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On trend analysis in climatic time series, with application to surface temperature

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ABSTRACT

The deletion of a trend as an initial step in the analysis of climatic time series may result in the elimination of low-frequency components which constitute an integral part of climatic variability. An example is given here showing that linear trend deletion from the time series of the World Ocean annual sea-surface temperature (1850–2009) reduces the low-frequency (from 0.02 year\textsuperscript{-1} to 0.001 year\textsuperscript{-1}) part of the time series spectrum by \(\sim 40\%\) to 80\% thus severely distorting the spectrum of climate. As an additional result, it is shown that the current warming can be explained in full within the framework of a stationary stochastic model fitted directly to the time series of annual sea-surface temperature (SST) from 1850 through 2009 with no trend deletion. According to the model, the recurrence time of runs of generally increasing temperature by \(\sim 0.5^\circ\text{C}\) lasting for several decades (as has been observed since about 1956) is about 500 years for the World Ocean. Thus, the current run of growing SST is not an extremely rare event in climate and can be explained as a part of the natural climatic variability. These results show that deleting linear trends requires a thorough preliminary analysis. It is suggested that the approach described here can be used to improve physical models of the World Ocean climate.

1. Introduction

Removing statistically significant linear trends from climatic time series is a standard technique in oceanography and other earth sciences (see e.g., Roden, 1963; Privalsky, 1983; Blender and Fraedrich, 2003; Knutson \textit{et al.}, 2006). Yet, instrumentally observed climatic time series are usually rather short because their total length is comparable to, or even less than, the largest time scale of interest. Therefore, it is possible that relatively long runs with generally increasing or decreasing values can be just time-domain manifestations of low-frequency components in the spectrum of respective quantity (see, e.g., Bloomfield and Nychka, 1992). The operation of trend removal from a relatively short time series will make it appear stationary but it will also affect the spectral density of the time series in a wide frequency band at the low-frequency end of the spectrum. Namely, this operation will

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reduce long-term variations of respective climatic characteristics and, therefore, it requires a thorough justification.

Numerous estimates of spectra of climatic time series show that the spectrum generally grows with the time scale (see e.g., Privalsky, 1983; Blender and Fraedrich, 2003; Fortus, 2010). Though the entire spectrum is not known, it is usually assumed to follow some stochastic model such as fractionally-rational with respect to the cosine or sine of frequency (ARIMA models of integer or fractional orders).

Thus, even if a linear trend is found to be statistically significant, its presence in the time series can be an artifact. When the time series is analyzed as is, one may arrive at a stochastic model for it which would show that the low-frequency variations regarded as a trend actually constitute a regular feature of respective random process. If such long runs of generally growing or decreasing values appear often enough in longer samples of the process simulated in accordance with the adopted stochastic model, there would be no ground to assume that the observed time series contains a deterministic trend.

In other words, showing that the trend rate estimated through commonly applied techniques is statistically significant does not necessarily serve as a reliable probabilistic proof of the presence of the trend and a more thorough analysis of the time series, including its stochastic modeling, is required. This step can be evaded if additional information is available that justifies the assumption of the presence of a trend. A typical example would be a time series of sea level variations in the presence of tectonic movements (Roden, 1963).

Therefore, the analysis of trends in oceanographic time series should include two stages:

1. Verifying the necessity to assume the presence of a trend, and
2. Estimating its parameters (in accordance with the stochastic model accepted for the time series).

This work is dedicated to the first task only.

2. Analysis and conclusions

The time series of the annual global SST from 1850 through 2009 is shown in Figure 1. If a linear trend is fitted to this data, the estimate of its rate will constitute about $0.004^\circ\text{C}\times\text{year}^{-1}$ and it will be statistically significant even at a high confidence level. Yet, to the best of our knowledge, the opposing hypothesis of a natural global warming due to the low-frequency climatic variability has not been properly tested within the framework of the theory of random processes. Having no reliable data about the temperature of the oceans prior to the instrumental age, we have no grounds to assume that the time series contains a deterministic linear trend without first checking the hypothesis of a natural origin of such or greater increases in the ocean temperature, with no contribution from any time-dependent external forcing. The inability of the global circulation models to reproduce the observed warming
without an additional anthropogenic forcing is used as part of the proof of the dominant role of human effects upon climate: “It is very unlikely that the 20th century warming can be explained by natural causes … The models fail to reproduce the observed warming when run using only natural factors.” (IPCC, 2007: p. 702). This latter statement can be equally applied to the physical models of climate rather than to the climate itself. Therefore, by fitting a model of some stationary random process to the available data and analyzing the properties of the process, we will try to verify whether the general warming of the SST since about 1956 can be fully explained by natural causes.

We will do so using the time series SST2 (Rayner et al., 2006) of observed annual sea-surface temperature anomalies from 1850 through 2009 which can be found at http://www.cru.uea.ac.uk/cru/data/temperature/ (see Fig. 1). The solid curve in the figure presents smoothed variations of SST obtained by applying an equal-weight running mean low-pass filter with the weight function’s half-length $M = 10$ years.

The recent increase in the (smoothed) temperature (1956–2008) amounts to approximately 0.48°C in 53 years while the growth of temperature earlier in the century (1912–1944) constituted 0.34°C in 33 yrs. Thus, the rate of growth of about 0.01°C per year that lasts a few decades is not uncommon, though the current run of increasing temperature is lasting longer than the previous one. Obviously, the human effects were not as strong in the first half of the previous century as they are assumed to be now.
Let $T_t$ be the surface temperature anomalies (zero mean annual temperature) in year $t$. The Wold decomposition of $T_t$ (see e.g., Box et al., 1994) is

$$T_t = a_t + \sum_{j=1}^{\infty} \psi_j a_{t-j}$$

(1)

where $\psi_j$ are the coefficients of the decomposition, $a_t$ is a sequence of identically distributed and mutually independent random variables (white noise) with a zero mean and a constant variance $\sigma_a^2$. In a specific case of the autoregressive model of order $p$, or AR($p$), Eq. (1) leads to

$$T_t = \sum_{j=1}^{p} \phi_j T_{t-j} + a_t.$$ (2)

Here, $\phi_j$ are the autoregressive coefficients which are estimated (along with the order $p$ of autoregression and the white noise variance $\sigma_a^2$) from the time series of the annual SST from 1850 through 2009. An expression for the spectral density $s(f)$ can be easily obtained from Eq. (2) as

$$s(f) = 2\sigma_a^2 \Delta t \left| 1 - \sum_{j=1}^{p} \phi_j e^{-i2\pi j \Delta t} \right|^{-2}$$

(3)

where $\Delta t = 1$ year, $f$ is frequency ($\text{year}^{-1}$) in the interval between 0 and $1/2 \Delta t$.

Methods used to estimate the autoregressive coefficients $\phi_j$ (for $j = 1, \ldots, p$) and the variance $\sigma_a^2$ are described in textbooks and monographs (see e.g., Box et al., 1994) while the order $p$ should be selected by using special criteria such as Akaike’s, Parzen’s, Schwarz-Rissanen’s, or Hannan-Quinn’s (see Privalesky, 1999).

For the SST data from 1850 through 2009, the estimate of the time series variance is $0.0562 \, (\circ \text{C})^2$, or standard deviation $0.237 \circ \text{C}$. All four of the above criteria indicate an AR model of order $p = 3$. The roots of the characteristic equation corresponding to the model (2) with $p = 3$ for the SST time series lie outside the unit circle, which means that the time series is stationary. The AR coefficients estimates (with estimate’s rms error values) are

$$\phi_1 = 0.8034 \pm 0.0800,$$

$$\phi_2 = -0.1992 \pm 0.1031,$$

$$\phi_3 = 0.3260 \pm 0.0800,$$

and the white noise variance $\sigma_a^2 = 0.0107 \, (\circ \text{C})^2$, or $\sigma_a = 0.1033 \circ \text{C}$.

The spectral density estimate calculated in accordance with Eq. (3) is shown in Figure 2; the spectrum of temperature is concentrated at lower frequencies and does not contain significant peaks.
If the trend is deleted, as is usually done in similar cases, the spectral density of the resulting time series will differ from that of the original time series (see Fig. 2). The differences are quite substantial at lower frequencies and negligibly small at higher frequencies. The loss of power spectral density at the frequencies 0.001, 0.005, 0.01, and 0.02 year\(^{-1}\) amounts to 82%, 76%, 63%, and 37%, respectively, and it tapers off to a few per cent and less at frequencies higher than 0.1 year\(^{-1}\). In other words, the operation of linear-trend removal from this climatic time series has resulted in a substantial loss of power density belonging to the long-term climatic variability. This shows that one should be extremely cautious when deciding to remove even statistically significant linear trends from oceanographic (or any other) climatic time series.

The knowledge of statistical parameters of the AR model fitted to the time series of SST allows one to simulate the sea-surface temperature of the World Ocean, thus extending the available data into the future and into the past for arbitrarily long time intervals. Basically, this can be done by using Eq. (2) to forecast and hindcast the time series \(p = 3\) steps into the future and into the past, and then continue in both directions with simulated pseudorandom white noise values \(a_t\). By changing the sequence \(a_t\), one would obtain different values of simulated temperature for values of \(t\) at \(t < 1850\) and \(t > 2009\) while the observed...
Anomalies will always be the same. Note that the spectral density of the simulated SST coincides with that of the original time series. The resulting simulated time series has a probability distribution function that is nearly Gaussian.

An example of simulated SST data is shown in Figure 3. The dashed horizontal lines show the approximate band, within which variations of SST ought to stay with probability 0.9. As seen from the figure, they indeed stay within that interval most of the time. Part of the simulated data between 2100BP and 2001BP is shown in Figure 4 for illustration along with the observed data from 1910 through 2009.

This experiment can be repeated as many times as needed and every time it will produce a sample realization of the above-described stationary random process which will, if need be, always contain the time series of SST observed between 1850 and 2009. The data are generated without any deterministic external forcing and, therefore, present simulations of the natural variability of climate of the World Ocean as described by our stochastic model. The simulations look quite realistic indicating that the temperature might have varied by 1.5°C in 1000 years. Such variations do not seem excessive and by themselves they can hardly be viewed as substantial changes of the current climate.

A Monte Carlo experiment with filtered data ($M = 10$) shows that runs of about 35-65 years during which the temperature increases by $\sim 0.5^\circ$C are not exceptional (406 occurrences in 200,000 years) and take about 10% of the total time span. The average recurrence
Figure 4. Anomalies of observed (1910–2009) and simulated (2100BP–2001BP) annual global sea-surface temperature.

time of such variations is 493 years which means that global warming events similar to what is being observed now in the World Ocean could have happened several times during the last two millennia. In other words, decades-long runs of general increase in the annual sea-surface temperature by $\sim0.5^\circ$C do not seem to be a very rare event in the climate of the World Ocean.

A similar study has been conducted for the annual global surface temperature and it produced basically the same results.

Note that using this specific type of stochastic model for the time series of temperature anomalies is hardly critical for the purpose of our study. For example, one could have assumed that the time series of SST presents a sample of a stationary random long-memory process such as described in Mandelbrot and Van Ness (1968) or Beran (1994) and arrived at a similar conclusion, probably, with even a shorter recurrence time because the spectral density of such processes tends to infinity as frequency tends to zero.

In conclusion,

- the warming of the World Ocean, which has been observed since the middle of the twentieth century, can be fully explained by the natural variability of climate;
the removal of a linear trend from a climatic time series just on the basis of the trend estimate’s statistical significance may substantially reduce the natural long-term climatic variability present in the time series; this operation requires additional justification.

We believe that both results could be used to improve physical models of the World Ocean climate.

REFERENCES


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