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By

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# Aggregation of Diverse Information with Double Auction Trading among Minimally-Intelligent Algorithmic Agents

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## Abstract

Information dissemination and aggregation are key economic functions of financial markets. How intelligent do traders have to be for the complex task of aggregating diverse information (i.e., approximate the predictions of the rational expectations equilibrium) in a competitive double auction market? An apparent ex-ante answer is: intelligent enough to perform the bootstrap operation necessary for the task—to somehow arrive at prices that are needed to generate those very prices. Constructing a path to such equilibrium through rational behavior has remained beyond what we know of human cognitive abilities. Yet, laboratory experiments report that profit motivated human traders are able to aggregate information in some, but not all, market environments (Plott and Sunder 1988, Forsythe and Lundholm 1990). Algorithmic agents have the potential to yield insights into how simple individual behavior may perform this complex market function as an emergent phenomenon. We report on a computational experiment with markets populated by algorithmic traders who follow cognitively simple heuristics humans are known to use. These markets, too, converge to rational expectations equilibria in environments in which human markets converge, albeit slowly and noisily. The results suggest that high level of individual intelligence or rationality is not necessary for efficient outcomes to emerge at the market level; the structure of the market itself is a source of rationality observed in the outcomes.

***JEL Codes:*** C92 · D44 · D50 · D70 · D82 · G14

***Keywords:*** Algorithmic traders, rational expectations, Structural rationality, Means-end heuristic, Information aggregation, Zero-intelligence agents

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 Comments are welcome: We are grateful to Martin Shubik for helpful conversations, and are responsible for the paper. Karim Jamal karim.jamal@ualberta.ca, Michael Maier msmaier@ualberta.ca, Shyam Sunder shyam.sunder@yale.edu.

*… social variables, not attached to particular individuals, are essential in studying the economy or any other social system and that, in particular, knowledge and technical information have an irremovably social component …*

Kenneth Arrow (1994, p. 8).

*To survive in a world where knowledge is limited, time is pressing, and deep thought is often an unattainable luxury, decision-makers must use bounded rationality. In this precis of Simple Heuristics That Make Us Smart, we explore fast and frugal heuristics—simple rules for making decisions with realistic mental resources. These heuristics enable smart choices to be made quickly and with a minimum of information by exploiting the way that information is structured in particular environments. Despite limiting information search and processing, simple heuristics perform comparably to more complex algorithms, particularly when generalizing to new data—simplicity leads to robustness.*

Peter Todd and Gerd Gigerenzer (1999).

## 1. Introduction

Hayek (1945) characterized markets as social mechanisms that disseminate and aggregate information dispersed among people and coordinate allocation of resources in the economy. Higher efficiency of market-driven allocations arises from the ability of markets to aggregate more information beyond the access or capability of a central planner. Yet the process by which markets carry out these aggregation and dissemination functions is not well understood. Muth's (1961) rational expectations (RE) framework has been influential in economic theorizing about market equilibrium by imposing a consistency requirement between expectations and outcomes generated by actions based on those expectations. The fixed point argument of RE models, however, depends on making the right conjectures; it does not propose a path for even perfectly rational optimizing agents to reliably arrive at equilibrium. Critics question the descriptive validity of the RE model in a world where individuals' cognitive capacities fall short of optimization and are better described by bounded rationality (Simon 1969).

In this study, we employ algorithmic trading to explore the apparent contradiction between sophisticated and efficient aggregate outcomes in markets populated with boundedly rational agents. We simulate markets with algorithmic traders who use simple heuristics to examine conditions under which the outcomes do or do not converge to the RE equilibria. We find that algorithmic markets do (and do not) converge under the same conditions in which laboratory markets populated with incentives-motivated human markets do (and do not) converge. Success of such simple algorithmic traders in replicating outcomes of human traders suggests that institutional properties of double auction markets is an important determinant of convergence to equilibria. Conversely, success of these simple heuristic strategies implies that the information processing necessary to attain theoretical equilibria in market institutions are far weaker than what is routinely assumed in deriving the equilibria mathematically.

Derivations of rational expectations equilibria require individuals to conjecture state-price correspondence which is fulfilled when they optimally act on those conjectures, learn others' preferences or strategies, and arrive at unbiased estimates of the underlying parameters of the economy by observing market variables. In theory, in order to disseminate information and to aggregate diverse information held by individuals, they are assumed to be able to bootstrap their way to make assessments necessary to arrive in equilibrium; yet such assessments require observation of equilibrium outcomes. This circular dependence gives rise to the importance of arriving at the right conjectures in Muth's RE model.

In *Sciences of the Artificial*, Simon (1969, p. 47) questioned the plausibility of human agents, with their limited cognition, forming rational expectations by intuition. Simon's boundedly rational individual traders are unlikely to have the memory or reasoning power required to act in a manner consistent with the requirements of rational expectations models.

Tversky and Kahneman (1974) reinforced the behavioral critique of economic models by identifying a series of simple judgment heuristics (e.g., representativeness, anchor-and-adjust) which people use to make decisions, though theses heuristics generate sub-optimal results in single-shot experiments.

In this paper we push the argument a step further to ask if rational outcomes at market level (specifically aggregation of information) are consistent with individuals using simple heuristics documented by Newell and Simon (1973) and Tversky and Kahneman (1974). Market experiments by Plott and Sunder (1988) and Forsythe and Lundholm (1990) with profit-motivated human agents revealed that they can aggregate diverse information to converge near RE equilibria, albeit slowly and imperfectly, under some circumstances but not in others. Likewise, prior experiments using simulated computer traders have reported that markets can yield efficient outcomes even when populated by minimally-intelligent traders in environments without uncertainty (Gode and Sunder 1993), with shared uncertainty (Jamal and Sunder 1996,) and even with asymmetric information (Jamal, Maier and Sunder 2016). The present paper examines markets with asymmetric distribution of information across traders typical of stock markets, and reports that simple biased heuristic behavior of individuals can also generate RE equilibria in double auctions through Hayekian aggregation of diverse bits of information in possession of individual traders.

We populate the markets with algorithmic traders who follow simple heuristics (means-ends analysis from Newell and Simon 1973, henceforth ME; and representativeness heuristic from Tversky and Kahneman 1974, henceforth R) which is widely considered to be suboptimal. In specified circumstances, outcomes of such markets also tend to converge to the neighborhood of the predictions of RE equilibria derived from assuming optimal agent behavior.

The present paper considers Plott and Sunder (1988, henceforth PS1988) double auction market environments in which individual agents are imperfectly but partially informed about the resolution of state uncertainty in a way that their aggregated information resolves all uncertainty. PS1988, using profit-motivated human traders, had reported that double auctions with 3 states of nature can also converge to rational expectations equilibria when either (1) they have homogeneous preferences and markets are incomplete, or (2) they have heterogeneous preferences provided that the markets are complete.[1] Forsythe and Lundholm (1990) experiment showed that even incomplete markets with heterogeneous preferences can aggregate information provided that the all preferences are common knowledge *and* traders get sufficient experience trading in that environment.

The paper is organized in four sections. Section 2 describes the minimally-intelligent algorithmic agents using two heuristics: a simple Means Ends heuristic (Newell and Simon 1973) and a representativeness heuristic (Tversky and Kahneman 1974) in combination with "zero-intelligence" behavior in a double auction market. In the third section, we implement these heuristics in incomplete and complete market environments in which all traders have diverse partial information, and compare the results with PS1988 data from parallel human experiments. The fourth section presents the implications of the findings and some concluding remarks.

[1] PS1988 report on three series of markets (A, B and C). In Series A markets, traders have diverse dividends in each of 3 states. If the state is *X*, half the traders get a ~*Y* signal, and half get a ~*Z* signal. Markets do not reliably converge to the predictions of rational expectations equilibria. In Series B markets, traders have diverse dividends but they can trade three state-contingent (i.e., Arrow-Debreu) securities simultaneously. These markets reliably converge to rational expectations predictions. In Series C markets, traders have homogenous dividends and rational expectations equilibria are attained; however, human subjects need considerable experience (learning) before results approach the RE equilibrium.

## 2.0 Design of Minimally-Intelligent Algorithmic Agents

Algorithmic agents employed in this paper are a combination of three simple elements: means-ends heuristic (ME) to iteratively adjust current aspiration levels (*CALs*) in light of observed market prices, representativeness heuristic (R) to use conjecture-the-state instead of expected value mode to set *CALs*, and zero-intelligence (ZI) to generate bids and asks using *CALs* as the anchor. We refer to this composite algorithm as a minimally-intelligent agent in the present context.

### 2.1 Means-Ends Heuristic (ME)

Simon (1955) proposed bounded rationality as a process model to understand and explain how humans, with their limited knowledge and cognitive capacity, behave in complex settings. Humans develop and use simple heuristics to attain satisfactory, not optimal, outcomes. To understand human problem solving Newell, Shaw and Simon (1957) developed a general problem solving program (GPS) of which means-ends heuristic of iteratively reducing differences is a key element. It can be summarized in four steps: (i) compare the current knowledge state *a* with a goal state *b* to identify difference *d* between them; (ii) find an operator *o* that will reduce the difference *d*; (iii) apply the operator *o* to the current knowledge state *a* to produce a new current knowledge state *a** that is closer to *b* than *a* was; and (iv) repeat this process until the current knowledge state *a** is satisfactorily close to the goal state *b*. Knowledge states of traders can be represented as aspiration levels (Simon 1956) that adjust in response to experience. An ME heuristic for a trader thus requires a mechanism for setting an initial aspiration level, and a method for adjusting aspiration levels based on experience (e.g., Jamal and Sunder 1996).

At the beginning of each period, the initial aspiration level of each trader ($CAL_0$) is set equal to the expected value of that trader's dividends conditional on the information the trader has. For a 3-state {$X, Y, Z$} security, if a trader has information Not $X$ {$\sim X$), it calculates the expected value from conditional probabilities of states $Y$ and $Z$.

$$E\ (D \mid \sim X) = \text{Prob}\ (Y \mid \sim X) * \text{Div}\ (Y) + \text{Prob}\ (Z \mid \sim X) * \text{Div}\ (Z)). \quad (1)$$

After each market transaction (generated by a process described below) the ME heuristic is activated to make gradual adjustments to each subject's price aspirations by placing a weight ($0 \geq \gamma \leq 1$) on new price $P_t$, and weight ($1- \gamma$) on the most recent $CAL_{t-1}$ . This can be represented as a first order adaptive process:

$$CAL_{t+1} = (1 - \gamma)\ CAL_t \ + \gamma\ P_t. \qquad (2)$$

Starting with $CAL_0$ as the initial value, through substitution we obtain:

$$CAL_{t+1} = (1 - \gamma)^{t+1}\ CAL_0 \ + \gamma\ ((1 - \gamma)^t P_1 + (1 - \gamma)^{t-1}\ P_2 + \ldots + (1 - \gamma)\ P_{t-1} + P_t). \ (3)$$

**2.2 Representativeness Heuristic (R)**

Maximization of expected values, expected utilities, or maximin are well-known approaches to deciding under uncertainty. Instead, the R heuristic (Tversky and Kahneman 1974) picks the most probable outcome under uncertainty as if it is the one which will occur. For example, if the state space $\Theta = \{X, Y\}$, and Prob. ($X$) > Prob. ($Y$), $X$ being more likely, is considered the representative outcome of the process. Suppose the subject receives an imperfect signal $s_i$ from set $S = \{s_1, s_2\}$ such that Prob. ($X \mid s_1$) > Prob ($Y \mid s_1$). A subject who sees signal $s_1$ infers the state to be $X$ because this state is more likely to generate the observed signal. The R heuristic is insensitive to base rates and uncertainty, rests on an extreme assumption about the learning or adjustment process, and is generally considered to be a cause of biased and irrational individual behavior. According to Tversky and Kahneman (1974), the R heuristic is used by individuals to make a conjecture about a state and to treat this conjecture as being certain. For any trader $i$ at any time $t$, the aspiration level generated by heuristic R is the dividend closest to the most recently observed transaction price $P_t$.

For example, consider a dividend vector for states X, Y and Z (0.1, 0.3, 0.6) with a last traded price of 0.4. Suppose that the true state is X. If the trader has been told that the true state is not Y, then the closest dividend corresponds to state Z, and so the trader would set their CAL to 0.6; however, if the trader were told that the true state is not Z, then they would select Y with a CAL of 0.3.

### 2.3 Zero-intelligence (ZI) Heuristic

Minimally intelligent algorithmic agents deployed in the markets reported here apply ZI heuristic (Gode and Sunder 1993) to *CAL* anchors set by either the ME and R heuristics (in a manner described below) for generating bids and asks —bidding below and asking above the *CAL*, using random numbers. When called upon to generate a bid, ZI picks a uniformly distributed random number ~*U* (0, *CAL*); when called upon to generate an ask, it picks a uniformly distributed random number ~*U* (*CAL*, *U*) when *U* is an exogenously specified constant (the upper limit of prices) for the entire simulation.

### 2.4 Integration of Three Heuristics into Market Simulation

Each period consists of *I* (= 5,000) iterations, and starts out with a clean slate (i.e., nothing is carried over from the past periods in the memory) The only difference among periods is the realized state of the world—*X*, *Y*, or *Z*—that determine the dividends the securities pay to various traders.

Algorithmic agents deployed in these markets use an ME and R heuristics to estimate their *CAL*, and use it to implement a Zero-Intelligence (ZI) strategy after Gode and Sunder (1993) consisting of bidding randomly below and asking above their aspiration levels. Traders draw a uniformly distributed random number between 0 and an upper limit of 1. If number drawn is less than or equal to 0.5, the trader will generate a bid. If the number drawn is greater than 0.5, an ask is generated. If the action is a bid, then the amount of the bid is determined by drawing a second randomly generated number between a lower bound of 0 and an upper bound of the individual trader's *CAL*. This bid is then compared to the highest bid that currently exists in the market. If the new bid is higher than the existing highest bid, the new bid becomes the new highest bid in the market. Correspondingly, if the action is an ask, then the amount of the ask is determined by generating a second random number in the range having a lower bound of the traders *CAL* and an upper bound of 1. This newly generated ask is then compared to the existing lowest ask in the market. If the new ask is less than the existing ask, then the new ask becomes the new lowest ask in the market. Bids and asks are generated randomly, distributed independently, identically, and uniformly over these ranges (see Figure 1). These algorithmic agents are myopic, making no attempt to anticipate, backward induct, or theorize about the behavior of other traders. They simply use the knowledge of observable past market events (transaction prices) to estimate their opportunity sets, and choose randomly from these sets.

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Insert figure 1 about here

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These markets are populated by traders of each payoff type who all have some partial information about the realized state of world. As shown in (1) and Figure 2, all traders of type $j$ use their expected dividend conditional on their partial information to set their initial *CAL* using the prior state probabilities. As each trade occurs, they update their *CALs* after each transaction using the ME heuristic (i.e., first order adaptive process) specified in (2) above and a given randomly chosen value of the adaptive parameter $\gamma$ for the simulation (see Section 3.4 below). Submission of bids and asks continues with the updated *CALs* serving as constraints on the opportunity sets of traders until the next transaction occurs, and this process is repeated for 500 iterations (phase 1 of each period).

In each iteration of phase 2 (iterations 501 to 5,000) which lasts until the end of the period, first a value of parameter $\rho$ is drawn independently $\sim U(0, 0.1)$. Second, the realized value of $\rho$ is used to make a binary random draw between heuristics R with probability $\rho$ and ME with probability (1- $\rho$). The heuristic chosen in this second step is used to set the *CAL* in this iteration. The two independent draws in each iteration in the second phase mean that at any stage in phase 2, the number of traders using *CAL* based on the R heuristic can vary between 0 and 12 traders (as compared to being fixed at 0 in phase 1).

At the end of each period, the realized state is revealed, dividends are paid to their accounts, and each trader's security endowment is refreshed for the following period. *CALs* of all traders are re-initialized to their respective expected values conditional on their new signal using (1); in the spirit of minimally intelligent algorithms, nothing is carried over from the preceding period [2]

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**Insert Figure 2 about here**

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### 3.1 Market Environments

PS1988 report results for three sets of markets labeled Series A (heterogeneous preferences, incomplete market), Series B (heterogeneous preferences, complete market), and Series C (homogeneous preferences, incomplete market). The state space $\Theta = \{X, Y, Z\}$ consists of three states. At the beginning of the period the state is realized, and the traders receive a private signal as follows: If the state is *X*, half the traders learn that the state is Not *Y*, and the other half learn that the state is Not *Z*. Similar information is provided under state *Y* (Not *X* and Not *Z*) and state *Z* (Not *X* and Not *Y*). No trader knows the state of nature but if they could pool their information, the state would be known with certainty. Traders are not allowed to communicate with one another except through trading. The current paper also reports the results for two markets of each of the same three series simulated with parameters given in Tables 1 and 2. We present single as

[2] At this stage, it would have been possible for the agents to keep track of the prices associated with each realized state and use this information in subsequent periods. In the spirit of minimal intelligence, our agents do not do so, and uninformed agents simply carry forward their *CAL* from the end of one period to the beginning of the next period. The *CAL* of informed agents responds to a perfect signal about the state realized in each period and is not dependent on experience in previous periods.

well as 100-replications charts for each of the six markets (Markets 6 and 11 from Series A, Markets 4 and 5 from Series B, and Markets 7 and 8 from Series C). In each market, there are 12 traders who trade three-state, single period assets. Before trading is started in each period, a random state of nature (*X, Y, or Z*) was drawn in PS1988 human subject experiments. In the simulations presented in the current paper, the same sequence of states is assumed to have been realized in each market session.

At the beginning of each period, each of the 12 algorithmic traders is endowed with *k* (= 2 or 8) securities of each type. Following PS1988, each security pays a single state-contingent dividend that varies across three groups within the 12 traders in Series A and B, and is identical in Series C. The vector of state-contingent dividends for each trader is known privately by each trader. These dividend values define the maximum (minimum) amount the trader should be willing to pay to buy (sell) a certificate.

As described above, in the simulated markets, the ME, R and ZI heuristics are combined to define the minimally intelligent algorithmic agents in a continuous double auction market. Again, the algorithmic traders are completely myopic and make no attempt to infer the knowledge of other traders. At the end of the period, the actual state (*X , Y, or Z*) is revealed, dividends are paid, and profit for each trader is computed.

In Series A and C, they trade a single security which pays the individual trader's *X* dividend if state *X* is realized, *Y* dividend if state *Y* is realized, and *Z* dividend if state *Z* is realized. Under state *X*, six traders learn information Not *Y* and six learn Not *Z*; and similarly for states *Y* and *Z*. In Series B, they simultaneously trade *X*, *Y*, and *Z*-contingent Arrow-Debreu securities in three different markets.

**3.2 Asset Markets Series A, B, and C**

In simulations of PS1988 all traders have some information, nobody has all the information, but the information dispersed among the traders, if aggregated, is perfect. Since Series A and C consist of single security markets, each trader *i* initializes its $CAL_i$ to one of the three possible signal-contingent expected dividend values as follows using (1):

$CAL_i = (E(D)/\text{ Signal not } X) = (\Pr(Y) * (D_{Yi}) + \Pr(Z) * (D_{Zi}))/(\Pr(Y) + \Pr(Z));$

or $\quad = (E(D)/\text{ Signal not } Y) = (\Pr(X) * (D_{Xi}) + \Pr(Z) * (D_{Zi}))/(\Pr(X) + \Pr(Z));$

or $\quad = (E(D)/\text{ Signal not } Z) = (\Pr(Y) * (D_{Yi}) + \Pr(X) * (D_{Xi})/(\Pr(X) + \Pr(Y)).$ (4)

Since Series B consists of three markets (for *X*-contingent, *Y*-contingent, and *Z*-contingent securities), each trader has memory for three signal-contingent *CAL*s (*CAL(X)*/ Signal not *X*, *CAL(X)*/ Signal not *Y*, and *CAL(X)*/ Signal not *Z for X*-contingent market) for each market for a total of nine *CAL*s. At the beginning of the first period of trading, each trader calculates its expected values of the securities as follows:

Signal Not *X*: $CAL_i(X) = 0;$

$CAL_i(Y) = \Pr(Y)/(\Pr(Y) + \Pr(Z))*D_{Yi};$

$CAL_i(Z) = \Pr(Z)/(\Pr(Y) + \Pr(Z))*D_{Zi};$

Signal Not *Y*: $CAL_i(X) = \Pr(X)/(\Pr(X) + \Pr(Z))*D_{Xi};$

$CAL_i(Y) = 0;$

$CAL_i(Z) = \Pr(Z)/(\Pr(X) + \Pr(Z))*D_{Zi};$

Signal Not *Z*: $CAL_i(X) = \Pr(X)/(\Pr(X) + \Pr(Y))*D_{Xi};$

$CAL_i(Y) = \Pr(Y)/(\Pr(X) + \Pr(Y))*D_{Yi};$

$$CAL_i\,(Z) = 0. \tag{5}$$

Starting with these signal-contingent *CAL*s at the beginning of the first period of the market, each trader updates them using ME heuristic in phase one and ME or R heuristic in phase 2 described after each transaction.

In Series B (Contingent Claims), representativeness is implemented in phase two with probability *ρ* in each iteration, by exploiting knowledge that at least some traders know that a particular state will not occur (and therefore set their *CAL* = 0 for that one state). No trader gets a signal to set the true state at 0, but some get a signal to set *CAL*= 0 for the other two states. Prices in these two (not true) states may drift downward under the pressure of market bids and offers. In this instance, traders view the vector of observed prices from trades in each security from the remaining two possible states and then choose the state which corresponds to the observed maximum price. This state is then treated as occurring with certainty (*CA*L= Dividend of this state). The representativeness *CAL* is used for the current iteration only; whether R is used in the next iteration is determined by a fresh independent draw with probability *ρ* (otherwise *CAL* from ME is used). This procedure is repeated for 5,000 iterations until the end of the period. At the end of the period, the state is revealed, dividends are credited to accounts according to the number of securities traders hold at the time. The number of securities are then reset to the starting endowments (either 2 or 8 per trader) for the next period.

## 4.0 Results

As benchmarks for comparison, PS1988 results with human traders revealed that (a) Series A (heterogeneous preferences and incomplete) markets did not aggregate information; (b) Series B (heterogeneous preferences and complete) markets and (c) Series C (homogeneous preferences and incomplete) markets did aggregate information to approximate RE equilibrium outcomes. In each of Figures 3-8, Panel A reproduces the price charts from PS1988.

### 4.1 Series A (Single Security, Heterogeneous Preferences, PS1988 Mkts 6 and 11)

Panels B and C of Figure 3 show the simulation results of Series A Market 6 for a single run and for 100 replications (with median price in red), respectively. The solid horizontal line in each period shows the RE equilibrium price for the period (depending on the realized state of the world). Similar to human markets in Panel A, in the single run Panel B, prices do not get anywhere near the REE price in any of the periods 3, 5, 6, and 13 when the realized state was *Z*. The same is true of the cloud of transaction prices from 100 independent replications shown in Panel C, suggesting that the failure of the single market in Panel B to approximate REE is not a fluke.

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**Insert Figure 3 about here**

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Forsythe and Lundhom (1990) reported that making heterogeneous preferences common knowledge *and* more trading experience resulted in even incomplete markets achieving REE in laboratory experiments with human traders. In our simulations, we cannot make information common knowledge, but we repeated Market 6 by increasing the endowment per trader from 2 to 8 in order to have a larger volume of trading each period. These results are shown in Figure 3 Panels D and E; they do not reveal a tendency of the simulated markets to get closer to REE with increased trading volume. Both human and simulated traders show little learning effects. Human subjects struggle throughout the experiment with incorrect price convergence (and low efficiency levels?) right to the end of the experiment.

Figure 4 shows the results for a second randomly chosen Market 11 from Series A of PS1988. These results are similar to the results for Market 6 in all respects for both Token 2 as well as Token 8 markets.

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**Insert Figure 4 about here**

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The middle four bars for each period in Fig. 9, show that the allocative efficiency of markets 6 and 11 with 2- as well as 8-token endowment simulations remains around 60%, falling far short of 100% observed in Series B and Series C markets.

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**Insert Figure 9 about here**

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### 4.2 Series B Complete Markets for Arrow-Debreu State-Contingent Securities (PS1988 Markets 4 and 5)

Three panels of Figure 5 show the results of eight periods of Series C Market 4 as follows: Human markets in Panel A, single-run simulation in Panel B and 100 replications of simulation (with median) in Panel C.

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**Insert Figure 5 about here**

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A review of Figure 5, panel B shows that in a single run all three state-contingent market prices converge near the respective RE equilibrium (which is equal to the highest of the state-contingent trader dividends in the market corresponding to the state which is realized, and zero in the other two markets). This is quite comparable to the behavior of markets populated with human traders in Panel A. The cloud of 100 replications of the simulation shown in Panel C (along with the median prices) confirms that this single-run result is not just a happenstance.

Three panels of Figure 6 (for PS1988 Series B Market 5) essentially support the same observations about Market 4 above.

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**Insert Figure 6 about here**

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Human subjects need the first two periods to learn about the market, but then in later periods converge reliably to the RE equilibria (see Figure 4, panel B). Our simulated markets converge faster to RE equilibria (right from Period 1) but then don't have any capacity to learn from period to period, simulated results are thus a noisier version of the human data.

The left two bars for each period in Fig. 9 show that the successful aggregation of information in Series B markets is also reflected in high allocative efficiency. With only a couple of exceptions where the efficiency falls short by two or three percent, all other periods achieve 100% allocative efficiency.

### 4.3 Series C (Single Security, Homogeneous Preferences, PS1988 Mkts 7 and 8)

Three panels of Figure 7 show the results of fourteen periods of Series C Market 7 from PS1988. Panel A shows the results from human traders, B for a single run of a market populated with minimally-intelligent traders, and Panel C shows combined data from 100 independent replications of minimally-intelligent market with identical sequence of state realizations (with median price).

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**Insert Figure 7 about here**

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As seen in Panel A, convergence to RE in human markets takes many periods of trading and occurs reliably only in the later part of the session. In the single-run chart in Panel B, the markets converge to RE in all except period 8. This difficulty in conjecturing the right state to arrive in REE is even clearer in Panel C which shows a cloud of 100 simulations. Even though the market ultimately converges to the REE price for the realized state for most of the runs, there are non-trivial number of runs in which the market initially conjectures the state to be *Y* when in fact it is *Z;* and the market initially conjectures the state to be *Z*, when in fact it is *X*. These results suggests that derivation of REE outcomes from these simple heuristics cannot be taken for granted, and a close scrutiny is needed about the interaction between the individual behavior defined by these heuristics and the market rules and environment in which trading takes place. However, it is noteworthy that, in most cases, the market is ultimately able to abandon the wrong conjectures in favor of the right conjectures by the end of the trading periods.

Figure 8 shows the results for seven periods of another Series C market 8. These simulated results are similar to the Plott and Sunder (1988) human data (panel A) though human subjects are able to converge more reliably and less noisily to RE equilibria across all 3 states. Human subjects need much less time to learn how to infer the correct state. The single run the market converges close to the RE equilibrium in all three states. These Series C simulated results are also less noisy than for Series A (diverse dividends) and there are no off-equilibrium trades occurring late in the period. For our simulated traders, Panel C shows that a 100-period run average exhibits the same pattern of behavior as a single run, and wrong conjectures about the state of the world are ultimately corrected by the end of the periods. Simulated markets converge close to RE equilibria in all states.

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**Insert Figure 8 about here**

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As seen in the two right-most bars for each period in Figure 9, allocative efficiency of the simulated incomplete markets with homogeneous preferences is always 100 percent, that is, all securities end up in the hands of the traders who value them most in the actually realized state of the world.

## 5. Concluding Remarks

The evidence presented in this paper shows that in fairly complex market environments, individual behavior described by simple heuristics can accomplish market-level outcomes whose formal derivation is based on strong optimization assumptions. Even if a key assumption (individual optimization) of the theory is descriptively invalid, it does not necessarily undermine the predictive value of the theory at the aggregate level. Our findings are consistent with Gigerenzer et al. (1999) who built on Simon's paradigm by proposing that individuals use "fast and frugal" heuristics to accomplish complex tasks.

The limited computational or other "cognitive" abilities with which these algorithmic traders are endowed do not exceed the documented abilities of human cognition. In fact, by most measures, they fall short of human faculties by a long shot. Yet, these simulated markets with dispersed information converge to close proximity of rational expectations equilibria and attain high allocative efficiency in Series B (heterogeneous preferences and complete markets) and Series C (homogeneous preferences and incomplete markets). Allocative efficiency is much lower (about 60%) in Series A markets (heterogeneous preferences and incomplete markets).

In the cognitive psychology literature (e.g., Tversky and Kahneman 1974), use of representativeness (R) and other such heuristics is often depicted as a counterpoint to the rationality assumptions frequently used in economic theory. In our simulations, we found it difficult for security markets to achieve REE *without* the use of the R heuristic. The leap of faith involved in this heuristic turned out to help make the right conjectures about the state of the world, and thus complete the bootstrap process necessary for arriving at the REE fixed point. In other words, in at least this market setting, the R heuristic is functional and assists the market to aggregate information and yield outcomes close to the REE.

We interpret the results to suggest that, even in these relatively complex environments, allocative efficiency of markets remains largely a function of their structure, not intelligence or behavior of agents. Perhaps it would be appropriate to recognize this structural rationality independent of rationality we attribute to individual agents. Out inability to construct a path from either rational or boundedly-rational individual behavior to efficient aggregate level outcomes does not mean that such outcomes cannot emerge from complex interactions among simple agents within the constraints of markets and other social institutions.

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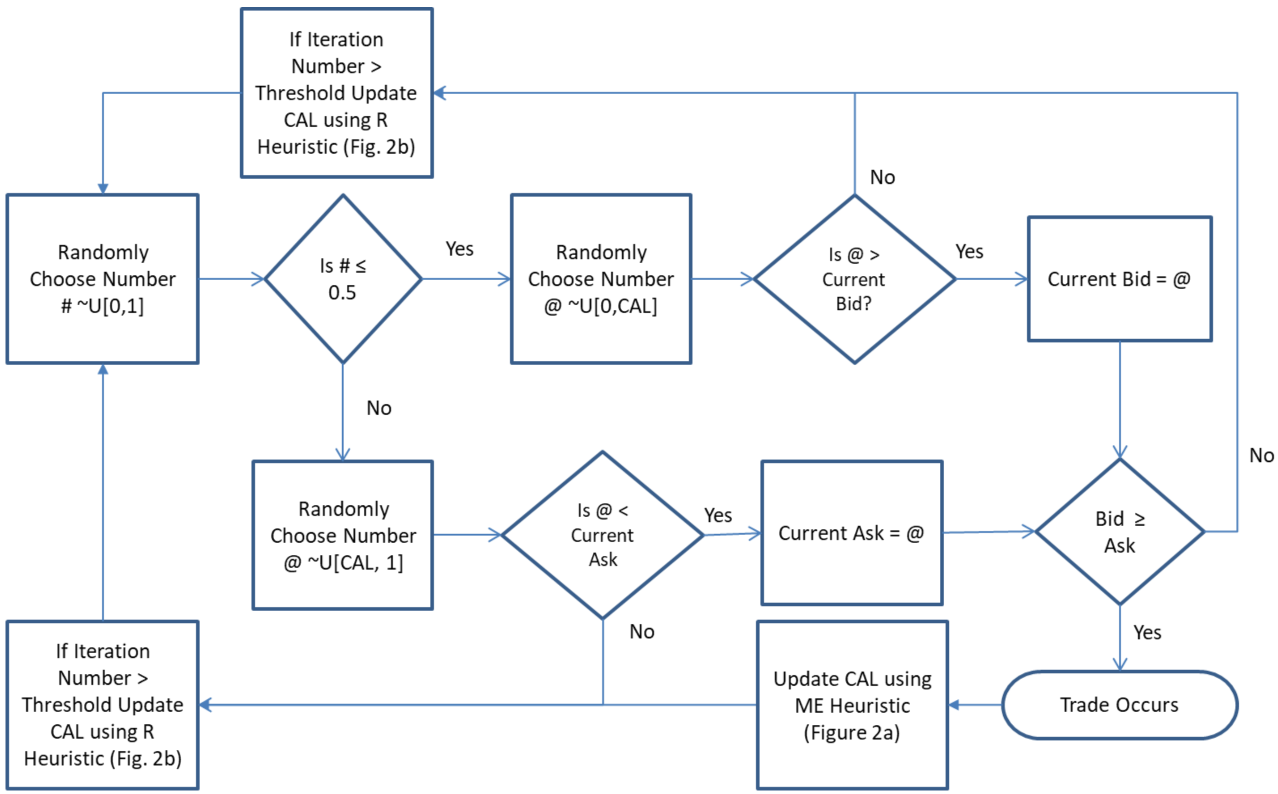
**Figure 1: Bid, Ask, and Transactions Algorithm**




The algorithmic traders in our simulated markets send a message by first drawing a random number from a uniform distribution bounded by 0 and 1. If the number is less than or equal (greater than) to 0.5, the message is a bid (ask). If the message is a bid, the trader draws a second random number from a uniform distribution bounded by 0 and the current aspiration level (*CAL*). If the message is an ask, the second random number is drawn from a uniform distribution between *CAL* and 1. If the trader's bid is more than the highest current bid in the market, the former becomes the current bid. If the trader's ask is less than the current (lowest) ask, then the former becomes the current ask in the market. When the current bid is equal to (or exceeds) the current ask, a trade occurs at the mid-point of the bid and ask. Visit www.zitraders.com for outline of the code.

**Figure 2a: Means-End (ME) Heuristic for Setting Current Aspiration Levels (CAL's)**

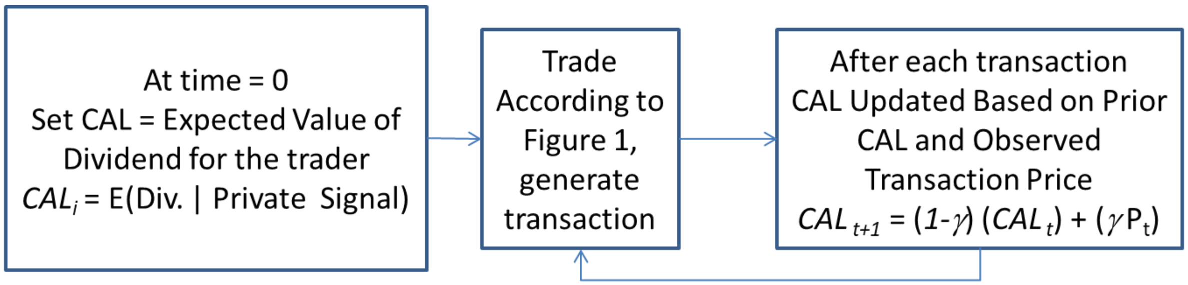
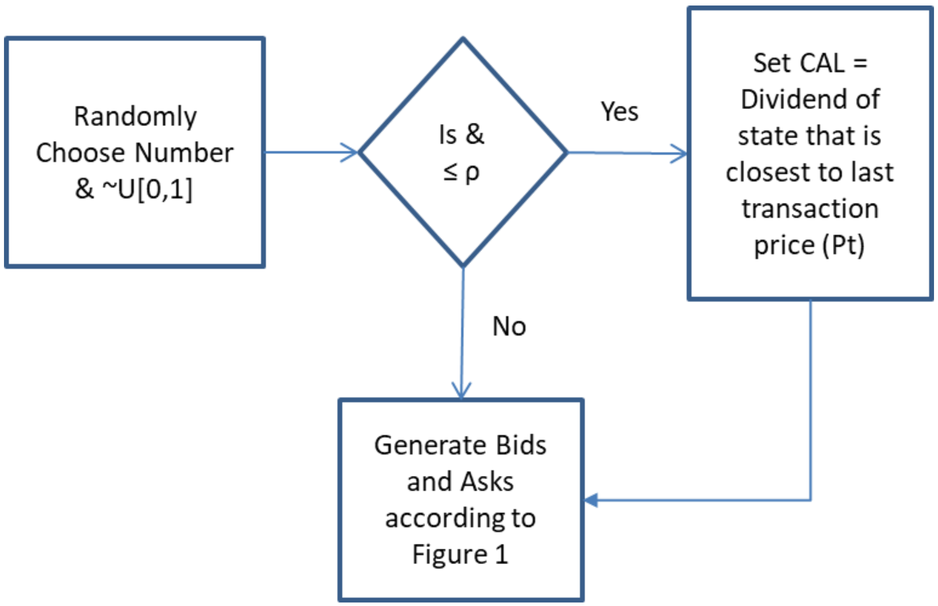



**Figure 2b: Representativeness (R) Heuristic for Setting Current Aspiration Levels (CAL's)**




Traders set a current aspiration level (*CAL*) to generate bids and asks. For the first phase of each period the traders use the ME algorithm (Panel a) exclusively to adjust their CAL after each trade. In the second phase, an individual trader may use the R algorithm (Panel b) on any iteration with probability ρ by setting CAL = closest possible dividend given the last transaction price, and then continue to use the ME algorithm after each transaction. Traders do not carry forward *CAL* from previous periods.

Figure 3: Information Aggregation in Series A Markets (Incomplete, Heterogeneous Preferences)
Panel A: PS1988 Human Market 6 (2 token endowment)


Fig. 3 Panel B: Simulated Market 6 Single Run (2-token endowment)

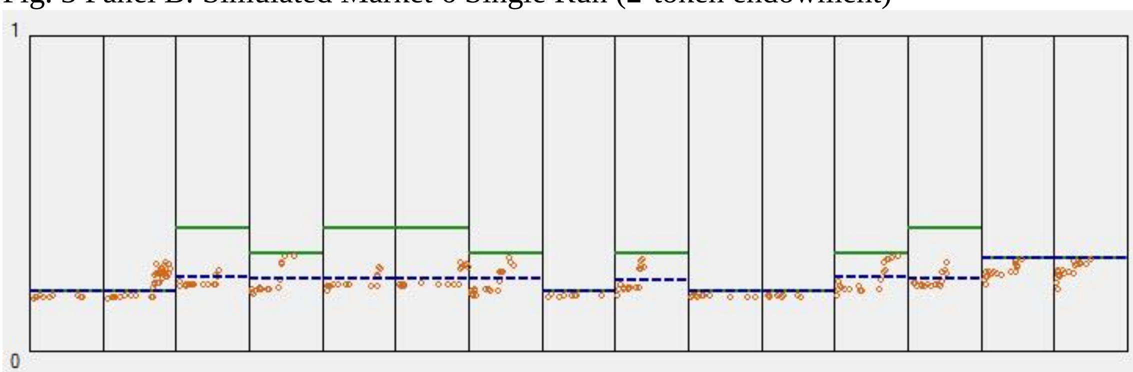


Fig. 3 Panel C: Simulated Market 6 100 runs plus median (2-token endowment)

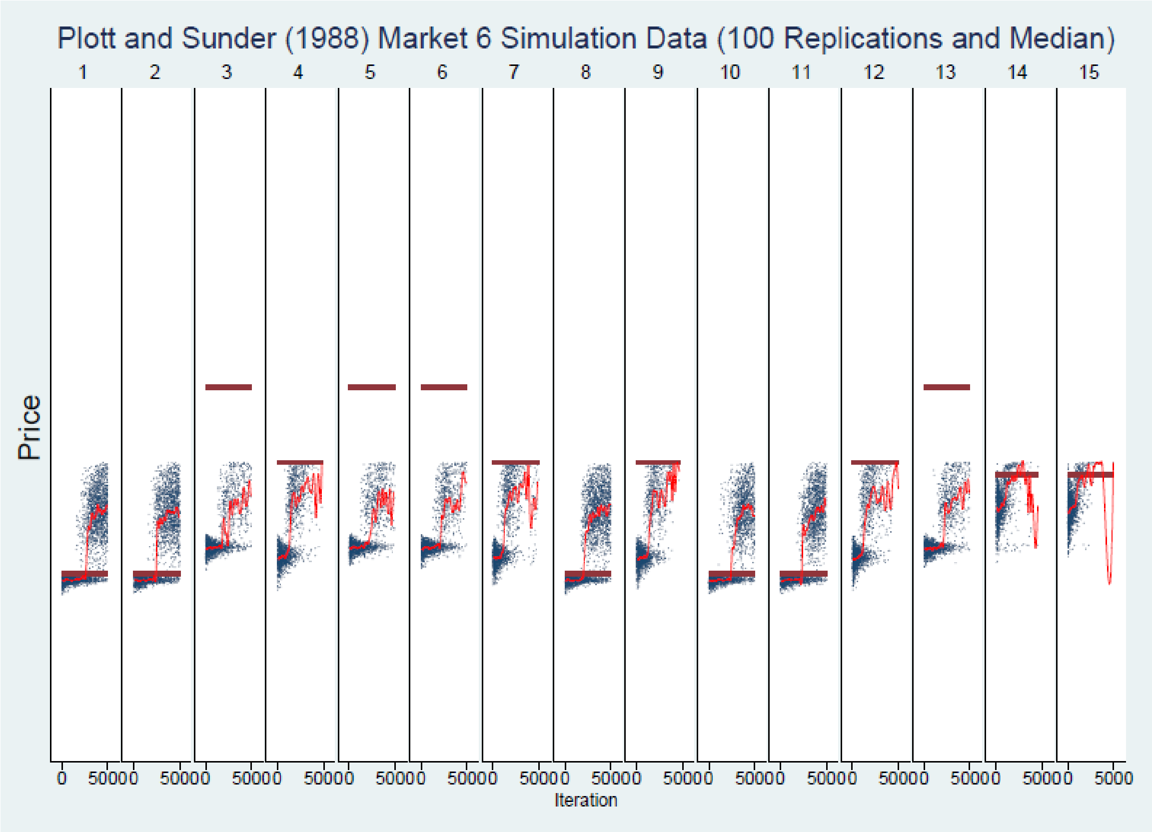


Fig. 3 Panel D: Simulated Market 6 Single Run (8-token endowment)

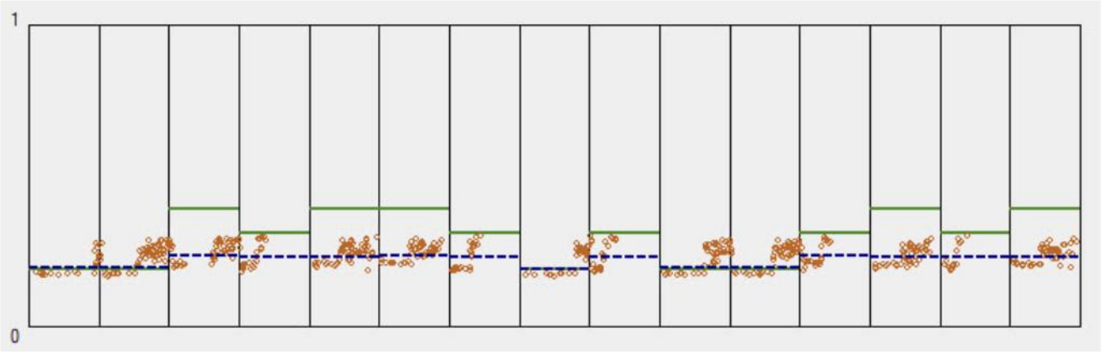



Fig. 3 Panel E: Simulated Market 6 100 runs plus median (8-token endowment)

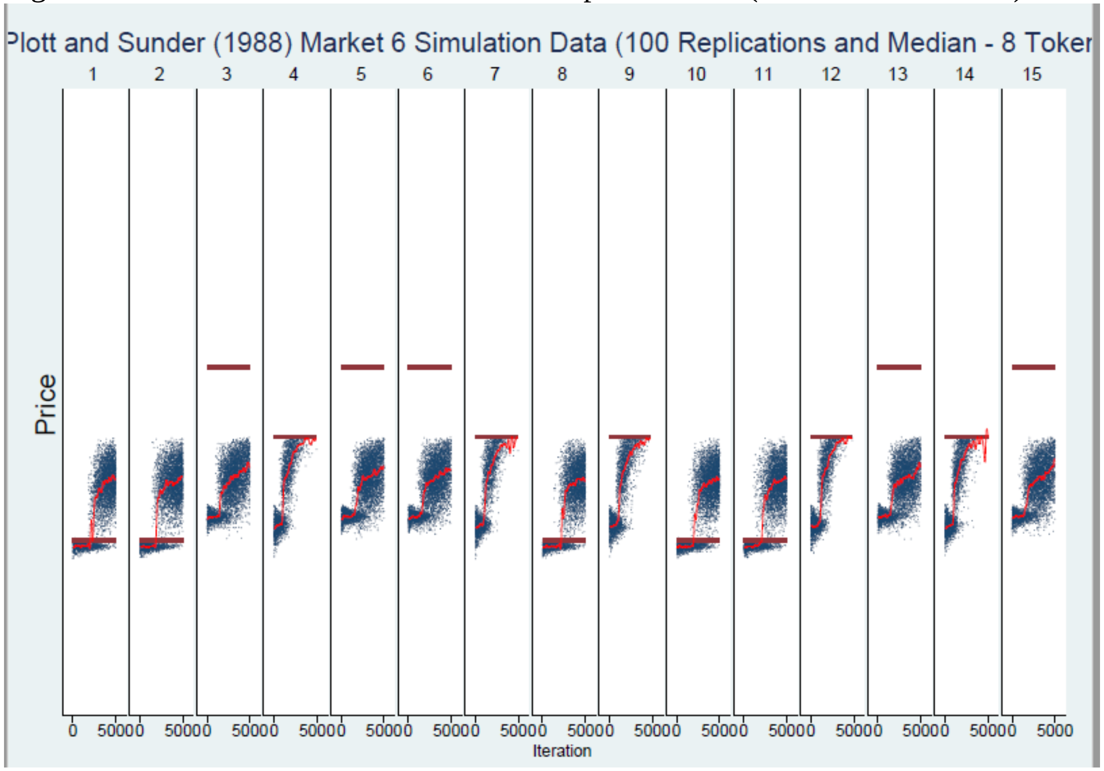


Figure 4: Information Aggregation in Series A Markets (Incomplete, Heterogeneous Preferences), Panel A: PS1988 Human Market 11 (2 token endowment)

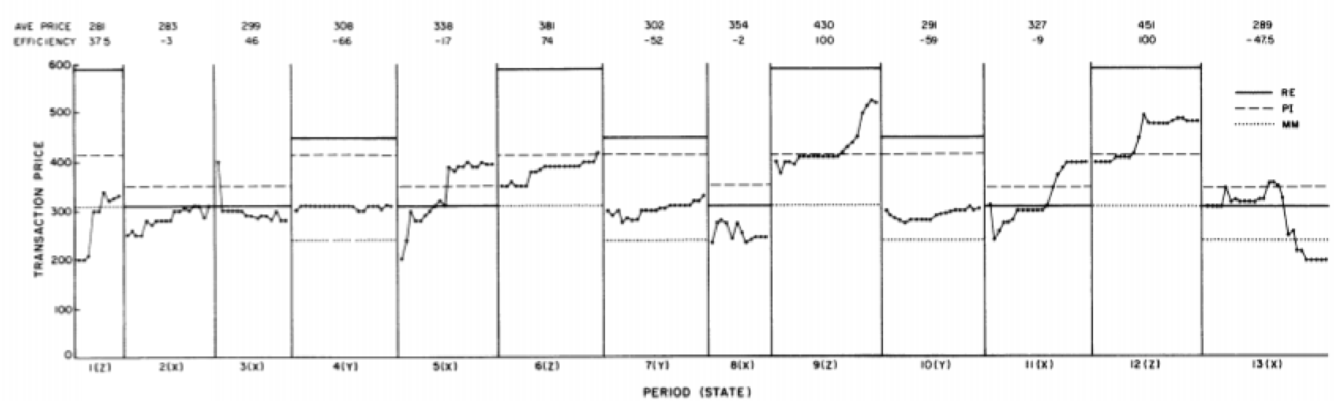



FIGURE 11.—Market 11 transaction prices.

Fig. 4 Panel B: Simulated Market 11 Single Run (2-token endowment)

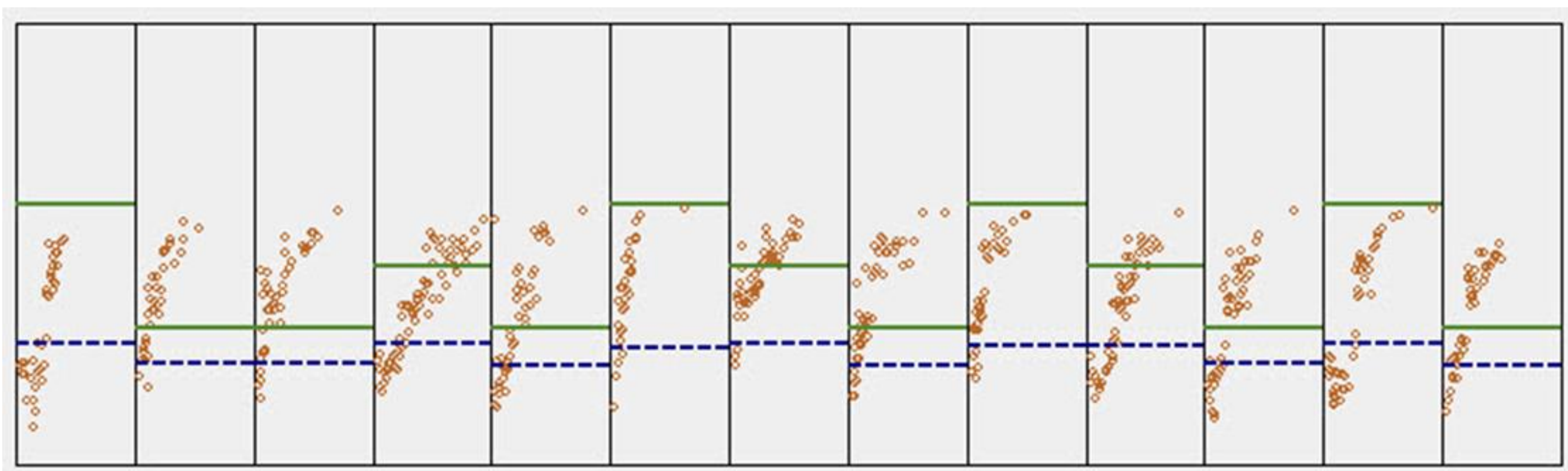


Fig. 4 Panel C: Simulated Market 11 100 runs plus median (2-token endowment)

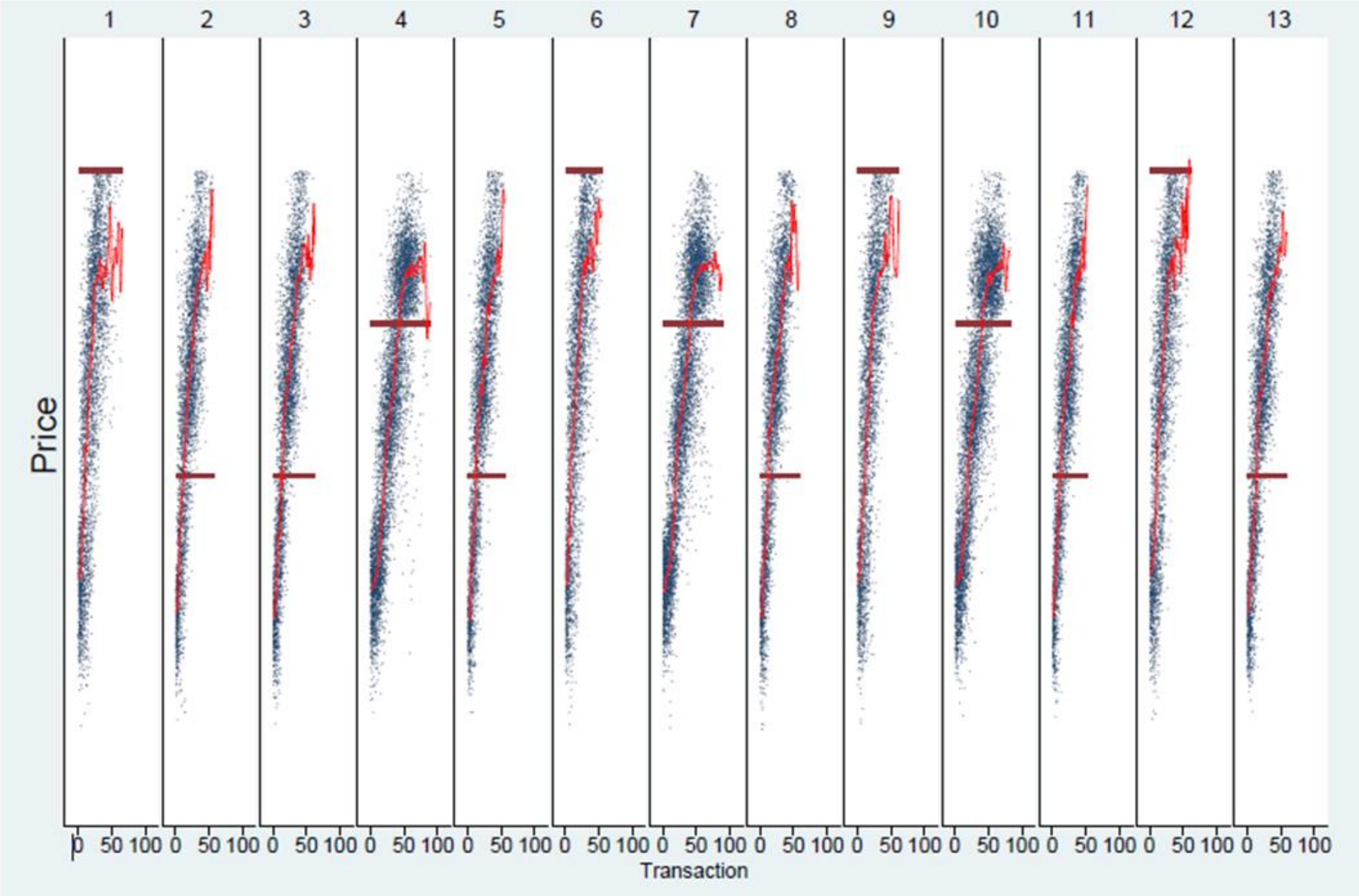


Fig. 4 Panel D: Simulated Market 11 Single Run (8-token endowment)

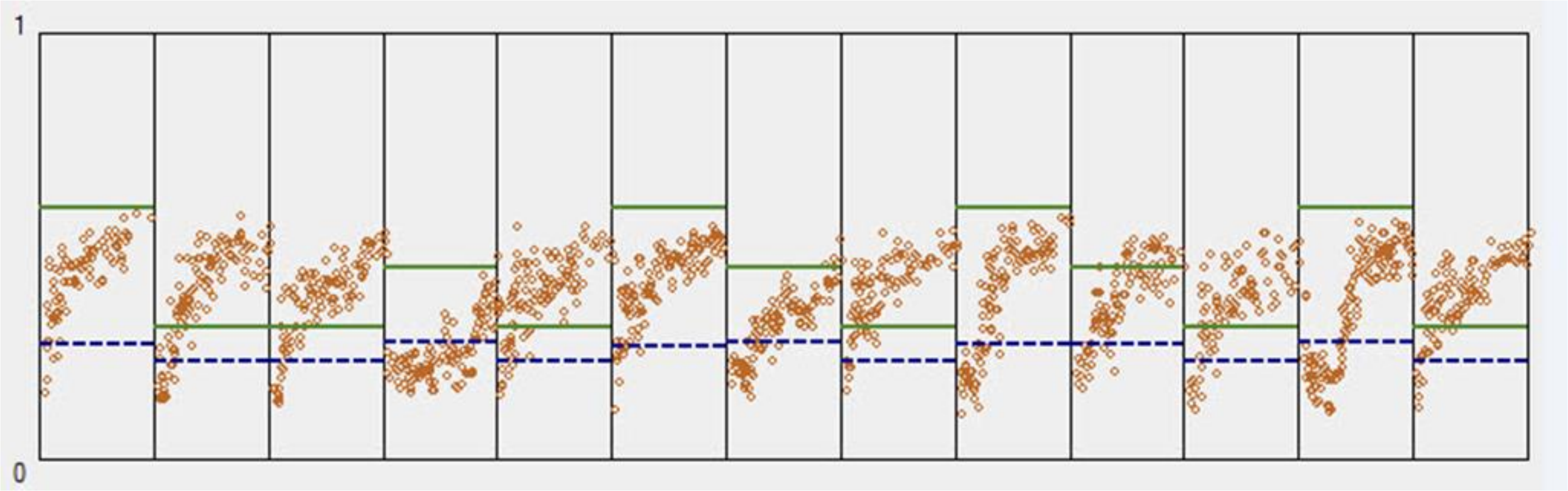


Fig. 4 Panel E: Simulated Market 11 100 runs plus median (8-token endowment)

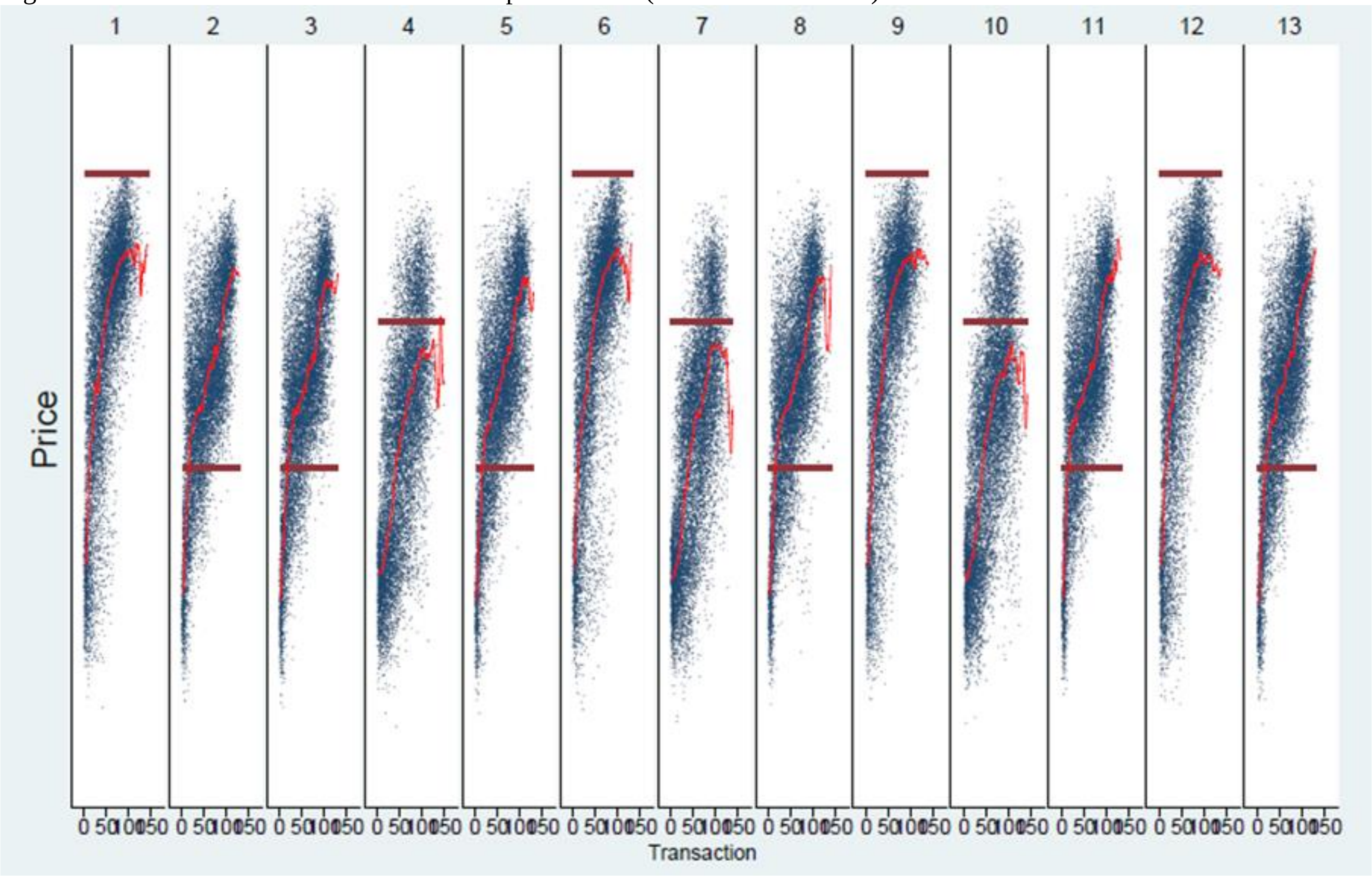


Figure 5: Information Aggregation in Series B Markets (Complete, Heterogeneous Preferences), PS1988 Market 4
Panel A: PS1988 Human Market 4 (2 token endowment)

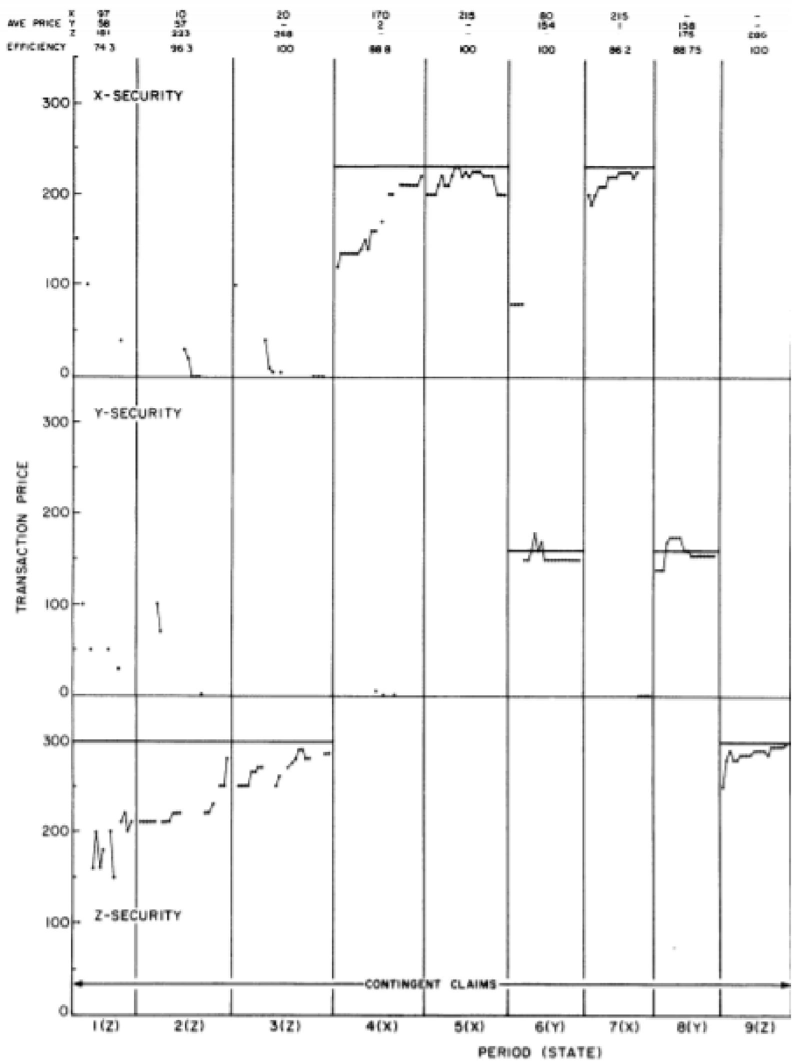



FIGURE 4.—Market 4-CC and 4-S transaction prices.

Fig. 5 Panel B: Simulated Market 4 Single Run (2-token endowment)

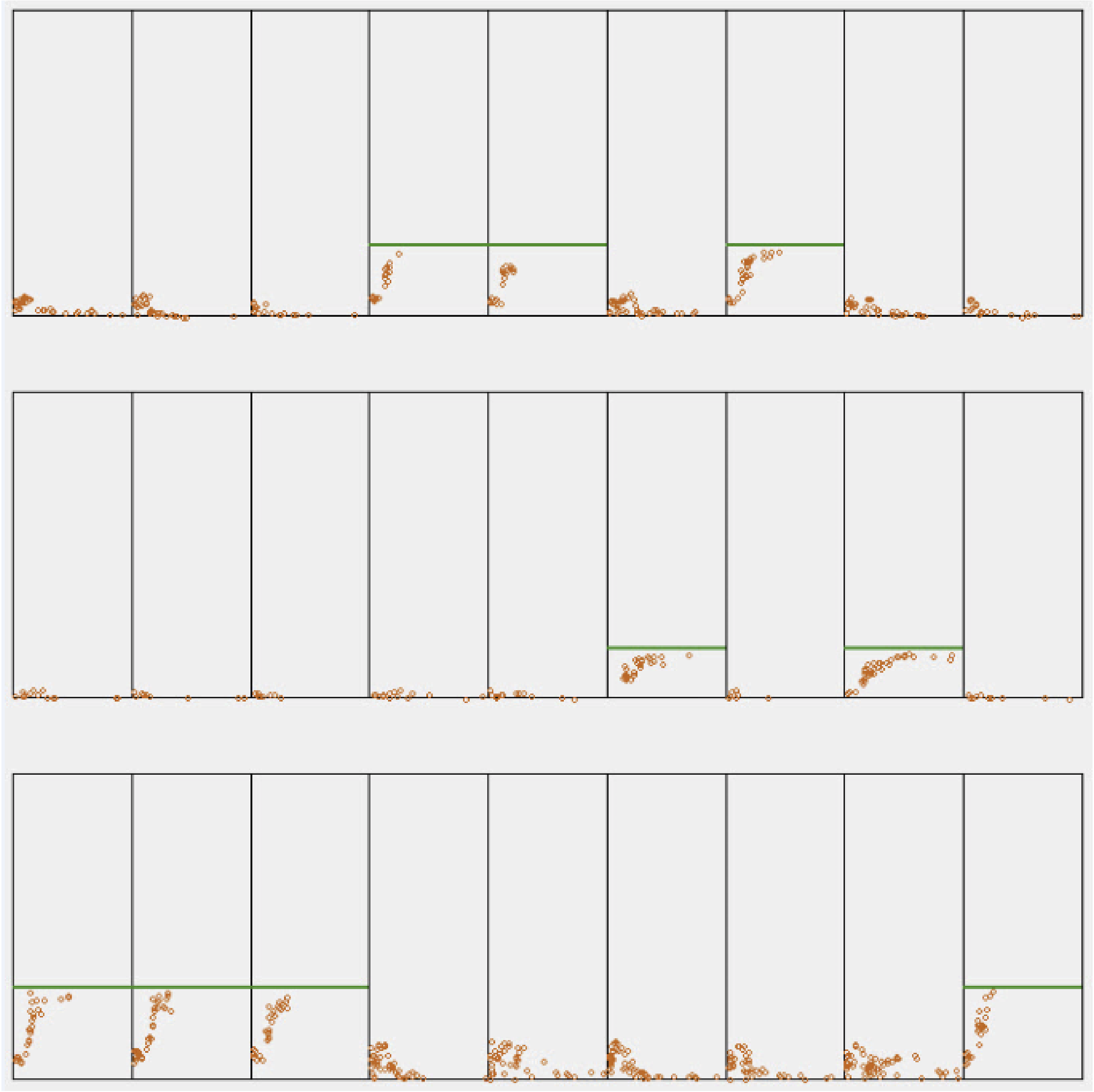


Fig. 5 Panel C: Simulated Market 4 100 runs plus median (2-token endowment)

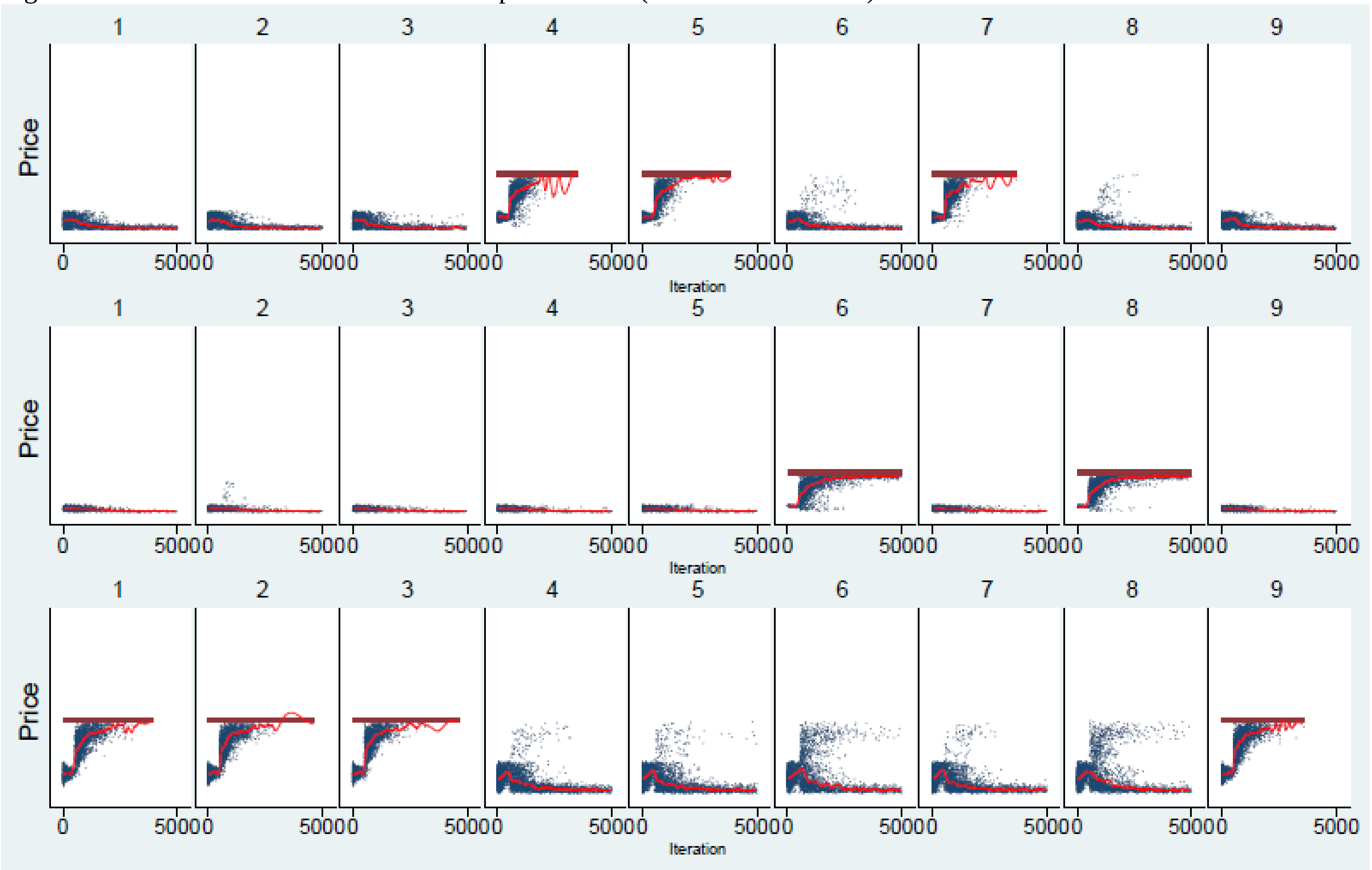


Figure 6: Information Aggregation in Series B Markets (Complete, Heterogeneous Preferences)
Panel A: PS1988 Human Market 5 (2 token endowment)

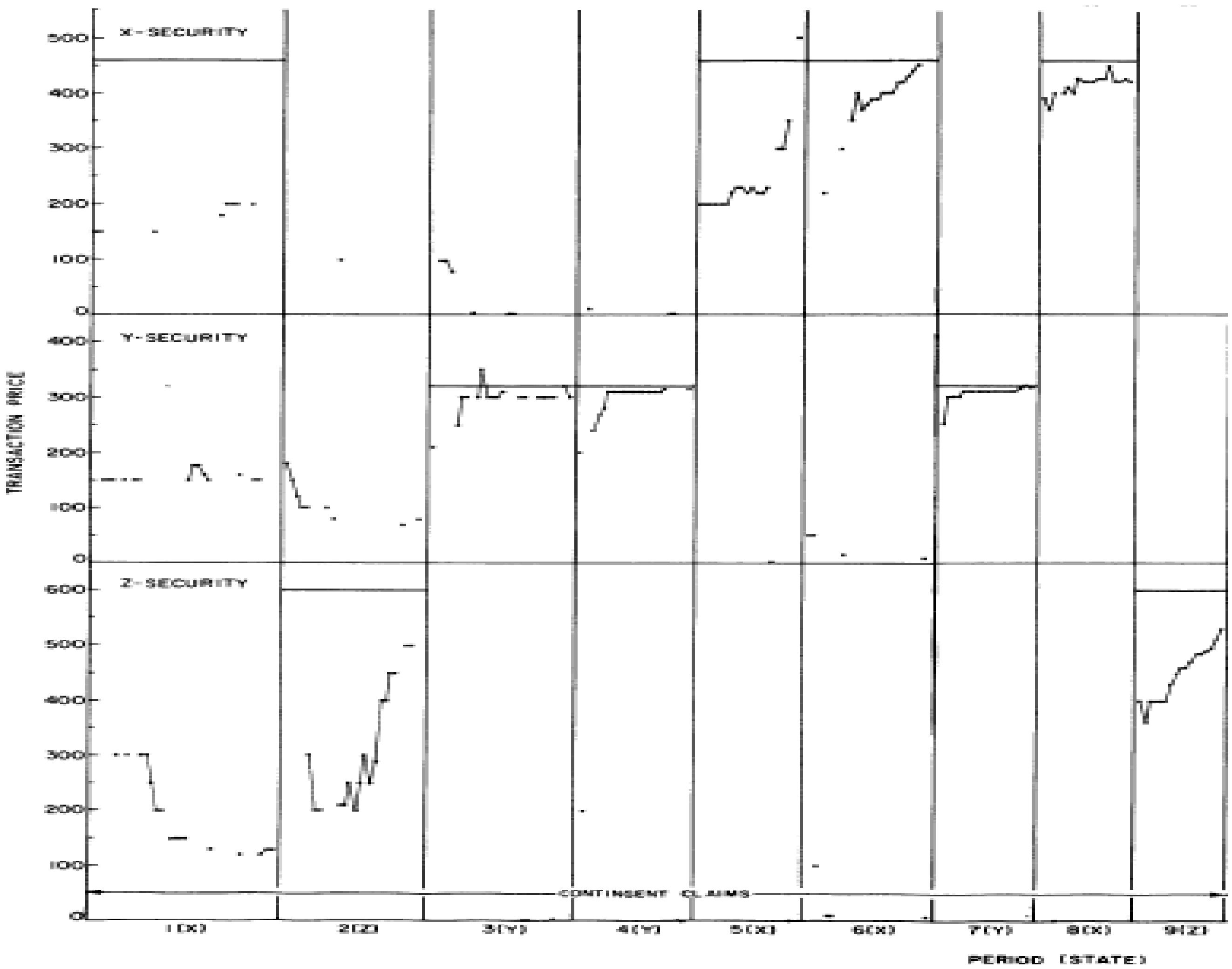


Fig. 6 Panel B: Simulated Market 5 Single run plus median (2-token endowment)

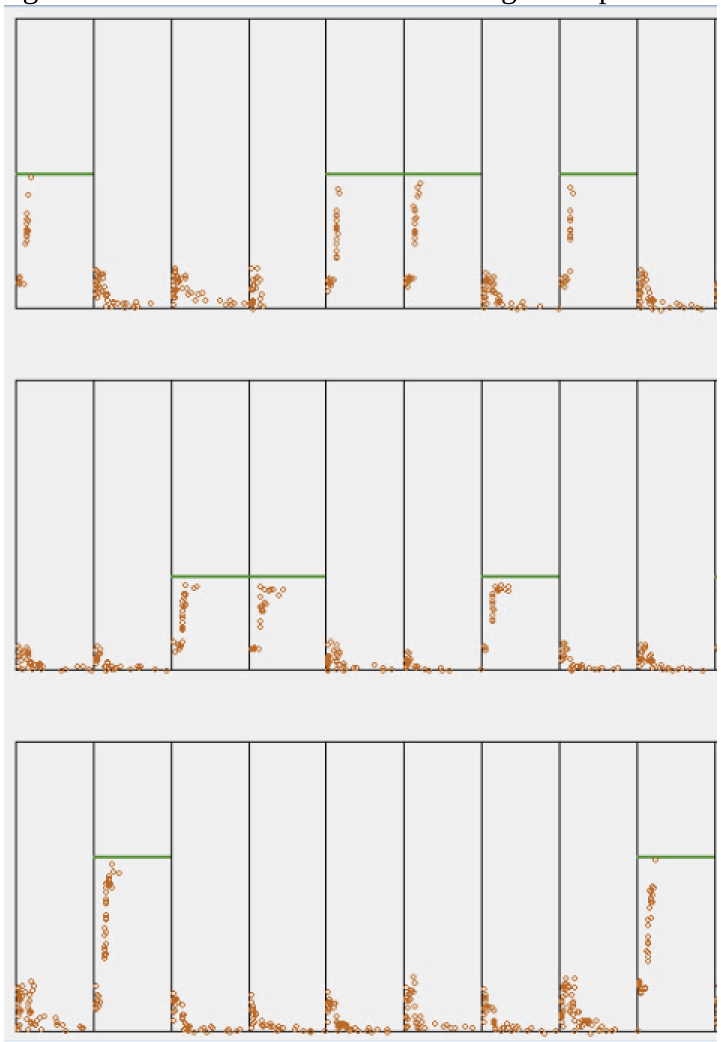


Fig. 6 Panel C: Simulated Market 5 100 runs plus median (2-token endowment)

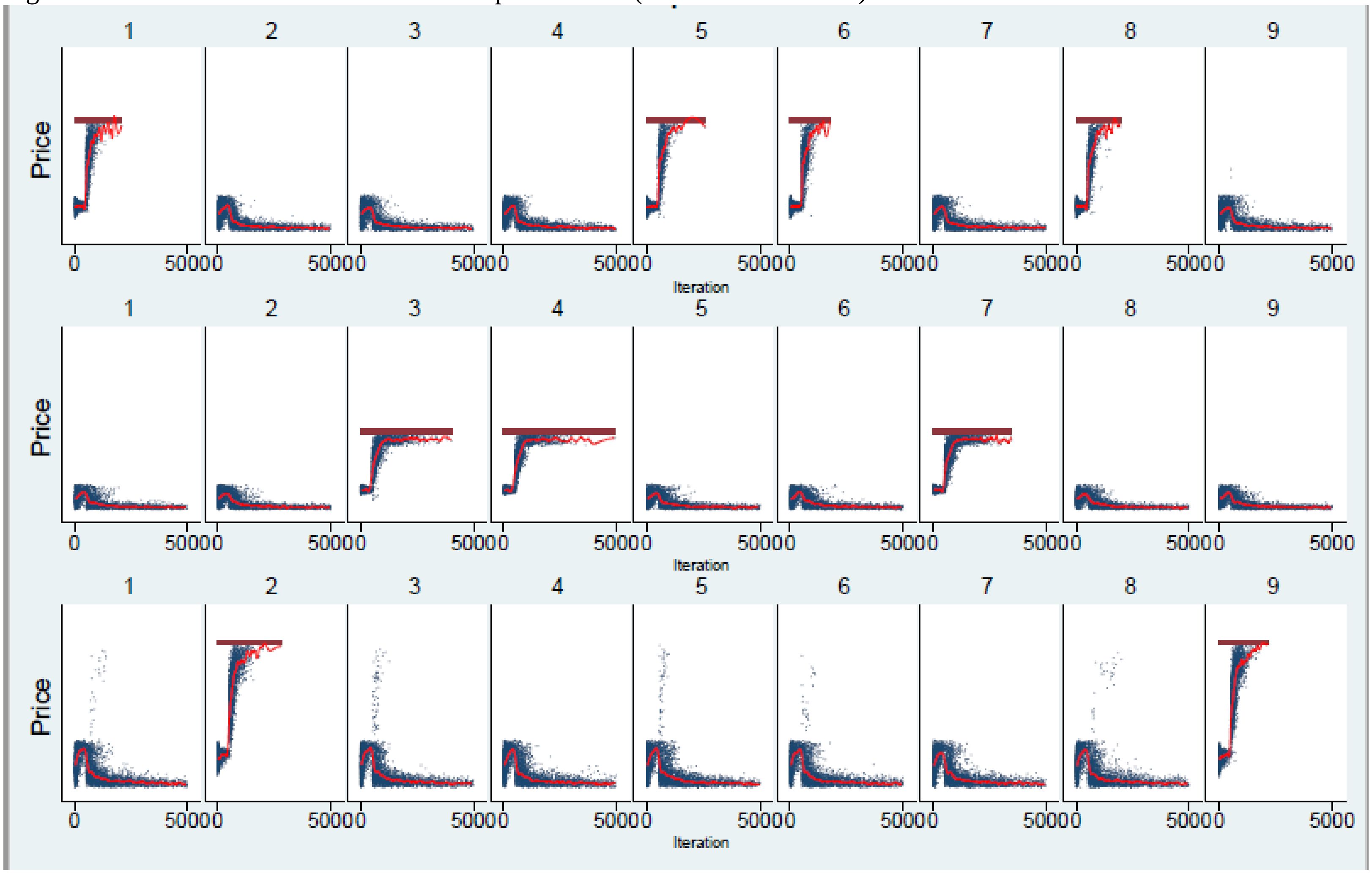


Figure 7: Information Aggregation in Series C Markets (Incomplete, Homogeneous Preferences)
Panel A: PS1988 Human Market 7 (2 token endowment)

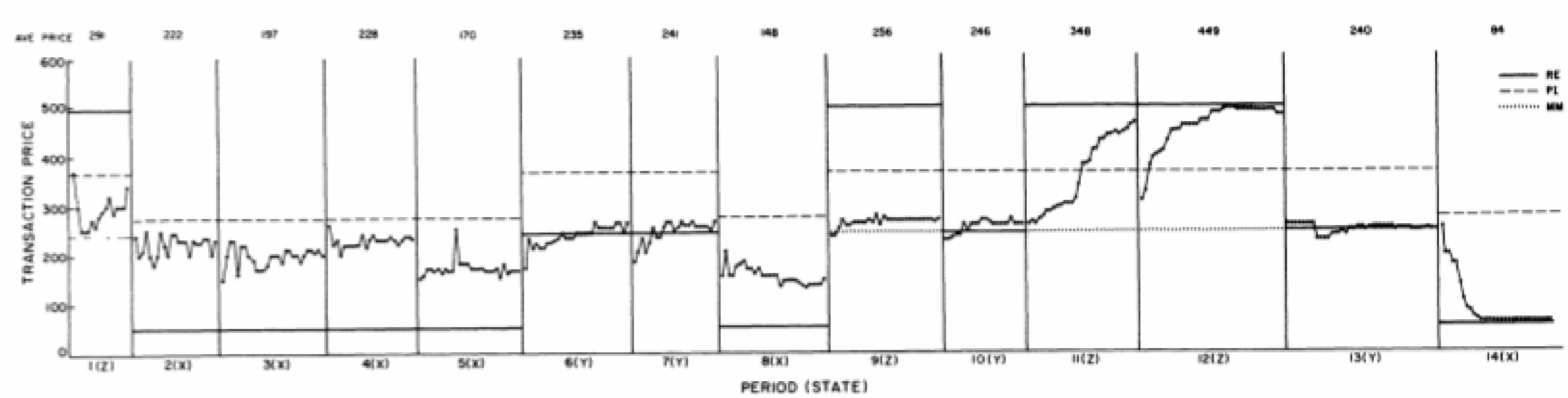



Fig. 7 Panel B: Simulated Market 7 Single Run (2-token endowment)

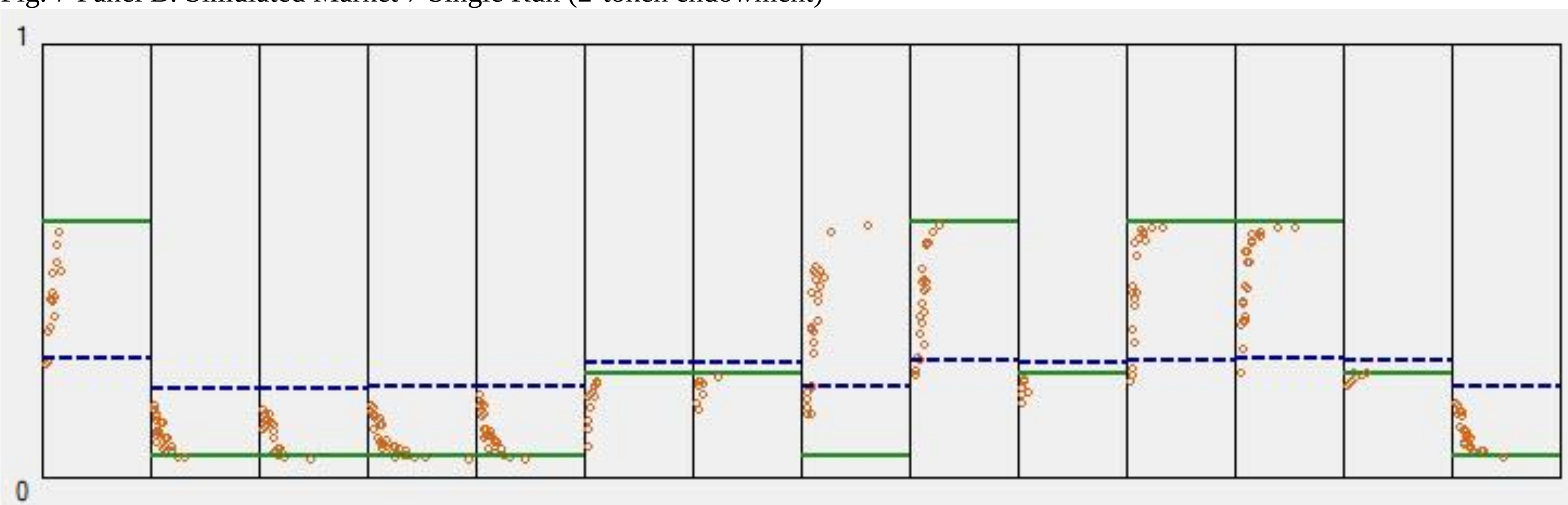


Fig. 7 Panel C: Simulated Market 7 100 runs plus median (2-token endowment)

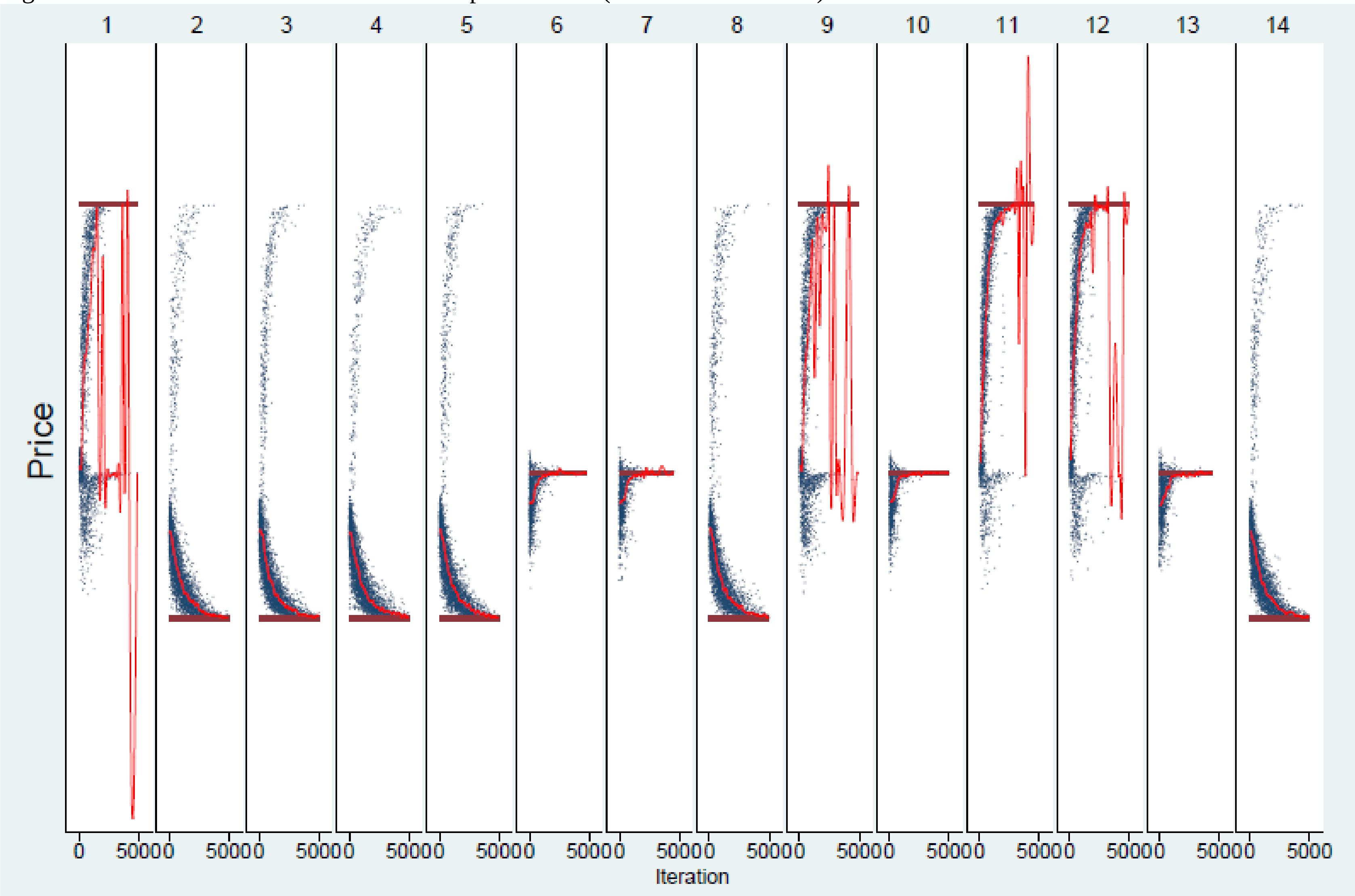


Figure 8: Information Aggregation in Series C Markets (Incomplete, Homogeneous Preferences

Panel A: PS1988 Human Market 8 (2 token endowment)

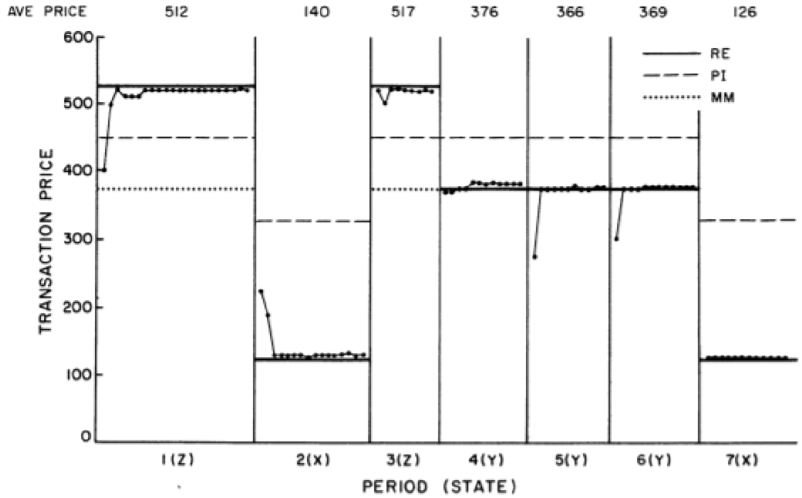



FIGURE 8.—Market 8 transaction prices.

Fig. 8 Panel B: Simulated Market 8 Single Run (2-token endowment)

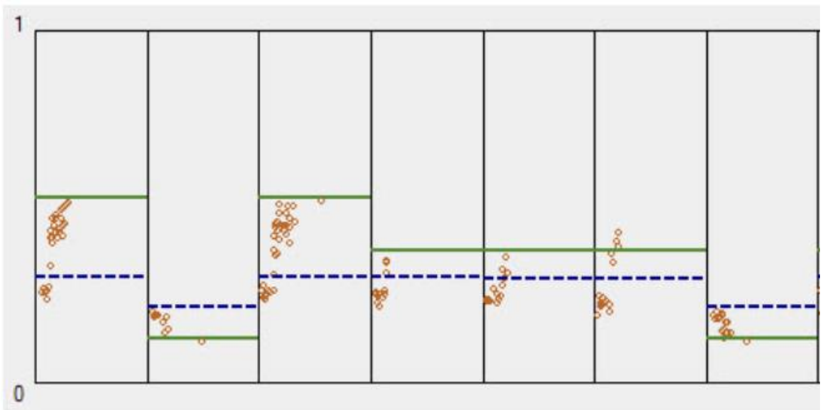
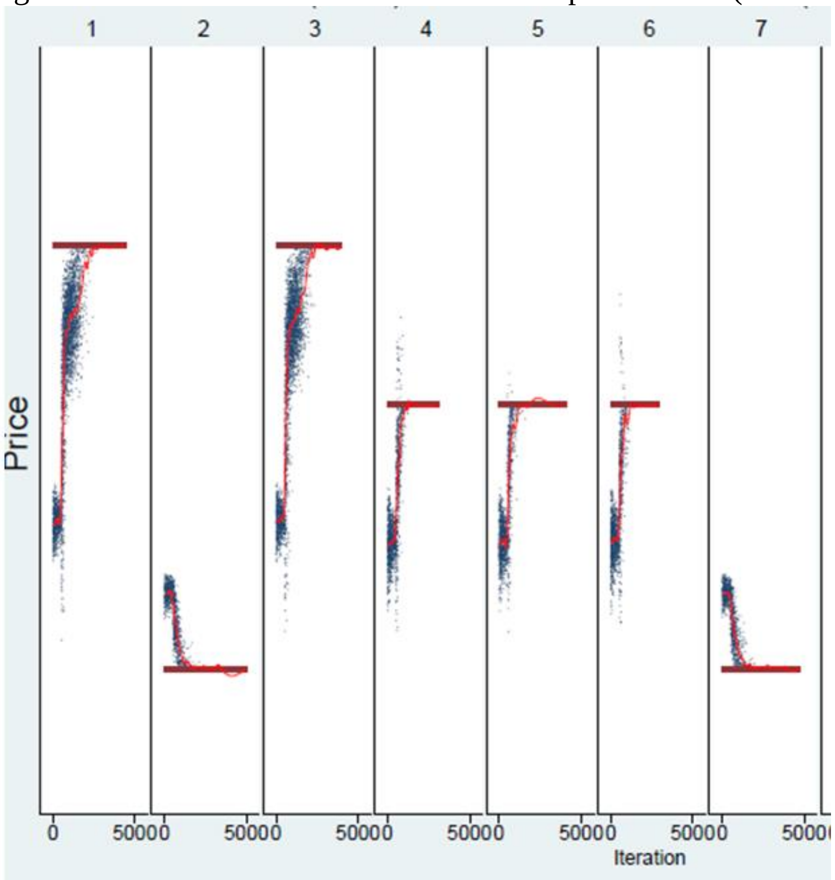


Fig. 8 Panel C: Simulated Market 8 100 runs plus median (2-tokens)



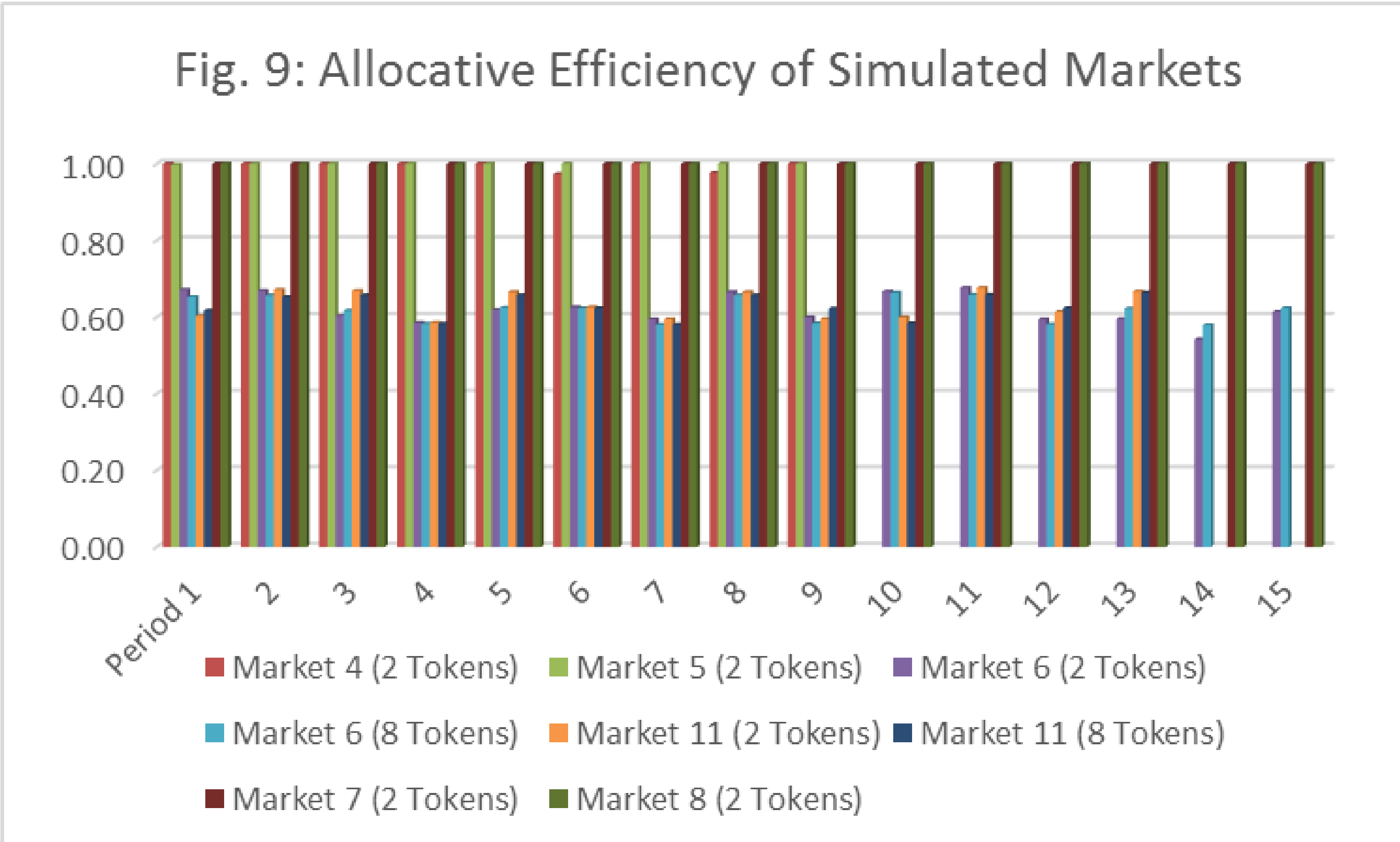


Fig. 9: Allocative Efficiency of Simulated Markets
1.00
0.80
0.60
0.40
0.20
0.00
Period 1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Market 4 (2 Tokens)
Market 5 (2 Tokens)
Market 6 (2 Tokens)
Market 6 (8 Tokens)
Market 11 (2 Tokens)
Market 11 (8 Tokens)
Market 7 (2 Tokens)
Market 8 (2 Tokens)

## Table 1 – Market Parameters

| Series | Corresponding Market | State | Probability | Dividends | | | RE Price Prediction | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Type I | Type II | Type III | 1 Security | *X* Security | *Y* Security | *Z* Security |
| Series B | Plott and Sunder 1988 Market 4 | *X* | 0.35 | 0.07 | 0.23 | 0.1 | | 0.23 | 0 | 0 |
| Complete | | *Y* | 0.2 | 0.13 | 0.09 | 0.16 | | 0 | 0.16 | 0 |
| Hetero. | | *Z* | 0.45 | 0.3 | 0.06 | 0.2 | | 0 | 0 | 0.3 |
| Series B | Plott and Sunder 1988 Market 5 | *X* | 1/3 | 0.14 | 0.46 | 0.2 | | 0.46 | 0 | 0 |
| Complete | | *Y* | 1/3 | 0.26 | 0.18 | 0.32 | | 0 | 0.32 | 0 |
| Hetero. | | *Z* | 1/3 | 0.6 | 0.12 | 0.4 | | 0 | 0 | 0.6 |
| Series A | Plott and Sunder 1988 Market 6 and 11 | *X* | 0.333 | 0.5 | 0.17 | 0.31 | 0.5 | | | |
| Incomplete | | *Y* | 0.333 | 0.24 | 0.45 | 0.19 | 0.45 | | | |
| Hetero. | | *Z* | 0.333 | 0.59 | 0.11 | 0.39 | 0.59 | | | |
| Series C | Plott and Sunder 1988 Market 7 | *X* | 1/3 | 0.05 | 0.05 | 0.05 | 0.05 | | | |
| Incomplete | | *Y* | 1/3 | 0.24 | 0.24 | 0.24 | 0.24 | | | |
| Homo. | | *Z* | 1/3 | 0.59 | 0.59 | 0.59 | 0.59 | | | |
| Series C | Plott and Sunder 1988 Market 8 | *X* | 1/3 | 0.125 | 0.125 | 0.125 | 0.125 | | | |
| Incomplete | | *Y* | 1/3 | 0.375 | 0.375 | 0.375 | 0.375 | | | |
| Homo. | | *Z* | 1/3 | 0.525 | 0.525 | 0.525 | 0.525 | | | |

**Table 2: Simulation Parameters**

| **Simulation Parameter** | **Value** |
|---|---|
| Number of Traders | 12 |
| Number of Endowed Tokens per Trader (at the beginning of each period) | 2 or 8 |
| Token Short Sale Constraint | Yes |
| Cash Endowment | 0 |
| Borrowing Constraint (i.e., Negative Cash Balance Constraint) | None |
| Total Number of Iterations per Period | 5,000 |
| Number of Iterations in Phase 1 and 2 of Each Period | 500 and 4,500 |
| Adaptive Parameter for ME Heuristic ($\gamma$) | U ~ (0,05, 0.15) |
| Probability ($\rho$) of R Heuristic in Phase 1 | 0 |
| Probability ($\rho$) of R Heuristic in Phase 2 (Drawn in Each Iteration) | $U$ ~ (0, 0.1) |
| | |