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QUANTITATIVE EASING, COLLATERAL CONSTRAINTS, AND FINANCIAL SPILLOVERS

By

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December 2018

COWLES FOUNDATION DISCUSSION PAPER NO. 2154



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Quantitative Easing, Collateral Constraints, and Financial Spillovers *

John Geanakoplos † Haobin Wang ‡

December 10, 2018

Abstract

The steady application of Quantitative Easing (QE) has been followed by big and non-monotonic effects on international asset prices and international capital flows. These are difficult to explain in conventional models, but arise naturally in a model with collateral. This paper develops a general-equilibrium framework to explore QE's international transmission involving an advanced economy (AE) and an emerging market economy (EM) whose assets have less collateral capacity. Capital flows arise as a result of international sharing of scarce collateral. The crucial insight is that private AE agents adjust their portfolios in different ways in response to QE, conditional on whether they are (i) fully leveraged, (ii) partially leveraged or (iii) unleveraged. These portfolio shifts of international assets can diminish or even reverse the effectiveness of ever-larger QE interventions on asset prices. The model provides a simultaneous interpretation of several important stylized facts associated with QE.

Keywords: Quantitative easing, collateral, leverage, financial spillovers, emerging markets, capital flows.

JEL Codes: D52, D53, E32, E44, E52, F34, F36, G01, G11, G12.

*We thank Michael Woodford, William Brainard, Stephen Roach, Giuseppe Moscarini, Yuichi Kitamura, Robert Shiller, Truman Bewley, Zhiwu Chen, Matteo Maggiori, Manmohan Singh, William English, Zhen Huo, Andy Atkeson, Costas Arkolakis and Gianluca Violante for helpful comments and discussions.

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1 Introduction

The so-called Quantitative Easing (QE) undertaken in the United States and Europe in the aftermath of the Great Recession has been the largest central bank intervention in history. Over the course of a few years, the Fed and the ECB purchased trillions of dollars of assets in an effort to raise their prices (lower their yields) and thereby to stimulate lagging economies. Prices did rise at first, but eventually they fell again even as QE continued to expand, creating a non-monotonic effect that is difficult to explain with conventional models.[1]

While QE was primarily intended to influence domestic economies, policy makers in emerging markets (EMs) have argued that QE policies in advanced economies (AEs) generated unprecedented cross-country effects on asset prices and cross-border financial flows. While a vast and lively empirical literature has identified and documented the financial spillover effects of QE[2], the economic explanations behind the asset price effects and the international transmission of QE remain relatively obscure (due to a lack of theoretical work).

This paper explains the non-monotonic price effects of QE on domestic prices and the effects of QE on international flows and prices in a two-country general equilibrium model with collateral. We show that all these effects emerge from the hypothesis that the assets purchased by QE in advanced economies function as good collateral, while similar assets in emerging countries cannot be easily used as collateral.

Collateral plays a key role in the model since all private borrowing must be secured by collateral, and only assets in AE can serve as collateral to secure financial claims. We introduce a central bank in AE that can implement QE in the form of open-market asset purchases (financed by issuing riskless debt). The model shows that the steady application of QE can generate non-monotonic effects on AE asset prices and international spreads; furthermore, it induces private portfolio shifts of international assets that can, in turn, feed back to affect the impact of QE on asset prices.

This paper focuses on a collateral channel of transmission. We feel that the collateral channel is essential for four reasons: (i) global demand for collateral and collateralized funding has risen sharply in the aftermath of the financial crisis for myriad reasons (e.g., increased risk aversion and regulatory changes)[3] and (ii) QE has involved massive purchases of high-quality collateral (that traditionally play an important role in facilitating collateralized cross-border funding) and (iii) collateral constraints can generate non-monotonic price effects from QE and (iv) AE countries have more collateral than EM

[1] Of course other factors, not connected with QE, may have caused the price reversal. But we seek to explain why QE itself might have been expected to generate non-monotonic effects in the scale of QE, even without changes in real growth or inflation.

[2] See, for example, Krishnamurthy and Vissing-Jorgensen (2011), Fratzscher et al. (2013), Chol and Rhee (2013), and Chen et al. (2015) (among many others).

[3] See, for example, Adrian and Shin (2010), Gorton and Metrick (2012), Singh (2013), and Bank of International Settlement CGFS Paper No.49 (2013) for further discussions on the evolving role of collateral after the recent financial crisis.

countries, and so QE can affect collateralized cross-border funding. The assets purchased by QE (e.g., government bonds and mortgage-backed securities (MBS)) are the most utilized for "collateral", measured by the amounts of collateralized borrowing obtained by pledging them (as collateral) in various collateral markets. For example, domestic and international holders of U.S. Treasury securities have easy access to collateralized funding by pledging the underlying securities as collateral in the repo, securities-lending, prime-brokerage, and derivatives markets. (According to Baklanova et al. (2016), U.S. Treasury securities make up the dominant share of collateral utilized in the multitrillion, bilateral, U.S. repo market and they also find that rates and haircuts vary significantly across asset classes.) [4]

By incorporating collateral and agent-heterogeneity, the model offers a simultaneous interpretation of the following four stylized facts observed during QE episodes in the United States since the fall of 2010 that are difficult to rationalize in conventional open macroeconomic models.

Fact 1(a): changes in long-term Treasury yields and the term premium. The Federal Reserve's large-scale purchase of long-term U.S. Treasury securities was accompanied by a decline in the 10-year Treasury yield[5] from 2.67% on November 3, 2010 (announcement date of QE2), to a low of 1.44% on July 24th, 2012. During this period, the Fed's holdings of long-term U.S. Treasury securities[6] rose from $146 billion to $346 billion.

Fact 1(b): The Federal Reserve continued to increase its holdings of long-term U.S. Treasury securities (which attained a high of $664 billion on November 12, 2014); however, the 10-year yield rose from 1.44% to 2.37% during this period.

As a side note to Fact 1 (a) and (b), estimates of the 10-year Treasury term premium[7] show similar patterns to the 10-year Treasury yield over the same period in consideration (Figure 1).

Table 1: Fact 1(a) and (b)

	11/3/2010	07/24/2012	11/12/2014
Fed's Holdings of Long-term Treasuries (bn)	146	346	664
10-Year Treasury Yield (%)	2.67	1.44	2.37

[4]Their data were collected from the U.S.-affiliated securities dealers of nine bank holding companies, under a voluntary pilot program run by the Office of Financial Research and the Federal Reserve System (with input from the Securities and Exchange Commission).

[5]Constant maturity yield from FRED.

[6]Data include U.S. Treasury securities with maturity over 10 years. Source: FRED.

[7]10-year Treasury term premium estimates by the ACM model, available on Federal Reserve Bank of New York website.

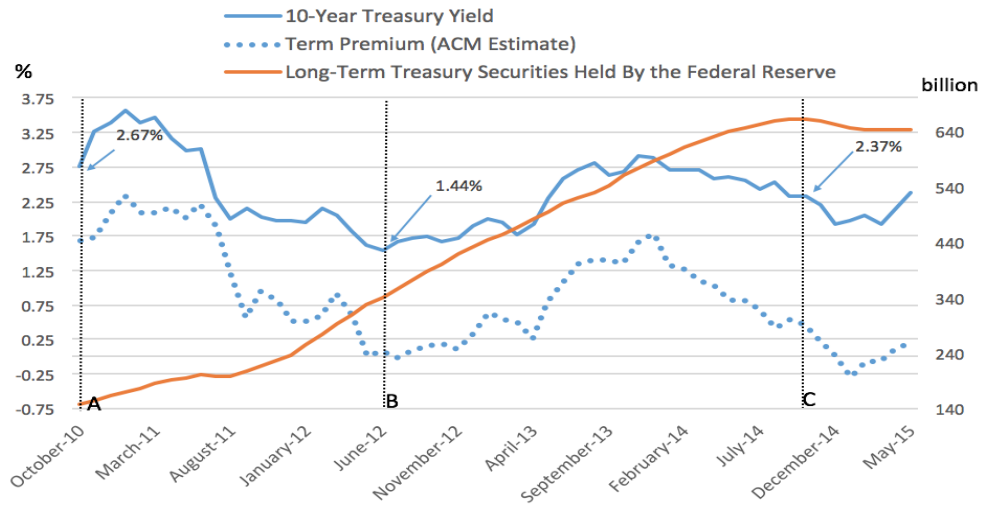


Figure 1: 10-Year Treasury Yield (L), ACM 10-Year Term Premium Estimates (L) and Fed's Holdings of Long-Term Treasuries (R)

Fact 2: changes in financial spillover effects. Long-term yield spreads[8] between EMs and the United States widened between November 2010 and July 2012; however, they subsequently shrank between July 2012 and November 2014.

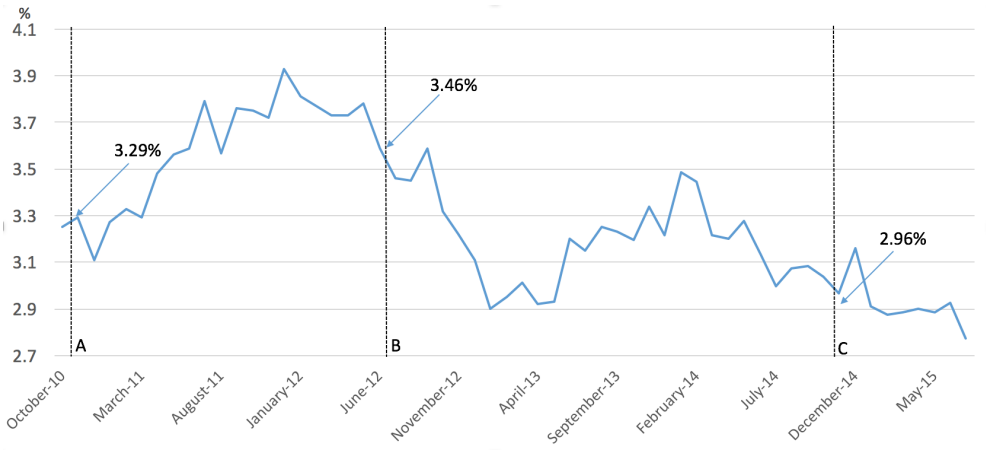


Figure 2: Yield Spread Between Emerging Markets and the U.S.

Fact 3: heterogeneous portfolio adjustments in the United States. During QE episodes, U.S. financial and non-financial organizations have responded differently in the context of international portfolio adjustments. For example, U.S. financial organizations increased their holdings of long-term foreign government bonds from $314 billion to $543 billion between September 2011 and November 2014. By contrast, non-financial organizations

[8]Spread between Barclays Emerging Market Local Currency Government Bond Index yield and U.S. Aggregate Bond Index yield.

reduced their holdings of long-term foreign government bonds from $179 billion to $112 billion over the same period.[9]

Table 2: Fact 3: U.S. Holdings of Long-Term Foreign Government Bonds

	Sept 2011	Nov 2014
U.S. Financial Organizations (bn)	314	543
U.S. Non-financial Organizations (bn)	179	112

Fact 4: Persistent global demand for U.S. Treasury securities. Overseas holdings of U.S. Treasuries have remained consistent in response to the Fed's asset purchases. In particular, foreign private demand for U.S. Treasury securities has been especially strong, even as Treasury yields fell.

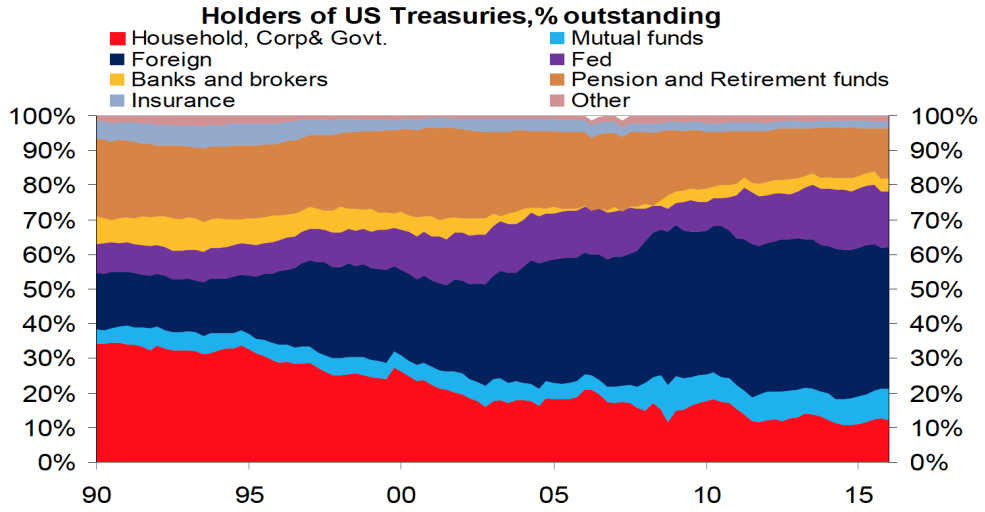


Figure 3: Persistent Global Demand for U.S. Treasury Securities[10]

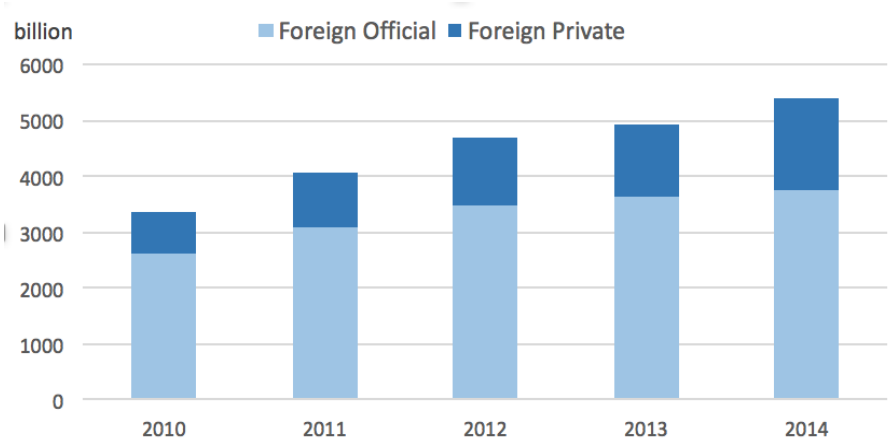


Figure 4: Foreign Holdings of U.S. Treasury Securities[11]

[9]Data source: U.S. Department of the Treasury. Starting date of dataset is September 2011.
[10]Source: Deutsche Bank.

The aforementioned facts are difficult to rationalize in conventional open macroeconomic frameworks (e.g., when the short-term interest rate is taken as the only dimension of monetary policy and monetary spillovers stem from simple interest-rate differentials). In conventional macroeconomic models, it is often assumed that the central bank can arbitrarily set the short-term nominal interest rate via an open-market operations; however, Fact 1 (a) and (b) suggest that the central bank has less control over other market rates of return. Figure 1 illustrates that the Federal Reserve's open-market purchases of long-term Treasury securities were accompanied by non-monotonic changes in long-term Treasury yield and its term premium. Thus, one needs to model a central bank's open-market purchases more explicitly in order to make sense of the possible effects of QE on asset prices.

The interest-rate differential in conventional open-economy models does not sufficiently clarify the kind of monetary spillovers observed in Fact 2 either, given that the U.S. policy rate has remained persistently near its effective lower bound. Thus, one needs to consider more explicitly how the central bank's open-market asset purchases may give rise to additional monetary spillovers (beyond those captured in conventional models). Fact 3 suggests that conventional models (with representative households) are unable to explain the heterogeneous asset-holding patterns within the United States during QE episodes. Lastly, Fact 4 suggests the need to capture certain distinct properties of U.S. Treasury securities in order to provide an economic foundation for the persistent global demand in response to QE. Thus, this paper develops a framework that rationalizes these patterns in the stylized facts as being collectively driven by QE. Of course, a myriad of other factors could have separately contributed to each of the above facts over this timeframe; however, we ignore them. In particular, our model suppresses credit easing (CE) policies. Instead, the model offers a way to jointly interpret the qualitative patterns in these facts with QE as the common driving force. [12]

As previously noted, collateral and agent-heterogeneity play key roles in the model. All private borrowing must be collateralized and only AE assets can be utilized internationally to secure privately issued financial claims. Capital flows arise as a result of international sharing of scarce collateral: collateral-constrained agents, in both economies, purchase AE assets with collateralized borrowing - bidding up the collateral value and price of AE assets while making the cheaper EM assets an attractive alternative to agents that do not demand collateral. Central banks (unlike private agents) can borrow to make asset purchases without collateral constraints; this type of superior financing ability does enable the AE central bank to have real effects on the global economy.

Importantly, the AE central bank's balance sheet is modeled as being indirectly owned by AE agents; thus, risks taken onto the AE central bank's balance sheet (via QE) will be distributed among AE agents (via the associated fiscal consequences). As a result,

[11] Source: Treasury International Survey (TIC).

[12] See Bernanke (2009) for a discussion on the distinction between CE and QE.

QE creates a hedging demand among AE agents to adjust their portfolios optimally. AE agents will respond to QE even if prices don't change, while EM households will respond only to price changes. The model simulation shows that the early phase of QE increases the price of AE assets and widens the spread between AE and EM asset prices. However, continued QE induces portfolio shifts (of AE agents) towards EM assets; this causes (i) a decline in AE asset prices and (ii) a tightening of the spread between AE and EM asset prices. Lastly, in the final phase of QE (i.e., extremely aggressive QE), AE asset prices and the international spread rise again.

The crucial insight is that *before prices have moved*, private AE agents adjust their international portfolios in response to QE in different ways, conditional on whether they are (i) fully leveraged, (ii) partially leveraged or (iii) unleveraged. The model characterizes three different types of QE-generated, private portfolio adjustments (of AE agents) that explain the aforementioned simulation patterns. First, fully leveraged agents partially offset QE via sales of some of their own AE assets (effectively selling them to the AE central bank). Second, partially leveraged agents respond to QE by concurrently (i) selling *more* AE assets than the AE central bank buys and (ii) buying some EM assets (with higher rates of return). Third, unleveraged agents respond to QE by concurrently selling existing holdings of EM assets and absorbing increases in riskless, central bank reserves. Such portfolio shifts of international assets can diminish or even reverse the effectiveness of ever-larger QE interventions on asset prices. As QE expands, fully leveraged agents become partially leveraged, and thus begin to reverse the effects of QE. EM agents have no incentive to change their portfolios in response to QE *before prices have moved*, because they are not directly affected by the AE central bank balance sheet. Even after prices adjust to QE, and despite radical cross-border portfolio adjustments, EM demand for AE assets remains relatively stable in response to QE because of persistent EM demand for collateral.

QE tends to raise the welfare of AE at the expense of EM, because the superiority of AE collateral means that EM will be a net buyer of AE collateral. By purchasing the same collateral, the AE central bank tends to raise its price, and thus benefit the AE sellers of the collateral at the expense of the EM buyers of AE collateral. Our model includes heterogeneous agents, so to get a Pareto improvement for all agents in AE, at the expense of EM, requires a calibrated intervention. AE agents who want to leverage will also be buyers of AE collateral, and thus hurt by the QE induced price rise of collateral. But they are also helped by QE, because the AE central bank is effectively purchasing the collateral on their behalf at 100% LTV levels, which they would like to do but which the market will not allow them to do because they cannot commit to repay. Thus we show that there is an interval of QE levels over which all AE agents benefit from QE, at the expense of EM agents.

The rest of this paper is organized as follows. Section 2 discusses related literature. Section 3 develops a two-country monetary model with endogenous collateral constraints.

The model characterizes three different types of private portfolio adjustment in response to QE. Section 4 illustrates the international transmission of QE via a simple numerical example and shows that under certain conditions, the aforementioned stylized facts emerge naturally in the collateral equilibrium of the model. Section 5 discusses the welfare implications of QE. Section 6 concludes.

2 Related Literature

This paper is related to a few different strands of literature. First, the general model setup builds upon the collateral equilibrium frameworks developed in Geanakoplos (1997), Geanakoplos and Zame (2014), and Fostel and Geanakoplos (2008, 2015 a,b), which emphasize the role of collateral and collateralization (e.g., leveraging and tranching) in driving asset prices and real investments. Second, the financial integration modeling is related to literature that studies global imbalances and emphasizes the role of heterogeneous financial developments (across countries). Specifically, (i) Caballero et al. (2008) show how the unique ability of the United States to produce and supply tradable financial assets (from real assets) can give rise to a persistent trade deficit, global imbalances, and low global interest rates; (ii) Mendoza et al. (2009) develop a model with idiosyncratic risks and limited contract enforceability and show that the United States can rationalize its asymmetric external balance sheet and relatively low interest rates due to a superior ability to enforce contracts; (iii) Maggiori (2013) provides a risk-based view of cross-country differences in financial development (and the associated model rationalizes global imbalances as a consequence of the asymmetric risk-absorbing capacities across countries); and (iv) Fostel, Geanakoplos, and Phelan (2017)—the most relevant work in this context—show, in a two-country model, that cross-border capital flow can arise between countries with different capacities to leverage or tranche assets. In their model, capital flows can arise as a way of sharing scarce collateral, even in the absence of interest rate differentials or risk-sharing motives. The financially sophisticated country has a superior ability to create contingent payoff streams from underlying collateral and induce cross-border asset holdings driven by demand for specialized cash flows. However, their model does not consider the effects of policy interventions on such capital flows.

Third, the modeling herein of the transmission mechanism of quantitative easing (QE) is related to literature that considers the central bank's asset purchases explicitly and how they may affect the private sector's financial conditions. Some recent work (e.g., Curdia and Woodford (2011) and Gertler and Karadi (2011, 2013)) develops models that capture how a central bank's direct lending to specific types of borrowers (at below-market rates) can help mitigate the malfunctioning of private financial intermediation (e.g., as we saw during the depths of the Great Recession). These models are appropriate for the study of central bank *credit-easing (CE) policies* during times of financial turmoil. Our focus differs from the aforementioned literature and is instead concentrated on central bank

QE policy that involves purchases of highly liquid assets as a way of influencing economic activity, even when all agents can trade all assets at the same prices.

In this regard, this paper is most closely related to Araujo, Schommer, and Woodford (2015); they consider central bank asset purchases when all market participants have equal access to the same set of traded assets at well-defined market prices (independent of the identity of the purchaser) and thus when the markets are efficient. However, the markets in their model are not completely frictionless since all privately issued financial claims must be secured by collateral. They note that this "collateral requirement" captures the kind of financial friction that exists—even in markets that are generally considered efficient [13] and is thus important for "unconventional" monetary policies that interacts with private intermediation. In a completely frictionless economy with rational expectations, the central bank's asset purchases do not have any real effects (since it is simply a financial intermediary; whatever it does, agents will undo the effects in their own portfolios). Thus, certain types of financial frictions must exist in order for the central bank's intervention to generate meaningful impacts. [14] They model the central bank's balance sheet as indirectly owned by domestic households, based on the assumption that any earnings or losses from the central bank's asset holdings will eventually be transferred to domestic households. They show that the central bank's purchases of collateral-like assets (e.g., government bonds and mortgage-backed securities) can generate real effects by affecting collateral constraints in the economy. However, their model does not consider how such central bank interventions may generate financial spillovers in open-economy contexts, or how it can create non-monotonic effects on spreads. The literature on the transmission of QE also includes other important studies (e.g., Vayanos and Vila (2009); Chen, Curdia, and Ferrero (2011); and Williamson (2012))—all of which provide key insights into possible mechanisms of QE's transmission.

Fourth, this paper is related to the literature that emphasizes the special role played by (i) U.S. assets in the international financial system and (ii) the U.S. dollar. Certain studies (e.g., Krishnamurthy and Jorgensen (2012), Matteo (2013), and Negal (2016)) provide empirical evidence on the safety and liquidity premium of the U.S. dollar and U.S. assets. On the theory side, both early and recent work (e.g., Despres, Kindleberger, and Salant (1966); Gourinchas and Rey (2007); and Farhi and Maggiori (2016)), provides insights into understanding the international financial architecture (with the United States characterized as the key country). This paper instead focuses on the superior collateral value of U.S. assets (e.g., U.S. Treasuries) relative to their emerging-marketing (EM) counterparts.

Last, but not least, this paper is also related to the literature on the portfolio balance

[13] As noted in Geanakoplos (1997), much of the lending in modern times is secured by some form of collateral (in contrast to a complete market setup with private agents borrowing arbitrarily with no concern over default).

[14] For instance, "Wallace Neutrality" suggests that a central bank's open market operation is neutral in the absence of any financial friction (i.e., when a market is complete). Backus and Kehoe (1989) suggest that under certain economic conditions, government portfolios are irrelevant (similar to the irrelevance of corporate liability structures under the Modigliani-Miller theorem).

channel of exchange rate determination.[15] This literature considers the exchange rate as the relative price of currencies that equilibrates the demand and supply of various international financial assets. While this paper assumes trivial fixed exchange rate, it does explicitly consider the demand and supply of different international assets (that resembles the portfolio balance channel mentioned above).

3 A Two-Country Monetary Model with Endogenous Collateral Constraints

3.1 Model Setup

Consider a simple endowment economy with two countries: AE and EM. There are two periods $t = 0, 1$, with one state 0 at time 0 and two possible states in $t = 1$: $\{U, D\}$, where U stands for the "Up" state while D stands for the "Down" state. Agents in both countries consume the same consumption good C. For simplicity, we ignore issues of stochastic inflation and fluctuating exchange rates: we normalize the price of consumption goods in all three states 0,U, D to be 1, and we assume that the exchange rate is always one. There is a riskless real government bond in each country, denoted by B_{AE} and B_{EM}. Each unit of riskless bond pays one unit of consumption good in both U and D, so B_{AE}=B_{EM}=B; thus there is only one type of riskless asset in the model. The riskless bond is meant to represent short-term, riskless government bonds (e.g., Treasury bills, or their EM counterparts).The payoffs (promises) of riskless real bonds will be paid by the government that issues them from tax revenues.

There is also a risky real asset in each country, denoted by Y_{AE} and Y_{EM}. Risky assets deliver consumption goods at $t = 1$[16], and their payoffs are given exogenously by (d_U^{AE}, d_D^{AE}) and (d_U^{EM}, d_D^{EM}).[17]Assume that $(d_U^{AE}, d_D^{AE}) = (d_U^{EM}, d_D^{EM})$, so that Y_{AE} and Y_{EM} deliver the same payoff vector. An important assumption is that *only* Y_{AE} can serve as collateral to back privately issued financial claims.[18] To distinguish Y_{AE} and Y_{EM} from a riskless asset, further assume $d_U^{AE} > d_D^{AE}$ and $d_U^{EM} > d_D^{EM}$ so that asset payoffs are strictly higher in the "Up" state. Y_{AE} is meant to represent assets acquired via QE (e.g., long-term Treasury securities, agency bonds or MBS); Y_{EM} is meant to represent the EM counterpart (of Y_{AE}) (e.g., long-term EM government bonds). Note that this paper does not seek to model the maturity structure of longer-term assets; instead, we focus on their

[15]Early work includes Kouri (1976), Tobin and de Macedo (1979), Driskill and McCafferty (1980a,b), Dornbusch and Fischer (1980), Henderson and Rogoff (1980) (among many others).

[16]Inflation does not play a crucial role in this paper, so it does not matter whether risky payoffs are money or consumption goods. Each risky asset is similar to a Lucas tree.

[17]These payoffs are meant to indicate the continuation values that the long term assets Y_{AE} and Y_{EM} would have at U and at D if the model had more than two periods. Even "riskless" long term U.S. Treasury Bonds have risky valuations in the short run.

[18]We could weaken this assumption. For example, we could equally assume Y_{EM} can also serve as collateral, but has smaller collateral capacity than Y_{AE}.

riskiness (i.e., risk premium) relative to riskless short-term assets.

While AE and EM assets can differ in many aspects in reality, we focus here on their distinct collateral properties, which will be the crucial properties for the international transmission of QE in the model.

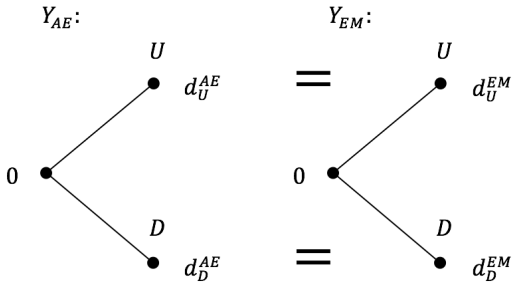


Figure 5: Payoff Vectors of Y_{AE} and Y_{EM}

There is a finite number of agents in each country, denoted by superscript $h \in H = 1, ..., H$ for AE, and $h^* \in H^* = 1, ..., H^*$ for EM. Agents $i \in H \cup H^*$ have von Neumann-Morgenstern utility function and common beliefs about future states.

$$u^i(c_0, c_U, c_D) = v^i(c_0) + \delta^i(prob_U \cdot v^i(c_U) + prob_D \cdot v^i(c_D)) \tag{1}$$

This paper focuses on heterogeneity that stems from agents' distinct endowment streams. However, heterogeneity can also arise from myriad other channels (e.g., heterogeneous beliefs about future states and heterogeneous risk aversions), which can generate qualitatively similar results.

Agents in each country are endowed with the consumption good C, their respective risky assets Y_{AE} and Y_{EM}, and riskless bond B. Agents' endowments, in the two countries, are summarized as follows:

$$\text{AE: } (e^h_{C_0}, e^h_{Y_{AE}}, e^h_B, \{e^h_{C_s}\}_{s\in\{U,D\}}) \quad \forall h \in H$$

$$\text{EM: } (e^{h^*}_{C_0}, e^{h^*}_{Y_{EM}}, e^{h^*}_B, \{e^{h^*}_{C_s}\}_{s\in\{U,D\}}) \quad \forall h^* \in H^*$$

e_{C_0} and $\{e_{C_s}\}_{s\in\{U,D\}}$ denote the endowment of consumption goods in each period and state; e_B denotes agents' initial endowment of riskless bonds; $e_{Y_{AE}}$ and $e_{Y_{EM}}$ denote agents' initial endowments of risky assets.

AE and EM are distinguished from each other in three important respects. First, Y_{AE} and Y_{EM} differ in their collateral capacities—only Y_{AE} can serve as collateral to secure privately issued financial claims. Second, agents in each country are only endowed with the risky asset of their own country at $t = 0$, and the AE assets are more valuable because they can be used as collateral. Third, only the AE central bank is active, and its balance sheet is indirectly owned by domestic agents only (as will be further discussed below).

3.2 Collateral and Privately Issued Financial Claims

In the Arrow-Debreu [A-D] model (or standard models of general equilibrium with incomplete asset markets), it is assumed that agents can issue arbitrary quantities of (existing) financial promises as long as they are able to deliver the promised amount in each possible state of the world. In reality, significant shares of privately issued financial claims are secured by some form of collateral.

Thus, we follow Geanakoplos (1997) and Geanakoplos and Zame (2014) and assume that in our two-country economy all private borrowing (i.e., issuances of financial claims) must be collateralized. Any privately issued financial claim at $t = 0$ specifies a quantity of (i) consumption good that the borrower has to repay (independent of state s) in $t = 1$ in order to extinguish the debt and (ii) a quantity of collateral, i.e., Y_{AE}, that can be seized by the lender in the event of default (i.e., non-payment of a promised amount of consumption good)[19]. The financial claim gives the holder (lender) no rights to the asset pledged as collateral, except in the event of default. In other words, the financial claim gives the issuer (borrower) the right to discharge the claim by paying the promised amount of consumption good, or, by giving up the collateral. The borrower chooses which is better for him. Such financial claims are referred to as no-recourse claims.

Specifically, assume there is trading in a set of privately issued financial claims $j \in J$ at $t = 0$. Each unit of financial claim j promises the delivery of j units of consumption good in both states at $t = 1$ (non-contingent financial claims) and is secured by one unit of Y_{AE} as collateral. Denote q_j as the price of one unit of financial claim j. A financial claim j can therefore be written as $\{(j, j); one\ unit\ of\ Y_{AE}\}$, and is sold at unit price q_j. Actual delivery of financial claim j is thus equal to $min(j, d_s^{AE})$, $\forall s \in \{U, D\}$. We shall assume that $j = d_D^{AE}$ is among the available $j \in J$.

Given such a set of financial claims, an agent can borrow amount q_j at $t = 0$ by issuing (selling) a unit of financial claim j. By doing so, the agent promises to repay (j, j) (in units of consumption goods) at $t = 1$ and must own one unit of Y_{AE} as collateral to secure the financial claim. Agents can buy or sell arbitrary quantities of each financial claim at a competitive per unit price.

3.3 Treasury Bonds, Central Bank Bonds, and Quantitative Easing:

We assume that the AE and EM Treasuries have already issued riskless bonds B promising one unit of the consumption good in each state, in the quantities $e_B^{AE} = \sum_{h \in H} e_B^h$ and $e_B^{EM} = \sum_{h^* \in H^*} e_B^{h^*}$, respectively. We suppose the AE central bank can acquire risky asset Y_{AE} by issuing riskless bonds b, also promising one unit of the consumption good. Denote the prices of b and Y_{AE} to be q_0 and π_{AE} respectively. If the AE central bank chooses to

[19] In this context, the borrower issues (sells) a financial claim to the lender; the lender pays the borrower today (i.e., at $t = 0$) in exchange for repayment of consumption good tomorrow (i.e., at $t = 1$).

acquire y_{AE}^{CB} units of Y_{AE} at $t = 0$, it will need to issue enough bonds b^{CB} so that

$$q_0 \cdot b^{CB} = \pi_{AE} \cdot y_{AE}^{CB} \tag{2}$$

The bonds B issued by the Treasury and the bonds b issued by the central bank reserves are perfect substitutes. In reality the central bank issues bank reserves, but these pay interest and so we shall call them central bank bonds.

In this finite-horizon model, assets recorded on the AE central bank's balance sheet must be sold in the terminal period $t = 1$ (at the same time the AE central bank redeems its outstanding liabilities). [20] However, the value of the AE central bank's assets can (i) differ across states of the world, (ii) exceed (or fall short of) the AE central bank's liabilities, and (iii) result in the AE central bank potentially accruing balance sheet earnings (or losses). As in Araujo, Schommer, and Woodford (2015), we assume that any such AE central bank's balance sheet surpluses would be transferred to the Treasury, thereby reducing the tax collection needed to retire government debt at $t = 1$; similarly, balance sheet losses would be offset by higher tax collection at $t = 1$. The revenues required to retire government debt (or pay off the AE central bank's balance sheet losses) are raised via lump-sum taxation on domestic agents. Thus, we can think of the AE central bank as a monetary-fiscal authority. Each agent's tax share is denoted by θ^h and is assumed to be the same in each state (where $\sum_{h=1}^{H} \theta^h = 1$).

Given this assumption, the tax obligation T_s^h of agent h in state s is given by

$$T_s^h = \theta^h(e_B^{AE} + b^{CB} - y_{AE}^{CB} d_s^{AE}) \tag{3}$$

Here, e_B^{AE} represents the total liabilities (bonds) issued by the Treasury and $y_{AE}^{CB} d_s^{AE} - b^{CB}$ represents the value of the AE central bank's assets at $t = 1$. The gap between $e_B^{AE} + b^{CB}$ and $y_{AE}^{CB} d_s^{AE}$ must be covered by tax revenues (generated by domestic agents).

The EM central bank does not engage in quantitative easing. Hence the tax obligation $T_s^{h^*}$ of each agent h^* in state s is given by

$$T_s^{h^*} = \theta^{h^*} e_B^{EM} \tag{4}$$

3.4 Agent Maximization and Equilibrium

To maximize utility, agent h decides on an optimal consumption stream, denoted by $c^h = (c_0^h, c_U^h, c_D^h)$, together with a period-0 asset portfolio$(\psi^h, \varphi^h, \mu^h, y_{AE}^h, y_{EM}^h)$. This would allow for the desired consumption stream, where $\psi^h = \{\psi_j^h\}_{j \in J}$ indicates the purchase of each type of financial claim $j \in J$; $\varphi^h = \{\varphi_j^h\}_{j \in J}$ indicates the sale (issuance) of each type of financial claim $j \in J$; μ^h indicates the holdings of riskless bonds[21]; y_{AE}^h and y_{EM}^h indicate the holdings of Y_{AE} and Y_{EM}. Agents are subject to (i) their budget constraints and (ii) collateral constraints.

[20] At the end of the terminal period, agents consume all the consumption goods.

[21] i.e., riskless Treasury bond or riskless central bank bond.

Denote $q = \{q_j\}_{j \in J}$ as the price of each privately issued financial claim $j \in J$; q_0 as the price of riskless assets; and $\pi = \{\pi_{EM}, \pi_{AE}\}$ as the prices of Y_{AE} and Y_{EM}.

The AE agent maximization problem can therefore be written as follows:

AE:

Given prices $\pi \in \mathbb{R}^2_{++}$, $q \in \mathbb{R}^J_+$, $q_0 \in \mathbb{R}_{++}$, and tax obligations $T^h \in \mathbb{R}^2_{++}$, agent h chooses a consumption stream and asset portfolio $(c^h, \psi^h, \varphi^h, \mu^h, y^h_{AE}, y^h_{EM})$ that solves the problem:

$$\max u^h(c^h) \qquad s.t. \tag{5}$$

$$\begin{aligned} c^h_0 + \sum_{j=1}^{J} q_j(\psi^h_j - \varphi^h_j) + \pi_{AE} y^h_{AE} + \pi_{EM} y^h_{EM} + q_0 \mu^h \leq \\ e^h_{C_0} + \pi_{AE} e^h_{Y_{AE}} + q_0 e^h_B, \end{aligned} \tag{6}$$

$$\begin{aligned} c^h_s \leq e^h_{C_s} + y^h_{AE} d^{AE}_s + y^h_{EM} d^{EM}_s + \sum_{j=1}^{J} (\psi^h_j - \varphi^h_j) min\{j, d^{AE}_s\} \\ + \mu^h - T^h_s, \forall s \in \{U, D\} \end{aligned} \tag{7}$$

$$y^h_{AE} \geq \sum_{j=1}^{J} \varphi^h_j, \tag{8}$$

where $c^h \geq 0, \psi^h \geq 0, \varphi^h \geq 0, \mu^h \geq 0, y^h_{AE} \geq 0, y^h_{EM} \geq 0$.

Equation (6) represents agent h's period-0 budget constraint, which says that the cost of (i) consumption c^h_0 and (ii) portfolio $(\psi^h, \varphi^h, \mu^h, y^h_{AE}, y^h_{EM})$ cannot exceed the value of initial endowment. (Note that when $\sum_{j=1}^{J} q_j(\psi^h_j - \varphi^h_j) > 0$, agent h is a net buyer of financial claims and thus a lender in the economy; similarly, when $\sum_{j=1}^{J} q_j(\psi^h_j - \varphi^h_j) < 0$, agent h is a net issuer of financial claims and thus a borrower in the economy.)

Equation (7) represents agent h's budget constraint in state s at $t = 1$, which says that the cost of consumption c^h_s cannot exceed the value of (i) consumption endowment in state s, (ii) payoff from Y_{AE} and Y_{EM}, (iii) net delivery from financial claims, (iv) delivery from riskless assets, and (v) net of tax obligation. As noted above, any losses or earnings from the AE central bank's asset purchases will be transferred to AE agents via tax collection at $t = 1$. Thus, one can think of the AE central bank's balance sheet as indirectly owned by AE agents.

Equation (8) represents the collateral constraint, which requires agent h to hold sufficient quantities of Y_{AE} (as collateral) to back its issuance of financial claims $\{\varphi^h_j\}_{j \in J}$.

The EM agent maximization problem can be similarly written as follows:

EM:

Given prices $\pi \in \mathbb{R}^2_{++}$, $q \in \mathbb{R}^J_+$, $q_0 \in \mathbb{R}_{++}$, and tax obligations $T^{h^*} \in \mathbb{R}^2_{++}$, agent h^* chooses a consumption stream and asset portfolio $(c^{h^*}, \psi^{h^*}, \varphi^{h^*}, \mu^{h^*}, y^{h^*}_{AE}, y^{h^*}_{EM})$ that solves the problem:

$$\max u^{h^*}(c^{h^*}) \qquad s.t. \tag{9}$$

$$\begin{aligned} c_0^{h^*} + \sum_{j=1}^{J} q_j(\psi_j^{h^*} - \varphi_j^{h^*}) + \pi_{AE} y^{h^*}_{AE} + \pi_{EM} y^{h^*}_{EM} + q_0 \mu^{h^*} \leq \\ e^{h^*}_{C_0} + \pi_{EM} e^{h^*}_{Y_{EM}} + q_0 e^{h^*}_B, \end{aligned} \tag{10}$$

$$\begin{aligned} c_s^{h^*} \leq e^{h^*}_{C_s} + y^{h^*}_{AE} d_s^{AE} + y^{h^*}_{EM} d_s^{EM} + \sum_{j=1}^{J} (\psi_j^{h^*} - \varphi_j^{h^*}) min\{j, d_s^{AE}\} \\ + \mu^{h^*} - T_s^{h^*}, \forall s \in \{U, D\} \end{aligned} \tag{11}$$

$$y^{h^*}_{AE} \geq \sum_{j=1}^{J} \varphi_j^{h^*}, \tag{12}$$

where $c^{h^*} \geq 0, \psi^{h^*} \geq 0, \varphi^{h^*} \geq 0, \mu^{h^*} \geq 0, y^{h^*}_{AE} \geq 0, y^{h^*}_{EM} \geq 0$.

Definition 1: Given the following endowment and AE central bank specification:

$$(e^h_{C_0}, e^h_{Y_{AE}}, e^h_B, \{e^h_{C_s}\}_{s\in\{U,D\}}) \text{ for } h \in H;$$

$$(e^{h^*}_{C_0}, e^{h^*}_{Y_{EM}}, e^{h^*}_B, \{e^{h^*}_{C_s}\}_{s\in\{U,D\}}) \text{ for each } h^* \in H^*;$$

$$y^{CB}_{AE},$$

a collateral equilibrium for the economy is a vector

$$[(\bar{c}, \bar{\psi}, \bar{\varphi}, \bar{\mu}, \bar{y}_{AE}, \bar{y}_{EM}); (\bar{c}^*, \bar{\psi}^*, \bar{\varphi}^*, \bar{\mu}^*, \bar{y}^*_{AE}, \bar{y}^*_{EM}); (\bar{\pi}, \bar{q}), \bar{T}, \bar{T}^*]$$

that satisfies the following:

(a) $(\bar{c}^h, \bar{\psi}^h, \bar{\varphi}^h, \bar{\mu}^h, \bar{y}^h_{AE}, \bar{y}^h_{EM})$ and $(\bar{c}^{h^*}, \bar{\psi}^{h^*}, \bar{\varphi}^{h^*}, \bar{\mu}^{h^*}, \bar{y}^{h^*}_{AE}, \bar{y}^{h^*}_{EM})$ respectively solve (5) and (9);

(b) $\sum_{h=1}^{H} \bar{c}^h_0 + \sum_{h^*=1}^{H^*} \bar{c}^{h^*}_0 = \sum_{h=1}^{H} e^h_{C_0} + \sum_{h^*=1}^{H^*} e^{h^*}_{C_0}$

(c) $\sum_{h=1}^{H} \bar{c}^h_s + \sum_{h^*=1}^{H^*} \bar{c}^{h^*}_s = \sum_{h=1}^{H} e^h_{C_s} + \sum_{h^*=1}^{H^*} e^{h^*}_{C_s} + \sum_{h=1}^{H} e^h_{Y_{AE}} d_s^{AE} + \sum_{h^*=1}^{H^*} e^{h^*}_{Y_{EM}} d_s^{EM}$
$\forall s \in \{U, D\}$;

(d) $\sum_{h=1}^{H} \bar{y}^h_{AE} + y^{CB}_{AE} + \sum_{h^*=1}^{H^*} \bar{y}^{h^*}_{AE} = \sum_{h=1}^{H} e^h_{Y_{AE}}$;

(e) $\sum_{h=1}^{H} \bar{y}_{EM}^{h} + \sum_{h^*=1}^{H^*} \bar{y}_{EM}^{h^*} = \sum_{h^*=1}^{H^*} e_{Y_{EM}}^{h^*}$;

(f) $\sum_{h=1}^{H} (\bar{\psi}_j^h - \bar{\varphi}_j^h) + \sum_{h^*=1}^{H^*} (\bar{\psi}_j^{h^*} - \bar{\varphi}_j^{h^*}) = 0 \quad \forall j \in J$;

(g) $\sum_{h=1}^{H} \bar{\mu}^h + \sum_{h^*=1}^{H^*} \bar{\mu}^{h^*} = \sum_{h=1}^{H} e_B^h + \sum_{h^*=1}^{H^*} e_B^{h^*} + b^{AE}$; where $b^{AE} = \frac{\pi_{AE}}{q_0} y_{AE}^{CB}$

The equilibrium can be characterized as a system of non-linear equations representing the agents' first order, boundary, and market-clearing conditions. Given numerical inputs, the system of equations can be solved numerically. (See the online appendix for the list of characterizing equations used for computation.)

3.5 Asymmetric Collateral Properties and Private Portfolio of International Assets

An underlying assumption in the model is that Y_{AE} (vs. Y_{EM}) is a superior form of collateral. In reality, the superior collateral capacity of Y_{AE} can be caused by a few factors: (i) better international credibility, or (ii) higher market liquidity or (iii) more reliable collateral properties (supported by highly advanced accounting and legal institutions). Indeed, one distinguishing feature of an advanced financial system is its superior ability to produce collateral-backed financial promises. This increases the collateral value of AE assets that can be pledged as collateral in the AE's sophisticated financial markets.

In the aftermath of the Great Recession, the use of collateral in the international financial system has become increasingly important due to (ii) more prudent risk management practices (adopted by financial institutions) and (ii) changing regulatory landscapes. This has partly contributed to a persistent demand for high-quality collateral (e.g., U.S. Treasury securities). AE assets tend to be preferred by financial institutions in need of collateralized borrowing. By contrast, EM assets generally have lower associated collateral properties, internationally, for a myriad of reasons (e.g., relatively low international credibility and less market liquidity as a result of underdeveloped collateral markets).

When Y_{AE} and Y_{EM} only differ in their collateral capacities (as we assume in the model), cross-border capital flows arise as a result of the international sharing of scarce collateral. Given that only Y_{AE} can serve as collateral to back privately issued financial claims, collateral-constrained agents, in both AE and EM, prefer holding Y_{AE} in order to obtain collateralized borrowing. Such global demand for collateral will increase the collateral value of Y_{AE} and its price. However, agents with no collateral constraints prefer relatively cheaper Y_{EM}. The following lemmas and propositions serve to characterize these types of cross-border asset-holding patterns.

Lemma 1 ***(Collateral Premium)*** *Suppose Y_{AE} and Y_{EM} deliver the same payoff vector at $t = 1$ (i.e., $d_s^{AE} = d_s^{EM} \quad \forall s \in \{U, D\}$). Despite the same payoff vector, in*

collateral equilibrium, Y_{AE} is always traded at a non-negative collateral premium over Y_{EM}, i.e. $\pi_{AE} \geq \pi_{EM}$. If some agent h holds $y^h_{EM} > 0$, and if the collateral constraint binds strictly for h (so that eliminating the constraint from her budget set would raise her utility), then $\pi_{AE} > \pi_{EM}$. The difference $\pi_{AE} - \pi_{EM}$ reflects the collateral value of Y_{AE}.

Lemma 1 is intuitively simple to understand: since Y_{AE} and Y_{EM} are assumed to deliver the same payoff vector, π_{AE} can not be lower than π_{EM} in equilibrium (or else an arbitrage opportunity exists). However, it is possible for π_{AE} to be greater than π_{EM}, since Y_{AE} can also serve as collateral, and thus is more valuable to agents with collateral constraints. Every agent with strictly binding collateral constraints would replace some of her y_{EM} with y_{AE} unless $\pi_{AE} > \pi_{EM}$. The collateral premium (i.e., $\pi_{AE} - \pi_{EM}$) reflects the "tightness" of the collateral constraints in the economy. [22]

No-Default Theorem (Fostel and Geanakoplos (2015)) *Assume there are only two possible states $s \in \{U, D\}$ at $t = 1$. Suppose that, in equilibrium, the financial claim j^*, where $j^* = min\{d^{AE}_U, d^{AE}_D\}$, is available to be traded; in this case, j^* will be the only financial claim that is actively traded.*[23]

The No-Default Theorem posits that when there are only two possible states at $t = 1$, agents endogenously choose a single, no-default financial claim j^*, where $j^* = min\{d^{AE}_U, d^{AE}_D\}$, to be actively traded in equilibrium[24]. Recall that a unit of financial claim j^* delivers payment (j^*, j^*) at $t = 1$, and is secured by one unit of collateral Y_{AE}. The condition $j^* = min\{d^{AE}_U, d^{AE}_D\}$ ensures that the payoff from Y_{AE} (i.e., the collateral) always weakly exceeds the promised payment in both states at $t = 1$ (so that default does not occur in equilibrium).

The No-Default Theorem simplifies our associated analysis since there is only one actively traded financial claim j^* to be considered (and equilibrium does not involve actual default). However, it must be emphasized that while actual default does not occur in equilibrium, the possibility of default (i) limits borrowing because it requires all privately issued financial claims to be fully secured by collateral and (ii) does have real consequences on the equilibrium price of collateral (for example, via the endogenously determined collateral premium $\pi_{AE} - \pi_{EM}$). Since default does not occur in equilibrium, holding a unit of financial claim j^* delivers the same payoff as holding j^* units of any other riskless asset, hence $q_{j^*} = q_0 d^{AE}_D$.

Figure 6 illustrates two possible types of privately issued financial claims, j and j^*. Each vector in the quadrant represents the payoff vector (i.e., payoff in $s \in \{U, D\}$) of the corresponding asset. $\overrightarrow{OB}$ (red) represents the payoff vector of one unit of collateral

[22] While many factors (e.g., credit risk, market liquidity, and underlying growth prospects) can contribute to variations in asset prices, Lemma 1 highlights collateral properties (e.g., haircuts) as a potential candidate.

[23] More precisely, the theorem states that if there is an equilibrium with positive trade in some $j \neq j^*$, then there is also an equilibrium with the same consumption in which only contract j^* is positively traded.

[24] Default can occur when the number of future states exceeds two.

Y_{AE}, while $\overrightarrow{OA}$ and $\overrightarrow{OA'}$ (green arrows) represent the payoff vector of one unit of j and j^*, respectively. Financial claim j involves a default in equilibrium because the promised payment j exceeds the value of collateral in state D, i.e., $j > d_D^{AE}$; however, financial claim j^* (where $j^* = min\{d_U^{AE}, d_D^{AE}\}$) does not involve any default since the promised payment does not exceed the value of collateral in both states. One observes that j^* is the maximum amount of promise an issuer can credibly make (against one unit of collateral) without invoking default in equilibrium. The No-Default Theorem shows that j^* will be the only actively traded financial claim in equilibrium.

Figure 6 also illustrates why collateral is important. An agent who holds Y_{AE} and wants money immediately could always sell the Y_{AE}. The reason to use Y_{AE} as collateral in order to borrow by selling j or j^* is that after repaying the loan, the agent is left with money exclusively in the state U. Thus agents who are especially desirous of money in U, either because they attach relatively high probability to the state or because they are relatively poor there, will be the ones who prefer to buy Y_{AE} (via borrowing) rather than Y_{EM}, which delivers dividends in both states.

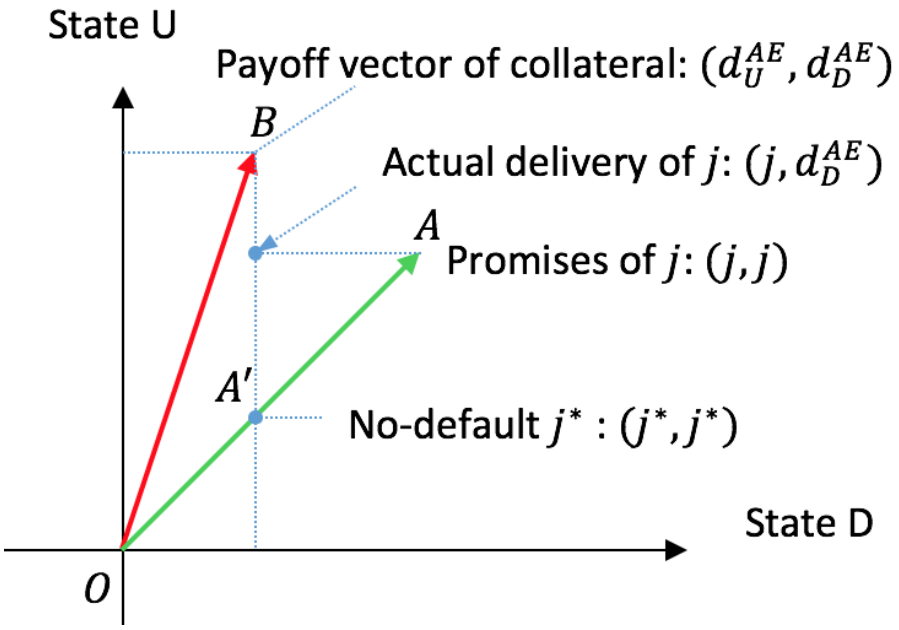


Figure 6: Default vs. No-Default Financial Claims

Lemma 2 ***(Maximum Leverage)*** *Suppose Y_{AE} and Y_{EM} deliver the same payoff vector at $t = 1$, i.e., $d_s^{AE} = d_s^{EM} \quad \forall s \in \{U, D\}$. If $\pi_{AE} > \pi_{EM}$, then in equilibrium any private portfolio necessarily satisfies: $y_{AE} = \varphi_{j^*}$.*

When collateral constraints bind in equilibrium, demand for collateral gives rise to the strictly positive collateral premium (i.e., $\pi_{AE} > \pi_{EM}$), as in Lemma 1. Lemma 2 says that given a cheaper alternative Y_{EM}, any holdings of Y_{AE} are necessarily collateralized (i.e., leveraged) to the maximum, otherwise the agent would replace some y_{AE} with cheaper y_{EM}.

While Lemma 2 depends upon the simplifying assumption that Y_{AE} and Y_{EM} only differ in terms of collateral properties, it does provide some practical insights: the lemma suggests that the existence of higher-return alternatives for AE assets (e.g., EM assets with high credit ratings) creates a compelling incentive for private agents who hold AE assets to make extensive use of the collateral properties (e.g., low haircuts) of AE assets (e.g., through leverage, maturity transformation, or other types of collateralization activities).

Lemma 3 ***(International Portfolio Choice)*** *Suppose Y_{AE} and Y_{EM} deliver the same payoff vector at $t = 1$, i.e., $d_s^{AE} = d_s^{EM} \quad \forall s \in \{U, D\}$, and suppose $\pi_{AE} - \pi_{EM} > 0$. Then there are three different types of portfolios held by agents in equilibrium:*

- *Type 1 (**fully leveraged portfolio**): $y_{AE} = \varphi_{j^*} > 0$, $y_{EM} = 0$, and $\psi_{j^*} = \mu = 0$.*
- *Type 2 (**partially leveraged portfolio**): $y_{AE} = \varphi_{j^*} > 0$, $y_{EM} > 0$, and $\psi_{j^*} = \mu = 0$.*
- *Type 3 (**unleveraged portfolio**): $y_{AE} = \varphi_{j^*} = 0$, $y_{EM} \geq 0$, and $\psi_{j^*} + \mu \geq 0$.*

Intuitively, the Type 1 portfolio consists exclusively of fully leveraged (i.e., collateralized) Y_{AE}; in other words, the entire portfolio is leveraged. (Note that the condition $y_{AE} = \varphi_{j^*} > 0$ indicates binding collateral constraints, so that the entire holding of Y_{AE} is pledged as collateral for the issuance of financial claims.) The Type 2 portfolio is a combination of fully leveraged Y_{AE}, together with a positive amount of Y_{EM}; this type of portfolio is considered partially leveraged. The Type 3 portfolio is a combination of Y_{EM} and riskless assets, which does not involve any leverage. Lemma 3 characterizes all possible types of portfolios that private agents will *actually* hold in an equilibrium when $\pi_{AE} > \pi_{EM}$.

Figure 7 illustrates the payoff vectors associated with these three types of portfolios. The grey (shaded) region represents the feasible set of portfolio payoffs that agents can attain with the available assets (i.e., Y_{AE}, Y_{EM} and riskless assets). Ray $\overrightarrow{OB}$ (red) represents the payoff vector of holding a positive quantity of risky assets (i.e., Y_{AE} and Y_{EM}). Ray $\overrightarrow{OA}$ (green) represents the payoff vectors that can be attained by holding different amounts of riskless assets. (Note that ray $\overrightarrow{OA}$ is clockwise relative to ray $\overrightarrow{OB}$ under the assumption that the payoff of risky assets (i.e., Y_{AE} and Y_{EM}) is strictly higher in state U (vs. state D).) Vector $\overrightarrow{BC}$ indicates the payoff vector associated with issuing financial claims (i.e., debt). (Note that $\overrightarrow{BC}$ is parallel to $\overrightarrow{OA}$, since both represent changes in the amounts of riskless assets being held.) Ray $\overrightarrow{OC}$ (blue) represents the payoff vector of fully leveraged Y_{AE}; it is achieved by (i) holding Y_{AE} ($\overrightarrow{OB}$) and (ii) simultaneously issuing debt ($\overrightarrow{BC}$). Note that point C is on the leftward boundary of the attainable region (grey) because $\overrightarrow{BC}$ represents the maximum amount of riskless debt that any private agent can issue against the holding of $\overrightarrow{OB}$ (i.e., the collateral). (Vector $\overrightarrow{OC}$ delivers zero payoff in

state D, indicating that the amount of collateral is just enough to allow the debt to be repaid in state D, so that default does not occur.)

As can be observed, the shaded region is spanned by rays $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$; moreover, these three rays are linearly dependent, implying that each point within the shaded region can be achieved via non-unique combinations of individual assets. Nevertheless, Lemma 3 suggests that under binding collateral constraints, each point in the attainable region is achieved via a unique combination of individual assets. The three types of portfolios correspond to the three regions shown in Figure 7. Denote (w_U, w_D) as the coordinates of portfolio payoffs, we can define the three regions (in Figure 7) formally as the follows:

- Region 1 (***fully leveraged portfolio***): $w_D = 0$ and $w_U > 0$ (i.e., ray $\overrightarrow{OC}$).
- Region 2 (***partially leveraged portfolio***): $\frac{w_U}{w_D} > \frac{d_U^{AE}}{d_D^{AE}}$ and $w_D > 0$ (i.e., in between ray $\overrightarrow{OC}$ and $\overrightarrow{OB}$).
- Region 3 (**unleveraged portfolio**): $\frac{d_U^{AE}}{d_D^{AE}} \geq \frac{w_U}{w_D} \geq 1$ (i.e., in between ray $\overrightarrow{OB}$ and $\overrightarrow{OA}$).

Payoffs in Region 1, 2 and 3 are attained by Type 1, 2 and 3 portfolios, respectively. Lemma 3 basically asserts that payoffs are never obtained by combining rays $\overrightarrow{OC}$ and $\overrightarrow{OA}$ when there is a positive collateral value. Such a portfolio can be improved by reducing borrowing by one unit of j^* and reducing Y_{AE} by one unit (that is by reducing $\overrightarrow{OC}$ by one unit) and increasing Y_{EM} by one unit (that is by extending $\overrightarrow{OB}$ by one unit) and decreasing μ by j^* units (that is by reducing $\overrightarrow{OA}$ by j^* units). This improvement gives the same terminal payoffs (w_U, w_D) while saving the collateral value cost $\pi_{AE} - \pi_{EM}$.

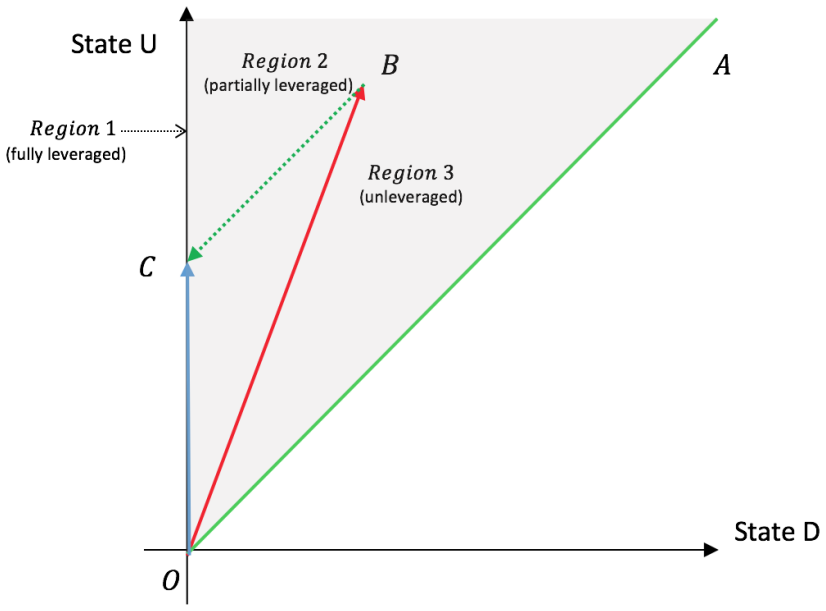


Figure 7: Payoffs of Type 1, 2 and 3 Portfolios

3.6 Effects of the AE Central Bank's Open-Market Purchases of Risky Assets (QE)

There are two primary channels that enable the AE central bank to generate real impacts in the model. The first channel is based on the fiscal consequences of the AE central bank's asset purchases. Since any earnings or losses from the AE central bank's balance sheet are ultimately transferred to domestic agents via taxation, one can think of the AE central bank's balance sheet as indirectly owned by AE agents.

The second channel is based on the AE central bank's ability to acquire Y_{AE} by borrowing without collateral constraints (i.e. with 100% LTV), because the central bank (unlike private agents) can credibly commit to repay (via tax collection) even if its promise exceeds collateral.

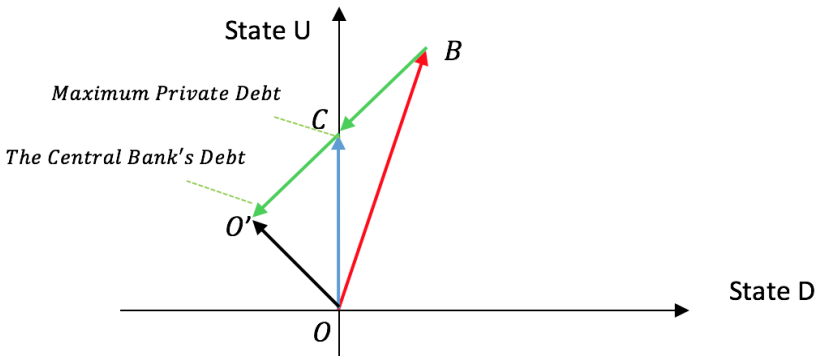


Figure 8: Portfolio Shift by QE

Figure 8 illustrates the effects of the AE central bank's purchase of Y_{AE} (which is entirely financed by issuing central bank reserves). As above, $\overrightarrow{OB}$ (red) represents the payoff vector of Y_{AE}, while $\overrightarrow{BO'}$ (green) represents the payoff vector of issuing central bank reserves. Thus, $\overrightarrow{OO'}$ (black) represents the effective payoff of the AE central bank's (i) purchase of Y_{AE} and (ii) simultaneous issuance of reserves. As previously noted, the AE central bank's balance sheet is indirectly owned by domestic agents; as a result, $\overrightarrow{OO'}$ will be indirectly forced upon the balance sheets of AE agents (based on their tax shares). ($\overrightarrow{OO'}$ points counter-clockwise relative to $\overrightarrow{OC}$ because the AE central bank can issue a greater amount of debt (vs. private agents) to finance its purchase of Y_{AE} (i.e., $|\overrightarrow{BO'}| > |\overrightarrow{BC}|$).) Note that the payoff of $\overrightarrow{OO'}$ in state D can be negative because the AE central bank is able to raise taxes in state D to pay for such losses.

Importantly, one can alternatively think of $\overrightarrow{OO'}$ as a combination of $\overrightarrow{OC}$ and $\overrightarrow{CO'}$: the AE central bank (i) purchases fully collateralized Y_{AE} (i.e., abiding by the same collateral requirement as the private agents), and (ii) issues additional debt (i.e., beyond the level of (private) collateralzied debt) to finance the entire purchase.

AE agents optimally adjust their portfolios in response to QE (i.e., when $\overrightarrow{OO'}$ is forced upon their balance sheets). Below, we illustrate three different situations in which AE

agents choose to (i) partially undo, (ii) overly undo or (iii) partially accommodate the AE central bank's asset purchases via QE.

Situation 1: Fully Leveraged AE Agents Partially Undo the AE Central Bank's Purchase of Y_{AE}

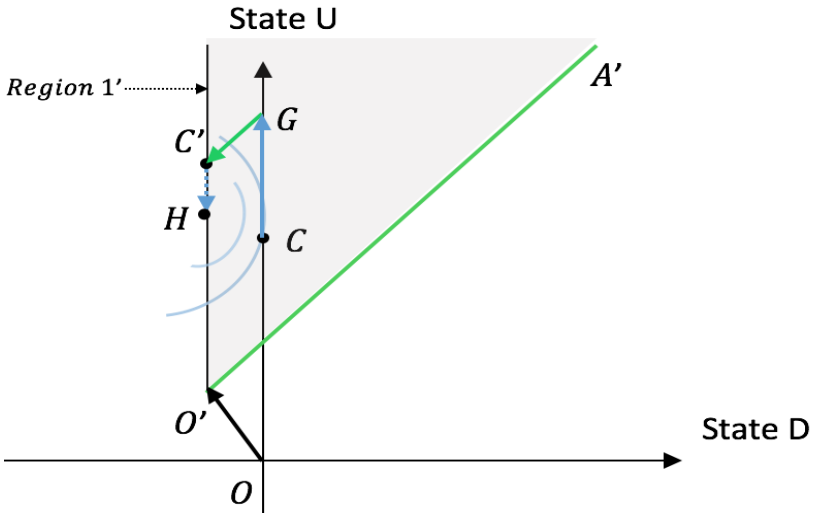


Figure 9: AE Agents Partially Undo the AE Central Bank's Purchase of Y_{AE}

Figure 9 illustrates the situation in which a (given) fully leveraged agent h adjusts its (Type 1) portfolio in response to QE. The indifference curves of (the indirect utility of) agent h are shown as ellipses in the figure. It is important to note that these indifference curves assume fixed prices and balanced budget (i.e., wealth is fully spent on consumption at time 0 and asset purchases). A move from point C to point G represents higher consumption in state U, the same consumption in state D, and (by balanced budget) lower consumption in state 0. Here, $\overrightarrow{OO'}$ represents agent h's share (i.e., based on its tax share) of the AE central bank's balance sheet. When forced upon agent h's balance sheet, $\overrightarrow{OO'}$ shifts the entire attainable set (of agent h) to the upper left.

In the case shown here, suppose that point C represents the portfolio payoff that gives agent h the highest possible (indirect) utility prior to the shift by $\overrightarrow{OO'}$. (Note that point C is not an unconstrained optimum; instead, it represents the highest indifference curve that agent h can attain before the shift (in Figure 7).) $\overrightarrow{OO'}$ moves agent h from point C to point C'. As previously noted, one can think of the move from point C to point C' as a result of (i) the AE central bank buying fully leveraged Y_{AE} ($\overrightarrow{CG}$) and (ii) issuing additional reserves ($\overrightarrow{GC'}$) to fully finance its purchase. With the shift created by $\overrightarrow{OO'}$, point H becomes the new constrained-optimum for agent h. Assuming fixed prices, agent h will adjust its portfolio by $\overrightarrow{C'H}$ in order to attain point H. The net trade $\overrightarrow{C'H}$ represents the hedging demand created by the AE central bank's asset purchases. Lemma 3 (below) shows that fully leveraged agent h will only partially undo the AE central bank's

purchase of Y_{AE} on its behalf (i.e., $|\overrightarrow{C'H}| < |\overrightarrow{CG}|$). In this particular situation, the AE central bank's purchases, together with the fully leveraged agent's responses, create an excess demand for Y_{AE}, thereby causing an upward pressure on π_{AE} in equilibrium.

Proposition 1 ***(Agents Partially Undo QE)*** *Assume fixed prices; if a given AE agent h's optimal portfolio is fully leveraged (Type 1), then one unit purchase of Y_{AE} by the AE central bank (on behalf of agent h) will lead to less than one unit sale of Y_{AE} by agent h (assuming that agent h's optimal portfolio remains fully leveraged after the AE central bank's purchases).*

Proof: Define agent h's marginal utility of income in state U as:

$$MUI_U^h = \frac{\partial u^h(c_U^h)}{\partial c_U^h} \cdot \frac{1}{p_U} \tag{13}$$

where

$$p_U = \frac{d_U^{AE} - d_D^{AE}}{\pi_{AE} - q_0 d_D^{AE}} \tag{14}$$

is the implicit price of consumption in state U (obtained as the ratio of the net payoff to the downpayment of the fully leveraged purchase of Y^{AE}). Similarly, define agent h's marginal utility of income at $t = 0$ as:

$$MUI_0^h = \frac{\partial u^h(c_0^h)}{\partial c_0^h} \tag{15}$$

At point C (prior to the shift by $\overrightarrow{OO'}$), agent h is unconstrained in state U; this implies that its marginal utility of income in state U is equal to that at $t = 0$ (i.e., $MUI_U^h = MUI_0^h$). Since Quantitative Easing (QE) trades are budget balanced, at fixed prices, $\overrightarrow{OO'}$ moves agent h's portfolio payoff from point C to C', resulting in an increase in agent h's consumption in state U but no change in consumption at time 0. This results in a decline in MUI_U^h relative to MUI_0^h; thus, agent h will optimally choose to (i) reduce its portfolio payoff in state U and (ii) simultaneously increase its period-0 income (in order to ensure the optimal condition $MUI_U^h = MUI_0^h$). By Lemma 3, agent h achieves such adjustment via selling fully leveraged Y_{AE} (i.e., $\overrightarrow{C'H}$), which lowers c_U^h and raises c_0^h. The new constrained-optimum H cannot be vertically as low as point C, since $MUI_U^h = MUI_0^h$ at point C. If point H were vertically as low or lower as C, then by budget balance consumption c_0^h at H would be higher than at C, which would result in $MUI_U^h > MUI_0^h$, which cannot be optimal for agent h. Thus, it follows that $|\overrightarrow{C'H}| < |\overrightarrow{GC}|$. □

Intuitively speaking, the central bank is doing for agent h what he would like to do, namely purchase Y^{AE} at 100% LTV; thus, agent h will only partially undo the Y_{AE} that is forced upon its balance sheet via QE.

Situation 2: Partially Leveraged AE Agents Overly Undo the AE Central Bank's Purchase of Y_{AE}, Selling More Y_{AE} than the Central Bank Buys

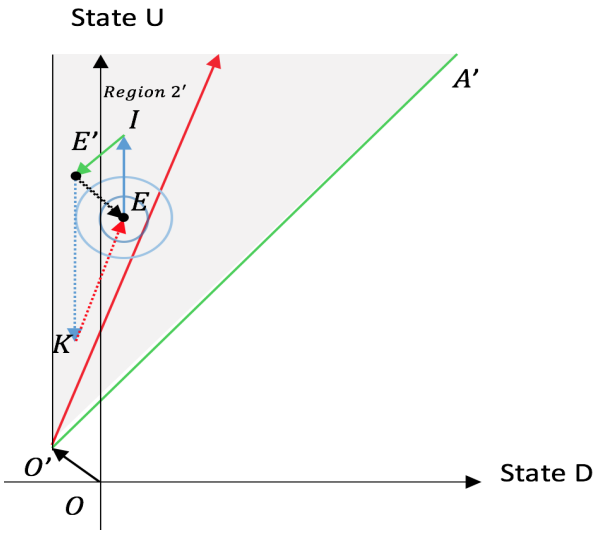


Figure 10: Agents Overly Undo the Central Bank's Purchase of Y_{AE}

Figure 10 illustrates an alternative case in which a given agent h chooses to "overly undo" the AE central bank's purchase of Y_{AE} (on behalf of agent h). In the case shown here, assume that point E represents the portfolio payoff that gives agent h the highest possible (indirect) utility prior to the shift by $\overrightarrow{OO'}$. (Note that point E (i) represents the payoff of a partially leveraged (i.e., Type 2) portfolio and (ii) is the unconstrained optimum of agent h, since it is strictly in the interior of the feasible set.) $\overrightarrow{OO'}$ shifts the entire feasible set to the upper left; in particular, it moves agent h from point E to point E'. To best visualize how agent h undoes the AE central bank's purchase of Y_{AE}, one can think of the move from point E to point E' as a combination of $\overrightarrow{EI}$ and $\overrightarrow{IE'}$.

Assume both prices and the wealth of agent h are fixed. Agent h can adjust its portfolio by $\overrightarrow{E'E}$ in order to return back to point E, thus fully hedging against the AE central bank's purchase of Y_{AE}, if he (i) sells some fully leveraged Y_{AE}, moving from E' to K, and (ii) buys some Y_{EM}, moving from K to E. In this particular situation, agent h overly undoes the AE central bank's purchase of Y_{AE} (i.e., by selling more Y_{AE} than the the AE central bank buys on behalf of agent h). This adjustment leaves agent h with extra wealth, which he will spend on increasing c_0, c_U, and c_D. In the end he will still overly undo the central bank purchase of Y_{AE}. This creates a downward pressure on π_{AE} in equilibrium.

In order to prove why this should be so, we recall the definition of loan to value (LTV) and collateral rate (CR) for a fully leveraged purchase of Y^{AE}:

$$LTV = \frac{q_0 d_D^{AE}}{\pi_{AE}} < 1;\ CR = \frac{1}{LTV} = \frac{\pi_{AE}}{q_0 d_D^{AE}} = 1 + \alpha > 1$$

Proposition 2 ***(Agents Overly Undo QE)*** *Assume fixed prices; if a given agent h's optimal portfolio is partially leveraged (Type 2), then one unit purchase of Y_{AE} by the AE central bank (on behalf of agent h) will lead to more than one unit sale of Y_{AE} by agent h and to a purchase of Y_{EM} (assuming that agent h's optimal portfolio remains partially leveraged after the AE central bank's purchases).*

Proof:

Step 1: If agent h returns to point E after the QE shift, then h must be selling $CV = 1 + \alpha > 1$ units of Y_{AE}.

The central bank is buying 1 unit of Y_{AE} and selling the same value of bonds, namely $\frac{\pi_{AE}}{q_0}$ bonds. Let agent h buy back $\frac{\pi_{AE}}{q_0}$ bonds from those he himself already issued. That frees him to sell $CV = 1+\alpha > 1$ units of Y_{AE}, since he no longer needs them as collateral for the bonds he no longer issues. Finally, let h buy α units of Y_{EM}. Agent h has thus in total sold 1 unit of risky assets, and purchased $\frac{\pi_{AE}}{q_0}$ bonds, precisely undoing what the central bank did, and returning him to point E. But notice that because the central bank operation broke even, agent h has actually saved $\alpha(\pi_{AE} - \pi_{EM}) > 0$ dollars by replacing α units of Y_{AE} with α units of Y_{EM}.

Step 2: Taking into account the wealth effects of portfolio adjustment, h must be selling more than one units of Y_{AE}.

We must check now that after returning to point E with extra money $\alpha(\pi_{AE}-\pi_{EM}) > 0$, agent h will not want to buy back at least α units of Y_{AE}. Because agent h has additively separable utility, at the fixed prices, he will spend the extra wealth in a way that will increase consumption beyond the levels at point E in all three states. By Lemma 3, if the agent remains partially leveraged in region 2, then agent h will spend a positive amount on c_0^h and a positive amount on Y_{EM}. It follows that agent h will spend strictly less than $\alpha(\pi_{AE} - \pi_{EM})$ on fully leveraged purchases of Y_{AE}. But each such purchase requries a downpayment of $\pi_{AE} - q_0 d_D^{AE} > \pi_{AE} - \pi_{EM}$. (The last inequality holds because Y_{EM} pays exactly the same as $d_D^{EM} = d_D^{AE}$ bonds in the down state, and strictly more in the up state, and hence must have greater value, $\pi_{EM} > q_0 d_D^{AE}$.) Hence agent h will buy strictly less than α units of Y_{AE} with his extra wealth of $\alpha(\pi_{AE} - \pi_{EM})$. Combing steps 1 and 2 leaves agent h selling more than one unit of Y_{AE} for every unit the central bank buys. □

Proposition 2 is a surprising result, as it shows the possibility of QE causing a downward pressure on AE asset prices, and upward pressure on EM asset prices. The intuition is as follows. Partially leveraged agents hold a portfolio that reflects a balance between the advantages of being able to leverage, by holding Y_{AE}, and getting higher return, by holding Y_{EM}. The central bank forces on them more 100% leveraged Y_{AE}, or equivalently, more leveraged (at private margins) Y_{AE} plus some borrowing. Partially leveraged agents can simply undo the new Y_{AE} purchases at private margins by unwinding the exact same number from their own portfolios. The additional borrowing forced upon them tips the balance by reducing the need for Y_{AE} in order to borrow. Therefore they sell even more

Y_{AE} and substitute higher yielding Y_{EM}.

Situation 3: Unleveraged AE Agents Partially Accommodate QE by Selling EM Assets and Absorbing the Increased Supply of Central Bank Reserves

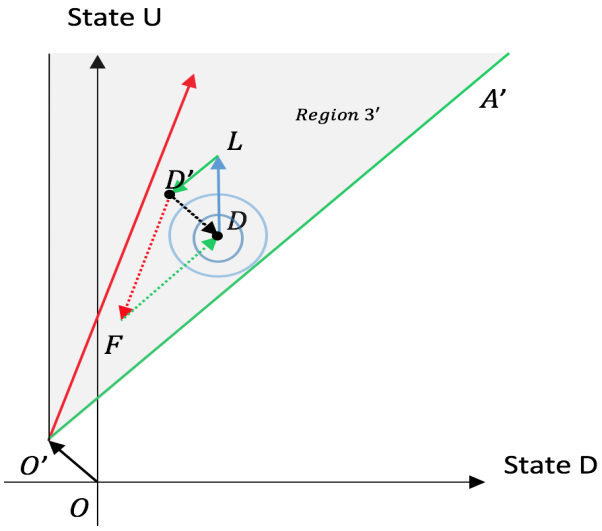


Figure 11: AE Agents Sell EM assets and Absorb Central Bank Reserves

Figure 11 illustrates the situation in which a given unleveraged agent h adjusts its (Type 3) portfolio in response to the AE central bank's purchase of Y_{AE}. In the case shown here, suppose that point D represents the portfolio payoff that gives agent h the highest (indirect) utility prior to the shift by $\overrightarrow{OO'}$. $\overrightarrow{OO'}$ moves agent h's portfolio payoff from point D to D'. As illustrated in Figure 8, the shift $\overrightarrow{DD'}$ can be thought of as a combination of $\overrightarrow{DL}$ and $\overrightarrow{LD'}$. Assuming prices are fixed, agent h can return back to point D via a combination of (i) selling one unit of Y_{EM} and (ii) buying a commensurate amount of riskless assets. But this will leave him with a budget shortfall. Proposition 3 shows that the upshot is that agent h sells more than one unit of Y_{EM}. In this particular situation, the AE central bank's asset purchases create (i) an excess demand for Y_{AE} (under fixed prices) and (ii) an excess supply of Y_{EM} and thus an upward pressure on $\pi_{AE} - \pi_{EM}$ and r in equilibrium.

Proposition 3 ***(Agents Partially Accommodate QE)*** *Assume fixed prices and assume that agent utilities $u^h(c_0^h, c_U^h, c_D^h)$ are generated by CRRA vNM utilities $v^h(c) = \frac{c^{\gamma_h}}{\gamma_h}, \gamma_h < 1$. If a given agent h's optimal portfolio is unleveraged (Type 3) and and he holds strictly positive amounts of Y_{EM}, then for every unit of Y_{AE} that the AE central bank buys (by issuing $\frac{\pi_{AE}}{q_0}$ units of riskless assets) on behalf of agent h, agent h will concurrently (i) sell more than one unit of Y_{EM} and (ii) buy less than $\frac{\pi_{AE}}{q_0}$ units of riskless assets (assuming that agent h's optimal portfolio remains unleveraged after the AE central bank's purchases).*

Proof:

At unchanged prices, the agent can get back to the same porfolio payoffs by purchasing $\frac{\pi_{AE}}{q_0}$ units of riskless assets and selling one unit of Y^{EM}. Since the central bank intervention broke even, that portfolio adjustment will leave agent h with a wealth loss of $\pi_{AE} - \pi_{EM}$. By homotheticity, he will deal with that by reducing all his holdings of c_0, Y^{EM}, B proportionately. Hence he will end up selling more than one unit of Y_{EM} and buying less than $\frac{\pi_{AE}}{q_0}$ units of riskless assets. □

Intuitively, unleveraged agent h is marginally indifferent between risky Y_{EM} and riskless assets (at its optimum) prior to QE; thus, for every unit of risky Y_{AE} that the AE central bank forces upon agent h's portfolio (via QE), agent h will optimally choose to sell some risky Y_{EM}, while buying some riskless assets. In this regard, unleveraged agents are the "natural" absorbers of the increased supply of central bank reserves. However, as the proof shows, agent h does not fully absorb the AE central bank's issuance of central bank reserves; thus, *ceteris paribus*, QE creates an excess supply of riskless assets and a simultaneous upward pressure on the real interest rate in equilibrium.

4 A Two-Country Monetary Model with Endogenous Collateral Constraints: Numerical Analysis

We illustrate the workings of the two-country collateral-equilibrium monetary model via a simple numerical example. The example displays the collateral channel for international capital flows and the heterogeneous responses of heterogeneous agents that is reminiscent of the financial and nonfinancial firms in the data. It also displays the reversal in the effects of QE as agents who were exclusively leveraged holders of AE assets eventually respond to the QE-induced yield advantage of EM assets over AE assets by diversifying into EM assets. A substantial number of AE agents begin with Type 1 portfolios, and as QE expands they shift into Type 2 portfolios, and then eventually Type 3 portfolios.

Section 4 is organized as the follows: Section 4.1 describes numerical specifications. Section 4.2 illustrates how capital flows arise as a result of the asymmetric collateral properties between Y_{AE} and Y_{EM}; this is attained by comparing the Autarky equilibrium with the Free-Trade equilibrium. Finally, Section 4.3 examines the effects of QE on cross-border asset prices and asset-holding patterns. We show that under certain conditions, the four stylized facts (described earlier) can emerge naturally in the collateral equilibrium of the model.

4.1 Numerical Specifications

As above, there are three states $0, U$, and D. Assume there are two types of agents in each country: $h = 1, 2$ in AE and $h^* = 3, 4$ in EM. All agents have a simple homothetic

utility function and common beliefs about future states:

$$u^h(c^h) = log(c_0^h) + \frac{1}{2}log(c_U^h) + \frac{1}{2}log(c_D^h) \tag{16}$$

Agent endowments are given as the following:

AE:

Agent 1: $(e^1_{C_0}, e^1_{Y_{AE}}, e^1_B, e^1_{c_U}, e^1_{c_D}) = (3.5, 0, 0.0005, 0, 3)$
Agent 2: $(e^2_{C_0}, e^2_{Y_{AE}}, e^2_B, e^2_{c_U}, e^2_{c_D}) = (3.5, 7, 0.0005, 15, 3)$

EM:

Agent 3: $(e^3_{C_0}, e^3_{Y_{EM}}, e^3_B, e^3_{c_U}, e^3_{c_D}) = (3.5, 0, 0.0005, 0, 3)$
Agent 4: $(e^4_{C_0}, e^4_{Y_{EM}}, e^4_B, e^4_{c_U}, e^4_{c_D}) = (3.5, 7, 0.0005, 15, 3)$

Payoff vectors of Y_{AE} and Y_{EM} are given by: $\{d_U^{AE}, d_D^{AE}\} = \{d_U^{EM}, d_D^{EM}\} = \{15/7, 6/7\}$.
Tax shares of agents are: $(\theta^1, \theta^2) = (0.9, 0.1)$ and $(\theta^3, \theta^4) = (0.9, 0.1)$.[25]

For ease of illustration, we introduce the additional notation ω, with $\omega = \frac{y_{AE}^{CB}}{e^1_{Y_{AE}} + e^2_{Y_{AE}}}$, which represents the share of Y_{AE} acquired by AE central bank via QE. Our focus is on how variations in ω affect equilibrium asset prices and asset-holding patterns.

Monetary specifications is therefore given by ω (variable for simulation)).

Under the aforementioned numerical specifications, heterogeneity among agents stems from their distinct endowment streams. However, as previously noted, heterogeneity can also arise from a myriad of other channels (e.g., heterogeneous beliefs about future states and heterogeneous risk aversions), which can generate qualitatively similar results.

In this particular example, both agent 1 (in the AE) and agent 3 (in the EM) are collateral-constrained and thus natural buyers of Y_{AE} for two main reasons. First, agent 1 and 3 have asymmetric endowments across the two states at $t = 1$ (i.e., they are much poorer in state U vs. state D). Risky assets deliver more in state U (vs. D), thus providing a good hedge against the endowment risk of agent 1 and 3. Second, agent 1 and 3 have no initial endowment of risky assets and thus want to purchase as many risky assets as possible via issuances of financial claims. Since only Y_{AE} can serve as collateral (to secure the issuance of financial claims), agent 1 and 3 strictly prefer purchasing Y_{AE} with collateralized borrowing than purchasing Y_{EM} with cash, so long as π_{AE} is not too much higher than π_{EM}. Agent 2 and 4, on the other hand, are unconstrained (since they are endowed with abundant risky assets).

[25] As in Araujo, Schommer and Woodford, asymmetric tax shares are meant to (i) amplify the effects of QE and (ii) facilitate the illustration of simulation results.

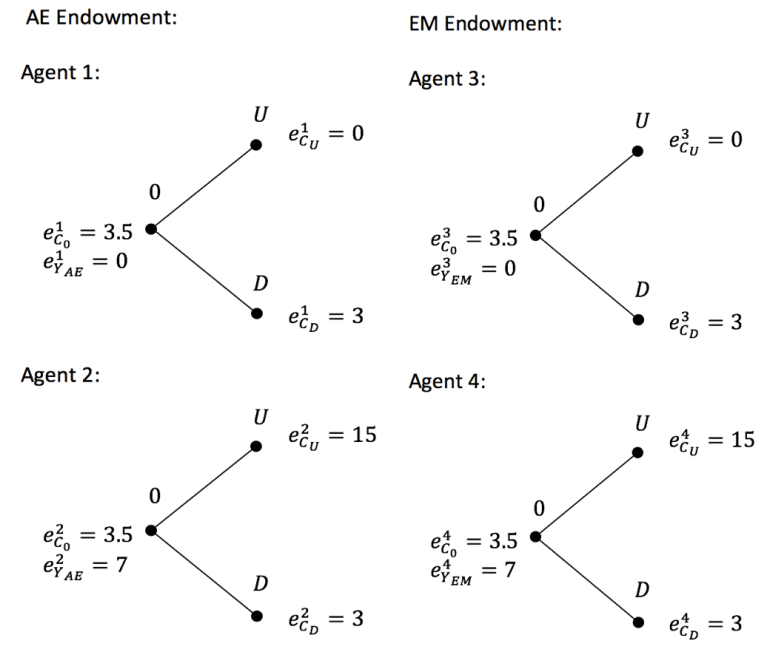


Figure 12: Graphical Illustration of Agent Endowments.

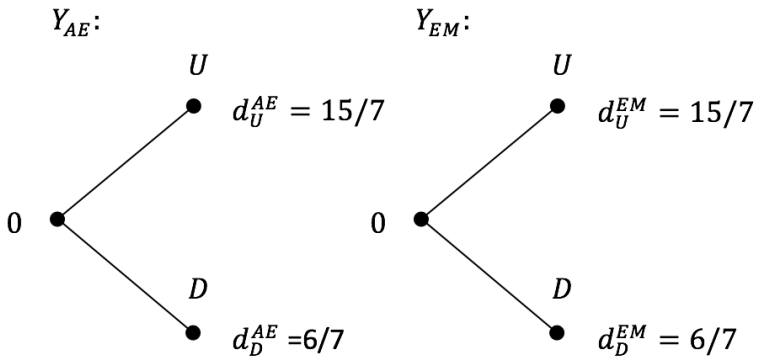


Figure 13: Graphical Illustration of Risky Asset Payoffs.

Note that EM endowments are symmetric to those in the AE (with agent 3 mirroring agent 1 and agent 4 mirroring agent 2); however, agent 2 and 4 are endowed with risky assets that have different collateral capacities.

4.2 Financial Integration and Capital Flows: $\omega = 0$

Prior to analyzing the financial spillover effects of QE, it is helpful to illustrate the underlying driving forces behind the capital flows in the model. We do so by comparing equilibrium outcomes (i.e., Autarky vs. Free Trade).

Table 3 shows the asset-holding patterns when AE and EM are not allowed to trade their assets. While both agent 1 and 3 are the natural buyers of risky assets, agent 1 (vs. agent 3) can afford to buy many more risky assets because only Y_{AE} can serve as

collateral (to secure a financial claim). Agent 3 can only purchase a small amount of Y_{EM} (in the absence of collateralized borrowing).

Table 3: Asset Holdings under Autarky

	Y_{AE}	Y_{EM}
Agent 1	3.89	-
Agent 2	3.11	-
Agent 3	-	1.23
Agent 4	-	5.77

Table 4 shows the asset-holding patterns when the AE and EM are open for trade. Under Free Trade, both agent 1 and 3 can purchase Y_{AE} with collateralized borrowing; thus, each holds half of the total supply of Y_{AE}. Unconstrained agent 2 and 4, on the other hand, split the supply of cheaper Y_{EM}. (Note that agent 2 is initially endowed with the more expensive Y_{AE} and thus has greater initial wealth than agent 4; this allows agent 2 to acquire a greater amount of Y_{EM} in equilibrium.)

Table 4: Asset Holdings under Free Trade

	Y_{AE}	Y_{EM}
Agent 1	3.5	0
Agent 2	0	3.58
Agent 3	3.5	0
Agent 4	0	3.42

Table 5 compares risky asset prices relative to bond prices under Autarky and Free Trade. Under Autarky, Y_{AE} is more expensive than Y_{EM} relative to their respective bond prices, because Y_{AE} can serve as collateral—resulting in additional collateral value[26]; with free trade, EM demand for collateral increases π_{AE}/q_0^{AE} further, while simultaneously making Y_{EM} more attractive to unconstrained agents. Surprisingly, free trade raises π_{EM}/q_0^{EM} more than π_{AE}/q_0^{AE}.[27]

4.3 Effects of QE on Cross-Border Asset Prices and Asset Holdings: $\omega > 0$

The aforementioned numerical exercises illustrate equilibrium outcomes in the absence of QE intervention (i.e., $\omega = 0$). We now consider the effects of the AE central

[26] We focus on the risky asset prices *relative to* bond prices because the ratio captures the cost of borrowing in different scenarios. Under Autarky, the riskless interest rates in the two economies are different; $q_0^{AE} < q_0^{EM}$ because Y_{AE} can serve as collateral to support the issuance of private financial promises, resulting in greater supply of bonds in AE.

[27] This is similar to the computational results in Fostel, Geanakoplos and Phelan (2017).

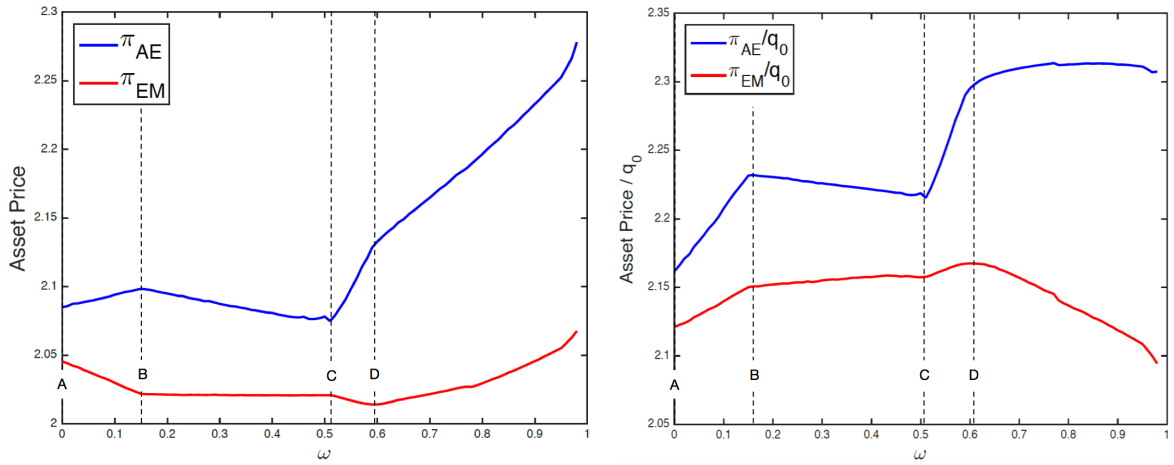
Table 5: Risky Asset Prices: Autarky vs Free Trade

	Autarky	Free Trade
π_{AE}/q_0^{AE}	2.332	2.378
π_{EM}/q_0^{EM}	2.186	2.333
π_{AE}	2.056	2.085
π_{EM}	2.105	2.045

bank's purchase of Y_{AE} with $\omega > 0$. The simulation results provide a way to jointly interpret the qualitative patterns of the empirical facts described earlier.

Interpreting Empirical Facts in the Model:

The model simulation in Figure 14 illustrates how free trade risky asset prices π_{AE} and π_{EM} respond to QE as ω increases linearly from 0 to 1. In Figure 14, as ω increases from point A to B, π_{AE} also increases. However, further increase in ω (beyond point B) results in a decline in π_{AE}. This is qualitatively consistent with Fact 1(a) and 1(b). In this scenario, the Federal Reserve's early purchase of long-term Treasury securities was accompanied by a decline in the long-term Treasury yield; however, ongoing purchases were conversely accompanied by an rise in the long-term Treasury yield. Below, we further clarify the mechanism that enables this pattern to arise in the model.



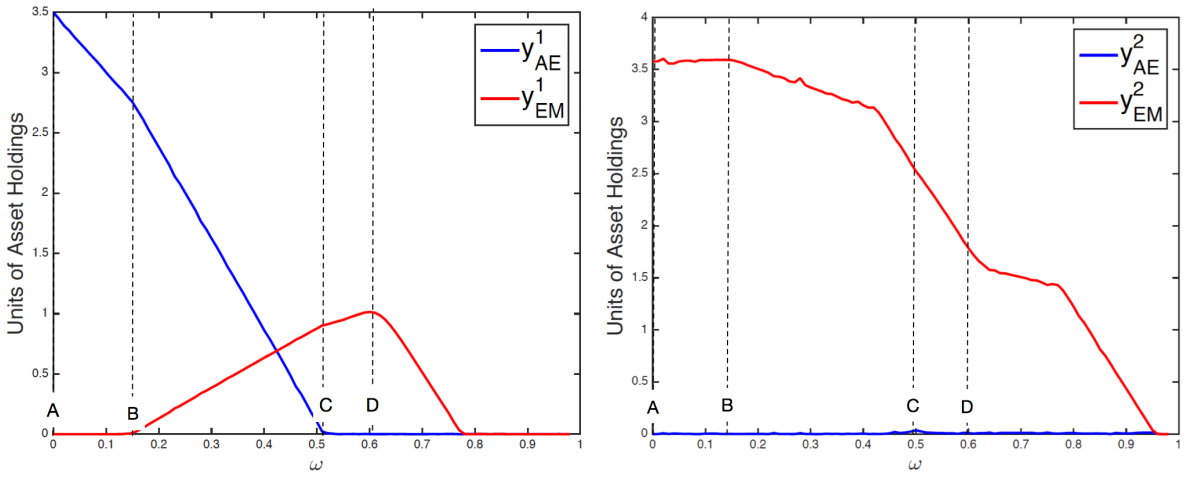
(a) (b)

Figure 14: Effects of QE on Risky Asset Prices

Rationalization of Fact 1(a): *increase in π_{AE} from point A to B*

As ω increases from point A to B, the AE central bank effectively buys Y_{AE} on behalf

of AE agents 1 and 2, leveraging[28] more than any private agents could because the AE central bank can commit to repay (even if its debt exceeds the collateral). This increases the demand for Y_{AE} and its price because at this stage, agent 1 is fully leveraged (with a Type 1 portfolio), and thus only partially undoes the Y_{AE} that the AE central bank forces upon its portfolio; this is shown by Situation 1 and Proposition 1. Overall, the AE central bank's purchases create an excess demand for Y_{AE}, thereby causing an upward pressure on π_{AE} in equilibrium. This gives rise to the increase in π_{AE} from point A to B. Meanwhile, agent 2 is unleveraged (with a Type 3 portfolio), and partially accommodates the AE central bank's purchase of Y_{AE} (on behalf of agent 2) by (i) selling its holdings of Y_{EM} and (ii) absorbing the increased supply of central bank reserves (as shown in Situation 3 and Proposition 3). This results in a decline in π_{EM} from point A to point B.



(a) AE: Agent 1

(b) AE: Agent 2

Figure 15: QE and Effects on the Asset Holdings of AE Agents

Rationalization of Fact 1(b): *decrease in π_{AE} from point B to C*

Fact 1(b) shows that the Federal Reserve's continued purchases of long-term Treasury securities (since July 2012) were accompanied by an increase in the long-term Treasury yield and its risk premium. The model suggests that one potential contributing factor (to the observed increase) in long-term Treasury yield and its risk premium could have been AE agents' portfolio shifts of international assets (in response to QE).

In the simulation, following a sufficiently large purchase of Y_{AE} (beyond point B) by the AE central bank, agent 1 switches from a fully leveraged (Type 1) portfolio into a partially leveraged (Type 2) portfolio. (This can be visualized in Figure 9: sufficiently

[28] As previously noted, the AE central bank's purchase of Y_{AE}, financed entirely by issuing reserves, is a form of leveraged purchase with a 100% loan-to-value ratio (LTV).

large shift by $\overrightarrow{OO'}$ can relocate the optimum of agent h from one region to another.) This puts agent 1 into Situation 2 (beyond point B); thus, agent 1 optimally chooses to overly undo the AE central bank's purchases by (i) selling more Y_{AE} than the AE central bank buys and (ii) buying cheaper Y_{EM} (i.e., by Proposition 2). Meanwhile, agent 2 continues to accommodate QE by (i) selling EM assets and (ii) absorbing some of the increased supply of central bank reserves. Overall, AE agents sell Y_{AE} sufficiently fast that it causes a decline in π_{AE} from point B to C.

Reinterpreting Treasury Yield Changes as a Result of Varying Risk Premium: *Variations in the Risk Premium of* Y_{AE}

One way that QE affects long-term interest rates is via direct impacts on the term (risk) premium[29]. Figure 1 suggests that changes in the 10-year Treasury yield (from November 2010 to November 2014) have been partly driven by variations in the term (risk) premium. The Federal Reserve's early purchases of long-term Treasury securities corresponded with a decline in the term premium of 10-year Treasury; however, subsequent purchases were conversely accompanied by a slight increase in the term premium.

In our model, one can similarly interpret the impacts of QE on π_{AE} as transmitted via its effects on the risk premium of Y_{AE}. Below, we compute the risk premium of Y_{AE} and show its non-monotonic response to QE in the model.

Define the expected real rate of return on riskless bond as:

$$1 + r = \frac{\frac{1}{2} + \frac{1}{2}}{q_0} \tag{17}$$

Similarly, define the expected real rate of return on Y_{AE} as:

$$1 + r^{AE} = \frac{\frac{1}{2} d_U^{AE} + \frac{1}{2} d_D^{AE}}{\pi_{AE}} \tag{18}$$

The risk premium of Y_{AE} can thus be defined as $r^{AE} - r$. Figure 16 shows QE's effects on the risk premium of Y_{AE}. QE lowers the risk premium of Y_{AE} (from point A to B) because AE agents only partially undo the AE central bank's purchase of Y_{AE}; this enables the AE central bank to (i) create an excess demand for Y_{AE} (under fixed prices) and (ii) push down the risk premium of Y_{AE} in equilibrium. However, continued QE (beyond point B) induces portfolio shifts towards cheaper Y_{EM} (that will more than offset the AE central bank's purchase of Y_{AE}); this results in an increase in the risk premium of Y_{AE}.

[29] See, for example, Krishnamurthy and Vissing-Jorgensen (2011) for a discussion on the risk premium channel of QE's transmission.

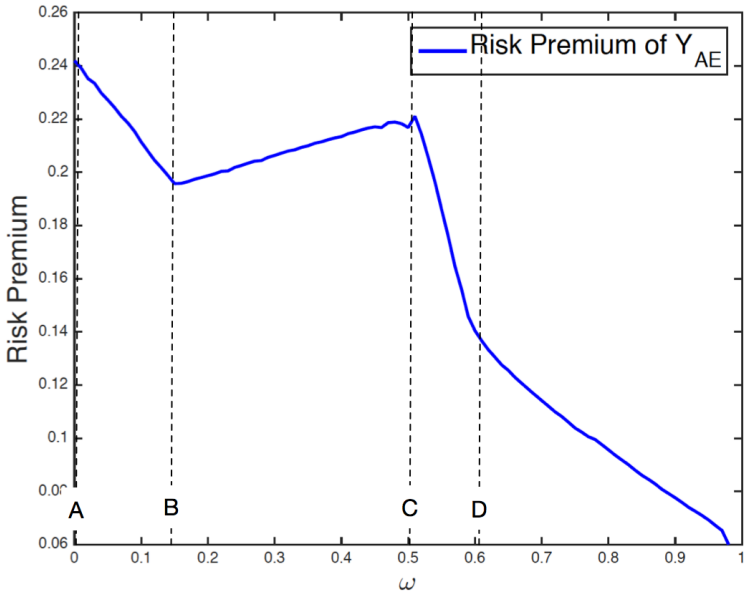


Figure 16: Effects of QE on the Risk Premium of Y_{AE}

Rationalization of Fact 2: ***variation in*** $\pi_{AE} - \pi_{EM}$ ***from point A to C;***

Fact 2 shows (i) a widening yield spread between the United States and EMs during the Federal Reserve's early purchases of long-term Treasury securities and (ii) a subsequent tightening of the spread in response to continued asset purchases. The model suggests that the private sector's portfolio shifts could have contributed to such financial spillover patterns.

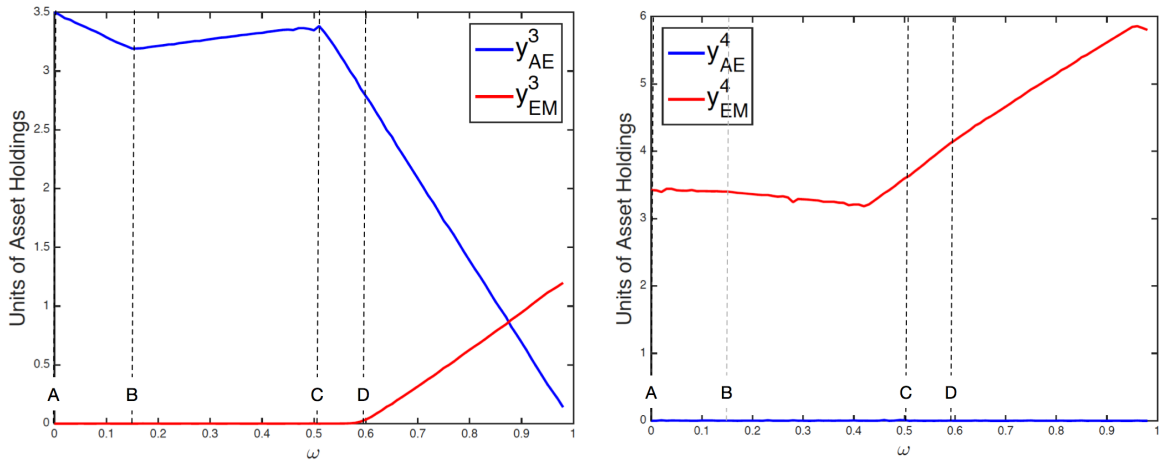
As previously noted, the AE central bank's early purchase creates an excess demand for Y_{AE} (from point A to B) and raises π_{AE} relative to π_{EM} in equilibrium (i.e., widening the spread between AE and EM asset prices). However, continued QE (beyond point B) induces AE agents to (i) sell Y_{AE} faster than the AE central bank buys and (ii) increase the net purchase of Y_{EM}. This type of portfolio adjustment increases the relative demand for Y_{EM} (vs. Y_{AE}) and thus tightens the spread.

Rationalization of Fact 3: ***heterogeneous variation in*** y^1_{EM} ***and*** y^2_{EM}

Fact 3 shows that U.S. financial and non-financial organizations had opposite responses to QE in their holdings of long-term foreign government bonds. Financial firms increased their purchases of foreign assets, while non-financial firms sold foreign assets. The model interprets such heterogeneous asset-holding patterns as driven by the distinct financial situations faced by U.S. entities (e.g., whether they are leveraged or unleveraged).

In this context, one can simplistically think of leverage-constrained agent 1 as representing U.S. financial organizations and agent 2 as representing U.S. non-financial organizations. In the simulation, agent 1 starts off (at $\omega = 0$) with a large share of Y_{AE} due

to demand for leverage. As seen in Figure 15, sufficiently large QE induces agent 1 to (i) quickly sell off leveraged Y_{AE} and (ii) buy significant amounts of Y_{EM}. (This mechanism is analytically shown by Proposition 2.) By contrast, agent 2 starts off (at $\omega = 0$) with a significant share of Y_{EM}, but responds to QE by (i) selling Y_{EM} and (ii) absorbing part of the increased supply of central bank reserves. Such model-based portfolio responses (of agent 1 and 2) are qualitatively consistent with Fact 3.



(a) EM: Agent 3 (b) EM: Agent 4

Figure 17: Effects of QE on Asset Holdings in EM

Rationalization of Fact 4: *stable* y^3_{AE} *from point A to C*

Fact 4 shows that throughout the QE episodes, foreign demand for U.S. Treasury securities have remained relatively stable. Our model rationalizes such persistent demand as driven by the continued EM demand for collateral in response to QE.

Recall that the AE central bank's balance sheet is indirectly owned by domestic agents; thus, the direct impacts of the AE central bank's asset purchases are upon AE agents. In the model, expectations about the fiscal consequences of QE induce AE agents to adjust their portfolios quite radically. By contrast, EM agents do not bear such fiscal consequences; as a result, collateral-constrained agent 3, in the EM, continues to have a strong demand for Y_{AE}. Figure 17(a) shows that y^3_{AE} declines initially from point A to B due to a rise in π_{AE}; however y^3_{AE} (i) increases again (from point B to C) as soon as π_{AE} declines and (ii) stays relatively stable from point A to C.

Beyond Point C: Hypothetical Scenarios

At point C, the AE central bank has acquired so much Y_{AE} (more than 50% of total Y_{AE}) that agent 1 in AE has completely sold off its holdings of Y_{AE} (the shift by $\overrightarrow{OO'}$ is sufficiently large that agent 1's optimum is located in Region 3'). Beyond point C, the AE central bank competes directly with agent 3 (in the EM) for Y_{AE}, resulting in (i) a monotonic decline in y^3_{AE} (due to shrinking supply of Y_{AE}) and (ii) a large increase in π_{AE} relative to π_{EM} (i.e., a sharp decline in the risk premium of Y_{AE}). At point D, the gap between π_{AE} and π_{EM} is so large that even agent 3 is induced to hold Y_{EM} beyond this point.

This implies that despite the possible decline in π_{AE} (e.g., from point B to C), sufficiently aggressive QE does eventually cause a rise in π_{AE} again. However, as the next section shows, too large of a QE can have dramatic consequences on (i) the welfare of private agents and (ii) the redistribution of wealth.

5 Welfare Consequences of QE

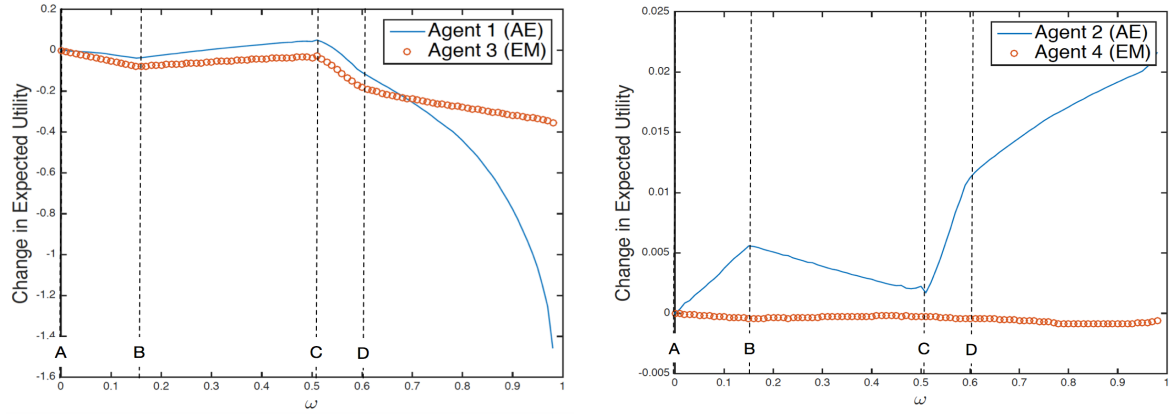
The above analysis highlights the heterogeneous responses of the private agents to QE, conditional on the particular type of portfolio they hold. Figure 18 tracks QE's welfare impacts in the model, which similarly shows significant heterogeneity across agents.

As shown in Figure 18(a), both agent 1 and 3 experience a decline in welfare from point A to B. Over this segment, both agents face binding collateral constraints and are unambiguously hurt by the rise in the price of collateral π_{AE}. However, the welfare of agent 1 (vs. agent 3) decreases to a lesser extent (despite the increase in π_{AE}) because a portion of the AE central bank's acquisition of Y_{AE} is indirectly owned by agent 1, which alleviates agent 1's collateral constraints. Overall there is net decline in the welfare of agent 1 due to the dominating price effect.[30] The welfare of agents 1 and 3 increases from point B to C as π_{AE} decreases over this segment.

Agent 2 (in the AE) enjoys a sharp increase in welfare from point A to B due to the rise in π_{AE} that results in a simultaneous increase in agent 2's wealth (via the value of initial endowments). However, the welfare of (i) agent 2 decreases from point B to C as a result of the decline in π_{AE} and (ii) agent 4 remains almost unchanged in response to QE.

Moderately aggressive QE (halfway between B and C) raises the welfare of both agents 1 and 2 in AE at the expense of agents in EM (compared to no QE: $\omega = 0$). This improvement of AE's welfare via its central bank intervention is not general; it only occurs over a limited range of interventions ω.

[30] This resembles the recent study by Rappoport (2016) that shows how mortgage subsides can possibly lower the welfare of borrowers: while debt subsidies ease borrowers' access to home loans, the associated increase in house price can counteract the beneficial effects of the subsidies.



(a) Agent 1 and 3

(b) Agent 2 and 4

Figure 18: Effects of QE on Welfare

Sufficiently aggressive QE (beyond point C) results in dramatic changes in the welfare of private agents. Agent 1 experiences a drastic decline in welfare because of (i) the large amounts of Y_{AE} that is forced upon its balance sheet via QE and (ii) short-selling of Y_{AE} is not allowed. Agent 2, on the other hand, experiences a sharp increase in welfare via the wealth channel; the rise in π_{AE} increases the value of agent 2's initial endowments substantially.

An implication of the aforementioned welfare analysis is that QE is likely to generate heterogeneous welfare effects and result in a redistribution of wealth. For instance, to the extent that QE does raise AE asset prices, QE may benefit owners of AE assets by increasing the value of their initial assets. However, higher AE asset prices as a result of QE may lower the welfare of investors who (i) demand AE assets as collateral for leverage or (ii) face financial constraints (e.g., short-selling constraints) to hedge against the AE central bank's purchases.

6 Conclusion

This paper proposes a framework to study potential channels for the international transmission of QE, with a particular emphasis on collateral and heterogeneity. The framework differs from conventional open macroeconomic models in several important aspects. First, all privately issued financial claims (i.e., private borrowing) must be secured by collateral and the collateral requirement is determined endogenously in equilibrium. Second, economies in the two-country setup (i.e., the AE and EM) differ in the collateral property of their assets; furthermore, capital flows arise as a result of international sharing of scarce collateral. Third, monetary policy consists of two dimensions: (i) conventional interest-rate policy and (ii) unconventional variation in the size and composition of the central bank's balance sheet. We consider the consequences of the AE central bank's pur-

chases of an asset that is utilized internationally as collateral to secure privately issued financial claims.

The model shows that it is indeed possible for the AE central bank's asset purchases (via QE) to increase AE asset prices in equilibrium; however, ever-larger QE can possibly result in a decline in AE asset prices due to dramatic portfolio shifts of international assets. Crucially, the model suggests that even within an AE, agents respond differently to QE due the the different kinds of financial situations that they face. While some AE agents choose to hedge against QE by replacing AE assets with EM counterparts, others voluntarily choose to absorb the increased supply of central bank reserves. The net effects of such portfolio adjustment matter significantly for the effectiveness of QE on asset prices. Overall, the study highlights the importance of understanding how private agents—both domestic and international—interact with central-bank interventions to generate equilibrium market outcomes.

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