Fiscal Policies and the Dollar/Pound Exchange: 1870-1984

Vittorio U. Grilli

Follow this and additional works at: https://elischolar.library.yale.edu/egcenter-discussion-paper-series

Recommended Citation
https://elischolar.library.yale.edu/egcenter-discussion-paper-series/556

This Discussion Paper is brought to you for free and open access by the Economic Growth Center at EliScholar – A Digital Platform for Scholarly Publishing at Yale. It has been accepted for inclusion in Discussion Papers by an authorized administrator of EliScholar – A Digital Platform for Scholarly Publishing at Yale. For more information, please contact elischolar@yale.edu.
FISCAL POLICIES AND THE DOLLAR/POUND EXCHANGE RATE: 1870 - 1984

Vittorio U. Grilli
Yale University

December 1987

Note: This research has been funded by NSF grant No. SES-8605866.

Center Discussion Papers are preliminary materials circulated to stimulate discussion and critical comments. References in publications to Discussion Papers should be cleared with the authors to protect the tentative character of these papers.

The author is an NBER Research Fellow.
Abstract

This paper investigates the consequences of fiscal policies for the exchange rate. After developing a simple theory of how government financing policies should effect the exchange rate, we test it using data on the dollar/pound exchange rate. Previous analyses have concentrated mainly on the post-Bretton Woods flexible exchange rate system, thus ignoring potentially useful information contained in fixed exchange rate periods or in previous flexible exchange rate periods. This paper shows that it is theoretically proper and econometrically feasible to merge evidence from different nominal exchange rate systems. The gain of this procedure is that we can extend the sample period back to the 1870's. Our results suggest that permanent government expenditures are the only fiscal variables that significantly affected the dollar/pound nominal exchange rate. Budget deficits appear to be irrelevant in this respect.
1. Introduction

That there exists a strong relationship between government policies and the nominal exchange rate has long been recognized. Monetary policies, in particular, have been the object of several theoretical and empirical attempts to explain exchange rate movements during periods of flexible exchange rate regimes. More recently, considerable effort has been devoted to the understanding of exchange rate discontinuities caused by the collapse of fixed exchange rate regimes. Most of this theoretical and empirical work has focused on the monetary causes of these exchange rate movements. They all stress the fundamental incompatibility of continuous inflationary policies with the maintenance of a fixed exchange rate.

Even if monetary policies have been a prime subject of investigation, little attention has been paid, either by the flexible exchange rate literature or by the collapsing exchange rate literature, to the underlying determinants of monetary policies. This paper, on the other hand, explicitly considers the rationale behind money supply decisions, by formalizing the link between fiscal and monetary policy. Inflation, in particular, is seen as the result of the optimal financing of an exogenous stream of government expenditures.

By stressing the role of inflation as a financing instrument, this analysis provides useful insights into the understanding of the evolution of the international monetary system. At the basis of this approach is the idea that both the dynamics of flexible exchange rates and the choice of exchange rate regime are endogenous variables. In this paper we take the view that government spending is the fundamental exogenous variable driving both the exchange rate (during flexible regime periods) and the switches between the
two alternative exchange rate regimes. A brief overview of the history of the dollar/pound exchange rate reveals, in fact, an alternation of periods of fixed exchange rates (during times of relative tranquillity in government spending), and periods of flexible rates (during periods of high and divergent level of expenditure).

The paper is organized as follows. Section 2 presents a simple model in which inflation, income taxes and deficits are the results of an optimal government budget decision. Section 3 derives the implications for exchange rate behavior, and estimate the model using ordinary least squares. Section 4 describes a variant of censored data techniques that can be fruitfully used in this circumstance, and provides maximum likelihood estimates following this approach. Section 5 concludes the paper with a summary of the results.

2. A Model of Optimal Seigniorage

In this section we describe a simple model in which the dynamics of income taxes, seigniorage and government debt are the result of a rational decision of the government seeking to finance a given flow of expenditure in an optimal manner. The model is close in spirit to work on optimal inflation tax by Phelps (1973). A similar approach is used by Mankiw (1987) to explain the post World War II behavior of nominal interest rates in the U.S.

The intuition behind the model is quite simple. The government can use different means of financing its expenditure: income taxes, monetization or deficits. The government has to choose the optimal mix of these instruments. If the problem has an interior solution, part of expenditure is going to be financed by money creation.

Consider an economy whose representative agent is interested in
maximizing his expected lifetime utility, $U$, which is given by

$$U = E_0 \left( \sum_{t=0}^{\infty} \beta^t u(c_t) \right)$$

where $c_t$ is the consumption at time $t$, and $0 < \beta < 1$ is the discount factor.

Randomness in this economy is the consequence of stochastic government spending and output, which will be introduced below. The agents of this economy have access to the international credit market, where real bonds $b_t$ are traded at the world given interest rate $r$ that, for simplicity, is postulated to be such that $\beta \frac{1}{1+r}$. The individuals are assumed to hold cash balances in order to economize on the transaction cost of exchange. Transactions are costly in the sense that a certain fraction of individual's real income is used up in the transaction process. This fraction, $v$, is a convex function of the ratio of real balances held by the agent to his real income, i.e.

$$v_t = v \left( \frac{M_t}{P_t y_t} \right)$$

$$v' < 0 \quad v'' > 0 \quad 0 < v_t < 1$$

where $P_t$ is the price level and $y_t$ the exogenous real income. In addition, agents are required to pay taxes $r_t$ in real terms. In order to capture in a simple way the distortionary effects of income taxes, we assume that a fraction of real income is absorbed in the tax collection process. The fraction, $z$, is a convex function of the ratio of taxes to real income:

1 A similar approach is used by Greenwood (1983) and Kimbrough (1986).
\[ z_t = z(t_t/y_t) \quad \begin{align*}
z' &> 0 \\
z'' &> 0 \\
0 &< z_t < 1
\end{align*} \]

The individual optimization problem is therefore given by:

\[
P-1 \max_{c_t, M_t} E_0 \left( \sum_{t=0}^{\infty} \beta^t u(c_t) \right) \]

s.t. \[
\sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \left\{ c_t + r_t + y_t + z_t y_t + \frac{M_t}{p_t} = y_t + (1+r)b_{t-1} + \frac{M_{t-1}}{p_t} \right\}
\]

where \( m_t \) are the real money balances, \( R_t \) is the nominal interest rate which satisfies the Fisher equation \((1+R_t) = (1+\pi_t)(1+r)\) and \( \pi_t \) is the rate of inflation.

In deriving problem P-1 we also assume that \( b_{-1} = M_{-1} = 0 \) and that the usual transversality condition holds.

The first order conditions of this problem imply:

\[
E_t u'(c_{t+1}) = u'(c_t) \quad \text{(2.1)}
\]
Equation (2.1) is the well known random walk property of the marginal utility of consumption, while (2.2) reveals that, since we assumed \( v'' > 0 \), increases in inflation are costly since they reduce desired cash balances as a fraction of income.

The next step is to endogenize inflation by considering the optimization problem of a government seeking to maximize the welfare of the representative agent in the economy. In this case the authorities will choose the paths of inflation, taxes and deficits which minimize the cost of raising the revenue necessary to finance their expenditures. The period \( t \) budget constraint for the government is given by:

\[
G_t + (1+r)B_{t-1} = \tau_t + B_t + \frac{(M_t - M_{t-1})}{P_t}
\]

Making use of the usual transversality condition, the government optimization problem can be written as:

\[
P - 2 \min_{\tau_t, \pi_t} \mathbb{E}_0 \left( \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \left( v\left( \frac{m_t}{y_t} \right)y_t + z\left( \frac{\tau_t}{y_t} \right)y_t \right) \right)
\]

s.t. \[
\sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \left( G_t - \tau_t - \left( \frac{R_t}{1+R_t} \right)m_t \right) = 0
\]

For simplicity, define seigniorage \( S_t = \frac{R_t}{1+R_t} m_t \). Note that the marginal
social cost of inflation is positive, i.e. \( \frac{\partial v(t)}{\partial \pi_t} > 0 \). Moreover, we require the objective function to be convex, which implies \( z'' > 0 \), as assumed, and

\[ \frac{\partial^2 v(t)}{\partial \pi_t^2} > 0. \]

The first order conditions for this problem are given by:

\[ \frac{\partial v(t)}{\partial \pi_t} = z'(t) \quad (2.3) \]

\[ E_t[z'(t+1)] = z'(t) \quad (2.4) \]

\[
E_t\left[ \begin{array}{c} \frac{\partial v(t+1)}{\partial \pi_{t+1}} \\ \frac{\partial s_{t+1}}{\partial \pi_{t+1}} \\ \frac{\partial s_{t}}{\partial \pi_{t}} \\ \frac{\partial s_{t}}{\partial \pi_{t}} \end{array} \right] = \frac{\partial v(t)}{\partial \pi_t} \quad (2.5)
\]

As (2.3) reveals, the solution of this problem involves equating the ratios of the marginal cost to the marginal revenue of the two alternative financing instruments (since output is exogenous, the marginal revenue of income taxes is one in this model). The optimum must be in the positively sloped side of the inflation tax Laffer's curve, where the marginal revenue from money creation is positive, i.e. \( \frac{\partial s_t}{\partial \pi_t} > 0 \). Equation (2.4) equates the ratio of marginal cost to marginal revenue of the income tax today and in the future. Equation (2.5) does the same for the inflation tax. In the special case in which taxes are not distortionary, i.e. \( z = 0 \), the solution to P-2
reduces

\[ \frac{\partial v(t)}{\partial \pi_t} = v'(t) \frac{\partial m_t}{\partial \pi_t} = 0 \]

Since condition (2.2) implies that in equilibrium \(-v'(\cdot) = \frac{R_t}{1 + R_t}\), the optimal policy is to set inflation in such a way that \(R_t = 0\). This is Friedman's optimum quantity of money rule. In this case seigniorage will be nil, and government expenditure will be financed only through taxes. The opposite extreme is one in which inflationary policy is non-distortionary, i.e. \(v = 0\). In this case, as long as the marginal revenue from money financing is positive, taxes will be set to zero. In the intermediate case in which both types of financing are distortionary, we would expect that both will be used to assure solvency.

In a recent paper, Kimbrough (1986) challenges the view that seigniorage should be part of an optimal tax policy. He argues that, even if taxes are distortionary, Friedman's optimal quantity of money rule should be followed. In his model, inflation decreases the potential output of the economy, by reducing the individual's time endowment (an hypothesis analogous to the one we made above). On the other hand, he assumes that the only alternative financing instrument is a consumption tax, which does not have the same direct negative effect on the production possibility frontier (it alters the marginal choice between consumption and leisure, but does not affect the total time endowment). Because of this asymmetry, in that model it is optimal to refrain from raising revenues through seigniorage.

In our model, instead, taxes and inflation lead to a similar contraction of potential output, so that they are both used as financing instruments.
Moreover, because of the deadweight losses imposed by taxes and inflationary finance, in our environment it is optimal to use deficits as a buffer for temporary deviations of government expenditure from its permanent level. Income taxes and inflation should be used to finance only the permanent part of government expenditure, a generalization of the well-known tax smoothing result obtained by Barro (1979). This result can be derived explicitly, by assuming that the ratio of seigniorage to output can be expressed as a linear function of inflation:

\[ \frac{S_t}{y_t} = b\pi_t \]

and that \( v(t) \) is a quadratic function of inflation:

\[ v = a_1 + a_2 \pi_t^2. \]

Then, inflation, as well as the nominal interest rate and seigniorage, is a martingale. The same is true for the income tax rate, under the assumption that \( z(t) \) is quadratic\(^2\). Making use of the expectation operator, and assuming for simplicity that the relevant covariance terms are equal to zero, we can manipulate the government budget constraint at period \( t \) to give:

\[
(1+r)B_{t-1} + \sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^j E_t \frac{G_{t+j}}{y_{t+j}} = \sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^j \left[ \left( E_t \frac{r_{t+j}^i}{y_{t+j}} + E_t \frac{S_{t+j}^i}{y_{t+j}} \right) E_t y_{t+j} \right]
\]

(2.6)

By assuming that output is a random walk and using the martingale properties

\(^2\) See Mankiw (1987) for a similar result.
of seigniorage and income tax, (2.6) can be written as:

\[
\frac{rB_{t-1} + G^P_t}{y_t} = \frac{\tau_t + b\pi_t}{y_t} \quad (2.7)
\]

where \( G^P_t = \left( \frac{r}{1+r} \right) \sum_{j=0}^{\infty} \left( \frac{1}{1+r} \right)^j E_t G_{t+j} \).

We can interpret the left-side ratio as the expected permanent fraction of output consumed by the government. As long as problem P-2 has an interior solution, increases in this ratio imply increases in the rate of inflation.

3. Implications for Exchange Rate Behavior and Empirical Application

Consider a foreign country which is in all ways analogous to the domestic economy.\(^3\) The exchange rate and the price levels in the two countries are assumed to be linked by:

\[
e_t = p_t - p^*_t + u_t \quad (2.9)
\]

e, p, p* are the logarithms of nominal exchange rate, domestic and foreign price level, respectively. In accord to a considerable body of empirical evidence, we also assume that the deviations from purchasing power parity are permanent in nature, i.e.:

\[\text{\footnotesize 3 Here we still maintain the assumption that the real interest rate is world determined and cannot be effected by either of the two economies.}\]
\[ u_t = u_{t-1} + \epsilon_{1t} \]  \hspace{1cm} (2.10)

where \( \epsilon_{1t} \) is iid. We can now express the rate of change of the exchange rate (\( \Delta e_t \)) as:

\[ \Delta e_t = \beta_{10} + \beta_{11} g^p_{us,t} + \beta_{12} g^p_{uk,t} + \epsilon_{1t} \]  \hspace{1cm} (2.11)

where \( g^p_{us} \) and \( g^p_{uk} \) represent the U.S. and U.K. ratios of permanent government expenditure to output, respectively.

We now investigate whether the effects of fiscal policy on exchange rate predicted by the above model can be detected for the dollar/pound rate. The data set used in this study is composed of annual observations, for the period 1870-1984. We argued above that temporary expenditure should be financed through budget deficits. Therefore, empirically we identify the temporary component of expenditure with the real deficit, defined as:

\[ D_t = \frac{B_t}{p_t} - \frac{B_{t-1}}{p_{t-1}} \]  \hspace{1cm} (3.1)

where \( B_t \) is the amount of government bonds outstanding at time \( t \). The permanent component is given by the difference between the actual real expenditure and its temporary element.\(^4\)

We included in the specification not only permanent expenditures but also

---

\(^4\) In this paper we used government expenditure of the central government, inclusive of interest payments and net of transfers. The results, however, are quite robust with respect to the choice of spending aggregate. Debt is total debt outside the central bank. Appendix B provides a detailed description of the data.
budget deficits as a percentage of output \((d_{us}, d_{uk})\). This allows us to test whether, contrary to the above theory, they have had any effect on the exchange rate. We also include government receipts as a percentage of output \((r_{us}, r_{uk})\). The above model predicts that, once we control for permanent spendings, government revenues should not have any independent effect on inflation (exchange rate). However, unexpected shifts in the relative welfare cost of the two financing instruments (i.e. changes in the \(z()\) and \(v()\) functions) would induce switches between income taxes and inflation, for given permanent government expenditure (see 2.3). Under these circumstances, increases in revenue would imply a reduction in inflation and have an appreciative effect on the exchange rate. The estimated exchange rate equation had, therefore, the following form:

\[
\Delta e = \beta_{10} + \beta_{11} g_{us} + \beta_{12} g_{uk} + \beta_{13} d_{us} + \beta_{14} d_{uk} \\
+ \beta_{15} r_{us} + \beta_{16} r_{uk} + \epsilon_{1t} \tag{3.2}
\]

where \(\epsilon_{1t}\) is assumed to be independently normally distributed.

The time series of the exchange rate for this sample is plotted in Figure 1. As it is well known, and as Figure 1 evidences, periods of fixed exchange regimes have alternated with periods of flexible rates. In table 1, we report the estimate of (3.2) obtained by using only data corresponding to periods of flexible exchange rates. Both permanent expenditures are significant and have the expected signs. Moreover, U.S. revenues appear to have had a significant (appreciative) effect on the dollar, indicating that shifts in the relative cost function may have occurred in the U.S..

Even if our data set extends over 115 years, the above estimate are based
only on 25 data points. This is because, for most of the sample, the exchange rate was fixed, making it difficult to detect the effects of changes in government expenditure. In the next section it will be shown how to improve our estimates by properly utilizing the information contained in periods of fixed exchange rates. For the moment, however, assume that one ignores the problem of the existence of periods of fixed exchange rates, and uses ordinary least squares to estimate equation (3.2) over the whole period 1870-1984. The results of this experiment are reported in Table 2. The results are very similar to the one obtained above. Both U.S. and U.K. permanent expenditures have the expected sign and are significantly different from zero at least at the 5% level. Of the other variables, U.S. government revenues seem to have a significant impact on the exchange rate.

4. Censored Regression Models With Unobserved Thresholds

During period of fixed exchange rates, variations in permanent expenditure would not be reflected in exchange rate changes. This does not imply, however, that the study of these periods cannot provide any useful information about our theory. On the contrary, the type of exchange rate system may be itself a function of the level of government expenditure. Periods of moderate spending (taking the conditions in the other country as given), and therefore of moderate monetization, can be compatible with a fixed parity. On the other hand, continuous or substantial increases in permanent expenditure may undermine the viability of a fixed exchange rate system, producing its collapse and a switch to a floating regime. For example, if we divide the observation into two groups depending on whether they belong to periods of fixed or flexible exchange rates, we notice that, while the U.S.
has essentially the same average level of expenditure in the two subsets (6.5% of output during fixed and 8.1% during flexible rates) the U.K.'s average expenditure is considerably higher during flexible rate periods (15.5% vs. 22.1%).

The issue of collapsing exchange rate regimes has been extensively analyzed by the speculative attack literature. The focus there is to determine the timing and the magnitude of a devaluation (revaluation), which is seen as the consequence of an attack on the official reserves by rational speculators. One of the main insights of this literature is that the crucial variable determining a switch from fixed to flexible exchange rate regime is the shadow exchange rate, i.e. the equilibrium exchange that would prevail in the post-collapse floating regime. Assuming that only a dollar revaluation is possible (as it has been the case in our sample), we can describe the condition for the viability of a fixed exchange rate as:

$$\Delta e^s_t > \Delta e^\text{min}_t$$

(4.1)

i.e., the rate of growth of the shadow exchange rate ($\Delta e^s_t$) must be above some minimum level ($\Delta e^\text{min}_t$). By definition, in a flexible exchange rate regime the shadow and the actual rate coincide, i.e. $\Delta e^s_t = \Delta e_t$. If the shadow rate were observable during the period of fixed rate, we could estimate (3.2) by using such data. The problem is that the floating shadow is not observable during fixed exchange rate regimes, since $\Delta e^s_t$ is observable only if:

---

5 See, for example, Flood and Garber (1984), Blanco and Garber (1986), Grilli (1986).
Therefore, in part of the sample \( \varepsilon_t \) does not have zero mean. This implies that the OLS estimates are both biased and inconsistent. The way in which we propose to estimate (3.2) is to consider

\[
\Delta e^s_t = \beta_1 x_{1t} + \varepsilon_{1t}
\]

as a censored regression \((x_{1t} \) is the \( 7 \times 1 \) vector of the fiscal variables, and \( \beta_1 \) is the \( 7 \times 1 \) vector of parameters) where the threshold \( \Delta e_{\text{min}}^\text{t} \), above which the data are censored, is itself unobservable. It is assumed, however, that we observe the variables determining it, that is:

\[
\Delta e_{\text{min}}^\text{t} = \beta_2' x_{2t} + \varepsilon_{2t}
\]

where \( x_{2t} \) is a vector of observable variables. It is useful to partition the sample observations into three distinct groups. The first group is composed of \( N_1 \) observations referring to fixed exchange rate periods. The only information that we have for these observations is that \( \Delta e^s_t > \Delta e_{\text{min}}^\text{t} \), i.e. \( (\varepsilon_1 - \varepsilon_2) > \beta_2' x_2 - \beta_1' x_1 \).

The second group is composed of \( N_2 \) observations referring to the periods in which a collapse of the system and a revaluation of the exchange rate occurred. In this case we know that a collapse occurred because \( \Delta e^s_t < \Delta e_{\text{min}}^\text{t} \), that is: \( (\varepsilon_1 - \varepsilon_2) < \beta_2' x_2 - \beta_1' x_1 \).

The last group is composed of \( N_3 \) observations referring to the flexible exchange rate periods. In this case we freely observe \( \Delta e_t \) which coincides with \( \Delta e^s_t \).
Assuming that \((\varepsilon_1, \varepsilon_2)\) are normally distributed with mean zero and covariance matrix \(\Omega = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}\), we can write the logarithm likelihood function for this problem as:

\[
\log(L(\beta_1', \beta_2', \Omega)) = \sum_{N_1} \log \left( 1 - \phi \left( \frac{\beta_2' x_{2t} - \beta_1' x_{1t}}{\sigma} \right) \right) + \sum_{N_2} \log \left( \phi \left( \frac{\beta_2' x_2 - \beta_1' x_1}{\sigma} \right) \right) + \sum_{N_3} \log \left( \phi \left( \frac{\Delta e_t - \beta_1' x_1}{\sigma_1} \right) \right)
\]

where \(\phi\) and \(\Phi\) are the density function and the distribution function, respectively, of the standard normal, and \(\sigma^2 = \sigma_1 + \sigma_2 - 2\sigma_{12}\). In order to obtain consistent estimates to use as starting values in the maximum likelihood procedure, we used a variant of a two-stage method described by Maddala (1983). A description of this estimator is given in Appendix A.

The estimates of the parameters obtained by maximizing (4.2) are given in Table 3, where \(x_2\) is composed of the lagged difference between the ratios of permanent expenditures to output of the two countries (\(\Delta g\)) and the fixed parity (\(\tilde{e}\)). These maximum likelihood estimates would seem to support the theory of optimal financing as a partial explanation of the movements of the Dollar/Pound exchange rate between 1870 and 1984. In fact, the two permanent expenditure ratios are both significant and have the expected sign. A one percent increase in the ratio of U.S. permanent expenditure to output induces
a three percent devaluation in the dollar. On the other hand, a one percent increase in the ratio of British permanent government expenditure to output revalues the dollar by almost one percent. U.S. revenue, on the other hand, seem to have lost part of their explanatory power.

5. Conclusions

A novel aspect of this paper is the choice of the time period in which the empirical investigation is conducted. Previous studies have mainly concentrated on the post Bretton Woods flexible exchange rate system. In this paper, we make use of econometric techniques that exploit information contained in data from periods of fixed exchange rates. Thus, our empirical analysis ranges from the 1870's to the 1980's. The study of this extended time period is of particular importance since major changes in the nominal exchange rates seem to be connected with major changes in government expenditure, like the ones occurring during war times.

The results of this paper suggest that the permanent components of public expenditures have been a crucial factor in driving the evolution of the dollar/pound rate in the last hundred and fifteen years. U.S. revenues also contribute to the explanation of exchange rate movements, indicating possible changes overtime of the welfare cost of alternative financing instruments. Moreover, the paper provides further evidence in favor of the thesis of irrelevance of budget deficits in the determination of nominal variables.
References


Board of Trade, Miscellaneous Statistics of the United Kingdom, London.


Evans, Paul, 1986, "Is the Dollar High Because of Large Budget Deficits?" Journal of Monetary Economics 18:227-249.


U.S. Board of Governors of the Federal Reserve System, Annual Statistical Digest, Washington, D. C.


Appendix A

First we obtained consistent estimate of $\beta_1'$ and $\sigma_1$ by estimating (3.2) by OLS using only the $N_3$ observations of flexible rates. Next, from the probit model based on the dichotomous variable $I$ (which takes value 1 for the $N_2$ observation and 0 for the $N_1$ observation) and $\frac{\beta_2'x_2 - \beta_1'x_1}{\sigma}$, and from using the OLS estimate of the $\beta_1'$s, we obtained consistent estimates for $\sigma$ and $\beta_2'$. The other two parameters, $\sigma_2$ and $\sigma_{12}$, do not appear in the log likelihood functions, so that they are not estimable by maximum likelihood. However, we can obtain consistent estimates in the following way. Consider the $N_2$ observations, i.e. the ones referring to a revaluation. In this sample we know that $\Delta e^s < \Delta e^\text{min}$, i.e.

$$\varepsilon_3 = \frac{\varepsilon_1 - \varepsilon_2}{\sigma} < \frac{\beta_2'x_2 - \beta_1'x_1}{\sigma} = \beta'x.$$

Therefore, if we run OLS on

$$\Delta e^s_t = \beta_1'x_1 + \varepsilon_1t$$

we would obtain biased estimates since $\varepsilon_1$ is not zero mean in this sample. However, we can control for this, by noting that:

$$E(\varepsilon_1 | \varepsilon_3 \leq \beta'x) = -\sigma_{13} \phi(\beta'x)$$

where $\sigma_{13} = \sigma_{1}^2 - \sigma_{12}$. Therefore, we can run OLS on
\[ \Delta e_t = \beta' x_t - \sigma_{13} \phi(\beta' x_t) + u_t \]

where \( u_t \) has now zero mean. From the estimate of \( \sigma_{13} \) we can derive \( \sigma_{12} \).

Finally, using \( \sigma = \sigma_1^2 + \sigma_2^2 - 2\sigma_{12} \) we can derive an estimate for \( \sigma_2 \).
Appendix B

This appendix provides a detailed description of the data sources. All data are centered on the end of June up to 1976, and the end of September thereafter.

1. GNP Statistics

1.A. United States

1.A.1 Real GNP, in 1929 dollars
- 1870-1888 Christina Romer [1986]
- 1889-1908 Historical Statistics of the United States Series F3
- 1909-1976 National Income and Product Accounts Table: Table 1.22 and 1.2

1.A.2 Deflator, 1929 = 100
- 1870-1975 Ratio of nominal to real income from Friedman and Schwartz (1982), Table 4.8

1.A.3 Nominal GNP
- 1870-1888 Estimated as real GNP* Deflator
- 1889-1908 Historical Statistics of the United States Series F1
- 1909-1976 National Income and Product Accounts Tables 1.22 and 1.1

1.B. United Kingdom

1.B.1 Nominal GNP
- 1870-1888 Capie and Webber (1985), Table III.12

1.B.2 Deflator, 1929=100
- 1870-1965 Capie and Webber, Table III.12

1.B.3 Real GNP, in 1929 pounds
- 1870-1984 Calculated as the ratio of Nominal GNP to Deflator
2. Public Finance Statistics

2.A. United States

2.A.1 Federal Government Expenditure
1870-1970 Historical Statistics of the U.S.
    Tables Y336, Y472
    Table #491

2.A.2 Federal Government Debt
1870-1970 Historical Statistics of the U.S. Table Y488
1971-1984 Statistical Abstract of the U.S. Table #491

2.B. United Kingdom

2.B.1 Central Government Expenditure
1870-1938 Mitchell and Dean
1939-1965 Mitchell and Jones
1966-1984 Annual Abstract of Statistics, V. 107 T320,
    V11 T352, V122 T16.4

2.B.2 Total National Debt
1870-1939 Mitchell and Dean, T5
1940-1966 Mitchell and Jones, T3

3. Monetary Aggregates

3.A. United States

3.A.1 Monetary Base
1870-1960 Friedman-Schwartz Table B-3
    Statistics, Series 14

3.A.2 Official Reserves
1878-1909 National Monetary Commission, T4
1910-1913 Commercial and Financial Chronicle
1914-1941 Banking and Monetary Statistics, V1, T160
1942-1970 Banking and Monetary Statistics, V2, T14.1
1971-1982 Annual Statistical Digest, various

3.B. United Kingdom

3.B.1 Monetary Base
1870-1982 Capie and Webber
3.B.2 Official Reserves
1870-1879 Miscellaneous Statistics of the UK, Board of Trade
1880-1887 Bankers Magazine
1888-1909 National Monetary Commission
1910-1918 Bankers Magazine
1928-1931 Banking and Monetary Statistics
1932-1939 Bankers Magazine
1946-1984 International Financial Statistics

4. Exchange Rate, Dollars per Pound
1870-1890 Commercial and Financial Chronicle
1891-1909 National Monetary Commission
1910-1953 Commercial and Financial Chronicle
1954-1984 OECD, Main Economic Indicators
Table 1
OLS Estimate: Flexible Exchange Period
(T-Statistic in Parenthesis)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>0.044</td>
<td>(0.023)</td>
</tr>
<tr>
<td>$g_{us}^P$</td>
<td>6.317</td>
<td>(2.381)</td>
</tr>
<tr>
<td>$g_{uk}^P$</td>
<td>-0.995</td>
<td>(2.579)</td>
</tr>
<tr>
<td>$d_{us}$</td>
<td>-2.304</td>
<td>(1.503)</td>
</tr>
<tr>
<td>$d_{uk}$</td>
<td>0.178</td>
<td>(0.573)</td>
</tr>
<tr>
<td>$r_{us}$</td>
<td>-2.752</td>
<td>(1.924)</td>
</tr>
<tr>
<td>$r_{uk}$</td>
<td>-0.164</td>
<td>(0.184)</td>
</tr>
</tbody>
</table>

Adj. $R^2 = 0.32$

$DW = 1.60$
Table 2
OLS Estimate: 1870-1984
(T-Statistic in Parenthesis)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>0.018</td>
<td>1.258</td>
</tr>
<tr>
<td>$g_{us}$</td>
<td>1.257</td>
<td>3.782</td>
</tr>
<tr>
<td>$g_{uk}$</td>
<td>-0.287</td>
<td>2.453</td>
</tr>
<tr>
<td>$d_{us}$</td>
<td>-0.392</td>
<td>1.751</td>
</tr>
<tr>
<td>$d_{uk}$</td>
<td>0.147</td>
<td>1.508</td>
</tr>
<tr>
<td>$r_{us}$</td>
<td>-0.759</td>
<td>2.479</td>
</tr>
<tr>
<td>$r_{uk}$</td>
<td>-0.056</td>
<td>0.238</td>
</tr>
</tbody>
</table>

Adj. $R^2 = 0.13$

$DW = 2.13$
Table 3  
Maximum Likelihood Estimates  
(T-Statistic in Parenthesis)

<table>
<thead>
<tr>
<th>$\beta_1$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>0.159</td>
<td>(1.130)</td>
</tr>
<tr>
<td>$g_{us}^p$</td>
<td>3.393</td>
<td>(2.064)</td>
</tr>
<tr>
<td>$g_{uk}^p$</td>
<td>-0.804</td>
<td>(2.128)</td>
</tr>
<tr>
<td>$d_{us}$</td>
<td>-0.244</td>
<td>(0.375)</td>
</tr>
<tr>
<td>$d_{uk}$</td>
<td>0.053</td>
<td>(0.188)</td>
</tr>
<tr>
<td>$r_{us}$</td>
<td>-1.476</td>
<td>(1.706)</td>
</tr>
<tr>
<td>$r_{uk}$</td>
<td>-0.330</td>
<td>(0.579)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.083</td>
<td>(1.399)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.095</td>
<td>(5.658)</td>
</tr>
</tbody>
</table>