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On Prices, Fiat Money, Credit and Transferable Utility. Part I

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PART I

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(PART I)

Martin Shubik*

1. INTRODUCTION

In this paper it is shown that in a competitive economy the acceptance of a fiat money, or credit system is equivalent to accepting the postulate that there exists a "utility pill" commodity in which the preferences of each individual are linear and furthermore there exists a set of weights which supply the ratios for interpersonal comparisons of utility.

Possibly much of our insight into the abstract properties of money can be obtained by reflecting upon an old American saying and a New York anecdote. The first is: "In God we trust, all others pay cash." The second concerns an importer who purchased a consignment of sardines, he in turn sold them to a distributor and finally they were sold by a grocer to his customers who almost died of food poisoning. The complaints went up the line until they reached the importer. Upon being told what had happened he said to the distributor: "You fool those were trading sardines not eating sardines."

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* I am indebted to H. Scarf and L. Shapley for their discussions with me on this topic.

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2. The Need for Money or Credit

As long as there are differentials in individual time preferences and/or individual production possibilities there will be a need for some monetary or credit mechanism so that the gains of intertemporal trading can be realized. Mathematically the existence of credit is equivalent to the relaxation of the budget constraint every period.

Suppose that there are m commodities in an economy together with a special $m+1$ st commodity called "money" which "is a legal tender at its face value for all debts public and private". Suppose that at time t the prices in the economy are $p_{1,t}, p_{2,t}, \dots p_{m,t}$ and 1 for money. An individual k trades $y^k_{i,t}$ units of the i^{th} commodity. Without money or credit the condition:

$$(1) \qquad \sum_{i=1}^{m} p_{i,t}\, y^k_{i,t} = 0$$

must hold for every time period. With money or credit we have the condition:

$$(2) \qquad \sum_{i=1}^{m} p_{it}\, y^k_{it} \geq - x^k_{m+1,t}$$

where $x^k_{m+1,t}$ is the amount of money or credit held by the k^{th} individual at the start of the t^{th} time period.

We ignore transactions convenience, risk, uncertainty and other dynamic features of money. Our interest is concentrated upon understanding the intertemporal maximization or "storage of wealth" property of money.

3. Models of Trade with Different Financial Restrictions

3.1 The General Problem and Some Simplifications

In an appropriately general formulation of the market it is desirable to consider both production and trade and an infinite time horizon. Apart from enlarging the strategy space of the individuals and presenting a considerably more difficult maximization problem the presence of production does not appear to introduce any extra conceptual problems concerning money. The absence of a finite time horizon removes certain terminal period difficulties which might otherwise present themselves; however the infinite time horizon requires that care be taken in the selection of an appropriate bounded utility measure. In the subsequent sections the general model is discussed and is then specialized and several models are constructed to illustrate different financial restrictions.

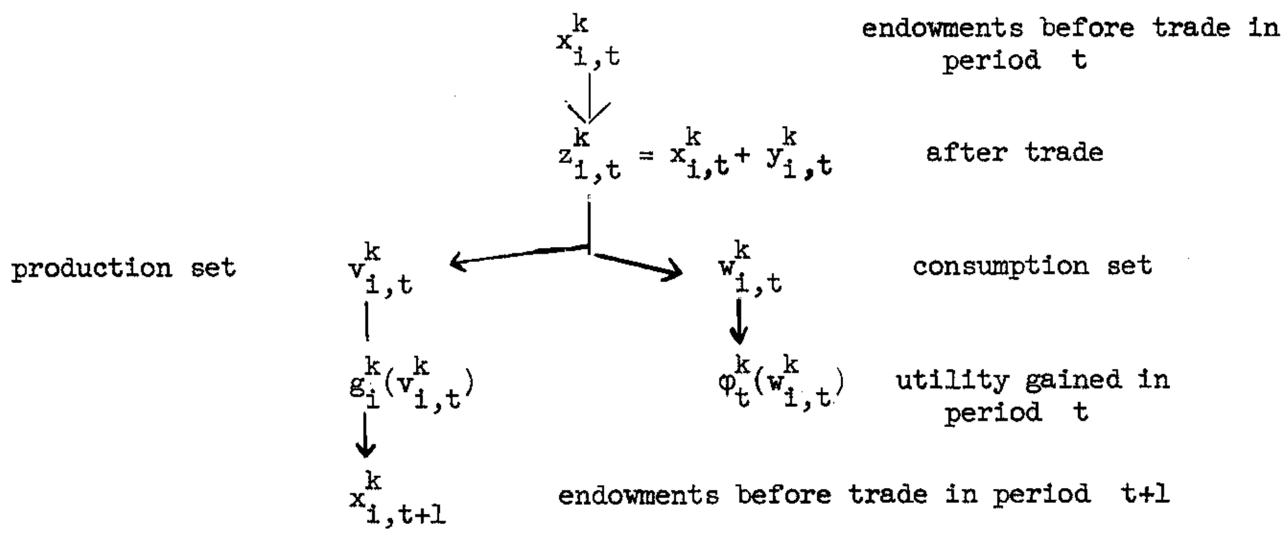
Let there be n individuals trading in m commodities. Assume that the preferences of the k^{th} individual for the consumption of the m commodities at time t are given by:

$$\varphi_t^k \left(w_{1,t}^k, w_{2,t}^k, \dots , w_{m,t}^k\right)$$

Suppose that the initial endowment of the k^{th} individual is $x_{1,1}^k, x_{2,1}^k, \dots x_{m,1}^k$. For every individual there is a production possibility set such that:

$$x_{i,t+1}^k \in g_i^k \left(v_{1,t}^k, v_{2,t}^k, \dots v_{m,t}^k\right)$$

We may consider a dynamic model in which the individuals first trade, after which they divide their resources into two sets, one of which they consume and the other they use to produce their new set of endowments for the next period.



If we consider an infinite trading period, the individual may attempt to maximize $\sum_{t=1}^{\infty} \varphi^k_t\ (w^k_{1,t}, \ldots w^k_{m,t})$ if it is bounded. Failing that it may be possible to maximize the average utility flow. We limit ourselves to a finite horizon τ , hence the individual may attempt to maximize:

(3) $$\sum_{t=1}^{\tau} \varphi^k_t\ (w^k_{1,t}, \ldots, w^k_{m,t}) .$$

At the end of the last period we may wish to specify a rule for the disposal and/or evaluation of any remaining intermediate goods which have no immediate value in consumption. If we abstract from production, the only intermediate goods with which we will be concerned are money and credit.

We abstract from production by assuming that at the start of each period a new endowment of the m goods $x^k_{1,t}, x^k_{2,t}, \ldots x^k_{m,t}$ appears for each individual k . All of the goods are assumed to enter directly into the utility function of each individual, and as a further simplification we assume that they have a life of only one period, i.e. they cannot be stored. During each period the individuals trade, then consume.

3.2 Barter

A strict barter economy inplies the lack of existence of not only fiat money and credit, but also forward contracts and other credit devices. Exchanges must be settled simultaneously with no further contractual agreements on the individuals once the exchange has taken place. If we assume the existence of a competitive market with prices $p_{i,t}$ in period t , the individual maximization problem can be expressed as:

$$\text{Maximize (3)} \quad U^k = \sum_{t=1}^{\tau} \varphi^k_t \left(w^k_{1,t}, w^k_{2,t}, \ldots w^k_{m,t}\right)$$

$$\text{subject to :} \qquad (1) \quad \sum_{i=1}^{m} p_{i,t}\, y^k_{i,t} = 0 \qquad \text{for } t = 1,2, \ldots \tau$$

This is equivalent to a series of τ independent maximizations as no opportunities are afforded to take advantage of intertemporal trades.

3.3 Forward Contracts

A natural relaxation of the barter conditions is direct barter arrangement through time in the form of a forward contract. Suppose forward contracts were only issued for a duration of r periods and only at the start of a new stretch of r periods. For example we might consider 90 day contracts issued only at the start of every new quarter.

The maximization model is the same as that for barter except the single constraint (1) is replaced by $\left[\frac{\tau}{r}\right] + 1$ constraints where $\left[\frac{\tau}{r}\right]$ is the integral part of this number. These conditions merely state that everyone's books must balance every r periods and also at the end at period τ.

This scheme allows for a limited advantage to be taken of trading through time. For example if the period were a year, the preferences of all individuals, the same, but there were seasonal production financing needs, the one year forward contract would suffice.

3.4 Fiat Money in Finite Supply

In order to introduce fiat money we specify the rules of a game for the n players. For the m commodities entering into the utility functions we assume the same supply conditions as before. However we introduce an $m + 1$ st commodity, fiat money which is given to all of the players at the first period in quantities $x^1_{m+1,1}, \ldots x^k_{m+1,1}, \ldots x^n_{m+1,1}$. The players are instructed that the fiat money is legal tender, there is no credit and no method for writing forward contracts available. At the end of the τ time periods, the players are each responsible to turn back to the referee as much fiat money as was issued to each. During the game fiat money is neither created nor destroyed. The referee defines the unit for the money and sets its price, by definition at 1. This is in keeping with such splendid prose as appears on various currencies; for example: "The United States of America will pay to the bearer on demand five dollars" is the inscription on a five dollar United States note.

The individual maximization problem becomes:

$$\text{Maximize (3)} \quad U^k = \sum_{t=1}^{\tau} \varphi_t^k (w_{1,t}^k, w_{2,t}^k, \ldots, w_{m,t}^k)$$

$$\text{subject to :} \quad (2) \quad \sum_{i=1}^{m} y_{i,t}^k p_{i,t} \geq - x_{m+1,t}^k \qquad t = 1,2, \ldots, \tau$$

$$(4) \quad \sum_{t=1}^{\tau} \sum_{i=1}^{m} y_{i,t}^k p_{i,t} = 0 ,$$

where $x_{m+1,t}^k$ is the amount of fiat money held by individual k at time t. There is also an upper bound for the left side of the inequalities in (2) this is given by

$$(5) \quad \sum_{i=1}^{m} y_{i,t}^k p_{i,t} \leq \sum_{j=1}^{n} x_{m+1,t}^j - x_{m+1,t}^k .$$

This states that the inflow of money cannot exceed the total available supply.

3.5 Credit

Credit is usually associated with individual trustworthiness, risk and other probabilistic features. Nevertheless it is possible to model it in a game context as follows. All individuals are given a credit rating which they maintain throughout the game. All their names are assumed to be equally prime. We assume however that at the whim of the referee he sets the size of the credit line which each may obtain. For example, he starts each player with an upper limit on credit of: $x^1_{m+1,1}, x^2_{m+1,1}, \dots, x^n_{m+1,1}$. There is no further method for the creation of credit beyond the initial authorization. The referee informs all players that at the end of the τ th time period there is to be no credit outstanding.

In this formulation credit and fiat money are operationally the same, as can be seen immediately if each individual were to issue i.o.u. notes to the extent of his credit line and to use them as fiat money. Perhaps the prime difference between fiat money and credit is that the first involves faith and trust in the government, while the second calls for faith and trust in lesser institutions and sometimes even individuals.

Were each individual allowed a credit line of unlimited size, but had to balance accounts at the end, the individual maximization would become:

Maximize (3) $$U^k = \sum_{t=1}^{\tau} \varphi^k_t (w^k_{1,t}, \dots w^k_{m,t})$$

subject to: (4) $$\sum_{t=1}^{\tau} \sum_{i=1}^{m} y^k_{i,t} \, p_{i,t} = 0 \; .$$

The inequalities of (2) and (5) would not be relevant.

3.6 Transferable Utility

We assume the existence of an abstract $m + 1$ st commodity which is valued by the players for its intrinsic worth. Say:

(6) $$\varphi^{*k}_t (w^k_{1,t}, \dots w^k_{m+1,t}) = \varphi^k_t (w^k_{1,t}, \dots, w^k_{m,t}) + \alpha^k w^k_{m+1,t} \; .$$

We may look upon the m+1 st commodity as consisting of "utility pills" which cannot be produced, but can be consumed by the players if they so wish. They are issued to the players at the start of the game in quantities $x^1_{m+1,1}$, $x^2_{m+1,1}$, ... $x^n_{m+1,1}$, they can be stored at no cost, and have their price fixed by the referee at 1 . There are n free parameters α^k, k = 1, ..., n which are the exchange rates for "utility pills" between the players. The interpersonal comparisons are fixed by these exchange rates.

The maximization conditions for the individual player now become:

$$(7) \qquad \text{Maximize} \sum_{t=1}^{\tau} \varphi^k (w^k_{1,t}, \dots w^k_{m,t}) + \alpha^k \sum_{t=1}^{\tau} w^k_{m+1,t}$$

subject to:

$$(8) \qquad \sum_{i=1}^{m} p_{it}\, y^k_{it} \geq - x^k_{m+1,t} \qquad t = 1,2 \dots \tau-1$$

$$(9) \qquad \sum_{i=1}^{m} p_{it}\, y^k_{it} \leq \sum_{j=1}^{n} x^j_{m+1,t} - x^k_{m+1,t} \quad t=1, \dots \tau-1$$

$$(10) \qquad \sum_{t=1}^{\tau} \sum_{i=1}^{m+1} p_{it}\, y^k_{it} = 0 \ .$$

The equality for the last period implies that all trades including trades in "utility pills" where $p_{m+1,t} = 1$ must balance.

4. General Equilibrium Conditions

4.1 Barter and Forward Contracts

As our main purpose is to compare fiat money, credit and transferable utility we do not go into mathematical detail for barter and forward contracts. As has already been noted, barter can be regarded merely as an iteration of the one period problem. If forward contracts have no limits on length or on periodicity of issue then it is easy to observe that they can serve as media of credit or as fiat money. The difference becomes one of law and the enforceability of contract. In our simplified models here these differences do not matter. They are of importance, however in the design of financial institutions in a world with theft, dishonesty and other threats to property and contracts.

4.2 Fiat Money and Credit

In Sections 3.4 and 3.5 we noted that in our "games" when there is no risk of the violation of individual or national trust, the models of a system with credit or with fiat money may be treated in the same mathematical framework.

We will assume convexity and twice-differentiability for the preference systems of the individuals. We further assume that initially for all individuals $x^k_{m+1,1} > 0$. In other words all individuals start with a positive amount of fiat money or credit. This implies that all the inequalities given in (3) and (5) can be satisfied by a suitable selection of a scale factor for any price system which is selected. This can be done as the price system is homogeneous of order zero. In these games where there are no probability features and fiat money or credit is used, any positive holdings are enough, or optimal to facilitate intertemporal trade. Money supply presents no problem. As the conditions of (2) and (5) do not constrain the optimization problem we may solve for the competitive equilibrium price system and trades by maximizing (3) subject to the single condition (4) for each player k . Introducing the Lagrangian Multiplier λ^k the problem becomes:

$$(11) \qquad \text{Maximize} \quad G^k = \sum_{t=1}^{\tau} \varphi_t^k (w_{1,t}^k, \ldots w_{m,t}^k) + \lambda^k \sum_{t=1}^{\tau} \sum_{i=1}^{m} p_{it} (x_{i,t}^k - w_{i,t}^k)$$

From this we obtain the following equations:

$$(12) \qquad \frac{\partial \varphi_t^k}{\partial w_{i,t}^k} = \lambda^k p_{i,t} \qquad n\ m\ \tau \ \text{equations}$$

$$(13) \qquad \sum_{t=1}^{\tau} \sum_{i=1}^{m} p_{it} (x_{it}^k - w_{it}^k) = 0 \qquad n \ \text{equations}$$

and then we have the material balance and the accounting equations:

$$(14) \qquad \sum_{k=1}^{n} y_{i,t}^k = \sum_{k=1}^{k} (x_{i,t}^k - w_{i,t}^k) = 0 \qquad (m+1)\ \tau \ \text{equations}$$

$$\text{and} \quad \sum_{i=1}^{m} p_{it}\, y_{it}^k + y_{m+1,t}^k = 0 \qquad n\ \tau \quad \text{equations}$$

from which we can determine the $n\ m\ \tau$ quantities $w_{i,t}^k$ which are consumed together with the $m\ \tau$ prices, the n values of λ^k and the $n\ \tau$ cash balances.

4.3 Transferable Utility

Unlike the "game" with fiat money or credit, the economy which uses "utility pills" to settle accounts does not have a price system which is homogeneous of order zero in the prices of the first m goods, it does however have a price system which is homogeneous of order zero in the prices of the m+1 goods which includes the "utility pills". This implies that even though everyone may start with a positive supply of the "utility pills" we cannot merely select a scale factor for the first m prices to ensure that the satisfaction of the boundary condition (10) will automatically take care of the inequalities of (8) and (9) . For the inequalities to cause no interference with the maximization problem we require a sufficiently large supply of "utility pills". As the pills are an embodiment of stored value and release the same amount of utility any time they are eaten, there is no incentive to eat them before the τ^{th} time period and there may well be an intertemporal trading incentive not to eat them but to use them in settling accounts each period. Suppose we have given each player a sufficiently large supply of "utility pills" that the inequalities of (8) and (9) will not interfere with their individual maximizations, then we may consider that each maximizes (7) subject to the single condition (10). Introducing the Lagrangian multiplier λ^k this becomes:

$$(15) \qquad \text{Maximize } G^{*k} = \sum_{t=1}^{\tau} \varphi_t^k (w_{1,t}^k, \dots w_{m,t}^k) + \alpha^k \sum_{t=1}^{\tau} w_{m+1,t}^k +$$

$$\lambda^k \sum_{t=1}^{\tau} \sum_{t=1}^{m+1} p_{i,t} (x_{i,t}^k - w_{i,t}^k)$$

where $p_{m+1,1} = 1$ by assumption, as we can arbitrarily fix one of the m+1 prices. From (15) we obtain the equations:

$$(16) \qquad \frac{\partial \varphi_t^k}{\partial w_{i,t}^k} = \lambda^k p_{i,t} \qquad n\ m\ \tau \text{ equations}$$

$$(17) \qquad \sum_{t=1}^{\tau} \sum_{i=1}^{m+1} p_{i,t} (x_{it}^k - w_{it}^k) \qquad n \text{ equations}$$

and

$$(18) \qquad \alpha^k = \lambda^k \qquad n \text{ equations}$$

The material balance and accounting equations give:

$$(19) \qquad \sum_{k=1}^{n} y_{i,t}^k = \sum_{k=1}^{n} (x_{i,t}^k - w_{i,t}^k) = 0 \qquad (m+1)\ \tau \text{ equations}$$

and

$$\sum_{i=1}^{m} p_{i,t}\, y_{i,t}^k + y_{m+1,t}^k = 0 \qquad n\ \tau \text{ equations}$$

We observe that equations (16) are identical with (12); (17) differ from (13) only in the presence of the m+1 st commodity; (18) have no counterpart, but immediately determine the values for the parameters α^k and (19) and (14) are identical. It follows immediately that any solution to the optimization problem with fiat money satisfies the conditions for optimization with transferable utility with the added specifications that (17) and (13) are satisfied by the values from (13) if $\sum_{t=1}^{\tau} y_{m+1,t}^k = 0$, i.e. over

τ time periods side payments balance out and furthermore the price system for the m commodities is no longer homogeneous of order zero, but is fixed in relation to the measure of the unit of transferable utility.

4.4 Joint Maximization with Transferable Utility

It follows immediately from 4.3 that if we normalize the utility functions of the individuals so that:

$$(20) \qquad \tilde{\varphi}_t^k \; (w_{1,t}^k, \ldots , w_{m+1,t}^k) = \frac{1}{\lambda^k} \varphi_t^k \; (w_{1,t}^k, \ldots w_{m,t}^k) + w_{m+1,t}^k$$

then the competitive equilibrium solution to the market in 4.3 is equivalent to the period by period jointly maximal solution to

$$(21) \qquad \sum_{k=1}^{n} \frac{1}{\lambda^k} \; \varphi_t^k \; (w_{1,t}^k, \ldots w_{m,t}^k) + \sum_{k=1}^{n} w_{m+1,t}^k$$

subject to

$$(22) \qquad \sum_{k=1}^{n} y_{i,t}^k + w_{i,t}^k = x_{i,t}^k \qquad\qquad i = 1, \ldots m+1$$

which gives:

$$(23) \qquad \text{Maximize} \;\; \tilde{G}_t = \sum_{k=1}^{n} \frac{1}{\lambda^k} \; \varphi_t^k \; (w_{1,t}^k, \ldots, w_{m,t}^k) + \sum_{k=1}^{n} w_{m+1,t}^k - \sum_{i=1}^{m+1} p_{i,t} \left(\sum_{k=1}^{n} (y_{i,t}^k + w_{i,t}^k - x_{i,t}^k) \right)$$

from which

$$(24) \qquad \frac{1}{\lambda^k} \frac{\partial \varphi_t^k}{\partial w_{i,t}^k} = p_{i,t} \qquad i = 1, \ldots m$$

and

$$(25) \qquad 1 = p_{m+1,t}$$

Hence we may say that the competitive functioning of an economy with fiat money yields the same results as if the individuals were jointly maximizing with a transferable utility available for side payments.

4.5 The Effect of the Infinite Time Horizon

In Section 1 a story was told concerning trading and eating sardines; the difference between the fiat money case and transferable utility is given in the difference between "trading sardines" and "eating sardines". At the end of τ periods the trading chips were returned to the referee in the first case, in the other case, they are eaten. In an infinite model the final return or destruction of the chips does not enter in. This does however have an effect on the price levels in the fiat money case, it brings about a multiplicative correction so that there will be "just enough" money for the price system. A simple example will illustrate this. Suppose two individuals trading in two goods x and y have the following preferences:

$\sqrt{x_1^1 y_1^1}$	0	period 1
0	$\sqrt{x_2^2 y_2^2}$	2
$\frac{1}{2}\sqrt{x_3^1 y_3^1}$	0	3
0	$\frac{1}{2}\sqrt{x_4^2 y_4^2}$	4
$\frac{1}{4}\sqrt{x_5^1 y_5^1}$	0	5
⋮	⋮	⋮

At each period the players have supplies of one of the commodities which each have a life of only one period and cannot be stored. Suppose that the first player has 10 - (1-1/t) units of the first commodity at periods 2t-1 or 2t for t = 1, 2, ... This gives him a supply gradually decreasing from 10 to 9 . The second player has 10 - (1-1/t) units of the second commodity. Hence supplies are:

(10, 0)	(0, 10)
(10, 0)	(0, 10)
(9-1/2, 0)	(0, 9-1/2)
(9-1/2, 0)	(0, 9-1/2)
(9-1/3, 0)	(0, 9-1/3)
⋮	⋮

We assume that prices are always $\Pi_{1,t} = \Pi_{2,t} = 1$ for both commodities at all periods. Suppose the initial distribution of fiat money is 20 units to the first player. Suppose that the "game" is to last for 4 periods at the end of which the first player must return the 20 chips to the referee. We can see that any price system where $\Pi_{1,t} = \Pi_{2,t} \leq 20/(10-(1-1/t))$ for $2t-1$, or $2t$ where $t = 1,2, \ldots$ will be in competitive equilibrium. For the 4 periods under consideration we have:

Period	Price	Money holdings of 1	Money holdings of 2
1	$1 \leq 2$	10	10
2	$1 \leq 2$	20	0
3	$1 \leq 2.105$	10	10
4	$1 \leq 2.105$	20	0
end		0	0

If there were no end to this process, the first player would not need to balance his budget with the 20 chips at the close of period 4 . This being the case, he would be motivated to use any spare fiat money to purchase more of the commodities and prices would rise to their upper bounds.

5. The Supply of Money, Risk, Frictions and Institutional Features

5.1 Institutional Features

Tobin has surveyed the many special features which must be covered by an adequate theory of money[1]. This paper touches on only a very few of the

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1/ Tobin, James "Money Capital and Other Stores of Value"

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cogent points raised there. However it is the aim here to devise a method for constructing a series of closed dynamic micro-economic models which separate out several of the different features of the monetary and financial system. A natural next step is to introduce the government as creator of fiat money and banks and other institutions as creators of credit into a model as explicit players. These institutions, however are primarily mechanisms designed to cope with the risk aspects of finance and with difficulties of transacting business. Hence it appears to be desirable first to construct models involving the different types of risk, failure, default, dishonesty and ill-fortune which create the need for the laws and institutions which constitute a financial system, then to consider the types of institutions needed to make the "game" have desired properties.

Even at the level of abstraction of the models in Section 3 it may be worthwhile to introduce a rudimentary Government Player with a policy goal and a well defined strategy set, in order to investigate the feasibility of the policy and to solve for the optimum policy. For example suppose we describe a game as follows, the government is the only player permitted to issue or call fiat money. Taxes must be paid in fiat money, there is no credit in the system. The government must state its goal and publish its strategy. Fiat money must be used for all transactions; there are no frictions and there are no risks or uncertainties. All individuals are constrained to act as maximizing pure competitors. For example let the government policy be to equalize (money) income using only graduated income tax and direct equal subsidies, for the next τ periods.

For the individual, his maximization problem becomes, modifying Section 3.4.

$$\text{Maximize (3)} \quad U^k = \sum_{t=1}^{\tau} \varphi_t^k \left(w_{1,t}^k, \ldots w_{m,t}^k\right)$$

subject to :

$$(26) \qquad \sum_i^m y_{it}^k \, p_{it} \geq - \left\{ x_{m+1,t}^k - b_t^k \sum_i^m p_{it} \, x_{it}^k + a_t \right\} \leq 0$$

where the right hand side of (26) consists of cash holdings minus income tax plus subsidy. For all individuals the extra (n-1) equations (27) must hold each period:

$$(27) \qquad (1-b_t^k) \sum_i^m p_{it} \, x_{it}^k = (1-b_t^j) \sum_i^m p_{it} \, x_{it}^j \quad .$$

5.2 Risk

The type of risk run in a financial community depends heavily upon the specific laws concerning debts, liens, contracts, liabilities and so forth as well as difficulties and costs of collection, the honesty of the population and the accuracy in prediction of markets and estimation of the success of processes.

One simple example modifying the model of credit in Section 3.5 is given. If there were no risk of default or failure there would be no difference between fiat money or credit, providing the rules of creation of credit and fiat money were appropriately framed. In Sections 3.4 and 3.5 equivalent models are constructed.

We now consider a credit model with risk. At the start of the game, the referee issues each player with a stack of notes which each player signs. They represent the extent of the credit extended to him. Each player is required to return his notes, or an equivalent sum (this calculation will be specified) to the referee at the end of the game. The referee announces that there is a probability of Π per period that the notes of player k will be devalued. The notes are all issued with the same face value; if there is a devaluation of k's notes, at the end of the game, he must return his own notes or $x^k_{m+1,1/b}$ of any other notes where $x^k_{m+1,1}$ was the original face value issued to him, and $\frac{1}{b}$ gives the factor of devaluation. The devaluation is announced at the start of any period if it is to take place during that period.

The type of risk described here might be termed as financial default. We can see immediately that if there were some sort of security in the form of transferable utility, gold, a useful intermediate good, valuables or fiat money which could be posted against the possibility of devaluation, the effect on trade could be ameliorated. This type of financial default will be examined further in Part II.

6. An Example

6.1 The Money Economy

Consider two players with utility functions

$$U_1 = \sqrt{x^1_{1,1} \; x^1_{2,1}} + \sqrt{x^1_{1,2} \; x^1_{2,2}}$$

$$U_2 = \log x^2_{1,1} \; x^2_{2,1} + \frac{1}{2} \log \; x^2_{1,2} \; x^2_{2,2} \quad .$$

Each period their endowments of the (perishable) commodities are (10,0), (10,0) and (0,10), (0,10) respectively. Player 1 starts with 10 units of fiat money, which must be returned at the end. In competitive equilibrium Player 1 will maximize:

(28) $$\sqrt{x^1_{1,1} \; x^1_{2,1}} + \sqrt{x^1_{1,2} \; x^1_{2,2}}$$

subject to

(29) $$p_{1,1} \, (10 - x^1_{1,1}) - p_{2,1} \, x^1_{2,1} + p_{1,2} \, (10 - x^1_{1,2}) - p_{2,2} \, x^1_{2,2} = 0$$

this gives

(30) $$\frac{1}{2} \sqrt{\frac{x^1_{2,1}}{x^1_{1,1}}} = \lambda_1 \; \Pi_{1,1}$$

(31) $$\frac{1}{2} \sqrt{\frac{x^1_{1,1}}{x^1_{2,1}}} = \lambda_1 \; \Pi_{2,1}$$

$$(32) \qquad \frac{1}{2}\sqrt{\frac{x^1_{2,2}}{x^1_{1,2}}} = \lambda_1 \Pi_{1,2}$$

$$(33) \qquad \frac{1}{2}\sqrt{\frac{x^1_{1,1}}{x^1_{2,1}}} = \lambda_1 \Pi_{2,1} \quad .$$

Player 2 will maximize

$$(34) \qquad \log \; x^2_{1,1} \; x^2_{2,1} \; + \frac{1}{2} \; \log \; x^2_{1,2} \; x^2_{2,2}$$

subject to

$$(35) \qquad - p_{1,1} \; x^2_{1,1} \; + (10 - x^2_{2,1}) \; p_{2,1} - p_{1,2} \; x^2_{1,2} + (10 - x^2_{2,2}) \; p_{2,2} = 0$$

this gives

$$(36) \qquad \frac{1}{x^2_{1,1}} = \lambda_2 \; \Pi_{1,1}$$

$$(37) \qquad \frac{1}{x^2_{2,1}} = \lambda_2 \; \Pi_{2,1}$$

$$(38) \qquad \frac{1}{2 \; x^2_{1,2}} = \lambda_2 \; \Pi_{1,2}$$

$$(39) \qquad \frac{1}{2 \; x^2_{2,2}} = \lambda_2 \; \Pi_{2,2}$$

At competitive equilibrium

(40) $x^2_{1,1} = 10 - x^1_{1,1}$ (41) $x^2_{2,1} = 10 - x^1_{2,1}$

(42) $x^2_{1,2} = 10 - x^1_{1,2}$ (43) $x^2_{2,2} = 10 - x^1_{2,2}$

The solutions to equations are given by :

$$x^1_{1,1} = x^1_{2,1} = 10/3 \qquad x^2_{1,1} = x^2_{2,1} = 20/3$$

$$x^1_{1,2} = x^1_{2,2} = 20/3 \qquad x^2_{1,2} = x^2_{2,2} = 10/3$$

$$\lambda_1 = 1/2 \qquad \lambda_2 = 3/20$$

$$0 < \Pi_{1,1} = \Pi_{1,2} = \Pi_{2,1} = \Pi_{2,2} \leq 3 \ .$$

We may easily check that if we had assumed that the utility functions of the players were

$$U^*_1 = \sqrt{x^1_{1,1}\ x^1_{2,1}} + \sqrt{x^1_{1,2}\ x^1_{2,2}} + \frac{1}{2}\,(x^1_{3,1} + x^1_{3,2})$$

and $$U^*_2 = \log x^2_{1,1}\ x^2_{2,1} + \frac{1}{2} \log\ x^2_{1,2}\ x^2_{2,2} + \frac{20}{3}\,(x^2_{3,1} + x^2_{3,2})$$

the same solutions satisfy.

We may make a third transformation, defining

$$\tilde{U}_1 = 2\left(\sqrt{x^1_{1,1}\, x^1_{2,1}} + \sqrt{x^1_{2,1}\, x^1_{2,2}}\right) + \left(x^1_{3,1} + x^1_{3,2}\right)$$

$$\tilde{U}_2 = \frac{20}{3}\left(\log x^2_{1,1}\, x^2_{2,1} + \frac{1}{2} \log x^2_{1,2}\, x^2_{2,2}\right) + \left(x^2_{3,1} + x^2_{3,2}\right)$$

With these utility functions let us assume that the players attempt to maximize $\tilde{U}_1 + \tilde{U}_2$ subject to the conditions that:

(44) $$x^1_{1,1} + x^2_{1,1} = 10$$

(45) $$x^1_{2,1} + x^2_{2,1} = 10$$

(46) $$x^1_{1,2} + x^2_{1,2} = 10$$

(47) $$x^2_{2,2} + x^2_{2,2} = 10$$

(48) $$x^1_{3,1} + x^2_{3,1} = x^1_{3,2} + x^2_{3,2} \quad .$$

The same solutions satisfy.