skew:	1.444	**Prob(JB):**	0.00
Kurtosis:	5.326	**Cond. No.**	237.

Table B.40: Min after Max Range

Dep. Variable:	min_post_max_mad	**R-squared:**	0.249
Model:	OLS	**Adj. R-squared:**	0.249
Method:	Least Squares	**F-statistic:**	8.798e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	06:00:35	**Log-Likelihood:**	-2.7920e+07
No. Observations:	8160800	**AIC:**	5.584e+07
Df Residuals:	8160796	**BIC:**	5.584e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	14.4527	0.011	1324.095	0.000	14.431	14.474
mu	-0.1424	8.92e-05	-1596.242	0.000	-0.143	-0.142
sigma	-0.0326	8.35e-05	-390.256	0.000	-0.033	-0.032
excess_states	-0.0294	0.000	-68.779	0.000	-0.030	-0.029

Omnibus:	2936185.546	**Durbin-Watson:**	0.695
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	10939134.176
Skew:	1.805	**Prob(JB):**	0.00
Kurtosis:	7.376	**Cond. No.**	237.

Table B.41: Min after Max Mean Absolute Deviation

Dep. Variable:	min_post_max_max	**R-squared:**	0.393
Model:	OLS	**Adj. R-squared:**	0.393
Method:	Least Squares	**F-statistic:**	1.838e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	06:00:37	**Log-Likelihood:**	-3.5887e+07
No. Observations:	8160800	**AIC:**	7.177e+07
Df Residuals:	8160796	**BIC:**	7.177e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	56.2682	0.025	2212.667	0.000	56.218	56.318
mu	-0.5212	0.000	-2296.112	0.000	-0.522	-0.521
sigma	-0.1260	0.000	-535.022	0.000	-0.127	-0.126
excess_states	-0.4277	0.001	-346.597	0.000	-0.430	-0.425

Omnibus:	1261804.705	**Durbin-Watson:**	0.648
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2049320.354
Skew:	1.059	**Prob(JB):**	0.00
Kurtosis:	4.243	**Cond. No.**	237.

Table B.42: Min after Max Max

Dep. Variable:	min_post_max_mean	**R-squared:**	0.432
Model:	OLS	**Adj. R-squared:**	0.432
Method:	Least Squares	**F-statistic:**	1.096e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	06:00:32	**Log-Likelihood:**	-3.0893e+07
No. Observations:	8160800	**AIC:**	6.179e+07
Df Residuals:	8160796	**BIC:**	6.179e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	33.8827	0.020	1708.976	0.000	33.844	33.922
mu	-0.2455	0.000	-1809.436	0.000	-0.246	-0.245
sigma	-0.0784	0.000	-610.539	0.000	-0.079	-0.078
excess_states	-0.9753	0.001	-1189.031	0.000	-0.977	-0.974

Omnibus:	3074832.478	**Durbin-Watson:**	0.352
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	14189321.999
Skew:	1.805	**Prob(JB):**	0.00
Kurtosis:	8.357	**Cond. No.**	237.

Table B.43: Min after Max Mean

Dep. Variable:	min_post_max_median	**R-squared:**	0.342
Model:	OLS	**Adj. R-squared:**	0.342
Method:	Least Squares	**F-statistic:**	6.281e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	06:00:33	**Log-Likelihood:**	-3.2356e+07
No. Observations:	8160800	**AIC:**	6.471e+07
Df Residuals:	8160796	**BIC:**	6.471e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	31.1906	0.024	1301.461	0.000	31.144	31.238
mu	-0.1917	0.000	-1190.411	0.000	-0.192	-0.191
sigma	-0.0644	0.000	-443.935	0.000	-0.065	-0.064
excess_states	-1.2564	0.001	-1275.759	0.000	-1.258	-1.255

Omnibus:	4217736.035	**Durbin-Watson:**	0.465
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	31157913.730
Skew:	2.414	**Prob(JB):**	0.00
Kurtosis:	11.266	**Cond. No.**	237.

Table B.44: Min after Max Median

Dep. Variable:	min_post_max_std	**R-squared:**	0.259
Model:	OLS	**Adj. R-squared:**	0.259
Method:	Least Squares	**F-statistic:**	9.364e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	06:00:34	**Log-Likelihood:**	-2.8838e+07
No. Observations:	8160800	**AIC:**	5.768e+07
Df Residuals:	8160796	**BIC:**	5.768e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	16.1835	0.012	1371.374	0.000	16.160	16.207
mu	-0.1641	9.92e-05	-1654.089	0.000	-0.164	-0.164
sigma	-0.0358	9.42e-05	-380.607	0.000	-0.036	-0.036
excess_states	0.0199	0.000	40.402	0.000	0.019	0.021

Omnibus:	2462173.981	**Durbin-Watson:**	0.698
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	6941368.668
Skew:	1.610	**Prob(JB):**	0.00
Kurtosis:	6.169	**Cond. No.**	237.

Table B.45: Min after Max Standard Deviation

B.1.11 Variance in Counts

Dep. Variable:	var_count_gap	**R-squared:**	0.165
Model:	OLS	**Adj. R-squared:**	0.165
Method:	Least Squares	**F-statistic:**	4.638e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:59:21	**Log-Likelihood:**	-5.8717e+07
No. Observations:	8160800	**AIC:**	1.174e+08
Df Residuals:	8160796	**BIC:**	1.174e+08
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	439.3987	0.452	971.164	0.000	438.512	440.286
mu	-4.3727	0.004	-1175.160	0.000	-4.380	-4.365
sigma	-2.1788	0.004	-563.467	0.000	-2.186	-2.171
excess_states	3.1709	0.020	160.128	0.000	3.132	3.210

Omnibus:	5038252.415	**Durbin-Watson:**	0.856
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	47858967.966
Skew:	2.938	**Prob(JB):**	0.00
Kurtosis:	13.307	**Cond. No.**	237.

Table B.46: Variance in Counts Range

Dep. Variable:	var_count_mad	**R-squared:**	0.151
Model:	OLS	**Adj. R-squared:**	0.151
Method:	Least Squares	**F-statistic:**	4.327e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:59:18	**Log-Likelihood:**	-4.9113e+07
No. Observations:	8160800	**AIC:**	9.823e+07
Df Residuals:	8160796	**BIC:**	9.823e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	139.4918	0.157	889.147	0.000	139.184	139.799
mu	-1.2778	0.001	-1121.610	0.000	-1.280	-1.276
sigma	-0.6559	0.001	-539.639	0.000	-0.658	-0.653
excess_states	-0.2293	0.006	-39.764	0.000	-0.241	-0.218

Omnibus:	6140055.056	**Durbin-Watson:**	0.853
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	114729588.508
Skew:	3.553	**Prob(JB):**	0.00
Kurtosis:	19.938	**Cond. No.**	237.

Table B.47: Variance in Counts Mean Absolute Deviation

Dep. Variable:	var_count_max	**R-squared:**	0.187
Model:	OLS	**Adj. R-squared:**	0.187
Method:	Least Squares	**F-statistic:**	5.588e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:59:19	**Log-Likelihood:**	-5.8674e+07
No. Observations:	8160800	**AIC:**	1.173e+08
Df Residuals:	8160796	**BIC:**	1.173e+08
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	493.5251	0.445	1108.024	0.000	492.652	494.398
mu	-4.7189	0.004	-1283.593	0.000	-4.726	-4.712
sigma	-2.3584	0.004	-609.915	0.000	-2.366	-2.351
excess_states	1.1493	0.020	58.312	0.000	1.111	1.188

Omnibus:	4927957.896	**Durbin-Watson:**	0.844
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	44833727.079
Skew:	2.869	**Prob(JB):**	0.00
Kurtosis:	12.946	**Cond. No.**	237.

Table B.48: Variance in Counts Max

Dep. Variable:	var_count_mean	**R-squared:**	0.216
Model:	OLS	**Adj. R-squared:**	0.216
Method:	Least Squares	**F-statistic:**	6.336e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:59:14	**Log-Likelihood:**	-4.9640e+07
No. Observations:	8160800	**AIC:**	9.928e+07
Df Residuals:	8160796	**BIC:**	9.928e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	202.6695	0.200	1015.254	0.000	202.278	203.061
mu	-1.5385	0.001	-1323.153	0.000	-1.541	-1.536
sigma	-0.7981	0.001	-586.395	0.000	-0.801	-0.795
excess_states	-4.1815	0.008	-535.340	0.000	-4.197	-4.166

Omnibus:	8187080.499	**Durbin-Watson:**	0.738
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	585613279.121
Skew:	4.924	**Prob(JB):**	0.00
Kurtosis:	43.314	**Cond. No.**	237.

Table B.49: Variance in Counts Mean

Dep. Variable:	var_count_median	**R-squared:**	0.136
Model:	OLS	**Adj. R-squared:**	0.136
Method:	Least Squares	**F-statistic:**	2.104e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:59:16	**Log-Likelihood:**	-5.0332e+07
No. Observations:	8160800	**AIC:**	1.007e+08
Df Residuals:	8160796	**BIC:**	1.007e+08
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	157.4870	0.238	661.011	0.000	157.020	157.954
mu	-0.9675	0.001	-777.412	0.000	-0.970	-0.965
sigma	-0.5254	0.001	-350.710	0.000	-0.528	-0.522
excess_states	-5.8125	0.009	-617.744	0.000	-5.831	-5.794

Omnibus:	11440162.164	**Durbin-Watson:**	0.822
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	3247985551.636
Skew:	8.376	**Prob(JB):**	0.00
Kurtosis:	99.288	**Cond. No.**	237.

Table B.50: Variance in Counts Median

Dep. Variable:	var_count_std	**R-squared:**	0.156
Model:	OLS	**Adj. R-squared:**	0.156
Method:	Least Squares	**F-statistic:**	4.436e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	05:59:17	**Log-Likelihood:**	-5.0438e+07
No. Observations:	8160800	**AIC:**	1.009e+08
Df Residuals:	8160796	**BIC:**	1.009e+08
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	162.4830	0.177	919.148	0.000	162.137	162.829
mu	-1.5358	0.001	-1144.108	0.000	-1.538	-1.533
sigma	-0.7831	0.001	-549.759	0.000	-0.786	-0.780
excess_states	0.2940	0.007	42.241	0.000	0.280	0.308

Omnibus:	5563347.461	**Durbin-Watson:**	0.849
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	72840227.319
Skew:	3.220	**Prob(JB):**	0.00
Kurtosis:	16.143	**Cond. No.**	237.

Table B.51: Variance in Counts Standard Deviation

B.2 Capacity 3 Regressions

B.2.1 Average Counts

Dep. Variable:	avg_count_gap	**R-squared:**	0.303
Model:	OLS	**Adj. R-squared:**	0.303
Method:	Least Squares	**F-statistic:**	5.055e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:45	**Log-Likelihood:**	-1.7445e+07
No. Observations:	4080400	**AIC:**	3.489e+07
Df Residuals:	4080396	**BIC:**	3.489e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	35.4688	0.037	970.107	0.000	35.397	35.541
mu	-0.3806	0.000	-1224.678	0.000	-0.381	-0.380
sigma	-0.0943	0.000	-336.773	0.000	-0.095	-0.094
num_cascade	0.1678	0.002	109.901	0.000	0.165	0.171

Omnibus:	722582.883	**Durbin-Watson:**	0.590
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1261698.351
Skew:	1.151	**Prob(JB):**	0.00
Kurtosis:	4.455	**Cond. No.**	267.

Table B.52: Average Counts Range Capacity 3

Dep. Variable:	avg_count_mad	**R-squared:**	0.277
Model:	OLS	**Adj. R-squared:**	0.277
Method:	Least Squares	**F-statistic:**	4.292e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:42	**Log-Likelihood:**	-1.3115e+07
No. Observations:	4080400	**AIC:**	2.623e+07
Df Residuals:	4080396	**BIC:**	2.623e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	12.9902	0.014	905.365	0.000	12.962	13.018
mu	-0.1220	0.000	-1122.877	0.000	-0.122	-0.122
sigma	-0.0362	9.72e-05	-372.226	0.000	-0.036	-0.036
num_cascade	-0.0701	0.001	-136.727	0.000	-0.071	-0.069

Omnibus:	1165865.220	**Durbin-Watson:**	0.617
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	3333348.709
Skew:	1.512	**Prob(JB):**	0.00
Kurtosis:	6.234	**Cond. No.**	267.

Table B.53: Average Counts Mean Absolute Deviation Capacity 3

Dep. Variable:	avg_count_max	**R-squared:**	0.418
Model:	OLS	**Adj. R-squared:**	0.418
Method:	Least Squares	**F-statistic:**	9.193e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:43	**Log-Likelihood:**	-1.7392e+07
No. Observations:	4080400	**AIC:**	3.478e+07
Df Residuals:	4080396	**BIC:**	3.478e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	51.9389	0.034	1527.429	0.000	51.872	52.006
mu	-0.4748	0.000	-1618.062	0.000	-0.475	-0.474
sigma	-0.1368	0.000	-478.355	0.000	-0.137	-0.136
num_cascade	-0.3575	0.002	-234.878	0.000	-0.360	-0.354

Omnibus:	392509.681	**Durbin-Watson:**	0.532
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	518726.846
Skew:	0.825	**Prob(JB):**	0.00
Kurtosis:	3.570	**Cond. No.**	267.

Table B.54: Average Counts Max Capacity 3

Dep. Variable:	avg_count_mean	**R-squared:**	0.442
Model:	OLS	**Adj. R-squared:**	0.442
Method:	Least Squares	**F-statistic:**	5.532e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:38	**Log-Likelihood:**	-1.4672e+07
No. Observations:	4080400	**AIC:**	2.934e+07
Df Residuals:	4080396	**BIC:**	2.934e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	31.4504	0.027	1160.800	0.000	31.397	31.503
mu	-0.2146	0.000	-1282.733	0.000	-0.215	-0.214
sigma	-0.0710	0.000	-454.404	0.000	-0.071	-0.071
num_cascade	-0.7632	0.001	-815.666	0.000	-0.765	-0.761

Omnibus:	1355354.608	**Durbin-Watson:**	0.275
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	5144653.244
Skew:	1.642	**Prob(JB):**	0.00
Kurtosis:	7.414	**Cond. No.**	267.

Table B.55: Average Counts Mean Capacity 3

Dep. Variable:	avg_count_median	**R-squared:**	0.335
Model:	OLS	**Adj. R-squared:**	0.335
Method:	Least Squares	**F-statistic:**	2.948e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:40	**Log-Likelihood:**	-1.5444e+07
No. Observations:	4080400	**AIC:**	3.089e+07
Df Residuals:	4080396	**BIC:**	3.089e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	29.7906	0.033	894.542	0.000	29.725	29.856
mu	-0.1705	0.000	-849.094	0.000	-0.171	-0.170
sigma	-0.0537	0.000	-307.533	0.000	-0.054	-0.053
num_cascade	-0.9808	0.001	-843.290	0.000	-0.983	-0.979

Omnibus:	2072003.138	**Durbin-Watson:**	0.393
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	13991714.020
Skew:	2.400	**Prob(JB):**	0.00
Kurtosis:	10.698	**Cond. No.**	267.

Table B.56: Average Counts Median Capacity 3

Dep. Variable:	avg_count_std	**R-squared:**	0.287
Model:	OLS	**Adj. R-squared:**	0.287
Method:	Least Squares	**F-statistic:**	4.568e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:41	**Log-Likelihood:**	-1.3618e+07
No. Observations:	4080400	**AIC:**	2.724e+07
Df Residuals:	4080396	**BIC:**	2.724e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	14.4049	0.015	933.616	0.000	14.375	14.435
mu	-0.1425	0.000	-1162.948	0.000	-0.143	-0.142
sigma	-0.0407	0.000	-370.421	0.000	-0.041	-0.041
num_cascade	-0.0224	0.001	-38.304	0.000	-0.024	-0.021

Omnibus:	926891.395	**Durbin-Watson:**	0.601
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2011073.392
Skew:	1.320	**Prob(JB):**	0.00
Kurtosis:	5.205	**Cond. No.**	267.

Table B.57: Average Counts Standard Deviation Capacity 3

B.2.2 Herfindahl Index

Dep. Variable:	counts_herfindahl_index	**R-squared:**	0.093
Model:	OLS	**Adj. R-squared:**	0.093
Method:	Least Squares	**F-statistic:**	1.835e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:49:12	**Log-Likelihood:**	2.3168e+06
No. Observations:	4080400	**AIC:**	-4.634e+06
Df Residuals:	4080396	**BIC:**	-4.634e+06
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	0.3015	0.000	1342.002	0.000	0.301	0.302
mu	-0.0006	2.18e-06	-274.211	0.000	-0.001	-0.001
sigma	-0.0005	2.81e-06	-193.038	0.000	-0.001	-0.001
num_cascade	-0.0066	1.22e-05	-544.736	0.000	-0.007	-0.007

Omnibus:	3312873.047	**Durbin-Watson:**	1.343
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	83610037.878
Skew:	3.854	**Prob(JB):**	0.00
Kurtosis:	23.794	**Cond. No.**	267.

Table B.58: Herfindahl Index Capacity 3

B.2.3 Kurtosis in Counts

Dep. Variable:	counts_kurtosis	**R-squared:**	0.116
Model:	OLS	**Adj. R-squared:**	0.116
Method:	Least Squares	**F-statistic:**	1.510e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:49:11	**Log-Likelihood:**	-1.0246e+07
No. Observations:	4080400	**AIC:**	2.049e+07
Df Residuals:	4080396	**BIC:**	2.049e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	-1.3523	0.004	-302.697	0.000	-1.361	-1.344
mu	0.0024	4.77e-05	50.281	0.000	0.002	0.002
sigma	-0.0055	5.31e-05	-104.159	0.000	-0.006	-0.005
num_cascade	0.1901	0.000	669.438	0.000	0.190	0.191

Omnibus:	2145007.226	**Durbin-Watson:**	1.625
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	16751242.641
Skew:	2.444	**Prob(JB):**	0.00
Kurtosis:	11.640	**Cond. No.**	267.

Table B.59: Kurtosis in Counts Capacity 3

B.2.4 Number of Zero Counts

Dep. Variable:	counts_zeros	**R-squared:**	0.384
Model:	OLS	**Adj. R-squared:**	0.384
Method:	Least Squares	**F-statistic:**	4.595e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:49:10	**Log-Likelihood:**	-1.1039e+07
No. Observations:	4080400	**AIC:**	2.208e+07
Df Residuals:	4080396	**BIC:**	2.208e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	0.9184	0.005	201.964	0.000	0.910	0.927
mu	0.0533	6.38e-05	835.520	0.000	0.053	0.053
sigma	-0.0763	7.33e-05	-1040.786	0.000	-0.076	-0.076
num_cascade	0.1597	0.000	458.887	0.000	0.159	0.160

Omnibus:	1089305.938	**Durbin-Watson:**	0.225
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2701324.330
Skew:	1.478	**Prob(JB):**	0.00
Kurtosis:	5.674	**Cond. No.**	267.

Table B.60: Number of Zero Counts Capacity 3

B.2.5 Decay Time

Dep. Variable:	decay_time_gap	**R-squared:**	0.261
Model:	OLS	**Adj. R-squared:**	0.261
Method:	Least Squares	**F-statistic:**	4.838e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:49:08	**Log-Likelihood:**	-1.8180e+07
No. Observations:	4080400	**AIC:**	3.636e+07
Df Residuals:	4080396	**BIC:**	3.636e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	34.1305	0.037	913.904	0.000	34.057	34.204
mu	-0.2870	0.000	-776.229	0.000	-0.288	-0.286
sigma	0.2827	0.000	753.744	0.000	0.282	0.283
num_cascade	0.7051	0.002	382.658	0.000	0.701	0.709

Omnibus:	214408.146	**Durbin-Watson:**	1.110
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	104811.825
Skew:	0.205	**Prob(JB):**	0.00
Kurtosis:	2.330	**Cond. No.**	267.

Table B.61: Decay Time Range Capacity 3

Dep. Variable:	decay_time_mad	**R-squared:**	0.234
Model:	OLS	**Adj. R-squared:**	0.234
Method:	Least Squares	**F-statistic:**	4.081e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:49:06	**Log-Likelihood:**	-1.2857e+07
No. Observations:	4080400	**AIC:**	2.571e+07
Df Residuals:	4080396	**BIC:**	2.571e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	12.6516	0.011	1118.942	0.000	12.629	12.674
mu	-0.0776	0.000	-762.171	0.000	-0.078	-0.077
sigma	0.0706	0.000	695.188	0.000	0.070	0.071
num_cascade	-0.1091	0.001	-211.718	0.000	-0.110	-0.108

Omnibus:	149708.195	**Durbin-Watson:**	1.151
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	166676.557
Skew:	0.491	**Prob(JB):**	0.00
Kurtosis:	3.128	**Cond. No.**	267.

Table B.62: Decay Time Mean Absolute Deviation Capacity 3

Dep. Variable:	decay_time_max	**R-squared:**	0.258
Model:	OLS	**Adj. R-squared:**	0.258
Method:	Least Squares	**F-statistic:**	4.653e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:49:07	**Log-Likelihood:**	-1.8216e+07
No. Observations:	4080400	**AIC:**	3.643e+07
Df Residuals:	4080396	**BIC:**	3.643e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	37.9136	0.038	988.846	0.000	37.838	37.989
mu	-0.3000	0.000	-803.101	0.000	-0.301	-0.299
sigma	0.2788	0.000	734.000	0.000	0.278	0.280
num_cascade	0.5940	0.002	318.493	0.000	0.590	0.598

Omnibus:	231072.892	**Durbin-Watson:**	1.084
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	106294.872
Skew:	0.192	**Prob(JB):**	0.00
Kurtosis:	2.309	**Cond. No.**	267.

Table B.63: Decay Time Max Capacity 3

Dep. Variable:	decay_time_mean	**R-squared:**	0.264
Model:	OLS	**Adj. R-squared:**	0.264
Method:	Least Squares	**F-statistic:**	5.094e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:49:02	**Log-Likelihood:**	-1.4131e+07
No. Observations:	4080400	**AIC:**	2.826e+07
Df Residuals:	4080396	**BIC:**	2.826e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	20.5466	0.017	1177.314	0.000	20.512	20.581
mu	-0.1266	0.000	-887.954	0.000	-0.127	-0.126
sigma	0.0830	0.000	586.934	0.000	0.083	0.083
num_cascade	-0.2466	0.001	-345.306	0.000	-0.248	-0.245

Omnibus:	437381.827	**Durbin-Watson:**	0.986
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	728437.999
Skew:	0.760	**Prob(JB):**	0.00
Kurtosis:	4.405	**Cond. No.**	267.

Table B.64: Decay Time Mean Capacity 3

Dep. Variable:	decay_time_median	**R-squared:**	0.185
Model:	OLS	**Adj. R-squared:**	0.185
Method:	Least Squares	**F-statistic:**	3.334e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:49:03	**Log-Likelihood:**	-1.4357e+07
No. Observations:	4080400	**AIC:**	2.871e+07
Df Residuals:	4080396	**BIC:**	2.871e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	17.6320	0.020	900.404	0.000	17.594	17.670
mu	-0.1070	0.000	-710.385	0.000	-0.107	-0.107
sigma	0.0551	0.000	387.913	0.000	0.055	0.055
num_cascade	-0.2985	0.001	-380.932	0.000	-0.300	-0.297

Omnibus:	1370658.511	**Durbin-Watson:**	1.318
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	6355956.000
Skew:	1.582	**Prob(JB):**	0.00
Kurtosis:	8.232	**Cond. No.**	267.

Table B.65: Decay Time Median Capacity 3

Dep. Variable:	decay_time_std	**R-squared:**	0.234
Model:	OLS	**Adj. R-squared:**	0.234
Method:	Least Squares	**F-statistic:**	3.925e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:49:04	**Log-Likelihood:**	-1.3535e+07
No. Observations:	4080400	**AIC:**	2.707e+07
Df Residuals:	4080396	**BIC:**	2.707e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	14.2624	0.013	1103.530	0.000	14.237	14.288
mu	-0.0921	0.000	-771.154	0.000	-0.092	-0.092
sigma	0.0863	0.000	719.056	0.000	0.086	0.087
num_cascade	-0.0471	0.001	-77.927	0.000	-0.048	-0.046

Omnibus:	101153.845	**Durbin-Watson:**	1.131
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	95621.576
Skew:	0.334	**Prob(JB):**	0.00
Kurtosis:	2.658	**Cond. No.**	267.

Table B.66: Decay Time Standard Deviation Capacity 3

B.2.6 Final Counts

Dep. Variable:	final_counts_sums_gap	**R-squared:**	0.303
Model:	OLS	**Adj. R-squared:**	0.303
Method:	Least Squares	**F-statistic:**	5.055e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:21	**Log-Likelihood:**	-3.6236e+07
No. Observations:	4080400	**AIC:**	7.247e+07
Df Residuals:	4080396	**BIC:**	7.247e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	3546.8850	3.656	970.107	0.000	3539.719	3554.051
mu	-38.0608	0.031	-1224.678	0.000	-38.122	-38.000
sigma	-9.4334	0.028	-336.773	0.000	-9.488	-9.378
num_cascade	16.7753	0.153	109.901	0.000	16.476	17.075

Omnibus:	722582.883	**Durbin-Watson:**	0.590
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1261698.351
Skew:	1.151	**Prob(JB):**	0.00
Kurtosis:	4.455	**Cond. No.**	267.

Table B.67: Final Counts Range Capacity 3

Dep. Variable:	final_counts_sums_mad	**R-squared:**	0.277
Model:	OLS	**Adj. R-squared:**	0.277
Method:	Least Squares	**F-statistic:**	4.292e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:18	**Log-Likelihood:**	-3.1906e+07
No. Observations:	4080400	**AIC:**	6.381e+07
Df Residuals:	4080396	**BIC:**	6.381e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1299.0193	1.435	905.365	0.000	1296.207	1301.832
mu	-12.2017	0.011	-1122.877	0.000	-12.223	-12.180
sigma	-3.6174	0.010	-372.226	0.000	-3.636	-3.598
num_cascade	-7.0134	0.051	-136.727	0.000	-7.114	-6.913

Omnibus:	1165865.220	**Durbin-Watson:**	0.617
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	3333348.709
Skew:	1.512	**Prob(JB):**	0.00
Kurtosis:	6.234	**Cond. No.**	267.

Table B.68: Final Counts Mean Absolute Deviation Capacity 3

Dep. Variable:	final_counts_sums_max	**R-squared:**	0.418
Model:	OLS	**Adj. R-squared:**	0.418
Method:	Least Squares	**F-statistic:**	9.193e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:20	**Log-Likelihood:**	-3.6183e+07
No. Observations:	4080400	**AIC:**	7.237e+07
Df Residuals:	4080396	**BIC:**	7.237e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	5193.8928	3.400	1527.429	0.000	5187.228	5200.558
mu	-47.4794	0.029	-1618.062	0.000	-47.537	-47.422
sigma	-13.6790	0.029	-478.355	0.000	-13.735	-13.623
num_cascade	-35.7455	0.152	-234.878	0.000	-36.044	-35.447

Omnibus:	392509.681	**Durbin-Watson:**	0.532
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	518726.846
Skew:	0.825	**Prob(JB):**	0.00
Kurtosis:	3.570	**Cond. No.**	267.

Table B.69: Final Counts Max Capacity 3

Dep. Variable:	final_counts_sums_mean	**R-squared:**	0.442
Model:	OLS	**Adj. R-squared:**	0.442
Method:	Least Squares	**F-statistic:**	5.532e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:15	**Log-Likelihood:**	-3.3463e+07
No. Observations:	4080400	**AIC:**	6.693e+07
Df Residuals:	4080396	**BIC:**	6.693e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	3145.0382	2.709	1160.800	0.000	3139.728	3150.349
mu	-21.4596	0.017	-1282.733	0.000	-21.492	-21.427
sigma	-7.1042	0.016	-454.404	0.000	-7.135	-7.074
num_cascade	-76.3159	0.094	-815.666	0.000	-76.499	-76.133

Omnibus:	1355354.608	**Durbin-Watson:**	0.275
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	5144653.244
Skew:	1.642	**Prob(JB):**	0.00
Kurtosis:	7.414	**Cond. No.**	267.

Table B.70: Final Counts Mean Capacity 3

Dep. Variable:	final_counts_sums_median	**R-squared:**	0.335
Model:	OLS	**Adj. R-squared:**	0.335
Method:	Least Squares	**F-statistic:**	2.948e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:16	**Log-Likelihood:**	-3.4235e+07
No. Observations:	4080400	**AIC:**	6.847e+07
Df Residuals:	4080396	**BIC:**	6.847e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	2979.0619	3.330	894.542	0.000	2972.535	2985.589
mu	-17.0455	0.020	-849.094	0.000	-17.085	-17.006
sigma	-5.3661	0.017	-307.533	0.000	-5.400	-5.332
num_cascade	-98.0805	0.116	-843.290	0.000	-98.308	-97.853

Omnibus:	2072003.138	**Durbin-Watson:**	0.393
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	13991714.020
Skew:	2.400	**Prob(JB):**	0.00
Kurtosis:	10.698	**Cond. No.**	267.

Table B.71: Final Counts Median Capacity 3

Dep. Variable:	final_counts_sums_std	**R-squared:**	0.287
Model:	OLS	**Adj. R-squared:**	0.287
Method:	Least Squares	**F-statistic:**	4.568e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:17	**Log-Likelihood:**	-3.2409e+07
No. Observations:	4080400	**AIC:**	6.482e+07
Df Residuals:	4080396	**BIC:**	6.482e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1440.4935	1.543	933.616	0.000	1437.469	1443.518
mu	-14.2517	0.012	-1162.948	0.000	-14.276	-14.228
sigma	-4.0728	0.011	-370.421	0.000	-4.094	-4.051
num_cascade	-2.2412	0.059	-38.304	0.000	-2.356	-2.127

Omnibus:	926891.395	**Durbin-Watson:**	0.601
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2011073.392
Skew:	1.320	**Prob(JB):**	0.00
Kurtosis:	5.205	**Cond. No.**	267.

Table B.72: Final Counts Standard Deviation Capacity 3

B.2.7 Max Counts

Dep. Variable:	max_count_gap	**R-squared:**	0.345
Model:	OLS	**Adj. R-squared:**	0.345
Method:	Least Squares	**F-statistic:**	6.237e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:29	**Log-Likelihood:**	-1.8492e+07
No. Observations:	4080400	**AIC:**	3.698e+07
Df Residuals:	4080396	**BIC:**	3.698e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	48.6908	0.046	1054.654	0.000	48.600	48.781
mu	-0.5331	0.000	-1360.558	0.000	-0.534	-0.532
sigma	-0.1516	0.000	-401.567	0.000	-0.152	-0.151
num_cascade	0.4059	0.002	205.994	0.000	0.402	0.410

Omnibus:	433749.392	**Durbin-Watson:**	0.591
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	585252.222
Skew:	0.895	**Prob(JB):**	0.00
Kurtosis:	3.491	**Cond. No.**	267.

Table B.73: Max Counts Range Capacity 3

Dep. Variable:	max_count_mad	**R-squared:**	0.300
Model:	OLS	**Adj. R-squared:**	0.300
Method:	Least Squares	**F-statistic:**	5.047e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:26	**Log-Likelihood:**	-1.4289e+07
No. Observations:	4080400	**AIC:**	2.858e+07
Df Residuals:	4080396	**BIC:**	2.858e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	17.8984	0.019	954.842	0.000	17.862	17.935
mu	-0.1716	0.000	-1214.453	0.000	-0.172	-0.171
sigma	-0.0548	0.000	-412.499	0.000	-0.055	-0.054
num_cascade	-0.0446	0.001	-65.352	0.000	-0.046	-0.043

Omnibus:	952679.673	**Durbin-Watson:**	0.617
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2103282.665
Skew:	1.347	**Prob(JB):**	0.00
Kurtosis:	5.261	**Cond. No.**	267.

Table B.74: Max Counts Mean Absolute Deviation Capacity 3

Dep. Variable:	max_count_max	**R-squared:**	0.453
Model:	OLS	**Adj. R-squared:**	0.453
Method:	Least Squares	**F-statistic:**	1.125e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:28	**Log-Likelihood:**	-1.8529e+07
No. Observations:	4080400	**AIC:**	3.706e+07
Df Residuals:	4080396	**BIC:**	3.706e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	73.0914	0.044	1655.633	0.000	73.005	73.178
mu	-0.6793	0.000	-1801.261	0.000	-0.680	-0.679
sigma	-0.1908	0.000	-478.655	0.000	-0.192	-0.190
num_cascade	-0.3299	0.002	-164.826	0.000	-0.334	-0.326

Omnibus:	168079.315	**Durbin-Watson:**	0.512
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	187507.874
Skew:	0.518	**Prob(JB):**	0.00
Kurtosis:	2.833	**Cond. No.**	267.

Table B.75: Max Counts Max Capacity 3

Dep. Variable:	max_count_mean	**R-squared:**	0.474
Model:	OLS	**Adj. R-squared:**	0.474
Method:	Least Squares	**F-statistic:**	7.702e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:22	**Log-Likelihood:**	-1.5852e+07
No. Observations:	4080400	**AIC:**	3.170e+07
Df Residuals:	4080396	**BIC:**	3.170e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	45.3930	0.034	1323.018	0.000	45.326	45.460
mu	-0.3256	0.000	-1511.609	0.000	-0.326	-0.325
sigma	-0.0836	0.000	-388.670	0.000	-0.084	-0.083
num_cascade	-0.9631	0.001	-793.858	0.000	-0.966	-0.961

Omnibus:	874882.403	**Durbin-Watson:**	0.285
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2234756.346
Skew:	1.174	**Prob(JB):**	0.00
Kurtosis:	5.762	**Cond. No.**	267.

Table B.76: Max Counts Mean Capacity 3

Dep. Variable:	max_count_median	**R-squared:**	0.361
Model:	OLS	**Adj. R-squared:**	0.361
Method:	Least Squares	**F-statistic:**	3.915e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:24	**Log-Likelihood:**	-1.6652e+07
No. Observations:	4080400	**AIC:**	3.330e+07
Df Residuals:	4080396	**BIC:**	3.330e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	43.6662	0.043	1008.505	0.000	43.581	43.751
mu	-0.2687	0.000	-1033.531	0.000	-0.269	-0.268
sigma	-0.0588	0.000	-244.622	0.000	-0.059	-0.058
num_cascade	-1.2837	0.002	-830.734	0.000	-1.287	-1.281

Omnibus:	1869888.366	**Durbin-Watson:**	0.438
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	10723795.729
Skew:	2.176	**Prob(JB):**	0.00
Kurtosis:	9.643	**Cond. No.**	267.

Table B.77: Max Counts Median Capacity 3

Dep. Variable:	max_count_std	**R-squared:**	0.315
Model:	OLS	**Adj. R-squared:**	0.315
Method:	Least Squares	**F-statistic:**	5.437e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:25	**Log-Likelihood:**	-1.4776e+07
No. Observations:	4080400	**AIC:**	2.955e+07
Df Residuals:	4080396	**BIC:**	2.955e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	19.9176	0.020	993.640	0.000	19.878	19.957
mu	-0.2005	0.000	-1267.478	0.000	-0.201	-0.200
sigma	-0.0628	0.000	-418.547	0.000	-0.063	-0.062
num_cascade	0.0271	0.001	34.847	0.000	0.026	0.029

Omnibus:	690305.065	**Durbin-Watson:**	0.604
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1155243.195
Skew:	1.132	**Prob(JB):**	0.00
Kurtosis:	4.290	**Cond. No.**	267.

Table B.78: Max Counts Standard Deviation Capacity 3

B.2.8 Time of Max

Dep. Variable:	max_time_gap	**R-squared:**	0.342
Model:	OLS	**Adj. R-squared:**	0.342
Method:	Least Squares	**F-statistic:**	6.318e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:37	**Log-Likelihood:**	-1.8556e+07
No. Observations:	4080400	**AIC:**	3.711e+07
Df Residuals:	4080396	**BIC:**	3.711e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	45.1581	0.037	1218.642	0.000	45.085	45.231
mu	-0.2809	0.000	-692.041	0.000	-0.282	-0.280
sigma	0.3749	0.000	824.736	0.000	0.374	0.376
num_cascade	1.6463	0.002	806.129	0.000	1.642	1.650

Omnibus:	291142.566	**Durbin-Watson:**	0.746
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	357152.115
Skew:	-0.711	**Prob(JB):**	0.00
Kurtosis:	3.285	**Cond. No.**	267.

Table B.79: Time of Max Range Capacity 3

Dep. Variable:	max_time_mad	**R-squared:**	0.263
Model:	OLS	**Adj. R-squared:**	0.263
Method:	Least Squares	**F-statistic:**	3.555e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:34	**Log-Likelihood:**	-1.3903e+07
No. Observations:	4080400	**AIC:**	2.781e+07
Df Residuals:	4080396	**BIC:**	2.781e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	16.2491	0.013	1267.422	0.000	16.224	16.274
mu	-0.0900	0.000	-702.630	0.000	-0.090	-0.090
sigma	0.1110	0.000	791.022	0.000	0.111	0.111
num_cascade	0.2320	0.001	350.135	0.000	0.231	0.233

Omnibus:	15914.735	**Durbin-Watson:**	0.933
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	15968.661
Skew:	-0.148	**Prob(JB):**	0.00
Kurtosis:	2.917	**Cond. No.**	267.

Table B.80: Time of Max Mean Absolute Deviation Capacity 3

Dep. Variable:	max_time_max	**R-squared:**	0.359
Model:	OLS	**Adj. R-squared:**	0.359
Method:	Least Squares	**F-statistic:**	4.190e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:36	**Log-Likelihood:**	-1.8638e+07
No. Observations:	4080400	**AIC:**	3.728e+07
Df Residuals:	4080396	**BIC:**	3.728e+07
Df Model:	3		

	coef	std err	z	P> |z|	[0.025	0.975]
const	67.4736	0.039	1749.289	0.000	67.398	67.549
mu	-0.3116	0.000	-750.807	0.000	-0.312	-0.311
sigma	0.5044	0.000	1065.852	0.000	0.504	0.505
num_cascade	0.4379	0.002	210.468	0.000	0.434	0.442

Omnibus:	488326.075	**Durbin-Watson:**	0.515
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	682007.092
Skew:	-0.976	**Prob(JB):**	0.00
Kurtosis:	3.449	**Cond. No.**	267.

Table B.81: Time of Max Max Capacity 3

Dep. Variable:	max_time_mean	**R-squared:**	0.397
Model:	OLS	**Adj. R-squared:**	0.397
Method:	Least Squares	**F-statistic:**	7.510e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:30	**Log-Likelihood:**	-1.7130e+07
No. Observations:	4080400	**AIC:**	3.426e+07
Df Residuals:	4080396	**BIC:**	3.426e+07
Df Model:	3		

	coef	std err	z	P> |z|	[0.025	0.975]
const	41.4900	0.031	1348.773	0.000	41.430	41.550
mu	-0.1492	0.000	-570.396	0.000	-0.150	-0.149
sigma	0.4008	0.000	1313.876	0.000	0.400	0.401
num_cascade	-0.6914	0.001	-467.160	0.000	-0.694	-0.689

Omnibus:	26783.373	**Durbin-Watson:**	0.656
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	27336.360
Skew:	0.198	**Prob(JB):**	0.00
Kurtosis:	3.068	**Cond. No.**	267.

Table B.82: Time of Max Mean Capacity 3

Dep. Variable:	max_time_median	**R-squared:**	0.354
Model:	OLS	**Adj. R-squared:**	0.354
Method:	Least Squares	**F-statistic:**	7.261e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:32	**Log-Likelihood:**	-1.7936e+07
No. Observations:	4080400	**AIC:**	3.587e+07
Df Residuals:	4080396	**BIC:**	3.587e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	39.3779	0.039	1020.523	0.000	39.302	39.454
mu	-0.1276	0.000	-404.081	0.000	-0.128	-0.127
sigma	0.4495	0.000	1271.386	0.000	0.449	0.450
num_cascade	-0.9015	0.002	-499.879	0.000	-0.905	-0.898

Omnibus:	79827.052	**Durbin-Watson:**	0.926
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	82664.959
Skew:	0.337	**Prob(JB):**	0.00
Kurtosis:	2.819	**Cond. No.**	267.

Table B.83: Time of Max Median Capacity 3

Dep. Variable:	max_time_std	**R-squared:**	0.269
Model:	OLS	**Adj. R-squared:**	0.269
Method:	Least Squares	**F-statistic:**	3.509e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:33	**Log-Likelihood:**	-1.4286e+07
No. Observations:	4080400	**AIC:**	2.857e+07
Df Residuals:	4080396	**BIC:**	2.857e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	18.7831	0.014	1347.660	0.000	18.756	18.810
mu	-0.1006	0.000	-710.182	0.000	-0.101	-0.100
sigma	0.1213	0.000	770.560	0.000	0.121	0.122
num_cascade	0.2881	0.001	395.942	0.000	0.287	0.290

Omnibus:	92302.166	**Durbin-Watson:**	0.863
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	98738.031
Skew:	-0.381	**Prob(JB):**	0.00
Kurtosis:	3.002	**Cond. No.**	267.

Table B.84: Time of Max Standard Deviation Capacity 3

B.2.9 Min after Max

Dep. Variable:	min_post_max_gap	**R-squared:**	0.296
Model:	OLS	**Adj. R-squared:**	0.296
Method:	Least Squares	**F-statistic:**	5.700e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:49:00	**Log-Likelihood:**	-1.8079e+07
No. Observations:	4080400	**AIC:**	3.616e+07
Df Residuals:	4080396	**BIC:**	3.616e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	43.2681	0.042	1019.606	0.000	43.185	43.351
mu	-0.4439	0.000	-1298.859	0.000	-0.445	-0.443
sigma	-0.0853	0.000	-257.011	0.000	-0.086	-0.085
num_cascade	0.0648	0.002	36.005	0.000	0.061	0.068

Omnibus:	917342.224	**Durbin-Watson:**	0.697
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1882238.866
Skew:	1.340	**Prob(JB):**	0.00
Kurtosis:	4.972	**Cond. No.**	267.

Table B.85: Min after Max Range Capacity 3

Dep. Variable:	min_post_max_mad	**R-squared:**	0.272
Model:	OLS	**Adj. R-squared:**	0.272
Method:	Least Squares	**F-statistic:**	4.762e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:58	**Log-Likelihood:**	-1.3935e+07
No. Observations:	4080400	**AIC:**	2.787e+07
Df Residuals:	4080396	**BIC:**	2.787e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	16.5112	0.018	927.906	0.000	16.476	16.546
mu	-0.1476	0.000	-1180.784	0.000	-0.148	-0.147
sigma	-0.0349	0.000	-293.421	0.000	-0.035	-0.035
num_cascade	-0.1459	0.001	-225.965	0.000	-0.147	-0.145

Omnibus:	1395185.642	**Durbin-Watson:**	0.711
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	5029951.006
Skew:	1.718	**Prob(JB):**	0.00
Kurtosis:	7.216	**Cond. No.**	267.

Table B.86: Min after Max Mean Absolute Deviation Capacity 3

Dep. Variable:	min_post_max_max	**R-squared:**	0.377
Model:	OLS	**Adj. R-squared:**	0.377
Method:	Least Squares	**F-statistic:**	8.721e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:59	**Log-Likelihood:**	-1.7953e+07
No. Observations:	4080400	**AIC:**	3.591e+07
Df Residuals:	4080396	**BIC:**	3.591e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	55.0374	0.038	1451.013	0.000	54.963	55.112
mu	-0.5074	0.000	-1577.507	0.000	-0.508	-0.507
sigma	-0.1233	0.000	-375.061	0.000	-0.124	-0.123
num_cascade	-0.3271	0.002	-189.083	0.000	-0.330	-0.324

Omnibus:	740031.785	**Durbin-Watson:**	0.676
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1316676.442
Skew:	1.165	**Prob(JB):**	0.00
Kurtosis:	4.521	**Cond. No.**	267.

Table B.87: Min after Max Max Capacity 3

Dep. Variable:	min_post_max_mean	**R-squared:**	0.443
Model:	OLS	**Adj. R-squared:**	0.443
Method:	Least Squares	**F-statistic:**	5.972e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:54	**Log-Likelihood:**	-1.4571e+07
No. Observations:	4080400	**AIC:**	2.914e+07
Df Residuals:	4080396	**BIC:**	2.914e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	30.8609	0.025	1227.564	0.000	30.812	30.910
mu	-0.2047	0.000	-1334.988	0.000	-0.205	-0.204
sigma	-0.0657	0.000	-446.045	0.000	-0.066	-0.065
num_cascade	-0.7888	0.001	-863.913	0.000	-0.791	-0.787

Omnibus:	1283793.239	**Durbin-Watson:**	0.389
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	4388106.506
Skew:	1.591	**Prob(JB):**	0.00
Kurtosis:	6.961	**Cond. No.**	267.

Table B.88: Min after Max Mean Capacity 3

Dep. Variable:	min_post_max_median	**R-squared:**	0.303
Model:	OLS	**Adj. R-squared:**	0.303
Method:	Least Squares	**F-statistic:**	2.634e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:55	**Log-Likelihood:**	-1.5852e+07
No. Observations:	4080400	**AIC:**	3.170e+07
Df Residuals:	4080396	**BIC:**	3.170e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	29.8461	0.035	843.549	0.000	29.777	29.915
mu	-0.1552	0.000	-755.945	0.000	-0.156	-0.155
sigma	-0.0501	0.000	-274.095	0.000	-0.050	-0.050
num_cascade	-1.0985	0.001	-843.329	0.000	-1.101	-1.096

Omnibus:	2649468.629	**Durbin-Watson:**	0.542
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	34307113.018
Skew:	3.006	**Prob(JB):**	0.00
Kurtosis:	15.870	**Cond. No.**	267.

Table B.89: Min after Max Median Capacity 3

Dep. Variable:	min_post_max_std	**R-squared:**	0.279
Model:	OLS	**Adj. R-squared:**	0.279
Method:	Least Squares	**F-statistic:**	5.051e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:56	**Log-Likelihood:**	-1.4441e+07
No. Observations:	4080400	**AIC:**	2.888e+07
Df Residuals:	4080396	**BIC:**	2.888e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	18.2046	0.019	961.306	0.000	18.167	18.242
mu	-0.1725	0.000	-1222.273	0.000	-0.173	-0.172
sigma	-0.0392	0.000	-291.015	0.000	-0.040	-0.039
num_cascade	-0.0836	0.001	-114.776	0.000	-0.085	-0.082

Omnibus:	1123888.943	**Durbin-Watson:**	0.698
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2915120.190
Skew:	1.503	**Prob(JB):**	0.00
Kurtosis:	5.849	**Cond. No.**	267.

Table B.90: Min after Max Standard Deviation Capacity 3

B.2.10 Variance in Counts

Dep. Variable:	var_count_gap	**R-squared:**	0.171
Model:	OLS	**Adj. R-squared:**	0.171
Method:	Least Squares	**F-statistic:**	2.402e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:52	**Log-Likelihood:**	-2.9422e+07
No. Observations:	4080400	**AIC:**	5.884e+07
Df Residuals:	4080396	**BIC:**	5.884e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	453.7000	0.690	657.613	0.000	452.348	455.052
mu	-4.5650	0.005	-846.843	0.000	-4.576	-4.554
sigma	-2.2448	0.005	-408.882	0.000	-2.256	-2.234
num_cascade	2.5361	0.029	88.405	0.000	2.480	2.592

Omnibus:	2394877.088	**Durbin-Watson:**	0.850
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	20119897.467
Skew:	2.796	**Prob(JB):**	0.00
Kurtosis:	12.331	**Cond. No.**	267.

Table B.91: Variance in Counts Range Capacity 3

Dep. Variable:	var_count_mad	**R-squared:**	0.158
Model:	OLS	**Adj. R-squared:**	0.158
Method:	Least Squares	**F-statistic:**	2.291e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:50	**Log-Likelihood:**	-2.4540e+07
No. Observations:	4080400	**AIC:**	4.908e+07
Df Residuals:	4080396	**BIC:**	4.908e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	150.2494	0.253	593.348	0.000	149.753	150.746
mu	-1.3017	0.002	-810.861	0.000	-1.305	-1.299
sigma	-0.6607	0.002	-387.389	0.000	-0.664	-0.657
num_cascade	-0.8756	0.009	-100.391	0.000	-0.893	-0.858

Omnibus:	3035126.103	**Durbin-Watson:**	0.851
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	56356062.900
Skew:	3.495	**Prob(JB):**	0.00
Kurtosis:	19.811	**Cond. No.**	267.

Table B.92: Variance in Counts Mean Absolute Deviation Capacity 3

Dep. Variable:	var_count_max	**R-squared:**	0.185
Model:	OLS	**Adj. R-squared:**	0.185
Method:	Least Squares	**F-statistic:**	2.699e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:51	**Log-Likelihood:**	-2.9392e+07
No. Observations:	4080400	**AIC:**	5.878e+07
Df Residuals:	4080396	**BIC:**	5.878e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	487.4694	0.674	722.758	0.000	486.148	488.791
mu	-4.7619	0.005	-895.288	0.000	-4.772	-4.751
sigma	-2.3563	0.005	-431.053	0.000	-2.367	-2.346
num_cascade	1.4979	0.028	52.795	0.000	1.442	1.554

Omnibus:	2394684.646	**Durbin-Watson:**	0.850
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	20131554.986
Skew:	2.796	**Prob(JB):**	0.00
Kurtosis:	12.335	**Cond. No.**	267.

Table B.93: Variance in Counts Max Capacity 3

Dep. Variable:	var_count_mean	**R-squared:**	0.215
Model:	OLS	**Adj. R-squared:**	0.215
Method:	Least Squares	**F-statistic:**	3.189e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:46	**Log-Likelihood:**	-2.4236e+07
No. Observations:	4080400	**AIC:**	4.847e+07
Df Residuals:	4080396	**BIC:**	4.847e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	186.8592	0.271	688.955	0.000	186.328	187.391
mu	-1.3396	0.001	-934.930	0.000	-1.342	-1.337
sigma	-0.6874	0.002	-420.500	0.000	-0.691	-0.684
num_cascade	-3.4859	0.009	-367.829	0.000	-3.505	-3.467

Omnibus:	3913067.874	**Durbin-Watson:**	0.776
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	217404570.685
Skew:	4.662	**Prob(JB):**	0.00
Kurtosis:	37.522	**Cond. No.**	267.

Table B.94: Variance in Counts Mean Capacity 3

Dep. Variable:	var_count_median	**R-squared:**	0.114
Model:	OLS	**Adj. R-squared:**	0.114
Method:	Least Squares	**F-statistic:**	7.996e+04
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:47	**Log-Likelihood:**	-2.4923e+07
No. Observations:	4080400	**AIC:**	4.985e+07
Df Residuals:	4080396	**BIC:**	4.985e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	151.2859	0.361	418.941	0.000	150.578	151.994
mu	-0.7993	0.002	-479.425	0.000	-0.803	-0.796
sigma	-0.4313	0.002	-220.622	0.000	-0.435	-0.427
num_cascade	-5.1348	0.013	-405.848	0.000	-5.160	-5.110

Omnibus:	5883263.806	**Durbin-Watson:**	0.876
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1857171242.211
Skew:	8.817	**Prob(JB):**	0.00
Kurtosis:	106.017	**Cond. No.**	267.

Table B.95: Variance in Counts Median Capacity 3

Dep. Variable:	var_count_std	**R-squared:**	0.163
Model:	OLS	**Adj. R-squared:**	0.163
Method:	Least Squares	**F-statistic:**	2.327e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:48:49	**Log-Likelihood:**	-2.5241e+07
No. Observations:	4080400	**AIC:**	5.048e+07
Df Residuals:	4080396	**BIC:**	5.048e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	172.5303	0.279	618.826	0.000	171.984	173.077
mu	-1.5891	0.002	-828.774	0.000	-1.593	-1.585
sigma	-0.7991	0.002	-398.130	0.000	-0.803	-0.795
num_cascade	-0.2163	0.010	-21.079	0.000	-0.236	-0.196

Omnibus:	2677892.151	**Durbin-Watson:**	0.844
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	32069157.179
Skew:	3.089	**Prob(JB):**	0.00
Kurtosis:	15.267	**Cond. No.**	267.

Table B.96: Variance in Counts Standard Deviation Capacity 3

B.3 Capacity 4 Regressions

B.3.1 Average Counts

Dep. Variable:	avg_count_gap	**R-squared:**	0.276
Model:	OLS	**Adj. R-squared:**	0.276
Method:	Least Squares	**F-statistic:**	4.459e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:04	**Log-Likelihood:**	-1.7317e+07
No. Observations:	4080400	**AIC:**	3.463e+07
Df Residuals:	4080396	**BIC:**	3.463e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	27.7729	0.033	832.742	0.000	27.708	27.838
mu	-0.3393	0.000	-1131.287	0.000	-0.340	-0.339
sigma	-0.0709	0.000	-265.813	0.000	-0.071	-0.070
num_cascade	0.4551	0.001	317.759	0.000	0.452	0.458

Omnibus:	912925.944	**Durbin-Watson:**	0.603
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1865088.908
Skew:	1.336	**Prob(JB):**	0.00
Kurtosis:	4.957	**Cond. No.**	267.

Table B.97: Average Counts Range Capacity 4

Dep. Variable:	avg_count_mad	**R-squared:**	0.246
Model:	OLS	**Adj. R-squared:**	0.246
Method:	Least Squares	**F-statistic:**	3.663e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:01	**Log-Likelihood:**	-1.3156e+07
No. Observations:	4080400	**AIC:**	2.631e+07
Df Residuals:	4080396	**BIC:**	2.631e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	9.9885	0.012	806.638	0.000	9.964	10.013
mu	-0.1136	0.000	-1029.392	0.000	-0.114	-0.113
sigma	-0.0308	9.54e-05	-323.306	0.000	-0.031	-0.031
num_cascade	0.1013	0.000	217.159	0.000	0.100	0.102

Omnibus:	1308941.848	**Durbin-Watson:**	0.586
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	4003549.587
Skew:	1.678	**Prob(JB):**	0.00
Kurtosis:	6.505	**Cond. No.**	267.

Table B.98: Average Counts Mean Absolute Deviation Capacity 4

Dep. Variable:	avg_count_max	**R-squared:**	0.442
Model:	OLS	**Adj. R-squared:**	0.442
Method:	Least Squares	**F-statistic:**	9.787e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:03	**Log-Likelihood:**	-1.7528e+07
No. Observations:	4080400	**AIC:**	3.506e+07
Df Residuals:	4080396	**BIC:**	3.506e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	57.8037	0.038	1535.201	0.000	57.730	57.878
mu	-0.5124	0.000	-1690.638	0.000	-0.513	-0.512
sigma	-0.1424	0.000	-469.925	0.000	-0.143	-0.142
num_cascade	-0.5317	0.002	-327.795	0.000	-0.535	-0.528

Omnibus:	237372.593	**Durbin-Watson:**	0.480
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	281462.356
Skew:	0.642	**Prob(JB):**	0.00
Kurtosis:	3.085	**Cond. No.**	267.

Table B.99: Average Counts Max Capacity 4

Dep. Variable:	avg_count_mean	**R-squared:**	0.440
Model:	OLS	**Adj. R-squared:**	0.440
Method:	Least Squares	**F-statistic:**	5.620e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:58	**Log-Likelihood:**	-1.6017e+07
No. Observations:	4080400	**AIC:**	3.203e+07
Df Residuals:	4080396	**BIC:**	3.203e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	43.2513	0.037	1164.495	0.000	43.179	43.324
mu	-0.2962	0.000	-1290.551	0.000	-0.297	-0.296
sigma	-0.0980	0.000	-459.623	0.000	-0.098	-0.098
num_cascade	-1.0611	0.001	-812.744	0.000	-1.064	-1.059

Omnibus:	1264458.488	**Durbin-Watson:**	0.277
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	4310526.661
Skew:	1.566	**Prob(JB):**	0.00
Kurtosis:	6.943	**Cond. No.**	267.

Table B.100: Average Counts Mean Capacity 4

Dep. Variable:	avg_count_median	**R-squared:**	0.383
Model:	OLS	**Adj. R-squared:**	0.383
Method:	Least Squares	**F-statistic:**	3.686e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:59	**Log-Likelihood:**	-1.6308e+07
No. Observations:	4080400	**AIC:**	3.262e+07
Df Residuals:	4080396	**BIC:**	3.262e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	41.0826	0.040	1019.550	0.000	41.004	41.162
mu	-0.2480	0.000	-992.688	0.000	-0.248	-0.247
sigma	-0.0836	0.000	-377.184	0.000	-0.084	-0.083
num_cascade	-1.2593	0.001	-897.304	0.000	-1.262	-1.257

Omnibus:	1436502.569	**Durbin-Watson:**	0.321
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	5118737.317
Skew:	1.782	**Prob(JB):**	0.00
Kurtosis:	7.173	**Cond. No.**	267.

Table B.101: Average Counts Median Capacity 4

Dep. Variable:	avg_count_std	**R-squared:**	0.257
Model:	OLS	**Adj. R-squared:**	0.257
Method:	Least Squares	**F-statistic:**	3.918e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:00	**Log-Likelihood:**	-1.3563e+07
No. Observations:	4080400	**AIC:**	2.713e+07
Df Residuals:	4080396	**BIC:**	2.713e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	11.1370	0.014	820.868	0.000	11.110	11.164
mu	-0.1291	0.000	-1066.317	0.000	-0.129	-0.129
sigma	-0.0330	0.000	-311.496	0.000	-0.033	-0.033
num_cascade	0.1310	0.001	241.517	0.000	0.130	0.132

Omnibus:	1116378.535	**Durbin-Watson:**	0.594
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2778532.032
Skew:	1.515	**Prob(JB):**	0.00
Kurtosis:	5.677	**Cond. No.**	267.

Table B.102: Average Counts Standard Deviation Capacity 4

B.3.2 Herfinhdahl Index

Dep. Variable:	counts_herfindahl_index	**R-squared:**	0.082
Model:	OLS	**Adj. R-squared:**	0.082
Method:	Least Squares	**F-statistic:**	1.398e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:32	**Log-Likelihood:**	2.4252e+06
No. Observations:	4080400	**AIC:**	-4.850e+06
Df Residuals:	4080396	**BIC:**	-4.850e+06
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	0.2678	0.000	1226.770	0.000	0.267	0.268
mu	-0.0003	2.1e-06	-132.349	0.000	-0.000	-0.000
sigma	-0.0005	2.78e-06	-178.060	0.000	-0.001	-0.000
num_cascade	-0.0065	1.18e-05	-548.068	0.000	-0.007	-0.006

Omnibus:	3607326.568	**Durbin-Watson:**	1.360
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	113885487.517
Skew:	4.310	**Prob(JB):**	0.00
Kurtosis:	27.404	**Cond. No.**	267.

Table B.103: Herfindahl Index Capacity 4

B.3.3 Kurtosis in Counts

Dep. Variable:	counts_kurtosis	**R-squared:**	0.028
Model:	OLS	**Adj. R-squared:**	0.028
Method:	Least Squares	**F-statistic:**	2.614e+04
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:31	**Log-Likelihood:**	-1.0019e+07
No. Observations:	4080400	**AIC:**	2.004e+07
Df Residuals:	4080396	**BIC:**	2.004e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	-0.1546	0.004	-35.955	0.000	-0.163	-0.146
mu	0.0050	4.47e-05	111.927	0.000	0.005	0.005
sigma	-0.0070	5.07e-05	-137.539	0.000	-0.007	-0.007
num_cascade	0.0722	0.000	268.779	0.000	0.072	0.073

Omnibus:	2513046.552	**Durbin-Watson:**	1.580
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	29656596.912
Skew:	2.828	**Prob(JB):**	0.00
Kurtosis:	14.935	**Cond. No.**	267.

Table B.104: Kurtosis in Counts Capacity 4

B.3.4 Number of Zero Counts

Dep. Variable:	counts_zeros	**R-squared:**	0.384
Model:	OLS	**Adj. R-squared:**	0.384
Method:	Least Squares	**F-statistic:**	4.593e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:30	**Log-Likelihood:**	-1.1039e+07
No. Observations:	4080400	**AIC:**	2.208e+07
Df Residuals:	4080396	**BIC:**	2.208e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	0.9168	0.005	201.639	0.000	0.908	0.926
mu	0.0533	6.38e-05	835.433	0.000	0.053	0.053
sigma	-0.0762	7.33e-05	-1040.668	0.000	-0.076	-0.076
num_cascade	0.1597	0.000	459.066	0.000	0.159	0.160

Omnibus:	1090276.559	**Durbin-Watson:**	0.226
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2706233.955
Skew:	1.479	**Prob(JB):**	0.00
Kurtosis:	5.678	**Cond. No.**	267.

Table B.105: Number of Zero Counts Capacity 4

B.3.5 Decay Time

Dep. Variable:	decay_time_gap	**R-squared:**	0.287
Model:	OLS	**Adj. R-squared:**	0.287
Method:	Least Squares	**F-statistic:**	5.835e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:28	**Log-Likelihood:**	-1.8289e+07
No. Observations:	4080400	**AIC:**	3.658e+07
Df Residuals:	4080396	**BIC:**	3.658e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	32.2058	0.038	854.502	0.000	32.132	32.280
mu	-0.3146	0.000	-835.077	0.000	-0.315	-0.314
sigma	0.2812	0.000	727.666	0.000	0.280	0.282
num_cascade	1.0231	0.002	539.618	0.000	1.019	1.027

Omnibus:	209083.954	**Durbin-Watson:**	1.063
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	92099.473
Skew:	0.141	**Prob(JB):**	0.00
Kurtosis:	2.320	**Cond. No.**	267.

Table B.106: Decay Time Range Capacity 4

Dep. Variable:	decay_time_mad	**R-squared:**	0.231
Model:	OLS	**Adj. R-squared:**	0.231
Method:	Least Squares	**F-statistic:**	3.999e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:25	**Log-Likelihood:**	-1.3026e+07
No. Observations:	4080400	**AIC:**	2.605e+07
Df Residuals:	4080396	**BIC:**	2.605e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	12.0139	0.011	1057.030	0.000	11.992	12.036
mu	-0.0871	0.000	-825.408	0.000	-0.087	-0.087
sigma	0.0686	0.000	649.872	0.000	0.068	0.069
num_cascade	0.0089	0.001	16.732	0.000	0.008	0.010

Omnibus:	119259.942	**Durbin-Watson:**	1.116
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	130050.741
Skew:	0.437	**Prob(JB):**	0.00
Kurtosis:	2.968	**Cond. No.**	267.

Table B.107: Decay Time Mean Absolute Deviation Capacity 4

Dep. Variable:	decay_time_max	**R-squared:**	0.281
Model:	OLS	**Adj. R-squared:**	0.281
Method:	Least Squares	**F-statistic:**	5.563e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:27	**Log-Likelihood:**	-1.8338e+07
No. Observations:	4080400	**AIC:**	3.668e+07
Df Residuals:	4080396	**BIC:**	3.668e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	36.0515	0.039	926.718	0.000	35.975	36.128
mu	-0.3298	0.000	-864.781	0.000	-0.331	-0.329
sigma	0.2753	0.000	701.542	0.000	0.275	0.276
num_cascade	0.9198	0.002	477.388	0.000	0.916	0.924

Omnibus:	230746.481	**Durbin-Watson:**	1.044
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	95478.335
Skew:	0.129	**Prob(JB):**	0.00
Kurtosis:	2.296	**Cond. No.**	267.

Table B.108: Decay Time Max Capacity 4

Dep. Variable:	decay_time_mean	**R-squared:**	0.253
Model:	OLS	**Adj. R-squared:**	0.253
Method:	Least Squares	**F-statistic:**	4.828e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:21	**Log-Likelihood:**	-1.4483e+07
No. Observations:	4080400	**AIC:**	2.897e+07
Df Residuals:	4080396	**BIC:**	2.897e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	20.4481	0.018	1115.063	0.000	20.412	20.484
mu	-0.1512	0.000	-990.205	0.000	-0.151	-0.151
sigma	0.0718	0.000	470.604	0.000	0.071	0.072
num_cascade	-0.0683	0.001	-89.050	0.000	-0.070	-0.067

Omnibus:	397635.306	**Durbin-Watson:**	0.985
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	603215.384
Skew:	0.741	**Prob(JB):**	0.00
Kurtosis:	4.162	**Cond. No.**	267.

Table B.109: Decay Time Mean Capacity 4

Dep. Variable:	decay_time_median	**R-squared:**	0.183
Model:	OLS	**Adj. R-squared:**	0.183
Method:	Least Squares	**F-statistic:**	3.345e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:22	**Log-Likelihood:**	-1.4708e+07
No. Observations:	4080400	**AIC:**	2.942e+07
Df Residuals:	4080396	**BIC:**	2.942e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	18.1801	0.020	887.066	0.000	18.140	18.220
mu	-0.1357	0.000	-838.159	0.000	-0.136	-0.135
sigma	0.0405	0.000	264.031	0.000	0.040	0.041
num_cascade	-0.1449	0.001	-175.561	0.000	-0.147	-0.143

Omnibus:	1298332.090	**Durbin-Watson:**	1.291
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	5105793.515
Skew:	1.552	**Prob(JB):**	0.00
Kurtosis:	7.517	**Cond. No.**	267.

Table B.110: Decay Time Median Capacity 4

Dep. Variable:	decay_time_std	**R-squared:**	0.240
Model:	OLS	**Adj. R-squared:**	0.240
Method:	Least Squares	**F-statistic:**	4.104e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:24	**Log-Likelihood:**	-1.3680e+07
No. Observations:	4080400	**AIC:**	2.736e+07
Df Residuals:	4080396	**BIC:**	2.736e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	13.5215	0.013	1038.779	0.000	13.496	13.547
mu	-0.1016	0.000	-826.995	0.000	-0.102	-0.101
sigma	0.0853	0.000	686.240	0.000	0.085	0.086
num_cascade	0.0743	0.001	118.555	0.000	0.073	0.076

Omnibus:	97882.648	**Durbin-Watson:**	1.098
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	84237.667
Skew:	0.290	**Prob(JB):**	0.00
Kurtosis:	2.602	**Cond. No.**	267.

Table B.111: Decay Time Standard Deviation Capacity 4

B.3.6 Final Counts

Dep. Variable:	final_counts_sums_gap	**R-squared:**	0.276
Model:	OLS	**Adj. R-squared:**	0.276
Method:	Least Squares	**F-statistic:**	4.459e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:41	**Log-Likelihood:**	-3.6108e+07
No. Observations:	4080400	**AIC:**	7.222e+07
Df Residuals:	4080396	**BIC:**	7.222e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	2777.2949	3.335	832.742	0.000	2770.758	2783.832
mu	-33.9341	0.030	-1131.287	0.000	-33.993	-33.875
sigma	-7.0929	0.027	-265.813	0.000	-7.145	-7.041
num_cascade	45.5075	0.143	317.759	0.000	45.227	45.788

Omnibus:	912925.944	**Durbin-Watson:**	0.603
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1865088.908
Skew:	1.336	**Prob(JB):**	0.00
Kurtosis:	4.957	**Cond. No.**	267.

Table B.112: Final Counts Range Capacity 4

Dep. Variable:	final_counts_sums_mad	**R-squared:**	0.246
Model:	OLS	**Adj. R-squared:**	0.246
Method:	Least Squares	**F-statistic:**	3.663e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:39	**Log-Likelihood:**	-3.1946e+07
No. Observations:	4080400	**AIC:**	6.389e+07
Df Residuals:	4080396	**BIC:**	6.389e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	998.8514	1.238	806.638	0.000	996.424	1001.278
mu	-11.3566	0.011	-1029.392	0.000	-11.378	-11.335
sigma	-3.0846	0.010	-323.306	0.000	-3.103	-3.066
num_cascade	10.1341	0.047	217.159	0.000	10.043	10.226

Omnibus:	1308941.848	**Durbin-Watson:**	0.586
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	4003549.587
Skew:	1.678	**Prob(JB):**	0.00
Kurtosis:	6.505	**Cond. No.**	267.

Table B.113: Final Counts Mean Absolute Deviation Capacity 4

Dep. Variable:	final_counts_sums_max	**R-squared:**	0.442
Model:	OLS	**Adj. R-squared:**	0.442
Method:	Least Squares	**F-statistic:**	9.787e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:40	**Log-Likelihood:**	-3.6319e+07
No. Observations:	4080400	**AIC:**	7.264e+07
Df Residuals:	4080396	**BIC:**	7.264e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	5780.3750	3.765	1535.201	0.000	5772.995	5787.755
mu	-51.2421	0.030	-1690.638	0.000	-51.302	-51.183
sigma	-14.2383	0.030	-469.925	0.000	-14.298	-14.179
num_cascade	-53.1660	0.162	-327.795	0.000	-53.484	-52.848

Omnibus:	237372.593	**Durbin-Watson:**	0.480
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	281462.356
Skew:	0.642	**Prob(JB):**	0.00
Kurtosis:	3.085	**Cond. No.**	267.

Table B.114: Final Counts Max Capacity 4

Dep. Variable:	final_counts_sums_mean	**R-squared:**	0.440
Model:	OLS	**Adj. R-squared:**	0.440
Method:	Least Squares	**F-statistic:**	5.620e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:35	**Log-Likelihood:**	-3.4808e+07
No. Observations:	4080400	**AIC:**	6.962e+07
Df Residuals:	4080396	**BIC:**	6.962e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	4325.1324	3.714	1164.495	0.000	4317.853	4332.412
mu	-29.6249	0.023	-1290.551	0.000	-29.670	-29.580
sigma	-9.7962	0.021	-459.623	0.000	-9.838	-9.754
num_cascade	-106.1086	0.131	-812.744	0.000	-106.364	-105.853

Omnibus:	1264458.488	**Durbin-Watson:**	0.277
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	4310526.661
Skew:	1.566	**Prob(JB):**	0.00
Kurtosis:	6.943	**Cond. No.**	267.

Table B.115: Final Counts Mean Capacity 4

Dep. Variable:	final_counts_sums_median	**R-squared:**	0.383
Model:	OLS	**Adj. R-squared:**	0.383
Method:	Least Squares	**F-statistic:**	3.686e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:36	**Log-Likelihood:**	-3.5099e+07
No. Observations:	4080400	**AIC:**	7.020e+07
Df Residuals:	4080396	**BIC:**	7.020e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	4108.2625	4.029	1019.550	0.000	4100.365	4116.160
mu	-24.7955	0.025	-992.688	0.000	-24.844	-24.747
sigma	-8.3571	0.022	-377.184	0.000	-8.400	-8.314
num_cascade	-125.9280	0.140	-897.304	0.000	-126.203	-125.653

Omnibus:	1436502.569	**Durbin-Watson:**	0.321
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	5118737.317
Skew:	1.782	**Prob(JB):**	0.00
Kurtosis:	7.173	**Cond. No.**	267.

Table B.116: Final Counts Median Capacity 4

Dep. Variable:	final_counts_sums_std	**R-squared:**	0.257
Model:	OLS	**Adj. R-squared:**	0.257
Method:	Least Squares	**F-statistic:**	3.918e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:37	**Log-Likelihood:**	-3.2354e+07
No. Observations:	4080400	**AIC:**	6.471e+07
Df Residuals:	4080396	**BIC:**	6.471e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	1113.6953	1.357	820.868	0.000	1111.036	1116.354
mu	-12.9060	0.012	-1066.317	0.000	-12.930	-12.882
sigma	-3.3026	0.011	-311.496	0.000	-3.323	-3.282
num_cascade	13.0992	0.054	241.517	0.000	12.993	13.205

Omnibus:	1116378.535	**Durbin-Watson:**	0.594
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2778532.032
Skew:	1.515	**Prob(JB):**	0.00
Kurtosis:	5.677	**Cond. No.**	267.

Table B.117: Final Counts Standard Deviation Capacity 4

B.3.7 Max Counts

Dep. Variable:	max_count_gap	**R-squared:**	0.318
Model:	OLS	**Adj. R-squared:**	0.318
Method:	Least Squares	**F-statistic:**	5.437e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:49	**Log-Likelihood:**	-1.8382e+07
No. Observations:	4080400	**AIC:**	3.676e+07
Df Residuals:	4080396	**BIC:**	3.676e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	38.1467	0.043	886.425	0.000	38.062	38.231
mu	-0.4737	0.000	-1244.434	0.000	-0.474	-0.473
sigma	-0.1261	0.000	-347.632	0.000	-0.127	-0.125
num_cascade	0.7864	0.002	418.728	0.000	0.783	0.790

Omnibus:	639910.110	**Durbin-Watson:**	0.604
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1002753.770
Skew:	1.110	**Prob(JB):**	0.00
Kurtosis:	3.985	**Cond. No.**	267.

Table B.118: Max Counts Range Capacity 4

Dep. Variable:	max_count_mad	**R-squared:**	0.267
Model:	OLS	**Adj. R-squared:**	0.267
Method:	Least Squares	**F-statistic:**	4.207e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:46	**Log-Likelihood:**	-1.4407e+07
No. Observations:	4080400	**AIC:**	2.881e+07
Df Residuals:	4080396	**BIC:**	2.881e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	13.9072	0.017	828.625	0.000	13.874	13.940
mu	-0.1600	0.000	-1098.593	0.000	-0.160	-0.160
sigma	-0.0496	0.000	-372.135	0.000	-0.050	-0.049
num_cascade	0.1844	0.001	285.322	0.000	0.183	0.186

Omnibus:	1177030.472	**Durbin-Watson:**	0.606
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	3070504.664
Skew:	1.576	**Prob(JB):**	0.00
Kurtosis:	5.851	**Cond. No.**	267.

Table B.119: Max Counts Mean Absolute Deviation Capacity 4

Dep. Variable:	max_count_max	**R-squared:**	0.473
Model:	OLS	**Adj. R-squared:**	0.473
Method:	Least Squares	**F-statistic:**	1.216e+06
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:47	**Log-Likelihood:**	-1.8565e+07
No. Observations:	4080400	**AIC:**	3.713e+07
Df Residuals:	4080396	**BIC:**	3.713e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	77.6734	0.046	1698.588	0.000	77.584	77.763
mu	-0.7117	0.000	-1879.463	0.000	-0.712	-0.711
sigma	-0.1977	0.000	-483.321	0.000	-0.199	-0.197
num_cascade	-0.4396	0.002	-214.948	0.000	-0.444	-0.436

Omnibus:	126051.266	**Durbin-Watson:**	0.475
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	122723.732
Skew:	0.387	**Prob(JB):**	0.00
Kurtosis:	2.650	**Cond. No.**	267.

Table B.120: Max Counts Max Capacity 4

Dep. Variable:	max_count_mean	**R-squared:**	0.474
Model:	OLS	**Adj. R-squared:**	0.474
Method:	Least Squares	**F-statistic:**	8.045e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:42	**Log-Likelihood:**	-1.6887e+07
No. Observations:	4080400	**AIC:**	3.377e+07
Df Residuals:	4080396	**BIC:**	3.377e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	57.8406	0.043	1336.468	0.000	57.756	57.925
mu	-0.4197	0.000	-1539.585	0.000	-0.420	-0.419
sigma	-0.1160	0.000	-426.368	0.000	-0.116	-0.115
num_cascade	-1.2136	0.002	-776.957	0.000	-1.217	-1.211

Omnibus:	731194.199	**Durbin-Watson:**	0.293
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1547731.005
Skew:	1.062	**Prob(JB):**	0.00
Kurtosis:	5.143	**Cond. No.**	267.

Table B.121: Max Counts Mean Capacity 4

Dep. Variable:	max_count_median	**R-squared:**	0.407
Model:	OLS	**Adj. R-squared:**	0.407
Method:	Least Squares	**F-statistic:**	4.992e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:44	**Log-Likelihood:**	-1.7204e+07
No. Observations:	4080400	**AIC:**	3.441e+07
Df Residuals:	4080396	**BIC:**	3.441e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	55.0846	0.048	1148.564	0.000	54.991	55.179
mu	-0.3575	0.000	-1192.133	0.000	-0.358	-0.357
sigma	-0.0934	0.000	-329.362	0.000	-0.094	-0.093
num_cascade	-1.4842	0.002	-872.683	0.000	-1.488	-1.481

Omnibus:	1099767.957	**Durbin-Watson:**	0.373
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2873593.055
Skew:	1.465	**Prob(JB):**	0.00
Kurtosis:	5.884	**Cond. No.**	267.

Table B.122: Max Counts Median Capacity 4

Dep. Variable:	max_count_std	**R-squared:**	0.282
Model:	OLS	**Adj. R-squared:**	0.282
Method:	Least Squares	**F-statistic:**	4.571e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:45	**Log-Likelihood:**	-1.4782e+07
No. Observations:	4080400	**AIC:**	2.956e+07
Df Residuals:	4080396	**BIC:**	2.956e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	15.5223	0.018	851.609	0.000	15.487	15.558
mu	-0.1817	0.000	-1148.408	0.000	-0.182	-0.181
sigma	-0.0547	0.000	-371.062	0.000	-0.055	-0.054
num_cascade	0.2327	0.001	313.623	0.000	0.231	0.234

Omnibus:	947679.939	**Durbin-Watson:**	0.610
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1952246.606
Skew:	1.384	**Prob(JB):**	0.00
Kurtosis:	4.956	**Cond. No.**	267.

Table B.123: Max Counts Standard Deviation Capacity 4

B.3.8 Time of Max

Dep. Variable:	max_time_gap	**R-squared:**	0.343
Model:	OLS	**Adj. R-squared:**	0.343
Method:	Least Squares	**F-statistic:**	7.091e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:56	**Log-Likelihood:**	-1.8626e+07
No. Observations:	4080400	**AIC:**	3.725e+07
Df Residuals:	4080396	**BIC:**	3.725e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	40.4393	0.038	1064.509	0.000	40.365	40.514
mu	-0.2723	0.000	-660.756	0.000	-0.273	-0.271
sigma	0.3478	0.000	757.286	0.000	0.347	0.349
num_cascade	1.9322	0.002	925.157	0.000	1.928	1.936

Omnibus:	255049.070	**Durbin-Watson:**	0.782
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	305353.539
Skew:	-0.657	**Prob(JB):**	0.00
Kurtosis:	3.266	**Cond. No.**	267.

Table B.124: Time of Max Range Capacity 4

Dep. Variable:	max_time_mad	**R-squared:**	0.257
Model:	OLS	**Adj. R-squared:**	0.257
Method:	Least Squares	**F-statistic:**	3.905e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:54	**Log-Likelihood:**	-1.3956e+07
No. Observations:	4080400	**AIC:**	2.791e+07
Df Residuals:	4080396	**BIC:**	2.791e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	14.5035	0.013	1117.808	0.000	14.478	14.529
mu	-0.0873	0.000	-674.429	0.000	-0.088	-0.087
sigma	0.1021	0.000	722.960	0.000	0.102	0.102
num_cascade	0.3411	0.001	507.284	0.000	0.340	0.342

Omnibus:	6581.486	**Durbin-Watson:**	0.958
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	6607.787
Skew:	-0.098	**Prob(JB):**	0.00
Kurtosis:	2.971	**Cond. No.**	267.

Table B.125: Time of Max Mean Absolute Deviation Capacity 4

Dep. Variable:	max_time_max	**R-squared:**	0.358
Model:	OLS	**Adj. R-squared:**	0.358
Method:	Least Squares	**F-statistic:**	4.270e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:55	**Log-Likelihood:**	-1.8669e+07
No. Observations:	4080400	**AIC:**	3.734e+07
Df Residuals:	4080396	**BIC:**	3.734e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	65.9427	0.039	1675.032	0.000	65.866	66.020
mu	-0.3020	0.000	-723.323	0.000	-0.303	-0.301
sigma	0.5105	0.000	1074.603	0.000	0.510	0.511
num_cascade	0.4990	0.002	236.903	0.000	0.495	0.503

Omnibus:	485168.584	**Durbin-Watson:**	0.526
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	676268.502
Skew:	-0.973	**Prob(JB):**	0.00
Kurtosis:	3.437	**Cond. No.**	267.

Table B.126: Time of Max Max Capacity 4

Dep. Variable:	max_time_mean	**R-squared:**	0.400
Model:	OLS	**Adj. R-squared:**	0.400
Method:	Least Squares	**F-statistic:**	7.666e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:50	**Log-Likelihood:**	-1.7336e+07
No. Observations:	4080400	**AIC:**	3.467e+07
Df Residuals:	4080396	**BIC:**	3.467e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	40.6855	0.032	1257.208	0.000	40.622	40.749
mu	-0.1348	0.000	-492.887	0.000	-0.135	-0.134
sigma	0.4346	0.000	1373.573	0.000	0.434	0.435
num_cascade	-0.6950	0.002	-444.646	0.000	-0.698	-0.692

Omnibus:	29082.825	**Durbin-Watson:**	0.675
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	29707.917
Skew:	0.209	**Prob(JB):**	0.00
Kurtosis:	3.028	**Cond. No.**	267.

Table B.127: Time of Max Mean Capacity 4

Dep. Variable:	max_time_median	**R-squared:**	0.362
Model:	OLS	**Adj. R-squared:**	0.362
Method:	Least Squares	**F-statistic:**	7.405e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:51	**Log-Likelihood:**	-1.8107e+07
No. Observations:	4080400	**AIC:**	3.621e+07
Df Residuals:	4080396	**BIC:**	3.621e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	37.3336	0.040	937.197	0.000	37.256	37.412
mu	-0.1058	0.000	-322.558	0.000	-0.106	-0.105
sigma	0.4913	0.000	1346.768	0.000	0.491	0.492
num_cascade	-0.8486	0.002	-453.090	0.000	-0.852	-0.845

Omnibus:	74988.236	**Durbin-Watson:**	0.914
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	76491.381
Skew:	0.319	**Prob(JB):**	0.00
Kurtosis:	2.795	**Cond. No.**	267.

Table B.128: Time of Max Median Capacity 4

Dep. Variable:	max_time_std	**R-squared:**	0.264
Model:	OLS	**Adj. R-squared:**	0.264
Method:	Least Squares	**F-statistic:**	3.897e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:50:53	**Log-Likelihood:**	-1.4353e+07
No. Observations:	4080400	**AIC:**	2.871e+07
Df Residuals:	4080396	**BIC:**	2.871e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	16.9699	0.014	1195.235	0.000	16.942	16.998
mu	-0.0981	0.000	-683.158	0.000	-0.098	-0.098
sigma	0.1113	0.000	701.267	0.000	0.111	0.112
num_cascade	0.4033	0.001	543.096	0.000	0.402	0.405

Omnibus:	67893.933	**Durbin-Watson:**	0.897
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	71347.342
Skew:	-0.324	**Prob(JB):**	0.00
Kurtosis:	3.016	**Cond. No.**	267.

Table B.129: Time of Max Standard Deviation Capacity 4

B.3.9 Min after Max

Dep. Variable:	min_post_max_gap	**R-squared:**	0.269
Model:	OLS	**Adj. R-squared:**	0.269
Method:	Least Squares	**F-statistic:**	5.107e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:20	**Log-Likelihood:**	-1.7912e+07
No. Observations:	4080400	**AIC:**	3.582e+07
Df Residuals:	4080396	**BIC:**	3.582e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	34.0628	0.039	864.432	0.000	33.986	34.140
mu	-0.3931	0.000	-1208.292	0.000	-0.394	-0.392
sigma	-0.0644	0.000	-202.615	0.000	-0.065	-0.064
num_cascade	0.4169	0.002	247.151	0.000	0.414	0.420

Omnibus:	1155525.611	**Durbin-Watson:**	0.730
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2946701.786
Skew:	1.558	**Prob(JB):**	0.00
Kurtosis:	5.762	**Cond. No.**	267.

Table B.130: Min after Max Range Capacity 4

Dep. Variable:	min_post_max_mad	**R-squared:**	0.237
Model:	OLS	**Adj. R-squared:**	0.237
Method:	Least Squares	**F-statistic:**	4.121e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:17	**Log-Likelihood:**	-1.3951e+07
No. Observations:	4080400	**AIC:**	2.790e+07
Df Residuals:	4080396	**BIC:**	2.790e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	12.6543	0.015	820.514	0.000	12.624	12.685
mu	-0.1371	0.000	-1087.075	0.000	-0.137	-0.137
sigma	-0.0303	0.000	-258.545	0.000	-0.031	-0.030
num_cascade	0.0828	0.001	144.448	0.000	0.082	0.084

Omnibus:	1500022.408	**Durbin-Watson:**	0.694
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	5574731.310
Skew:	1.851	**Prob(JB):**	0.00
Kurtosis:	7.369	**Cond. No.**	267.

Table B.131: Min after Max Mean Absolute Deviation Capacity 4

Dep. Variable:	min_post_max_max	**R-squared:**	0.409
Model:	OLS	**Adj. R-squared:**	0.409
Method:	Least Squares	**F-statistic:**	9.744e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:18	**Log-Likelihood:**	-1.7927e+07
No. Observations:	4080400	**AIC:**	3.585e+07
Df Residuals:	4080396	**BIC:**	3.585e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	60.3208	0.040	1523.374	0.000	60.243	60.398
mu	-0.5350	0.000	-1675.637	0.000	-0.536	-0.534
sigma	-0.1287	0.000	-382.207	0.000	-0.129	-0.128
num_cascade	-0.5145	0.002	-292.899	0.000	-0.518	-0.511

Omnibus:	526205.783	**Durbin-Watson:**	0.622
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	784796.255
Skew:	0.952	**Prob(JB):**	0.00
Kurtosis:	3.996	**Cond. No.**	267.

Table B.132: Min after Max Max Capacity 4

Dep. Variable:	min_post_max_mean	**R-squared:**	0.443
Model:	OLS	**Adj. R-squared:**	0.443
Method:	Least Squares	**F-statistic:**	5.807e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:13	**Log-Likelihood:**	-1.5966e+07
No. Observations:	4080400	**AIC:**	3.193e+07
Df Residuals:	4080396	**BIC:**	3.193e+07
Df Model:	3		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	43.3572	0.036	1214.908	0.000	43.287	43.427
mu	-0.2864	0.000	-1317.839	0.000	-0.287	-0.286
sigma	-0.0912	0.000	-438.857	0.000	-0.092	-0.091
num_cascade	-1.1319	0.001	-875.750	0.000	-1.134	-1.129

Omnibus:	1208872.097	**Durbin-Watson:**	0.345
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	3751364.540
Skew:	1.532	**Prob(JB):**	0.00
Kurtosis:	6.560	**Cond. No.**	267.

Table B.133: Min after Max Mean Capacity 4

Dep. Variable:	min_post_max_median	**R-squared:**	0.377
Model:	OLS	**Adj. R-squared:**	0.377
Method:	Least Squares	**F-statistic:**	3.736e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:14	**Log-Likelihood:**	-1.6407e+07
No. Observations:	4080400	**AIC:**	3.281e+07
Df Residuals:	4080396	**BIC:**	3.281e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	41.0901	0.040	1016.549	0.000	41.011	41.169
mu	-0.2282	0.000	-937.884	0.000	-0.229	-0.228
sigma	-0.0787	0.000	-352.635	0.000	-0.079	-0.078
num_cascade	-1.3951	0.001	-961.826	0.000	-1.398	-1.392

Omnibus:	1651099.514	**Durbin-Watson:**	0.417
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	7618581.026
Skew:	1.962	**Prob(JB):**	0.00
Kurtosis:	8.423	**Cond. No.**	267.

Table B.134: Min after Max Median Capacity 4

Dep. Variable:	min_post_max_std	**R-squared:**	0.246
Model:	OLS	**Adj. R-squared:**	0.246
Method:	Least Squares	**F-statistic:**	4.385e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:16	**Log-Likelihood:**	-1.4371e+07
No. Observations:	4080400	**AIC:**	2.874e+07
Df Residuals:	4080396	**BIC:**	2.874e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	14.1015	0.017	834.170	0.000	14.068	14.135
mu	-0.1557	0.000	-1124.288	0.000	-0.156	-0.155
sigma	-0.0324	0.000	-246.822	0.000	-0.033	-0.032
num_cascade	0.1171	0.001	174.506	0.000	0.116	0.118

Omnibus:	1323213.367	**Durbin-Watson:**	0.709
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	3995900.650
Skew:	1.705	**Prob(JB):**	0.00
Kurtosis:	6.445	**Cond. No.**	267.

Table B.135: Min after Max Standard Deviation Capacity 4

B.3.10 Variance in Counts

Dep. Variable:	var_count_gap	**R-squared:**	0.159
Model:	OLS	**Adj. R-squared:**	0.159
Method:	Least Squares	**F-statistic:**	2.242e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:12	**Log-Likelihood:**	-2.9291e+07
No. Observations:	4080400	**AIC:**	5.858e+07
Df Residuals:	4080396	**BIC:**	5.858e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	404.0840	0.662	609.944	0.000	402.786	405.382
mu	-4.1803	0.005	-815.444	0.000	-4.190	-4.170
sigma	-2.1127	0.005	-387.790	0.000	-2.123	-2.102
num_cascade	3.7112	0.028	134.673	0.000	3.657	3.765

Omnibus:	2649253.981	**Durbin-Watson:**	0.864
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	28630731.817
Skew:	3.090	**Prob(JB):**	0.00
Kurtosis:	14.411	**Cond. No.**	267.

Table B.136: Variance in Counts Range Capacity 4

Dep. Variable:	var_count_mad	**R-squared:**	0.146
Model:	OLS	**Adj. R-squared:**	0.146
Method:	Least Squares	**F-statistic:**	2.056e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:09	**Log-Likelihood:**	-2.4567e+07
No. Observations:	4080400	**AIC:**	4.913e+07
Df Residuals:	4080396	**BIC:**	4.913e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	130.5630	0.224	581.728	0.000	130.123	131.003
mu	-1.2540	0.002	-776.617	0.000	-1.257	-1.251
sigma	-0.6511	0.002	-375.953	0.000	-0.654	-0.648
num_cascade	0.3990	0.008	50.792	0.000	0.384	0.414

Omnibus:	3090826.499	**Durbin-Watson:**	0.858
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	57528545.565
Skew:	3.592	**Prob(JB):**	0.00
Kurtosis:	19.934	**Cond. No.**	267.

Table B.137: Variance in Counts Mean Absolute Deviation Capacity 4

Dep. Variable:	var_count_max	**R-squared:**	0.189
Model:	OLS	**Adj. R-squared:**	0.189
Method:	Least Squares	**F-statistic:**	2.902e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:11	**Log-Likelihood:**	-2.9280e+07
No. Observations:	4080400	**AIC:**	5.856e+07
Df Residuals:	4080396	**BIC:**	5.856e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	491.4161	0.657	748.086	0.000	490.129	492.704
mu	-4.6759	0.005	-921.196	0.000	-4.686	-4.666
sigma	-2.3604	0.005	-431.491	0.000	-2.371	-2.350
num_cascade	0.8102	0.028	29.364	0.000	0.756	0.864

Omnibus:	2535958.142	**Durbin-Watson:**	0.839
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	25033310.289
Skew:	2.946	**Prob(JB):**	0.00
Kurtosis:	13.607	**Cond. No.**	267.

Table B.138: Variance in Counts Max Capacity 4

Dep. Variable:	var_count_mean	**R-squared:**	0.222
Model:	OLS	**Adj. R-squared:**	0.222
Method:	Least Squares	**F-statistic:**	3.268e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:05	**Log-Likelihood:**	-2.5252e+07
No. Observations:	4080400	**AIC:**	5.050e+07
Df Residuals:	4080396	**BIC:**	5.050e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	246.0784	0.351	701.850	0.000	245.391	246.766
mu	-1.7374	0.002	-955.487	0.000	-1.741	-1.734
sigma	-0.9089	0.002	-419.026	0.000	-0.913	-0.905
num_cascade	-4.7433	0.012	-390.394	0.000	-4.767	-4.719

Omnibus:	4031959.224	**Durbin-Watson:**	0.723
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	266027967.358
Skew:	4.831	**Prob(JB):**	0.00
Kurtosis:	41.358	**Cond. No.**	267.

Table B.139: Variance in Counts Mean Capacity 4

Dep. Variable:	var_count_median	**R-squared:**	0.156
Model:	OLS	**Adj. R-squared:**	0.156
Method:	Least Squares	**F-statistic:**	1.334e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:07	**Log-Likelihood:**	-2.5370e+07
No. Observations:	4080400	**AIC:**	5.074e+07
Df Residuals:	4080396	**BIC:**	5.074e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	203.5536	0.400	509.129	0.000	202.770	204.337
mu	-1.1356	0.002	-617.995	0.000	-1.139	-1.132
sigma	-0.6194	0.002	-273.384	0.000	-0.624	-0.615
num_cascade	-6.4245	0.014	-463.501	0.000	-6.452	-6.397

Omnibus:	5583841.535	**Durbin-Watson:**	0.784
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1434596063.767
Skew:	8.025	**Prob(JB):**	0.00
Kurtosis:	93.445	**Cond. No.**	267.

Table B.140: Variance in Counts Median Capacity 4

Dep. Variable:	var_count_std	**R-squared:**	0.150
Model:	OLS	**Adj. R-squared:**	0.150
Method:	Least Squares	**F-statistic:**	2.116e+05
Date:	Fri, 25 Feb 2022	**Prob (F-statistic):**	0.00
Time:	23:51:08	**Log-Likelihood:**	-2.5194e+07
No. Observations:	4080400	**AIC:**	5.039e+07
Df Residuals:	4080396	**BIC:**	5.039e+07
Df Model:	3		

	coef	**std err**	**z**	**P> |z|**	**[0.025**	**0.975]**
const	150.8106	0.256	588.836	0.000	150.309	151.313
mu	-1.4824	0.002	-789.721	0.000	-1.486	-1.479
sigma	-0.7670	0.002	-379.370	0.000	-0.771	-0.763
num_cascade	0.7696	0.010	79.823	0.000	0.751	0.789

Omnibus:	2881259.717	**Durbin-Watson:**	0.856
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	41093818.883
Skew:	3.350	**Prob(JB):**	0.00
Kurtosis:	17.029	**Cond. No.**	267.

Table B.141: Variance in Counts Capacity 4