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The purpose of this paper is to show that the "shadow wage", which is used in evaluating government industrial projects, should be close to the actual wage in manufacturing under more general conditions than those assumed by Harris and Todaro (1970). Harris and Todaro argued that because of induced migration, the use of a shadow wage significantly below the actual wage would be harmful.[1] Their model can be criticised on the grounds that, for several reasons, induced migration is not as great as they predicted.[2] This naturally raises the possibility that the shadow wage should be substantially lower than the market wage.

However, the work until now has assumed that the aim of government policy is to maximize the value of measured national income.[3] This objective neglects other elements of individuals' utilities, such as the extent of risk and a possible preference for remaining in a rural area. It is precisely these elements of utility that will cause the level of migration to be lower than that predicted by Harris and Todaro. The present paper reformulates the Harris and Todaro model in terms of utility: workers decide to migrate on the basis of their expected utilities rather than their expected incomes, and the government's objective is to maximise the sum of utilities. Thus, in this paper, utilities replace income and the rest of the mdoel is unchanged. It is demonstrated that, with this modification, the shadow wage in manufacturing should be close to the market wage however strong are the forces which reduce migration. The intuition behind this result can

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[1]This argument is also made in Harberger (1971) and is expressed particularly clearly in Sen (1975).

[2]See, for example, Fields (1975).

[3]This assumption is either explicit or implicit in Bhagwati and Srinivasan (1974), Harberger (1971), Harris and Todaro (1970), Little and Mirrlees (1974) and UNIDO (1972).

be obtained by reformulating Sen's (1975)[4] argument to show that the gain in utility from transfering an extra worker from agriculture to the manufacturing sector is exactly balanced by the loss in utility from those leaving agriculture and failing to find employment.[5] Thus total utility is not increased by employment in manufacturing even though the value of the national income will have risen if the level of migration is less than that predicted by Harris and Todaro. The reason for this conflict between output and utility is that the additional output was bought at the cost of inducing more people to take the risk of migrating and of moving people from the countryside into the towns. This implies that the mere observation of unemployment rates lower than those predicted by Harris and Todaro does not imply that the shadow wage in manufacturing should be significantly lower than the actual wage.

Before proceeding, two points should be noted. First, this paper neglects that component of the shadow wage which is due to the reduction in government funds available for investment as a result of the wage payment.[6] Second, although the result presented here is consistent with the results of Bhagwati and Srinivasan (1974), the problem considered here is different.

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[4]Sen does not mention that, as is demonstrated below, it is necessary to assume a constant marginal product in agriculture for the result to apply exactly.

[5]For example, suppose that the utility level in manufacturing is two, that in agriculture one, and that the unemployed have zero utility. In this case, half of the urban population would be unemployed. If one worker is than transferred from agriculture to manufacturing, that worker will gain one unit of utility. However, this move will induce an additional worker to migrate in order to maintain the urban unemployment rate. This second worker will lose one unit of utility, cancelling out the gain of the first worker.

[6]As pointed out by Sen (1960), this component of the shadow wage can be important. However, this can only increase the shadow wage (see UNIDO (1972) and Little and Mirrlees (1974)) and so does not affect the main point of this paper.

Bhagwati and Srinivasan consider the optimal government policy towards the economy as a whole, while this paper looks at the problem of a project evaluator who can choose the shadow wage but cannot influence other aspects of government policy.

In the next section, the basic model will be described and the main result will be obtained. The model will be the simplest possible that can demonstrate the result. It will also incorporate assumptions about the search mechanism and the payment of marginal products in agriculture that will not exactly correspond to reality in any country. These points and their relation to the basic result are discussed in the conclusion. These qualifications mean that the result should not be interpreted as a definite injunction to use the manufacturing wage as the shadow wage. Rather, the result implies that a government which is concerned with more than just maximizing its measured national income should use a shadow wage that is higher than the agricultural output foregone as a result of induced migration.

The Basic Model

The model is almost identical to that used by Harris and Todaro. The economy comprises two sectors, Agriculture and Manufacturing, each of which produce a single internationally traded good.[7] We will choose units so that the price of each good is unity. Each sector has a production function:

$$X_A = F_A(L_A) \tag{1}$$

$$X_M = F_M(L_M) \tag{2}$$

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[7] The assumption that the goods are traded is simply a device to fix the relative output prices. It does not affect the results in any way.

where L_A, L_M are the quantities of labor employed in agriculture and manufacturing. X_A, X_M are the quantities of goods produced by agriculture and manufacturing.

The fundamental distortion in this model, and the one that causes the migration, is the payment of a fixed real wage, $\bar{W}_m$, in the manufacturing sector that is higher than the wage paid in agriculture. The latter is determined by the marginal product of labor in agriculture:

$$W_A = F'_A \tag{3}$$

where W_A is the wage in agriculture

F'_A is the marginal product of labor in agriculture.

In order to avoid complications associated with the distributional impact of rents and profits, it is assumed that the government receives all the profits from manufacturing and distributes them equally to the whole population. We also assume that land is equally distributed and that people continue to receive rent after they have migrated. Thus, if we normalise the population size to equal unity, each person receives an unearned income, D, given by:

$$D = F_A - L_A F'_A + F_M - L_M \bar{W}_M \tag{4}$$

We can now describe the migration mechanism. This paper follows Harris and Todaro in assuming that the probability of a migrant obtaining employment in manufacturing is given by the proportion of the urban workforce that are employed. However, it is the purpose of this paper to incorporate three factors that reduce migration below the level predicted by Harris and Todaro. These are:

1) Workers are risk averse.

2) The cost of living is higher in the cities than in the countryside.

3) There are non-pecuniary advantages to living and working in rural areas.

Items (2) and (3) imply that income is not a satisfactory indicator of the 'true utility' of living in each area. Thus workers in agriculture will have a different utility function (in terms of income) from those who have migrated to the towns. In addition it is possible that the unemployed migrants will have a different utility function from those who find employment. Item (1) requires that such 'true utility' indicators will have to be transformed into von Neumann-Morgenstern utility functions (which incorporate attitudes towards risk) if it is to be claimed that workers maximise expected utility. Thus the unemployed will have a (von Neumann-Morgenstern) utility given by:

$$V_u = V_u(D) \tag{5}$$

The utilities of the agricultural and manufacturing workers are given by:

$$V_A = V_A(D + F'_A) \tag{6}$$

$$V_M = V_M(D + \bar{W}_M) \tag{7}$$

The migration equilibrium condition is that the average utility level in the economy is equal to the utility level in agriculture, for otherwise expected utility maximising workers will migrate:

$$V_A = \sum_i L_i V_i \tag{8}$$

$$\sum_i L_i = 1 \tag{9}$$

where i is an index that runs over u,A,M, L_u is the number of unemployed migrants.

The utility functions can differ in their origins (representing differences in non-pecuniary advantages) and in their slopes (representing differences in both non-pecuniary advantages and the cost of living). Also, if the workers are risk averse, each function will be concave. It will

be shown that none of these three characteristics, which affect migration, have any role in determining whether or not the shadow wage equals the actual wage. This implies that the level of migration is irrelevant to the question of whether the shadow wage should equal the actual wage.

It is now necessary to specify the government's objective function. First, consider the case of the migrants. The ex ante expected utility of each migrant is given by:

$$(L_u V_u + L_M V_M)/(L_u + L_M)$$

However, ex post some migrants have the utility V_u and some the utility V_m. The question is whether the government should be interested in the ex ante or the ex post utility levels. This paper will assume that the government is concerned only with the ex ante expected utility. The view that is embodied in this judgement is that the government should not mind if people voluntarily take risks and lose: the government should not be paternalistic. If, instead, it was thought that the government should discourage risk taking and thus be especially concerned about the migrants who fail to find jobs, it is straightforward to show that the shadow wage should be even higher than the conclusions of this paper would indicate.[8]

We know from the migration condition that the ex ante expected utility of migrants will equal the certain utility of non migrants. There is, therefore, no reason to put more weight on the expected utility of migrants rather than non-migrants or vice versa. Thus the government's objective function is:[9]

[8]The higher shadow wage would reduce employment and thus the inducement to migrate.

[9]The results below do not, of course, depend on the additive separability of (10) as any differentiable function can be taken to be additively separable in a given neighbourhood.

$$L_A V_A + (L_u + L_M)\left(\frac{L_u V_u + L_M V_M}{L_u + L_M}\right) = \sum_i L_i V_i \qquad (10)$$

It should be noted that, in contrast to the national income, the objective function (10) embodies those elements of individual utilities that reduce the level of migration below that predicted by Harris and Todaro. The risk aversion is represented by the fact that the ex ante expected utility of migrants is less than the utility of the expected wage, as a result of the concavity of V_u and V_M. The cost of living and non-pecuniary advantages of agricultural life are represented by a higher value of V_A than V_M or V_u for any given level of income.

The problem for the government project evaluator is to choose the employment level in manufacturing (and thus the shadow wage) so as to maximise (10) subject to equations (3) - (9).[10] In order to derive necessary conditions for the solution of this problem, we form the Lagrangian.

$$L = \sum_i L_i V_i + \lambda(V_A - \sum_i L_i V_i) + \phi(1 - \sum_i L_i)$$

$$+ \psi(F_A - L_A F'_A + F_M - L_M \bar{W}_M - D) \qquad (11)$$

The first-order conditions are:

$$\frac{\partial L}{\partial L_A} = V_A(1-\lambda) + L_A(1-\lambda)V'_A F''_A + \lambda V'_A F''_A - \phi - \psi L_A F''_A = 0 \qquad (12)$$

$$\frac{\partial L}{\partial L_M} = V_M(1-\lambda) - \phi + \psi(F'_M - \bar{W}_M) = 0 \qquad (13)$$

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[10] This formulation assumes that the government controls all of the manufacturing sector. However, this is not essential to the result.

$$\frac{\partial L}{\partial L_u} = V_u(1-\lambda) - \phi = 0 \tag{14}$$

$$\frac{\partial L}{\partial D} = (1-\lambda) \sum_i L_i V'_i + \lambda V'_A - \psi = 0 \tag{15}$$

where primes denote first-order derivatives and double primes denote second-order derivatives.

In order to interpret thoses conditions, note that the migration condition, equation (8), can be written as:

$$(1-L_A)V_A = L_M V_M + L_u V_u \tag{16}$$

Substituting (12), (13), (14) into (16), we obtain:

$$(1-L_A)\ (\psi L_A F''_A + \phi - V'_A F''_A(\lambda + L_A(1-\lambda))) =$$

$$L_M\phi - L_M\ \psi(F'_M - \bar{W}_M) + L_u\phi \tag{17}$$

Remembering equation (9), we can rewrite equation (17):

$$\frac{L_M}{1-L_A}\ (\bar{W}_M - F'_M) = L_A F''_A - \frac{V'_A F''_A(\lambda + L_A(1-\lambda))}{\psi} \tag{18}$$

It is equation (18) that provides the basic result of this paper. In order to simplify the interpretation, assume first that $F''_A = 0$ so that the marginal product of labor in agriculture is constant. In this case the right-hand side of the equation is zero. The left hand side of the equation is the difference between the wage in manufacturing and the marginal product, the wage subsidy in manufacturing, multiplied by the proportion of the urban work force that is employed in manufacturing. Thus, this equation states that the subsidy to employment should be zero. This means that the shadow wage in manufacturing should equal the market wage.

The basic intuition behind this result is that, when there are no subsidies, the average marginal product of labor in each sector plus the equally distributed profits and rents are supporting the same average utility level. Therefore, if people are moved from one sector to another, diminishing returns in manufacturing will ensure that this level of average utility cannot be maintained.

In order for this argument to work precisely, it is necessary to equate the average marginal product in the urban sector - $L_M F'_M/(L_M + L_u)$ - with the "marginal product of a migrant": the extra output produced in the urban sector when one extra person migrates. It is this condition that is upset when there is a diminishing marginal product in agriculture, because migration out of agriculture will raise the agricultural wage and thus, via the migration condition, reduce the proportion of migrants who remain unemployed. In this case, the "marginal product of a migrant" will be greater than the average marginal product in the urban sector and migration would be beneficial. It is this effect that is represented by the right hand side in equation (18).

This argument suggests that the right hand side of equation (18) is positive of F''_A is negative. This can be confirmed by writing equation (15) as:

$$V'_A(\lambda + L_A(1-\lambda)) = \psi - (1-\lambda) \sum_{i \neq A} L_i V'_i \qquad (19)$$

Substituting (19) into (18), we obtain:

$$\frac{L_M}{1-L_A}(\bar{W}_M - F'_M) = -(1-L_A)F''_A + F''_A \frac{(1-\lambda)}{\psi} \sum_{i \neq A} L_i V'_i \qquad (20)$$

From equations (13) and (14) note that:

$$\frac{(1-\lambda)}{\psi} = \frac{\bar{W}_M - F'_M}{V_M - V_u} \tag{21}$$

Substituting (21) into (20), we obtain:

$$\frac{L_M}{1-L_A}(\bar{W}_M - F'_M) = -(1 - L_A)F''_A + F''_A \sum_{i \neq A} L_i V'_i \frac{\bar{W}_M - F'_M}{V_M - V_u} \tag{22}$$

As expected, the first term on the right hand side of equation (22) requires a subsidy in manufacturing. However, the final term reduces the size of the subsidy in manufacturing, although it can never eliminate it.

Thus equation (18) does call for a subsidy to manufacturing. However, the advisability of such a subsidy is unrelated to the modifications that have been introduced in this paper to explain a lower level of migration than that predicted by Harris and Todaro. This is shown by the fact that even if individuals' utilities equalled their wages (the assumption used by Harris and Todaro) equation (22) would still require a subsidy. Also, no subsidy would be required if $F''_A = 0$ however low the level of migration. Thus we see that the level of migration has no effect on whether or not there should be a wage subsidy in manufacturing.

Conclusion

This paper has shown that under circumstances considerably more general than those of the original Harris-Todaro model, but more restrictive in requiring a constant marginal product in agriculture, the shadow wage should equal the market wage in manufacturing. The importance of this result is not that it prescribes a particular shadow wage. After all, if the marginal product in agriculture is not constant, a lower

shadow wage is indicated. What is important is that it has been shown that mere observation of unemployment rates lower than those predicted by Harris and Todaro does not imply that the shadow wage in manufacturing should be lower than the market wage.

The implication of this is that people should look more closely at why unemployment is lower than predicted by Harris and Todaro. If the reason is that the migration mechanism is fundamentally different from that of Harris and Todaro, the results of this paper may not apply. If, however, it is risk aversion or differences in the cost of living that are reducing migration, the argument for a low shadow wage is incorrect. Thus, to the extent that people have not distinguished between different causes of low migration, the shadow wage should be higher than is often thought.

All of the formal analysis rested on the assumption that workers in agriculture are paid their marginal products. Strictly speaking, this is not essential. What is necessary is that individuals should make their migration decisions on the basis of their marginal products.[11] Thus, in a family farm where everybody receives the average product, the family will still consider an individual member's marginal product before deciding whether he should migrate. Similarly, it is the marginal product that counts if the whole family moves and can sell the land. However, it is possible to construct examples where workers' migration decisions are not based on their marginal product. In such a case, the result of this paper

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[11] It is for this reason that we assumed that workers continued to receive rent from their land after they migrated.

would have to be modified.

Finally, it is worth considering two modifications to the migration mechanism in order to test how robust the result is. One simple modification is to assume that unsuccessful migrants find jobs in the unorganized sector but at much lower wages than those in modern manufacturing. So long as the marginal product of labor in these activities does not diminish, this modification really just consists of raising the value of V_u for any value of D. Thus the result is unchanged. However, if the marginal product does diminish, a subsidy is required.[12]

The other modification is to assume, following Sen (1975), that migrants do not have the same probability of employment as somebody who has been an urban worker for some time. Instead, they have to "queue" for a job. Stiglitz (1974) shows that the equilibrium conditions for this sort of model are very similar to those of the Harris-Todaro model, unless the labor force is growing fast or the workers have high rates of time discount. Thus our result is approximately correct.

These two examples show that slight modifications to the migration mechanism will produce slight modifications to the formal results. However, the main point of this paper remains: project evaluators should look more closely at the cause of low migration before using it as a justification for low shadow wages. The unwarranted use of low shadow wages can reduce the well-being of the population even if it increases the value of the national income.

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[12] The reason for this is that, with a diminishing marginal product in the unorganized sector, an increase in the urban workforce will lower the wage in the unorganized sector. This will reduce migration and increase $L_M/(L_M + L_u)$, in the same manner as an increased wage in agriculture when F''_A is negative.

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