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Incorporating the Distribution of Income and the Duration of Life into a Measure of Economic Growth: Some New Suggestions

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INTRODUCTION

In the growing literature on the measurement of economic growth two important developments are certainly worth emphasizing. Firstly there have been several attempts of incorporating income distribution considerations into the computation of growth rates (e.g., Chenery *et al*., 1974). Secondly, there have been various suggestions of going beyond the measurement of income when determining the welfare of nations (e.g., Morris' and Liser's Physical Quality of Life Indicator).

The purpose of this paper is to propose computational methods which could improve the estimation of growth rates. In a first section dealing with the inclusion of income inequality in the measurement of growth, we suggest to apply Donaldson's and Weymark's (1980) generalization of the Gini index. In a second section which relates growth to changes in the duration of life we argue in favor of the use of a new indicator, called the Equivalent Length of Life or E.L.L. which emphasizes the problem of Inequality before Death and which can be computed in various ways, depending on how one views such an Inequality. The numerical examples in both sections are based on Canadian data.

I. Incorporating the Distribution of Income into a Measure of Economic Growth:

As Usher (1980) explains it, one can incorporate the Distribution of Income into a measure of growth in either of two ways. One may impose a social welfare function $W(y^1, y^2 . . . ., y^n)$ where y^i is the income of individual i, compute a socially equivalent income* y_E and observe its rate of change over time. y_E is essentially defined by the relation:

$$y_E = W^{-1} [W (y^1, y^2. . . ., y^n)]$$

It is usually assumed that the social welfare function is biased towards equality so that

$$W (\bar{y}, . . . \bar{y}) \geq W(y^1, . . . y^n)$$

where $\bar{y}$ is the average of the incomes $y^1 . . . y^n$. Naturally one does not generally specify to whom the welfare function applies (Parliament? dictator?. . .). Such a computation of y_E has been made for example by Sen (1976) who used a social welfare function derived from the Gini coefficient which amounts to weight incomes according to their **rank** order.

Another way of incorporating the distribution of income into a measure of economic growth is to imagine that one **asks** an individual to determine which certain equivalent income y^c would make him as happy as having an equal chance of enjoying the income of each person in the community $(y^1,. . .y^n)$. It has been shown (Usher, 1980) that, subject to certain assumptions, this certainty equivalent income y^c depends on a parameter ε which measures the elasticity of the utility of income with respect to income. Once an individual knows the value of his ε, he can use it to compute for example the certainty equivalent income in his community for two different years and derive the rate of growth of this

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* y_E is called the socially equivalent income because if everybody in the society earned this income y_E, the social welfare of the society would be the same as the one existing presently with an unequal income distribution $y^1 . . . . y^n$.

equivalent income. Such a computation has been made by Usher (1980) with Canadian data for the period 1965-1973 and it turns out that, depending on the value of ε chosen, the annual rate of economic growth varies from 2.73% to 5.18%. In this section we shall take essentially the first approach but the socially equivalent incomes that we compute are not derived from the ordinary Gini coefficient but from a generalization of the Gini index proposed recently by Donaldson and Weymark (1980). They have shown that the socially equivalent income y_E could be written as

$$y_E = \frac{\sum_{i=1}^{n} a_i \tilde{y}_i}{\sum_{i=1}^{n} a_i} \qquad \text{where } \{\tilde{y}_1, \tilde{y}_2 \ldots \tilde{y}_i \ldots \tilde{y}_n\}$$

is a sequence of ordered incomes and $\{a_1 \ldots a_i \ldots a_n\}$ are coefficients which may be expressed as functionsof i. The authors distinguished between two cases. In the first one the sequence $\{\tilde{y}_1 \ldots \tilde{y}_n\}$ is welfare ranked, that is $\tilde{y}_1 > \tilde{y}_2 > \ldots > \tilde{y}_n$ and then $a_1 < a_n < \ldots < a_n$. They proved that in this case the only function a_i which satisfies Dalton's (1920) principle of population (that is which gives an inequality measure which does not vary when the income distribution is replicated) is the function $a_i = i^\delta - (i-1)^\delta$ with $\delta > 1$ so that

$$y_E = \frac{\sum_{i=1}^{n} [i^\delta - (i-1)^\delta]\tilde{y}_i}{n^\delta} \qquad \text{where } n = \Sigma i .$$

When $\delta = 2$, y_E corresponds to the rank weighting procedure implicit in the use of the Gini index of inequality (Sen, 1976)

In the second case, the incomes are ill-fare ranked, that is

$$\hat{y}_1 \leq \hat{y}_n \ldots \leq \hat{y}_n$$

The authors then prove that the equally distributed equivalent level of income y_E can be written as

$$y_E = \frac{\sum_{i=1}^{n} b_i \hat{y}_i}{\sum_{i=1}^{n} b_i} \quad \text{where } b_1 > b_2 > \ldots > b_n$$

If Dalton's principle is to hold, b_i is of the form $i^\delta - (i-1)^\delta$ with $0 < \delta \leq 1$.

We have applied this computing procedure to Usher's (1980) data (Table 3.1, page 44) on Canada's family income distribution in 1965 and 1973, that is, we have estimated the equivalent incomes y_E for both years, for various values of δ, as well as the corresponding growth rates of y_E during the period 1965-1973. Naturally, as Usher himself warns us, one should be cautious in using such data and be aware of the assumptions required before making use of them. In particular one has to assume that, within each decile, all incomes are equal. Moreover as pointed out by Usher the apparent widening of the Canadian income distribution over the period could be only a consequence of institutional and demographic change such as changes in the age distribution of the population, in the labor force participation of women. . . which the data do not take into account. Finally, whereas Donaldson and Weymark's procedure is based on a symmetric social welfare function of the incomes of individuals, we applied it to family incomes but were not able to adjust the data (using for example a system of adult equivalents for children) for differences in family size.

Despite all these difficulties we believe that it may still be a useful exercise to apply the Donaldson and Weymark procedure to Usher's data. The results are presented in Table 1. When δ is < 1, incomes are illfare ranked so that the lower δ , the more weight one gives to low incomes. This is naturally true for 1965 as well as 1973. However, since in 1973, for a given set of weights, the share of high incomes was higher, (cf. Usher's data) we expect that in 1973 the overall weight (including weights and income shares) of high incomes decreased at a lower rate, as $\delta(<1)$ increases so that one should find that the rate of growth for the period 1969-73 should increase as $\delta(<1)$ increases. This is very clear in Table 1: when $\delta = 0.01$, the rate of growth of the equivalent income was 26.7%. When $\delta = 0.90$, it was 46.9%.

The relations are naturally inversed when incomes are welfare ranked. In this case ($\delta > 1$), the higher δ, the higher the relative weight of low incomes, this being true for both years. However since in 1973 low incomes, for a given weight, had a lower share than in 1965 we expect the rate of growth of the equivalent income to decrease as δ increases. In Table 1 we see that the rate of growth is equal to 42.2% when $\delta = 2$ (the case corresponding to the ordinary Gini measure) and to 24.4% only when $\delta = 70$. Evidently the highest growth rates are obtained when $\delta = 1$, which gives equal weight to all incomes.

To summarize we see that the growth rate is very sensitive to changes in incomes as well as to the way we rank and weigh them. If we choose the illfare ranking method, the weight we give to low incomes is independent of changes in the income distribution at high incomes levels; if we use the welfare ranking method, the weight we give to high incomes is independent of changes in the income distribution at low income levels.

This measure of growth assumes therefore that one's welfare depends not only on one's income but also on the income of others and this assumption has already been emphasized, among others by Duesenberry (1949), Easterlin (1972), Hirsch (1979), Layard (1980) and Runciman (1966).

There is another way of improving the measurement of growth and development. It would take into account not the distribution of incomes but changes in the duration of life. This possibility will be discussed in the following section.

II. Measuring Economic Growth by Imputing for Changes in the Duration of Life:

Such an attempt has been made by Usher (1980, chapter 11). His ingenious method proposes the following expression for real income inclusive of the imputation for longevity.

$$\hat{Y}(t) = Y(t) \left[\frac{L(t)}{L(b)}\right]^{\frac{1}{\beta}}$$

where Y(t) is real income at time t, $\hat{Y}(t)$ is corrected (for longevity changes) income and $L(t) = \sum_{j=0}^{n-1} \frac{S_{j+1}}{(1+r)^j}$,

S_j is the probability of surviving up to age j, n is the upper limit to the length of life, r is a rate of discount, b is the base year and β is the elasticity of annual utility with respect to consumption.

As explained by Usher, this method uses hypotheses similar to those of the expected utility approach (the probabilities here refer to the various possible lengths of life) but it assumes also separability insofar as the utility derived from a stream of consumption for a fixed length of life is equal to the sum of the discounted annual consumptions. Usher's "real income inclusive of the imputation for longevity" is therefore defined as "an amount of consumption such that a representative consumer who enjoyed that consumption in the year t and every subsequent year, together with the mortality rates of the base year, would be as well off as he would be with the actual sustainable consumption and the actual mortality rates of the year t."

Usher computed then $\hat{Y}(t)$ for 1926 and 1974, using 1961 as base year for various values of r and β, obtaining annual growth rates of $\hat{Y}(t)$ varying from 2.96% (r=5% and $\beta = 1$) to 7.27% (r=1%, $\beta = 0.05$). As pointed out by Usher the separability assumption may be quite unrealistic, since it places no value on longevity itself, independently of the level of consumption.

In fact it has been argued recently (Hicks and Streeten, 1979) that longevity measured by the life expectancy, was a better measure of development than any income related index since a good indicator of development should measure outputs, not inputs, and life expectancy could be considered as the best available output of development efforts.

Life expectancy, however, is an average, based on life table data, which weights equally all age groups. It has been suggested (Silber, 1980) to give different weights to various age groups, in order, in particular, to emphasize clearly the existence of Inequality before Death. In the income distribution literature two main approaches to the measurement of inequality have appeared. A measure of income inequality could be invariant to proportional increases of all incomes (the "rightist" approach) or to equal additions to all incomes (the "leftist" approach)

Similarly, using life tables, one could say that the inequality before death would not change if everybody's length of life increased by the same percentage or if it increased by the same number of years. This is the main basis of what we have called E.L.L., the Equivalent Length of Life, that is the duration of life which would give society the same level of welfare as the one it derives actually from an unequal distribution of life durations. If one is a "rightist", one could use Atkinson's view (1970) on inequality and then E.L.L. is defined as

$$x_E = [\frac{1}{n} \sum_{i=1}^{n} x_i^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}$$

where ε is a given parameter (the elasticity of utility with respect to the length of life), x_i is the average number of years lived by the individual belonging to category i (these categories are usually defined as "individuals dying between age a and a + 5")and n the total number of individuals in the population.

Similarly, if one is a "leftist," one would use Kolm's (1976) leftist measure called also the Kolm-Pollak measure (Blackorby and Donaldson, 1980) and then E.L.L. is defined as

$$x_E = -\frac{1}{\sigma} \ln [\frac{1}{n} \sum_{i=1}^{n} e^{-\sigma x_i}]$$

where σ is a parameter defining the marginal utility of life duration.

It should be clear that there are important differences between inequality of incomes and inequality before death. Inequalities of income are to a far greater degree inherited than inequalities in the duration of life. Secondly, it is not possible to redistribute life expectancy as one may redistribute income. Moreover at any moment of time, there are some people who have high incomes and there are some people who have low incomes but we are all alive. Finally, if we take the certainty equivalent approach risk aversion may vary with age, whereas Atkinson's and Kolm's measures assume respectively constant relative or absolute risk aversion.

Despite these problems we believe that the computation of an equivalent lentgh of life (E.L.L.) is a useful **exercise** , in particular when a social welfare function approach is taken (rather than the certainty equivalent method). It certainly makes sense for policy makers to prefer, for example, having two individuals dying at age 50 than having one die at age 15 and the other at age 85. Straight averaging (which is what life expectancy does) would seem in the second case to overstate the society's performance.

Naturally we assume here that society has a priori a preference towards equity but our approach still leaves room for other ethical choices since we postulate two possible views of inequality, one represented by homothetic social welfare functions (Blackorby and Donaldson, 1978), one by translatable social welfare functions (Blackorby and Donaldson, 1980). Atkinson's "rightist" measure obviously belongs to the first category, the Kolm-Pollak "leftist" one to the second. We may even use a social welfare function which is both homothetic and translatable, that would give us a measure of inequality before death which would be invariant to both equal absolute as well as proportional increases in the duration of life. Such a property has been shown to hold for Donaldson and Weymark's generalization of the Gini coefficient (1980).

Let us now present results of the computation of E.L.L. in Canada for the years 1926 and 1974, and of its rate of growth during this period, using Atkinson's and Kolm's approach (Table 2) as well as Donaldson and Weymark's generalization (Table 3). We notice in Table 2 that the rate of growth of E.L.L. is very sensitive to the way we view inequality (rightist versus leftist approach) as well as to the value we assign to the parameters used in Atkinson's and Kolm's function. When $\varepsilon = 0$, E.L.L. is simply the usual life expectancy. As we indicated elsewhere (Silber, 1980), E.L.L. is simply a generalized mean using as weighting function in one case $e^{-\alpha \ln x_i}$ (Atkinson's approach with $\alpha = \varepsilon - 1$), in the other $e^{-\alpha x_i}$ (Kolm's approach with $\alpha = \sigma$).

It appears, from what has been written in the first section of this paper, that when Donaldson's and Weymark's method is used, we are weighting the durations of life according to their ranking in the "life duration distribution" (life-table). This may seem a strange procedure because whereas in an income distribution an income i refers to a given individual i, in a life table a duration of life x does not refer to a specific individual x. However, we know that there will be in general an individual in the population who will live x years and x years only. By applying Donaldson's and Weymark's method to life tables we do not argue that status is conferred by ranking (as was argued for income distribution data). We just suggest that this method corresponds to another weighting procedure which has the interesting property of being equity oriented in the leftist as well as in the rightist sense (cf. supra). The results of these computations, as indicated, are presented in Table 3.

When δ was less than 1, we used the life table as it is usually published since it is classified by increasing duration of life. When δ was greater than 1, we simply reversed the order of the data. We also had to extend Donaldson's and Weymark's formula for the case where more than one individual has the same duration of life, but this was no problem since their method assumes that each income (here duration of life) is greater (if δ is >1, lower if $\delta < 1$) or equal to the following one. As expected E.L.L. increases with δ when the illfare ranking method is used ($\delta<1$) and decreases with δ when the welfare ranking method is used. For $\delta=1$, E.L.L. is naturally equal to the life expectancy. We observe also that since most of the mortality decrease concerned low age groups (infant mortality decrease), the rates of growth of E.L.L. during the period 1926-1974 are highest when a high weight is given to young age groups, that is for high values of δ when $\delta > 1$, and for low values of δ when $\delta < 1$.

Conclusion

In this paper we first reviewed the problem of incorporating income distribution data when measuring economic growth. We suggested the use of a computational method, proposed by Donaldson and Weymark, which is a generalization of the Gini index of inequality and which weights income according to their ranks.

Then we looked at the possibility of incorporating life duration changes, when measuring economic growth. This led us to propose the use of an indicator, the Equivalent Length of Life or E.L.L., which has quite interesting properties. Firstly, it is of the output, not of the input type, since the duration of life is the result of many factors. Secondly, it does not ignore distributional considerations since it uses the whole life table, not only the mean of the distribution (called life expectancy). Thirdly, the computation of E.L.L. is consistent with "ethical flexibility" insofar as it may be estimated using a relative (Atkinson's rightist) or absolute (Kolm-Pollak leftist) approach to inequality before death, or even a combination of both approaches (Donaldson's and Weymark's method). As a numerical example, Canadian life table data for the years 1926 and 1974 were used and they indicated clearly how different computed rates of growth could be, when based on different views of inequality.

Table 1: "Equally distributed equivalent levels of income" in Canada in 1965 and 1973, using Donaldson's and Weymark's method

Ranking procedure	Value of the parameter δ	Equivalent Income in 1965	Equivalent Income in 1973	Rate of Growth in 1965-1973 of the Equivalent Income
	0.01	317.4	402.2	26.7%
	0.05	452.8	601.4	32.8%
	0.10	612.1	836.5	36.7%
Ill-fare ranking	0.25	1032.5	1460.9	41.5%
	0.50	1583.8	2289.7	44.6%
	0.75	2003.6	2929.5	46.2%
	0.90	2210.6	3248.1	46.9%
Utilitarian rule*	1.00	2334	3439	47.3%
	2	1487.7	2115	42.2%
	3	1140.1	1574.9	38.1%
	4	941.5	1273.5	35.3%
	5	809.5	1078	33.2%
	7.50	613.5	797.2	29.9%
	10	506.4	649.2	28.2%
	15	396.7	501.5	36.4%
Welfare ranking	20	345	433.1	25.5%
	30	302.9	377.7	24.7%
	40	289.4	360	24.4%
	50	284.9	354	24.3%
	60	283.3	351.9	24.2%
	70	282.7	351.2	24.2%

*It should be clear that when = 1, $y_E = \bar{y}$ where $\bar{y}$ is the average income, hence the name given to the procedure.

TABLE 2: Measuring the rate of growth of the Equivalent Length of Life (E.L.L.) in Canada Between 1926 and 1974, using Atkinson's and Kolm's method

Atkinson's Social Welfare Function				Kolm's Social Welfare Function			
Value of the parameter ε	E.L.L. in 1926	E.L.L. in 1974	Growth rate of E.L.L between 1926 & 1974	Value of the parameter σ	E.L.L. in 1926	E.L.L. in 1974	Growth rate of E.L.L. between 1926 and 1974
-4	72.60	78.69	8.4%	-0.10	20.9	38.3	82.8%
-3	71.06	77.88	9.6%	-0.09	22.9	41.7	81.8%
-2	68.99	76.87	11.4%	-0.05	35.5	58.6	65.3%
-1	65.84	75.51	14.7%	-0.01	55.3	71.5	29.3%
0	59.60	73.35	23.1%	0.01	63.1	74.8	18.5%
0.25	56.65	72.48	27.9%	0.05	71.4	78.4	9.8%
0.50	52.31	71.27	36.3%	0.25	81.6	83.4	2.1%
0.75	45.50	69.36	52.4%				

TABLE 3: Measuring the rate of growth of E.L.L., using Donaldson's and Weymark's method

Ranking Procedure	Value of the parameter δ	Value of E.L.L. in Canada in 1926	Value of E.L.L. in Canada in 1974	Rate of growth of E.L.L during the period 1926-1974
Ill-fare ranking	0.25	24.36	38.63	58.6%
	0.50	40.61	57.49	41.6%
	0.75	51.75	67.42	30.3%
	0.90	56.74	71.14	25.4%
Utilitarian rule*	1.00	59.53	73.04	22.7%
Welfare ranking	2	44.69	64.77	44.9%
	3	35.09	58.75	67.4%
	4	28.20	54.15	92.0%
	5	22.99	50.45	119.4%

*Here obviously E.L.L. is simply the life expectancy, as computed also in Table 2 when $\varepsilon = 0$. (The slight differences in the results are a consequence of differences in rounding up.)

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