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ABSTRACT

In sharp contrast to most previous crisis episodes, the Treasury market experienced severe stress and illiquidity during the COVID-19 crisis, raising concerns that the safe-haven status of U.S. Treasuries may be eroding. We document large shifts in Treasury ownership during this period and the accumulation of Treasury and reverse repo positions on dealer balance sheets. To understand the pricing consequences, we build a model in which balance sheet constraints of dealers and demand/supply shocks from habitat agents determine the term structure of Treasury yields. A novel element of our model is the inclusion of levered investors' repo financing as part of dealers' intermediation activities. Both direct holdings of Treasuries and reverse repo positions of dealers are subject to a regulatory balance sheet constraint. According to the model, Treasury inconvenience yields, measured as the spread between Treasuries and overnight-index swap (OIS) rates, as well as spreads between dealers' reverse repo and repo rates, should be increasing in dealers' balance sheet costs. Consistent with model predictions, we find that both spreads are large and positive during the COVID-19 crisis. We further show that the same model, adapted to the institutional setting in 2007-2009, also helps explain the opposite signs of repo spreads and Treasury convenience yields during the financial crisis.

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1 Introduction

U.S. Treasury bonds are generally viewed as some of the most liquid and safe assets in the world. Their safety and liquidity is reflected in a price premium (Longstaff (2004); Krishnamurthy and Vissing-Jorgensen (2012)). During periods of financial market turmoil when prices of risky and illiquid assets fall dramatically due to a flight-to-safety and flight-to-liquidity, the price premium of Treasuries typically rises (Nagel (2016); Adrian, Crump, and Vogt (2019)). More generally, Treasury bonds have had negative beta in recent decades, rising in price when stock prices fall (He, Krishnamurthy, and Milbradt (2019); Baele, Bekaert, Inghelbrecht, and Wei (2019); Campbell, Pflueger, and Viceira (2019)); Cieslak and Vissing-Jorgensen (2020)).

Events in March 2020 during the COVID-19 crisis did not follow this established crisis playbook. Like in many previous periods of financial market turmoil, stock prices fell dramatically, the VIX of implied stock index return volatility spiked, credit spreads widened, the dollar appreciated, and prime money market funds experienced outflows. Yet, in sharp contrast to previous crisis episodes, prices of long-term Treasury securities fell sharply. From March 9 to 23 when the stock market experienced four halts, the 10-year Treasury yield increased up to 60 basis points, resulting in a striking and unusual positive correlation between stock and bond returns (see Figure 1). Widening bid-ask spreads and collapsing order book depth indicated market illiquidity in the Treasury bond market (Fleming and Ruela (2020)). In direct response, the Federal Reserve (Fed) offered essentially unconstrained short-term financing to primary dealers for their Treasury positions, but the take-up was very low.¹

Why was it different this time? Given the Treasury bond market’s outsized role in the financial system, this stunning deviation from historical correlations in recent decades calls for an explanation. Are the events in March 2020 the canary in the coal mine indicating a fundamental change in the properties of Treasury bonds away from being a negative-beta flight-to-safety target asset? Or can the surprising price movements of Treasury bonds be attributed to market dysfunctionality induced by frictions? Our goal in this paper is to shed light on this question, both empirically and theoretically.

Notes: This figure plots daily series of the constant-maturity Treasury (CMT) yields in percent, of 3-month and 10-year maturities (left panel), and of the VIX (right panel), from January 1, 2020 to March 27, 2020.

We start by characterizing major features of asset price movements and investor flows during the crucial weeks in March 2020. A simple explanation of the rise in long-term Treasury yields would be that the enormous fiscal burden of the COVID-19 pandemic triggered a shift in inflation expectations and inflation uncertainty. However, asset price data suggest that this is unlikely. Prices of inflation-protected bonds (TIPS) fell along with the prices of nominal Treasuries. Inflation-swaps show no increase in risk-neutral inflation expectations. Prices of inflation caps and floors do not provide evidence of an increase in inflation uncertainty either.

An alternative explanation would be that the cyclicality of real interest rates has changed. As Campbell, Sunderam, and Viceira (2017) emphasize, the negative beta of Treasuries in the decades leading up to 2020 partly reflects a positive correlation between stock prices and real interest rates. That March 2020 represents a regime-shift towards a negative correlation cannot be ruled out at this point. But it would be difficult to come up with an economic mechanism that explains this shift. In the post-WWII history examined by Campbell, Sunderam, and Viceira (2017), the only major episode with pro-cyclical bond prices (or, counter-cyclical real interest rates) was the Volcker disinflation of the early 1980s where the rise of real interest rates in a recession was induced by contractionary monetary policy intended to crush inflation. This is clearly not what happened in March 2020.

We therefore turn to an examination of investor flows to understand whether supply and demand...
balances may have interacted with intermediation frictions to give rise to the unusual price movements in the Treasury market. Foreign investors, including foreign central banks and investors in tax havens, sold about $300 billion (bn) worth of Treasuries, mutual funds sold around $15bn, and the U.S. Treasury issued about $150bn net. Much of this supply was temporarily accommodated by broker-dealers, partly through somewhat higher direct holdings, but also indirectly through a massive expansion of $400bn in repo financing that primary dealers provided to levered investors. Eventually, starting at the end of March, the Federal Reserve came in and purchased $700bn worth of Treasury notes and bonds, and the expansion in dealer balance sheets reverted back. The selling pressure and its eventual accommodation by the Federal Reserve were concentrated in long-term Treasuries. There was little net selling by foreigners in T-bills and the Federal Reserve did not expand their T-bill holdings.

While these are big shifts in the ownership of Treasuries, it is far from obvious that they could induce substantial increases in Treasury yields in a market that is usually thought to be extremely liquid. Why didn’t financial intermediaries and agile institutional investors financed by intermediaries accommodate this supply more elastically? We argue that balance sheet constraints of dealers played a key role.

We build on the preferred habitat model of Vayanos and Vila (2020) and Greenwood and Vayanos (2014) to understand how a negative demand shock for long-term Treasuries can affect the term structure of Treasury yields. In the preferred habitat model, dealers intermediate the exogenous demands of habitat investors. Since repo financing was an important part of dealers’ intermediation activities in March 2020, we extend this model to allow levered investors (hedge funds) to take positions in Treasuries financed by borrowing from dealers in the repo market. Moreover, dealers are subject to a balance sheet constraint, consistent with the supplementary leverage ratio (SLR) that dealers are subject to following the reforms adopted after the 2007-09 financial crisis. Importantly, both direct holdings of Treasuries and reverse repo positions that finance levered investments by hedge funds take up balance sheet space.

Dealers therefore demand compensation for the shadow cost of balance sheet expansion via direct holdings (with compensation in the form of higher yield) or repo (with compensation in the form of higher repo rates). As a consequence, when habitat agents’ desire for direct holdings of Treasuries drops, the degree to which a rise in yields makes levered investments in Treasuries...
attractive to hedge funds is limited by the simultaneous rise in repo rates charged by dealers. The repo friction hurts the demand for Treasury bonds in two ways, first through a more costly current repo funding, and second via the anticipation of a reduced collateral value of Treasuries in future financing. Therefore, dealers’ direct holdings and yields need to rise even more in order to clear the market. In equilibrium then, the yield curve steepens and repo rates rise above frictionless risk-free rates.

Empirically, we find support for these predictions. To measure repo rates at which dealers lend to levered investors, we use the General Collateral Finance (GCF) repo rates (see Fleming and Garbade (2003)). Much of the activity in the GCF repo market involves large dealers lending to smaller ones. In this sense, these rates reflect the conditions at which large dealers are willing to lend against general Treasury collateral (Baklanova, Copeland, and McCaughrin (2015)). We compare the GCF rate to Triparty repo rates. Most of the financing flow in the Triparty market involves cash lenders like money market funds lending to large dealers (Baklanova, Copeland, and McCaughrin (2015)). Consistent with the balance sheet cost explanation, we find that GCF repo rates substantially exceeded Triparty repo rates at the time when Treasury yields spiked.

In the model, the rise in long-term yields has two components. The first component is a heightened risk premium because dealers demand compensation for the interest-rate risk exposure that the expansion of their direct Treasury holdings entails. The second component reflects the inconvenience yield induced by the balance sheet cost due to the SLR constraint. To isolate this second component, we use the dealers’ pricing kernel to price a derivative asset that offers exactly the same cash-flows as physical Treasury bonds, but without the balance sheet cost. We think of this derivative asset as an overnight index swap (OIS).² Practically, the weight imposed by the SLR constraint on interest rate derivative contracts is about two orders of magnitude smaller than the weight on Treasury securities, so zero balance sheet cost is a good approximation.

In line with the model’s predictions, we find that during the two weeks of turmoil, Treasury yields rose substantially above maturity-matched OIS rates, reflecting the inconvenience yield. Other measures of the Treasury convenience yield have been eroding since the Great Recession (Du, Im, and Schreger (2018)). Based on the findings in Krishnamurthy and Vissing-Jorgensen (2012), the rise in the supply of U.S. Treasuries since the Great Recession could also have contributed to

²The OIS rate can be interpreted as the risk-neutral expectation of the expected Federal Funds Target Rate.
a disappearing convenience yield. Viewed from this perspective, the rise of Treasury yields relative to OIS rates in March 2020 is a further extension of this phenomenon.

The inconvenience yield of Treasuries during March 2020 is particularly striking in contrast with the financial crisis in 2007–09 (or, the Great Recession). Flight-to-safety and liquidity during the early stages of the financial crisis until mid-2008 pushed Treasury yields below OIS rates. Dealers came into the financial crisis with a short position in Treasuries. Rather than having to absorb a supply of Treasuries like in March 2020, dealers were scrambling to obtain more Treasuries. As a consequence, dealers were willing to lend cash to obtain Treasury collateral in the GCF repo market at much lower rates—about 70 bps—than they were charged by cash lenders in the triparty repo market. Unlike in COVID-19 crisis in March 2020, the Federal Reserve took action in 2008 to increase the supply of Treasuries in the market, for example allowing dealers to obtain Treasuries against non-Treasury collateral in the Term Securities Lending Facility (TSLF). This seems to have alleviated the shortage of Treasuries, leading to a closing of the Treasury-OIS and triparty-GCF rate spreads.

These empirical patterns in 2007-09 financial crisis (or, the Great Recession) are consistent with our model, too, but under different conditions. If the dealer sector is short in Treasuries, and habitat investors demand more direct holdings of Treasuries, consistent with the widespread flight-to-safety during the financial crisis, then the Treasury-OIS and triparty-GCF rate spreads should switch signs compared with March 2020, consistent with our empirical observations. Our model therefore provides a unified account of the very different Treasury and repo market dislocations in the 2007-09 financial crisis and COVID-19 crisis.

In summary, the observed movements in Treasury yields and spreads in March 2020 can be rationalized as a consequence of selling pressure that originated from large holders of Treasuries interacted with intermediation frictions, including regulatory constraints such as the SLR. Evidently, the current institutional environment in the Treasury market is such that it cannot absorb large selling pressure without substantial price dislocations or intervention by the Federal Reserve as the market maker of last resort. Indeed, the Fed announced that it would directly purchase Treasuries “to support the smooth functioning of markets” on March 15 and 23, which alleviated market stress (as seen in Figure 1). Consistent with our model particularly, the Fed also announced that it would
temporarily exempt Treasuries from the SLR on April 1.\textsuperscript{3}

Our theory and evidence can explain why selling pressure had such a strong price impact, but it does not answer the question of what motivated some large holders of U.S. Treasuries to sell in March 2020.\textsuperscript{4} Nor does it answer the question of why dealers, levered investors financed by dealers, and ultimately the Federal Reserve, had to absorb this additional supply, while other long-term investors stayed away.

The fact that the Federal Reserve was able to alleviate the dislocations by substantially tilting the maturity structure of U.S. government liabilities away from the long-term securities it purchased towards very short-term liabilities it created (reserves) invites comparisons with emerging-markets crises where shortening of maturities by sovereign issuers is a typical response to investors’ concerns about issuers’ ability to repay (Broner, Lorenzoni, and Schmukler (2013)). But since neither inflation nor default risk concerns are apparent in derivatives prices in March 2020, there is little to suggest that concerns about the U.S. fiscal situation are the underlying cause, although the “safe” asset status of U.S. Treasuries should be disciplined by fundamental fiscal capacities (e.g., He, Krishnamurthy, and Milbradt (2019)).

Related literature. Two related works, Duffie (2020) and Schrimpf, Shin, and Sushko (2020), also provide some evidence on how the Treasury market has been stressed by COVID-19 recently. They mainly focus on raising policy proposals that can potentially make the Treasury market robust to shocks. A number of empirical studies document the change of corporate bond price and liquidity during the COVID-19 period, including D’Amico, Kurakula, and Lee (2020), Haddad, Moreira, and Muir (2020), Kargar, Lester, Lindsay, Liu, Weill, and Zuniga (2020), Qiu and Nozawa (2020), and O’Hara and Zhou (2020).

An expanding literature studies safe assets’ supply, demand, and convenience premia, including early studies by Bansal and Coleman (1996), Duffee (1996), and Longstaff (2004), and recent studies by Krishnamurthy and Vissing-Jorgensen (2012), Bansal, Coleman, and Lundblad (2011),}\textsuperscript{3}

\textsuperscript{3}See https://www.federalreserve.gov/newsevents/pressreleases/monetary20200315a.htm for the announcement of direct Treasury purchases and https://www.federalreserve.gov/newsevents/pressreleases/bcreg20200401a.htm for the announcement of the temporary change to the SLR rule.

\textsuperscript{4}The shock that initially triggered a large selling pressure of Treasury bonds during the COVID-19 crisis was reportedly caused by a scramble for cash. For example, cash is pursued by corporations for payroll and operation, by foreign central banks for potential fiscal stimuli, and so on. As the prices of Treasury bonds tanked, levered investors like hedge funds who had been taking arbitrage positions (e.g., between the Treasury cash and futures markets) conducted “fire-sales” and amplified the initial shock (Schrimpf, Shin, and Sushko (2020)).

We build our model based on Vayanos and Vila (2020) and Greenwood and Vayanos (2014), who study the equilibrium Treasury pricing where a risk-averse intermediary sector absorbs exogenous demand/supply shocks from the habitat agents. We introduce Poisson events which capture temporary (and potentially large) supply shocks in their model, and more importantly the levered hedge fund sector that borrows from the dealers via the repo market. The endogenous repo spread (i.e., the difference between the GCF repo lending rates and the Triparty repo borrowing rates) and its connection to the broad demand/supply in the Treasury market are the major contributions of this paper. We further provide strong empirical support for these theoretical predictions.


2 Motivating Evidence and Institutional Background

To set the stage for our main analysis, we first provide further evidence on the Treasury market disruption during the COVID-19 crisis, in addition to Figure 1. The evidence motivates us to examine the interaction of supply and demand balances with intermediation frictions as the potential economic mechanism. Accordingly, we also introduce institutional background that is useful for the development of our institutional trading model of the Treasury market.

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5The theoretical literature has modeled several economic channels, such as on information sensitivity, complexity, and coordination (Gorton and Pennacchi (1990), Caballero and Simsek (2013), He, Krishnamurthy, and Milbradt (2019), and Farhi and Maggioir (2017)). See Gorton (2017) and Caballero, Farhi, and Gourinchas (2017) for two broad surveys on safe assets.
2.1 Motivating Evidence

A simple explanation of the soaring 10-year Treasury yield would be that the expected fiscal burden of the COVID-19 pandemic can trigger a shift in inflation expectations and inflation uncertainty. The left panel of Figure 2 plots weekly time series of two market-based measures of inflation expectation, the 10-year breakeven inflation rate that equals the difference between the 10-year CMT nominal and TIPS yields and the 10-year inflation swap rate.\(^6\) Both of them fell throughout the COVID-19 period, especially in March 2020. The right panel plots weekly series of measures of the inflation uncertainty and the (risk-neutral) probability of a large increase in inflation (of more than 3%), extracted from 5-year inflation caps and floors by the Federal Reserve Bank of Minneapolis. We observe that the inflation uncertainty dropped significantly until mid-March, and increased afterwards. The probability of a large increase in inflation has dropped and stayed low throughout April, pointing to concerns about deflation rather than inflation.

Figure 2: Inflation Expectation and Uncertainty During the COVID-19 Crisis

Notes: The left panel plots weekly time series of the 10-year breakeven inflation rate (that equals the difference between the 10-year CMT nominal and TIPS yields) and the 10-year inflation swap rate, both in percentage. The right panel plots weekly series of the (risk-neutral) standard deviation and probability of a more than 3% increase in inflation that are estimated using 5-year inflation caps and floors. The sample period is from January 1, 2020 to April 13, 2020.

We then examine investor flows in the Treasury market to understand the changes in supply and demand balances, and their potential association with price movements. From the top left

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\(^6\) As in Figure 1, we consider three event dates, 1/30/2020 when the World Health Organization declared the outbreak of coronavirus a global health emergency, 3/9/2020 when the S&P 500 index declined by 7%, triggering the first market-wide circuit breaker trading halt, and 3/23/2020 when the Fed announced “its full range of tools” to support the U.S. economy including unlimited purchases of Treasury securities.
panel of Figure 3, we observe a net positive issuance of $150bn in March by the U.S. Treasury. However, foreign investors, including foreign central banks and investors in tax havens, sold about $300bn worth of long-term Treasuries (middle left panel), while mutual funds sold around $15bn (bottom left panel). Much of this supply was temporarily accommodated by broker-dealers, partly through somewhat higher direct holdings (top right panel), but much of it indirectly through a massive expansion of $300bn in repo financing that primary dealers provided to levered investors (middle right) before 3/9.\textsuperscript{7}

Yet, during the week from March 9 to 15 when the 10-year Treasury yield increased most sharply (see Figure 1), primary dealers’ direct Treasury holdings remained almost flat, while their reverse repo lending increased by another $120bn. The Fed offered $1.5 trillion repo funding to primary dealers on March 12 (i.e., they would be able to access the funding at low repo rate), but the taking was abysmally low. Hence, dealers seem to face certain balance sheet constraints. Eventually, starting from March 15, the Federal Reserve came in and purchased $700 billion worth of Treasury notes and bonds, and the expansion in dealer balance sheets reverted back. On balance, primary dealers’ net cash Treasury positions and reverse repo amounts decreased in March. The selling pressure and its eventual accommodation by the Federal Reserve were concentrated in long-term Treasuries. There was little net selling by foreigners in T-bills and the Federal Reserve did not expand their holdings of T-bills.

In sum, examinations of asset price movements and investor flows during the crucial weeks in March 2020 suggest that shifts of fundamental risks are unlikely to have caused the unusual price movements in the Treasury market. Instead, the large shifts in the supply of Treasuries, primary dealers’ balance sheet movements, and the nature of the Federal Reserve’s interventions point to an economic interpretation based on institutional trading with frictions. To understand the relevant frictions, we briefly introduce some institutional background.

### 2.2 Institutional Background

In this section, we first outline institutional features of the Treasury and repo markets, especially the important role of dealers as intermediaries in both markets. We then discuss the key regulations

\textsuperscript{7}Moreover, primary dealers are net borrowers in the repo market. Hence, they take positive market-making portfolios of Treasuries and fund them with repos. See Appendix B.2 for further details.
Figure 3: Investor Flows and Positions of Treasuries during the COVID-19 Crisis

Notes: The left panels plot monthly series of the net issuance amount of Treasury notes and bonds (top left panel), the changes in holdings of long-term Treasuries of foreign investors (middle left panel), and the net flows of long-term government mutual funds (bottom left panel). The right panels plot weekly series of the primary dealers’ net positions of Treasuries (top right panel), the primary dealers’ gross reverse repo amount (middle right panel), and the amount of the Fed’s holdings of Treasuries (bottom right panel). The units are all in billions of U.S. dollars.
that were introduced in the U.S. since the 2007-09 financial crisis and their potential impacts on the Treasury markets and dealers.\textsuperscript{8}

### 2.2.1 The Treasury and Repo Markets

The U.S. Treasury market is one of the largest fixed-income markets with an outstanding balance held by the public of about $17 trillion as of March 2020. The major component is coupon-bearing Treasuries, about $12.5 trillion, while T-bills comprise the second largest fraction, about $2.6 trillion.\textsuperscript{9} The secondary cash market of Treasuries maintains an average daily total trading volume of about $575bn over the period of August 2017 to July 2018, of which coupon Treasuries and T-bills account for the largest bulk, about 82% and 15%, respectively. About 70% of the trading volume is concentrated in on-the-run securities, which are most recently auctioned of a given tenor, while the rest is in off-the-run securities, which consist of all the previously issued securities.\textsuperscript{10}

Off-the-run securities account for the major fraction—over 95%—of outstanding Treasuries (Clark, Cameron, and Mann (2016)) and were hit hardest by the COVID-19 market disruption, though on-the-run securities were also notably affected (Fleming and Ruela (2020)). In contrast to on-the-run securities that are mainly traded on electronic exchange-like platforms, investors of off-the-run Treasuries mostly trade over-the-counter with broker-dealers as readily available counterparties.

Broker-dealers, especially the primary dealers that are trading counterparties of the Federal Reserve Bank of New York in its implementation of monetary policy, are important participants in both the primary and secondary markets of Treasuries. In particular, primary dealers are expected to participate in all issuance auctions of Treasuries, and have traditionally been the predominant purchasers at these auctions.\textsuperscript{11} Dealers are also key intermediaries of the Treasury cash market,


\textsuperscript{9}The rest are Treasury Inflation-Protected Securities (TIPS), floating rate notes (FRNs), and Separate Trading of Registered Interest and Principal Securities (STRIPS). The outstanding balance is obtained from the SIFMA (https://www.sifma.org/resources/research/us-fixed-income-issuance-and-outstanding/).

\textsuperscript{10}The summary of trading volume can be found at https://libertystreeteconomics.newyorkfed.org/2018/09/unlocking-the-treasury-market-through-trace.html.

\textsuperscript{11}The primary dealers’ share of purchases at issuance auctions has been declining over time, though; see Fleming and Myers (2013) for further discussions. He, Kelly, and Manela (2017) document the role of primary dealers in understanding the cross-sectional returns of asset prices outside the equity class.
accounting for about 75% of all transactions. In fact, all client transactions, which account for half of the total $575 billion daily trading volume, go through dealers.\footnote{The so-called principal trading firms that specialize in electronic and automated intermediation only participate in the inter-dealer segment.}

When intermediating Treasury trades of clients, dealers need to use their balance sheet to hold inventories. For example, when investors with Treasuries receive a liquidity shock, they sell holdings to dealers who take them on their balance sheet as inventories before finding ultimate buyers. Carrying inventory entails risk for which dealers demand compensation, as in classical modes of market-making (Ho and Stoll (1981) and Amihud and Mendelson (1980)). However, given the safety of Treasury securities, the inventory risk is unlikely to be severe. The cost that particularly interests us arises from balance sheet constraints associated with post-crisis regulations, which we will review below.

In addition to selling them outright in the Treasury cash market, investors often post Treasuries as collateral to borrow cash on a short-term basis, particularly in the repo market. The U.S. repo market is comprised of two segments, triparty repo and bilateral repo. A triparty repo involves a third party known as a clearing bank who provides clearing and settlement services such as keeping the repo on its books and ensuring the execution according to repo terms.\footnote{In the U.S., tri-party repo services have been offered by Bank of New York Mellon and JP Morgan Chase. The latter announced a plan to exit this business in 2019 (https://www.wsj.com/articles/j-p-morgan-to-exit-part-of-its-government-securities-business-1469135462).} Differently, in bilateral repo, the clearing and settlement are managed by each counterparty’s custodian bank. Furthermore, within the triparty repo market, a special General Collateral Financing Repo Service (GCF Repo) allows securities dealers registered with the Fixed Income Clearing Corporation (FICC) as netting members to trade repos among themselves. That is, the GCF repo is mainly an inter-dealer market, while in the non-GCF triparty repo market (referred to as triparty repo hereafter), broad cash lenders including money market mutual funds (MMFs), banks, and securities lenders lend cash to dealers.

As in the cash Treasury market, broker-dealers are also key intermediaries in the repo market, transmitting funds from lenders to borrowers who cannot directly deal with each other for certain reasons.\footnote{Dealers also obtain funding for their market-making inventories in the triparty repo market by posting Treasuries as collateral assets. This is effectively similar to the strategy of levered investors like hedge funds who finance their proprietary portfolios through repo funding. The hedge fund sector in our model includes this strategy of dealers (see Section 3 for details).} In particular, large high-credit-profiles dealers borrow cash in the triparty market from
cash lenders and lend to small low-credit-profiles dealers in the GCF market. Large dealers also borrow cash in the triparty market and lend to levered investors especially hedge funds in bilateral repo markets.\textsuperscript{15} Hence, both the GCF and bilateral repo markets are used by levered investors to finance their cash Treasury positions, where the funds are transmitted by large dealers from the triparty market.

In addition to the intermediation activity on a cash basis, dealers also conduct intermediation in the (mainly bilateral) repo market on a security basis, which is often used to source securities and facilitate short-selling. For example, dealers can lend cash to providers of Treasury securities, and use the received collateral assets (or provide them to hedge funds) to cover short sales or establish hedging positions in the cash Treasury market.\textsuperscript{16} Typical providers of Treasury securities include pension funds, mutual funds, and insurance companies who hold large portfolios of Treasuries and seek to earn extra yield by lending them out. Although securities-driven repo often targets specific Treasuries, denoted as special repo as in Duffie (1996), it is also used widely to source general securities when Treasuries are in shortage as a whole. In either case, short-selling incurs costs of searching for securities and failing to deliver, which can lead to large market disruptions (Duffie, Garleanu, and Pedersen (2002), Fleming and Garbade (2004), and Garbade, Keane, Logan, Stokes, and Wolgemuth (2010)). The implementation of the Fed’s TSLF program from March 2008 to June 2009 that lent Treasuries to dealers against non-Treasury collateral suggests that such costs can be too high for market participants.

\subsection*{2.2.2 Post-Crisis Regulations}

As discussed above, dealers’ intermediation activities in the Treasury market are unlikely to pose severe risks to their balance sheets because of Treasuries’ great safety and liquidity. However, since the 2007-09 financial crisis, various regulatory reforms of financial institutions and markets have been proposed, some of which impose constraints on dealers’ balance sheet capacity (Duffie (2018)). Among them, the most relevant for the Treasury market is the so-called supplemental leverage ratio

\textsuperscript{15}In recent years, repo transactions cleared through the FICC Delivery-versus-Payment (DVP) repo service have increased, which facilitate cash flow from the triparty repo market to hedge funds. See \url{https://www.jpmorgan.com/global/research/sponsored-repo} for details.

\textsuperscript{16}In this regard, securities lending contracts are economically equivalent to repo for borrowing and lending securities (Baklanova, Copeland, and McCaughrin (2015)). Dealers who facilitate a hedge fund customer’s short positions, often known as prime brokers, can rehypothecate collateral assets from other customers’ long positions in the repo market, or borrow from securities lenders such as pension funds.
To strengthen the resilience of the global banking system, the Basel III regulatory framework introduced the SLR as a backstop to risk-based capital regulation. It is defined as the Tier I capital divided by total leverage exposure irrespective of its riskiness, which is distinct from the conventional risk-weighted-asset (RWA) capital requirement. The total leverage exposure includes both on-balance-sheet assets and off-balance-sheet exposures to derivatives. The Basel Committee proposed a 3% minimum leverage ratio, while U.S. regulators require global systemically important institutions (G-SIBs) to maintain an SLR of at least 5% on a consolidated basis and at least 6% for their depository subsidiaries. The rule of SLR was finalized in 2014 and was phased in from 2015 with G-SIBs and other large banking institutions required to make public disclosures related to the SLR, and was largely implemented in 2018.

The leverage exposure in the SLR includes the total notional of all cash and repo transactions, regardless of which securities are used as collateral, hurting the bank dealers’ intermediation activities in both cash and repo markets of Treasury securities greatly. As argued by Duffie (2018) based on a variant of debt overhang proposed in Andersen, Duffie, and Song (2019), “the SLR increases ‘rental cost’ for the space on a bank’s balance sheet.” The announcement of the temporary exemption of Treasuries from the SLR on April 1, 2020 by the Fed in response to the Treasury market disruption suggests that this is indeed the case.

Compared with the constraint on Treasury cash and repo positions, the constraint imposed by the SLR on standard interest-rate derivatives is minor. Specifically, the off-balance-sheet exposures to derivatives included in the SLR total leverage exposure are calculated based on the Current Exposure Method and consisting of the Current Exposure and Potential Future Exposure (Polk (2014)). For standard interest-rate derivatives like vanilla Libor swaps and OIS that have become centrally cleared, the CE is effectively zero because the variation margin is posted on a daily basis. The PFE is defined using a combination of net and gross risk exposures, equal to \( PFE = 0.4 \times A_{\text{gross}} + 0.6 \times NGR \times A_{\text{gross}} \), where \( A_{\text{gross}} \) is the adjusted gross notional equal to gross notional multiplied by a maximum of 1.5%, and \( NGR \) is the net-to-gross ratio equal to the net current mark-to-market value and gross current mark-to-market value. That is, the constraint imposed by SLR on interest-rate derivatives is about two orders of magnitude smaller than that on the Treasury positions.
In addition, we briefly discuss two other regulations—the Liquidity Coverage Ratio (LCR) and Volcker rule—that have been progressively put into effect since 2014. The LCR is to ensure that potential cash outflows over a 30-day period are sufficiently covered by liquid assets. Specifically, it assumes that dealers may lose all of their collateralized funding with terms of less than 30 days and hence stipulates that they need to hold sufficient cash and high-quality liquid assets to cover this loss of funding. The Volcker rule prohibits proprietary trading of banks (or financial institutions with access to FDIC insurance or the Federal Reserve’s discount window) that are financed by low-cost deposits of the affiliated bank branch. The Volcker Rule exempts hedging, market-making, and various financial instruments such as foreign exchange and government securities. Yet, the difficulty in drawing a clear-cut distinction between market-making and proprietary trading has the potential unintended consequences of reducing bank dealers’ market-making activities. Both the LCR and Volcker rule may lead to higher funding costs to dealers.

3 The Model

We now show within a model how supply shocks can interact with intermediation frictions to give rise to the observed Treasury market disruptions. Moreover, the model provides additional empirical predictions about spreads between different repo rates, swap rates, and Treasury yields that we examine subsequently.

The economy in this section is an extension to Greenwood and Vayanos (2014). In their model, preferred habitat agents trade with arbitrageurs. We separate the arbitrageurs in Greenwood and Vayanos (2014) into hedge funds and dealers, and introduce a repo market in which the former group borrows from the latter. Throughout, we use lowercase letters to denote an individual agent’s choices while uppercase letters denote aggregate quantities.

3.1 Aggregate Shocks and Assets

There are two sources of aggregate risk in this model, following Greenwood and Vayanos (2014). The first is the stochastic evolution of short interest rate $r_t$, which follows an Ornstein–Uhlenbeck process.
process

\[ dr_t = \kappa (\bar{r} - r_t) \, dt + \sigma dZ_t, \]

where \( \{Z_t : 0 \leq t < \infty\} \) is a standard Brownian motion defined on a complete probability space, \( \kappa \) is the mean-reverting parameter, and \( \sigma \) is the volatility of short-rate.

The second aggregate shock is Treasury demand/supply shock \( \tilde{\beta}_t \), which follows a Markov chain \( \tilde{\beta}_t \in \{0, \beta\} \). The jump intensity from \( \tilde{\beta}_t = 0 \) (\( \tilde{\beta}_t = \beta \)) to \( \tilde{\beta}_t = \beta \) (\( \tilde{\beta}_t = 0 \)) is denoted by \( \xi_0 \) (\( \xi_\beta \)). We interpret \( \tilde{\beta}_t = 0 \) (\( \tilde{\beta}_t = \beta \)) as the normal (stress) state; and our model can capture both demand and supply shocks depending on the sign of \( \beta \).

We consider zero-coupon Treasury bonds which mature at the tenor \( \tau \in [0, T] \). Denote by \( P_t(\tau) \) their endogenous price to be solved in equilibrium.

### 3.2 Habitat Agents

Following Greenwood and Vayanos (2014) we consider an exogenous demand/supply shock from habitat agents, so that their holdings of bonds with tenor \( \tau \) is

\[ H_t(\tau) = -\theta(\tau) \tilde{\beta}_t, \]

where \( \theta(\tau) \) captures the exposure to the shock. Without loss of generality, let \( \theta(\tau) \geq 0 \). The case of \( \beta > 0 \) corresponds to an exogenous supply shock to the economy, while the case of \( \beta < 0 \) it represents a demand shock. These habitat agents represent insurance companies, pension funds, and/or foreign central banks in practice.

Let \( \Theta \equiv \int_0^T \theta(\tau) \, d\tau \). We first analyze the model without specifying the function \( \theta(\cdot) \). Depending on applications, we later specialize the function \( \theta(\cdot) \) either to

\[ \theta(\tau) = \begin{cases} 
1, & \text{for } \tau > \hat{\tau}, \\
0, & \text{otherwise}, 
\end{cases} \]

so that the (negative) demand shock hits the long-end of the curve, or let \( \theta(\tau) = 1 \) for all \( \tau \in [0, T] \) so that the demand shock applies to the entire curve.
3.3 Hedge Funds and Repo

A unit measure of hedge funds in this economy can borrow from dealers in the repo market to exploit the investment opportunity created by aggregate demand/supply shocks $\tilde{\beta}$. When a hedge fund borrows one dollar from a dealer by pledging one dollar of Treasury bonds as collateral, she needs to pay an endogenous repo financing rate of $R_t dt$.

Following Greenwood and Vayanos (2014) we assume that each hedge fund solves the following instantaneous mean-variance objective at time $t$

$$\max_{q^h_t(\tau) \geq 0} \mathbb{E}_t \left[ dG^h_t \right] - \frac{1}{2\rho_h} \text{Var}_t \left[ dG^h_t \right]. \quad (3)$$

where $dG^h_t$ is the hedge fund’s trading gain, and $\rho_h > 0$ is the hedge fund’s risk-bearing capacity (or risk tolerance, the inverse of their absolute risk-aversion). Given the repo financing cost $R_t$ which will be determined in equilibrium, the dynamics of her wealth is given by

$$dG^h_t = \int_0^T q^h_t(\tau) d\tau \begin{pmatrix} \frac{dP_t(\tau)}{P_t(\tau)} - R_t dt \end{pmatrix} d\tau. \quad (4)$$

For each $\tau \in [0, T]$, the hedge fund sector’s repo demand, denoted by $q^h_t(\tau)$, in general depends on the repo financing cost $R_t$; the higher the $R_t$, the greater the demand. This price-dependence of repo demand is one of our key contributions relative to Greenwood and Vayanos (2014).

3.4 The Dealer Sector

A unit measure of risk-averse dealers absorb the residual Treasury supply/demand shocks and provide overnight repo funding to the hedge fund sector.

3.4.1 Dealer’s problem

Following Greenwood and Vayanos (2014) we assume that each dealer solves the following instantaneous mean-variance objective at time $t$:

$$\max_{x_t(\tau), q^d_t(\tau) \geq 0} E_t \left[ dw^d_t \right] - \frac{1}{2\rho_d} \text{Var}_t \left[ dw^d_t \right], \quad (5)$$
where $\rho_d > 0$ is the dealer’s risk-bearing capacity, and the dynamics of his wealth, $dw_t - w_t r_t dt$, is given by

$$\int_0^T x_t(\tau) \left( \frac{dP_t^r}{P_t^r} - r_t dt - \frac{\Lambda_t dt}{B/S \text{ cost}} \right) d\tau + \int_0^T q_t^d(\tau) \left( \frac{\Delta_t}{\text{repo wedge}} - \frac{\Lambda_t}{B/S \text{ cost}} \right) dtd\tau$$  \hfill (6)

subject to

$$x_t(\tau) + q_t^d(\tau) \geq 0.$$  \hfill (7)

Here, $x_t(\tau)$ is the direct holdings of bond $\tau$ (in terms of dollars, which could be negative), and $q_t^d \geq 0$ is the dealer’s reverse repo position (i.e., repo funding provided to hedge funds).

In Eq. (6), we have defined

$$\Delta_t \equiv R_t - r_t,$$  \hfill (8)

which captures the wedge between the collateralized borrowing rate $R_t$ in the repo market and the instantaneous risk-free lending rate $r_t$. We do not specify in more detail the funding markets in which dealers borrow at rate $r_t$. Empirically, we approximate $r_t$ with the repo rate from the triparty market because funding flows in this market are mainly from cash lenders like money market funds to large dealers. We proxy for $R_t$ with the GCF repo rate because the funding flows in the GCF are mainly from large dealers to smaller dealers. The wedge between GCF and Triparty repo rates therefore captures the wedge between the rates at which large dealers lend and borrow in collateralized funding markets and it corresponds to $\Delta_t$ in our model. This GCF-Triparty repo wedge is a riskless profit earned by the dealer sector in equilibrium. In the next section, we explain the additional balance sheet cost $\Lambda_t$ in the dealer’s budget equation (6) that allows this wedge to exist.

In the baseline model we rule out “naked” short-selling for our dealers, as required by (7). We envision that repo is the only channel through which dealers borrow the bonds and then short-sell them to habitat agents. As a result, $x_t(\tau)$, whenever it is negative, has to be bounded by the dealer’s reverse repo position $q_t^d \geq 0$; this gives rise to the constraint $x_t(\tau) + q_t^d(\tau) \geq 0$ in (7). In practice, dealers can also resort to borrowing bonds in the costlier securities lending market, or even engage in “naked” short sales by failing to deliver the short-sold bond (delivery failures were quite common around the time of 2007-09 financial crisis). We relax the no-naked-short-selling condition
(7) later in Section 4.3 when we apply our model to the episode of 2007-09 financial crisis.

### 3.4.2 Balance sheet cost

Compared with Greenwood and Vayanos (2014), we study the repo market in which dealers provide repo services $q_t^d(\tau)$ in (6), in addition to their portfolio choice $x_t(\tau)$. What is more, dealers face an additional balance sheet cost—denoted by $\Lambda_t$—in their portfolio choices in our model. Each individual dealer takes the marginal cost $\Lambda_t$ as given, so the total effect is linear in their own positions $x_t(\tau)$ and $q_t^d(\tau)$. The balance sheet cost $\Lambda_t$ depends on the aggregate holdings only; one can think of a frictionless inter-dealer market which equalizes these costs across dealers.

Recall that for each tenor $\tau$, the aggregate bond holdings in the dealer sector is $X_t(\tau)$, and the aggregate reverse repo $Q_t^d(\tau)$. What is the balance sheet size for tenor $\tau$? There are two cases to consider.

1. If $X_t(\tau) \geq 0$, then the accounting is straightforward. The balance sheet occupied by tenor-$\tau$ bonds, denoted by $B_t(\tau)$, is simply

$$B_t(\tau) = X_t(\tau) + Q_t^d(\tau).$$

Denote $Q_t^d \equiv \int_0^T Q_t^d(\tau) d\tau$. Integrating over $\tau \in [0, T]$, we can calculate the balance sheet size of the entire dealer sector as

$$B_t \equiv \int_0^T X_t(\tau) d\tau + Q_t^d.$$

(9)

Note, all equilibrium aggregate variables, $X_t(\tau)$, $Q_t^d$ and $B_t$, will depend on the aggregate bond demand shock $\tilde{\beta}_t$.

2. When $X_t(\tau) < 0$ (i.e., when the dealer is short selling some tenor-$\tau$ bonds), then

$$B_t(\tau) = Q_t^d(\tau).$$

To see this, recall that we have ruled out naked short selling in Section (3.4). To sell short a bond, the dealer therefore must first borrow the bond via the repo market. The dealer’s
short sale may be smaller than than his entire reverse repo position; in this case, the dealer’s liability side books both the short sale and his tri-party repo market borrowing.\footnote{As an example, suppose that a dealer engages in \( q^d (\tau) = 3 \) repo lending, and at the same time holds a short position \( x = -1 \) by having (short) sold in the Treasury market. This implies that the dealer first takes 3 dollars worth of bonds that he receives through reverse repo; he then passes 2 dollars’ worth of bonds as collateral in the Tri-party repo market (where the collateral would have rested without being available to anyone else for purchase), and sells the remaining 1 dollar of bonds to the Treasury market. On the dealer’s balance sheet, the dealer’s obligation to return the borrowed and short-sold Treasuries to the habitat agents is recorded as a liability of 1 as “Financial instruments sold but not yet purchased.” Adding the 2 dollars of liability in the form of Tri-party repo, the balance sheet size of this dealer is 3, which is his total repo position.}

Combining these two cases, we can write the aggregate size of the dealer’s balance sheet as

\[
B_t \equiv \int_0^T \max (X_t (\tau), 0) \, d\tau + Q_t^d.
\]

As we will see, the particular supply shock during the COVID-19 crisis implies that Case 1 prevails in equilibrium.

We assume that the marginal balance sheet cost \( \Lambda_t \equiv \Lambda (B_t) \) is linear in the balance sheet size \( B_t \):

\[
\Lambda_t = \lambda B_t, \text{ with } \lambda > 0.
\]

In words, the dealer is bearing a marginal cost from taking on an extra dollar of Treasuries (whether directly held by the dealer or indirectly by financing hedge funds’ positions) onto the balance sheet, and this cost is increasing in the aggregate balance sheet size \( B_t \).

As explained in the introduction, the balance sheet cost captures the Supplementary Leverage Ratio (SLR) constraint combined with some cost of raising equity (with an upward sloping equity supply curve, say as in He and Krishnamurthy (2012, 2013b)).

### 3.5 Equilibrium

Without loss of generality, we follow Greenwood and Vayanos (2014) by normalizing the aggregate bond supply for each tenor \( \tau \in [0, T] \) to be zero. We focus on symmetric equilibrium in which individual agents (hedge funds and dealers) are employing the same strategy as their own groups, respectively. In aggregate, for tenor \( \tau \), the dealer sector has \( X_t (\tau) = x_t (\tau) \) amount of direct (cash) Treasury holdings and provides \( Q_t^d (\tau) = q_t^d (\tau) \geq 0 \) amount of reverse repo.

We define the equilibrium in the standard way:
Definition 1. Given the demand by habitat agents $H_t = -\theta (\tau) \tilde{\beta}_t$ in (1), a (symmetric) equilibrium is a collection of quantities $\{q^h_t (\tau), Q^h_t (\tau)\}$ by hedge funds, $\{x_t (\tau), X_t (\tau), q^d_t (\tau), Q^d_t (\tau)\}$ by dealers, and prices $\{P_t (\tau), \Lambda_t, \Phi_t (\tau)\}$, so that

1. Each hedge fund solves the problem in (3);
2. Each dealer solves the problem in (5);
3. Allocations are symmetric and consistent: $q^h_t (\tau) = Q^h_t (\tau)$, $x_t (\tau) = X_t (\tau)$, and $q^d_t (\tau) = Q^d_t (\tau)$;
4. Both Treasury and repo markets clear for $\tau \in [0, T]$, i.e.,

$$0 = H_t (\tau) + Q^h_t (\tau) + X_t (\tau),$$
$$0 = Q^h_t (\tau) - Q^d_t (\tau).$$

Since $Q^h_t (\tau) = Q^d_t (\tau)$ in equilibrium, we will use $Q$ to denote them whenever without risk of confusion. Figure 4 illustrates the model setting with $X > 0$. There is an aggregate bond (risk) supply $S$ to be borne by habitat agents (via direct holdings $X$ and $Q$) and dealers (direct holdings $X$). But as shown, dealers are serving as the counterparty for repo transactions $Q$, which occupy their balance sheet—hence a balance sheet size of $B = Q + X$. We will come back to Figure 4 in Section 4.1.5 to offer a full illustration of how this simple fact drives the interesting interactions between the repo market and Treasury market in our model.

4 Model Solution and Implications

We first consider a special case of our model and illustrate its economic mechanism in detail. We then show that this case is sufficiently rich to deliver interesting empirical patterns during the Treasury market breakdown when the pandemic hit the U.S. in mid-March 2020.

4.1 Model Solution and Mechanism

As the main focus our paper is on the 2020 COVID-19 crisis, in this section we consider $\beta > 0$ which entails a positive supply shock of Treasuries to be absorbed by dealers and hedge funds.
As in Greenwood and Vayanos (2014), the demand from habitat agents is price inelastic, and hence exogenously determined by the aggregate demand shock as \( H_t(\tau) = -\theta(\tau) \hat{\beta}_t < 0 \) given in (1). Invoking the market clearing condition (10) we immediately know that in equilibrium

\[
Q_t(\tau) + X_t(\tau) = \theta(\tau) \hat{\beta}_t \geq 0 \text{ for all } \tau \in [0, T]. \tag{11}
\]

This implies that the naked-short-selling constraint (7) never binds in equilibrium. We study the alternative case in Section (4.3).

### 4.1.1 Equilibrium balance sheet size and repo wedge

We will show soon in equilibrium that

\[
X_t(\tau) \geq 0 \text{ for all } \tau \in [0, T]. \tag{12}
\]

As explained in Section (3.4.2), under (12) the dealer’s balance sheet size for bond \( \tau \) is \( B_t(\tau) = Q_t(\tau) + X_t(\tau) \). This is particularly convenient because then the market clearing \( 0 = H_t(\tau) + Q_t(\tau) + X_t(\tau) \) in (10) implies that the equilibrium balance sheet size is independent of the equilibrium repo size \( Q_t(\tau) \):

\[
B_t(\tau) = -H_t(\tau) \geq 0.
\]

To see this, imagine the hedge fund sector buys one dollar’s worth of bonds from the dealer, but using leverage through repo. While this reduces the risk the dealer has to bear, it does not relax the dealer’s balance sheet.

We now use (1) to calculate the equilibrium aggregate balance sheet (recall \( \Theta = \int_0^T \theta(\tau) d\tau \))

\[
B_t = \int_0^T B_t(\tau) d\tau = \hat{\beta}_t \Theta. \tag{13}
\]

Equation (11) and \( Q_t(\tau) \geq 0 \) imply that the dealer is taking an interior repo position in equilibrium. Because the dealer’s objective is linear in the repo funding supply \( Q_t(\tau) \), with a marginal benefit...
of $\Delta_t - \Lambda_t$, in equilibrium it must be that

$$\Delta_t = \Lambda_t = \lambda B_t > 0.$$ 

This result has important implications for our problem. Per unit of Treasury bond, the dealer’s holding cost is the balance sheet cost $\Lambda_t$ while the hedge fund’s holding cost is the repo financing wedge $\Delta_t$. In equilibrium they must be the same.

### 4.1.2 Optimal risk sharing within the intermediary sector

The same holding cost for both dealers and hedge funds imply that they are facing the same problem. To see this, because the dealer’s problem is linear in the reverse repo $q^d$, the profit from repo provision must be zero.\(^1\) Plugging in $\Delta_t = \Lambda_t$ one can show that the wealth dynamics of a dealer and that of a hedge fund are identical.

In our model, dealers and hedge funds only differ in their risk-bearing capacity $a_d$ and $a_h$. However, the standard asset pricing result implies that, in equilibrium, the optimal risk sharing has dealers (hedge funds) directly hold

$$X_t(\tau) = -\frac{\rho_d}{\rho_d + \rho_h} H_t(\tau),$$

and the hedge fund sector to hold ($Q_t^h(\tau) = Q_t(\tau)$)

$$Q_t(\tau) = -\frac{\rho_h}{\rho_d + \rho_h} H_t(\tau).$$

### 4.1.3 Euler equation and equilibrium asset pricing

We now study equilibrium bond pricing based on the standard Euler equation. The derivation in this section is an extension of Greenwood and Vayanos (2014) but with Poisson jumps. What is more, in our model the equilibrium pricing kernel—which is determined by $\{X_t(\tau) : \tau \in [0, T]\}$—is endogenous and pinned down by the dealer’s equilibrium holdings (14) under the optimal risk sharing just shown.

\(^1\)Recall that the no-naked-short-selling constraint (7) never binds in equilibrium due to (12).
Denote $P_{\beta t}^\tau \equiv P_{\beta t}^\tau (\tau)$ and guess that the equilibrium prices take the form of

$$P_{\beta t}^\tau = \exp \left[- \left( A(\tau) r_t + C_{\beta t} (\tau) \right) \right], \text{ with } \tilde{\beta}_t \in \{0, \beta\}$$

(16)

where $A(\tau), C_0(\tau),$ and $C_\beta (\tau)$ are endogenous functions of $\tau$. We only outline the key steps for the dealer’s Euler equation; for detailed derivations and numerical method, see Appendix A.

For illustration, suppose that $\tilde{\beta}_t = \beta$—i.e., the demand shock has occurred to the economy. Applying Ito’s Lemma we have

$$\frac{dP_{\beta t}^\tau}{P_{\beta t}^\tau} = \mu_{\beta t} dt + \left\{ e^{C_\beta(\tau) - C_0(\tau)} - 1 \right\} (dN_t - \xi_\beta dt) - A(\tau) \sigma dZ_t,$$

where $dN_t$ denotes the Poisson shock with intensity $\xi_\beta$, and the drift of the bond return is given by

$$\mu_{\beta t} \equiv A'(\tau) r_t + A(\tau) \kappa (r_t - \tau) + C'_\beta (\tau) + \frac{1}{2} A^2(\tau) \sigma^2 + \xi_\beta \left( e^{C_\beta(\tau) - C_0(\tau)} - 1 \right).$$

(17)

Recall that the dealer is maximizing his mean-variance objective over (as the equilibrium profits from repo service is zero)

$$dw_{\beta t}^d - w_{\beta t}^d r_t dt = \int_0^T x_{\beta t} (\tau) \left( \frac{dP_{\beta t}^\tau}{P_{\beta t}^\tau} - r_t dt - \Lambda_{\beta t} dt \right) d\tau,$$

so that

$$\mathbb{E}_t \left[ dw_{\beta t}^d - w_{\beta t}^d r_t dt \right] = \int_0^T x_{\beta t} (\tau) \left( \mu_{\beta t} - r_t - \Lambda_{\beta t} dt \right) d\tau dt,$$

$$\text{Var}_t \left[ dw_{\beta t}^d - w_{\beta t}^d r_t dt \right] = \left( \int_0^T x_{\beta t} (\tau) \left( e^{C_\beta(\tau) - C_0(\tau)} - 1 \right) d\tau \right)^2 \xi_\beta dt + \left( \int_0^T x_{\beta t} (\tau) A(\tau) d\tau \right)^2 \sigma^2 dt.$$

As a result, we can derive the representative dealer’s Euler equation from his first-order condition.
with respect to $x_{\beta t}(\tau)$:

$$\mu_{\beta t}^T - r_t - \Lambda_{\beta t} = A(\tau) \frac{\sigma^2}{\rho_d} \int_0^T X_{\beta t}(u) A(u) \, du + \text{risk premium for Brownian interest rate risk } \sigma_d Z_t$$

$$\left(e^{C_0(\tau) - C_\beta(\tau)} - 1\right) \cdot \frac{\xi_{\beta t}}{\rho_d} \int_0^T \left\{X_{\beta t}(u) \left(e^{C_\beta(\tau) - C_0(\tau)} - 1\right) \, du\right\},$$  \hspace{1cm} (18)

where we have invoked the dealer’s equilibrium holdings $x_{\beta t}(\tau) = X_{\beta t}(\tau)$, which are given by (14).

Note, one can also write the Euler equation from the entire intermediary sector’s perspective: it absorbs the whole supply shock $-H_t(\tau)$ but with an effective risk bearing capacity of $\rho_I \equiv \rho_d + \rho_h$.

Now plugging in (17), one can derive $A(\tau) = \frac{1 - e^{-\kappa \tau}}{\kappa}$ and an ordinary differential equation (ODE) for $C_\beta(\cdot)$. Repeating the same exercise for $\tilde{\beta}_t = 0$ (i.e., before the economy is hit by the demand shock), we obtain another ODE for $C_0(\cdot)$. Appendix A gives the details as well as the numerical algorithm to solve for \{C_0(\cdot), C_\beta(\cdot)\}.

Equation (18) is the standard Euler equation for the representative risk-averse dealer. The left-hand side gives the bond $\tau$’s expected effective excess return, net the balance sheet cost. In equilibrium it equals the risk premium that compensates the dealer for bearing the additional Brownian interest rate risk $\sigma_d Z_t$ and Poisson demand risk $\tilde{\beta}$, as shown on the right-hand side of (18).

Equation (18) also makes it clear that the entire dealer’s equilibrium portfolio \{X_{\tau} \mid \tau \in [0, T]\} pins down the pricing kernel in this model that prices each bond with tenor $\tau$. Similar to Greenwood and Vayanos (2014), demand shocks affect the dealer’s equilibrium holdings and hence the pricing kernel. Dealers who absorb an increase in the supply of long-term bonds bear more interest rate risk in their portfolio, and hence in equilibrium require all bonds—both long-term and short-term—to offer higher expected returns in excess of the short rate.

### 4.1.4 Shadow price for OIS curves

One of the key empirical objectives in our paper is to track changes in the (in-)convenience yield of Treasuries during the COVID-19 crisis episode. For this reason, we seek a maturity-matched benchmark for comparison that isolates the (in-)convenience yield. In our model, we can define as benchmark a derivative asset with the exact same cash flows as physical Treasury bonds, but free
from balance sheet or inventory concerns and hence free from inconvenience yields. Empirically, we proxy for the yield of this derivative asset with overnight index swap (OIS) rates. In practice, the weight imposed by the SLR constraint on interest rate derivative contracts is about two orders of magnitude smaller than the underlying bonds, and dealers can in principle write any derivative contracts in case there is a demand, free from the physical “inventory” constraints.

Similar to Garleanu and Pedersen (2011), suppose that dealers in our model are quoting prices for OIS contracts, which are zero-net supply in equilibrium. Denote by \( P_{\beta t}^{OIS,\tau} \) the price of an OIS contract with tenor \( \tau \), which takes the following functional form (one can show that \( A(\tau) = \frac{1-e^{-\kappa \tau}}{\kappa} \) as in Section 4.1.3)

\[
P_{\beta t}^{OIS,\tau} = \exp \left[ - \left( A(\tau) r_t + C_{OIS}^{\beta}(\tau) \right) \right], \text{ with } \tilde{\beta}_t \in \{0, \beta\}. \tag{19}
\]

Then the drift of \( dP_{\beta t}^{OIS,\tau} / dt \) satisfies the standard Euler equation but without the balance sheet and inventory adjustment:

\[
\mu_{\beta t}^{OIS,\tau} - r_t = A(\tau) \frac{\sigma^2}{\rho_d} \int_0^T X_{\beta t}(u) A(u) du + \left( e^{C_0^{\beta}(\tau) - C_{\beta}(\tau)} - 1 \right) \frac{\xi_{\beta}}{\rho_d} \int_0^T \left( e^{C_0^{\beta}(\tau) - C_{\beta}(\tau)} - 1 \right) dX_{\beta t}(u). \tag{20}
\]

One can solve for \( \{C_0^{OIS}(\cdot), C_{\beta}^{OIS}(\cdot)\} \) following the same technique as in Section (4.1.3) (for details, see Appendix A.3).

The Treasury-OIS spread at \( \tau \) roughly captures the Treasury’s extra holding cost \( \Lambda_t \) during the remaining time-to-maturity \( \tau \), discounted by the equilibrium pricing kernel which takes into account aggregate demand shock. Hence through the lens of our model, Treasury-OIS spreads observed in the empirical data capture the Treasury (in)convenience yields.

### 4.1.5 Summary and model mechanism

The following proposition summarizes what we have shown.

**Proposition 1.** Consider the scenario of a potential supply shock, i.e., \( \beta > 0 \), from the habitat agents. Recall the habitat agents random demand \( H_t(\tau) = -\theta(\tau) \tilde{\beta}_t \). The equilibrium is character-

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20 The Potential Future Exposure of derivative contracts equals the effective notional principal amount, times the add-on factor. The add-on factor for interest rate derivatives is 0 for bonds with remaining maturity of one-year or less; 0.5% over 1 to 5 years; and 1.5% over 5 years.
1. The dealer’s balance sheet size and the equilibrium repo rate spread is given by

\[ B_t = \tilde{\beta}_t \Theta, \quad \text{and} \quad \Delta_t = \Lambda_t = \lambda B_t = \lambda \tilde{\beta}_t \Theta; \]

2. The holdings of Treasury bonds by hedge funds via repo financing are

\[ Q_t(\tau) = \frac{\rho_h}{\rho_d + \rho_h} \theta(\tau) \tilde{\beta}_t; \]

3. The direct holdings of Treasury bonds with tenor \( \tau \) by the dealer sector are

\[ X_t(\tau) = \frac{\rho_d}{\rho_d + \rho_h} \theta(\tau) \tilde{\beta}_t; \]

4. The equilibrium bond (OIS) prices \( P_{\tilde{\beta}_t}^\tau \) (\( P_{\tilde{\beta}_t}^{OIS,\tau} \)) in Eq. (16) (Eq. (19)) satisfy the Euler equation Eq. (18) (Eq. (20)), with solution \( A(\tau) = \frac{1-e^{-\kappa \tau}}{\kappa} \) and the ODE system of \{\( C_0(\cdot), C_\beta(\cdot) \}\} \{\( C_{OIS}^0(\cdot), C_{OIS}^\beta(\cdot) \)\} in Eq. (35) (Eq. (36)) given in Appendix A.3.

Figure 4 illustrates the workings of our model; the mechanism applies to tenor-\( \tau \) bonds as well as the entire maturity spectrum. Say a supply shock hits habitat agents (\( \tilde{\beta}_t \) jumps from 0 to \( \beta > 0 \)). As illustrated by Figure 4, the dealers’ balance sheet \( B = -H \) expands due to market clearing. The greater the (negative) demand shock size \( \beta \), the larger the dealers’ balance sheet size, and hence the higher the balance sheet cost \( \Lambda(B) = \lambda B \). This effect is stronger for a larger balance sheet cost parameter \( \lambda \), which presumably reflects the scarcity of intermediary capital (He and Krishnamurthy (2012, 2013b)) and/or debt overhang (Andersen, Duffie, and Song (2019)).

Hedge funds step in to absorb the supply shock from habitat agents via collateralized repo borrowing from dealers. Since reverse repo takes up space on the dealers’ balance sheet, in equilibrium the dealers pass the balance sheet cost \( \Lambda \) through to hedge funds via the repo wedge \( \Delta \) on a one-to-one basis, adversely affecting the repo demand \( Q \) from the hedge fund sector. In equilibrium both dealers and hedge funds achieve the optimal risk sharing.

The dealers’ direct holding \( X \) pins down their pricing kernel and hence equilibrium bond prices via their Euler equation (18). As noted in Figure 4, the balance sheet cost \( \Lambda \), as the holding cost or
Dealers’ Balance Sheet \( B = Q + X = -H \)
\( B (\uparrow) \) drives up B/S cost \( \Lambda = \lambda B (\uparrow) \)

Hedge fund demand via Repo, decreasing in \( \Delta = \Lambda (\uparrow) \)
Dealers’ direct holding \( Q (\uparrow) \) and pricing kernel

\( X (\uparrow) \)

Optimal Risk Sharing

Hedge funds’ Risk Exposure \( Q \)
Dealers’ Risk Exposure \( X \)

\( \tilde{\beta} \) shock to Habitat agents’ holdings \(-H (\uparrow)\)

Notes: A schematic representation of the model when the economy suffers from a supply shock \( \tilde{\beta} > 0 \) from habitat agents. Increasing one-to-one to absorb the supply shock, the dealers’ balance sheet (with a size \( B = -H \)) accommodates not only dealers’ direct holdings (\( X \)) but also repo financing (\( Q \)) from hedge funds.

4.2 Treasury Market Breakdown in the COVIC-19 Crisis in 2020

We show that a simple special case of our model, which features a supply shock to long-term Treasuries, can explain the market turmoil experienced in March 2020. Consider \( \theta (\tau) = 1_{\{\tau > \hat{\tau}\}} \) and \( \beta > 0 \), so that long-term Treasuries are being sold by habitat agents. We then have \( \Theta = \int_0^T \theta (\tau) d\tau = T - \hat{\tau} \) in all the equilibrium objects in Proposition 1. In our illustrative numerical example, we set \( \hat{\tau} = 5 \).

4.2.1 Model implications: Treasury inconvenience yield

Figure 5 Panel A (left) plots the equilibrium yield curves at the normal state \( \tilde{\beta} = 0 \) and the stressed state \( \tilde{\beta} = \beta > 0 \). We observe that the yield at the long-end rises when the hedge fund sector and
dealers are absorbing the aggregate supply shock from the habitat agents. As in Greenwood and Vayanos (2014), the equilibrium yield at the short-end also rises because all Treasury bonds are priced by the same marginal investors (hedge funds and dealers). Importantly, we observe that the yield curve steepens in the stressed state, consistent with yield curve movements in March 2020 (see Figure 1).

To express the new implication of our paper, Figure 5 Panel B (right) plots the equilibrium repo GCF-Triparty wedge \( \Delta \) (left axis) and 10-year Treasury-OIS spread (right axis) in the stress state \( \tilde{\beta} = \beta \), both as a function of the supply shock size \( \beta \). The equilibrium repo wedge \( \Delta \) equals the balance sheet cost \( \Lambda_t = \lambda \Theta \tilde{\beta}_t \), which is linear in the supply shock size \( \beta \) in the stress state. This represents the extra holding cost of Treasury bonds on the dealer’s balance sheet (i.e., the inconvenience yield of Treasuries). This inconvenience yield is state-dependent, and drives about a 5-bps wedge between the instantaneous Treasury yield and risk-free short rate \( r_t \) in Panel A in the stress state.

In Panel B we see that our model generates a positive implied 10-year Treasury-OIS spread, thanks to the balance sheet cost. The model implied 10-year Treasury-OIS spread is increasing in the shock size \( \beta \), just like the repo wedge \( \Delta \). These theoretical predictions will be confirmed in the data, as shown in the next section.

### 4.2.2 Empirical evidence of Treasury inconvenience yield

Figure 6 plots daily series of the 10-year and 3-month Treasury-OIS spreads (in the left panel) during the COVID-19 crisis. Consistent with the model prediction (Figure 5 Panel B), the 10-year Treasury-OIS spread is indeed positive. Moreover, it decreased before 3/9/2020, consistent with a standard flight-to-safety to long-term Treasuries, but shot up since then amid a selling pressure by investors who scrambled for cash. On 3/15 right before the Fed announced direct purchases of Treasuries, the 10-year Treasury-OIS spread jumped up by about 30bps, consistent with an increase of \( \Lambda_t \) in our model. In contrast, the 3-month Treasury-OIS spread decreased on 3/15, consistent with the supply shock being concentrated at the long end. The 10-year Treasury-OIS spread began to decrease afterwards likely because the Fed’s direct purchases weakened the supply shock.

The right panel of Figure 6 plots daily series of the GCF-Triparty repo spread during the COVID-19 crisis. Also consistent with the model prediction (Figure 5 Panel B), the repo spread
Figure 5: Supply Shock: Yield Curves, Repo Spreads, and Treasury-OIS Spreads

Notes: Panel A (left) plots model-implied Treasury yield curves in both states (normal $\tilde{\beta} = 0$ and stressed $\beta = \beta$); Panel B (right) plots model-implied GCF-Triparty Repo spreads and Treasury–OIS spreads. The model captures a supply shock for Treasury bonds from habitat agents so that $\beta > 0$, just like in 2020 COVID-19 crisis. Parameters: $\bar{r} = 0.055$, $\kappa = 0.201$, $\rho_h = \rho_d = 1/57$, $\sigma = 0.017$, $\xi_0 = 0.1$, $\xi_\beta = 0.4$, $\lambda = 0.01$, and $r_t = 0$.

is mostly positive. It spiked up as high as 60 basis points during the two-week period from 3/9 to 3/23, driven by the increase of the balance sheet cost $\Lambda_t$ associated with the supply shock. Holders of long-term Treasury securities found it hard to post them as collateral in the repo market to raise cash.

Together with the results in Section 2.1, we find that during the COVID-19 crisis, both the long-term Treasury yield and repo spread increased considerably. Consistent with our model, these increases are associated with the sale of long-term Treasuries. Primary dealers only take a limited amount of them, and also provide limited repo funding for levered investors to take them until the Fed steps in with direct purchases.

4.3 Excess Treasury Demand and 2007–09 Financial Crisis

While our focus in this paper is on the COVID-19 crisis in 2020, we now show that one can also understand, through the lens of the same model, the movements of yields and yield spreads during the 2007-09 financial crisis. This serves as a useful additional validation of the model.

We have so far focused on the case with a supply shock from habitat agents, i.e., $\beta > 0$. But in the 2007-09 financial crisis, the shock was a different one. In the early stages of the financial crisis,
there was a positive demand shock for Treasuries. To accommodate such a demand shock, the intermediary sector has to short-sell some bonds to meet that excess demand from habitat agents. In this section we first introduce costly naked short-selling into our base model, and then apply this model to understand the empirical patterns in the 2007-09 financial crisis. Since this is an episode prior to the SLR regulation, the analysis in this section ignores the balance sheet cost (it is trivial to add back the balance sheet cost).

As there is no clear term structure pattern in 2007-09 financial crisis, for exposition purposes we eliminate the tenor-dependence of demand shocks by setting $\theta(\tau) = 1$ in this section; hence $\Theta = T$. We therefore drop $\tau$ whenever appropriate, especially for aggregate quantities and prices.

### 4.3.1 Excess Treasury Demand and Naked Short-Selling

When Treasury demand surges, $\tilde{\beta}_t = \beta < 0$ implies that $H_t > 0$. In this case, the no-naked-short-selling constraint in (7) must fail, simply because

$$Q_t + X_t = -H_t < 0.$$  \hfill (21)

We introduce naked short-selling to accommodate this empirically relevant case. We assume that naked short-selling is costly, which captures reputation loss or regulatory penalty when the
dealer fails to deliver. We could equivalently introduce a securities lending market in which dealers engage in a search for Treasury bonds to borrow from some habitat agents, and then sell to other habitat agents who demand these bonds, as long as these security lending activities entail some cost (for instance, it is difficult to locate the securities).

For each tenor \( \tau \), denote by \( n^d_t(\tau) \geq 0 \) the amount of naked short-selling that the dealer is engaged in, with an additional marginal cost \( \Gamma_t \geq 0 \) (to be specified shortly). The dealer’s wealth dynamics \( dw_t - rw_t dt \) now equal

\[
\int_0^T x_t(\tau) \left( \frac{dP^\tau_t}{P^\tau_t} - r_t dt \right) d\tau - \int_0^T n^d_t(\tau) \Gamma_t d\tau dt + \int_0^T q^d_t(\tau) \Delta_t d\tau \tag{22}
\]

s.t. \( x_t(\tau) + n^d_t(\tau) + q^d_t(\tau) \geq 0, q^d_t(\tau) \geq 0, n^d_t(\tau) \geq 0 \).

The dealer is facing the exact same problem as before in (6), except one difference: now he can short-sell \( n_t(\tau) \) dollars of bonds to relax his short-selling constraint (23) at a cost of \( n^d_t(\tau) \Gamma_t \). \(^{21}\)

For simplicity, we assume that hedge funds are endowed with the same technology in naked short-selling as dealers. More specifically, if the hedge fund short-sells \( n^h_t(\tau) \) amount of bonds, then its wealth dynamics \( dw^h_t - w^h_t r_t dt \) read

\[
\int_0^T q^h_t(\tau) \left( \frac{dP^\tau_t}{P^\tau_t} - \left( R_t \mathbf{1}_{q^h_t(\tau) \geq 0} + r_t \mathbf{1}_{q^h_t(\tau) < 0} \right) dt \right) d\tau - \int_0^T n^h_t(\tau) \Gamma_t d\tau dt \\
\text{s.t. } q^h_t(\tau) + n^h_t(\tau) \geq 0.
\]

Here, if the hedge fund is holding a short position \( q^h_t(\tau) < 0 \), then it earns a short risk-less rate \( r_t \) on the sales proceeds.

Suppose that in equilibrium the hedge fund sector is offering \( N^h_t(\tau) \) amount of naked short-selling. To close the model, we assume that the naked short-selling cost \( \Gamma_t \) is increasing in the

\(^{21}\)For instance, \( x_t(\tau) = -1.5 \), \( q^d_t(\tau) = 1 \), and \( n^d_t(\tau) = 0.5 \) imply that the dealer short-sells 1 unit of Treasury via repo, and short-sells another 0.5 through either naked positions or security lending.
equilibrium aggregate naked short selling \( N_t \equiv N_t^h + N_t^d \). For simplicity we assume that

\[
\Gamma_t \equiv \Gamma(N_t) = \gamma N_t \text{ for some } \gamma > 0.
\]

### 4.3.2 Equilibrium repo wedge and bond pricing

Naked short-selling allows us to connect its cost \( \Gamma_t \) to the repo wedge \( \Delta_t \) in equilibrium. Some positive naked short-selling \( N_t > 0 \) must occur in equilibrium given a positive excess Treasury demand \( -H_t < 0 \) (recall Eq. (21)). Then, a strictly positive cost of naked short-selling \( \Gamma_t = \lambda N_t > 0 \) implies that (23) holds with equality, with

\[
N_t = -Q_t - X_t = -\tilde{\beta}_t > 0.
\]

In fact, because both hedge funds and dealers are endowed with the same naked-short-selling constraint, in equilibrium they will absorb the habitat agents’ demand shock in proportion to their risk-bearing capacity as in Section (4.1.2):

\[
X_t = -N_t^d = \frac{\rho_h}{\rho_h + \rho_d} \tilde{\beta}_t < 0
\]

\[
Q_t = -N_t^h = \frac{\rho_d}{\rho_h + \rho_d} \tilde{\beta}_t < 0.
\]

In this case, in equilibrium both hedge funds and dealers are in a short position, and hence the equilibrium repo volume is zero (note, in our model repo can only be used to finance long positions).

However, the shadow price of repo financing is linked to the cost of naked short-selling. To see this, each dealer in equilibrium is solving the following problem

\[
\max_{q_t^d \geq 0, n_t^d = N_t^d \geq 0} \quad -n_t^d \cdot \Gamma_t + q_t^d \cdot \Delta_t,
\]

with a linear constraint (23). Because dealers are free to obtain one unit of security via repo (and then sell it short), in equilibrium they will offer an attractive repo wedge to just offset the naked
short-selling cost:\textsuperscript{22}  
\[ \Delta_t = -\Gamma_t = -\gamma N_t = \gamma T \beta_t < 0. \]  

(24)

The negative repo wedge can be viewed as a natural consequence of a Treasury bond shortage given a demand shock. In our model, the Triparty repo rate is fixed by the short-term risk-free rate \( r_t \). But dealers are desperate to borrow and short-sell Treasury bonds to satisfy the habitat agents’ surging demand. Intuitively, dealers are bidding down the GCF repo rate aggressively and willing to eat the loss of GCF-Triparty repo wedge \( \Delta_t \), as long as in equilibrium this loss can be covered from the saved cost of cutting an extra dollar of naked short-selling \( \Gamma_t \). This mechanism is similar to Treasury being “special” in Duffie (1996).

Given this, one can solve for equilibrium bond pricing as in Section 4.1.5, recognizing that now the holding cost of an additional Treasury bond is negative with \( \Delta_t = \gamma T \beta_t < 0 \)—i.e., a convenience yield.

Figure 7: Demand Shock: Yield Curves, Repo Spreads, and Treasury-OIS Spreads

Notes: Panel A (left) panel plots model-implied Treasury yield curves in both states (normal \( \tilde{\beta} = 0 \) and stressed \( \tilde{\beta} = \beta \)); Panel B (right) plots model-implied GCF-Triparty Repo spreads and Treasury-OIS spreads. The model captures a demand shock for Treasury bonds from habitat agents so that \( \beta < 0 \), just like in the 2007–09 financial crisis. Parameters: \( \bar{\tau} = 0.055, \kappa = 0.201, \rho_h = \rho_d = 1/57, \sigma = 0.017, \xi_0 = 0.1, \xi_\beta = 0.4, \gamma = 0.01, \) and \( r_t = 0.03 \).

\textsuperscript{22}We have assumed away tenor-dependent demand shocks in this section, by setting \( \theta (\tau) = 1 \) for all \( \tau \in [0, T] \). For general \( \theta (\tau) \), in equilibrium \( N_t (\tau) = H_t (\tau) \) as well as the cost of naked short-selling \( \gamma N_t (\tau) \) will be \( \tau \)-dependent. This would be inconsistent with our General Collateral (GC) repo market setting, in which the repo wedge \( \Delta_t \) is uniform across all tenor \( \tau \). The special repo market studied in Duffie (1996) precisely captures this tenor-\( \tau \) dependent demand, and would be an interesting direction for future research.
4.3.3 Treasury convenience yield in 2007–09 financial crisis: Theory and evidence

As before Figure 7 Panel A (left) plots the equilibrium yield curves at the normal state $\tilde{\beta} = 0$ and the stressed state $\tilde{\beta} = \beta < 0$ (demand shock). Overall the yield curve is downward sloping. This is because the dealers (hedge funds) are in a short position in equilibrium; long-term bonds provide a hedge benefit for the marginal investor, hence demand a lower premium.

In contrast to the supply shock we studied in Section 4.2, the demand shock $\tilde{\beta} = \beta < 0$ pushes down the entire yield curve. Figure 7 Panel B then plots the equilibrium GCF-Triparty repo wedge $\Delta$ (left axis) and 10-year Treasury-OIS spread (right axis) after the demand shock, both as a function of the shock size $\beta < 0$. The equilibrium repo wedge $\Delta$, which is now negative, equals the naked short-selling cost $\Gamma_t = \lambda T \tilde{\beta}_t < 0$. This implies that Treasuries enjoy a convenience yield—a holding benefit—which is widely documented in the safe asset literature. This explains a negative implied 10-year Treasury-OIS spread, that is increasing in the shock size $\beta$, as plotted in the same figure.

Turning to empirical analysis, Figure 8 plots weekly series of the holdings of Treasuries by foreign investors, Fed, and primary dealers, as well as the repo amounts of primary dealers, from January 1, 2007 to December 31, 2008. The three event dates considered are 7/31/2007 when Bear Stearns filled for bankruptcy, 9/15/2008 when Lehman Brothers filled for bankruptcy, and 11/25/2008 when the Fed announced the direct purchases of agency MBS. We observe a flight-to-safety to long-term Treasuries in the 2007–09 crisis, in sharp contrast to their selling pressure during the COVID-19 crisis (see Figure 3). In particular, the top left panel shows a large increase in foreign investors’ holdings of long-term Treasuries by about $350bn since July 31, 2007, while the top right panel shows a decrease in the Fed’s holdings. In stark contrast to March 2020, the Fed reduced its holdings of Treasuries to help accommodate the safety demand. Primary dealers were net short in Treasuries, especially coupon Treasuries, and were on the net lending side in the repo market. That is, they maintained short positions in their market-making portfolios of Treasuries and often borrowed Treasuries in the repo market. Their net short cash positions and net repo lending amounts trended lower towards the end of 2008, suggesting an easing of the shortage of Treasuries.

The excess demand of Treasuries as evidenced in Figure 8 would imply an effectively positive
Figure 8: Investor Flows and Positions of Treasuries during the 2007-09 Financial Crisis

Notes: The top left panel plots monthly series of the changes in holdings of long-term Treasuries by foreign investors. The top right panel plots weekly series of the amount of the Fed’s Treasury holdings. The bottom left panel plots weekly series of the primary dealers’ net positions of Treasuries. The bottom right panel plots weekly series of the primary dealers’ gross repo amount, gross reverse repo amount, and net reverse repo amount. The units are all in billions of U.S. dollars. The sample period is from January 1, 2007 to December 31, 2008.
shorting cost $\Gamma_t$, which affects the repo spread and Treasury-OIS spread negatively in our model (as shown in Figure 7 Panel B). The top panel of Figure 9 plots monthly series of the GCF-Triparty repo spread from January 2007 to December 2008. Indeed, the repo spread is mostly negative, and reached as low as $-80$ basis points in December 2007. From the middle panel, the 10-year Treasury-OIS spread also stays below zero mostly. That is, amid a flight-to-safety to long-term Treasuries (with decreasing 10-year Treasury yield throughout the period as in the bottom panel), the large shorting cost pushed the repo spread and Treasury-OIS spread to negative levels, consistent with our model. Both moved up notably after the Fed's TSLF program introduced in March 2008, which likely eased the shortage of Treasuries and decreased the shorting cost $\Gamma_t$. In addition, we also observe that the 3-month Treasury-OIS spread stays slightly more negative, showing that the excess demand for T-bills is even stronger.

5 Regression Analysis of Dealer Constraints and Costs

The empirical evidence we have presented so far, in Sections 2.1, 4.2, and 4.3, following an event-study approach like Krishnamurthy and Vissing-Jorgensen (2011), reveals that (a) the shock to the demand of long-term Treasuries is negative in the COVID-19 crisis and positive in the 2007-09 financial crisis, (b) dealers incur a balance sheet cost in holding long-term Treasuries and intermediating repos in the COVID-19 crisis, and (c) dealers incur a cost in short-selling Treasuries in the 2007–09 crisis. In this section, we conduct formal regression analysis using the full sample from 2006 to 2020 and provide further supporting evidence to our model.

5.1 Dealer Constraints and Costs across Sub-periods

Guided by the model, we first break up the full sample period according to variations of dealers' balance sheet cost and shorting cost ($\Lambda_t$ and $\Gamma_t$ in the model, respectively). As shown in Section 4, the former positively affects the repo spread and Treasury-OIS spread, while the latter affects them negatively when there is excess demand for Treasuries.

The post-crisis implementation of SLR that drives dealers’ balance sheet cost phased in from 2015 onward (see Section 2.2.2). Hence, we define pre-SLR and post-SLR as before and after
Figure 9: Treasury-OIS and GCF-Triparty Repo Spreads during the 2007–09 Crisis

Notes: This figure plots monthly series of the GCF-Triparty repo spread (top panel), the 3-month and 10-year Treasury-OIS spreads (middle panel), and the constant maturity Treasury (CMT) yields of 3-month and 10-year maturities (bottom panel), from January 1, 2007 to December 31, 2008.
July 2015, respectively, and expect $\Lambda_t$ to be effectively positive post-SLR but negligible pre-SLR.\(^2\)

Moreover, according to the NBER Business Cycle classifications, the Great Recession (GR) ended in June 2009, so we define pre-GR and post-GR as before and after June 2009, respectively. Given that primary dealers entered the GR with large net short positions of Treasuries, we expect $\Gamma_t$ to be effectively positive. However, the shorting cost likely decreased significantly in the period of March 2008–June 2009 during which the Fed increased the supply of Treasuries by allowing dealers to obtain Treasuries against non-Treasury collateral in the Term Securities Lending Facility (TSLF). We hence define pre-TSLF and post-TSLF as before and after March 2008, respectively.

In sum, our sample period consists of four segments roughly together the respective interpretation of our model: (1) pre-TSLF, when $\Gamma_t$ is positive and $\Lambda_t$ is zero, (2) post-TSLF and pre-GR, when $\Gamma_t$ is reduced substantially but still positive and $\Lambda_t$ is zero, (3) post-GR and pre-SLR when both $\Gamma_t$ and $\Lambda_t$ are zero, and (4) post-SLR when $\Gamma_t$ is zero and $\Lambda_t$ is positive.

### 5.2 Empirical Results

![Average GCF-Triparty Repo and Treasury-OIS Spreads of Sub-periods](image)

**Figure 10: Average GCF-Triparty Repo and Treasury-OIS Spreads of Sub-periods**

The left panel of Figure 10 reports the average GCF-Triparty repo spread over these four sub-periods. Consistent with our characterizations of these sub-periods based on the costs $\Gamma_t$ and $\Lambda_t$, the effects

\(^2\) Though not in effect officially until 2018, banks could have had reputation concerns due to the disclosures and also begin to prepare for the final adoption long before, which would make the SLR exert influence much earlier.
the GCF-triparty repo spread is indeed negative pre-TSLF because of the positive shorting cost, about −23 bps, which was greatly reduced to only −3 bps with the Fed’s TSLF program that eased the shortage of Treasuries. The average GCF-triparty repo spread turns positive post-GR, reaching only about 5 bps before the SLR formally phased in but about 11 bps afterwards when \( \Gamma_t \) turns positive. Column (1) of Table 1 reports regressions of the repo spread on the sub-period dummies to formally test the significance of its change:

\[
GCF - Triparty_t = \alpha + \beta_1 D_{Post-TSLF} + \beta_2 D_{Post-GR} + \beta_3 D_{Post-SLR} + \varepsilon_t, \tag{25}
\]

where the full sample is used (note that the repo spread is monthly from October 2006 to July 2012 and daily from August 1, 2012 to March 31, 2020, as detailed in Appendix B.1). The significant coefficients match the changes of the GCF-triparty spread across the sub-periods and confirm the significance of these changes.

Regarding the balance sheet cost \( \Gamma_t \), the effect of the SLR rule is likely to come in over an extended period of time during which dealers gradually adjust their policies rather than as an immediate effect, so the post-SLR dummy in column (1) of Table 1 captures its average effect accordingly. However, the change over an extend period of time may be confounded by other factors. We explore a variation based on quarter-end shocks to mitigate this concern and further quantify the potential effect of SLR on repo spread. In particular, foreign bank dealers’ repo intermediation activities contract at quarter ends when snapshots of their balance sheet are used to calculate leverage ratio (Duffie (2018)). In consequence, post-SLR when \( \Lambda_t \) becomes positive, U.S. dealers receive higher demand for repo intermediation, resulting effectively in a higher \( \lambda \) on quarter ends than other days within the same quarter. This difference on quarter ends versus other days should be weak pre-SLR.

Columns (2) and (3) of Table 1 report regressions of the repo spread on quarter-end dummies \( D_{QuarterEnd} \), for both post-SLR and pre-SLR respectively. We control for quarter fixed effects so that the coefficient \( D_{QuarterEnd} \) captures the difference of the repo spread between the quarter-end and other days within the same quarter. Indeed, the repo spread is about 23 bps higher on quarter-ends post-SLR, but negative and insignificant pre-SLR. Column (4) reports regressions on the interaction term of \( D_{QuarterEnd} \) and \( D_{Post-SLR} \), where the difference in quarter-end effects before
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<td>0.444</td>
<td>0.548</td>
<td>0.367</td>
</tr>
<tr>
<td>Quarter FE</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Column (1) reports regressions of the GCF-Triparty repo spread on dummy variables for the sub-periods over the full sample period of October 2006 to March 2020. Columns (2) and (3) report regressions of the GCF-Triparty repo spread on the dummy for quarter ends ($D_{QuarterEnd}$) for post-SLR and pre-SLR samples, respectively, while column (4) reports regressions on $D_{Post-SLR}$, $D_{QuarterEnd}$, and their interaction term for the full sample. Columns (5) and (6) report regressions similar to those in columns (3) and (4), respectively, but on the post-GR sample. Quarter fixed effects are controlled in all regressions except that in column (1). The $t$-statistics based on standard errors clustered at calendar quarters are reported in parentheses. The significance levels are represented by *$p<0.1$, **$p<0.05$, ***$p<0.01$.

and after the SLR phases in is confirmed to be significant. In addition, the pre-SLR period examined in column (3) includes the pre-TSLF period when primary dealers hold large short positions of Treasuries and $\Gamma_t$ is positive. In columns (5) and (6), we regress repo spread on quarter end dummies using the post-TSLF sample. The coefficient on $D_{QuarterEnd}$ is positive but insignificant, likely reflecting some effects of SLR before 2015. The coefficient on the interaction term of $D_{QuarterEnd}$ and $D_{Post-SLR}$ is positive and significant, confirming the difference in quarter-end effects before and after the SLR phases in.

Turning to the Treasury-OIS spread, the right left panel of Figure 10 reports the average Treasury-OIS spread over the four sub-periods. Again, consistent with our characterizations of these sub-periods based on the shorting and balance sheet costs, the average Treasury-OIS spread is about −45 bps, which was greatly reduced to only -9 bps with the Fed’s TSLF program. It
Table 2: Regressions of Treasury-OIS Spread and its Relation to Repo Spread

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Pre-GR</th>
<th>Post-GR &amp; Pre-SLR</th>
<th>Post-SLR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Treasury-OIS 10y</td>
<td>GCF-Triparty</td>
<td>GCF-Triparty</td>
<td>GCF-Triparty</td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.455***</td>
<td>-0.024</td>
<td>0.057***</td>
<td>0.043</td>
</tr>
<tr>
<td></td>
<td>(-24.736)</td>
<td>(-1.093)</td>
<td>(3.409)</td>
<td>(1.481)</td>
</tr>
<tr>
<td>$D_{Post-TSLF}$</td>
<td>0.367***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.017)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{Post-GR}$</td>
<td>0.246***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.725)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{Post-SLR}$</td>
<td>0.206***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(9.501)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treasury-OIS 10y</td>
<td>0.380***</td>
<td>-0.039</td>
<td>0.188**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.276)</td>
<td>(-0.429)</td>
<td>(2.161)</td>
<td></td>
</tr>
<tr>
<td>Obs</td>
<td>1,956</td>
<td>30</td>
<td>766</td>
<td>1,160</td>
</tr>
<tr>
<td>Adj $R^2$</td>
<td>0.719</td>
<td>0.194</td>
<td>0.002</td>
<td>0.034</td>
</tr>
</tbody>
</table>

Notes: The first column reports regressions of the 10-year Treasury-OIS spread on dummy variables for sub-periods over the full sample period of October 2006 to March 2020. The next three columns report regressions of the GCF-Triparty repo spread on the 10-year Treasury-OIS spread for the sample of pre-GR, of post-GR and pre-SLR, and of post-SLR, respectively. The t-statistics based on standard errors clustered at calendar quarters are reported in parentheses. The significance levels are represented by *$p<0.1$, **$p<0.05$, ***$p<0.01$.

also turns positive post-GR, reaching about 16 bps before the SLR formally phased in but about 36 bps afterwards. The first column of Table 2 reports a regression similar to (25) of the 10-year Treasury-OIS spread on dummies for sub-periods. The significant coefficients match the changes of the Treasury-OIS spread across the sub-periods and confirm the significance of these changes.

The last three columns of Table 2 report contemporaneous time-series regressions of the repo spread on the 10-year Treasury–OIS spread for the three sub-periods featuring different combinations of $\Gamma_t$ and $\Lambda_t$. As shown in Figure 5 and 7, our model implies a positive relation between them for the cases of both positive (positive $\Lambda_t$) and negative supply shocks (positive $\Gamma_t$). Indeed, the regression coefficient is positive and significant for both the pre-TSLF and post-SLR periods. For the sample period after the TSLF program was implemented but before the SLR phased in, the regression coefficient is insignificant, as both $\Gamma_t$ and $\Lambda_t$ are negligible and there should be no detectable association between the repo spread and Treasury-OIS spread.
6 Conclusion

In sharp contrast to most previous crisis episodes, the Treasury market experienced severe stress and illiquidity in March 2020 during the COVID-19 pandemic, raising concerns that the safe-haven status of U.S. Treasuries could be eroding. We document that some large owners of Treasuries substantially reduced their holdings during March 2020 and the intermediary sector struggled to absorb this demand shock.

To understand the inelastic response of the intermediary sector, we build a model in which balance sheet constraints of dealers and demand shocks from habitat agents interact with each other, affecting the term structure of Treasury yields. A novel element of our model is to introduce repo financing as an important part of dealers’ intermediation activities, through which levered investors obtain leverage. Both direct holdings of Treasuries and reverse repo positions of dealers are subject to a balance sheet constraint related to regulation reforms since the 2007–09 crisis such as the supplementary leverage ratio. Consistent with model implications, the spread between the Treasury yield and overnight-index swap rate (OIS) and the spread between dealers’ reverse repo and repo rates are both highly positive in the COVID-19 crisis, and both greatly negative in the 2007–09 crisis. Over the whole sample of 2006–2020, the Treasury-OIS spread and repo spread increased after 2015 when the regulatory reforms phased, and their correlation is significantly positive.
References


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Appendix

A Euler Equations for Treasury Pricing

We derive the ODE system for \( \{ C_0 (\cdot), C_\beta (\cdot) \} \) based on the dealer’s Euler equation first. We then offer a detailed numerical procedure to solve the ODE system, and then give the results on the equilibrium OIS curves.

A.1 Euler Equation and ODE System

Guess that

\[
P^\tau_{\beta t} = \exp \left[ - (A (\tau) r_t + C_0 (\tau)) \right]
\]

\[
P^\tau_{\beta t} = \exp \left[ - (A (\tau) r_t + C_\beta (\tau)) \right]
\]

and recall the yield is defined as

\[
y_t = \frac{- \log P^\tau_{\beta t}}{\tau} = \frac{A(\tau) r_t + C_\beta (\tau)}{\tau}.
\]

Return dynamics. One can show that (where \( dN_t \) denotes the Poisson shock with intensity \( \xi_\beta \))

\[
\frac{dP^\tau_{\beta t}}{P^\tau_{\beta t}} = \left[ A' (\tau) r_t + A (\tau) \kappa (r_t - \tau) + C_0' (\tau) + \frac{\sigma^2}{2} A^2 (\tau) \right] dt + \left\{ e^{C_0 (\tau) - C_\beta (\tau)} - 1 \right\} dN_t - A (\tau) \sigma dZ_t,
\]

(26)

\[
\frac{dP^\tau_{\beta t}}{P^\tau_{\beta t}} = \left[ A' (\tau) r_t + A (\tau) \kappa (r_t - \tau) + C_\beta' (\tau) + \frac{\sigma^2}{2} A^2 (\tau) \right] dt + \left\{ e^{C_\beta (\tau) - C_0 (\tau)} - 1 \right\} dN_t - A (\tau) \sigma dZ_t.
\]

(27)

We perform the calculation for \( \frac{dP^\tau_{\beta t}}{P^\tau_{\beta t}}, \frac{dP^\tau_{\beta t}}{P^\tau_{\beta t}} \) follows similarly. First,

\[
\frac{dP^\tau_{\beta t}}{P^\tau_{\beta t}} = \exp \left[ -(A (\tau) r_t + C_\beta (\tau)) \right] \left[ A' (\tau) r_t dt + A (\tau) \kappa (r_t - \tau) dt + C_\beta' (\tau) dt - A (\tau) \sigma dZ_t \right]
\]

\[
+ \exp \left[ -(A (\tau) r_t + C_\beta (\tau)) \right] \frac{1}{2} A^2 (\tau) \sigma^2 dt + \left[ e^{-(A (\tau) r_t + C_0 (\tau))} - e^{-(A (\tau) r_t + C_\beta (\tau))} \right] dN_t,
\]
which implies that

\[
\frac{dP_{\beta t}^\tau}{P_{\beta t}^\tau} = A' (\tau) r_t dt + A (\tau) \kappa (r_t - \tau) dt + C'_{\beta} (\tau) dt + \frac{A^2 (\tau) \sigma^2}{2} dt - A (\tau) \sigma dZ_t + \left\{ e^{C_{\beta}(\tau)-C_0(\tau)} - 1 \right\} dN_t.
\]

It is easy to calculate that \( \mathbb{E}_t \left[ \frac{dP_{\beta t}^\tau}{P_{\beta t}^\tau} \right] = \mu_{\beta t}^\tau dt \) with

\[
\mu_{\beta t}^\tau \equiv A' (\tau) r_t + A (\tau) \kappa (r_t - \tau) + C'_{\beta} (\tau) + \frac{1}{2} A^2 (\tau) \sigma^2 + \xi_{\beta} \left( e^{C_{\beta}(\tau)-C_0(\tau)} - 1 \right).
\]

**Euler equations.** As before let us focus on the distress state. The arbitrager is maximizing the mean-variance objective over

\[
\int_0^T x_{\beta t}^\tau \left( \frac{dP_{\beta t}^\tau}{P_{\beta t}^\tau} - r_t dt - \Lambda_{0 t} dt \right) d\tau.
\]

Given (26), we have

\[
\mathbb{E}_t \left[ \int_0^T x_{\beta t}^\tau \left( \frac{dP_{\beta t}^\tau}{P_{\beta t}^\tau} - r_t dt - \Lambda_{\beta t} dt \right) d\tau \right] = \int_0^T x_{\beta t}^\tau (\mu_t^\tau - r_t - \Lambda_{\beta t}) d\tau dt
\]

\[
Var_t \left[ \int_0^T x_{\beta t}^\tau \left( \frac{dP_{\beta t}^\tau}{P_{\beta t}^\tau} - r_t dt - \Lambda_{\beta t} dt \right) d\tau \right] = \left( \int_0^T \left\{ x_{\beta t}^\tau \left( e^{C_{\beta}(\tau)-C_0(\tau)} - 1 \right) d\tau \right\} \right)^2 \xi_{0 t} dt + \left( \int_0^T x_{\beta t}^\tau A (\tau) d\tau \right)^2 \sigma^2 dt
\]

Therefore the dealer’s FOC with respect to \( x_{\beta t}^\tau \) is (we omit \( dt \) from now on)

\[
\mu_{\beta t}^\tau - r_t - \Lambda_{\beta t} = A (\tau) \frac{\sigma^2}{\rho_d} \int_0^T x_{\beta t}^u A (u) du + \left( e^{C_{\beta}(u)-C_0(u)} - 1 \right) \cdot \frac{\xi_{\beta}}{\rho_d} \int_0^T \left\{ x_{\beta t}^u \left( e^{C_{\beta}(u)-C_0(u)} - 1 \right) du \right\}.
\]

Expanding \( \mu_{\beta t}^\tau \) in (28) and collecting terms we have

\[
(A' (\tau) - 1) r_t - \Lambda_{\beta t} + A (\tau) \kappa (r_t - \tau) + C'_{\beta} (\tau) + \frac{1}{2} A^2 (\tau) \sigma^2 r_t + \xi_{\beta} \left( e^{C_{\beta}(u)-C_0(u)} - 1 \right)
\]

\[
= A (\tau) \cdot \frac{\sigma^2}{\rho_d} \left\{ \int_0^T x_{\beta t}^u A (u) du \right\} + \left( e^{C_{\beta}(u)-C_0(u)} - 1 \right) \cdot \frac{\xi_{\beta}}{\rho_d} \int_0^T \left\{ x_{\beta t}^u \left( e^{C_{\beta}(u)-C_0(u)} - 1 \right) du \right\}.
\]

Because this equation holds for all \( r_t \) we must have \( A (\tau) \kappa + A' (\tau) = 1 \); with initial condition \( A (0) = 0 \) we have

\[
A (\tau) = \frac{1 - e^{-\kappa \tau}}{\kappa}.
\]
Moving on to the function $C_0(\cdot)$, and collecting terms, we have

$$
C'_\beta(\tau) + \xi_\beta \left( e^{C_\beta(u) - C_0(u)} - 1 \right) \left( 1 - \frac{1}{\rho d} \int_0^T \left\{ x_{\beta t} \left( e^{C_\beta(u) - C_0(u)} - 1 \right) du \right\} \right)
= A(\tau) \cdot \frac{\sigma^2}{\rho d} \int_0^T x_{\beta t} A(u) \, du + \Lambda_\beta + A(\tau) \kappa \tau - \frac{1}{2} A^2(\tau) \sigma^2.
$$

This is the equation when $\bar{\beta}_t = \beta$.

For the normal state $\beta_t = 0$ we have

$$
\frac{dP_0^\tau}{P_0^\tau} = \mu_0^\tau dt + \left\{ e^{C_0(\tau) - C_\beta(\tau)} - 1 \right\} d\hat{N}_t - A(\tau) \sigma dZ_t
$$

where

$$
\mu_0^\tau \equiv A'(\tau) r_t + A(\tau) \kappa (r_t - \bar{r}) + C'_0(\tau) + \frac{1}{2} A^2(\tau) \sigma^2 + \xi_0 \left( e^{C_0(\tau) - C_\beta(\tau)} - 1 \right).
$$

And the ODE for $C_0(\cdot)$ based on the dealer’s Euler equation at the stress state can be derived analogously:

$$
C'_0(\tau) + \xi_0 \left( e^{C_0(\tau) - C_\beta(\tau)} - 1 \right) \left( 1 - \frac{1}{\rho d} \int_0^T \left\{ x_{t_0} \left( e^{C_0(\tau) - C_\beta(\tau)} - 1 \right) du \right\} \right)
= A(\tau) \cdot \frac{\sigma^2}{\rho d} \int_0^T x_{t_0} A(u) \, du + \Lambda_\beta + A(\tau) \kappa \tau - \frac{1}{2} A^2(\tau) \sigma^2.
$$

We hence have arrived at the ODE system (30) and (31) for $\{C_0(\cdot), C_\beta(\cdot)\}$, with boundary conditions

$$
C_0(0) = C_\beta(0) = 0, C'_0(0) = \Lambda_0 = \lambda B_0, C'_\beta(0) = \Lambda_\beta = \lambda B_\beta.
$$

### A.2 Numerical Methods

This section outlines the numerical procedure in solving for (30) and (31). Denote

$$
D_0(\tau) \equiv A(\tau) \cdot \frac{\sigma^2}{\rho d} \int_0^T x_{t_0} A(u) \, du + \Lambda_0 + A(\tau) \kappa \tau - \frac{1}{2} A^2(\tau) \sigma^2,
$$

$$
D_\beta(\tau) \equiv A(\tau) \cdot \frac{\sigma^2}{\rho d} \int_0^T x_{t_\beta} A(u) \, du + \Lambda_\beta + A(\tau) \kappa \tau - \frac{1}{2} A^2(\tau) \sigma^2,
$$

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which have been solved in closed-form given \( x_0^0 \) and \( A(\tau) = \frac{1-e^{-e\tau}}{\kappa} \) (recall Eq. (29)).

The numerical procedure is as follows.

1. Start with \( K_0^{(0)} = K_\beta^{(0)} = 1 \).

2. With \( \left(K_0^{(n)}, K_\beta^{(n)}\right) \), define

\[
\hat{D}_0(\tau) \equiv D_0(\tau) + K_0^{(n)}, \quad \hat{D}_\beta(\tau) \equiv D_\beta(\tau) + K_\beta^{(n)}.
\]

We now obtain the solution by solving the ODE system (32)

\[
\begin{align*}
C_0'(\tau) + K_0^{(n)}e^{C_0(\tau) - C_\beta(\tau)} &= \hat{D}_0(\tau) \\
C_\beta'(\tau) + K_\beta^{(n)}e^{C_\beta(\tau) - C_0(\tau)} &= \hat{D}_\beta(\tau)
\end{align*}
\]

with the initial conditions

\[
C_0(0) = C_\beta(0) = C_0'(0) = C_\beta'(0) = 0,
\]

by following these steps.

(a) First of all, from the first equation in (32) we have

\[
e^{C_0(\tau) - C_\beta(\tau)} = \frac{\hat{D}_0(\tau) - C_0'(\tau)}{K_0^{(n)}}.
\]

Taking derivative with respect to \( \tau \) on both sides of the first equation in (32), and plugging in \( e^{C_0(\tau) - C_\beta(\tau)} \), we get

\[
C_0''(\tau) + \left(\hat{D}_0(\tau) - C_0'(\tau)\right)\left(C_0'(\tau) - C_\beta'(\tau)\right) = \hat{D}_0'(\tau).
\]

(b) Now, using \( C_\beta'(\tau) = \hat{D}_\beta(\tau) - K_\beta^{(n)}e^{C_\beta(\tau) - C_0(\tau)} \) from the second equation, and plugging in (33), one can get,

\[
C_0''(\tau) + \left(\hat{D}_0(\tau) - C_0'(\tau)\right)\left(C_0'(\tau) - \hat{D}_\beta(\tau)\right) + K_\beta^{(n)}K_0^{(n)} = \hat{D}_0'(\tau).
\]
(c) Letting $z(\tau) \equiv C_0'(\tau)$, we have a Riccati equation for $z(\tau)$

$$z'(\tau) = \left( \hat{D}_0'(\tau) + \hat{D}_\beta(\tau) \hat{D}_0(\tau) - K_0^{(n)}K_0^{(n)} \right) - \left( \hat{D}_\beta(\tau) + \hat{D}_0(\tau) \right) z(\tau) + z^2(\tau)$$

with a boundary condition $z(0) = 0$. Once we numerically solve for $z(\tau)$, we can calculate $C_0(\tau) = \int_0^\tau z(u) \, du$ and derive $C_\beta(\tau)$ according to Eq. (33).

3. Calculate $K_0^{(n+1)}, K_\beta^{(n+1)}$ based on the new solution using (34):

$$
\begin{align*}
K_0^{(n+1)} &= \xi_0 \left( 1 - \frac{1}{\rho d} \int_0^T \left\{ x_{00}^u \left( e^{C_0(u)} - C_\beta(u) - 1 \right) \right\} du \right)
K_\beta^{(n+1)} &= \xi_\beta \left( 1 - \frac{1}{\rho d} \int_0^T \left\{ x_{\beta\beta}^u \left( e^{C_\beta(u)} - C_0(u) - 1 \right) \right\} du \right)
\end{align*}
$$

(34)

4. If $\left\| (K_0^{(n+1)} - K_0^{(n)}, K_\beta^{(n+1)} - K_\beta^{(n)}) \right\| < \epsilon$ then terminate. Otherwise set $n = n + 1$ and go to Step 2.

### A.3 Derivation of Equilibrium OIS Curves

We guess and verify that the equilibrium OIS prices take the following forms with $A(\tau) = \frac{1-e^{-\kappa \tau}}{\kappa}$

$$
\begin{align*}
P_{0t}^{\text{OIS},\tau} &= \exp \left[ - \left( A(\tau) r_t + C_0^{\text{OIS}}(\tau) \right) \right] \\
P_{\beta t}^{\text{OIS},\tau} &= \exp \left[ - \left( A(\tau) r_t + C_\beta^{\text{OIS}}(\tau) \right) \right]
\end{align*}
$$

where $\left\{ C_0^{\text{OIS}}(\cdot), C_\beta^{\text{OIS}}(\cdot) \right\}$ are to be determined endogenously. Note that the Treasury-OIS spread at tenor $\tau$, denoted by $\Delta y^\tau$, then can be calculated as

$$
\Delta y^\tau = \frac{\ln P^\tau - \ln P_{0t}^{\text{OIS},\tau}}{\tau} = \frac{C_0^{\text{OIS}}(\tau) - C(\tau)}{\tau}.
$$
Based on similar derivations as in Section A.1, we arrive at

\[ C^{OIS}_{0}(\tau) + \xi_0 \left( e^{C^{OIS}_{0}(\tau)} - C^{OIS}_{0}(\tau) - 1 \right) \left( 1 - \frac{1}{\rho_d} \int_0^T \{ x^u \left( e^{C^{OIS}_{0}(\tau)} - C^{OIS}_{0}(\tau) - 1 \right) du \} \right) = A(\tau) \kappa T - \frac{1}{2} A^2(\tau) \sigma^2 + A(\tau) \frac{\sigma^2}{\rho_d} \int_0^T x^u A(u) du \]  

(35)

\[ C^{OIS}_{\beta}(\tau) + \xi_{\beta} \left( e^{C^{OIS}_{\beta}(\tau)} - C^{OIS}_{\beta}(\tau) - 1 \right) \left( 1 - \frac{1}{\rho_d} \int_0^T \{ x^u \left( e^{C^{OIS}_{\beta}(\tau)} - C^{OIS}_{\beta}(\tau) - 1 \right) du \} \right) = A(\tau) \kappa T - \frac{1}{2} A^2(\tau) \sigma^2 + A(\tau) \frac{\sigma^2}{\rho_d} \int_0^T x^u A(u) du \]  

(36)

with initial conditions \( C^{OIS}_{0}(0) = C^{OIS}_{0}(0) = C^{OIS}_{\beta}(0) = C^{OIS}_{\beta}(0) = 0 \).

**B Data and Additional Evidence**

In this appendix, we first provide details of the data and variables used in empirical analysis. We then present two sets of additional empirical evidence, one highlighting the flight-to-cash nature of the COVID-19 shock and the other breaking down primary dealers’ repo positions into different tenor buckets.

**B.1 Data**

We obtain daily series of constant-maturity Treasury (CMT) yields from the H.15 reports of the Federal Reserve, which are equal to the coupon rates on par bonds. We obtain daily series of overnight index swap (OIS) rates from Bloomberg. The OIS is a fully collateralized interest rate swap contract that exchanges a constant cash flow against a flow of floating payment indexed to the geometric average of the daily effective federal funds rate. OIS contracts with maturities of up to one year have only one final payment, while cash payments for those with maturities of over one year are made quarterly. Hence, OIS rates are effectively zero-coupon yields for maturities of up to one year and par yields for maturities of over one year, both comparable to the CMT yields. We then take the difference between the CMT yield and the maturity-matched OIS rate as the Treasury-OIS spread.

We use overnight repo rates of Treasury securities of both the tri-party market and the GCF market. Daily series of GCF repo rates are provided by the Depository Trust & Clearing Corpo-
ration (DTCC), available starting from 2005, calculated as the average interest rate across repo transactions weighted by volume within a day. The tri-party repo rates are from multiple sources. First, we obtain daily series of the Tri-party General Collateral Rate (TGCR) computed by the Federal Reserve Bank of New York, available from August 22, 2014. The TGCR is calculated as the volume-weighted median of transaction-level tri-party repo data, excluding GCF Repo transactions and transactions to which the Federal Reserve is a counterparty. Second, we obtain daily series of tri-party repo rates from August 1, 2012 to August 21, 2014 calculated by the Bank of New York Mellon, the largest of the two clearing banks. These repo rates are also calculated as volume-weighted medians on each business day for new, overnight repo trades with U.S. Treasuries (excluding Strips) as collateral assets. Third, for November 2010–July 2012, we obtain monthly series of tri-party repo rates using the overnight tri-party repo trades between MMFs and dealers reported in the N-MFP fillings with the SEC, similar to Hu, Pan, and Wang (2019). Specifically, we calculate the volume-weighted medians of all overnight tri-party repo trades on the last business day of each month. Fourth, for October 2006 – April 2010, we use the month-end value-weighted average overnight repo rates (weighted by notional amounts) constructed in Krishnamurthy, Nagel, and Orlov (2014) based on quarterly N-CSR, N-CSRS, and N-Q fillings with SEC. MMFs file these reports at different month-ends throughout each quarter, so monthly series of repo rates can be calculated.

The weekly series of primary dealers’ net positions and financing amounts of Treasury securities are obtained from the FR2004 data collected by the Federal Reserve Bank of New York. The data are reported on a weekly basis, as of the close of business each Wednesday. The reported series are netted and aggregated across all primary dealers, available for four categories, including T-bills, coupon-bearing nominal securities (coupons), Treasury inflation-protected securities (TIPS), and Floating Rate Notes (FRNs) that began to be issued in January 2014 and reported from 2015. The

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25 The TGCR is one of the three overnight repo rates provided by the Federal Reserve Bank of New York as important reference rates to financial markets, together with the Broad General Collateral Rate (BGCR) and the Secured Overnight Financing Rate (SOFR). The calculation of BGCR includes all trades used in the calculation of TGCR plus the GCF Repo transactions. The calculation of SOFR includes all trades used in the calculation of BGCR plus the bilateral Treasury repo transactions cleared through the Delivery-versus-Payment (DVP) service offered by the FICC but filtered to remove a portion of transactions considered “specials.” For further details, see [https://www.newyorkfed.org/markets/treasury-repo-reference-rates-information](https://www.newyorkfed.org/markets/treasury-repo-reference-rates-information).

26 The data can be found at [https://repoindex.bnymellon.com/repoindex/](https://repoindex.bnymellon.com/repoindex/).

27 For details, see [https://www.newyorkfed.org/markets/gsds/search](https://www.newyorkfed.org/markets/gsds/search).
net positions include both spot cash positions and Treasury derivatives like futures (Fleming and Rosenberg (2007)).

Regarding financing amounts, the FR2004 data separate repo from other financing activities like security lending contracts from April 3, 2013, but repo and securities lending contracts are blended together earlier. Hence, we use the amounts of repo and reverse repo from April 3, 2013 onward, but total financing amounts of cash in and cash out before, which will be referred to as repo and reverse repo for convenience. Note that the financing contracts are defined from the perspective of dealers, so dealers borrow cash through repo and lend cash out through reverse repo. For both repo and reverse repo, the amounts are available for overnight and term contracts separately. We mainly focus on the total amount by adding the overnight and term financing amounts together, but will briefly discuss the breakdown in the next section.

The daily VIX series are obtained from the Chicago Board Options Exchange (CBOE). We obtain daily series of constant-maturity yields of TIPS also from the H.15 reports of the Federal Reserve. We compute the breakeven inflation rate as the difference between the CMT nominal yield and TIPS yield of the same maturity, which is a market-based measure of expected inflation. We also obtain daily series of inflation swap rates from Bloomberg as an alternative measure of expected inflation. To measure the uncertainty and tail event probability of inflation, we obtain weekly series of the standard deviation and the probability of a large increase in inflation (of more than 3%) based on inflation density estimates using 5-year inflation caps and floors.\(^{28}\)

We obtain the amounts of Treasury holdings and issuance from various sources. First, the monthly series of net issuance amount (gross minus retirement) of Treasury notes and bonds are obtained from the SIFMA.\(^{29}\) Second, series of net new cash flows into long-term government bond mutual funds are obtained from the Investment Company Institute (ICI), available at the monthly frequency through March 2020 and at the weekly frequency afterwards.\(^{30}\) Third, monthly series of foreign net purchases of U.S. long-term Treasury securities are obtained from the Treasury International Capital (TIC) system.\(^{31}\) We group them into four categories: Europe, Asia, Caribbean


\(^{29}\)The series can be found at [https://www.sifma.org/resources/research/us-marketable-treasury-issuance-outstanding-and-interest-rates/](https://www.sifma.org/resources/research/us-marketable-treasury-issuance-outstanding-and-interest-rates/).

\(^{30}\)The series can be found at [https://www.ici.org/research/stats/flows](https://www.ici.org/research/stats/flows).

\(^{31}\)The series can be found at [https://www.treasury.gov/resource-center/data-chart-center/tic/Pages/ticsec.aspx](https://www.treasury.gov/resource-center/data-chart-center/tic/Pages/ticsec.aspx).
(including a lot popular tax haven countries), and Other (including all other countries and international organizations). The nonmarketable Treasuries are excluded. Fourth, weekly series of the total face value of U.S. Treasury securities held by the Federal Reserve are provided in their H.4.1 release.\textsuperscript{32} We group these series into T-bills, nominal coupons, and TIPS.

We obtain weekly series of total net assets (TNA) of prime, government, and tax-exempt money market funds from the ICI. We obtain weekly seasonally adjusted series of the commercial and industrial loans (C&I), cash assets, treasury and agency securities, fed funds sold and cash lent out through reverse repo,\textsuperscript{33} and deposits, of all commercial banks, provided in the H.8 release of the Federal Reserve.\textsuperscript{34}

\subsection*{B.2 Additional Empirical Evidence}

We provide two sets of additional empirical evidence. First, Figure B1 presents series of fund flows of MMFs and banks. The top panels show that there are negative net flows out of prime MMFs and positive net flows into government MMFs, while the middle panels show that bank deposits increased significantly, in both the COVID-19 and 2007–09 crises. The bottom panels show that banks allocate the incoming deposits into cash assets significantly and into long-term Treasuries and agency securities slightly. All these features are characteristic of flight-to-safety and flight-to-liquidity.

Second, Figure B2 provides a breakdown of primary dealers’ repo and reverse repo amounts into overnight and term contracts. The net reverse repo amounts show that primary dealers conduct maturity transformation in their Treasury repo intermediation, borrowing overnight and lending term funds in both the COVID-19 and 2007-09 crises. Yet, the net borrowing amount through overnight repo is much larger in 2020 than in 2007–09, while the net lending amount through term reverse repo is similar in these two time periods. That is, primary dealers become more of a net cash borrower in the repo market post-crisis.

\textsuperscript{32}The series can be found at \url{https://www.federalreserve.gov/releases/h41/}.
\textsuperscript{33}These include total federal funds sold to, and reverse repos with, commercial banks, brokers and dealers, and others, including the Federal Home Loan Banks.
\textsuperscript{34}The series can be found at \url{https://www.federalreserve.gov/releases/h8/}.
Figure B1: Fund Flows of MMFs and Banks During the COVID-19 and 2007–09 Crises

Notes: The top panels plot weekly series of the flows of money market funds (MMFs). The middle panels plot weekly series of the amounts of deposits of commercial banks. The bottom panels plot weekly series of the amounts of commercial and industrial loans (C&I), cash assets, Treasuries and agency securities, and funds lent in the federal funds and repo markets, on the asset side of commercial banks. The units are all in billions of U.S. dollars. The sample period is from January 1, 2007 to December 31, 2008 for the left panels, and from January 1, 2020 to April 13, 2020 for the right panels.
Figure B2: Breakdown of Primary Dealers’ Treasury Repo Positions

Notes: This figure plots weekly series of the primary dealers’ gross repo amount, gross reverse repo amount, and net reverse repo amount for the overnight (left panels) and term (right panels) contracts. The sample period is from January 1, 2020 to April 13, 2020 for the top panels, and from January 1, 2007 to December 31, 2008 for the bottom panels. The units are all in billions of U.S. dollars.