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CHARITY, TAXATION AND DISTRIBUTIONAL WEIGHTS IN BENEFIT-COST ANALYSIS

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CHARITY, TAXATION AND DISTRIBUTIONAL WEIGHTS

IN BENEFIT-COST ANALYSIS

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1. Introduction

The use of differential weighting schemes for aggregating the returns from public projects across income groups has become commonplace in benefit-cost analysis. In the economic literature, this practice has received extensive support[1] and somewhat less extensive criticism.[2] Obviously, the use of weighting schemes favouring poorer groups is motivated by the concern of many, if not all, economists for the alleviation of economic distress; but economists are not alone in feeling this way. Private charity is a common feature of economic life, although the degree to which it reduces income inequality is not always impressive.

The present paper aims to show that the existence of private charity has important implications for benefit-cost analysis. In particular, it argues that the net returns from "small" public projects accruing to different income groups connected by private charitable donations from one to the other should be weighted equally in benefit-cost analysis. This holds regardless of the degree to which the existing distribution of income is judged to be non-optimal, whether this judgement is made before or after the impact of private charity is taken into account, and regardless of the detailed form of the government's "social welfare" function. The basic model is outlined in Section 2 and the central argument is presented in Section 3. Section 4 then examines the degree to which this result is affected by allowing charitable donations to be tax-deductible.

2. Private Charity

Consider an economy containing three income groups: high, medium and low. Since our aim is to focus on the distribution of income between rather than within groups, we shall treat each group as consisting of a single individual. Each group receives a lump sum income determined outside the model and y^i will denote the lump sum income of group i. We number the groups 1 through 3 and arrange them such that $y^1 > y^2 > y^3$. Each group may consume a part of its income and make voluntary donations to other groups with the remainder. The consumption of group i is denoted c^i and the utility of group i will be expressed as a function of the consumption of all groups. So $U^i = U^i(c^1, c^2, c^3)$. The functions U^i are strictly concave, twice differentiable and non-decreasing in all arguments. For the derivatives of U^i we shall use the familiar notation $U^i_j \equiv \partial U^i/\partial c^j$.

We shall assume that preferences are such that donations always take the form of transfers to poorer groups, although this assumption may easily be relaxed without disturbing the results. Group 1 (the richest) donates to both groups 2 and 3, group 2 donates only to group 3, and group 3 (the poorest) makes no donations. The consumption of each of the three groups is given by

$$c^1 = y^1 - v^1, \tag{1}$$

$$c^2 = y^2 - v^2 + v^{12} \tag{2}$$

and

$$c^3 = y^3 + v^2 + v^1 - v^{12}, \tag{3}$$

where v^i denotes the total voluntary donations of group i and v^{12} denotes that part of group 1's donations going to group 2.

Groups 1 and 2 make their charitable donations separately. Utility maximization in group 1 requires, for an interior solution ($v^1 > 0$), that

$$U_1^1 = \lambda U_2^1 + (1 - \lambda)U_3^1 \quad , \tag{4}$$

where λ is the marginal share of group 1's donations going to group 2 and $0 \leq \lambda \leq 1$. If λ is determined entirely by group 1 (is a control variable for them) then, for $0 < \lambda < 1$, λ will be set such that $U_1^1 = U_2^1 = U_3^1$; but it is possible that λ will be at least partly outside the control of group 1, as in the case where charity is administered institutionally. For utility maximization in group 2 we require, for $v^2 > 0$, that

$$U_2^2 = U_3^2 \ . \tag{5}$$

Obviously (4) and (5) state simply that charitable donations are made up to the point where the marginal cost of a donation is equal to its marginal reward, from the point of view of the donor.

It is important to stress at this point that private charity need not generate an income distribution that is "optimal" from the point of view of the government. Suppose government planners possess a "social welfare function $W(U^1, U^2, U^3)$. Then W need not be maximized by private charity. Indeed, the redistribution of income resulting from charity may reduce W if the "social welfare" function is such that greater, rather than less, inequality is desired by the government. It seems difficult to reject the hypothesis that there are governments for whom this is so. Private charity need not even generate a distribution that is Pareto optimal in the usual sense. In fact, in the present model it will virtually never do so, provided charitable donations are made atomistically, as we have assumed.

To see this in its strongest form, let group 1's preferences be such that for $v^{12} = 0$ and $v^{1} > 0$, $U^1_2 < U^1_3$. This implies that group 1's utility is maximized by setting $\lambda = 0$ and that in equilibrium $U^1_1 = U^1_3 > U^1_2$. For group 2, we have from our earlier discussion $U^2_2 = U^2_3 > U^2_1$ and for group 3, $U^3_3 > U^3_1$ and $U^3_3 > U^3_2$. Now imagine groups 1 and 2 to have chosen their optimal levels of voluntary donations, as above, and consider a contract between them committing each to raise its donation to group 3 by one unit. The effect on the utility of each group is

$$dU^i = -\,dU^i_1 - dU^i_2 + 2dU^i_3 > 0, \quad i = 1,2,3, \tag{6}$$

Such a contract would be unanimously approved; the atomistic determination of charitable donations generates an equilibrium that is not Pareto optimal.[3]

3. Benefit-Cost Analysis

Now imagine the introduction of a public project having the net effect B^i on the income of group i. The effects on the consumption of the three groups are by definition given by

$$dc^1 = -dv^1 + B^1, \tag{7}$$

$$dc^2 = \lambda dv^1 - dv^2 + B^2 \tag{8}$$

and

$$dc^3 = (1 - \lambda)dv^1 + dv^2 + B^3. \tag{9}$$

Simply summing these equations we obtain

$$\sum_{i=1}^{3} dc^i = \sum_{i=1}^{3} B^i \equiv B^*. \tag{10}$$

We assume that the project is sufficiently "small" that equations (4) and (5) hold both before and after the project is adopted. That is, the adjustment by groups 1 and 2 to the income changes caused by the project does not lead either to move from an interior solution to its utility maximization problem to a corner solution. Equations (4) and (5) may now be totally differentiated, and together with (10) this gives the system

$$\begin{bmatrix} 1 & 1 & 1 \\ F_1 & F_2 & F_3 \\ G_1 & G_2 & G_3 \end{bmatrix} \begin{bmatrix} dc^1 \\ dc^2 \\ dc^3 \end{bmatrix} = \begin{bmatrix} B^* \\ 0 \\ 0 \end{bmatrix}, \tag{11}$$

where $F_i = (U^1_{1i} - \lambda U^1_{2i} - (1 - \lambda)\, U^1_{3i})$ and $G_i = (U^2_{2i} - U^2_{3i})$. Solving this system we obtain

$$dc^i = B^* H^i, \qquad i = 1, 2, 3, \tag{12}$$

where $H^1 = (F_2G_3 - F_3G_2)/D$, $H^2 = (F_3G_1 - F_1G_3)/D$, $H^3 = (F_1G_2 - F_2G_1)/D$ and $D = F_2G_3 - F_3G_2 + F_3G_1 - F_1G_3 + F_1G_2 - F_2G_1$.

The equilibrium change in the consumption of group i resulting from the proje is a constant, H^i, multiplied by B^*, the unweighted sum of the net benefits of the project. It is easily seen from above that the H^i terms must sum to unity. For group i, H^i gives the share of the aggregate benefits of the project which accrue, in equilibrium, to the consumption of group i. The point is that these shares depend on preferences and not the initial distributional impact of the project. Relatively weak additional assumptions will ensure the strict positively of each of the H^i terms. For example, if the utility functions are each additively separable, this result is guaranteed. Nevertheless, the strict positivity of the H^i terms is not implied by our earlier assumptions. Suppose $H^i < 0$ for some i. It is easy to show that an increase in any or all of the lump sum income terms, y^j, then implies a fall in the equilibrium consumption of group i; in particular, an increase in y^i, holding all other lump sum income terms constant causes group i's consumption to decline. This is clearly an extreme case. Obviously, since $\Sigma H^i = 1$, it is impossible for all the H^i terms to be negative, and intuitively, this would violate our assumption that utility functions are non-decreasing in all arguments. Increasing all the y^i terms would then reduce both the consumption and the welfare of all groups.

The change in the utility of group i is given by

$$dU^i = \sum_{j=1}^{3} U^i_j \, dc^j = B^* \sum_{j=1}^{3} U^i_j H^j \equiv B^* K^i \quad . \tag{13}$$

The negativity of one or more of the H^j terms in (13) is a necessary but not sufficient condition for $K^i < 0$. Clearly, $K^i < 0$ implies that group i is made worse off by an increase in any or all of the y^j terms, including y^i, and this is clearly a pathological case. Provided each of the H^i terms is positive,

$$\text{sign}\ (dU^i) = \text{sign}\ (B^*)\ , \qquad i = 1, 2, 3. \qquad (14)$$

A project benefits or harms each group according to whether the unweighted sum of project net benefits is positive or negative, regardless of the distributional impact of the project. Projects may be ranked, from the point of view of each individual, according to B^*. The possibility that a project could benefit one group while harming another exists, but this possibility rests entirely on the nature of group preferences (the remote possibility that one or more of the K^i terms is negative) and not at all on the distributional impact of the project.

Suppose that $K^i < 0$ for some i. Now consider a Bergson-Samuelson social welfare function $W = W(U^1, U^2, U^3)$, with derivatives $W_j > 0$ for all j. Then the welfare impact of a project is given by

$$dW = \sum_{j=1}^{3} W_j dU^j = B^* \sum_{j=1}^{3} W_j H^j \equiv B^* V. \qquad (15)$$

If $V < 0$ then a project for which $B^* > 0$ reduces welfare. Similarly, raising each of the y^i terms reduces welfare. This case seems scarcely worth considering but in this instance projects can be ranked inversely according to B^*. Otherwise, with $V > 0$, as we expect, projects may be ranked according to B^* even in the seemingly unlikely case where one or more of the H^i terms, or even the K^i terms, is negative.

Intuitively, the existence of private benevolence implies that the fortunes of the various income groups are linked independently of the distributional impact of the project. The equilibrium welfare impact of a project on each group depends on the aggregate net wealth generated by the project and not at all on the proportion of that aggregate net wealth accruing directly to that particular group. This implies that projects may be ranked, according to their welfare impact, by comparing the unweighted sums of their net benefits. That is,

$$\text{sign } (dW) = \text{sign } (B^*)$$

for each project. If there are two or more mutually exclusive projects for which $B^* > 0$, then the one for which B^* is greatest has the most favourable welfare impact. This result is independent of the detailed nature of the social welfare function. It holds regardless of the degree to which the existing distribution of income is judged to be undesirable, whether this judgement is made before or after the effects of private charity are taken into account. It holds even in the case, discussed at the end of the previous section, where some other distribution of income is Pareto superior to the existing one and where a redistribution of income would be unanimously approved. Finally, it holds even when private benevolence redistributes income in the opposite direction from the redistribution that the government would wish.

4. Tax-Deductible Charity

Charitable donations are frequently tax-deductible. Since this changes the marginal optimality conditions determining the levels of charitable donations, the question arises of whether the introduction of a tax system possessing this feature will alter our conclusion that the distributional impact of a project should be disregarded. The answer to this question proves to depend on the way tax revenues are treated and on whether the receipts of charitable donations and project benefits are taxed. We look at the question from the point of view of a project evaluator for whom the tax system must be treated as given, regardless of its merits. The discussion is confined to an examination of the implications of a fixed tax schedule for benefit-cost analysis.

We shall suppose, for now, that tax revenues represent a pure loss from the point of view of each income group. While there is presumably some other, governmental group benefiting from these revenues, for the time being their interests will be treated as lying outside the scope of our welfare analysis. This assumption will presently be relaxed. Let the marginal proportional tax rates applying to groups 1, 2 and 3 be fixed at t^1, t^2 and t^3, respectively. Three alternative tax systems will be considered:

Table 1: Three Alternative Tax Systems

	Charitable donations	Charitable receipts	Project returns
Case (1)	deductible	taxed	taxed
Case (2)	deductible	taxed	not taxed
Case (3)	deductible	not taxed	not taxed

Case (1): Charitable receipts and project returns taxed

The change in the consumption of each group, previously given by equations (7) to (9) is now given by

$$dc^1 = - dv^1 (1 - t^1) + B^1 (1 - t^1), \tag{16}$$

$$dc^2 = \lambda dv^1 (1 - t^2) - dv^2 (1 - t^2) + B^2 (1 - t^2), \tag{17}$$

$$dc^3 = (1 - \lambda) dv^1 (1 - t^3) + dv^2 (1 - t^2) + B^3 (1 - t^3) \cdot \tag{18}$$

Dividing (16), (17) and (18) by $(1 - t^1)$, $(1 - t^2)$ and $(1 - t^3)$, respectively and summing we obtain

$$\sum_{i=1}^{3} dc^i/(1 - t^i) = \sum_{i=1}^{3} B^i \equiv B^*, \tag{19}$$

The marginal optimality conditions applying to donations by groups 1 and 2 (previously equations (4) and (5)) now become

$$U_1^1 (1 - t^1) = \lambda U_2^1 (1 - t^2) + (1 - \lambda) U_3^1 (1 - t^3) \tag{20}$$

and

$$U_2^2 (1 - t^2) = U_3^2 (1 - t^3) \; . \tag{21}$$

The analysis now proceeds exactly as before. The equilibrium solution corresponding to (12) is now

$$dc^i = H^i_{(1)} B^*, \qquad i = 1, 2, 3 \tag{22}$$

We can omit detailed discussion of the $H^i_{(1)}$ terms since they differ from the H^i terms in (12) only by the presence of some multiplicative tax rates.

Their qualitative form, interpretation and likely sign in the same. The important feature of (22) is that the multiplicative term B^* is, as before, the unweighted sum of net project returns. Projects may be ranked, from the point of view of each group, according to their aggregate net benefits, regardless of the initial distribution of project benefits or of the distribution of marginal tax rates.

Case (2): Charitable receipts taxed; project returns not taxed

We now suppose the net benefits from projects to ta[illegible] form which places them outside the tax system. We now have

$$dc^1 = - dv^1 (1 - t^1) + B^1 , \tag{23}$$

$$dc^2 = \lambda dv^1 (1 - t^2) - dv^2 (1 - t^2) + B^2 \tag{24}$$

and

$$dc^3 = (1 - \lambda) dv^1 (1 - t^3) + dv^2 (1 - t^3) + B^3. \tag{25}$$

The marginal optimality conditions (20) and (21) are unchanged. We eliminate the variables dv^1 and dv^2 from (23) to (25)

$$\sum_{i=1}^{3} dc^i = \sum_{i=1}^{3} B^i/(1 - t^i) \equiv B'. \tag{26}$$

Proceeding as before, the resulting system gives the solution

$$dc^i = H^i_{(2)} B' , \qquad i = 1, 2, 3 . \tag{27}$$

Again the $H^i_{(2)}$ terms need not be discussed, but now the distributional impact of the project becomes important. If, as is frequently but not always the case, wealthier groups incur higher marginal tax rates ($t^1 > t^2 > t^3$) then the appropriate weights to apply to the projects benefits received by richer groups exceed those applying for poorer groups.

This counterintuitive, not to mention reactionary, result is even stronger in case (3).

Case (3): <u>Charitable receipts and project returns not taxed.</u>

The technique of analysis is now sufficiently clear that we can pass directly to the main result, namely

$$dc^i = H^i_{(3)} \quad B'' , \qquad i = 1, 2, 3 \tag{28}$$

where

$$B'' = \frac{B^1(1 - \lambda + \lambda/(1 - t^2)}{(1 - t^1)} + \frac{B^2}{(1 - t^2)} + \frac{B^3}{(1 - t^3)} \quad . \tag{29}$$

Again, the detailed form of the $H^i_{(3)}$ terms need not concern us, but since the term $(1 - \lambda + \lambda/(1 - t^2))$ exceeds unity the premium applying to the returns of group 1 is now even higher than that obtained in case (2).

These results may be understood intuitively by noting that the tax-deductibility of donations means that a dollar sacrificed by the donor generates more than a dollar for the recipient. At the margin, these quantities are, from the point of view of the donor, equally valuable. So far as groups 1 to 3 are concerned, this "magnification" at the margin is analogous to the return generated by an intertemporal investment. The impact of a new infusion of funds (a public project) on the net wealth of the three groups is measured by discounting the returns received by recipient (poorer) groups. This is analous to discounting the returns generated by public projects in later time periods, the discount rate depending on the rate of return available on intertemporal investments. When project returns are themselves tax-deductible (case (1)), these effects cancel out, but not otherwise.

The above results for cases (2) and (3) should be treated with caution. This is especially clear when we relax the separation between groups 1 to 3 and the recipients of tax revenues. Define a new group,

group G, which receives tax revenues and which in addition either makes charitable donations to, or receives them from, some subset of the other three groups. Then we immediately return to our earlier result. Projects may be ranked, from the point of view of each of the four groups, according to the unweighted sum of their net benefits.

To show this, we shall suppose that group G makes voluntary donations to group 3. We will also allow the possibility that group G also receives benefits from public projects. The utility functions of the four groups are now expressed as $U^i = U^i(c^1, c^2, c^3, c^G)$, $i = 1, 2, 3, G$, where c^G denotes the consumption of group G. We shall suppose that the tax system operates as in case (2) above and, for simplicity, that group G is untaxed. Group G's utility maximization gives

$$U^G_G = U^G_3 (1 - t^3) \quad . \tag{30}$$

Equations (20), (21) and (23) to (25) apply as before, except that $dv^G(1 - t^3)$ must now be added to the right-hand side of (25), where dv^G is the change in the donations of group G, and in addition we have

$$dc^G = - dv^1 (t^1 - \lambda t^2 - (1 - \lambda)t^3) - dv^2 (t^2 - t^3) - dv^G(1 - t^3) + B^G , \tag{31}$$

Simply summing equations (23) to (25) and (31) we obtain

$$\sum_{i=1}^{3} dc^i + dc^G = \sum_{i=1}^{3} B^i + B^G \equiv B^{**} \quad . \tag{32}$$

We totally differentiate equations (20), (21) and (30) and together with (32) these give a four equation - four variable system with the solution

$$dc^i = J^i B^{**} , \qquad i = 1, 2, 3, G. \tag{33}$$

The terms J^i sum to unity and again we expect $J^i > 0$ for all i, but the multiplicative term B^{**} is again the unweighted sum of project returns; projects should be ranked without regard to their distributional impact. This result is also obtained in each of the other possible variants of the tax system.

5. Summary and Conclusions

The focus of this paper is on the distribution of income between groups connected by private charitable donations from one to the other. It studies the impact of public projects on the consumption of the various groups, taking into account the equilibrium adjustment of private charitable donations to the income effects of public projects. It is concluded that the returns from "small" public projects accruing to different groups connected in this way should be weighted equally in benefit-cost analysis, regardless of the degree to which their incomes differ and regardless of the precise form of the government's "social welfare" function. These results do not in any way require private charity to have equalized the distribution of income or to have moved substantially in that direction, and in fact they continue to apply when private charity redistributes income in the opposite direction from the redistribution the government would wish.

Although these results are complicated by allowing charitable donations to be tax-deductible, these complications vanish once the group receiving tax revenues (the government) is connected, via charitable donations, to other groups in the society. From a methodological point of view, these results are similar to those of the "rational expectations" literature; taking account of the equilibrium impact that public policy decisions have on the decisions of individuals does not represent a mere "refinement" to the analysis, but alters its conclusions fundamentally.

Footnotes

*This research was conducted while the author was Visiting Fellow, Economic Growth Center, Yale University, on leave from Monash University, Melbourne. The paper has benefitted substantially from the author's discussions with Brian D. Wright, but the author is responsible for the views presented and any errors.

[1]The many advocates of differential weighting schemes include Weisbrod (1968), Dasgupta, Marglin and Sen (1972), Little and Mirrlees (1974), Lal (1974) and Squire and Van der Tak (1975).

[2]Critics include Harberger (1971) and (1978), Parish (1976) and Ng (1978). Harberger (1978) provides a more extensive bibliography.

[3]For a fuller development of this line of argument see Warr and Wright (1979).

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