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Fire Sales, the LOLR, and Bank Runs with Continuous Asset Liquidity

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Abstract

Banks' asset fire sales and recourse to central bank credit are modeled with continuous asset liquidity, allowing us to derive the liability structure of a bank. Both asset sales liquidity and the central bank collateral framework are modeled as power functions within the unit interval. Funding stability is captured as a strategic bank run game in pure strategies between depositors. Fire sale liquidity and the central bank collateral framework determine jointly the ability of the banking system to deliver maturity transformation without endangering financial stability. The model also explains why banks tend to use the least liquid eligible collateral with the central bank and why a sudden unanticipated reduction of asset liquidity, or a tightening of the collateral framework, can trigger a bank run. The model also shows that the collateral framework can be understood, beyond its aim to protect the central bank, as a financial stability and unconventional monetary policy instrument.

Keywords: asset liquidity, bank intermediation spreads, bank run, capital structure, collateral, fire sales, lender of last resort

JEL Classifications: C61, E43, E58, G21, G32, G33

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1. Introduction

The model proposed in this paper sheds new light on how asset liquidity and the central bank collateral framework affect the liability structure of banks, financial stability, and monetary policy. The Global Financial Crisis of 2007–2009 (GFC) is said to also have been triggered by the insufficient asset liquidity buffers of banks relative to their short-term liabilities. These insufficient buffers would have led to an (at least temporarily) excessive reliance on central bank funding (BCBS 2013). The economic trade-offs between the efficiency of the banking system in delivering maturity transformation and financial stability is also crucial when assessing the net benefits of regulation for society. In the words of the *Turner Review*:

> [T]here is a tradeoff to be struck. Increased maturity transformation delivers bene- fits to the non-bank sectors of the economy and produces term structures of interest rates more favorable to long-term investment. But the greater the aggregate degree of maturity transformation, the more the systemic risks and the greater the extent to which risks can only be offset by the potential for central bank liquidity assistance. (FSA 2009)

Although the central bank collateral framework has drawn relatively limited attention in academic writing, it is one of the most complex and economically significant elements of monetary policy implementation (for example, Bindseil, Corsi et al. 2017; Bindseil, Dragu et al. 2017; Nyborg 2016). Unencumbered central bank eligible collateral is potential liquidity, as it can, in principle, be swapped into central bank money. It is therefore not an exaggeration to argue that the collateral framework must be an important ingredient of any theory of liquidity crises (as noted in Bagehot 1873) and of any monetary theory. The Markets Committee of the Bank for International Settlements (BIS) summarizes various measures central banks took during the Lehman Brothers crisis:

> During the height of the financial crisis in 2008–09, a number of central banks introduced, to varying degrees, crisis management measures such as a temporary acceptance of additional types of collateral, a temporary lowering of the minimum rating requirements of existing eligible collateral or a temporary relaxation of haircut standards. (BIS Markets Committee 2013, 8–9)

The COVID-19 crisis again led central banks to relax their collateral frameworks, despite the fact that credit quality of issuers and counterparties did not improve. The European Central Bank (ECB) announced an "unprecedented" package on April 7, 2020:

> The Governing Council of the European Central Bank (ECB) today adopted a package of temporary collateral easing measures to facilitate the availability of eligible collateral for Eurosystem counterparties to participate in liquidity providing operations . . . the Eurosystem is increasing its risk tolerance to support the provision of credit via its refinancing operations, particularly by lowering collateral valuation haircuts for all assets consistently. . . . [T]he Governing Council decided to temporarily increase its risk tolerance level in credit operations through a general reduction of collateral valuation haircuts by a fixed factor of 20%. (ECB 2020)

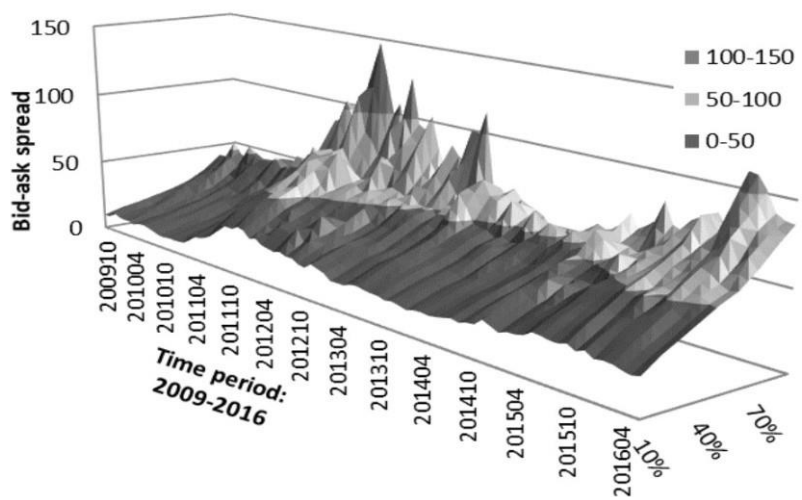
Three strands of academic literature are relevant to the present paper. First, Rochet and Vives (2004) is close to the present paper in the sense that it also models the role of fire sales and the central bank lender-of-last-resort (LOLR) function for banks' funding stability, liquidity, and solvency. The model of Rochet and Vives, however, takes strong simplifying assumptions regarding asset liquidity (only two types of assets are distinguished: cash and nonliquid assets). Beyond Rochet and Vives, there is an extensive more general multiple funding equilibrium literature such as represented, for example, by Morris and Shin (2001) under the headline of "global games." This literature uses more general and sophisticated equilibrium concepts than the present paper, which limits itself to pure and dominant strategies of investors/depositors, and to the existence or not of a strict Nash equilibrium in the sense of Fudenberg and Tirole (1991). Again, none of these papers, however, models explicitly fire sales and the central bank LOLR together, nor do they allow us to derive the bank's liability structure from this modeling. Finally, there is a relatively recent bank run literature integrating bank runs and the LOLR into macroeconomic models, such as Gertler and Kiyotaki (2015) and De Fiore, Hoerova, and Uhlig (2021). These models are more ambitious in terms of integrating bank runs in a general equilibrium macro context, but the modeling of the limits of banks to issue short-term deposits is rather ad hoc (a moral hazard problem of the bank manager is postulated to limit deposit issuance; namely, bank managers would be able to divert assets from the bank to enrich themselves personally). In contrast, in the model proposed here, the bank's ability to issue short-term deposits follows endogenously from the threat of a bank run.

Second, Ashcraft, Garleanu, and Pedersen (2010) relates to the present model in the sense that central bank haircut policies are identified and modeled as a monetary policy instrument (see also Chapman, Chiu, and Molico 2011). Ashcraft, Garleanu, and Pedersen assumes that banks refinance assets at the central bank and that the haircut determines the leverage ratio and thus the funding costs of assets, being a weighted average of the risk-free rate and the shadow cost of equity (see also Brunnermeier and Pedersen 2009).

Third, the present paper explains the spread between the risk-free rate (which is close to the rate of central bank credit operations and the rate of remuneration of overnight deposits of households with the banking system) and the actual funding costs of the real economy (or the effective monetary conditions for the economy). In this sense, the paper contributes to enrich the analysis of monetary policy in particular by capturing more explicitly the spread between the central bank credit operations rate and the actual monetary conditions as they are felt by participants in the real economy when seeking bank or market funding. Indeed, the present paper shows that a drop in bank asset liquidity and/or an increase in central bank haircuts both tighten effective monetary conditions in the sense that they reduce the ability of the banking system to undertake maturity transformation, and hence, everything else equal, will increase the share of "expensive" bank funding sources such as long-term bonds and equity, implying that also the lending rates that a competitive banking system is able to offer have to increase.

Empirical evidence of the different degrees of banks' asset liquidity, focusing on the case of the euro area, is provided by Bindseil, Dragu et al. (2017). The paper provides a cross-section analysis of liquidity properties and central bank collateral haircuts of the euro area fixedincome universe. That asset liquidity is continuous, and that it fluctuates over time, has been described empirically in the finance literature, such as recently in Dötz and Weth (2019), which also argues that liquidation will be carried out in a liquidity pecking order style and that marginal liquidation costs increase in redemptions (9–11), that is, as assumed in the present paper. The authors construct a sample of corporate bond fund asset liquidity data covering the 80 months before June 2016, referring to about 700,000 security holdings positions (12). The liquidity measure consists of monthly averages of bid-ask spreads. Figure 1 shows continuous portfolio liquidity, put at any moment in time in a "liquidity pecking order" (that is, securities ranked from the most to the least liquid). Obviously, the least liquid assets held by a corporate bond fund will still be more liquid than many other bank assets, such as in particular loan portfolios. Still, Dötz and Weth illustrates the idea of continuous asset liquidity and the changes of asset liquidity over time.

Figure 1: Liquidity Structure of Corporate Bond Funds across Time



Note: Bid-ask spread in basis points.

Source: Dötz and Weth 2019.

The paper also sheds new light on related debates. For example, first, Acosta-Smith et al. (2019), which studies the relationship between bank capital and liquidity transformation, finds that "banks engage in less liquidity transformation when they have higher capital," but our paper suggests that there are alternative causalities that could explain this empirical pattern. Second, the literature on fire sales, as summarized by Shleifer and Vishny (2011), generally regards fire sales as entailing "systemic risk and significant [negative] externalities (43). Our paper models potential fire sales as being an integral part of maturity transformation by banks, whereby indeed, actual fire sales would not materialize in the absence of negative shocks on asset liquidity or asset values. Third, the present model provides further insights into empirical phenomena, such as those observed by Boyson, Helwege, and Jindra (2014), which investigates "which factor, liquidity or solvency, is more important for financial crisis." Several further empirical hypotheses supported in that paper are captured also in our model, and therefore are supported by additional theoretical explanations. Finally, the paper also sheds new light on why both liquidity regulation and the lender of last resort are needed, extending the work of Carlson, Duygan-Bump, and Nelson (2015), among others.

2. A Model of Funding Stability with Continuous Asset Liquidity and Haircuts

Throughout this paper, the following stylized bank balance sheet is assumed, with total length set to unit. Assets are heterogeneous in a continuous sense, while there are three types of liabilities that are each separately homogenous equity (e), long-term debt (t), and short-term deposits held by two depositors (each depositor holding therefore $(1 - t - e)/2$ short-term deposits), with $e, t \geq 0$, $e + t \in [0, 1]$.

Figure 2: Stylized Bank Balance Sheet

Assets		Liabilities	
Assets	1	Short-term debt 1	$(1 - t - e)/2$
		Short-term debt 2	$(1 - t - e)/2$
		Long-term debt ("term funding")	t
		Equity	e

Source: Created by the authors.

Short-term deposits can be subject to a bank run (Diamond and Dybvig 1983, 15). Banks could address this by not relying, or at least not extensively relying, on short-term funding. However, in general, investors prefer to hold short-term debt instruments over long-term debt instruments, and hence face a higher interest rate on long-term debt. In other words, long-term debt is associated with higher funding costs for the bank. Banks could also hold sufficient amounts of liquid assets, both in the sense of being able to liquidate these assets in case of need and to be able to pledge them with the central bank at limited haircuts. However, on average, liquid assets generate lower returns than illiquid ones. We next consider in more detail the different balance sheet positions.

2.1 Bank Assets

Assets are continuous with regard to two liquidity characteristics: (i) Asset liquidity as measured by the "fire sale" discount to be accepted if an asset is to be sold in the short run; (ii) valuation haircut if submitted as central bank collateral.

Bank Assets as Central Bank Collateral

Assume that assets are ranked from those the central bank considers will require the lowest haircuts to those requiring the highest ones. The central bank collateral haircut function is set to be from the asset unit interval [0, 1] into the possible haircut unit interval [0, 1]. Assume that it has the following functional form with $\delta \geq 0$:

$$h(x) = x\delta \tag{1}$$

The power function in the unit interval captures broadly the properties of a typical central bank haircut framework: haircuts for the most liquid assets will be close to zero, while haircuts for the least liquid assets accepted will be high, and an often-significant part of assets will not be accepted at all, which is equivalent to a 100% haircut. If δ is close to 0, then the haircuts increase and converge quickly towards 1. If in contrast δ is large (say 10) then haircuts stay at close to zero for a while and only start to increase in a convex manner when approaching the least liquid assets. The total haircut (and the average haircut) if all assets are pledged is $\frac{1}{\delta+1}$, and hence potential central bank credit is $\frac{\delta}{\delta+1}$. This is obtained from the integration rule $\mathcal{J}\, x^{\delta} dx = \frac{1}{\delta+1} x^{\delta+1}$. For example, in the case of the Eurosystem, out of EUR 30 trillion of aggregated bank assets, the value of central bank eligible collateral after haircuts that could be used at any moment in time is about EUR 5 trillion. The eligibility criteria and haircut matrices are provided by the ECB, and one can match this information in principle with an informed guess of banks' assets holdings. This implies that the effective average haircut applied by the Eurosystem to (the entirety of) bank assets is about 83% (EUR 25 trillion/EUR 30 trillion), and central bank refinancing power is about 17% of eligible assets, which approximately implies, if one assumes a power function as done above, a parameter value δ = 0.2. ECB (2015, 31) provides an overview of the ECB's haircut scheme.

Liquidity of Bank Assets

Now consider asset liquidity in the sense of the ability of banks to sell assets in the short term without this inflicting a loss for the bank. Assume again that assets are ranked from the most liquid to the least liquid, and that the fire sale discount function is a function from [0, 1] to [0, 1] with the following function form, for $\Theta \geq 0$:

$$d(x) = x^{\Theta} \tag{2}$$

The smaller Θ, the faster fire sale losses increase toward 100% when moving from the most liquid to the least liquid assets. If a certain share x of the assets has to be sold, then the fire sale discounts will have to be booked as a loss and reduce equity. Assuming that the bank starts with the most liquid assets, the loss will be $\frac{1}{\Theta+1}\, x^{\Theta+1}$. Empirical estimates of default costs in the corporate finance literature vary between 10% and 44% (see, for example, Davydenko, Strebulaev, and Zhao 2011; and Glover 2016). This cost can be interpreted as the liquidation cost of assets, captured in the parameter Θ. Liquidation of all assets will lead to a damage of 1, so the remaining asset value will be Θ. If default cost is 10%, this would mean that Θ = 9, and if default cost is 44%, then Θ = 1.27. For a value of default costs in the middle of the empirical estimates of say 25%, one obtains Θ = 3.

2.2 Bank Liabilities

Four types of liabilities are distinguished: (i) *Short-term liabilities* are equally split to two ex ante identical depositors; (ii) *Long-term debt* does not mature within the period considered and is ranked pari passu with short-term debt; (iii) *Equity* is junior to all other liabilities and cannot flow out either; and (iv) *Central bank borrowing* is zero initially but can substitute for outflows of short-term liabilities in case of need. It is collateralized, and therefore the central bank acquires in case of default ownership of the assets pledged as collateral. Apart from this, the central bank claim ranks pari passu; that is, remaining claims after collateral liquidation are treated in the same way as an initial unsecured deposit.

2.3 Timeline

The model is based on the following timeline:

- Initially, the bank has the balance sheet composition as shown in Figure 1.

- Short-term depositors/investors play a strategic game with two alternative actions: to run or not to run. "Running" means to withdraw deposits. If successful, the value of claims is afterwards equal to the initial value minus a small cost, E, capturing the transaction cost of withdrawing the deposits.

- It is not to be taken for granted that depositors can withdraw all their funds. Outflows need to be funded by the bank in some way: (i) recourse to central bank credit, assuming that the bank has sufficient eligible collateral; (ii) quick liquidation of assets ("fire sales"); or (iii) if it is impossible to pay out the depositors that want to withdraw their deposits, illiquidity-induced default will occur. In such case, all (remaining) assets need to be liquidated and corresponding default-related losses occur.

- If the bank is not closed due to illiquidity, still its solvency needs verification after it undertakes fire sales. If capital is negative, the bank is resolved. Again, it is assumed in this case that the full costs of immediately liquidating all assets materialize.

2.4 Strict Nash No-Run (SNNR) Equilibrium

The decision set of depositor i (i = 1, 2) from which he will choose his decision D_i consists of $\{K_i, R_i\}$, whereby "K" stands for "keeping" deposits and "R" stands for "run." Let $U_i = U_i(D_1, D_2)$ be the payoff function of depositor i. Note that the strategic game is symmetric, that is: $U_1(K_1, K_2) = U_2(K_1, K_2)$, $U_1(K_1, R_2) = U_2(R_1, K_2)$, $U_1(R_1, K_2) = U_2(K_1, R_2)$, $U_1(R_1, R_2) = U_2(R_1, R_2)$. This allows us in the rest of the paper to express conditions with reference to only one of the two players, say depositor 1. A strict Nash equilibrium is defined as a strategic game in which each player has a unique best response to the other players' strategies (see Fudenberg and Tirole 1991). A strict Nash no-run (SNNR) equilibrium in the run game is therefore one in which the no-run choice dominates the "run" choice regardless of what the other depositors decide, that is, an SNNR equilibrium is defined by:

$$U_1(K_1, K_2) > U_1(R_1, K_2) \text{ and } U_1(K_1, R_2) > U_1(R_1, R_2) \tag{3}$$

3. Pure Reliance on Either Central Bank Credit or Asset Fire Sales

3.1 Pure Reliance on Central Bank Funding

Assume first that asset liquidation is not an option, say, because markets are totally frozen, that is, $\Theta = 0$. In this case, the analysis can focus on the sufficiency or not of buffers for central bank credit. The following proposition states the necessary condition for funding stability of banks in this case.

> **Proposition 3.1.** If $\Theta = 0$, and assuming a small transaction cost E of withdrawing deposits, an SNNR equilibrium prevails if and only if $\frac{\delta}{\delta+1} \geq \frac{1-t-e}{2}$, that is, the liquidity buffer based on recourse to the central bank is not smaller than one-half of the short-term deposits.

Proof. To prove this result (and similar subsequent propositions), it is sufficient to calculate through the payoffs for the alternative decisions of depositors under the possible parameter combinations and establish the frontiers of parameter combinations under which the conditions of an SNNR equilibrium apply. We now distinguish the three possible cases: $(1 - t - e) < \frac{\delta}{\delta+1}$ (which we will denote by (1)), $\frac{(1-t-e)}{2} < \frac{\delta}{\delta+1} < (1 - t - e)$ (which we will denote by (2)), and $\frac{\delta}{\delta+1} < \frac{(1-t-e)}{2}$ (denoted by (3)). These represent, respectively, the cases in which liquidity buffers provided by central bank collateral are sufficient to compensate for the withdrawal of all deposits, for the withdrawal of (not more than) one depositor, or for not even the withdrawal of one depositor. It may be noted that in the case assumed here that $\Theta = 0$, illiquidity of the bank means complete destruction of asset value as the liquidation value of assets is assumed to be zero. The proposition follows from verifying the condition $U_1(K_1, K_2) > U_1(R_1, K_2)$ and $U_1(K_1, R_2) > U_1(R_1, R_2)$ for the three different cases distinguished above. One obtains the scenarios shown in Figure 3.

Figure 3: Payoffs of Depositors in Three Cases (see Proof of Proposition 3.1)

Case	$U_1(K_1, K_2)$	$U_1(R_1, K_2)$	$U_1(K_1, R_2)$	$U_1(R_1, R_2)$
(1)	$\frac{(1-t-e)}{2}$	$\frac{(1-t-e)}{2} - E$	$\frac{(1-t-e)}{2}$	$\frac{(1-t-e)}{2} - E$
(2)	$\frac{(1-t-e)}{2}$	$\frac{(1-t-e)}{2} - E$	$\frac{(1-t-e)}{2}$	$\frac{1}{2}\frac{\delta}{\delta+1}$
(3)	$\frac{(1-t-e)}{2}$	$\frac{\delta}{\delta+1}$	0	$\frac{1}{2}\frac{\delta}{\delta+1}$

Source: Authors' calculations.

It is easily verified that the conditions for an SNNR equilibrium are given if and only if $\frac{(1-t-e)}{2} < \frac{\delta}{\delta+1}$.

3.2 Pure Reliance on Asset Fire Sales

Now consider the case in which the central bank does not offer any credit to banks, that is, accepts no collateral at all. In this case, $\delta = 0$, such that addressing deposit outflows will have to rely exclusively on asset liquidation. Assume that the bank does whatever it takes in terms of asset liquidation to avoid illiquidity-induced default. The total amount of liquidity that the bank can generate through asset fire sales is $\frac{\Theta}{\Theta+1}$. With full reliance on central bank lending, the question was whether the related liquidity buffers would be sufficient. In the present case, two default-triggering events need to be considered. Indeed, even if the bank has survived a liquidity withdrawal, it may afterwards be assessed as insolvent and thus be liquidated at the request of the bank supervisor. As noted above, for a given liquidity withdrawal x, the fire sale related loss is $\frac{1}{\Theta+1}\, x^{\Theta+1}$. The latter default event occurs if this loss exceeds initial equity.[4]

Proposition 3.2. If $\delta = 0$, an SNNR equilibrium exists if and only if:

$$\frac{(1-t-e)}{2} \leq \frac{\Theta}{(\Theta+1)} \text{ and } e \geq \frac{1}{(\Theta+1)}\,\frac{(1-t-e)^{\Theta+1}}{2}$$

The proposition can be verified by again establishing the strategic game payoffs and showing under which circumstances the SNNR conditions are met. The proof has been provided elsewhere (Bindseil 2013). In sum, to ensure financial stability in the case of absence of central bank credit, minimum liquidity and capital buffers are needed in some appropriate

[4] Note that it is assumed that equity is never sufficient to absorb the losses resulting from a bank default; that is, it is assumed that $e \leq 1/(\Theta + 1)$. Of course, one could also calculate through the opposite case, but it is omitted here as it does not seem to match reality.

combination to ensure the stability of a given amount of short-term funding. The lower the asset liquidity, the lower the amount of short-term funding that can be sustained for a given level of equity.

4. Cases in Which the Banks Rely on Both Types of Liquidity Buffers

Now we consider the cases in which both $\Theta, \delta > 0$. It is assumed that the ordering of assets is the same for both forms of liquidity generation; that is, if asset i is subject to lower fire sale discounts than asset j, then also asset i will have a lower central bank collateral haircut than asset j. Proposition 4.1 narrows the actual range of mixed cases, or cases in which both liquidity sources play a role in the planning of the bank.

> **Proposition 4.1.** If either $\delta \geq \Theta$, or both $\Theta > \delta$ and $\delta/(\delta + 1) \geq (1- t- e)/2$ are satisfied, then banks will only rely on central bank credit to address possible deposit withdrawals, and hence the conditions established in Proposition 3.1 apply to the existence of an SNNR equilibrium.

Taking recourse to the central bank does not cause a loss, while fire sales cause one. If, in addition, central bank recourse yields more liquidity (that is, $\delta > \Theta$), then central bank credit strictly dominates asset fire sales as a source of emergency liquidity. If $\Theta > \delta$ and central bank liquidity buffers allow us to address liquidity outflows relating to one depositor, that is, $\delta/(\delta + 1)$ $(1- t- e)/2$, which, as shown previously, allows us to sustain the SNNR equilibrium, then again relying only on central bank credit dominates strategies that rely on both sources.

The cases in which the bank wants to rely potentially on both funding sources therefore appear to be limited to the ones in which $\Theta > \delta$ and $(1- t - e)/2 > \delta/(\delta + 1)$. Again, a number of cases have to be distinguished. There will generally be a trade-off between the maximum liquidity generation and the ability to avoid losses, under the optimal use of the two funding sources. For example, the maximum generation of liquidity is achieved through fire sales only and will be equal to $\Theta/(1 + \Theta)$. However, this also leads to the highest possible fire sales losses and damage to equity, $1/(1 + \Theta)$, and it is realistic to assume that this extent of losses would exceed equity. Regardless, if all the assets of the bank are sold, it has ceased to exist. The lowest generation of liquidity is achieved if all assets are pledged through central bank credit, and in this case, liquidity generation is $\delta/(1 + \delta)$ and fire sale losses are 0. Between these two extreme pairs of liquidity generation and fire sale losses, the set of efficient combinations of the two can be calculated. The following proposition addresses the question of whether the bank's strategy should foresee the need to fire sale the most liquid assets and pledge the rest with the central bank, or the other way around.

> **Proposition 4.2.** In funding strategies to address withdrawals of short-term deposits rely on both funding sources, the bank should always foresee the need to fire sale the most liquid assets and pledge the less liquid assets.

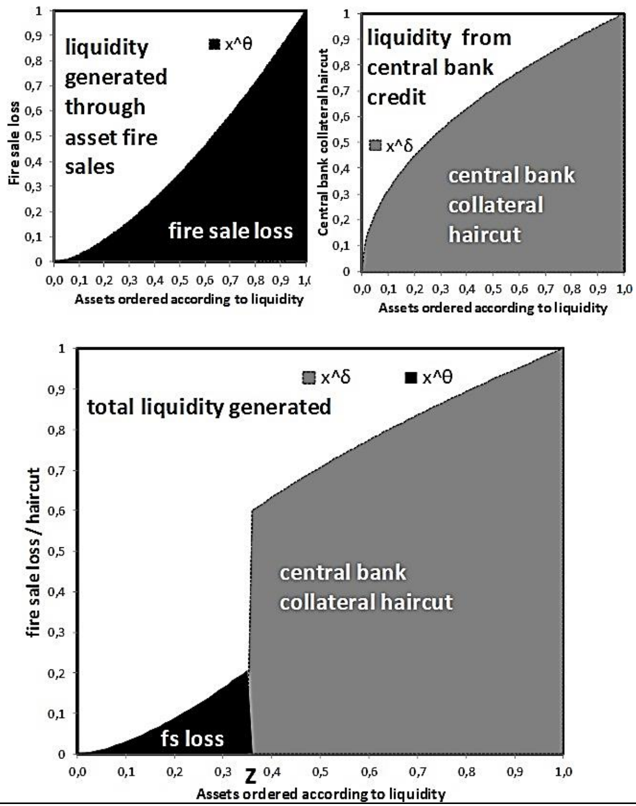
The proof of this proposition is provided in Bindseil (2013). The proof relies on showing that with the strategy to fire sale the most liquid assets and pledge the rest, the bank can achieve combinations of liquidity generation and fire sale cost, which are always superior to the combinations under the reverse strategy. The following Proposition 4.3 provides the condition in the case of strategies relying on both funding sources for an SNNR equilibrium, depending on the initial liability structure of the bank and the parameters Θ and δ.

Proposition 4.3. Let $z \in [0, 1]$ determine which share of its assets is foreseen by the bank to be used for fire sales (that is, the less liquid share $1 - z$ of assets is foreseen for pledging with the central bank). Let $k = h(z)$ be the fire sale losses from fire selling the z most liquid assets and let $y = f(z)$ be the total liquidity generated from fire selling the most liquid assets z and from pledging the least liquid assets $1 - z$. Then, an SNNR equilibrium exists if and only if $\exists z \in [0, 1]$, such that:

$$y = f(z) = \frac{\delta}{(\delta+1)} + \frac{z^{\delta+1}}{(\delta+1)} - \frac{z^{\Theta+1}}{(\Theta+1)} \geq \frac{(1-\mathrm{t}-e)}{2}, \qquad k = \frac{z^{\Theta+1}}{\Theta+1} \leq e$$

The proof of this proposition is provided Bindseil (2013). Figure 3 illustrates the generation of liquidity and fire sale losses under strategy z. The figure reflects that the bank plans to fire sale the most liquid part of its assets z, and pledge with the central bank the least liquid part of assets $1 - z$. Therefore, total liquidity y that could be generated corresponds to the sum of y_1, the surface above the fire sale loss curve x^{Θ} up to z, and y_2, the surface above the haircut curve x^{δ}, starting at z. Fire sale losses k will be equal to the surface below the fire sale loss curve between 0 and *z.*

Figure 4: Liquidity Generation and Fire Sale Losses If Most Liquid Assets (from 0 to z) Are Sold, while Least Liquid Assets (from z to 1) Are Pledged with Central Bank



Source: Created by the authors.

5. Stable Funding Structure with the Lowest Possible Cost

In the previous sections, it was assumed that the initial bank balance sheet was given and the conditions for stability of short-term funding were established. It was shown that, depending on the bank's liability structure, the haircut δ, and asset liquidity *Θ*, short-term bank funding was stable or not. This section makes the liability structure endogenous in a simple setting. It is assumed that different liabilities require different remuneration rates but are at these rates perfectly elastic. For given, deterministic *δ* and *Θ*, competing banks can be assumed to choose the cheapest possible liability structure as determined by the conditions in the strategic depositor game, such that the single no-run equilibrium applies. Assume that the cost of remuneration of the three asset types are r_e for equity, r_t for term funding, and 0 for short term deposits. Also assume that $r_e > r_t > 0$, and that $\Theta > \delta$. In this setting, what will be the composition of the banks' liabilities? The objective of the liability structure will be to minimize the average funding cost subject to maintaining a stable short-term funding basis.

- One strategy could be to rely only on central bank credit and thus aim to have $\delta/(\delta + 1) \geq (1 - e - t)/2$, such that fire sales will not be needed at all as backstop. If fire sales are not needed, then term funding is superior to equity and equity will be set to zero; that is, liabilities will consist only in term funding t and short-term deposits s = 1 – t. Therefore, the condition for stable short-term funding will be $\delta/(\delta + 1) \geq (1\ t)/2$, which implies $t^* = 1 - 2\delta/(\delta + 1)$. The average remuneration rate of bank funding will be $t^* r_t$.

- A second strategy would be to rely only on the fire sales approach but to hold the necessary equity. This would mean that the two minimum conditions to be fulfilled are $z - z^{\Theta}/(1 + \Theta) \geq (1 - t - e)/2$ and $e \geq (z^{(1+\Theta)}/(1 + \Theta)$. The funding costs $tr_t + er_e$ can be minimized subject to these two constraints to obtain a unique optimum $t^*(\Theta, r_e, r_t)$ and $e^*(\Theta, r_e, r_t)$, to obtain the minimum average funding costs of the bank liabilities $t^* r_t + e^* r_e$. This case corresponds to $\delta = 0$ in the following more general formulation.

- A third, general strategy is to rely on both sources of funding, to plan to fire sell the most liquid assets from 0 to z, and to pledge the least liquid assets from z to 1, as described in Propositions 4.2 and 4.3 and in the following optimization problem.

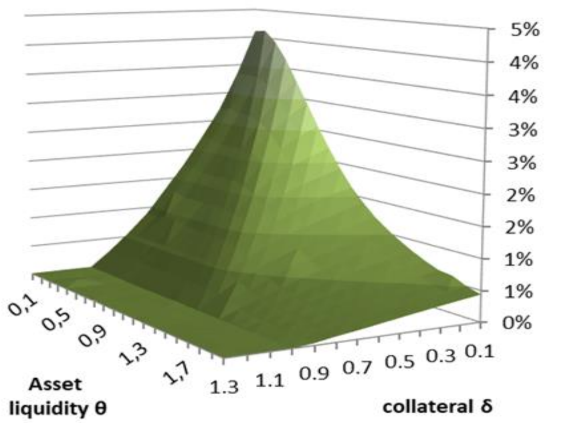
Problem 5.1. Suppose a bank relies on both sources of liquidity. The general problem of optimal liquidity management is to minimize through the choice of t, e, z ∈ [0, 1], with t + e ∈ [0, 1] the average remuneration rate of the banks' liabilities $t{*}r_t + e{*}r_e$, and with parameters subject to the two constraints:

$$\frac{\delta}{(\delta + 1)} + \frac{z^{\delta + 1}}{(\delta + 1)} + \frac{z^{\Theta + 1}}{(\Theta + 1)} \geq \frac{(1 - t - e)}{2} \tag{4}$$

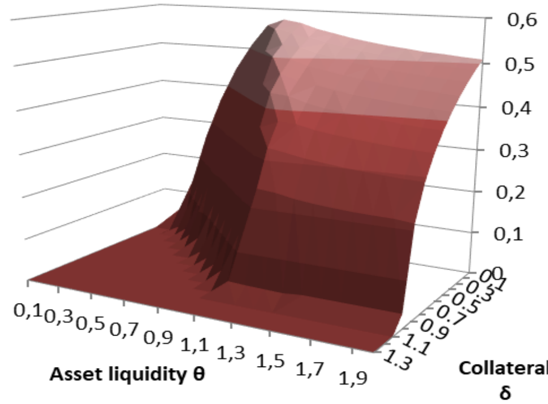
$$e \ \geq \ \frac{z^{\Theta + 1}}{(\Theta + 1)} \tag{5}$$

We provide the mathematical solution to this optimization problem in the Appendix. Figure 4 illustrates the functional relationships among the given parameters (Θ, δ, r_e, r_t), and the optimum values of $e{*}$, $t{*}$, $s{*}$, $z{*}$, and $r{*}$ (with $s{*} = 1 - e{*} - t{*}$ being the implied share of short-term funding). Key findings are: the minimum bank intermediation cost $r{*}$ falls monotonously in Θ and in δ before reaching zero (Figure 4.a). Both the share of assets foreseen for fire selling, $z{*}$, and the equity ratio fall monotonously in δ, but both first increase and then decrease again in Θ (unless δ is high enough to allow for $z{*} = 0$ and $e{*} = 0$) (Figure 4.b and 4.c). The share of long-term debt, $t{*}$, falls monotonously in Θ and in δ before reaching zero (Figure 4.d). The share of short-term deposits, s∗, increases monotonously in Θ and in δ before reaching 100% (Figure 4.e).

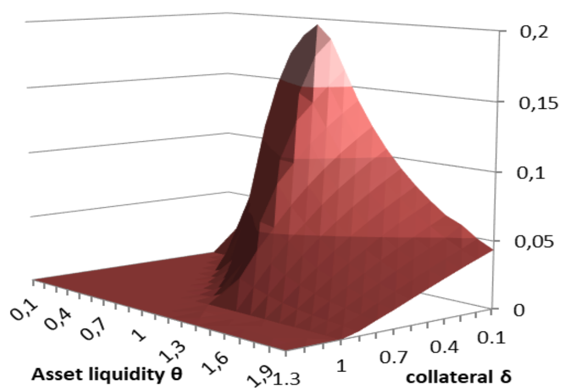
Figure 5: Minimum Funding Costs (r^*), Cost-Minimizing Liquidity-Generating Strategy (z), and Liability Mix (e^*, t^*, s^*), as Functions of δ and Θ, for r_t = 5%, r_e = 10%



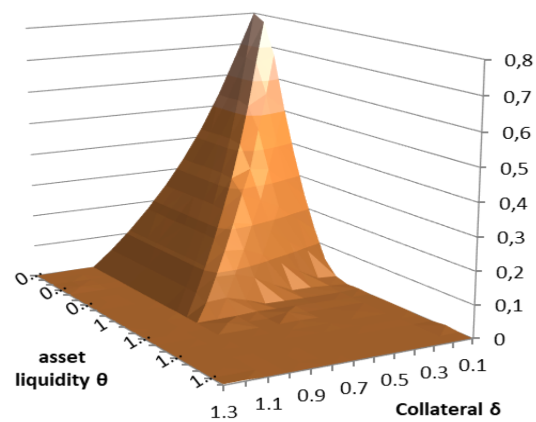
a. Minimum bank intermediation cost r^* as function of θ and δ



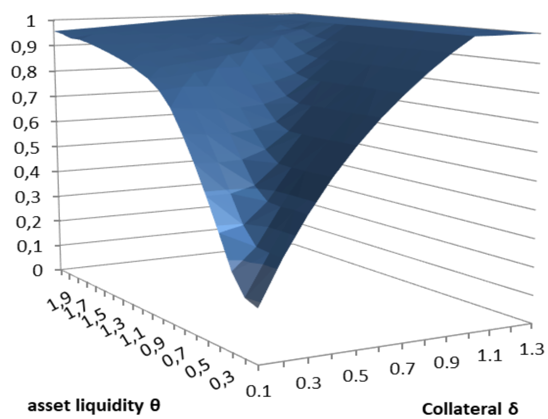
b. Funding-cost minimizing share of assets foreseen for fire sales, z^*, as function of θ, δ



c. Funding-cost minimizing share of equity funding, e*, as function of θ and δ



d. Funding-cost minimizing share of long-term funding, t^*, as function of θ and δ

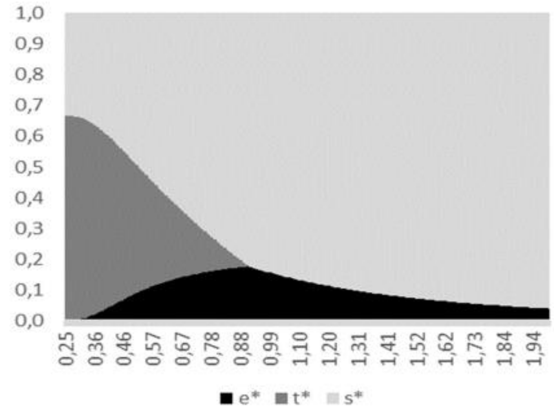


e. Funding-cost minimizing share of short-term funding, s^*, as function of θ and δ

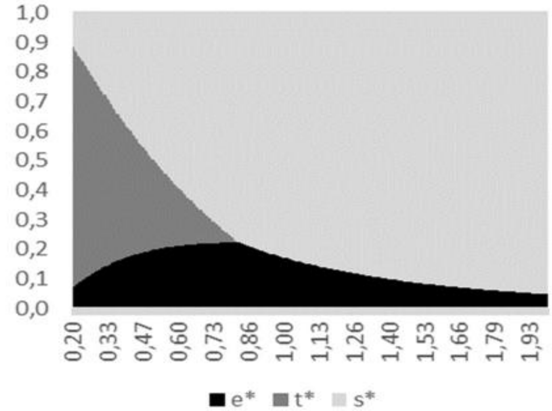
Source: Created by the authors.

Figure 6.a shows how the liability mix minimizing the funding cost evolves as a function of asset liquidity Θ, for given collateral liquidity δ = 0.2 (and again r_t = 5%, r_e = 10%). It illustrates that the optimal equity share is not a monotonous function of the asset liquidity but reaches a maximum for about Θ = 0.9. In contrast, the optimal long- and short-term funding shares decline monotonously with an improving asset liquidity (Figure 6.b shows the same relations, but for δ = 0). Figure 6.c illustrates how the liability mix minimizing funding cost evolves as function of the equity premium r_e, for a given term funding premium r_t = 2%, for given (Θ, δ) = (0.5, 0.2). The equity share $e*$ falls monotonously in the equity premium r_e, while the share of long-term debt $t*$ increases monotonously in r_e. The share of short-term funding $s*$ also declines monotonously in r_e. Figure 6.d shows the same relationships for δ = 0.

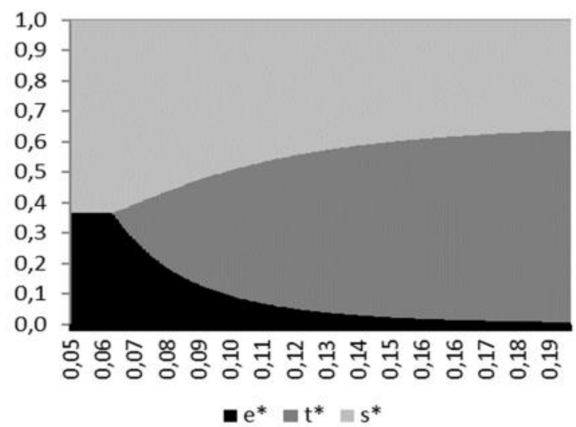
Figure 6: Cheapest Stable Funding Mix (e^*, t^*, s^*), Depending on Asset Liquidity, Access to the Central Bank, and Relative Size of Equity Premium



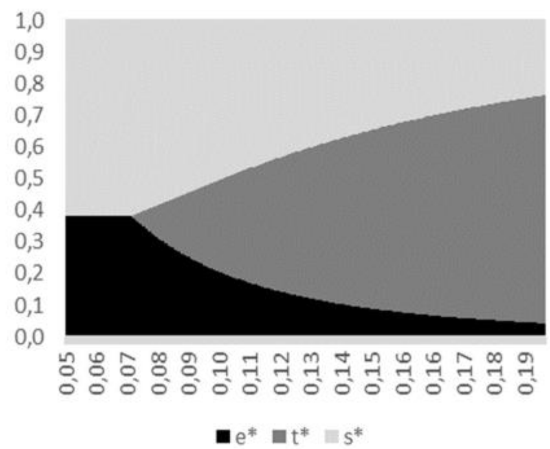
a. (e^*, t^*, s^*) as functions of Θ, for $\delta = 0.2$; $r_t = 5\%$, $r_e = 10\%$. X-axis: value of Θ; Y-axis: liability composition



b. (e^*, t^*, s^*) as functions of Θ, for $\delta = 0$; $r_t = 5\%$; $r_e = 10\%$. X-axis: value of Θ; Y-axis: liability composition



c. (e^*, t^*, s^*) as functions of r_e, for $r_t = 2\%$, and (Θ, δ) = (0.5, 0.2). X-axis: value of r_e; Y-axis: liability composition



d. (e^*, t^*, s^*) as functions of r_e, for $r_t = 2\%$, and (Θ, δ) = (0.5, 0). X-axis: value of r_e; Y-axis: liability composition

Source: Created by the authors.

6. The Central Bank Collateral Framework as a Policy Tool

Although the primary purpose of the central bank collateral framework is risk protection, the observed collateral policy measures of central banks during the GFC and in 2020 raise the question of what exactly the intentions of the central banks have been to widen collateral availability (and hence potential central bank recourse), in particular in a context of deteriorating asset liquidity. The model proposed in this paper allows interpreting the relaxation of the collateral framework as a policy measure:

(1) First, when Θ (asset liquidity) suddenly declines, increasing δ (by decreasing haircuts and broadening collateral eligibility) is a way to preserve the no-run equilibrium (the SNNR) and thereby is a necessary condition to prevent increases in central bank reliance, fire sales, and/or defaults. In this sense, it benefits all banks and financial stability in general, and not only those banks that already experience a run. Moreover, it may be noted that the model provides support to Bagehot's "inertia principle," according to which, the central bank should not tighten its collateral framework in a financial crisis as a reaction to the deterioration of asset liquidity: "If it is known that the Bank of England is freely advancing on what in ordinary times is reckoned a good security—on what is then commonly pledged and easily convertible—the alarm of the solvent merchants and bankers will be stayed . . . " (Bagehot 1873). Lowering δ when Θ declines anyway would mean decreasing particularly strongly the amount of sustainable short-term funding and thereby maximizing the probability of a destabilization of bank funding, contributing, instead of preventing, large central bank recourse and fire sales of assets.

(2) Second, assuming that a deterioration of Θ can be anticipated as a crisis is building, one could imagine that banks can adjust their liability structures in time. It would come at a high cost because, in such a context, investors also will have a strong preference for short-term assets and the collective attempt of all banks to increase the maturity of their liabilities will therefore lead to a steep increase of bank funding costs (and hence of bank lending rates). This would be procyclical, and an adjustment of the collateral framework parameter δ could be seen as a policy tool to prevent such a steep increase of funding costs.

(3) Third, although the effect described in the previous point could in theory also be addressed by conventional monetary policy, that is, a lowering of central bank interest rates, this has limits as far as the zero lower bound is reached (as it was the case for most central banks in 2020). When this limit is reached, a widening of collateral availability may become relevant as an alternative approach to lowering effective bank funding costs or at least preventing their increase.

In the context of the model, assume that (Θ_1, δ_1) are the pre-crisis asset parameters and (for a given equity and term funding premia r_t, r_e) the minimum funding cost is $r_1^* = r^*(\Theta_1, \delta_1)$, and the related liability composition is $e_1^* = e^*(\Theta_1, \delta_1)$, $t_1^* = t^*(\Theta_1, \delta_1)$, $s^*_1 = s^*(\Theta_1, \delta_1)$. Assume that a market liquidity crisis materializes with Θ shifting from Θ_1 to $\Theta_2 < \Theta_1$. This implies that $r_2^* = r^*(\Theta_2, \delta_1) > r_1^*$. Moreover, for (Θ_2, δ_1), the funding structure e^*_1, t^*_1, s^*_1 does not allow for a single no-run equilibrium, but the banks are now in the multiple equilibrium case in which a bank run can occur. Now call δ_2 the value of δ for which $r^*(\Theta_2, \delta_2) = r_1^*$. In other words, and assuming that the central bank is constrained by the zero lower bound, the collateral framework δ_2 is the one that allows us to restore the monetary conditions (in the sense of the bank funding costs, and thus the bank lending rates) that prevailed before the liquidity crisis—however, only under the assumption that the banks can rapidly adjust their funding structures toward $e^*_2 = e^*(\Theta_2, \delta_2)$, $t^*_2 = t^*(\Theta_2, \delta_2)$, $s^*_2 = s^*(\Theta_2, \delta_2)$. Then, monetary conditions and financial stability have been restored. Alternatively, the central bank may immediately want to restore the single no-run equilibrium and not take the risk of a run in the possibly lengthy process of the adjustment of banks' liability structures. This will require choosing another value of δ, δ_2. Call δ_3 the collateral framework that is obviously sufficient to restore immediately a no-run equilibrium, in the sense that $e^*_1 \le e^*(\Theta_2, \delta_3)$, $t^*_1 \le t^*(\Theta_2, \delta_3)$, $s^*_1 \ge s^*(\Theta_2, \delta_3)$. However, δ_3 is not strictly necessary for restoring a single no-run equilibrium. Call δ_4 the smallest value of δ for which the funding structure (e^*_1, t^*_1, s^*_1) implies a no-run equilibrium for (Θ_2, δ_4), that is, for which the sufficient equity condition (5) and the sufficient liquidity condition (4) are both fulfilled, whereby banks can of course recalibrate the parameter z without any delay.

In what follows, we refer the reader to the example Figure 7, with r_t = 5% and r_e = 10%. Short-term funding costs are set at 0%, consistent with the idea that the zero lower bound is constraining. Figure 7 shows, for these given funding cost parameters, how the optimum values r^*, z^*, e^*, t^*, and s^* depend on the asset fire sale liquidity parameter Θ and the collateral framework parameter δ.

Assume that (Θ_1, δ_1) = (0.7, 0.2) and that the banks have chosen the cheapest sustainable funding structures. Following Figure 7, we have (e^*_1, t^*_1, s^*_1) = (67%, 15%, 19%), to obtain funding costs of r_1* = 2.5%. Let the crisis-related asset liquidity be Θ_2 = 0.2. The table indicates that δ_2 = 0.4, as $r^*(\Theta_2, \delta_2)$ = 2.2% ≤ r_1^*. A more radical move by the central bank to δ_3 = 0.7 also obviously and immediately solves the issue of the bank run, as the optimal and thus sustainable funding mix for (Θ_2, δ_3) = (0.2, 0.7) is less demanding for sure than the pre-crisis funding mix, since the table indicates that $e^*_1 \le e^*(\Theta_2, \delta_3)$, $t*_1 \le t^*(\Theta_2, \delta_3)$, $s^*_1 \ge s^*(\Theta_2, \delta_3)$. Finally, one may check if δ = 0.6 or even δ = 0.5 would also ensure the fulfilment of the sufficient equity and sufficient liquidity conditions (5) and (4). Note that for (Θ_1, δ_1), z^* = 44%, while for (Θ_2, δ_3), like for any (Θ, δ) with $\Theta \le \delta$, z^* = 0. It turns out that the bank simply needs to set z = 0 and both δ_4 = 0.6 and δ_4 = 0.5 become sufficient from a funding stability perspective, while δ_4 = 0.4 is indeed insufficient. Therefore, to immediately restore both an accommodative monetary policy stance and financial stability, after the liquidity crisis (after the deterioration of Θ from 0.7 to 0.2), the central bank in this case should move its collateral framework δ from 0.2 to 0.5. If this would be too accommodating from the monetary policy perspective, then the central bank could of course raise the short-term risk-free interest rate.

Figure 7: Optimum Values of *r**, *s**, *e**, *t**, and *z** for δ and θ Each Taking Values {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8}, in %, with r_t = 5% and r_e = 10%

*r** (minimum funding cost)	δ = 0.1	δ = 0.2	δ = 0.3	δ = 0.4	δ = 0.5	δ = 0.6	δ = 0.7	δ = 0.8
θ = 0.1	4,1%	3,3%	2,7%	2,2%	1,7%	1,3%	0,9%	0,6%
θ = 0.2	4,1%	3,3%	2,8%	2,2%	1,7%	1,3%	0,9%	0,6%
θ = 0.3	3,9%	3,3%	2,7%	2,2%	1,7%	1,3%	0,9%	0,6%
θ = 0.4	3,7%	3,2%	2,7%	2,2%	1,7%	1,3%	0,9%	0,6%
θ = 0.5	3,3%	3,0%	2,6%	2,1%	1,7%	1,3%	0,9%	0,6%
θ = 0.6	2,9%	2,7%	2,4%	2,1%	1,7%	1,3%	0,9%	0,6%
θ = 0.7	2,6%	2,4%	2,2%	2,0%	1,6%	1,3%	0,9%	0,6%
θ = 0.8	2,2%	2,1%	1,9%	1,8%	1,5%	1,2%	0,9%	0,6%
*s** (short-term funding ratio)	δ = 0.1	δ = 0.2	δ = 0.3	δ = 0.4	δ = 0.5	δ = 0.6	δ = 0.7	δ = 0.8
θ = 0.1	18%	33%	46%	57%	67%	75%	82%	89%
θ = 0.2	20%	33%	46%	57%	67%	75%	82%	89%
θ = 0.3	27%	34%	46%	57%	67%	75%	83%	89%
θ = 0.4	38%	40%	47%	57%	67%	75%	83%	89%
θ = 0.5	49%	49%	51%	58%	67%	75%	82%	89%
θ = 0.6	58%	58%	58%	61%	67%	75%	82%	89%
θ = 0.7	67%	67%	66%	67%	69%	75%	82%	89%
θ = 0.8	75%	74%	74%	74%	75%	77%	82%	89%
*e** (equity ratio)	δ = 0.1	δ = 0.2	δ = 0.3	δ = 0.4	δ = 0.5	δ = 0.6	δ = 0.7	δ = 0.8
θ = 0.1	0%	0%	0%	0%	0%	0%	0%	0%
θ = 0.2	2%	0%	0%	0%	0%	0%	0%	0%
θ = 0.3	6%	0%	0%	0%	0%	0%	0%	0%
θ = 0.4	11%	4%	1%	0%	0%	0%	0%	0%
θ = 0.5	15%	9%	3%	1%	0%	0%	0%	0%
θ = 0.6	17%	12%	7%	4%	0%	0%	0%	0%
θ = 0.7	19%	15%	11%	7%	2%	0%	0%	0%
θ = 0.8	20%	16%	13%	9%	5%	1%	0%	0%
*t** (long-term funding ratio)	δ = 0.1	δ = 0.2	δ = 0.3	δ = 0.4	δ = 0.5	δ = 0.6	δ = 0.7	δ = 0.8
θ = 0.1	82%	67%	54%	43%	33%	25%	18%	11%
θ = 0.2	78%	67%	54%	43%	33%	25%	18%	11%
θ = 0.3	68%	66%	54%	43%	33%	25%	17%	11%
θ = 0.4	51%	56%	53%	43%	33%	25%	17%	11%
θ = 0.5	37%	42%	46%	42%	33%	25%	18%	11%
θ = 0.6	24%	30%	34%	35%	33%	25%	18%	11%
θ = 0.7	14%	19%	23%	26%	29%	25%	18%	11%
θ = 0.8	5%	9%	13%	17%	20%	22%	17%	11%
*z** (assets for fire sales)	δ = 0.1	δ = 0.2	δ = 0.3	δ = 0.4	δ = 0.5	δ = 0.6	δ = 0.7	δ = 0.8
θ = 0.1	0%	0%	0%	0%	0%	0%	0%	0%
θ = 0.2	5%	0%	0%	0%	0%	0%	0%	0%
θ = 0.3	13%	2%	0%	0%	0%	0%	0%	0%
θ = 0.4	26%	13%	3%	0%	0%	0%	0%	0%
θ = 0.5	36%	26%	13%	4%	0%	0%	0%	0%
θ = 0.6	44%	36%	26%	16%	4%	0%	0%	0%
θ = 0.7	51%	44%	36%	27%	13%	3%	0%	0%
θ = 0.8	56%	51%	44%	36%	26%	13%	3%	0%

Note: Numbers in gray fields are those referred to in text.

Source: Created by the authors.

7. Conclusion

This paper provides a model of the role of the central bank collateral framework for the LOLR, bank intermediation costs, banks' optimal funding structure, and monetary policy at the zero lower bound. Its innovation relative to the existing literature consists in specifying asset liquidity and the collateral framework as continuous functions in the unit interval, which allows building an integrated model encompassing all these dimensions and deriving an analytical solution of this model. We have shown how broadening the collateral framework, such as a number of central banks did in, for example, during the GFC and in 2020, can be considered to restore financial stability and adequate monetary conditions after a negative asset liquidity shock. Bank asset liquidity and the central bank collateral framework jointly determine financial stability and the ability of the banking system to deliver maturity transformation, which is one of its key functions. The model also shows that a widening of the central bank collateral set can prevent large recourse to central bank credit by banks suffering from a deterioration of asset liquidity. In this sense, the paper provides further illustration of Bagehot's conjecture that only the "brave plan" of the 19th century Bank of England would be a "safe" plan. In other words, by being "brave" and increasing δ after an exogenous drop of Θ, the central bank will preserve the banks' funding stability and thereby minimize the recourse of stressed banks to its credit facilities, and hence be on the "safe" side also in terms of financial exposures. The model also allows us to identify the impact of asset liquidity and of the central bank collateral framework on funding costs of banks and thereby on monetary policy conditions: first, policymakers need to be aware that a tightening of any of the two emergency liquidity sources of banks needs to be, everything else unchanged, compensated by a lowering of the monetary policy interest rate to maintain unchanged funding costs of the real economy. Second, when the central bank has reached the zero lower bound, and therefore cannot use standard interest rate policies any longer, it can consider using its collateral framework to counteract a further increase of actual funding costs of banks (and hence of the real economy depending on banks), which would otherwise result from the deteriorated asset liquidity because of the implied more expensive funding structure.

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9. Appendix: Solution to Problem 5.1

The problem we want to solve consists of minimizing the function $r: (t, e)$

$r_t t + r_e e$ for given parameters $r_e \geq r_t$, over the subspace of triples $(t, e, z) \in [0,1]^3$, subject to the following constraints:

(1) $\frac{\delta}{\delta+1} + \frac{z^{\delta+1}}{\delta+1} - \frac{z^{\Theta+1}}{\Theta+1} \geq \frac{(1-t-e)}{2}$

(2) $e \geq \frac{z^{\Theta+1}}{\Theta+1}$

(3) $t + e \leq 1$

where $\Theta > \delta \geq 0$ are given parameters.

Let $W \subset [0,1]^3$ be the domain of admissible triples (t, e, z). Clearly, W is a compact subspace of R^3; therefore, $r: W \rightarrow R$ admits a minimum. Since the gradient $\nabla r = (r_t, r_e, 0)$ is non-vanishing everywhere on W, the minimum point lies on the boundary ∂W. We now analyze the behavior of r over a decomposition of such a boundary. Note that $\partial W = \cup^5{}_{i=1} W_i$, where we set:

(1) $W_1 = \{(t, e, z) \in W: e = 0, \text{or } t = 0, \text{or } z = 0\}$

(2) $W_2 = \{(t, e, z) \in W: e = 1, \text{or } t = 1, \text{or } z = 1\}$

(3) $W_3 = \{(t, e, z) \in W: t + e = 1\}$

(4) $W_4 = \{(t, e, z) \in W: e = \frac{z^{\Theta+1}}{\Theta+1}\}$

(5) $W_5 = \{(t, e, z) \in W: \frac{\delta}{\delta+1} + \frac{z^{\delta+1}}{\delta+1} - \frac{z^{\Theta+1}}{\Theta+1} = \frac{(1-t-e)}{2}\}$

Let's study r on W_1. If $e = 0$, then necessarily $z = 0$, which implies:

$$\frac{\delta}{\delta+1} \geq \frac{(1-t)}{2}$$

If $\delta \leq 1$, we minimize W by choosing $(t^*, e^*, z^*) = \frac{1-\delta}{1+\delta}, 0,0$, which yields $r^* = r_t \frac{1-\delta}{1+\delta}$. Note that in this case, $t \leq 1$, and equality holds if and only if $\delta = 1$. If instead $\delta > 1$, then $(t^*, e^*, z^*) = (0,0,0)$ is admissible, so we obtain $r^* = 0$. From now on, we will assume $\delta < 1$ to avoid the trivial solution.

Suppose $t = 0$, then we set $e = \frac{z^{\Theta+1}}{\Theta+1}$, subject to the condition:

$$h(z) = \frac{\delta - 1}{\delta + 1} + 2\frac{z^{\delta+1}}{\delta + 1} - \frac{z^{\Theta\,+\,1}}{\Theta\,+\,1} \geq 0$$

We have $h(0) < 0, h(1) > 0$, and $h'(z) > 0$ for every $z \in [0,1]$. Therefore, $\exists!\, z^* \in [0,1]$ with $h(z^*) = 0$. By construction, z^* is the smallest value of z for which $0, \frac{z^{\Theta\,+\,1}}{\Theta\,+\,1}, z$ satisfies all constraints. Thus, we get a candidate triple $0, \frac{(z^*)^{\Theta\,+\,1}}{\Theta\,+\,1}, z^*$, which yields $r^* = r_e \frac{(z^*)^{\Theta\,+\,1}}{\Theta\,+\,1}$. Finally, $z = 0$ can be reduced to an already analyzed case.

On W_2 and W_3, it is easy to prove there is no better contribution from any possible configuration. Turning to W_4, we have $e = \frac{z^{\Theta\,+\,1}}{\Theta\,+\,1}$, and t is subject to the constraint:

$$t \geq \frac{1 - \delta}{1 + \delta} + \frac{z^{\Theta\,+\,1}}{\Theta\,+\,1} - 2\frac{z^{\delta+1}}{\delta + 1} = -h(z)$$

To minimize, we set $t = -h(z)$, and we study $r = \varphi(z) = -r_t h(z) + r_e \frac{z^{\Theta\,+\,1}}{\Theta\,+\,1}$. By solving $\varphi'(z) = 0$, we see that φ has a minimum at:

$$\bar{z} = \frac{(2r_t)^{\frac{1}{\Theta - \delta}}}{r_t + r_e}$$

If $h(\bar{z}) \leq 0$, we have a new candidate triple $(t^*, e^*, z^*) = -h(\bar{z}), \frac{\bar{z}^{\Theta\,+\,1}}{\Theta\,+\,1}, \bar{z}$. Finally, the case of W_5 can be reconducted to this last one, so the analysis is complete. The solution to the original problem is obtained by comparing the two (or three, depending on the cases) candidates selected by the algorithm we have just described.

Remark. If we fix δ and let Θ vary, we can observe that:

(1) $z^*(\Theta) \leq z^*(\Theta')$ if $\Theta > \Theta'$, where $z^*(\Theta)$ denotes the unique solution to $\frac{\delta-1}{\delta+1} + 2\frac{z^{\delta+1}}{\delta+1} - \frac{z^{\Theta\,+\,1}}{\Theta\,+\,1} = 0$ over $[0,1]$

(2) $\lim_{\Theta \to +\infty} z(\Theta) = 1$, for $z(\Theta) = \frac{(2r_t)^{\frac{1}{\Theta - \delta}}}{r_t + r_e}$

Thus, for $\Theta - \delta$ big enough, we obtain $\bar{z}(\Theta) > z^*(\Theta)$, which forces us to pick $(t^*, e^*, z^*) = 0, \frac{z^*(\Theta)}{\Theta+1}, z^*(\Theta)$ over $(t^*, e^*, z^*) = -h(\bar{z}), \frac{\bar{z}^{\Theta\,+\,1}}{\Theta\,+\,1}, \bar{z}$. On the other hand, for Θ close enough to δ, we have that $(t^*, e^*, z^*) = -h(\bar{z}), \frac{\bar{z}^{\Theta\,+\,1}}{\Theta\,+\,1}, \bar{z}$ is admissible, so that $(t^*, e^*, z^*) = 0, \frac{z^*(\Theta)}{\Theta+1}, z^*(\Theta)$ no longer runs among candidates for the minimization of r.