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EMBEDDING COOPERATION IN GENERAL-EQUILIBRIUM MODELS

By

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"Embedding cooperation in general-equilibrium models"

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Abstract. Humans cooperate a great deal in economic activity, but our two major models of equilibrium – Walrasian competitive in markets and Nash in games – portray us as only non-cooperative. In earlier work, I have proposed a model of cooperative decision making (Kantian optimization); here, I embed Kantian optimization in general-equilibrium models and show that 'Walras-Kant' equilibria exist and often resolve inefficiencies associated with income taxation, public goods and bads, and non-traditional firm ownership, which typically plague models where agents are Nash optimizers. In four examples, introducing Kantian optimization in one market – often the labor market – suffices to internalize externalities, generating Pareto efficient equilibria in their presence. The scope for efficient decentralization via markets appears to be significantly broadened with cooperative behavior.

Key words: Kantian optimization, cooperation, general equilibrium, market socialism, global emissions control, worker-owned firms, externalities, public goods

JEL codes: D50, D60, D62, D70, D91, E19, H21, H23, H41

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1. Introduction

Economic theory has studied *par excellence* how economic agents compete with each other, the two magisterial models being general-equilibrium theory and game theory. However, what distinguishes our species' economic behavior from that of the other great apes is our ability to cooperate with each other. A series of recent books, by economists, anthropologists, and evolutionary psychologists focus upon our cooperative abilities (Bowles and Gintis [2011], Tomasello [2014, 2016], Henrich and Henrich [2007]). It is a serious lacuna in our economic modeling not to have attempted to introduce a model of cooperation at the level of abstraction of Nash equilibrium into general-equilibrium theory, to rectify the one-sided view of economic decision-making that Nash equilibrium represents.

In recent work, I (2010, 2015) have proposed how cooperation of economic agents can be formalized as a mathematical first cousin of Nash optimization, a protocol I've called Kantian optimization. In a variety of non-market games, Kantian optimization delivers equilibrium allocations that are Pareto efficient when Nash optimization fails to do so. In the present paper, I insert Kantian optimization into Arrow-Debreu general-equilibrium models of market economies, and show that the same result holds. To wit, if there is Kantian optimization in a single market (usually the labor market), this suffices to resolve a number of classical inefficiencies, dubbed 'market failures.' Examples presented below show that Kantian optimization delivers decentralized, Pareto efficient allocations when there are public and private goods; that it can eliminate the dead-weight loss of income taxation; that it provides a decentralized, efficient solution to the problem of global carbon emissions; and that it gives Pareto efficient equilibria in an economy with worker-owned firms. In most of these cases, there are degrees of freedom in the equilibrium income distribution that can be exogenously chosen, while preserving Pareto efficiency. Thus, in a word, cooperative economic behavior, modeled as Kantian optimization, enables us to separate efficiency from distributional considerations.

The notable advantage of the Kantian approach is that it does not rely on the ubiquitous practice of behavioral economics (to date) in explaining cooperative behavior,

which is to insert 'exotic' arguments into preferences – arguments like altruism, or a warm glow, or a preference for equality or fairness – and then to derive the cooperative solution as a *Nash* equilibrium of the game with altered preferences. Granted, a moral view motivates Kantian optimization, but that morality is captured in how people optimize, rather than by amending their preferences. I claim that to get the 'right equilibrium' using the technique of inserting exotic arguments into preferences usually requires that the modeler know what the cooperative equilibrium is *ex ante* – then preferences can be constructed (reverse engineering) so that the Nash equilibrium of the game with the altered preferences is the desired (cooperative) strategy profile. But in many cases – including all the cases I study below—it is not obvious what the cooperative (i.e., Pareto efficient) equilibrium *is*, ex ante, and so this technique cannot be used. Do we know what the frontier of Pareto efficient allocations in the problem of global emissions is? In the absence of knowledge of all the information about preferences and technologies, I assert we do not: but the Kantian model I propose below finds a point -- in fact a multi-dimensional manifold of such points -- on this frontier in a decentralized manner. In the models below preferences are classical and self-interested, containing only traditional (non-exotic) arguments. Parsimony in modeling preferences is achieved by varying the optimization protocol.

Indeed, these results suggest that our nomenclature of 'market failures' may be off-base: for the efficiency results I demonstrate are all achieved in market economies. The failure of efficiency appears to be due, not to the market as such, but to Nash optimization.

In the next section, I review the definition of three kinds of Kantian optimization in games, and their moral motivation. Section 3 presents a model of greenhouse gas emissions by a set of countries, where Pareto efficient equilibrium allocations exist, and the income distribution among *m* countries at equilibrium has $m-1$ degrees of freedom. Section 4 decentralizes the efficient allocation of a public and private good in a semi-market economy. Section 5 presents a model of market socialism, where Pareto efficient Walras-Kant equilibria exist at almost any degree of income equality. Section 6 presents a model of worker-owned firms, where production involves labor in *m* occupations, and there are Pareto efficient equilibria with $m-1$ degrees of freedom in the income

distribution. Section 7 concludes by briefly addressing the skepticism that many will have with regard to the realism of Kantian optimization as a human behavioral protocol. An appendix presents the proofs of the existence theorems.

I do not present the most general model in each of the four substantive cases I discuss: on the contrary, I try to present the simplest models that make the point, so as not to distract the reader with complexity that would render less transparent the central arguments. The third substantive model, in section 5, is presented in most detail.

2. Kantian equilibrium in games[1]

We consider games in normal form among *n* players, whose payoff functions are denoted $V^i : S^1 \times .... \times S^n \to \Re$, where the strategy spaces S^i are intervals of non-negative real numbers. Strategies will be usually denoted $E^i \in S^i$; a strategy profile is denoted **E**; for any vector $\mathbf{z} = (z^1,...,z^n)$, denote $z^S = \sum z^i$. Denote the j^{th} partial derivative of any function *f* by f_j.

Definition 1 A game $\{V^i, S^i, n\}$ is *strictly monotone decreasing (increasing)* if the payoff of each player is strictly decreasing (increasing) in the strategies of the other players. A game is *strictly monotone* if it is either strictly monotone increasing or decreasing.

Consider, first, symmetric games. It will suffice, for our purposes here, to consider symmetric games where the payoff function of player *i* is $V(E^i, \sum_j E^j)$, for some function *V*. The supposition that the game is of this form is that all players 'are in the same boat,' and being in the same boat induces a kind of solidarity, which suggests the question "What is the strategy I'd like all of us to play?" In this case, each player chooses the strategy that maximizes $V(E, nE)$. Clearly players unanimously agree on the answer. This motivates:

[1] This section reviews material presented in Roemer (2010, 2015, in press).

Definition 2 A *simple Kantian equilibrium* is a strategy $E^* \in \bigcap_{i=1}^{n} S^i$ such that:

$$(\forall i)(E^* = \arg\max_{E \in S^i} V^i(E,...,E)). \quad (2.1)$$

Typically, simple Kantian equilibria do not exist. However, if the game is symmetric, they do. More generally, they exist if the game has a 'common diagonal': that is, if every player orders the strategy profiles associated with the 'main diagonal' of the payoff matrix in the same way.

It is noteworthy that solidarity is defined (in the American Heritage Dictionary) as as 'a union of purpose, sympathies, or interests among the members of a group.' This is most easily interpreted as a symmetric game. Thus, solidarity is *not* the characterization of the actions that the members of the group take (e.g., a simple Kantian equilibrium), but rather of the state of the world that places them 'in the same boat.' I am suggesting that the Kantian question proposed in the last paragraph is induced by the commonality of the members' interests[2].

The aphorism that summarizes this kind of thinking is "either we all hang together, or we each hang separately," a phrase first uttered by Benjamin Franklin at the signing of the Declaration of Independence in 1776. Franklin was urging his co-signers *not* to reason according to the Nash protocol, which would have produced too many free riders – too few signers of the Declaration.

The nomenclature 'Kantian equilibrium' is borrowed, with some apology, from Kant's categorical imperative: that morality requires taking that action that one would wish be universalized.

When games are not symmetric, simple Kantian equilibria rarely exist. We generalize in two ways. Imagine, now, a game with heterogeneous preferences V^i and suppose the existing strategy profile is $\mathrm{E} = (E^1,...,E^n)$. Agent 1 is considering increasing his effort by 10%. But he understands that there are externalities to the choice of effort levels, and he asks, "How would *I* like it if everyone increased his effort by 10%?" If there are negative externalities (a monotone decreasing game), then he might

[2] That humans, and not chimpanzees, have the capacity to think this way is certainly related to what Tomasello calls our ability to conceptualize 'joint intentionality.'

well answer that he would dislike this change (unlike a Nash optimizer who might find increasing only his *own* effort by 10% attractive). This motivates the definition:

Definition 3 $(E^{1*},...,E^{n*})$ is a *multiplicative Kantian equilibrium* if no player would prefer to re-scale the entire vector by any non-negative scalar. That is:

$$(\forall i)(1 = \arg\max_{r} V^i(rE^{1*},...,rE^{n*})) . \quad (2.2)$$

To be precise, for agent *i*, the domain of *r* in the 'argmax' function is bounded by the requirement that $rE^{i*} \in S^i$. It is not necessary, however, that $rE^{j*} \in S^j$ for $j \neq i$.[3]

In other words, a multiplicative Kantian equilibrium is a stable point with respect to optimization protocol "change my effort by a factor *r* only if I would prefer the strategy profile where everyone changes his effort by the factor *r*." The first-cousin relationship to Nash equilibrium is evident – players evaluate the counterfactual using their own preferences, and need not know the preferences of others.

The motivation here is, again, a kind of solidarity. By contemplating the effect of a general re-scaling of the strategy profile, the agent is forced to consider the externalities others impose on him. He is not an altruist – that is, the protocol does not force him to consider the externalities his behavior has for others. But, indeed, it turns out that the internalization of externalities that this kind of Kantian reasoning induces suffices, in many cases, to resolve inefficiencies characteristic of Nash equilibrium. (When a small child throws her candy wrapper on the sidewalk, the parent may say "How would *you* like it if *everyone* threw his candy wrapper on the sidewalk?" This query assumes or encourages a Kantian morality in the child. In contrast, exploiting altruism, the parent would say, "How do you think others feel when you throw your candy wrapper…." The altruistic approach may be ineffective, if the child understands that her candy wrapper has a miniscule effect on the environment.)

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[3] To be still more precise, the domain condition on *r* when considering agent *i* is that $rE^i \in S^i$ and $(rE^1,....,rE^n)$ defines a payoff for *i*. In all the games in this paper, it's in fact the case that $V^i(E^1,...,E^n) = \hat{V}^i(E^i,E^S)$ -- that is, the payoff depends on one's own contribution and the total contribution – and there is a given domain D^i for the function $\hat{V}^i$. The proper domain specification for *r* in (2.2) is $\{r \mid (rE^i, rE^S) \in D^i\}$.

Re-scaling the strategy profile is only one kind of symmetric change in the profile. Another kind of counterfactual an agent could consider is *translating* the strategy profile by a constant. This induces another kind of Kantian equilibrium:

Definition 4 $(E^{1*},...,E^{n*})$ is an *additive Kantian equilibrium* if no player would prefer to translate the entire vector by any scalar; that is:

$$(\forall i)(0 = \underset{\{r|(E^{i*}+r)\in S^i\}}{\arg\max}\ V^i(E^{1*}+r,...,E^{n*}+r)). \qquad (2.3)$$

The sense in which Kantian optimization models cooperation is suggested by:

Proposition 1[4] *If the game* $\{V^i, S^i, n\}$ *is a strictly monotone game, then any simple or additive Kantian equilibrium is Pareto efficient in the game, and any positive multiplicative Kantian equilibrium is Pareto efficient in the game.*

Thus, Kantian optimization resolves tragedies of the commons (monotone decreasing games) and the inefficiency of public-good provision/ free rider problems (monotone increasing games).

An example

Consider a fishing economy where the fishers have utility functions $u^i(x,E)$ where *x* is fish consumed and *E* is efficiency units of labor expended fishing. The production function of the lake where people fish is $X = G(E^S)$, *G* increasing and concave. The allocation rule is 'each keeps his catch,' so (apart from noise) the allocation of fish at an effort vector is proportional to effort:

$$x^i = \frac{E^i}{E^S} G(E^S).$$

This induces a game defined by:

$$V^i(E^i, E^S) = u^i(\frac{E^i}{E^S} G(E^S), E^i).$$

If *G* is strictly concave, this is a monotone decreasing game. Proposition 1 tells us that any multiplicative Kantian equilibrium is Pareto efficient *in the game*. But something

[4] Roemer (in press, chapters 2 and 3).

stronger is true: such an equilibrium is also Pareto efficient *in the economy*. To see this, we write the characterizing condition for a multiplicative Kantian equilibrium in the differentiable case:

$$\text{For all } i \qquad \left.\frac{d}{dr}\right|_{r=1} u^i\left(\frac{rE^i}{rE^S}G(rE^S), rE^i\right) = 0\,. \tag{2.4}$$

Expanding this expression, and assuming all efforts are positive, yields:

$$\text{for all } i \quad \frac{-u_2^i}{u_1^i} = G'(E^S)\,,$$

which is just the statement that the marginal rates of substitution of all agents between labor and consumption equal the marginal rate of transformation, the condition for Pareto efficiency in the economy.

In other words, efficiency in the *game* means that, among all allocations achievable according to the 'each-keeps-his-catch' rule, the Kantian equilibrium is efficient. Efficiency in the *economy* means that there is no other feasible allocation, under any rule, that Pareto dominates the Kantian equilibrium of the game. □□

Mathematically, we can contrast Nash (non-cooperative) behavior with Kantian (cooperative) behavior in this way. At a given strategy profile $\mathbf{E} \in \prod_{i=1}^{n} S^i$, each Nash optimizer contemplates changing the strategy profile by varying *his own dimension only* of the profile **E**. Thus the *n* counterfactual *strategy profiles* are chosen from *different* unidimensional line segments of the profile space $\prod_{i=1}^{n} S^i$. However, in Kantian optimization, all players contemplate deviating within the *same* unidimensional line segment of the profile space – in the case of multiplicative Kantian along the ray through the profile **E**, and in additive Kantian, along the 45 degree line (in the two player case) in the profile space containing **E**. Restricting individual deviations to a common space of strategy profiles is the mathematical representation of cooperation.

There are many other 'Kantian variations' besides the additive and multiplicative ones (see Roemer[2015]), but they will not be needed in the present analysis.

I do not claim that Kantian optimization is rational in one-shot games. What I argue is that in many situations, the same players counteract with each other all the time, and a morality of solidarity may develop that induces players to optimize in the Kantian manner, if they come to feel a sense of solidarity. It is not my task here to show how this emerges; rather, I want to show that if Kantian optimization is a moral protocol in a population, then many inefficiencies *in market economies* can be resolved. I suggest readers think of Kantian optimization as a moral code. In the last section of the paper, I address the accessibility of Kantian reasoning to human beings in somewhat more detail.

I amplify on my remark about behavioral economics in the Introduction. An experimenter observes in the lab that subjects do not play what the experimenter believes is the Nash equilibrium of the game (think of trust games, public-good games, the ultimatum and dictator games). So the experimenter looks for 'exotic' preferences, which would, if held by the subjects, produce the observed outcome as a *Nash* equilibrium. This is done by inserting arguments like altruism, a concern for fairness, a concern for equality, etc., *as arguments of preferences*. In contrast, my approach is to keep preferences classical, but to alter the way that agents optimize.

I believe my approach is superior, because it decentralizes the cooperative solution even when it is not obvious what that solution is. Contrast this with the lab games that I listed above: in all those cases, we can immediately see what the cooperative solution is, and so it is not so hard to design preferences that will make that solution a Nash equilibrium of the game so defined. But in many cases, the cooperative solution is not obvious. Take the fishing example above: it is far from obvious what the Pareto efficient allocation in which each keeps his catch *is.* (Under general conditions of convexity, such an allocation exists, and is locally unique: see Roemer and Silvestre [1993].) But the multiplicative Kantian equilibrium locates it without resorting to inventing new preferences for the players. How would you go about assigning new preferences to the fishers so that the *Nash* equilibrium of the new game is the Pareto efficient allocation in which each keeps his catch? In other work, I have shown there is no natural way to do this (Roemer [in press]).

To reiterate: my earlier papers on Kantian equilibrium have investigated its ability to resolve inefficiencies in non-market games, or in very simple market games. The

contribution of the present paper is to embed Kantian optimization in general-equilibrium market economies, and to show how it can resolve inefficiencies there that are prevalent with Nash optimization.

3. A model of global carbon emissions

Country *i* operates a single firm, whose production function is $G^i(K,E)$ where *K* is capital and *E* is carbon emissions. All firms produce a single consumption good, called *x*. (We could generalize and have many goods, but that is just a distraction.) Labor is implicit. Capital is purchased on an international capital market, but labor is immobile: hence, the entire labor supply of country *i* works in the firm of the country. We therefore do not display explicitly the dependence of the technology on labor, nor do we display labor in the utility function of each country.

There is a representative agent in each country, with utility function $u^i(x,E^S)$, where $E^S \equiv \sum E^i$ is the global emissions. Utility is increasing in *x* and decreasing in E^S. We assume that these agents care about the future citizens of their country, and they have internalized this in their preferences through the dependence of utility on global emissions.

Country *i* has a capital endowment of $\bar{K}^i$. It is easiest to assume that capital does not depreciate. (It also has a labor endowment, but as I remarked, we need not display that explicitly.)

An allocation $\{(x^i,K^i,E^i) \mid i=1,...,n\}$ is *feasible* if:

$$\sum x^i \le \sum G^i(K^i,E^i)$$
$$\sum K^i \le \bar{K}^S \equiv \sum \bar{K}^i \quad .$$

By standard methods, one shows the following:

Fact. An interior allocation is Pareto efficient if and only if:

(i) for all *i*,*j* $G_1^i = G_1^j$

(ii) for all i, $G_2^i = -\sum_{j=1}^{n} \frac{u_2^j}{u_1^j}$, (3.1)

as well as the material balance conditions. Thus, efficiency requires equalization across countries of both marginal products, and a Samuelson condition relating the marginal product of emissions to the marginal rates of substitution.

There are three prices, (p,r,c) for the good, capital, and a unit of emissions, respectively. Each firm will maximize profits, which are given by $pG^i(K,E) - rK - cE$ if the firm 'demands[5]' (K,E).Of course, profits include neoclassical profits and labor income. We need not distinguish between these, since workers in each country offer their labor inelastically to the firm, and all profits net of capital costs and emissions payments redound to the citizenry.

Capital will be supplied on the global market by the citizenry that owns it.

The citizenry of each country must 'supply' the universe of firms with emissions. It is the determination of the global emission supply that is unconventional.

A firm will pay cE^iinto a global fund if it emits E^i, and these revenues will be distributed to the global citizenry, according to a share vector $(a^1,...,a^n)$, non-negative and summing to one, which emerges endogenously as part of the equilibrium. Thus the income of country *i* from this demogrant will be $a^i cE^S$, while the country's firm pays cE^i into the fund.

Consider the following game whose *n* players are the citizenries of each country (i.e., the *n* representative agents). The strategy space for each player is $\Re_+$. Given a capital and emissions demand by its firm (K^i,E^i) , prices, and a vector $a=(a^1,...,a^n)\in\Delta^{n-1}$ the payoff function for player *i* is:

$$V^i(\hat{E}^S) = u^i\left(\frac{r\bar{K}^i + pG^i(K^i,E^i) - rK^i - cE^i + a^i c\hat{E}^S}{p}, \hat{E}^S\right), \qquad (3.2)$$

where $\hat{E}^S$ is a proposal, by county *i,* for *global* emissions.

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[5] To say the firm 'demands' emissions E^i means it proposes to emit that many tons of carbon.

Note the first argument in *u* is the amount of the good that country *i* can purchase given its income, which comes from three sources – its capital income, its profit income, and its demogrant.

Note that a *simple Kantian equilibrium for* $(p,c,r,a,K^1,E^1,....,K^n,E^n)$ is a number $\hat{E}^S \in \Re_+$ such that, for all *i*, $\hat{E}^S$ maximizes $V^i(\cdot)$.

The *ethical appeal* of the simple Kantian equilibrium of the game defined, if it exists, is that it is a *unanimous decision* of the global level of the public bad, which maximizes the utility of countries, subject to the existence of a feasible share rule for allocating the sum total of emissions taxes.

Definition 5. A *global Walras-Kant equilibrium with emissions* is a price vector (p,r,c) and a share vector $(a^1,...,a^n)$, summing to one, demands for capital and emissions (K^i,E^i) by each firm G^i, a vector of consumptions $(x^1,...,x^n)$, and a *total supply* of global emissions $\hat{E}^S$, such that:

- For each *i*, $(K^i,E^i) = \underset{K,E}{\arg\max}\, pG^i(K,E) - rK - cE$
- The number $\hat{E}^S$ is a simple Kantian equilibrium of the game *V* defined in (3.2), given prices, $\{K^1,...,K^n\}$, $(E^1,...,E^n)$ and $\{a^i\}$. Countries *unanimously* agree on the global emissions supply.
- for all *i*, $x^i = \dfrac{r(\bar{K}^i - K^i) + pG^i(K^i,E^i) - cE^i + a^i cE^S}{p}$)
- $\sum x^i = \sum G^i$, $\sum K^i = \bar{K}$, and $\hat{E}^S = \sum E^i$.

Note, especially, that there are no *ex ante* limits on emissions, and no *ex ante* allocation of emissions credits to countries. So these two contentious problems in the discussion of global emissions' control are solved by the use of Kantian optimization – that is to say,

by cooperation in the choice of emission 'supplies.' The citizens supply the permission to the countries *in toto* to emit. The *agreement* among countries specifies that firms may not emit until it is verified that total emissions will be no greater (in fact equal to) the citizenry-determined total supply of emissions.

Proposition 2 *Any global W-K equilibrium with global emissions is Pareto efficient.*

Proof:

1. By profit-maximization, we have:

$$\forall i \quad \frac{r}{p} = G_1^i, \frac{c}{p} = G_2^i \ . \tag{3.3}$$

2. It follows from the Fact (3.1) characterizing Pareto efficiency that condition (i) holds. What remains to prove for condition (ii) is that

$$\frac{c}{p} = -\sum \frac{u_2^i}{u_1^i} \ .$$

3. A simple Kantian equilibrium of the game *V*, satsifies

$$(\forall i) \qquad \frac{d}{d\hat{E}^s} u^i\left(\frac{r(\bar{K}^i - K^i) + pG(K^i, E^i) - cE^i + a^i c\hat{E}^s}{p}, \hat{E}^s\right) = 0 \ . \tag{3.4}$$

Compute this says:

$$\text{for all } i \qquad u_1^i \frac{1}{p}(a^i c) + u_2^i = 0,$$

which reduces to:

$$\text{for all } i \qquad \frac{a^i c}{p} = -\frac{u_2^i}{u_1^i} \ . \tag{3.5}$$

Summing the last equations over *i* proves the claim, by step 2. □□

I demonstrate existence under an assumption that is simplifying but probably not necessary:

Assumption QL. All the utility functions are quasi-linear of the form $u^i(x, E^S) = x - h^i(E^S), h^i$ convex and increasing and $-h^i$ satisfies the Inada conditions.

Nevertheless, the Assumption QL is perhaps not a bad one since the agents are countries: it says countries wish to maximize GDP minus the costs of global warming.

Proposition 3 *Under Assumption QL, and standard concavity assumptions on the functions* G^i *including Inada conditions, a Walras-Kant equilibrium with global emissions exists.*

Proof: See Appendix, Parts A and B.

Remark. Without the assumption of quasi-linearity, the determination of the $\{a^i\}$ and $\hat{E}^S$, in step 2 of the above proof, is not so easy.

I have stated the ethical appeal of the simple Kantian equilibrium for global emissions. It is this appeal that would motivate countries to agree to this procedure.

Now the income allocation, which is locally unique and that emerges from this equilibrium, may not be so desirable. This can be amended by adding fixed transfers to T^i the incomes of all countries, which sum zero. These transfers will not alter the analysis. Thus, we have an $n-1$ manifold of efficient equilibria. Of course, there will be a political problem in agreeing upon what the transfers will be.

This equilibrium concept decentralizes the problem. There is no need for a centralized decision on the allocation of permits. The scheme is reminiscent of 'cap-and-trade,' where the *global* cap on emissions is set by the world's citizenry. The market for emission permits is replaced by the requirement (agreed to by the community of countries) that total emissions do not exceed the 'supply' of global emission permits, which is the simple Kantian equilibrium of the game defined in (3.2).

Skepticism with regard to the proposal may be due, first, to whether the simple Kantian equilibrium of the game defined is indeed ethically attractive to the participants, and second, in having confidence that the preferences of countries will properly take future effects of global emissions into account. However, if countries are not willing to

take the effects of emissions on future generations into account, there is no satisfactory solution to the global emissions problem under any procedure.

4. An economy with a private good and a public good

Consider an economy with a private good (*x*), a public good (*y*), and labor (*E*). Citizens have concave, differentiable utility functions $u^i(x,y,E)$ of the usual kind. There is a private firm that produces the private good with production function G using labor as the only input. There is a cooperative firm that produces the public good from labor, using production function *H*. The private firm is owned by citizens. Each citizen is endowed with $(\bar{E}^i,\theta^i)$, a positive amount of labor in efficiency units and a share of the private firm. The cooperative firm will be organized along a cooperative principle.

Let $E^i=(E_1^i,E_2^i)$ be a supply of labor by agent *i* to firm 1 (private) and firm 2 (cooperative), respectively. There are *n* citizens. A feasible allocation satisfies:

$$G(E_1^S)\geq x^S, H(E_2^S)\geq y,$$
$$(\forall i)(E_1^i+E_2^i\leq \bar{E}^i).$$

Fact. An interior[6] allocation in the differentiable case is Pareto efficient if and only if:

$$\text{(A)}\quad (\forall i)(G'(E_1^S)=-\frac{u_3^i(x^i,y,E^i)}{u_1^i(x^i,y,E^i)}\text{, and (B)}\quad \frac{G'(E_1^S)}{H'(E_2^S)}=\sum_i\frac{u_2^i}{u_1^i}\text{,}\qquad (4.1)$$

in addition to the material balance equations.

We define a notion of equilibrium, which is semi-market. The private firm maximizes profits facing prices for the private good and labor (p,w). Citizens supply labor to both the private firm and the cooperative firm. Workers are paid wages by the private firm, but not by the cooperative firm, which operates outside the market. The vector of labor supplies and demands for the private good are conventional. But the vector of labor supplies to the cooperative firm must be an additive Kantian equilibrium of a game to be defined.

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[6] An *interior* allocation is one where (x^i,y,E^i) is positive for all *i* and $E^i<\bar{E}^i$. It is not necessary that both E_1^i and E_2^i be positive for every *i*.

Suppose we are given a vector of labor supplies to the private firm $\hat{E}_1 = (\hat{E}_1^1,...,\hat{E}_1^n)$ and private-good consumptions $\hat{x} = (\hat{x}^1,...,\hat{x}^n)$. Define a game among the *n* players with the following payoff functions:

$$V^i(E_2^1,...,E_2^n) = u^i(\hat{x}^i, H(E_2^S), \hat{E}_1^i + E_2^i) \qquad (4.2)$$

(Recall $E_2^S = \sum_i E_2^i$.)

We can now define a *Walras-Kant equilibrium with a public and private good* as follows. It consists of a price vector (p,w) , an allocation of goods $(\hat{x}^1,...,\hat{x}^n,\hat{y})$, *n* effort vectors $\hat{E}^i = (\hat{E}_1^i, \hat{E}_2^i)$, and a supply of the good and demand for labor by the private firm $(\hat{X},\hat{D})$ such that:

(a) The vector $(\hat{X},\hat{D})$ maximizes Firm 1's profits , i.e., $pX - wD$. Denote these profits by $\Pi(\hat{X},\hat{D})$;

(b) Given $(\hat{y},\hat{E}_2^1,....,\hat{E}_2^n)$, for each *i*, the choice $(\hat{x}^i,\hat{E}_1^i)$ maximizes $u^i(x^i,\hat{y},E_1^i + \hat{E}_2^i)$ over the budget set:

$$\{(x^i,E_1^i) \mid px^i \le wE_1^i + \theta^i\Pi(\hat{X},\hat{D})\} ;$$

(c) Given $(\hat{x}^1,...,\hat{x}^n,\hat{E}_1^1,...,\hat{E}_1^n)$, the vector $\hat{E}_2 = (\hat{E}_2^1,...,\hat{E}_2^n)$ is an additive Kantian equilibrium of the game *V* defined in (4.2) above;

(d) Markets clear: $\sum \hat{x}^i = \hat{X}$, $\sum \hat{E}_1^i = D$, and in addition $H(E_2^S) = y$.

In other words, every worker may participate in both the private and cooperative economy; his choices in the private economy are optimal for him, given prices, and given the labor he expends in the cooperative firm and the value of the public good, and the levels of participation of workers in the cooperative firm form an additive Kantian equilibrium for them, given the consumption and labor they receive in the private/market sector. So in the private economy, workers behave as they do under capitalism, but when producing the public good, they optimize in a cooperative fashion.

Proposition 4 *Any Walras-Kant equilibrium with a public and private good is Pareto efficient.*

Proof:

1. I will assume the equilibrium is interior for simplicity, although the proof extends to corner solutions.

2. By profit maximization, $G'(E_1^S) = \frac{w}{p}$. By utility maximization over (x^i, E_1^i), it is easy to check that $\frac{w}{p} = -\frac{u_3^i}{u_1^i}$ where the argument of u^i is $(\hat{x}^i, \hat{y}, \hat{E}_1^i + \hat{E}_2^i)$. It follows that condition (A) of the characterization of Pareto efficiency in (4.1) holds.

2. By concavity, the following first-order condition characterizes the additive Kantian equilibrium of the game *V*:

for all *i*, $\left.\frac{d}{dr}\right|_{r=0} V^i(r+E_2^1,...,r+E_2^n) = \left.\frac{d}{dr}\right|_{r=0} u^i(\hat{x}^i, H(nr+E_2^S), \hat{E}_1^i + r + E_2^i) = 0.$

Expanding this condition we have:

For all *i*, $u_2^i H'(E_2^S)n + u_3^i = 0$. (4.3)

Using the fact that $-G'(E_1^S)u_1^i = u_3^i$, proved in step 2, we can write (4.3) as:

$$u_2^i H'(E_2^S)n - G'(E_1^S)u_1^i = 0. \qquad (4.4)$$

Now, since $E_2^S > 0$, by interiority of the equilibrium, $H'(E_2^S)$ is well-defined and positive; rewrite equation (4.4) as:

$$\frac{G'(E_1^S)}{H'(E_2^S)n} = \frac{u_2^i}{u_1^i}. \qquad (4.5)$$

Now add condition (4.5) over *i*, giving condition (B) in (4.1) of Pareto efficiency. □□

What happens if we substitute for condition (c), condition (c*):

(c*) Given $(\hat{x}^1,...,\hat{x}^n, \hat{E}_1^1,...,\hat{E}_1^n)$ the vector $\hat{E}_2 = (\hat{E}_2^1,...,\hat{E}_2^n)$ is a *multiplicative* Kantian equilibrium of the game *V* defined in (2) above.

We have:

Proposition 5 *Any Walras-Kant (multiplicative variant) equilibrium in which for all i* $E_2^i > 0$ *is Pareto efficient.*

Proof:

1. Same step 2 in Prop. 4.

2. We now require:

$$\text{for all } i, \quad \left.\frac{d}{dr}\right|_{r=1} u^i(\hat{x}^i, H(rE_2^S), \hat{E}_1^i + rE_2^i) = 0 ,$$

or: $u_2^i H'(E_2^S)\frac{E_2^S}{E_2^i} + u_3^i = 0$.

Again, we substitute $-G'(E_1^S)u_1^i$ for u_3^i, giving:

$$\frac{u_2^i}{u_1^i} = \frac{E_2^i}{E_2^S}\frac{G'(E_1^S)}{H'(E_2^S)} .$$

Adding over i gives the required condition (B) for Pareto efficiency. □

We have the existence result:

Proposition 6 *If G obeys the Inada conditions and is strictly concave, and the utility functions are strictly concave, then an additive Walras-Kant equilibrium with a public and private good exists.*

See Appendix, Parts A and C.

Remark. The proof of existence of a *multiplicative* Walras-Kant equilibrium should also be true, but will be more delicate. This is because the zero vector is always a multiplicative Kantian equilibrium of the game *V*, but we want to show the existence of a Walras-Kant equilibrium where the vector $\mathbf{E}_2$ is strictly positive (or else we lose Pareto efficiency). Doing this requires cutting out a small piece of the domain $\prod_{i=1}^{n}[0,\bar{E}^i]$ near the origin, and then some conditions on the derivatives of u^i are needed to guarantee that Φ^2 maps this slightly smaller domain into itself. To avoid this complication, I have

elected to prove existence for the additive Kantian version, which does not suffer from this problem.

5. A design for market socialism

This model is somewhat more complicated, for it is the only one in which I assume there are two produced private commodities and labor. I have chosen to introduce this complication to show that the usefulness of Kantian optimization is not restricted to economies with a single private commodity. It will be clear that the arguments will hold for economies with any number of private, produced commodities.

A. Introduction

Market-socialist models to date have not modeled cooperation or solidarity among citizens although these features are at the heart of the socialist ideal. Despite the importance of cooperation to the socialist vision, existing models present no explicit conception of how people would *behave* differently (cooperatively) in a socialist society from how they behave (non-cooperatively) in a capitalist economy. In market-socialist models heretofore (e.g., Lange and Taylor [1938], Roemer [1994]), agents are presumed to optimize in the same way that Arrow-Debreu agents optimize, maximizing a self-regarding preference order subject to constraints. One might suppose that socialist citizens would possess preferences with an altruistic element in them. However, I have not seen any market-socialist models with this property – and in any case, if an agent is small in the economy, it is unclear whether his having a preference order with an altruistic character would produce equilibria any different from one in which agents are entirely self-regarding. (See Dufwenberg et al [2011].) After all, if an agent is small, what difference would his altruistic contribution make, and would this small contribution outweigh the personal cost he sustains by making it? The preferences of agents are standard and self-regarding in my proposal.

Income taxation is the redistributive mechanism here. The key observation is that Kantian (as opposed to Nash) optimization in the labor-supply decision nullifies the usual deadweight loss incurred with income taxation. Any degree of post-fisc income equality can be achieved without sacrificing Pareto efficiency. The economic mechanism is

decentralized, efficient, and as equal as citizens choose it to be, through a presumably democratic choice of the tax rate.

B. The economic environment

There are two produced private goods and a homogeneous kind of labor, measured in efficiency units. There are two firms, each of which produces one of the goods from inputs of labor and capital, using production functions *G* and *H* respectively, which map $\Re_+^2 \to \Re_+$. Worker *i* is endowed with $\bar{E}^i$ units of labor in efficiency units, and receives a profit share θ^{il} from Firm *l*, for $l = 1,2$. The state owns fractions θ^{0l} of firm $l = 1,2$, and is endowed with K_0 units of the capital good. Good 1 is used both for consumption and capital, and Good 2 is a pure consumption good. The state uses its capital to finance investment in the two firms, and the private agents spend their incomes on consumption of the two goods. Private agent *i* has preferences over the two consumption goods and labor expended (in efficiency units) represented by a utility function $u^i : \Re_+^2 \times [0, \bar{E}^i] \to \Re_+$. All activity takes place in a single period.

Firms are traditional – they are price-takers and demand capital and labor and supply commodities to maximize profits. A linear tax at an exogenous rate $t \in [0,1]$ will be levied on all private incomes, with the tax revenues returned to the population as an equal demogrant. Given their incomes (which consist of after-tax wages, profit income and the demogrant) and their labor supply, producer-consumers choose the optimal commodity bundle in the classical way. However, the determination of labor supply, and hence of income, is non-traditional – that is to say, the worker does not choose her labor supply in the Nash manner. A vector of labor supplies must be an *additive Kantian equilibrium* of a game to be defined below.

C. The game

Let (p_1, p_2, w, r) be a price vector where p_l is the price of commodity *l*, for $l = 1, 2$, *w* is the wage rate for labor in efficiency units, and *r* is the interest rate on capital. Let (E^{i1}, E^{i2}) be a labor supply vector by agent *i* to Firms 1 and 2. Thus the vector of labors

supplied to Firm G is $\mathbf{E}^1 = (E^{11},...,E^{n1})$ and the vector of labors supplied to Firm H is $\mathbf{E}^2 = (E^{12},...,E^{n2})$. Fix the capital levels K^l, $l = 1,2,$ of the two firms. Define the *income* of private agent i at $(\mathbf{E}^1, \mathbf{E}^2)$ under a linear income tax at rate t as:

$$I^i(E^i, E^{S1}, E^{S2}) = (1-t)wE^i + (1-t)(\theta^{i1}\Pi^1(K^1, E^{S1}) + \theta^{i2}\Pi^2(K^2, E^{S2})) + \frac{t}{n}\left(p_1 G(K^1, E^{S1}) + p_2 H(K^2, E^{S2}) - \theta^{01}\Pi^1(K^1, E^{S1}) - \theta^{02}\Pi^2(K^2, E^{S2}) - rK^1 - rK^2\right) \quad (5.1)$$

where the profits of the two firms are defined by:

$$\Pi^1(K^1, E^{S1}) \equiv p_1 G(K^1, E^{S1}) - wE^{S1} - rK^1, \quad \Pi^2(K^2, E^{S2}) \equiv p_2 H(K^2, E^{S2}) - wE^{S2} - rK^2 \quad (5.2)$$

and E^{Sl} is the labor supplied to Firm l. The last term on the r.h.s. of (5.1) is the value of the demogrant, equal to the per capita share of total tax revenues (where taxes are levied on all private incomes but not on the state's income).

The income of the state is:

$$I^0 = \theta^{01}\Pi^1(K^1, E^{S1}) + \theta^{02}\Pi^1(K^2, E^{S2}) + r(K^1 + K^2). \quad (5.3)$$

That is, the state receives its share of firms' profits plus the return on its investment, but this is not taxed, which explains the specification of the demogrant in equation (5.1).

Now suppose that every (private) agent were to increase her total labor by a constant ρ, positive or negative. Then i's hypothetical income would be:

$$\begin{aligned} &I^i(E^i + \rho, E^{S1} + \lambda n\rho, E^{S2} + (1-\lambda)n\rho) = (1-t)w(E^i + \rho) + \\ &(1-t)(\theta^{i1}\Pi^1(K^1, E^{S1} + \lambda n\rho) + \theta^{i2}\Pi^2(K^2, E^{S2} + (1-\lambda)n\rho)) + \\ &\frac{t}{n}\left(\begin{array}{l} p_1 G(K^1, E^{S1} + \lambda n\rho) + p_2 H(K^2, E^{S2} + (1-\lambda)n\rho) - \\ \theta^{01}\Pi^1(K^1, E^{S1} + \lambda n\rho) - \theta^{02}\Pi^2(K^2, E^{S2} + (1-\lambda)n\rho) - r(K^1 + K^2) \end{array}\right) \end{aligned} \quad (5.4)$$

where fraction λ of the total increase in labor $n\rho$ is allocated to Firm 1, and fraction $(1-\lambda)$ to Firm 2. We need not adopt a rule for how each agent would allocate her additional labor ρ between the two firms, as this will turn out not to matter. It is assumed that workers are price takers: in particular, they take the wage w as given.

A comment on the logic behind equation (5.4) is in order. A *Nash player*, who chooses his labor supply while assuming all other labor supplies remain fixed, need not consider the effect of his labor-supply decision on either the profits of firms in which he

works or owns equity, or upon the demogrant, if the economy is large. Hence, our practice in Nash-type analysis is to ignore these effects. But in Kantian optimization, the counterfactual the worker envisages is that *all* workers change their labor supplies in the same amount as the change he is contemplating, and hence consistency in the thought experiment requires that we alter the labor supplies to firms, and the value of the demogrant, accordingly. Hence, the formulation of equation (5.4)[7].

At this counterfactual labor supply by worker *i*, $E^{i1} + E^{i2} + \rho$, given her income as specified by (5.4) , let the agent compute her commodity demands, which are the solution of the program:

$$\begin{aligned} &\max_{x,y} u^i(x, y, E^i + \rho) \\ &\text{subj. to} \\ &p_1 x + p_2 y = I^i(E^i + \rho, E^{S1} + \lambda n\rho, E^{S2} + (1-\lambda)n\rho) \end{aligned} \tag{5.5}$$

Denote the solution to this program by $(x^i(I^i[\rho], E^i + \rho), y^i(I^i[\rho], E^i + \rho))$, where I abbreviate with the notation $I^i[\rho] \equiv I^i(E^i + \rho, E^{S1} + \lambda n\rho, E^{S2} + (1-\lambda)n\rho)$.

We now define the payoff functions of a game. The payoff to agent *i* is his utility at prices (p_1, p_2, w, r) if the capital invested in the firms is (K^1, K^2) , and the vector of labor supplies $(E^1, ..., E^n)$ were to determine wage income, profit income, and the value of the demogrant, that is:

$$V_+^i(E^1, ..., E^n) = u^i(x^i(I^i[0], E^i), y^i(I^i[0], E^i), E^i) \ . \tag{5.6}$$

Incorporated in the payoff function is the assumption that at her personal part of the community effort vector, agent *i* has chosen her *commodity* demands optimally, given the income generated.

Thus, given a vector of prices $\mathbf{p} = (p_1, p_2, w, r)$, and the ownership shares of firms, a game whose strategies are effort/labor supplies is defined, denoted $\mathbf{V}_+$. We can define

[7] Saying that workers are price-takers means they do not contemplate the change in the wage that would be forthcoming were the aggregate labor supply to change. Strictly speaking, their taking the wage as fixed must be regarded as an illusion. This contrasts with price-taking under Nash optimization, which is rational if the economy is large.

its additive Kantian equilibrium, which is a vector of labor supplies $\mathbf{E} = (E^1,...,E^n)$ satisfying (2.3): that is to say, a vector $\mathbf{E}$ such that:

$$(\forall i)(\arg\max_{\rho} u^i(x^i(I^i[\rho], E^i + \rho), y^i(I^i[\rho], E^i + \rho), E^i + \rho) = 0). \quad (5.7)$$

D. Walras-Kant equilibrium market-socialist equilibrium with taxation

The data of the economy are $(u^1,...,u^n; G, H; \bar{E}^1,..., \bar{E}^n; \{\theta^{il}, i = 0,...,n; l = 1,2\}; K_0)$. It is useful, for conceptualizing Pareto efficiency, to define the utility function of the state, which is:

$$u^0(x,y) = x. \quad (5.8)$$

That is, the state cares only about Good 1, which it uses for investment.

We now define a *Walras-Kant (additive) equilibrium at tax rate t,* to consist of:

i. a price vector (p_1, p_2, w, r),

ii. labor and capital demands by the two firms of D^1, D^2 and K^1, K^2, respectively,

iii. labor supplies (E^{i1}, E^{i2}) by all workers *i* to Firms 1 and 2,

iv. for all private agents *i*, commodity demands (x^i, y^i) for the outputs of Firms 1 and 2, resp., and a demand for the first good by the state of x^0,

such that:

v. at given prices, (K^l, D^l) maximizes profits of Firm *l*, for *l* = 1, 2,

vi. the labor supply vector $\mathbf{E} = (E^1,...,E^n)$, where $E^i = E^{i1} + E^{i2}$, constitutes an additive Kantian equilibrium at the given prices of the game $\mathbf{V}_+$, as defined in (5.6),

vii. (x^i, y^i) maximizes the utility of agent *i*, given prices, her labor supply, and her income, given by (5.1),

viii. x^0 maximizes the state's utility u^0 subject to its budget constraint $px^0 \leq \theta^{01}\Pi^1 + \theta^{02}\Pi^2 + rK_0$, and

ix. all markets clear; that is, $D^l = \sum_i E^{il}$ for $l = 1,2$, $x^S = G(K^1, D^1)$

$y^S = H(K^2, D^2)$, and $K_0 = K^1 + K^2$.

The depreciation rate of capital is set at zero. Thus, at the beginning of the next period the state's endowment of the capital good will be $K_0 + \frac{I^0}{p_1}$ (see eqn. (5.3)).

E. The first theorem of welfare economics for market socialism

The appropriate concept of Pareto efficiency will be called *investment constrained Pareto efficiency* (ICPE). An allocation is ICPE if there is no other feasible allocation that makes at least one agent better off without harming any agent, where the state is included as an agent. Since the model is not intertemporal, it is important to qualify the kind of Pareto efficiency that can be realized: citizens cannot trade off present against future consumption in the model, and hence we cannot speak of efficiency in the full sense. To say this more straightforwardly: the state's investment is determined by its endowment of capital, not by any considerations of the population's future welfare. We know that both the Soviet Union and post-1949-China probably invested too much, committing their populations to excessively low consumption. Such can happen in this model, too.

It is easy to show that, with differentiability, an interior allocation[8] is ICPE exactly when:

[8] An allocation is called *interior* if all private agents consume positive amounts of both commodities and leisure, and all supply positive amounts of labor (but it is not necessary that any agent supplies labor to both firms).

$$
\begin{aligned}
&(a)\sum_{i=0}^{n} x^i = G(K^1, E^{1S}) \quad (b)\sum_{i=1}^{n} y^i = H(K^2, E^{2S}) \\
&(c)\sum_{i=1}^{n} E^i = E^{1S} + E^{2S} \quad (d) K^1 + K^2 = K_0 \\
&(e)\,(\forall i)\; G_1(K^1, E^{1S}) = -\frac{u_3^i(x^i, y^i, E^i)}{u_1^i(x^i, y^i, E^i)} \quad (f)\,(\forall i)\; H_1(K^2, E^{2S}) = -\frac{u_3^i(x^i, y^i, E^i)}{u_2^i(x^i, y^i, E^i)} \\
&(g)\frac{G_2}{G_1} = \frac{H_2}{H_1}
\end{aligned}
\tag{5.9}
$$

where $G_1 \equiv \dfrac{\partial G}{\partial K}$, etc.

Conditions (a)-(d) specify feasibility; conditions (e)-(g) specify efficiency.

Proposition 7 *Assume differentiability of the production functions and the utility functions. Assume that the production functions are concave and the utility functions are strictly concave. Let* $(p_1, p_2, w, r, \mathbf{E}^1, \mathbf{E}^2, D^1, D^2, K^1, K^2, \mathbf{x}, \mathbf{y})$ *comprise a Walras-Kant (additive) equilibrium at any income tax rate* $t \in [0,1]$. *Then the induced allocation is investment-constrained Pareto efficient.*

Proof of Proposition 7:

0. Although the theorem's statement assumes the equilibrium is interior, this is easy to relax, with a concomitant alteration of the first-order conditions.

1. At a Walras-Kant equilibrium at tax rate *t*, profit-maximization gives:

$$
p_1 G_2(K^1, E^{S1}) = w = p_2 H_2(K^2, E^{S2}) \text{ and } p_1 G_1 = r = p_2 H_1\,, \tag{5.10}
$$

and clearing of the capital market tells us that $K^1 + K^2 = K_0$. Therefore, it follows from (5.9) that an interior equilibrium is ICPE if and only if:

$$
(\forall 1 \le i \le n)\left(-\frac{u_3^i(x^i, y^i, E^i)}{u_1^i(x^i, y^i, E^i)} = \frac{w}{p_1} \;\&\; -\frac{u_3^i(x^i, y^i, E^i)}{u_2^i(x^i, y^i, E^i)} = \frac{w}{p_2}\right) \tag{5.11}
$$

.

2. Consider the program:

$$
\begin{aligned}
&\max_{x,y} u^i(x, y, E) \\
&\text{subj. to} \\
&p_1 x + p_2 y = I
\end{aligned}
$$

where E and I are fixed. Denote the solution $(x^i(I;E),y^i(I;E))$. The f.o.c.s for the solution of the program are:

$$\begin{aligned} &p_2u_1^i(x^i(I;E),y^i(I;E),E)-p_1u_2^i(x^i(I;E),y^i(I;E),E)=0\\ &p_1x^i(I;E)+p_2y^i(I;E)-I=0 \end{aligned} \tag{5.12}$$

By the implicit function theorem, the functions (x^i,y^i) are differentiable and their derivatives are given by:

$$x_1^i(I,E)=\frac{p_1u_{22}^i-p_2u_{12}^i}{(p_2,-p_1)\mathbf{U}^i(p_2,-p_1)^T}, \tag{5.13}$$

$$y_1^i(I,E)=\frac{p_2u_{11}^i-p_1u_{12}^i}{(p_2,-p_1)\mathbf{U}^i(p_2,-p_1)^T}, \tag{5.14}$$

$$x_2^i(I,E)=\frac{p_2(p_1u_{23}^i-p_2u_{13}^i)}{(p_2,-p_1)\mathbf{U}^i(p_2,-p_1)^T}, \text{ and} \tag{5.15}$$

$$y_2^i(I,E)=\frac{p_1(p_2u_{13}^i-p_1u_{23}^i)}{(p_2,-p_1)\mathbf{U}^i(p_2,-p_1)^T}, \tag{5.16}$$

where $\mathbf{U}^i$ is the leading principal sub-matrix of order two of the Hessian of the function u^i, and the superscript T indicates 'transpose.' Note that the implicit function theorem indeed applies because $\mathbf{U}^i$ is negative definite by the strict concavity of u^i, and so the denominators of equations (5.13)-(5.16) do not vanish.

4. Now the labor-supply vector is an interior additive Kantian equilibrium of the game $\mathbf{V}_+$ if and only if :

   for all $i \geq 1$:

$$\left.\frac{d}{d\rho}\right|_{\rho=0} u^i(x^i(I^i[\rho],E^i+\rho),y^i(I^i[\rho],E^i+\rho),E^i+\rho)=0\,. \tag{5.17}$$

This statement reduces to:

$$u_1^i\cdot\left(x_1^iI^{i\prime}[0]+x_2^i\right)+u_2^i\cdot\left(y_1^iI^{i\prime}[0]+y_2^i\right)+u_3^i=0\,, \tag{5.18}$$

where $I^{i\prime}[0] \equiv \left.\dfrac{dI^i(E^i+\rho, E^{S1}+\lambda n\rho, E^{S2}+(1-\lambda)n\rho)}{d\rho}\right|_{\rho=0}$.

5. From (5.4), calculate that:

$$I^{i\prime}[0] = (1-t)w + (1-t)\left(\theta^{i1}\Pi_2^1(K^1, E^{S1})\lambda n + \theta^{i2}\Pi_2^2(K^2, E^{S2})(1-\lambda)n\right) + \frac{t}{n}\left(\begin{array}{l} p_1 G_2(K^1, E^{S1})\lambda n + p_2 H_2(K^2, E^{S2})(1-\lambda)n - \\ \left(\theta^{01}\Pi_2^1(K^1, E^{S1})\lambda n - \theta^{02}\Pi_2^2(K^2, E^{S2})(1-\lambda)n\right) \end{array}\right) \quad (5.19)$$

Since the four partial derivatives $(\Pi_1^1, \Pi_2^1, \Pi_1^2, \Pi_2^2)$ of the firms' profit functions are zero, by profit maximization, and $p_1 G_2 = p_2 H_2 = w$, (5.19) reduces to:

$$I^{i\prime}[0] = (1-t)w + tw = w, \quad (5.20)$$

for any *t*. It is now evident why we did not have to specify how workers allocate the increment ρ in labor between the two firms: that allocation does not affect the validity of (5.20).

We therefore write the condition for Kantian equilibrium of labor supplies, equation (5.18), as:

$$u_1^i \cdot \left(x_1^i w + x_2^i\right) + u_2^i \cdot \left(y_1^i w + y_2^i\right) + u_3^i = 0 . \quad (5.21)$$

6. We now expand equation (5.21) by making a sequence of substitutions: (i) substitute the expressions for the four derivatives of the x^i and y^i functions from (5.13) through (5.16), and (ii) eliminate p_1 via the substitution $p_1 = \dfrac{p_2 u_1^i}{u_2^i}$, the f.o.c. from (5.12). So doing reduces (5.21) to:

$$\begin{array}{l} \dfrac{u_1^i}{d}\left(w\dfrac{p_2 u_1^i}{u_2^i}u_{22}^i - wp_2 u_{12}^i + p_2\left(\dfrac{p_2 u_1^i}{u_2^i}u_{23}^i - p_2 u_{13}^i\right)\right) + \\ \dfrac{u_2^i}{d}\left(wp_2 u_{11}^i - wp_2\dfrac{u_1^i}{u_2^i}u_{12}^i + \dfrac{p_2 u_1^i}{u_2^i}\left(p_2 u_{13}^i - \dfrac{p_2 u_1^i}{u_2^i}u_{23}^i\right)\right) + \\ u_3^i = 0, \end{array} \quad (5.22)$$

where $d = (p_2, -p_1)\mathbf{U}^i(p_2, -p_1)^T = p_2^2\left(\frac{u_1^i}{u_2^i}, -1\right)\mathbf{U}^i\left(\frac{u_1^i}{u_2^i}, -1\right)^T$, which is a negative number.

Finally, divide both sides of equation (5.22) by the positive number u_2^i, simplify, and calculate that that equation reduces to:

$$\frac{w}{p_2} = -\frac{u_3^i}{u_2^i}, \tag{5.23}$$

which is one of the two required efficiency conditions for agent *i*.

7. Now substitute for p_2 in the last equation using $p_2 = \frac{p_1 u_2^i}{u_1^i}$, yielding:

$$\frac{w}{p_1} = -\frac{u_3^i}{u_1^i}. \tag{5.24}$$

By equations (5.23), (5.24) and (3.2), the proposition is proved. □□

The key move in the proof is to show that, regardless of the tax rate, when a worker thinks of all workers as varying their labor supplies in the amount she is contemplating varying her own, she internalizes the externality generated by her labor-supply choice – a choice that affects firm profits and tax revenues. Her own action causes a negligible change in these magnitudes, but of course the aggregate effect of many small changes is significant. The *additive counterfactual* in the universal change in labor supplies and *linear income taxation* combine in such a way as to exactly cancel the deadweight loss of taxation that afflicts Nash optimization in the labor-supply decision. (This is the meaning of equation (5.20), the key to the proof.) This kind of pairing – associating a specific cooperative optimization protocol with a particular allocation rule, where the two together deliver Pareto efficiency – is a feature of Kantian equilibrium in simpler (non-market) environments. What's new here is combining additive Kantian optimization with markets.

A remark on why the incentive problem, causing deadweight losses in the standard model, does not bite here. Consider, for dramatic effect, an income tax rate of one, and suppose every worker is supplying zero labor (as she would in the standard model at this

tax rate). But here, by using the Kantian optimization protocol, a worker balances her share of an increase in income that would occur if *all* workers increased their labor supply from zero to some small positive ρ against her (very small) disutility of labor at zero. The trade-off is usually worth it. Consequently, at the Kantian equilibrium, even at a tax rate of unity, (most) workers will supply a positive amount of labor.

E. An example of Walras-Kant (additive) equilibrium

Because capital allocation is passive in this model, let's simplify by studying an economic environment where the capital inputs are fixed, there is no state, and we model production as a function of labor only:

$$G(E) = E - \frac{a}{2}E^2,\ H(E) = E - \frac{b}{2}E^2$$
$$u^i(x,y,E) = x^{1/3}y^{1/3}(\bar{E}^i - E)^{1/3},\ \text{for } i = 1,...,n \tag{5.25}$$

There are *n* agents, and the total endowment of labor is $\bar{E}^S = \sum_{i=1}^{n} \bar{E}^i$. We let $\theta^{i1} = \theta^{i2} = \frac{1}{n}$ for all $1 \le i \le n$. We set $\theta^{01} = \theta^{02} = 0$. We normalize the price vector by choosing $w = 1$. There is no market for capital and hence no interest rate.

An interior allocation is a Walras-Kant (additive) equilibrium at income tax rate *t* when the allocation is Pareto efficient, the income of *i* is given by (5.1), and markets clear. (The critical condition that the labor supplies comprise a Kantian equilibrium of the game $\mathbf{V}_+$ is embedded in the efficiency conditions, as the proof of Proposition 1 shows.) We write these conditions as:

$$1 - aE^{1S} = \frac{1}{p_1},\ 1 - bE^{2S} = \frac{1}{p_2}\ \ (\text{MRT}^i = w/p_i)$$
$$\frac{x^i}{\bar{E}^i - E_i} = \frac{1}{p_1},\quad \frac{y^i}{\bar{E}^i - E_i} = \frac{1}{p_2}\ \ (\text{MRT}^i = MRS) \tag{5.26}$$

and (5.1) holds for all *i*. By (5.26), the post-fisc income of agent *i* is given by $I^i = p_1x^i + p_2y^i = 2(\bar{E}^i - E_i)$. Hence, (5.1) can be written:

$$2(\bar{E}^i - E^i) = (1-t)E^i + \frac{1-t}{n}(\Pi^1 + \Pi^2) + \frac{t}{n}(G(E^{S1}) + H(E^{S2})) = (1-t)E^i + \frac{p_1 G(E^{S1}) + p_2 H(E^{S2})}{n} - \frac{(1-t)}{n}(E^{S1} + E^{S2}) \quad . \tag{5.27}$$

By adding up the equations over all *i* in (5.26), we have:

$$\begin{aligned} p_1 x^S = p_1 G(E^{S1}) = \bar{E}^S - (E^{S1} + E^{S2}), \\ p_2 y^S = p_2 H(E^{S2}) = \bar{E}^S - (E^{S1} + E^{S2}) \end{aligned} \tag{5.28}.$$

Now using the expressions for commodity prices in (5.26), we write these equations as:

$$\frac{G(E^{S1})}{1-aE^{S1}} = \bar{E}^S - (E^{S1} + E^{S2}), \quad \frac{H(E^{S2})}{1-bE^{S2}} = \bar{E}^S - (E^{S1} + E^{S2}). \tag{5.29}$$

System (5.29) comprises two equations in the two unknowns E^{S1} and E^{S2}; the solution must be a vector $(E^{S1}, E^{S2}) \in (0, \frac{1}{a}) \times (0, \frac{1}{b})$. Thus total production at Walras-Kant equilibrium for this economy, if such exists, is independent of the tax rate *t*. Profits are also independent of *t*. Taxation simply redistributes a fixed output of commodities.

Parameterize the example with $(a,b) = (0.1, 0.2)$, $\bar{E}^S = 10, n = 100.$ We have not yet specified the individual endowments $\bar{E}^i$. We solve (5.29):

$$E^{S1} = 3.28, \quad E^{S2} = 2.63 \ . \tag{5.30}$$

Profits are positive for both firms, and comprise 28% of national income.

To complete the analysis, we must specify the $\{\bar{E}^i\}$ and solve for $\{E^i\}$. Rewrite equation (5.27) as:

$$2\bar{E}^i + E^i(t-3) = \frac{\Pi^1 + \Pi^2}{n} + \frac{t}{n}(E^{S1} + E^{S2}) \ . \tag{5.31}$$

Examination shows that equation (5.31) possesses an interior solution in which $E^i \in (0, \bar{E}^i]$ for all *i* exactly when:

$$\text{for all } i, \quad \bar{E}^i > \frac{1}{2n}(\Pi^1 + \Pi^2 + t(E^{S1} + E^{S2})) \ . \tag{5.32}$$

If, on the other hand, (5.32) is false for some *i*, then there is no interior equilibrium.

It is of interest to compute the lower bound on the labor endowment, call it ω, that will guarantee an interior Walras-Kant equilibrium at tax rate *t*. From (5.32) , this depends upon the tax rate. We compute this lower bound for various tax rates for our example:

t	min ω
0.	0
0.1	0
0.2	0
0.3	0
0.4	0
0.5	0.00344757
0.6	0.0109318
0.7	0.0184161
0.8	0.0259003
0.9	0.0333846
1.	0.0408689

Table. ω is the minimum value of $\bar{E}^i$ supporting an interior Walras-Kant equilibrium as a function of the tax rate

Recall that the average labor endowment with our parameterization is $\frac{\bar{E}^s}{n} = 0.1$. From the table, a Walras-Kant (additive) equilibrium exists where all agents work regardless of the distribution of individual labor endowments, as long as $t \leq 0.4$. But as the tax rate rises, the restriction on the distribution of labor endowments bites.

For tax rates larger than 40%, equilibrium still exists, but workers who are insufficiently skilled do not work. We illlustrate with a second paramaterization. The utility functions and production parameters are as before, but we examine an economy with two agents (*n* =2), where $\bar{E}^1 = 9, \bar{E}^2 = 1$. If both agents work, then E^{s1} and E^{s2} are given by (5.30). Let us look for an equilibrium where $t = 1$. Both agents must then have the same after-tax income. Inequality (5.32) is false for agent 1, so there is no equilibrium at $t = 1$ where both agents work. We therefore set agent 2's labor supply to zero: $E^2 = 0$. The other equations characterizing a Walras-Kant equilibrium are:

$$\begin{aligned}
&\frac{x^1}{\bar{E}^1-(E_1^1+E_1^2)}=\frac{1}{p_1},\ \frac{y^1}{\bar{E}^1-(E_1^1+E_1^2)}=\frac{1}{p_2},\\
&G'(E^{11})=\frac{1}{p_1},\quad H'(E^{12})=\frac{1}{p_2}\\
&x^1=x^2,\quad y^1=y^2,\\
&G(E^{11})=x^1+x^2,\ H(E^{12})=y^1+y^2,\\
&\frac{x^2}{\bar{E}^2}\geq\frac{1}{p_1},\ \frac{y^2}{\bar{E}^2}\geq\frac{1}{p_2},
\end{aligned}\tag{5.33}$$

The two equations in the first line say the marginal rates of substitution for the agent with positive labor supply equal the correct price ratios; the second line says the marginal rates of transformation equal the correct price ratios; the third line is true because when the tax rate is 1, both agents have the same (post-fisc) income, and so consume the two commodities identically; the fourth line expresses market-clearing for the two commodities; and the fifth line expresses the efficiency condition for the agent who supplies zero labor. The solution is given by:

$$\begin{aligned}
&x^1=x^2=1.513,\quad y^1=y^2=1.024,\\
&p^1=1.592,\quad p^2=2.352,\ E_1^1=3.717,\ E_1^2=2.874
\end{aligned}\tag{5.34}$$

F. Existence of Walras-Kant market socialist equilibrium

We first note:

Proposition 8 *Let* $(p_1,p_2,w,r,\mathbf{E}^1,\mathbf{E}^2,D^1,D^2,K^1,K^2,\mathbf{x},\mathbf{y})$ *be a Walrasian equilibrium at* $t=0$. (The state is simply another agent who desires to consume only the first good, and possesses no labor endowment.) *Then it is also an additive Walras-Kant market socialist equilibrium at* $t=0$.

Proof:

We know the allocation is ICPE by the usual first welfare theorem for private-ownership economies. The income equation (5.1) holds by definition of Walrasian equilibrium. We need only show that the labor supplies comprise a Kantian equilibrium, which is to say, that equation (5.21) holds. But we have shown that this is equivalent to the

efficiency conditions that $MRS^i = MRT$. These conditions hold by hypothesis, since the allocation is Walrasian and therefore Pareto efficient, and the claim is proved. □

We assume:

Assumption A

(i) G, H are unbounded, concave, homothetic, and the Inada conditions hold, and

(ii) all consumer preferences are representable by strictly concave, differentiable utility functions, and both commodities are normal goods for all consumers.

Proposition 9 *Let an economic environment* $\{\mathbf{u}, \theta, G, H, \bar{E}, K_0\}$ *be given and let Assumption A hold. Suppose that* $\bar{E}^i > 0$ *for all (private) agents and that* $\theta^{01} + \theta^{02} < 2$. *Then a Walras-Kant equilibrium exists for any* $0 \leq t < 1$.

Proof: See Appendix, Parts A and D.

A comment on investment in the model is called for. In the approach I've taken, only the state invests. Could private agents invest in the firms as well, and preserve the efficiency result? The answer is yes, if the profile of investments is also an additive Kantian equilibrium. I elected not to follow this route here, both for reasons of simplicity, and because it strikes me as more credible that workers can learn to adopt Kantian optimization in their labor-supply decisions than in their investment decisions. Perhaps I am here influenced by the observation that workers have a history of cooperation, and investors do not, at least to the same extent.

6. An economy with worker-owned firms

Traditionally, worker-owned firms are modeled as having the objective of maximizing value-added per worker. Here, I propose instead that firms maximize profits, pay wages to workers, and then profits are divided among workers in proportion to their labor. There are, however, workers from different occupations, whose labor is incommensurate, and so one must determine, somehow, what fraction of profits will be distributed to the set of workers in each occupation. We will show these fractions can be specified exogenously.

There is an economy with one good. There are two kinds of labor – two occupations. The good is produced by a concave production function $G(E,D)$ where *E* and *D* are the levels of the two occupational labor supplies. We simplify here by ignoring the capital input.

There are *n* citizen-workers, partitioned into two elements:

$$I_1 = \{i \mid \bar{E}^i > 0 \text{ and } \bar{D}^i = 0\}$$
$$I_2 = \{i \mid \bar{D}^i > 0 \text{ and } \bar{E}^i = 0\}$$

where $\bar{E}^i (\text{or } \bar{D}^i)$ is the endowment of labor the agent has in the *E* (or *D*) occupation.

Individuals have utility functions of the form $u^{1i}(x,E)$ or $u^{2i}(x,D)$ depending upon the kind of labor they possess.

The economy uses markets, with three prices, (p,w,d), *p* being the price of the good, *w* the wage of *E* labor and *d* the wage of *D* labor. There is one firm, utilizing the production function *G*. The firm maximizes profits. The profits accrue to workers in proportion to their labor supplies, as follows. A fraction λ of profits will be divided among the *E* workers in proportion to their labor contributions, while fraction $1-\lambda$ of the profits are divided among the *D* workers in proportion to their labor contributions. λ is an exogenous parameter of the model. Thus, for instance, the income of a worker of type 1 (that is, $i \in I_1$) will be:

$$wE^i + \frac{E^i}{E^S}\lambda\Pi, \tag{6.1}$$

where Π is the firm's profits, and $E^S \equiv \sum_{i \in I_1} E^i$. The analogous express holds for workers of type 2.

Given prices, consider a game V^1 whose players are the *E* workers. We are *given* a total labor supply $\hat{D}^S$ by the type-2 workers. The payoff functions for the *E*-workers are:

$$V^{1i}(E^1,...,E^n;\hat{D}^S) = u^{1i}\left(\frac{wE^i + \frac{E^i}{E^S}\lambda(G(E^S,\hat{D}^S) - wE^S - d\hat{D}^S)}{p}, E^i\right); \tag{6.2}$$

Analogously, given a total labor supply by the *E* workers of $\hat{E}^S$, consider a game among the *D* workers whose payoff functions are:

$$V^{2i}(D^1,...,D^n;\hat{E}^S)=u^{2i}\left(\frac{dD^i+\frac{D^i}{D^S}(1-\lambda)(pG(\hat{E}^S,D^S)-w\hat{E}^S-dD^S)}{p},D^i\right). \quad (6.3)$$

Definition 6 A *Walras-Kant worker-ownership equilibrium with profit-share parameter* $\lambda\in[0,1]$ is

- a price vector (p,w,d)
- consumption bundles (x^i,E^i) for all $i\in I_1$ and (x^i,D^i) for all $i\in I_2$

such that:

- the vector (x^S,E^S,D^S) solves the firm's profit maximization problem:
  - $\max_{x,E,D} px-wE-dD$
    $s1.\quad x=G(E,D)$
- given D^S, $(E^1,...,E^{n_1})$ is a multiplicative Kantian equilibrium of the game $V^1(\cdot;D^S)$ defined in (6.2) for the type 1 workers,
- given E^S, $(D^1,...,D^{n_2})$ is a multiplicative Kantian equilibrium of the game $V^2(\cdot;E^S)$ defined in (6.3) for the type 2 workers.

Notice that all markets clear by the definition of equilibrium.

Conceptually, the main difference between this conception of an economy with worker-owned firms and Drèze's (1965) model of worker-owned firms is that here, workers receive a wage and then a share of profits whereas in Drèze's model, workers do not receive wages, but divide up value-added net of the cost of capital. In the present economic environment, since there is no payment to capital, this means that total firm revenues would be divided up among workers. Drèze also gives weights to the shares that workers of different occupations receive, but they emerge *endogenously*, whereas in my model, the weights $(\lambda,1-\lambda)$ are exogenous – a policy variable.

Proposition 10 *Any Walras-Kant worker-ownership equilibrium such that the two occupational labor vectors are strictly positive is Pareto efficient.*

Proof:

1. By profit-maximization, we have:

$$\frac{w}{p} = G_1(E^S, D^S), \quad \frac{d}{p} = G_2(E^S, D^S) . \tag{6.4}$$

2. The condition that the vector $\mathbf{E} = (E^1, ..., E^{n_1})$ be a multiplicative Kantian equilibrium of the game V^1 is:

$$(\forall i \in I_1) \quad u_1^{1i} \cdot \frac{1}{p}(wE^i + \frac{E^i}{E^S}\lambda \Pi_1 E^S) + u_2^{1i} E^i = 0 , \tag{6.5}$$

where Π_1 is the derivative of the profit function w.r.t. the labor supply of type 1. By profit maximization, at the equilibrium allocation, $\Pi_1 = 0$, and so (6.5) reduces to:

$$u_1^{1i} \frac{w}{p} E^i + u_2^{1i} E^i = 0 ; \tag{6.6}$$

invoking the fact that $E^i > 0$, we have:

$$\frac{w}{p} = -\frac{u_2^{1i}}{u_1^{1i}} \text{ for all } i \in I_1 . \tag{6.7}$$

3. In like manner, we have

$$\frac{d}{p} = -\frac{u_2^{2i}}{u_1^{2i}} \text{ for all } i \in I_2 . \tag{6.8}$$

4. By (6.4),(6.7) and (6.8), the allocation is Pareto efficient. □□

Proposition 11 *Under standard conditions*[9]*, there exists a WKWO equilibrium for any* $\lambda \in [0,1]$.

Proof: See Appendix, Parts A and E.

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[9] The main assumption that deserves mention is that leisure and consumption are normal goods for all preference orders.

Remark. Society is free to choose the share λ. More generally, suppose there are *m* occupations, and $G:\Re_+^m \to \Re_+$. Then *there will exist equilibria for any profit-share vector* $\Lambda = (\lambda^1,...,\lambda^m)$ *in the* $(m-1)$ *unit simplex.* For instance, one could choose Λ so as to divide profits equally among all occupations, by letting λ^j be proportional to the number of workers of occupational type *j*. (Within each occupation, the profits will be divided in proportion to effort.) Thus, in this economy, we can achieve an approximation to equality of distribution of capital income.

Of course, we have avoided the question of capital inputs, and so have not had to worry about paying interest to investors. I do not think there would be any problem adding capital to the model; however, workers would then have to pay interest to investors before dividing the remaining profits among themselves.

7. The psychology of Kantian optimization

The *differentia specifica* of the models here proposed is Kantian optimization in the labor-supply decision (or in the emissions decision). Having a formal definition of cooperation is, obviously, a pre-condition to embedding cooperation in equilibrium models.

It will likely be the case that skepticism regarding my proposal will focus upon the realism of supposing that a large population of producers can learn to optimize their labor-supply (or emissions) decisions in the Kantian manner. There are, I think, three necessary conditions for the psychological accessibility of such behavior: desire, understanding, and trust. Citizens/workers must *desire* to cooperate with each other, they must view themselves as part of a solidaristic society, whose members believe that cooperation in economic decisions is the *modus operandi*. But why should the Kantian optimization protocol appeal to people as the preferred mode of cooperation? I think the motivation must be in the *conception of fairness or solidarity* embodied in the statement, "I should only reduce (increase) my labor supply if I would like all others to reduce

(increase) their labor supplies in like manner[10]." Our brains love symmetry, and fairness always, I believe, involves a conception of symmetrical treatment. Secondly, people must *understand* that cooperation in the labor-supply decision internalizes the externalities that are improperly treated with Nash optimization. That's what Benjamin Franklin was appealing to when he uttered his famous phrase about hanging together, which later became a motivational slogan in the American labor movement. For instance, in the market-socialist model, each must understand that if all increase their labor supply by a small increment, each person's *income* increases by the wage times that increment, because what a worker loses in the tax on her wage, she gains back in the increased value of the demogrant. Thirdly, individuals must *trust* that others will behave cooperatively as well, and will not take advantage of their own cooperative behavior, by optimizing in the Nash manner. If these three conditions are met, then the *method* of implementing cooperative behavior is not difficult: for instance, in the market-socialist model, each worker should choose his labor supply to equalize his marginal rates of substitution between commodities and labor to his *gross* real wage, rather than his after-tax wage. Rather than thinking "Is the disutility of an extra day's work worth to me the *after-tax* wage increment?" the worker should ask whether it is worth the *gross* wage increment. If we believe people are capable of optimizing in the Nash manner, optimizing in the Kantian manner is no more cognitively demanding, if the necessary conditions are met.

To return to my earlier comment, these results suggest that the market (conjoined with price-taking behavior) is an even more powerful allocation mechanism than standard theory suggests. In many cases -- I do not have a complete characterization of them – inefficiencies of market equilibrium are due not to 'the market,' but to the behavioral protocol of Nash optimization. This is a mathematical claim, based on the efficiency theorems above, which is true regardless of the *realism* of Kantian optimization. As I said, I do not challenge the claim that in truly one-shot games, Nash optimization is

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[10] But why should workers conceive of symmetric treatment as a *translation* of the labor vector rather than a *rescaling* of it (multiplication by positive constant)? I have no good answer to this question, except to say that there are examples of it in history. 'Doing one's bit' in the Second World War in Britain arguably involved a translation of the labor vector, not a rescaling. Unfortunately, there is no simple income tax function that will combine with 'rescaling' as the Kantian protocol to produce Pareto efficient allocations with any degree of income equality.

rational. In games played by members of a society with a common culture, where members have learned to trust each other, due to a history of repeated interactions characterized by cooperative behavior, Kantian optimization may, however, become a moral norm. How that comes to be is another story.

I believe there are many examples in real life of simple Kantian equilibrium: many people recycle their garbage, even if penalties for failing to do so do not exist; voting can be viewed as a simple Kantian equilibrium; the British 'doing their bit' in the two world wars is a simple Kantian equilibrium or perhaps an additive Kantian equilibrium; the degree of tax compliance in most advanced democracies is far greater than can be rationalized by reasonable risk preferences and existing penalties, and is perhaps better understood as due to Kantian optimization (I pay taxes because that's the action I'd like all to take); participation in labor strikes and demonstrations may be more convincingly explained as Kantian behavior than Nash behavior (à la Olson [1965]). I have discussed these examples in more detail elsewhere (Roemer [in press]). I do not claim to have airtight proof that people are, indeed, optimizing according to a Kantian protocol in the above examples (and many others), but observation is consistent with this explanation. And let us not belittle the suggestive role of theory: once we have a precise model of a behavior, we may be stimulated to look for it in history and in the laboratory, and be surprised at how often it turns up[11].

Appendix: Proofs of existence theorems

A. An important correspondence[12]

We will use the correspondence Φ defined below in a number of proofs, and the fact, shown here, that at a fixed point of Φ, all markets clear.

Let Δ be a price simplex of dimension $n-1$ for an economy with *n* markets. Let $z : \Delta \to \Re^n$ be the excess demand function of the economy, which obeys Walras's

[11] I would be interested in knowing how many economists thought Nash equilibrium was a crazy idea when Nash first proposed it. As rumor has it, apparently John von Neumann did.

[12] See Mas-Colell, Whinston and Green (1995), Proposition 17.C.1.

Law: for all $\mathbf{p} \in \Delta$, $\mathbf{p} \cdot z(\mathbf{p}) = 0$. Define the correspondence $\Phi : \Lambda \to \Lambda$ as follows. On $\mathbf{int}\,\Lambda$, define:

$$\Phi(\mathbf{p}) = \{\mathbf{q} \in \Lambda \,|\, z(\mathbf{p}) \cdot \mathbf{q} \geq z(\mathbf{p}) \cdot \mathbf{q}', \text{ for all } \mathbf{q}' \in \Lambda\} .$$

On $\partial\Lambda$, define:

$$\Phi(\mathbf{p}) = \{\mathbf{q} \in \Lambda \,|\, \mathbf{p} \cdot \mathbf{q} = 0\} .$$

Suppose $\mathbf{p}^*$ is a fixed point of Φ. It must lie by definition in $\operatorname{int}\Delta$. Thus $z(\mathbf{p}^*) \cdot \mathbf{p}^* = 0$ and the definition of Φ on $\operatorname{int}\Delta$ tells us that $z(\mathbf{p}^*) \leq 0$. It follows that $z(\mathbf{p}^*) = 0$: for if $z(\mathbf{p}^*)$ had a negative component, Walras's Law would be contradicted.

Therefore all markets in the economy clear at $\mathbf{p}^*$.

B. Essentials of the proof of Proposition 3

1. Let $(p,r,c) \in \mathbf{int}\,\Delta$, the price 2-simplex. By profit maximization, assuming the Inada conditions hold for the production functions, we have the demands for capital and emissions in each country, (K^i, E^i) satisfy $pG_2(K^i,E^i) = c, \quad pG_1(K^i,E^i) = r.$

2. The *total supply* of emissions by the *n* countries $\hat{E}^S$ must satisfy:

for all *i* $$\frac{d}{d\hat{E}^S}\frac{r(\bar{K}^i - K^i) + pG(K^i,E^i) - cE^i + a^i c\hat{E}^S}{p} - h^i(\hat{E}^S) = 0. \qquad (8.1)$$

This says $$\frac{a^i c}{p} = h^{i\prime}(\hat{E}^S) \text{ for all } i, \qquad (8.2)$$

where the a^i sum to one.

Let's define functions $a^i(\cdot)$ by the equations:

$$a^i(X)\frac{c}{p} = h^{i\prime}(X) .$$

Obviously $a^i(X)$ are increasing functions, and since $h^{i\prime}(0)=0$, and the $h^{i\prime}$ increase without bound, there is a unique value z^* such that $\sum a^i(z^*)=1$. Let $\hat{E}^S \equiv z^*$. Thus, at these values of $\{a^i\}$ there is unanimity among countries regarding the global supply of emissions.

3. We have now defined the demands and supplies of capital, emissions, and the good at any interior price vector, and the vector *a*. Note that Walras' Law holds:

$$p(\sum x^i - \sum G^i) + r(\sum K^i - \sum \bar{K}^i) + c(\sum E^i - \hat{E}^S) = 0.$$

This uses the fact that $\sum a^i = 1$; the $\{x^i\}$ are defined by $x^i = \Pi^i + r\bar{K}^i$, where Π^i are the profits of the country *i*'s firm at the given prices.

Define the excess demand function $z(\mathbf{p}) = (\Delta x(\mathbf{p}), \Delta K(\mathbf{p}), \Delta E^S(\mathbf{p}))$ where $\Delta x(\mathbf{p}) = \sum x^i - \sum G^i$, etc.

4. Construct the correspondence Φ as in Part A above. At a fixed point of Φ, all markets clear. The shares $\{a^i\}$ are given by step 2 above.

5. It is left to verify that the conditions of Kakutani's FPT hold. Φ is convex-valued on $\mathbf{int}\,\Lambda$ and u.h.c. here by the Maximum Theorem. It is obviously convex valued on $\partial\Lambda$, and standard argument shows it is u.h.c. here. Thus a fixed point exists. □

C. Proof of Proposition 6

1. Denote by Λ the 1-simplex of prices (p,w). Define the compact, convex set:

$$\Omega = \Lambda \times \prod_{i=1}^{n} [0, \bar{E}^i].$$

We are given $(p, w, E_2^1, ..., E_2^n) \in \Omega$. Define the supply of the private good and the demand for labor for the *G* firm by profit-maximization:

$$(\hat{X},\hat{D}) = \underset{(X,D)}{\arg\max}(pX - wD) \text{ s.t. } X = G(D).$$

Denote the profits by $\Pi(\hat{X},\hat{D})$.

2. Next, define $\hat{y} = H(E_2^S)$, and define $(\hat{x}^i,\hat{E}_1^i)$ as the solution of

$$\begin{aligned} &\max_{(x,E)} u^i(x,\hat{y},E_2^i + E) \\ &s.t. \\ &px \le wE + \theta^i\Pi(p,w) \end{aligned} \qquad (8.3)$$

3. Define:

$$\text{for all } i \qquad r^i = \underset{r\in[-\hat{E}_1^i - E_2^i,\ \bar{E}^i - \hat{E}_1^i - E_2^i]}{\arg\max} V^i(r + E_2^1,....,r + E_2^n). \qquad (8.4)$$

Note, by the domain over which the maximization occurs, $r^i + \hat{E}_1^i + E_2^i \in [0,\bar{E}^i]$. By strict concavity of utility, the solution of (8.4) for a given *i* is unique.

Now define for all *i*, $\hat{E}_2^i = r^i + E_2^i$.

4. Note that Walras' Law holds by adding up the budget constraints:

$$p(\sum \hat{x}^i - X) + w(D - \sum \hat{E}_1^i) = 0.$$

Denote the excess demand function by:

$$z(p,w;\hat{y}) = (\Delta x, \Delta E_1) \text{ where}$$

$\Delta x = \sum \hat{x}^i - X, \quad \Delta E_1 = D - \sum \hat{E}_1^i$. Recall that $(\hat{x}^i,\hat{E}_1^i)$ depends on $\hat{y}$.

Now define $\Phi^1 : \Omega \to \Delta$ as in Part A above. That is, if $\mathbf{p} \equiv (p,w) \in \text{int}\,\Delta$ define:

$$\Phi^1(\mathbf{p},E_2^1,...,E_2^n) = \{\mathbf{q} \in \Delta \mid \mathbf{q}\cdot z(\mathbf{p};H(E_2^S)) \ge \mathbf{q}'\cdot z(\mathbf{p};H(E_2^S)), \text{ for all } \mathbf{q}' \in \Delta\}.$$

For $\mathbf{p} \in \partial\Delta$, define $\Phi^1(\mathbf{p},E_2^1,...,E_2^n) = \{\mathbf{q} \in \Delta \mid \mathbf{p}\cdot\mathbf{q} = 0\}$.

Define $\Phi^2(\mathbf{p},E_2^1,...,E_2^n) = (r^1 + E_2^1,...,r^n + E_2^n)$, which is single-valued.

Finally, define $\hat{\Phi} = \Phi^1 \times \Phi^2$, noting that $\hat{\Phi}$ maps Ω into itself.

5. Suppose $(p^*, w^*, E_2^{1*}, ..., E_2^{n*})$ is a fixed point of $\hat{\Phi}$. By Part A above, $\Delta x = 0 = \Delta E_1$. Now it also follows from the fixed point property that for all *i*, $r^i = 0$. Therefore $(E_2^{1*}, ..., E_2^{n*})$ is an additive Kantian equilibrium of the game *V* defined in (4.2). This shows that the fixed point is a Walras-Kant equilibrium with a public and private good.

6. It is left only to verify that $\hat{\Phi}$ is upper hemi-continuous and convex-valued. Convex-valuedness is immediate, as is u.h.c. on $\text{int}\,\Delta$. The u.h.c. of $\hat{\Phi}$ on $\partial\Delta$ is a standard argument which we skip. □

D. Proof of a lemma, and then Proposition 9

Let Δ be the 3-simplex of price vectors $\mathbf{p} = (p_1, p_2, w, r)$. We define a correspondence on the domain $\text{int}\,\Delta$. Let *Q* can be any real number, and $Q^{i*}(\cdot)$ be positive continuous functions on $\text{int}\,\Delta$. Let:

$$A^i(p_1, p_2, w; Q) = \arg\max_{x,y,E} \{u^i(x, y, E) \mid p_1 x + p_2 y = wE + Q\}$$

$$B^i(p_1, p_2, w; Q^{i*}) = \{(x, y, E) \in \Re^2_+ \times [0, \bar{E}^i] \mid p_1 x + p_2 y = (1-t)wE + Q^{i*}(\mathbf{p})\} \qquad (8.5)$$

Now define $\Gamma^i : \text{int}\,\Delta \to\!\!\to \Re^3_+$ by:

$$\Gamma^i(p_1, p_2, w; Q^*(\mathbf{p})) = \bigcup_{\infty > Q \geq \tilde{Q}} A^i(p_1, p_2, w; Q) \bigcap B^i(p_1, p_2, w; Q^{i*}(\mathbf{p})) \ , \qquad (8.6)$$

where $\tilde{Q} = Q^{i*}(\mathbf{p}) - tw\bar{E}^i$. $\tilde{Q}$ may be positive, zero, or negative. Finally, define:

$$\Gamma(p_1, p_2, w; Q^*(\mathbf{p})) = \Gamma^1(p_1, p_2, w; Q^{1*}(\mathbf{p})) \times ... \times \Gamma^n(p_1, p_2, w; Q^{n*}(\mathbf{p})). \qquad (8.7)$$

Lemma *Let* $t \in (0, 1]$ *and* $(p_1, p_2, w, r) \in \text{int}\,\Delta$. *Let* $Q^{i*} : \text{int}\,\Delta \to \Re_{++}$ *be continuous functions for all i. If Assumption A(ii) holds then* Γ *is a (non-empty) continuous function mapping* $\text{int}\,\Delta \to \prod_{i=1}^{n} (\Re^2_+ \times [0, \bar{E}^i])$.

Proof of lemma:

1. It suffices to show that Γ^i is single-valued and continuous for any *i*. By strict concavity of preferences, the correspondence A^i is single-valued and continuous on $\text{int}\,\Delta$.

Suppose that Γ^i contains two elements; i.e., there are allocations $(x_\nu, y_\nu, E_\nu) \in A^i(p_1, p_2, w; Q_\nu) \cap B^i(p_1, p_2, w; Q^*(\mathbf{p}))$, for $\nu = 1,2$, with $Q_2 > Q_1$. It follows that:

$$\begin{aligned} p_1(\delta x) + p_2(\delta y) &= w(\delta E) + (\delta Q) \\ p_1(\delta x) + p_2(\delta y) &= (1-t)w(\delta E) \end{aligned}, \tag{8.8}$$

where $\delta x \equiv x_2 - x_1$, etc. Therefore the quantities on the right-hand sides of the two equations in (8.8) are equal, implying that:

$$\delta Q = -tw\delta E, \tag{8.9}$$

and so $\delta E < 0$ (note $t > 0$ by assumption). Therefore :

$$p_1 x_2 + p_2 y_2 = (1-t)wE_2 + Q^*(\mathbf{p}) < (1-t)wE_1 + Q^*(\mathbf{p}) = p_1 x_1 + p_2 y_1 \tag{8.10}$$

and so either $x_2 < x_1$ or $y_2 < y_1$. But since $(x_\nu, y_\nu, E_\nu) \in A^i(p_1, p_2, w; Q_\nu)$ for $\nu = 1,2$, it must be that $x_2 > x_1$ and $y_2 > y_1$ because both commodities are normal goods, and the consumer's wealth (check the definition of A^i) is greater at $\nu = 2$ than at $\nu = 1$. This contradiction proves that Γ^i contains at most one element.

2. Next we show Γ^i contains at least one element. $B^i(p_1, p_2, w; Q^{i*})$ is a *planar segment.* We say a point (x, y, E) lies *above* (resp. *below*) the planar segment $B^i(p_1, p_2, w; Q^{i*})$ if it lies in the positive orthant and $p_1 x + p_2 y < (1-t)wE + Q^{i*}$ (resp., $p_1 x + p_2 y > (1-t)wE + Q^{i*}$). Note that the points on planar segment

$$p_1 x + p_2 y = wE + \tilde{Q}, \quad (x, y, E) \in \Re^2_+ \times [0, \bar{E}^i]$$

lie entirely below (or, at one point, on) the planar segment $B^i(p_1, p_2, w; Q^{i*})$ because:

$$wE + \tilde{Q} = wE + Q^{i*} - tw\bar{E}^i \leq wE + Q^{i*} - twE = (1-t)wE + Q^{i*}. \tag{8.11}$$

It therefore follows that $A^i(p_1, p_2, w; \tilde{Q})$ lies below (or possibly on) the planar segment $B^i(p_1, p_2, w; Q^{i*})$. On the other hand, for large values of *Q*, the points of

$$p_1 x + p_2 y = wE + Q, \quad (x, y, E) \in \Re^2_+ \times [0, \bar{E}^i]$$

must lie entirely above B^i. Since $A^i(p_1, p_2, w; Q)$ is a continuous function of *Q*, by the Berge maximum theorem, it follows that there exists at least one value of *Q* such that $A^i(p_1, p_2, w; Q) \cap B^i(p_1, p_2, w; Q^{i*}) \neq \varnothing$. Thus, Γ^i is a well-defined function.

3. Continuity of Γ^i follows from Berge's maximum theorem. □

Proof of Proposition 9:

0. The theorem is true for $t = 0$ by Proposition 8, since a Walrasian equilibrium exists at $t = 0$ under the stated premises. Henceforth, we assume $0 < t < 1$.

1. Let Δ be the 3-simplex of prices. Given a price vector $(p_1, p_2, w, r) \in \text{int}\,\Delta$ define (D^1, D^2, K^1, K^2) to be the solution of:

$$\begin{aligned}(K^1, D^1) &= \arg\max_{(K,E)}(p_1 G(K,E) - wE - rK)\\ (K^2, D^2) &= \arg\max_{(K,E)}(p_2 H(K,E) - wE - rK)\end{aligned} \tag{8.12}$$

Note that, by Assumption *A*(i) the solution exists and satisfies:

$$G_2(K^1, D^1) = \frac{w}{p_1}, \quad H_2(K^2, D^2) = \frac{w}{p_2}, G_1(K^1, D^1) = \frac{r}{p_1}, H_1(K^2, D^2) = \frac{r}{p_2}.$$

2. The profits of the two firms and the value of the demogrant are defined at (D^1, D^2, K^1, K^2). Profits are positive for any price vector $(p_1, p_2, w, r) \in \text{int}\,\Delta$.

We now consider the budget constraints of individuals:

$$\begin{aligned}&p_1 x + p_2 y = (1-t)wE + (1-t)\left(\theta^{i1}\Pi^1(K^1, D^1) + \theta^{i2}\Pi^2(K^2, D^2)\right) +\\ &\frac{t}{n}(p_1 G(K^1, D^1) + p_2 H(K^2, D^2) - \theta^{01}\Pi^1(K^1, D^1) - \theta^{02}\Pi^2(K^2, D^2) - r(K^1 + K^2))\end{aligned} \tag{8.13}$$

and the budget constraint of the state at the firms' demands:

$$p_1 x^0 = \theta^{01}\Pi^1(K^1, D^1) + \theta^{02}\Pi^2(K^2, D^2) + r(K^1 + K^2). \tag{8.14}$$

Let $Q^{i*}(\mathbf{p})$ equal the sum of the last two terms on the r.h.s. of equation (8.13). By the theorem's premise, all private agents have positive income at any $(p_1, p_2, w, r) \in \text{int}\,\Delta$, because the state does not receive all the firms' profits by assumption, and the tax rate is positive. $Q^{i*}(\cdot)$ are positive continuous functions, and so the premises of the Lemma hold; therefore the functions $\Gamma^i(p_1, p_2, w; Q^{i*}(\mathbf{p}))$ are defined and continuous. Henceforth, we write $\Gamma^i(p_1, p_2, w; Q^{i*}) \equiv \Gamma^i(\mathbf{p})$. Let $(x^i, y^i, E^i) = \Gamma^i(\mathbf{p})$ for $i \geq 1$.

4. Define the excess demand functions at a vector $\mathbf{p} = (p_1, p_2, w, r) \in \text{int}\,\Delta$:

$$\Delta E = D^1 + D^2 - \sum E^i,\ \Delta x = \sum_{i=0}^{n} x^i - G(K^1, D^1),$$
$$\Delta y = \sum_{i=1}^{n} y^i - H(K^2, D^2),\quad \Delta K = K^1 + K^2 - K_0 \qquad (8.15)$$

Define the excess demand function for the economy by:

$$z(\mathbf{p}) = (\Delta x, \Delta y, (1-t)\Delta E, (1-t)\Delta K)\,. \qquad (8.16)$$

Next, define the correspondence Φ on Δ as in Part A above.

6. By summing the budget constraints in (8.13) and (8.14) we calculate Walras's Law for this economy, defined on $\operatorname{int}\Delta$:

$$p_1\Delta x + p_2\Delta y + (1-t)w\Delta E + (1-t)r\Delta K = z(\mathbf{p})\cdot\mathbf{p} = 0. \qquad (8.17)$$

7. At a fixed point $\mathbf{p}$ of Φ, by Part A above, $z(\mathbf{p}) = 0$. Consequently $\Delta x = \Delta y = (1-t)\Delta E = (1-t)\Delta K = 0$, and all markets clear. We deduce $\Delta E = \Delta K = 0$ from the premise that $1-t > 0$.

8. Associated with these prices is an allocation $(\mathbf{x},\mathbf{y},\mathbf{E})$, with $(x^i, y^i, E^i) \in \Gamma^i(\mathbf{p})$ for all $1 \le i \le n$. We must show that $(E^1,...,E^n)$is an additive Kantian equilibrium at prices $\mathbf{p}$. This follows immediately from the definition of the functions Γ^i , because the first-order conditions for Kantian equilibrium, which were derived in steps 4,5,6, and 7 of the proof of Proposition 7, follow from the definition of Γ^i, given that

$$G_2(K^1, D^1) = \frac{w}{p_1} \text{ and } H_2(K^2, D^2) = \frac{w}{p_2}.$$

9. Thus, a fixed point of Φ is a Walras-Kant equilibrium at tax rate *t.* To show the existence of a fixed point, we need to check that the premises of Kakutani's fixed point theorem hold. Φ is obviously convex-valued. Upper-hemi-continuity of Φ at any point in $\operatorname{int}\Delta$ follows quickly.

Finally, we examine u.h.c. of Φ at points on the boundary of the simplex. Suppose $\mathbf{p}^j = (p_1^j, p_2^j, w^j, r^j) \to \mathbf{p} \in \partial\Delta$. Suppose the sign pattern of $\mathbf{p}$ is (+,+,0,+). We have $\Phi(p_1, p_2, w, r) = \{(0,0,1,0)\}$. . Eventually p_1^j, p_2^j, r^j are positive and bounded away from zero, and $w^j \to 0$.

We must show that $\lim_{j\to\infty}\Phi(\mathbf{p}^j)=(0,0,1,0)$. Without loss of generality, we may assume that $\mathbf{p}^j\in\text{int}\,\Delta$ for all *j*. Denote the excess demands at $\mathbf{p}^j$ by $\Delta x(j),\Delta y(j),\Delta E(j)$ and $\Delta K(j)$. We will show that, for *j* sufficiently large

$$\Delta E(j)>\max[\Delta x(j),\Delta y(j),\Delta K(j)], \tag{8.18}$$

and this will imply that, for sufficiently large *j*, $\Phi(\mathbf{p}^j)=(0,0,1,0)$. To show (8.18), we will show that $\frac{\Delta z(j)}{\Delta E(j)}\to 0$, for $z\in\{x,y,K\}$.

We show that $\frac{\Delta K(j)}{\Delta E(j)}\to 0$. We know $\Delta E(j)\to\infty$, because $w^j\to 0$, and so the firms will demand unbounded amounts of labor, while the supply of labor is bounded. If $\Delta K(j)$ were bounded above, we would be done. So we suppose that $\Delta K(j)$ is unbounded. It follows that for at least one firm – say the *G* firm -- $K^{1j}\to\infty$ and $D^{1j}\to\infty$. But by profit maximization, $\frac{G_2(K^{1j},D^{1j})}{G_1(K^{1j},D^{1j})}=\frac{w^j}{r^j}\to 0$. By homotheticity of *G* (Assumption *A*(i)), the points (K^{1j},D^{1j}) must eventually lie below any ray in the positive quadrant of (K,D) space. This implies that $\frac{\Delta K(j)}{\Delta E(j)}\to 0$, as required.

To show $\frac{\Delta z(j)}{\Delta E(j)}\to 0$ for $z\in\{x,y\}$, it suffices to show $\frac{\Pi(j)}{D(j)}\to 0$ because the demand for the two commodities cannot grow faster than total profits (wage income goes to zero). We show $\frac{G(K^1(j),D^1(j))}{D^1(j)}\to 0$. Let *j* be large and $J>j$. Then:

$$G(K^1(J),D^1(J))-G(K^1(j),D^1(j))\le G_1(K^1(j),D^1(j))(\delta K)+G_2(K^1(j),D^1(j))(\delta D), \tag{8.19}$$

by concavity of *G*, where $\delta K\equiv K(J)-K(j)$, etc., and so:

$$\frac{G(K^1(J),D^1(J))-G(K^1(j),D^1(j))}{\delta D}\le G_1(K^1(j),D^1(j))\frac{\delta K}{\delta D}+G_2(K^1(j),D^1(j)). \tag{8.20}$$

Now let $j, J \to \infty$, but *j* more slowly than *J*. We know from above that $\frac{\delta K}{\delta D} \to 0$, and $G_1(K^1(j), D^1(j)) = \frac{r^j}{p_1^j} \to \frac{r^*}{p_1^*}$, and $G_2(K^1(j), D^1(j)) = \frac{w^j}{p_1^j} \to 0$. Therefore the right-hand side of (8.20) approaches zero, and so $\frac{G(K^1(J), D^1(J))}{D^1(J)} \to 0$, as was to be proved.

We examine one more case on the boundary of the simplex. Suppose the sign pattern of $\mathbf{p}^*$ is (0,+,0,+). Then $\Phi(\mathbf{p}^*) = \{(a, 0, 1-a, 0) \mid 0 \le a \le 1\}$. We know that $p_1^j, w^j \to 0$ and eventually p_2^j **and** r^j are bounded away from zero. If eventually $\Delta E(j)$ is greater than $\Delta y(j), \Delta x(j)$ **and** $\Delta K(j)$, then eventually $\Phi(\mathbf{p}^j) = (0,0,1,0) \in \Phi(\mathbf{p}^*)$. Firm 2 eventually demands huge amounts of labor, because the wage goes to zero but the price of output is significantly positive. The profits of Firm 1 go to zero since $p_1^j \to 0$. These facts imply that $\frac{\Pi(j)}{\Delta E(j)} \to 0$ and so, as in the first case examined above, $\Delta E(j)$ dominates the other excess demands, as required.

The other cases of points on $\partial\Delta$ yield to similar analysis. Hence, the premises of Kakutani's theorem hold, and a fixed point in $\mathbf{int}\,\Delta$, which is a Walras-Kant market socialist equilibrium, exists. □

E. Essentials of proof of Proposition 11

1. Let Δ be the 2-simplex. Let $\mathbf{p} = (p, w, d) \in \mathbf{int}\,\Delta$. Let (X, A, B) be the profit maximizing supply of output, demand for labor of occupation 1 and demand for labor of occupation 2 by the firm. This exists and is unique if *G* is strictly concave, differentiable, and satisfies the Inada conditions, since the f.o.c.s are then:

$$\frac{w}{p} = G_1, \quad \frac{d}{p} = G_2 .$$

2. Given *B*, we show the existence of a unique vector $((x^1, E^1), ,..., (x^{n_1}, E^{n_1}))$ such that:

(i) for each $i \in I_1$, $\frac{w}{p} = -\frac{u_2^{1i}(x^i, E^i)}{u_1^{1i}(x^i, E^i)}$. and

(ii) for each $i \in I_1$, $px^i = wE^i + \frac{E^i}{A}\lambda\Pi(A,B)$.

It is easiest to see this claim is if we define worker *i's* utility function over consumption and *leisure* (measured in efficiency units):

$$U^{1i}(x^i, \bar{E}^i - E^i) \equiv u^{1i}(x^i, E^i).$$

Write $\ell^i \equiv \bar{E}^i - E^i$. (i) and (ii) above now become:

(i') $\frac{w}{p} = \frac{U_2^{1i}(x^i,\ell^i)}{U_1^{1i}(x^i,\ell^i)}$ and (ii') $px^i + \ell^i(w + \frac{\lambda\Pi(A,B)}{A}) = w\bar{E}^i + \frac{\bar{E}^i}{A}\lambda\Pi(A,B)$

.

The locus of points (x^i,ℓ^i) described by (i') is an expansion path for the utility function U^i in the non-negative quadrant of the (x^i,ℓ^i)plane, and the locus of points (x^i,ℓ^i) described by (ii') intersects the positive quadrant of that plane in a non-empty straight-line segment of negative slope. The intersection of these two loci exists and is a unique point *if consumption and leisure are both normal goods,* for this guarantees that the expansion path begins at the origin, is a monotone increasing path, and eventually lies entirely above the line segment of (ii'), so it intersects that line segment in a single point.

Hence the point defined by (i) and (ii) exists and is unique.

3. In like manner, given *A*, there exists a unique vector $((x^1,D^1),,...,(x^{n_1},D^{n_1}))$ such that:

(i) for each $i \in I_2$, $\frac{d}{p} = -\frac{u_2^{2i}(x^i,D^i)}{u_1^{2i}(x^i,D^i)}$. and

(ii) for each $i \in I_2$, $px^i = dD^i + \frac{D^i}{B}(1-\lambda)\Pi(A,B)$.

4. We now define an excess demand function for this economy on $\mathbf{int}\,\Delta$. Denote:

$$\Delta x(\mathbf{p}) = \sum_{i\in I} x^i - X, \quad \Delta E(\mathbf{p}) = A - \sum_{i\in I_1} E^i, \ \Delta D(\mathbf{p}) = B - \sum_{i\in I_2} D^i, \tag{8.21}$$

where x^i, E^i, D^i are the quantities defined in steps 2 and 3.

Define the excess demand function on $\mathbf{int}\,\Delta$:

$$z(\mathbf{p}) = (\Delta x(\mathbf{p}), \Delta E(\mathbf{p}), \Delta D(\mathbf{p})) \tag{8.22}$$

6. The reader may now verify that Walras's Law holds:

$$p\Delta x + w\Delta E + d\Delta D = 0 . \qquad (8.23)$$

7. We note that *z* is single-valued. From here on, we proceed as in earlier sections. Define the correspondence $\Phi : \Delta \to \Delta$ as in Part A above.

8. If $\mathbf{p}^*$ is a fixed point of Φ then $\mathbf{z}(\mathbf{p}^*) = \mathbf{0}$. All three markets clear at $\mathbf{p}^*$. It is left only to observe that the conditions (i) and (ii) in steps 2 and 3, which define the supplies of the two occupational labor vectors, and the demand for the consumption good, exactly characterize what it means for those vectors to be multiplicative Kantian equilibria of the games V^1 and V^2. This is true, because condition (i) is the f.o.c. for the vector **E** being a multiplicative Kantian equilibrium of the game V^1, and condition (ii) is the budget constraint of the worker (and likewise for the game V^2). This shows that the allocation is indeed a WKWO equilibrium and **p*** is an equilibrium price vector.

9. It is left to verify the premises of the Kakutani theorem for Φ. On $\mathbf{int}\,\Delta$, upper-hemi continuity follows from Berge's maximum theorem. The correspondence is single-valued on the interior, so it is convex valued. We skip the verification of these properties on the boundary of the simplex. □

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