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FOOTNOTES

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[2]Bank's net free reserves are equal to excess reserves less debt to the central bank; the model does not attempt to explain the two items separately.

[3]Public is assumed not to hold currency.

[4]For example, consider the following trivial model:

$$X_D^*(t) = B_o - B_1 P(t)$$

$$\Delta X_D(t) = \alpha(X_D^*(t) - X_D(t-1))$$

$$\overline{X}_S(t) = X_D(t)$$

where X_D^* is the desired quantity of a commodity, P its price, ΔX_D the change in actual demand, $\alpha \neq 0$ the speed of adjustment, and $\overline{X}_S$ the exogenously determined supply. Irrespective of the speed of adjustment α, the system will be in long run equilibrium two period after a change in the supply.

PITFALLS IN FINANCIAL MODEL-BUILDING[1]

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Most monetary economists agree that the financial system is a complex of interrelated markets for assets and debts. The prices and interest rates determined in these markets, and the quantities to which they refer, both influence and are influenced by the "real economy," the complex of markets for currently produced goods and services. These interdependences are easy to acknowledge in principle but difficult to honor in practice, either in theoretical analysis or in empirical investigation. All of us seek and use simplifications to overcome the frustrating sterility of the cliché that everything depends on everything else. But we all know that we do so at some peril.

In this paper we argue for the importance of explicit recognition of the essential interdependences of markets in theoretical and empirical specifications of financial models. Failure to respect some elementary interrelationships -- for example those enforced by balance sheet identities -- can result in inadvertent but serious errors of econometric inference and of policy. This is true equally of equilibrium relationships and of dynamic models of the behavior of the system in disequilibrium.

We will try to illustrate the basic point with the help of computer simulations of a fictitious economy of our own construction. This procedure guarantees us an Olympian knowledge of the true structure that is generating the observations. Therefore it can exhibit some implications of specifications and misspecifications that are inaccessible both to analytical inspection and to econometric treatment of actual data.

We fully realize, of course, that this procedure cannot tell us anything about the real world. You can't get something for nothing. We realize further that lessons derived or illustrated by simulations of our particular structure will not be very convincing or even interesting to people who believe that the model bears no resemblance to the processes which generate actual statistical data. We have tried to formulate a model we believe in qualitatively, though of course the numerical values of the parameters are arbitrary.

I. An Equilibrium System

We begin by setting forth the equations of a static equilibrium of a simple financial system. The model contains the following six assets: currency and bank reserves, Treasury securities, private loans, demand deposits, time deposits, equities. With each asset is associated an interest rate; some rates are market determined, some are policy variables, some are institutional constants. There are three sectors: government, commercial banks, public. The constituents of their balance sheets, and the symbols used for them in the paper, are given in Table 1.

The interest rates involved in the model are:

r_F central bank discount rate

r_S Treasury security rate

r_L loan rate

$\bar{r}_D$ demand deposit rate, legal ceiling (generally zero)

$\bar{r}_T$ time deposit rate, legal ceiling

r marginal efficiency of real investment

r_K market yield on equity

TABLE 1.

Debts of \ Assets of	Government	Banks	Public	Total Debts
Government		S^B Treasury Bills	S^P Treasury Bills	G-R Treasury Bills
		E Required Reserves Net Free Reserves	C Currency	R Reserves of Currency
Banks			D Demand Deposits T Time Deposits	D Demand Deposits T Time Deposits
Public		-L Loans		-L Loans
Equities in Physical Capital			V Equities	V Equities
Net Worth Assets-Debts	-G Government Debt	0	W^P Equities + Government Debt	

$\hat{r}^P$ the vector of interest rates relevant to public portfolio decisions, $(r_S, r_L, \bar{r}_D, \bar{r}_T, r_K)$.

$\hat{r}_B$ the vector of interest rates relevant to banks' asset choices, (r_F, r_S, r_L) .

In addition to the interest rates and the accounting variables of Table 1, the following symbols are used:

p the market valuation of equities; the replacement value of the physical assets to which the equities give title is taken to be 1 and serves as the numeraire of the system;

K the stock of capital at replacement cost;

Y national income;

k_D and k_T required reserve ratios for demand and time deposits respectively.

$I = \Delta K$ net investment at replacement cost

H private saving

GP government purchases

$\overline{tx}$ the marginal tax rate.

The equations are:

Public asset holdings and debts

[$C = C^P(\hat{r}^P, Y)W^P$ Currency] (Assumed zero in simulation model)

(1) $D = D^P(\hat{r}^P, Y)W^P$ Demand deposits

(2) $T = T^P(\hat{r}^P, Y)W^P$ Time deposits

(3) $S^P = S^P(\hat{r}^P, Y)W^P$ Treasury securities

(4) $-L = -L^P(\hat{r}^P, Y)W^P$ Borrowing

[$V = V^P(\hat{r}^P, Y)W^P$ Equities

$= (1 - L^P - S^P - T^P - D^P - C^P)W^P$] [implied by other equations]

Bank asset holdings

(5) $E = E^B_D(\hat{r}^B)(1 - k_D)D + E^B_T(\hat{r}^B)(1 - k_T)T$ Net free reserves

(6) $S^B = S^B_D(\hat{r}^B)(1 - k_D)D + S^B_T(\hat{r}^B)(1 - k_T)T$ Treasury securities

(7) $L = L^B_D(\hat{r}^B)(1 - k_D)D + L^B_T(\hat{r}^B)(1 - k_T)T$ Loans

$= (1 - E^B_D - S^B_D)(1 - k_D)D + (1 - E^B_T - S^B_T)(1 - k_T)T$

Balance equations

(8) $k_D D + k_T T + E + C = R$ Currency and bank reserves

(9) $S^P + S^B = G - R$ Treasury securities

[$L^P + L^B = 0$ implied by (4) and (7)] Loans

(10) $V = pK$ Market value of equity

(11) $W^P = G + V$ Public wealth

(12) $pr_K = r$ Yield and value of equity

(13) $r = \alpha_0 + \alpha_1 \frac{Y}{K}$ Relation of marginal to average product of capital.

In addition, two inequalities must be satisfied in order for the ceiling rates on deposits to be effective. Banks must be willing to accept demand and time deposits at prevailing interest rates in at least as large volume as the public wishes to hold.

The thirteen equations (leaving aside public currency holdings) determine 7 quantities $(D, T, S^P, S^B, L, E, V)$, four rates (r_S, r_L, r, r_K), the market value of equity p, and of wealth W^P. Exogenous variables are of two kinds: policy variables r_F, $\bar{r}_T$, R, k_D, k_T and other variables G, K, α_0, Y. Alternative interpretations are possible, depending on the _modus operandi_ or objectives of the central bank. Although the supply of reserves R is one of the quantities the central bank directly controls, it may nevertheless be an endogenous variable and r_S an exogenous one if the central bank supplies whatever reserves are needed to peg the market interest rate at some target level.

A number of the features of this model need explanation:

1. The structure of the balance sheet desired by the public is taken to depend on the vector of relevant interest rates and on its net worth W^P in a special way. Desired holdings of the various assets and debts are homogeneous in wealth; a change in W^P with given interest rates changes all items in the balance sheet in the same proportion. With respect to interest rate effects, the assets are assumed to be gross substitutes. An increase in the rate on a particular asset increases the public's demand for the asset but diminishes or leaves unchanged its demand for any other.

2. Similar behavior is assumed of banks with regard to the allocation of their "disposable assets" -- deposits less required reserves -- among net

free reserves,[2] government securities, and loans. However, allowance is made for possible differences between the allocations of disposable demand deposits and disposable time deposits. Since time deposits are, from the individual banker's viewpoint, less volatile than demand deposits, they may be more adventurously invested.

The vector of interest rates relevant to the banks is somewhat different from the one relevant to the public. It includes the central bank discount rate, which is irrelevant to the public, but excludes the rate on equities, which the banks do not hold. It is also assumed, though this is not essential, that asset allocations of deposits are independent of the rates that are paid to depositors.

3. In each case, banks and public, the entire list of relevant interest rates occurs in each equation. The reason for this is as follows: The total effect of an interest rate change, summed over the whole portfolio or balance sheet, must be zero. Thus if a particular rate is entered only as a positive factor in the demand for its own asset and not included in any other equation, the offsetting negative effect is being implicitly assigned to the missing equation. (In the above model, bank demand for loans and public demand for capital play this residual role.)

It is always important to check the specification of the unwritten equation that is implied by the explicit specification of the others. For example, one might be tempted, either because it is theoretically convenient or because of econometric results and significance tests, to regard the time deposit interest rate as important for time deposits but of negligible importance in public demand for any other particular asset -- demand deposits, currency, securities, loans. But to drop it out of those equations is to assume that all the funds attracted into time deposits come from equities. If this is an assumption one would not make deliberately, neither should he make is inadvertently. It is quite possible that cross-effects are so diffused that none of them appears significant in empirical regressions. Yet it is a mistake to drop them out, because their sum is not zero but equal in absolute value to the own-effect.

4. The same observation applies to other variables affecting balance sheet or portfolio choice. In the model, income Y is entered to represent the standard influence of transactions volume on desired holdings for demand deposits and for currency. By the same token, Y belongs in the other asset demand functions of the public. If an increase in income induces the public to add to their money holdings, it induces them to diminish their holdings of something else. If this something else is not specified, the implicit assumption is that all the movement into cash is at the expense of the residual asset, the one whose equation is not written down.

5. The influence of Y on asset choice is one causal link from the real economy to financial markets. An additional link is the influence of r , the marginal efficiency of capital, another variable exogenous to the financial sector. An increase in r , for example, will raise either the market value of equities, and with it the public's wealth, or the market yield of equities, or both. In any event it will lead to a general reshuffling of portfolios, and a new structure of rates. The marginal efficiency of capital itself is linearly related to its average product Y/K ; both Y and K

are exogenous to the financial sector.

6. One of the basic theoretical propositions motivating the model is that the market valuation of equities, relative to the replacement cost of the physical assets they represent, is the major determinant of new investment. Investment is stimulated when capital is valued more highly in the market than it costs to produce it, and discouraged when its valuation is less than its replacement cost. Another way to state the same point is to say that investment is encouraged when the market yield on equity r_K is low relative to the real returns to physical investment.

An increase in p, the market valuation, can occur as a result of an increase in the marginal efficiency of capital r, i.e., as a result of events exogenous to the financial sector. But an increase in p may also occur as a consequence of financial events that reduce r_K, the yield that investors require in order to hold equity capital. Indeed this is the sole linkage in the model through which financial events, including monetary policies, affect the real economy. In other words, the valuation of investment goods relative to their cost is the prime indicator and proper target of monetary policy. Nothing else, whether it is the quantity of "money" or some financial interest rate, can be more than an imperfect and derivative indicator of the effective thrust of monetary events and policies. As some of our examples below will show, such indicators can be quite misleading.

In the actual economy, of course, the single linkage just described is a multiple one. There are many kinds of physical capital and many markets where existing stocks are valued -- not just markets for equities, but other markets for operating businesses and for houses, other kinds of real estate, cars and other durable goods, etc. The value of these stocks then helps to determine the profitability of new production of the same kind of capital or of close substitutes. Here this variety is ignored by aggregating all capital and attributing to it a single market price and a single replacement cost.

7. The effects of changes in Regulation Q ceiling rates on time deposits have been much debated in recent years, among both monetary theorists and men of affairs. In our view this discussion has not paid enough attention to the general equilibrium effects of such regulatory measures and has been too preoccupied with the effects on commercial bank loans or deposits. A reduction in the ceiling may in some circumstances be deflationary, but the fact that it drives funds out of banks and forces them to contract their loans is no proof at all of this assertion. Erstwhile depositors will be looking for places to invest their funds, and they may be glad to acquire, either directly or through other intermediaries, the assets the banks have to sell and to accomodate the borrowers the banks turn away. Whether the ultimate result is to bid interest rates and equity yields down or up is a complicated question: the answer depends, among other things, on whether time deposits are in wealth-owners' portfolios predominantly substitutes for demand deposits and currency or for loans and equities. The former substitution pattern tends to make a reduction in time deposit rates deflationary, the latter pattern, expansionary. The answer depends also, of course, on what is assumed about the supply of unborrowed reserves and other instruments of monetary control.

For some purposes it will be useful to make explicit the connections between the financial system and the real economy, extending the model to encompass endogenous determination of income, investment, and the marginal productivity of capital. Our extensions are of the most primitive sort; our purpose is not to build a complete model but include the linkages necessary to illuminate the problems of constructing a model of the financial sector. The explicit equations, (14), (15), and (16), are given below in section III-7. Net investment depends, for the reasons already stated, on the market value of capital, p. The model is a stationary one -- alternatively, it could be interpreted to describe deviations from trend. In an equilibrium with $p = 1$, net investment will be zero. Government expenditures are exogenous; tax revenues and saving are linear functions of income; the level of income is determined by the usual multiplier process. The marginal productivity of capital has an exogenous component but also varies directly with income. Both income and the marginal productivity of capital feed back into the equations of the financial sector in the manners already described. The model does not determine a commodity price level; everything is expressed in terms of newly produced capital goods, the numeraire.

II. Dynamics of Adjustment

No one seriously believes that either the economy as a whole or its financial subsector is continuously in an equilibrium. Equations like those of the model described above do not hold every moment of time. Consequently analysts and policy-makers can hope to receive no more than limited guidance from comparative static analysis of the full effects of "changing" exogenous variables, including the instruments of policy. They need to know also the laws governing the system in disequilibrium. Since there are many dynamic specifications that have the same static equilibrium, the model-builder has great freedom. Moreover, economic theory, although it imposes some a priori constraints on specification of equilibrium models, has almost nothing to say on mechanisms of adjustment. The burden on empirical testing and estimation is very heavy, but it is precisely in the estimation of lag strictures and autoregressive effects that statistical and econometric techniques encounter greatest difficulties.

There are, of course, some identities -- e.g. balance sheet or income identities -- that apply out of equilibrium as well as in. Our structures in Part I on the need for model-builders to pay explicit attention to these identities apply with equal force to dynamic specifications. A common and useful dynamic equation is that the deviation of a variable from its "desired level" -- i.e., its value according to one of the equations of the equilibrium model -- is diminished by a certain proportion each unit of time. This specification is incomplete when the model includes a number of such variables constrained to add up to a given total, the same total for actual values and desired values. Deviations of actual from desired values must always add up to zero. If, for example, the public is raising its holdings of demand deposits to bring them closer to the quantity desired at current levels of income and interest rates, the public must also be reducing its holdings of some other assets, taking those holdings either toward or away from equilibrium.

In general, the adjustment of any one asset holding depends not only on its own deviation but also on the deviations of other assets. The public might have exactly the right amount of demand deposits and yet change this holding in the course of adjusting other holdings to their desired levels. Failure to specify explicitly these dynamic cross-adjustment effects has the unintended consequence that they are all thrown into the omitted equation. In the model of Part I, for example, the equity equation happens to be the one which is arbitrarily omitted, since by "Walras's law" its specification is implicit in the other equations. If no cross-effects were allowed in the explicit equations of adjustment of the other asset demands, then the counterparts of all the own-adjustments specified would be loaded into the implicit adjustment equation for equities. The assumption would be, for example, that when people increase their demand deposits to bring them up to desired levels they get all the funds by selling equities. It is doubtful that a model-builder would want to make an assumption of this sort, but he might do so inadvertently.

The necessity for the effects of a change in a variable to sum to zero across an exhaustive list of asset holdings applies separately to every lagged value introduced as an explanatory variable. Model-builders are tempted, of course, to choose for each equation, one at a time, the lag structure that seems best to fit their common sense judgments and the data. They should remember that they are implicitly building the reflection of this lag structure into other equations. For example, it would be hard to make sense of a model that relates one asset holding to interest rates lagged two and three quarters and relates a close substitute to the same interest rate lagged one and four quarters.

We are pleading, in short, for a "general disequilibrium" framework for the dynamics of adjustment to a "general equilibrium" system. This is the spirit in which the simulation model, described in the next section, has been constructed.

III. Description of the Structure of the Model

The model which has been simulated is as follows:

1. Public's desired balance sheet. Each desired asset holding is of the form $X^P = (a_0 + a_1 r_T + a_2 r_S + a_3 r_L + a_4 r_K + a_5 Y)W^P$. The assumed coefficients of the linear forms are given in Table 2; we do not attempt to defend the realism of these numbers or the ones in later tables. We shall designate by $X^{**}(t)$ the value of X^P which this function yields for contemporaneous r's , Y , and W^P . The sum of X^{**}/W^P must be identically equal to 1; therefore, the constant terms must add up to one and the other coefficients to 0. Own-rate coefficients in each case are shown in squares.

2. Public's adjustment behavior. This is assumed to take the following form $\Delta X_i(t) = X_i(t) - X_i(t-1) = \Sigma_j \alpha_{ij}(X_j^{**}(t) - X(t-1)) + \beta_i H(t) +$

$$\gamma_i K(t-1)\Delta p(t) .$$

The first terms simply represent the stock adjustment terms previously discussed, including "cross" as well as "own" terms. The last two terms represent initial allocations of new saving $H(t)$ and of capital gains on equities $K(t-1)\Delta p(t)$. Together these two variables account for the change in public wealth $\Delta W^P(t)$. As the column sums of Table 3 indicate, the sum of the reactions to a particular deviation, with wealth constant, must be zero,

TABLE 2.

Desired Balance Sheet of Public

Coefficients of:		Const.	r_T (time dep.)	r_S (Securities)	r_L (Loans)	r_K (Equities)	Y (Income)	
	C^{**}/W^P	0	0	0	0	0	0	Currency[3/]
(1)	D^{**}/W^P	.55	-.20	-.10	0	0	.10	Demand Deposits
(2)	T^{**}/W^P	-.05	.40	-.20	0	0	-.03	Time Deposits
(3)	S^{P**}/W^P	.20	-.15	.40	0	-.025	-.05	Treasury Securities
(4)	L^{**}/W^P	0	0	0	.20	-.125	-.05	Loans (in negative sense)
	V^{**}/W^P	.30	-.05	-.10	-.20	+.15	.03	Capital
	TOTALS	1.00	0	0	0	0	0	

and the sum of the reactions to a change in wealth must be one.

There are five deviations $X^{**}(t) - X(t-1)$ and two wealth increments $H(t)$ and $K(t-1)\Delta p(t)$. But they are linearly dependent: the sum of the five deviations must equal the sum of the two wealth increments. Therefore, there are only six indentifiable coefficients, not seven, in each ΔX adjustment equations. We have chosen to leave out $V^{**}(t) - V(t-1)$, which can be derived as the sum of the last two variables -- column headings in Table 3 -- less the sum of the first four. Therefore, each of the first four columns of Table 3 describes the pattern of reactions to a unit deviation in the designated variable <u>offset by a unit deviation of opposite sign in equity holdings</u>. Likewise, each of the last two columns describes the pattern of reactions to a unit increment of wealth matched by a unit deviation of the same sign in equity holdings.

The numerical values in the table embody some preconceptions of the authors. One is that new saving is initially accumulated as demand deposits, later to be distributed among other assets if holdings of demand deposits are too large. Another is that capital gains are initially held in the assets that gave rise to them; later they may be at least partially realized and distributed across the whole portfolio. The fourth column has the following interpretation: If people are in debt more than they like, (and have equivalently more equity capital than they would like) they repay 40% of the excess, selling equities for 1/4 of the repayment and using demand deposits for the other 3/4. Conversely if their debt is less than desired they borrow 40% and divide the newly borrowed funds in the same one-to-three ratio between equities and cash. Subsequently the borrowed money finds its way into equities, which the equilibrium equations tell us is the purpose of incurring debt.

3. <u>Banks' desired allocation of deposited funds</u>. As explained in Part I, banks accept as given and beyond their control the quantities of time and demand deposits forthcoming at the ceiling rates. They allocate these depostis, after meeting the reserve requirements upon them, among excess reserves, securities, and loans. These allocations are not the same for the two kinds of deposits; banks are more willing to lend out their time deposits, which are regarded as less likely to be withdrawn. The form of the equation for banks' desired holding of an asset is $X^B = (1 - k_D)D\{a_D + a_1(r_S - r_F) + a_2(r_L - r_F)\} +$

TABLE 3.

Adjustment Behavior of Public

Coefficient of:	Deviation from Desired Stocks				Changes in Wealth	
	Demand Dep. $D^{**}(t) - D(t-1)$	Time Dep. $T^{**}(t) - T(t-1)$	Securities $S^{P**}(t) - S^{P}(t-1)$	Loans $L^{**}(t) - L(t-1)$	Saving $H(t)$	Capital Gains $K_{t-1}(p_t - p_{t-1})$
(1') $\Delta D(t)$	+.30	-.08	-.08	-.30	1.00	0
(2') $\Delta T(t)$	-.10	.20	-.10	0	0	0
(3') $\Delta S^{P}(t)$	-.15	-.10	.20	0	0	0
(4') $\Delta L(t)$	0	0	0	.40	0	0
$\Delta V(t)$	-.05	-.02	-.02	-.10	0	1.00
TOTALS	0	0	0	0	1.00	1.00

$(1 - k_T)T\{a_T + a_1(r_S - r_F) + a_2(r_L - r_F)\}$. We shall call the value of X^B for contemporaneous values of interest rates and deposits $X^*(t)$.

TABLE 4.

Desired Portfolio of Banks

		Constants		Coefficients of differentials above discount rate		
		a_D demand deposits	a_T time deposits	a_1 securities rate $r_S - r_F$	a_2 loan rate $r_L - r_F$	
(5)	E^*	.01	0	-.04	-.01	Net free Reserves
(6)	S^{B*}	.67	.34	+.06	-.09	Treasury Securities
(7)	$-L^*$	.32	.66	-.02	+.10	Loans
	TOTALS	1.00	1.00	0	0	

4. Banks' adjustment behavior. The dynamics of bank behavior are similar in structure to the dynamics of public portfolio adjustment. Changes in bank portfolio allocations depend, on the one hand, on deviations from desired allocations and, on the other hand, on changes in disposable deposits. The assumed structure of the former responses is given in the first two columns of Table 5, for net free reserves and securities. A unit deviation in either of these has as its counterpart a unit deviation of opposite sign in loans.

The structure of responses to changes in disposable assets is given in columns three and four; very simply, all changes are initially absorbed in net free reserves. As indicated in those columns, disposable assets may change either because deposits change or because reserve requirements are altered.

Reserve requirement changes also figure in column one: banks are assumed to realize, for example, that net free reserves of last period are already less excessive if reserve requirements have meanwhile been increased.

TABLE 5.

Adjustment Behavior of Banks

	Coefficients of: Deviation from Desired Stocks		Changes in Disposable Assets		Changes in Loan Demand
	Net free Reserves	Treasury Securities	Demand Deposits	Time Deposits	
	$E^*(t) - E(t-1) + \Delta k_D D(t-1) + \Delta k_T T(t-1)$	$S^{B*}(t) - S^B(t-1)$	$(k-k_D)\Delta D - \Delta k_D D(t-1)$	$(1-k_T)\Delta T - \Delta k_T T(t-1)$	$-\Delta L$
(5') ΔE	.5	-.5	1	1	-1
(6') ΔS^B	-.5	.5	0	0	0
$-\Delta L$	0	0	0	0	1
TOTALS	0	0	1	1	0

Finally, the last row and column of Table 5 recognize that in the short run banks meet from excess reserves whatever loan demand comes their way at the established interest rate. However, banks adjust the loan rate up or down, depending on whether $L^*(t) - L(t-1)$ is greater or smaller than zero:

$$\Delta r_L = 10 \left\{ \frac{L^*(t) - L(t-1)}{(1-k_D)D(t-1) + (1-k_T)T(t-1)} \right\} \qquad (7')$$

This is the *modus operandi* of the loan market, and determines the loan rate. There are two other balance equations, one for bank reserves (currency) and one for interest-bearing government debt. These equations determine the two remaining interest rates, on securities and equities. These must adjust contemporaneously as necessary to clear these markets.

(8) $$E(t) + k_D D(t) + k_T T(t) = R(t)$$ Reserves

(9) $$S^P(t) + S^B(t) = G(t) - R(t)$$ Securities

As in the static model, we have equations for the market value of the capital stock and for total public wealth:

(10) $$V(t) = p(t)K(t)$$

(11) $$W^P(t) = G(t) + V(t)$$

5. Market value of equity. As explained in Part I, there is an inverse relation (12) between the market value of equity and the return it bears. Their product is equal to the marginal productivity of capital, r . This in turn was assumed to be positively and linearly related to the average product capital; in the dynamic version this relation is lagged.

(12) $$r_K(t)p(t) = r(t)$$

(13') $$r(t) = \alpha_0 + \alpha_1 \frac{Y(t-1)}{K(t-1)}$$

In some simulation runs α_0 is varied in a cyclical pattern in order to "drive" the economy. Two pairs of normal values of (α_0, α_1) are used -- one is (9,2.5) and the other (8,5). The second gives a more powerful endogenous determination of r. Since the equilibrium value of the average product of capital is taken to be .4, the equilibrium value of marginal productivity is in both cases 10 (percent).

6. Changes in wealth. Equation (11) implies that $\Delta W^P(t) = \Delta V(t) = \Delta G(t)$. Likewise, equation (10) says that $\Delta V(t)$ may be the result either of real investment $I(t) = \Delta K(t)$ or of capital gains or losses on existing capital. The allocation of changes in wealth between saving $(H(t) = \Delta G(t) + p(t)I(t))$ and capital gains makes a difference in the adjustment process -- see Table 3.

The 13-equation static model has now been augmented by the seven adjustment equations (1') through (7'). Correspondingly, actual values of the seven quantities are augmented by seven desired levels D^{**}, T^{**}, S^{P**}, L^{**}, E^{*}, S^{B*}, L^{*}.

The model so far described tells how the financial system operates in response to monetary policy changes or to other shocks arising either inside the financial sector or in the real economy. This model can trace the effects of these shocks on time paths of interest rates, financial quantities, and the market valuation of capital. Among the variables whose time paths are treated as exogenous to the financial system are income Y, the exogenous component of marginal efficiency of capital α_0, the real capital stock K, the government debt G.

In a rough sense, this model is analogous to the LM sector of the textbook Keynes-Hicks macro-economic model. That is, it tells what interest rates will be associated -- via monetary and financial institutions, markets, and behavior -- with different states and paths of income and other "real economy" variables.

7. <u>The model extended to endogenous determination of income</u>. As noted in Part I, we have also constructed a primitive extension of the model to allow for endogenous determination of income. The dynamic version of this extension consists of the following equations:

$$Y(t)(1 - c(1 - \overline{tx})) = c_0 + \Delta K(t) + GP(t) \tag{14}$$

This is the conventional multiplier relation. Here c is the marginal propensity to consume from disposable income, c_0 is the consumption intercept, $\overline{tx}$ is the marginal tax rate, and GP is government purchases. No lags are introduced; (14) holds for contemporaneous values of the variables.

$$\Delta G(t) = GP(t) - \overline{tx}Y(t) - \overline{tx}_0 \tag{15}$$

The increase in government debt is identical to the budget deficit, which is the excess of government purchases over tax revenue. Tax revenue is a linear function of income.

$$\Delta K(t) = \gamma_0(p(t) - 1) + \gamma_1(p(t-1) - 1) \tag{16}$$

As explained in Part I, the valuation of equity is the channel through which financial policies and events are transmitted to the real economy. Equation (16) expresses this linkage. In one numerical version (γ_0, γ_1) is $(1.5,0)$; in an alternative version (γ_0,γ_1) is $(1.5,1.5)$. These three equations convert Y, G, and K into endogenous variables and introduce GP,

tx_0 and $\overline{tx}$ as new policy parameters.

The extended model can be driven by three kinds of shocks (a) exogenous changes in r ---i.e., changes in the α_0 part of r (b) monetary instruments, in particular changes in R , the supply of bank reserves, and (c) fiscal policy, represented by variation of government purchases GP while tax rates remain constant.

IV. Description of Simulations

The dynamic systems described in section III are systems of simultaneous non-linear first order difference equations in 20 or 23 variables. There are three such systems, one for the financial sector alone, and two variants of the extended model, with "weak investment" and "strong investment" responses to changes in income Y . Simulations of the following types have been run:

(a) Unit impulses: the system is displaced from equilibrium by a once-for-all increase of 10% in a single exogenous variable, holding all others at their initial equilibrium values, and the paths of the endogenous variables to the new equilibrium are traced.

(b) Exogenous cycles. The system is displaced from equilibrium by a sinusoidal fluctuation in a single exogenous variable, with a period of 24 units of time. At its peaks the variable is 5% above, at its troughs 5% below, its initial value.

There are both monetary cycles, in which the driving force is R , the supply of unborrowed reserves, and non-monetary cycles. In the several non-monetary cycles, the driving forces are GP , government purchases, and r , themarginal efficiency of capital or its exogenous component α_0 . There are two kinds of non-monetary cycles, corresponding to alternative assumptions about the behavior of the central bank. In one set of simulations, the monetary authority holds R constant and lets interest rates fluctuate. In another set, the monetary authority desires to peg the Treasury security rate, and therefore engages heavily in open market operations designed to keep the rate on target.

The results of these simulations are summarized in the Appendix Tables. They form the basis for some observations in the subsequent sections of this paper.

V. Equilibrium Responses, Financial Sector

The comparative static properties of the model, a number of which were discussed qualitatively in Part I, are illustrated in Table A-1. How to read it may be explained by reference to the first column, which concerns the ultimate effects of a 10% or .17 change in unborrowed bank reserves R , accomplished by open market operations. Note that the public eventually sold not only .17 securities to the central bank but another .34 to the banks. With the reserve requirements in force, the increase in reserves could legally have supported an expansion of 1.13 in demand deposits or 3.40 in time deposits, or any linear combination. However, this does not happen. Both demand and time deposits have elasticities less than one (.43 and .69) with respect to reserve changes, and their total increase is only .84. Banks keep .10 of the new reserves idle. Even so, the public has considerably reshuffled its wealth, selling securities and borrowing as the counterpart of their increased deposits. Their portfolio shifts, and their counterparts in the banks' portfolios, are induced by a general reduction in interest rates, with which goes an increase in the valuation of equity capital p . Thus the column says that open market purchases have an effect in the expected expansionary direction.

The other columns are to be interpreted similarly. A number of properties of the model worth noting are illustrated in Table A-1:

(a) In several instances D and p move in opposite directions, and increases in D accompany increases rather than reductions in interest rates. Thus column 3 concerns an increase in wealth which takes the form entirely of equity capital; no government debt in monetary or other form is provided to balance it. As might be expected, this is highly deflationary. But the public does acquire more bank deposits, and the banks are induced by higher interest rates to cut their excess reserves drastically.

Columns 4 to 7 concern 10% increases in demand deposits as a result of autonomous changes in asset preferences, the shift in each case coming from the asset indicated. All such shifts are of course deflationary, even though demand deposits increase and satisfy partially the public's desire to hold more of them. Banks are again induced to economize reserves by increases in interest rates.

(b) Changes in excess reserves are also an unreliable guide to the thrust of the financial system, as measured by p . When monetary policy is expansionary, excess reserves go up along with p . When, as in column 8, nonmonetary events are raising both p and the demands on the banking system, net free reserves fall.

(c) Although interest rates move together in all the cases in Table A-1, they too can be misleading indicators. Consider, for example, an autonomous shift from securities into capital, whose effects could be calculated by subtracting column 6 from column 5. Then r_S would rise by .04, r_K would fall by .21 and p would rise.

In Table A-2 the equilibrium responses of the endogenous variables to three exogenous variables, Y , G , and R are compared to the relative amplitudes of the same endogenous variables in cycles driven by the same three exogenous variables. Thus demand deposits had a relative amplitude 1.07 times as large as income in an income-driven cycle; this compares with a comparative-statics elasticity of 1.03. The table shows that for some variables such elasticities are a misleading indicator of cyclical sensitivities. The magnitude of the cyclical fluctuations in T , for example, is on the order of two-thirds its equilibrium response for both the Y and R cycles. On the other hand, in the Y cycle, the security holdings of both the banks and the public fluctuate more than two and a half times their equilibrium response. Similarly, fluctuations in bank reserves cause bigger fluctuations in r_K and p than would be expected from the corresponding once-for-all elasticity. This suggests that it may be difficult to obtain accurate estimates of the demand relationships from cyclical data.

VI. Adjustment Speeds

The speed with which a simultaneous difference equation model returns to equilibrium when subjected to a change in an exogenous variable cannot be inferred by inspection of individual behavioral equations. Systems with slow adjustment in individual behavioral equations may move quickly to a new equilibrium, and systems incorporating rapid adjustment in individual equations may be slow to reach a new equilibrium. This reflects two features of a "general disequilibrium" system.

First, some variables can be taken as given by individual decision-makers or in particular markets but are endogenous to the system as a whole. Slow response of individuals in one dimension may merely result in a compensating large and rapid adjustment of other endogenous variables. This process may get the system to equilibrium in a short time.[4/]

Second, adjustments made in one market, while moving it towards equilibrium, may move other markets away from equilibrium. Even for the relatively simple model of the financial sector we have constructed, the dynamics of adjustment would be extremely difficult to obtain analytically. Although the system is non-linear, one might expect the endogenous variables to exhibit behavior similar to that generated by a high order linear difference equation. Hence we should not be surprised to find that the speed with which particular variables adjust to their new equilibrium depends on the particular exogenous variable which is changed. Moreover, there is no simple way to infer from the speeds of adjustment to each of two or more individual shocks how fast the system would adjust to a combination of shocks, either simultaneous or sequential.

In our simulations (see Table A-4) it appears that most variables are relatively slow to reach a new equilibrium following an increase in the supply of real capital or an increase in the marginal product of capital, and adjust relatively fast to an injection of reserves. Similarly, on the basis of the analogy with linear difference equations, we would expect to find some variables responding relatively fast to some shocks and relatively slow to others. Demand deposits, for example, complete 75% of their adjustment to a change in income within two periods, whereas loans require 18 periods for a similar adjustment. In response to a change in the marginal product of capital, however, loans adjust 75% of the way in 5 periods whereas the similar adjustment requires 9 periods in the case of demand deposits.

In spite of the fact that relative speeds of adjustment depend on which exogenous variable is changing, some endogenous variables seem to adjust relatively slowly for almost all of the shocks we have considered. Even though individuals always hold the desired quantity of loans, L is frequently among the last of the variables to come within 25% of its new equilibrium value. In two thirds of the cases, the loan rate, which banks adjust "slowly," achieves 75% adjustment before the quantity of loans. With the exception of the adjustment to a change in the marginal product of capital, demand deposits adjust more rapidly than time deposits.

VII. Cyclical Timing Patterns

In a highly interdependent dynamic system, the chronological order in which variables reach cyclical peaks and troughs proves nothing whatever about directions of causation. Although few people would seriously claim that cyclical lead-lag patterns are a reliable guide to direction of causal influence, believers in the causal primacy of monetary variables have offered the timing order of variables in business cycles as partial evidence for their position. Simulation of cycles of known exogenous or causal source is a good way to show that observed timing order can be very misleading.

The dangers involved in relying on the timing of peaks and troughs as an indication of causality are illustrated in Tables A-5 and A-6. In every case considered, some endogenous variable leads the exogenous variable driving the system. In each of the reserve cycles, for example, free reserves lead the total supply of reserves. Similarly, an exogenous cycle in the marginal product of capital generates a cycle in income which leads it in both variants of the extended model.

Even though leads and lags do not provide information about causation, if they could be depended on they would be extremely useful in forecasting the future course of the economy. Unfortunately, the tables provide numerous examples of variables which lead another endogenous variable when the economy is driven by one exogenous variable and lag it when driven by another exogenous variable.

For example, in the extended model with reserves fixed, loans lead income when government purchases are the driving force, but lag income in cycles driven by fluctuations in the marginal product of capital.

Similarly, in the financial sector simulations, the rate on securities leads the rate on equities by two periods when income alone varies exogenously, but lags it by one period when fluctuations in investment accompany the variations in income.

Not surprisingly, the leads and lags are also sensitive to the policy actions of the monetary authority. If the supply of reserves is fixed and government purchases cause fluctuations, free reserves trough when income peaks. When the monetary authority pegs the rate on securities, however, free reserves actually peak with income. Similarly, in the "strong investment" variant, loans lead government purchases by two periods when the rate on securities is endogenous and lag government purchases when it is pegged.

Table A-1

Financial Sector Model

Equilibrium Responses
to Once-for-all 10% Increases in Single Variables
(units in upper left of cell; elasticity in lower right)

Variable \ Shock, and amount	Reserves	Gov. Debt	Real Cap.	Preferences for Demand Deposits				Mrg. Prod	Income	Reserve Requirements		Ceiling Rate
				from T**	from S**	from K**	from L**			Demand Dep.	Time Dep.	
	R	G	K					r	Y	k_D	k_T	R_T
	.17	.75	1.25	.70	.70	.70	.70	1.0	0.5	1.5%	0.5%	.25
Demand Deposits D	.30	.15	.57	.53	.45	.41	.46	.19	.73	-.18	-.07	-.94
	.43	.21	.82	.76	.65	.59	.66	.27	1.05	-.26	-.10	-1.35
Time Deposits T	.54	.10	.52	-.88	-.38	-.36	-.31	.20	-.71	-.30	-.12	2.04
	.69	.13	.66	1.12	-.48	-.46	-.40	.25	-.90	-.38	-.15	2.54
Security Holdings S^P	-.51	.67	.47	.08	-.09	.08	.16	.18	.05	.26	.09	-.31
	-1.92	2.52	1.76	.30	-.34	.30	.60	.68	.19	.97	.34	-1.18
Loans -L	.33	.18	1.58	-.26	-.01	.14	.32	.57	.08	-.22	-.10	.79
	.33	.18	1.58	-.26	-.01	.14	.32	.57	.08	-.22	-.10	.79
Excess Reserves E	.10	-.02	-.11	-.04	-.05	-.04	-.05	-.04	-.07	-.06	-.02	.04
	(n.a.)	(n.a.)	(n.a.)	(n.a.)	(n.a.)	(n.a.)	(n.a.)	(n.a.)	(n.a.)	(n.a.)	(n.a.)	(n.a.)
Bank Securities S^B	.34	.08	-.47	-.09	.09	-.09	.16	-.18	-.05	-.26	-.09	.31
	1.09	.26	-1.50	-.29	.29	-.29	.51	-.58	-.16	-.83	-.29	.99
Security rate r_S	-.09	.06	.09	.02	.08	.04	.04	.03	.09	.05	.02	.04
	-.46	.30	.44	.10	.39	.20	.20	.15	.44	.24	.10	.20
Yield on Capital r_K	-.25	-.03	1.16	.27	.18	.49	.38	.46	.32	.20	.08	-.31
	-.24	-.03	1.13	.26	.18	.48	.37	.45	.31	.20	.08	-.30
Loan interest r_L	-.20	.04	.63	.20	.10	.20	.28	.24	.28	.14	.05	-.34
	-.38	.08	1.22	.38	.19	.39	.54	.47	.54	.27	.10	-.66
Equity value[1] p	.02	+.00	-.13	-.03	-.02	-.05	-.04	.05	-.03	-.02	-.01	.03
	.20	+.00	-1.30	-.30	-.20	-.50	-.40	.50	-.30	-.20	-.10	.30

1 Numbers are inexact because changes in p were reported only to one significant figure.

Table A-2

Financial Sector Model

Amplitudes (Peak less Trough) relative to

Amplitude of Driving Force (10% of equilibrium value),

(Equilibrium Elasticities (absolute values) from Table 1 in parentheses)

Exogenous cycles in:

Variable	Income Y	Income Y, Investment ΔK 1/	Govt Debt G	Reserves R
Demand Deposits D	1.07 (1.05)	3.20	.40 (.21)	.46 (.43)
Time Deposits T	.60 (.90)	.34	.34 (.13)	.39 (.69)
Security Holdings S^P	.53 (.19)	3.24	2.34 (2.52)	1.73 (1.92)
Loans -L	.25 (.08)	2.31	.06 (.18)	.17 (.33)
Excess Reserves E 2/	.09 (.07)	.33	.03 (.02)	.13 (.10)
Banks' Reserves S^B	.45 (.16)	2.78	.48 (.26)	1.03 (1.09)
Security Rates r_S	.39 (.44)	.49	.44 (.30)	.39 (.46)
Yield on Capital r_K	.32 (.31)	2.92	.17 (.03)	.37 (.24)
Loan Interest r_L	.45 (.54)	2.04	.08 (.08)	.41 (.38)
Equity Value p	.20 (.30)	2.70	.20 (±.00)	.40 (.20)

1/ In this simulation ΔK was always 1/2 (actual Y - equilibrium Y), corresponding to a multiplier of 2; the capital stock varied accordingly, whereas in column 1 the capital stock is held constant.

2/ Amplitudes given in units.

TABLE A-3

Extended Model Cycles
Amplitudes (Peak less Trough) relative to Amplitude of Driving Force
(10% of Equilibrium Value)

		Strong investment variant					Weak investment variant				
		Exogenous cycle in:									
		GP, Govt. Purchases		r, Marg. Product		Reserves R	Reserves R	GP Govt. Purchases		r, Marg. Product	
		r_S pegged	R fixed	r_S pegged	R fixed			r_S pegged	R fixed	r_S pegged	R fixed
Demand Deposits	D	1.42	.83	1.31	1.16	.66	.63	1.32	.79	1.08	1.03
Time Deposits	T	.33	.52	.41	.25	.42	.41	.34	.54	.40	.23
Security Holdings	S^P	1.24	.94	.56	.79	1.73	1.73	1.20	.90	.53	.75
Loans	-L	.38	.20	1.02	.95	.37	.34	.28	.17	.91	.85
Excess Reserves	E[1]	.10	.07	.09	.12	.09	.11	.04	.06	.07	.11
Banks' Securities	S^B	.83	.22	.61	1.09	.87	.90	.90	.16	.61	.94
Security Rates	r_S	.05	.39	.00	.20	.39	.39	.05	.39	.00	.20
Yield on capital	r_K	.10	.16	.91	.93	.06	.13	.03	.13	.76	.81
Loan Interest	r_L	.14	.33	.70	.80	.21	.27	.10	.29	.58	.70
Equity Value	p	.00	.00	.20	.20	.00	.20	.00	.00	.40	.40
Reserves	R	1.00	.00	.41	.00	1.00	1.00	.93	.00	.41	.00
Income	Y	.80	.68	.34	.30	.12	.10	.76	.26	.68	.24

[1] Amplitude given in units.

Table A-4

Financial Sector Model

Speeds of Adjustment to

Once-for-All 10% Increases in Single Variables

(Smallest number of periods after which variable's distance from new equilibrium is 25% or less of full equilibrium response. Starred entries (*) designate adjustment paths that over-shoot and oscillate.)

Shock / Variable		Reserves R	Govt. Debt G	Real Cap. K	Preferences for Demand Dep. from T**	from S**	from K**	from L**	Mang. Prod. r	Income Y	Reserve Req. k_D	k_T
Demand Deposits	D	5	5*	10*	2	3	2	2	9	2	4	5
Time Deposits	T	7	14*	10*	6	7	6	5	7	6	8	8
Security Holdings	S^P	4	1	12*	5	1*	8	8	10	8	4	4
Loans	-L	4	11	7*	13*	20*	11*	3	5	18*	10	9
Excess Reserves	E	4*	5*	5*	5	2	4	5	7	5	4*	1
Banks' Securities	S^B	5	4*	12*	6	6*	9	8	10	8*	5	4
Security rates	r_S	1	2*	2*	5	1	2	3	6	3	2	3
Yield on Capital	r_K	3*	15*	10*	15*	2	5*	7	8	4	2	2
Loan Interest	r_L	4	4*	9*	4*	4	5	5	7	5	4	3
Equity value	p	3*	15*	9*	15*	2	5*	7	6*	6	2	2

Table A-5

Lag (+) or Lead (-), Compared with Exogenous Cycles (24 periods)

Financial Sector

Endogenous Variables	Exogenous Variables			
	Y	R	G	Y, ΔK
D	1/2	2 1/2	-2 1/2	0
T	(2 1/2)	3	(-2 1/2)	(4)
S^P	(2)	(2)	1/2	(-5 1/2)
-L	-1	5 1/2	(-4 1/2)	2
E	(0)	-1	(-2)	(0)
S^B	-4	3 1/2	-1	-5 1/2
r_S	1	(1/2)	-2 1/2	3 1/2
r_K	3	(2 1/2)	(1)	2 1/2
r_L	2 1/2	(3)	-4 1/2	4
p	(4)	2 1/2	1	(2 1/2)
R	-	0	-	-
Y	0	-	-	0
ΔK	-	-	-	0

Comparison is with second cyclical peak of cyclically fluctuating exogenous variable in simulation run. Numbers in parentheses refer to timing of a trough in comparison with this reference cycle's peak; this comparison is made for variables that move counter cyclically.

Table A-6

Lag (+) or Lead (-), compared with Exogenous Cycles (24 periods)

	Extended Model - Strong Investment					Extended Model - Weak Investment				
	r_S pegged R endogenous			r_S endogenous R fixed		r_S pegged R endogenous			r_S endogenous R fixed	
	GP-cycle	r-cycle		GP-cycle	r-cycle	GP-cycle	r-cycle		GP-cycle	r-cycle
D	2	-1	1	1	- 1/2	2	-1/3	1 1/2	1	-2
T	(-1)	4 1/2	3 1/2	(2)	5 1/2	(-1/2)	5	3 1/2	(2 1/2)	4
S^P	(+1)	(-1 1/2)	(1 1/2)	(-5 1/2)	(-5 1/2)	(1)	-1	(1 1/2)	(-5 1/2)	6 or (-6)
-L	2	1 1/2	3 1/2	-2	2	2 1/2	2	4	-2 1/2	1/2
E	0	(-2 1/2)	- 1	(0)	(0)	0	(-1 1/2	-1 1/2	(1/2)	(-2)
S^B	3	(5 1/2)	2	-3	-6 or (6)	3 1/2	-6 or (6)	2	-1 1/2	(4 1/2)
r_S	–	–	(1)	1 1/2	3	- 4	–	(1)	1 1/2	1
[illegible]	1/2	2	(-1/2)	1	2 1/2	-3 1/2	2	(1/2)	1	1
r_L	1/2	2	(1 1/2)	2	3 1/2	- 1	3	(1)	2	2
P	–	- 2	0	–		–	- 2	1/2	–	- 3
R	1 1/2	1 1/2	0	–	–	1 1/2	1 1/2	0	–	–
Y	1/2	- 2	0	0	-1 1/2	1/2	-1 1/2	1/2	0	- 3
ΔK	1	- 2	0	0	-1 1/2	3	-1 1/2	1/2	–	- 3

Comparison is with second cyclical peak of cyclically fluctuating exogenous variable in simulation run. Numbers in parentheses refer to timing of a trough in comparison with this cycle's peak; this comparison is made for variables that move counter cyclically.