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Game Theory, Behavior and the Paradox of the Prisoners Dilemma — 3 Solutions

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by

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1. Introduction

The Prisoners' Dilemma game is a deceptively simple 2 x 2 matrix game which can be used to illustrate the value and the limitations of game theoretic thinking. Its simplicity makes it most attractive as a paradigm to explain human behavior. Furthermore it is easy to experiment with.

The very simplicity of this game is a danger. Anologies between it and human affairs are best employed to study their inadequacies and to pinpoint what has been left out rather than to claim how much of the world can be packed into a 2 x 2 matrix.

2. The Prisoners' Dilemma: 1 and k Periods

Consider two players A and B each with a choice involving the selection of one of two moves with payoffs as indicated in Table 1. Here we observe that if each selects his first move they obtain 5 each (symmetry as evinced by the sizes of the payoffs is not necessary but at this point it makes the discussion somewhat easier to follow). If A

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chooses 1 and B 2 or viceversa this results in one player profiting and the other losing with outcomes worth (-5, 10) or (10, -5) respectively. If both select 2 the outcome is worth (0, 0) .

TABLE 1

		B's choice 1		B's choice 2	
A's choice	1	5,	5	-5,	10
	2	10,	-5	0,	0

A different way of illustrating the payoff structure is by means of a graph with the payoffs to Player A on the abcissa and Player B on the ordinate. This is shown in Figure 1.

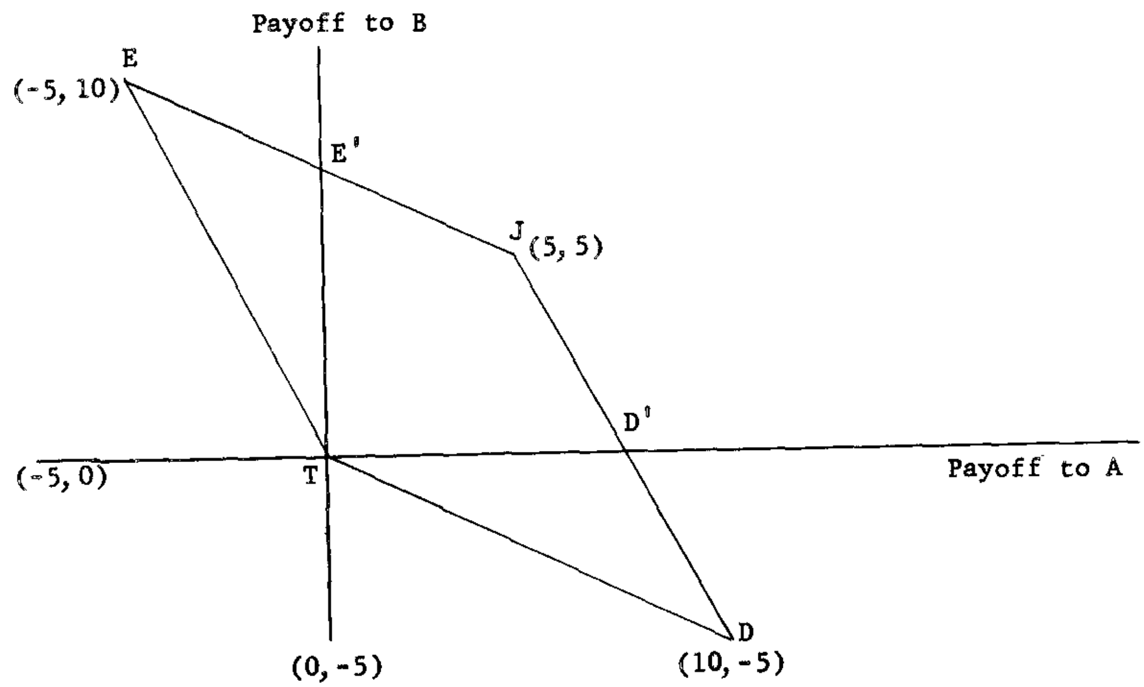

FIGURE 1

The points J , D , E and T show the four pairs of values given in the matrix. The area enclosed by joining JDTED gives every possible payoff pair that could arise if the players decided to use mixed strategies, i.e. introduce probabilities into their decisions.

The Pareto optimal surface in this case is EJD . The paradox associated with the Prisoners' Dilemma arise when we consider two individuals involved in playing this game once without any communication. The argument goes that the first will face a payoff matrix as is shown in Table 2.

TABLE 2

	1	2
1	5	-5
2	10	0

Individual self-interest is such that his second choice always is better than his first (10 is preferred to 5 and 0 to -5). The same analysis holds true of the second player. Thus each uses his second strategy with the result of (0, 0) shown as point T in Figure 1. This is not Pareto optimal.

The paradox is that each player by following his self-interest arrives at a payoff of 0 where both could have obtained 5 .

If we wish to elaborate the example we might imagine two prisoners who are being questioned separately. If both keep silent the worst that can happen is that they can be held for a short time on a vagrancy charge. If one turns state's evidence he is released (value 10) and the other draws a heavy sentence (-5). If both confess they each draw a moderately heavy sentence (value (0, 0)).

The game may be studied in its extensive form[1] and in its characteristic function form.[2] In its extensive form it appears as is shown in Figure 2. Here Player 1 (A) selects his move 1 or 2, then Player 2 (B)

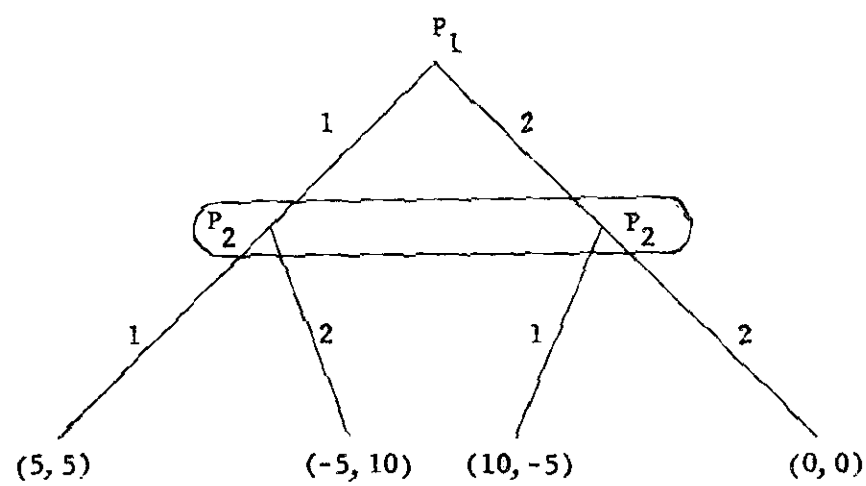

FIGURE 2

makes his move without knowledge of the act of Player 1. After his move the game is over and the payoffs are indicated at the four terminal points of the tree. The fact that Player 2 moves with lack of information concerning the action of Player 1 is illustrated by enclosing the two nodes which represent his choice points in a curve we call an information set.[3]

The characteristic function is a set function which describes the amount that any subset of players can guarantee for itself. When there are only two players there are only the two single players and the two player sets to consider.*

$$V(\overline{1}) = 0$$

$$V(\overline{2}) = 0$$

$$V(\overline{12}) = 10$$

If each player uses his second move regardless of the action of the other he can guarantee that he obtains no less than zero. If they both use their first move they obtain (5, 5) or 10 together.** Thus the region of the Pareto optimal surface bounded by considerations of individual rationality is $E'JD'$. This region, in a somewhat more general form is known as <u>the core</u> of the game.[5] When there is more than two players the core consists of those outcomes on the Pareto optimal surface which yield to any set of players as much or more than they could possibly obtain by themselves.

It seems to be depressing and possibly silly that two individuals who could obtain (5, 5) take (0, 0) instead. Is life that competitive, nasty and brutish? Do people in fact fail to cooperate when this game is played once? There is a fair amount of evidence that people do select the <u>noncooperative equilibrium</u>[6] outcome of (0, 0). If the game is to be played only once without face-to-face communication and without contracts which can be enforced it may not be too unreasonable to find the lack of cooperation. Should this happen if the play of the game is repeated several

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*The notation $\overline{1}$ or $\overline{12}$ is used to stand for a set consisting of Player 1 or Players 1 and 2.

**If we did not wish to add the payoffs of the players we could use a slightly more general form called a <u>characterizing function</u>.[4]

times? We might hope that the length of play should make a difference. People should learn how to cooperate.

We examine the Prisoners' Dilemma played twice. In order to study it fully the extensive and matrix forms of this game are presented completely in Figure 3 and Table 3. The first part of Figure 3 is the same as in Figure 2. However after both have moved they are informed about the result of the first game (this is shown by the one element information sets); then they play again.

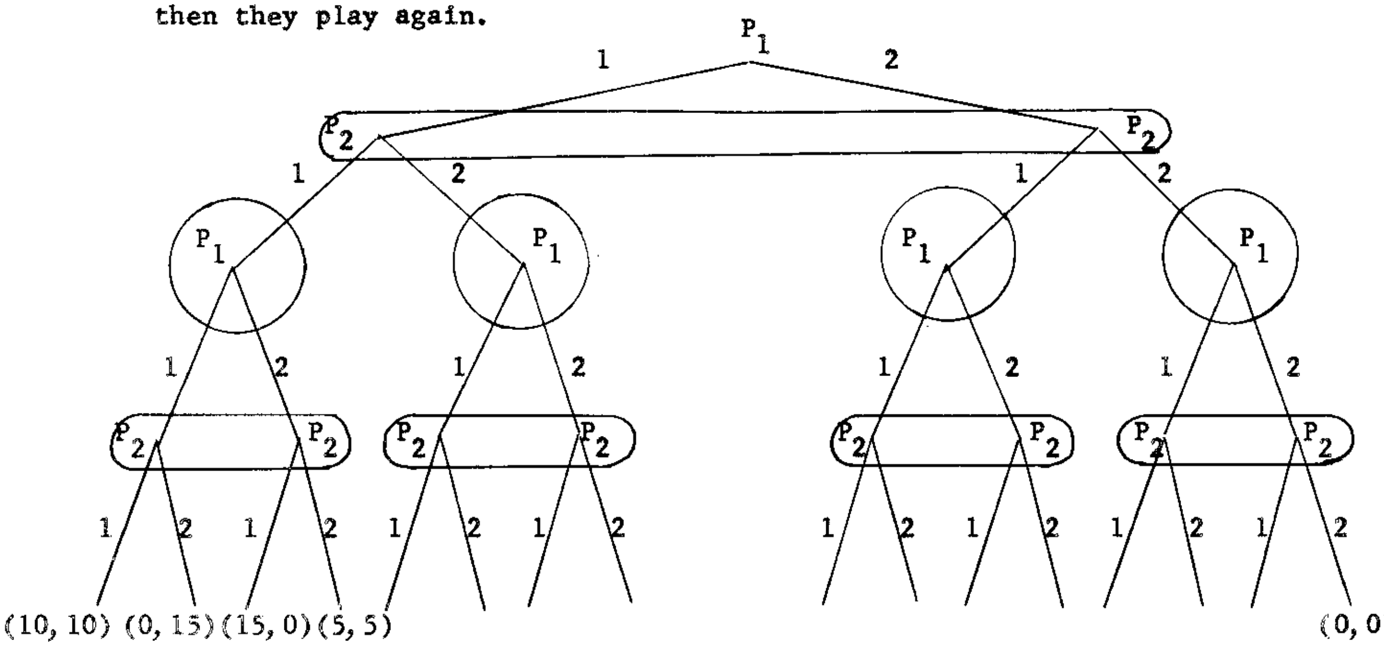

FIGURE 3

A strategy for a player is a plan which dictates the play of a player as a function of his information. A simple way of viewing a strategy is that it is the book of instructions that you might leave to a representative who is going to play a game for you! Each player has eight strategies of the following form:

To begin select move	1 or 2	2 possibilities
If the opponent selects	1 or 2	2 possibilities
then on the second round select	1 or 2	2 possibilities

There are 2 x 2 x 2 = 8 different plans which can be used. We denote each by a pentad of numbers such as (1; 1, 2; 2, 1) which may be read as follows: "choose 1 to start; then if the opponent chooses 1 select 2; if he chooses 2 select 1."

We observe that even though the game is played twice the only equilibrium point is given by (2; 1, 2; 2, 2) played by each. This gives a payoff of (0, 0) . In order to see why this is so we may assume that we are at the last play of the game. This is merely the one period Prisoners' Dilemma as is shown in Figure 2. Each player is motivated to doublecross the other. We may then work a backward induction arguing that at the ultimate play of the game each player will doublecross, hence we may replace this part of the game by the payoffs and examine the penultimate play (in this case, the first play). However as the penultimate play is now strategically the last (we know they are going to doublecross on the ultimate play) they might as well doublecross on the penultimate play. Hence in the two period game the only equilibrium point yields (0, 0) and they each play (2, 2) each time.

It is easy to see that this backward induction can be carried out for any finite k periods, replacing the game by one of k-1 periods, then k-2 periods and so forth.

The logical utterly silly conclusion is that even if the players

TABLE 3

	(1;1, 1;2, 1)	(1;1, 1;2, 2)	(1;1, 2;2, 1)	(1;1, 2;2, 2)	(2;1, 1;2, 1)	(2;1, 1;2, 2)	(2;1, 2;2, 1)	(2;1, 2;2, 2)
(1;1, 1;2, 1)	10, 10	10, 10	-5, 15	-5, 15	-5, 15	-5, 15	-10, 20	-10, 20
(1;1, 1;2, 2)	10, 10	10, 10	-5, 15	-5, 15	-5, 15	5, 5	-5, 10	-5, 10
(1;1, 2;2, 1)	15, -5	15, -5	5, 5	5, 5	-5, 15	-5, 15	-10, 20	-10, 20
(1;1, 2;2, 2)	15, -5	15, -5	5, 5	5, 5	5, 5	5, 5	-5, 10	-5, 10
(2;1, 1;2, 1)	15, -5	5, 5	15, -5	5, 5	5, 5	-5, 10	5, 5	-5, 10
(2;1, 1;2, 2)	15, -5	5, 5	15, -5	5, 5	10, -5	0, 0	-5, 10	0, 0
(2;1, 2;2, 1)	20, -10	10, -5	20, -10	10, -5	5, 5	-10, 5	5, 5	-5, 10
(2;1, 2;2, 2)	20, -10	10, -5	20, -10	10, -5	10, -5	0, 0	10, -5	0, 0

played 1,000 times with a possible joint gain of $5,000 each they will end up with (0, 0) having "out-psyched" each other a thousand times!!

People do not play this way. Depending upon the number of periods,[7] the information conditions[8] and the briefing, different levels of cooperation are encountered. Can we match theory, experimental results and our casual observations of human affairs? Is all that is missing merely better mathematical analysis of the same model or is a different model called for?

3. Game Theoretic Resolutions of the Prisoners' Dilemma

The Prisoners' Dilemma comes about when both players attain a noncooperative equilibrium. A noncooperative equilibrium point has the property that neither player can gain from violating the equilibrium unilaterally. Referring back to Tables 1 and 3 we can see that the strategies 2, 2 and (2; 1, 2; 2, 2) , (2; 1, 2; 2, 2) form equilibrium pairs in the two games respectively. If one player knows the strategy of the other he will be motivated to leave his strategy unchanged.

There would be no dilemma if the players were allowed to talk face-to-face before they played and to sign enforceable agreements. The interest in the problem comes about in studying the implications of the relative lack of communication and lack of enforceable agreement between them.

Keeping the properties of low communication (in particular communication takes place only through seeing the moves played); and no enforceable agreements three modifications have been suggested for "resolving" the dilemma. They are the Metagames of Nigel Howard,[9] a treatment of the repeated Prisoners' Dilemma as a game of infinite length[10] and the Games of Economic Survival or Social Survival of Shubik.[11]

The infinite game modification involves the least extra modeling; while Shubik's calls for an additional economic assumption and Howard's for an additional socio-psychological assumption.

3.1 The Pure Infinite Game

Suppose that the game shown in Table 1 is repeated endlessly. Then the payoffs could become unboundedly large. However the average per period payoff will remain bounded (and could never be greater than 10). It can be argued that it is reasonable to assume that the players will consider their per period payoffs and also take into account that if they manage to achieve any temporary stationary state better than (0, 0) this can be enforced as an equilibrium point because if one player violates the stationary state then the other can choose his strategy 2 in every subsequent play and thereby has a threat of punishment greater than the gain from violation.

In Figure 4 below the whole shaded area TD'JE' becomes a set of equilibrium points. Even the joint maximum with payoff (5, 5) becomes an equilibrium point. Suppose each has chosen his first move in the initial subgame, then if each thought that the other would switch to his second move for ever in retaliation to a violation of the status quo this would be enough to turn the joint maximum into an equilibrium.

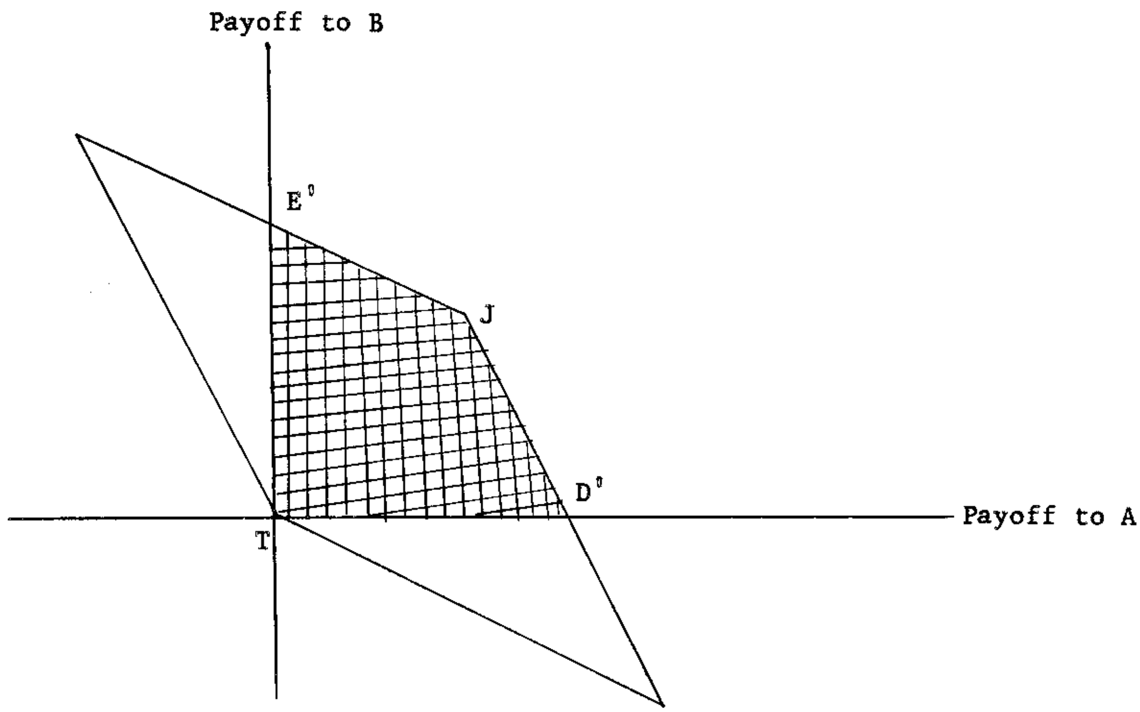

FIGURE 4

Mathematically these results are impeccable, however the conversion of every outcome that satisfies individual rationality (i.e. gives each individual at least as much as he can guarantee for himself) into an equilibrium point leaves something to be desired and points to a weakness in the model itself.

3.2 Games of Economic and Social Survival: Preliminary Models

A different approach had been adopted previously by Shubik in his treatment of a class of games called Games of Economic Survival[12] or more generally Games of Social Survival.[13] In a game of social survival, the objectives of the individual are modified to take into account both his gain and his concern for survival. The type of game that is considered is one which may contain one or more of three features additional to being of infinite length:

(a) There may be a discount rate ρ ; which means that the present value of payoffs in the future decreases with distance into the future.

(b) There may be an exogenous probability that the game will terminate after a finite time regardless of the state of the fortunes of the players.

(c) The game may terminate for an individual player owing to his ruin.

Two examples are given, they are closely related and differ operationally not much more than in the names of two variables. These are aimed at the specific problem of "resolving" the Prisoners' Dilemma. In the first instance although the game we consider is possibly of infinite length at every period there is a probability p that the game (which we denote by Γ) will be played at least once more and a probability of $(1 - p)$ that the game is over. This is illustrated in Figure 5a. This figure is not a game tree in the strict sense of Figure 2 or 3. Each vertex is marked by a Γ which means that the Prisoners' Dilemma game shown in Table 1 is played once at that point. If the game lasts five times <u>in toto</u> the payoffs to the players will be

$$\left(\sum_{t=1}^{5} P_{1,t}, \sum_{t=1}^{5} P_{2,t} \right) .$$

After every single play "Nature" decides or a probability determines if the game is going to continue. The overall game (of possibly infinite length) now will have a finite expected value given by:

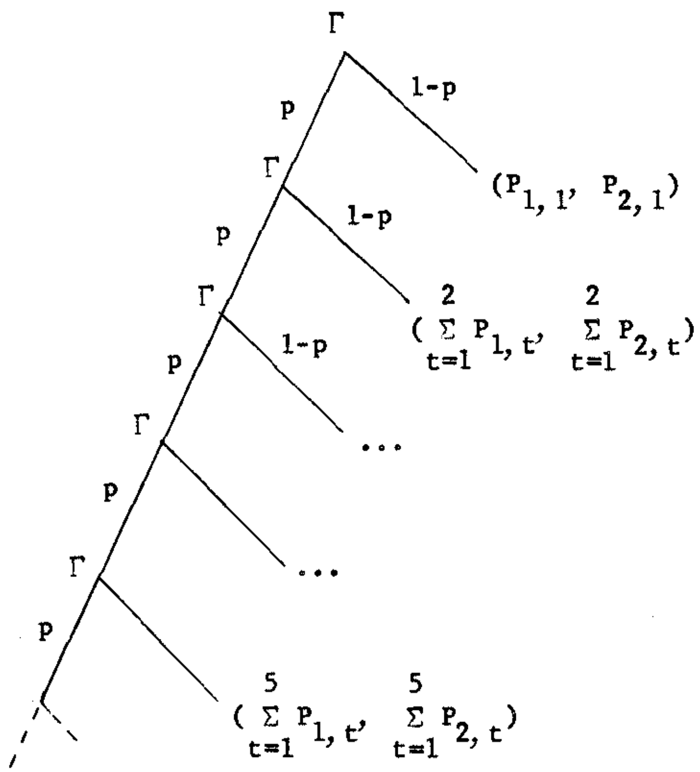

Figure 5a

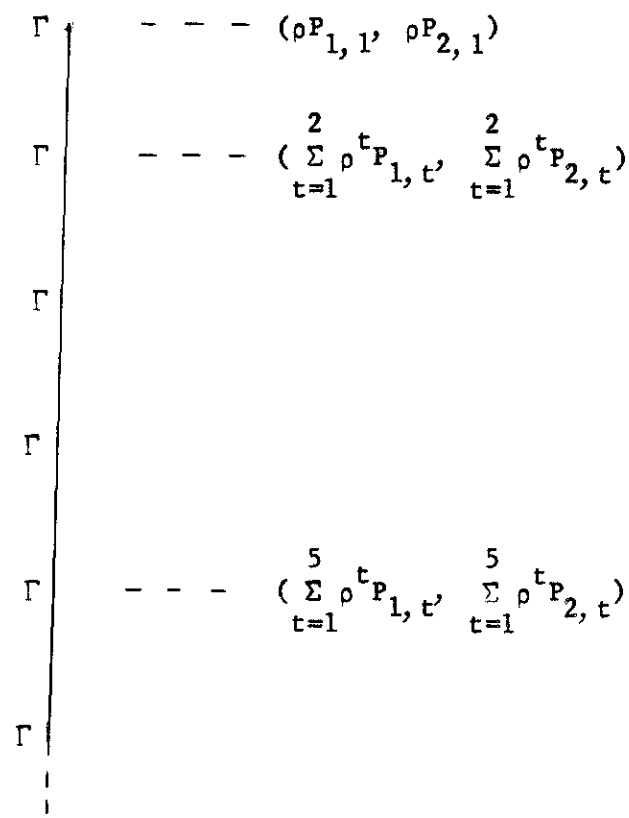

Figure 5b

$$\left(\sum_{t=1}^{\infty} p^t P_{1,t}, \sum_{t=1}^{\infty} p^t P_{2,t}\right)$$

where $0 \leq p < 1$.

In the second example the game is always of infinite length, however payoffs in future games are discounted. Figure 5b shows the structure of the start of this infinite game tree. The amounts of the payoffs accruing to the players after any period are also shown in Figure 5b. As with the first example even though the game is of infinite length the value of the payoffs is finite. In the previous example the expected payoff was finite; here it is the discounted income stream that is finite. It is given by:

$$\left(\sum_{t=1}^{\infty} \rho^t P_{1,t},\ \sum_{t=1}^{\infty} \rho^t P_{2,t}\right)$$

where $0 \leq \rho < 1$. This is virtually the same expression as the previous one.

We consider the infinite game with the discount (leaving the game with an ending probability to be considered by the reader). If both players cooperate and select their first move, each time they play the payoff to each is the income of 5 per period. As there is a discount rate in effect

TABLE 4

	1	2
1	5, 5	-5, 10
2	10, -5	0, 0

this has a present worth of:

$$5(\rho + \rho^2 + \rho^3 + \ldots) = \frac{5\rho}{1-\rho} .$$

A strategy[14] (as we have previously noted) in a finite game may be regarded as a book of instructions which a player could give to a stand-in or representative who is going to play the game for him. It contains rules to cover all contingencies. When the game is of infinite length the definition of a strategy still goes through but we must be careful to cover the infinities of alternatives that arise. Fortunately in the case of this game some of the strategies we would like to examine can be stated more or less in words. For example a strategy which may be regarded as containing a threat is as follows:

> Player 1 "I will play my move 1 to begin with and will continue to do so, so long as my information shows that the other player has chosen his move 1. If my information tells me he has used move 2, then I will use move 2 for the immediate k subsequent periods. After which I will resume using move 1. If he uses his move 2 again after I have resumed using move 1, then I will switch to move 2 for the k+1 immediately subsequent periods ... and so on increasing my retaliation by an extra period for each departure from the (1, 1) steady state."

This boils down to a bid for (1, 1) as the steady state which on escalation of the degree of retaliation depending upon the number of times the other side "violates" the steady state.

In this context we can interpret a threat as a strategy containing a bid for a steady state solution to the repeated play game and also containing a commitment to a set of actions to be taken if the steady state is violated.

We add two qualifiers to our concept of threat they are "sensible" and "plausible." A threat strategy is said to be sensible if the steady state it advocates is Pareto optimal. We rule out efforts to enforce outcomes which are unfavorable to both, relative to other outcomes. We also rule out blatently "irrational" situations that could arise were we to consider pathological behavior. Thus we rule out strategies which might invoke the reply: "Is that a threat or a promise?"

The concept of plausibility is at the crux of the relationship between a strict game theoretic formulation of a conflict situation and the treatment of conflict by a mixture of "gamesmanship," bargaining and strategic theories and behavioral models of man.

In a strict game theoretic formulation of conflict, strategies are viewed as absolute commitments. There is no verbal byplay involving changing one's mind after the strategy has been stated. It is as though all of the play were turned over to a Doomsday Machine once both players have chosen their strategies. Thus all strategies are equally plausible in a formal game model.

In particular the sending of messages, meetings, notes, conferences or discussions are all ruled out of the formal game model except for the single simultaneous move of sending the strategies to the Doomsday Machine. Thus a strategy in the sense of game theory is always conpletely plausible.

In actual games even of a relatively formal variety verbal exchanges play an important role. In negotiations words are often moves. Strategy in a negotiation calls for a plan for changing offers and "threats." A strategy in the sense of game theory which is a complete statement of plan is merely a move in a bargaining or negotiation situation. In the first instance as there is total precommitment the strategy indeed holds for the course of the whole game. In the bargain there may be no precommitment whatsoever, thus a plan, offer or threat invoked this period can be changed next period.

Even when there is only one message sent from one player to the other preliminary to the repeated play of a game such as the Prisoners' Dilemma, unless it is explicitly stated by the referee that any message is absolutely binding, the players attach a degree of plausibility to the contents. Plausibility is a function of the formality of the structure of interpersonal interaction. Third parties and written documents as well as the avoidance of individual face-to-face confrontation are often used to remove socio-psychological factors. Although total precommitment cannot be obtained in human affairs other than in actual games (where a violation of the rules is tantamount to not playing the game), third parties, giving hostages, putting items in escrow and many other devices are used to increase the plausibility of a commitment.

Returning to our specific analysis of the Prisoners' Dilemma when we consider the area of equilibrium points in Figure 4 we rule out those which are not Pareto optimal as being, in this case, not <u>sensible</u> (we will return to a discussion of the meaning of "sensible" and to a discussion of pathological behavior later in Section 5). Limiting ourselves to sensible outcomes we wish to examine the plausibility of threats which bring about sensible outcomes. The <u>least</u> plausible threat is that which promises to punish the other player (and yourself) for ever, by playing Move 2 every period in revenge for one violation of a desired steady state. Let us examine the threat strategy suggested for Player 1 on page 15. For a discount rate of .9 (i.e. an interest rate of 10%) this strategy is sufficient to enforce (5, 5) as an equilibrium point with $k = 2$. In other words Player 1 needs to "punish" twice for the first violation, three times for the second and so forth... . $k = 1$ is not quite sufficient as is seen below.

If Player 2 conforms with the desires of Player 1 his payoff is $5\left(\frac{\rho}{1-\rho}\right) = 45$ when $\rho = .9$. If he violates the equilibrium in an optimal manner his payoff is given by $10(\rho + \rho^3 + \rho^6 + \rho^{10} + \rho^{15} + \ldots \rho^{\frac{n(n+1)}{2}} + \ldots)$. This has a value of 29.42 for $\rho = .9$ which is less than 45. We can see that one double cross may pay when $k = 1$ but a second will not as $10 < 5 + 4.5 + 4.05$. Hence $k = 2$ gives immediate stability. This threat is far more plausible than the other and even incorporates a "teaching" or reinforcement device that shows Player 2 that retribution increases with every violation.

We have not offered a <u>measure</u> for the <u>degree of plausibility</u> of a threat, however at least in the repeated Prisoners' Dilemma game with a discount or termination probability we have examined two threat strategies and have argued that one is not very plausible whereas the other is extremely plausible. The "resolution" or "solving" of the Prisoners' Dilemma came about through the enlarging of the model rather than the invocation of new mathematical properties of the original model.

We have not made use in the two examples given above of the role of assets and the ruin and survival conditions encountered in a game of social survival. The complete details have been described elsewhere[15] and as our major purpose is to concentrate on "resolutions of the Prisoners' Dilemma" we do not pursue our discussion of games of social survival further at this point.

Prior to returning to the discussion of threat we turn to the third way in which the Prisoners' Dilemma has been resolved.

3.3 Metagames

Howard's solution for the Prisoners' Dilemma is based upon a concept of "metagame" which allows several levels of conditional strategies. Instead of considering only two alternatives for Player 2, let us imagine four:

1. Play alternative 1 regardless
2. Play alternative 1 or 2 if you think Player 1 will play 1 of 2
3. Play alternative 1 or 2 if you think Player 1 will play 2 or 1
4. Play alternative 2 regardless

If Player 1 is informed of the choice problem facing Player 2, he has sixteen alternatives or strategies or plans. They can be denoted by 1111, 1112, 1121, 1211, 2111, 1122, 1212, ..., 2222 ; which stand for:

1111, Play 1 regardless.

1112, Play 1 unless you think he selects 2 regardless of you, in which case select 2.

⋮

2222, Play 2 regardless.

A 16 x 4 matrix results from these considerations and as can be seen in Table 5 there are now three equilibrium points, two of which are jointly optimal and one is the old jointly unsatisfactory (0, 0) . Howard shows that there is nothing more to be gained by going to the higher regressions of the metagame.[16] Rapaport regards this work[17] as "the resolution" of the Prisoners' Dilemma; although I fail to see why. Experimental evidence does however indicate that with this type of strategy enlargement

TABLE 5

	1	2	3	4
1111	5, 5	5, 5	-5, 10	-5, 10
1112	5, 5	5, 5	-5, 10	0, 0
1121	5, 5	5, 5	10, -5	10, -5
1211	5, 5	0, 0	-5, 10	-5, 10
2111	10, -5	5, 5	-5, 10	-5, 10
1122	5, 5	[5, 5]	10, -5	0, 0
1212	5, 5	0, 0	-5, 10	0, 0
2112	10, -5	5, 5	-5, 10	0, 0
1221	5, 5	0, 0	10, -5	-5, 10
2121	10, -5	5, 5	10, -5	-5, 10
2211	10, -5	0, 0	-5, 10	-5, 10
2221	10, -5	0, 0	10, -5	-5, 10
2212	10, -5	0, 0	-5, 10	0, 0
2122	10, -5	[5, 5]	10, -5	0, 0
1222	5, 5	0, 0	10, -5	0, 0
2222	10, -5	0, 0	10, -5	[0, 0]

possible[18] the degree of cooperation is increased considerably. An interpretation of the metagame in terms of a formalized way of conveying intent as well as immediate behavior indicates why it is reasonably to expect a fair amount of cooperation from the players. The four strategies of Player 2 divide into two types: 1 and 4 are "inner-directed: whereas 2 and 3 are "other-directed." Strategy 2 is a statement of an intent to coordinate. Strategy 3 conveys the intent to avoid coordination.

The metagame can be regarded as an attempt to build complete plausibility for a restricted set of intents within an extension of the usual Prisoners' Dilemma game. Although it may be mathematically interesting it by no means offers a general resolution to the problem of threats or to the many problems concerning strategies, signalling, communication and coding which must be considered in the study of the iterated game.

4. Complications and Relevance

Although in 3.2 by introducing a discount factor and an ending probability we resolved the Dilemma; I claim that for most problems of interest the model is still not rich enough to capture a useful abstraction of human affairs. It misses on two counts. The first concerns survival and can be dealt with in a formal mathematical structure as is done elsewhere. The second concerns the problems of coding, language and communication and is discussed in Section 5. Sometimes the resoltion to a paradox comes about when it is observed not that the answer is wrong, but that the question is wrong.

Under the force of gravity in a vacuum a feather and a lump of lead fall at the same speed. This is a paradox to most people who are not physicists and who have seen a feather and an object of lead fall through the atmosphere. The analogy with the Prisoners' Dilemma is direct. In a one-shot game where individuals understand the game, do not know each other and have something to gain or lose it is no paradox whatsoever that many will arrive at the equilibrium pair of (2, 2) . It is also not surprising that not all pairs of individuals will play this way because even under the most sterile *in vitro* conditions the experimenter still is forced to use individuals who retain their memories and their status as socialized people. In the iterated game it is not surprising that a wide variety of results can be obtained by varying the experiment very slightly. It is virtually impossible to run the iterated experiment totally *in vitro*. The best that can be hoped for is as though we were trying to perform an experiment with a feather and an object of lead in a vacuum apparatus with a leak of undetermined size letting in a gas of unknown viscosity.

It may be easier to control an experiment by introducing a more complex environment than by making the error of assuming that models with the fewest parameters are always the best for studying human behavior. When all other things are equal this may be so, but here they are not equal.

It is conceivable that complicated problems require complicated answers. A sense of aesthetics and a feeling for mathematical beauty may provide a valuable motivation to keep models parsimonious. However the requirements of relevance and the aesthetic desire for elegant simple models may easily be in conflict.

There are many models that can be formulated and many experiments that can be performed with a broad class of games called by loose association "the iterated Prisoners' Dilemma." Many of the experiments are undoubtedly worth performing (see Rapaport and Orwant[19] for example); however the only paradox (if there is any) is that so much concern has been lavished on mistaking a class of games for a single game and on trying to use too simple a constraint to explain too much.

5. Words, Threats, Incentives, Briefings and People

It would require a paper many times the length of this one to even start to do justice to the concept of threat. However, some of the salient features which cause the difficulties in interpreting the varied results of simple game experiments can be noted and their analysis will provide the first steps in understanding the basic yet elusive phenomenon of threat.

Tied in with these must come an appreciation of both the considerable power and the extreme limitations of formal game theoretic models. The confusion concerning the role of game theory is elegantly exemplified by the book of Tom Schelling, "The Strategy of Conflict."[20] This book was simultaneously a perceptive and stimulating discourse on the "gamesmanship" of bargaining, negotiation and conflict and a monument to a profound misunderstanding of the formal theory of games. Reviews by otherwise usually responsible individuals such as Bishop[21] merely helped to illustrate the depth of misunderstanding.

A major difficulty that is encountered in attempting to apply game theory to the study of the dynamics of bargaining or negotiation is that it was not designed in any operational way to deal with words as moves. Many of the "plays" and counterplays in bargaining involve words and gestures whose meanings are often deliberately ambiguous. Verbal offers and threats are made and withdrawn during the course of negotiation. In terms of game theory they pose two distinct and unresolved problems. They are: (1) the problem of coding and (2) the problem of commitment.

When a mathematical model of the game of chess is constructed there is no ambiguity in the description of the moves that can be made. "Pawn to White's King four" has only one meaning. When an individual says to another: "If you do not keep smiling I'll bash your nose into a bloody pulp" we may need much more context in order to be able to evaluate the specific meanings to be attached to "smiling" and possibly even to "bloody pulp." Furthermore even if those were unambiguously defined, a truth content must be attached to the statement as it is not a formal precommitment.

Language contains a high degree of ambiguity. It is precisely the presence of this ambiguity that gives rise to much of the art of "gamesmenship." Skilled negotiators and lawyers thrive on the ambiguity. In a formal sense this feature poses the coding problem inasmuch as a sentence if it is to be interpreted as a move can have many more than one meaning.

Formally the ambiguity of language can be handled easily by game theory. Practically or fruitfully it cannot. All one has to do is to treat *any* sentence used as a move as belonging to a set of meanings where the author has secretly decided on the meaning he intends to use. For example the statement: "If you do that you may not even live to regret it" contains a thinly veiled but unspecified threat. If we call the statement T we can call the possible meanings of the statement $T_1, T_2, \ldots T_n$. Thus the part of the game tree which represents an individual using that sentence as a move is as shown in Figure 6.

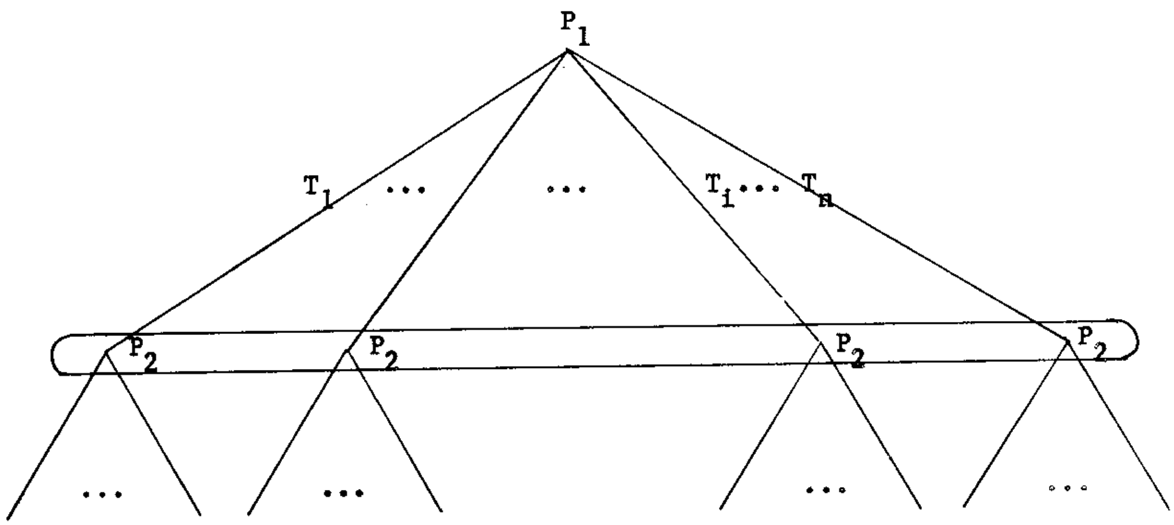

FIGURE 6

Similarly we could enlarge each T_i to include a random move with say m outcomes and probabilities $p_{i,j}$ where $j = 1, \ldots, m$ and $\sum_{j=1}^{m} p_{ij} = 1$. These can be regarded as modeling behavioristic limitations on the individual.

The interpretation of the above is: He says sentence T ; the meaning he attaches to it is T_i and the actions he is going to follow are given by T_{ij} with probability p_{ij} .

The additional enlargement takes care of plausibility and pre-commitment. It is not necessary to introduce probabilities we could equally well have increased the number of moves from n to m·n assuming that the individual consciously selects the way he will react.

By increasing the size of the game by many times both the problems of coding and commitment can be covered by formal game theory. The trouble is that in all except the most simple instances we do not know how to do so in practice.

The briefing or the preliminary initial one or two way sending of a message in Prisoners' Dilemma experiments enlarge the game this way and virtually no attention has been paid to interpreting how the game is enlarged.

Formal game theory at this stage of development is as poor as is economic theory in socio-psychological variables. There is a great deal that can be done utilizing the concept of the individual rational man; thus as is argued elsewhere[22] applications to certain problems in competition, oligopoly and welfare economics are not hampered by the rationalistic assumptions concerning the individual. Applications to bargaining, negotiations and to most gaming experiments are limited.

Leaving aside the difficulties of coding and commitment and assuming that information conditions can be controlled[23] there remains the problems of controlling for:

(1) Intelligence and ability of the individuals to completely understand the game

(2) Motives; i.e. what is the real payoff

(3) Learning

(4) Personality factors and other socio-psychological variables.

The first point is not too hard to control for. The second causes a great deal of trouble. Unless monetary or other rewards are high enough to make them clearly crucial many experiments must be viewed as studies in reaction to boredom or cutthroat behavior.[24]

The other two points call for enlarging our thoughts about the explanation of behavior viewed in games such as the iterated Prisoners' Dilemma. It would be extremely interesting and valuable to know how psychotics play such a game. The effect of madmen in negotiations has not been studied seriously. There is an anectodal literature on the subject as can be seen in the writings of Rapoport,[25] Schelling[26] and Shubik.[27] But it is to be hoped that none of these authors would confuse their often ingenious but basically casual examples with even a serious start on the development of an adequate theory.

It is particularly strange to observe the split personality in the writings of Rapoport. On the one hand he writes a glowing popular article in Scientific American[28] comparing the work of Nigel Howard with the solution of Zeno's paradox. Yet on the other hand the book of Rapoport and Orwant[29] devoted to the Prisoners' Dilemma adopts a completely different approach and starts to do precisely what I believe is necessary and that is to enlarge the framework of the analysis to include learning and socio-psychological factors.

6. Conclusions

We are a long way from "the theory of threats." In a very limited sense formal game theory can model a highly special concept of threat;[30, 31] but in most cases of interest we must go beyond the formal theory and add other features. It is in all probability premature to look for a single theory. Different fields of investigation may be able to utilize special definitions; thus the threat concept of a psychiatrist need not match those of an ethologist or a diplomatic historian.

The iterated Prisoners' Dilemma game and its various solutions serves as an extremely useful starting point to understand the power and limitations of game theory.

The paradox of the Prisoners' Dilemma will never be solved (or has already been solved) because it does not exist.

FOOTNOTES

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[2]Ibid., Chapter 1; Chapter 8.

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[5]Ibid., Chapter 1.

[6]Ibid.

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[14]Shapley, L.S. and M. Shubik, Competition, Welfare and the Theory of Games, Chapter 1.

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[30] Nash, J.F., Jr., "Two-Person Cooperative Games," in *Econometrica*, 21: pp. 128-140, 1953.

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