The Effective Exchange Rate, Employment and Growth in a Foreign Exchange Constrained Economy

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by

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I. INTRODUCTION

The "decade of development" bodes turning into a decade of despair. In place of the optimism regarding the development prospects of the less developed countries that marked the speeches of the early 1960's there is evolving a sense of helplessness on the part of economists and policy makers concerned with development both in the poor and advanced countries. The despair and felt helplessness is rooted, first, in growing awareness that forecasts and promises of large flows of assistance from the rich to the poor countries will not be fulfilled. It is rooted, second, in a developing belief, sharpened by the urgency of the problem and failure of past attempts to cope, that without massive assistance the less developed countries can do little on their own about lagging growth and rising unemployment.

Colombia's present economic problems or, more saliently, the way these problems and possible remedies are perceived, reveal the evolving syndrome of felt futility. Growth rates have lagged below plans. A major urban unemployment problem is developing. Vanek reaches discouraging conclusions regarding Colombia's ability to do anything about the lagging growth problem on her own.2

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He argues that, given the high import intensity of investment, a substantial increase in the growth rate requires more import capacity (more foreign aid) and that increased saving would simply increase unemployment while freezing little additional imports for investment. Currie's proposals to increase employment through public works appear, on their face, to be infeasible because of the indirect and direct requirements for imports. The foreign exchange constraint thus is viewed as blocking domestic policies to deal with growth and employment problems.

Colombia's problems may be extreme, but the same syndrome of problems and pessimistic analysis exists in Brazil, India, and many other countries. While, to lend concreteness to the discussion, in the following analysis I shall occasionally refer to Colombia, the problem and the analysis are quite general.

There is no denying that countries like Colombia will have vastly greater difficulty in coping with their problems when foreign assistance is scarce than when it is plentiful. It is the contention of this paper, however, that at least part of the felt futility resides in the persistence of looking at the constraints and policy options in terms of a model that is better suited to revealing the payoffs from more assistance than to exploring what can be done without more assistance. The blinders of the "two gap" model keep attention away from the central question of how an economy can learn to cope with an environment where imports must largely be financed through exports. In particular, it represses a powerful instrument of policy—the effective exchange rate.

Section II will examine the basic factors and assumptions that lie behind

the impotency of domestic policy alone to increase employment and investment
that is an implication of the two gap model. Section III will present a model
that incorporates factor substitution possibilities and factor prices. Section IV considers how the consumption-investment-employment choice set is
influenced by substitution possibilities and the role of prices in determining
what is achievable. Section V reconsiders the policy options and in particular considers the effective exchange rate as an instrument of policy.
II. THE BASIC TWO GAP MODEL AND THE IMPLIED POLICY DILEMMA

Considering fine structure as well as basics, the variety of "gap" models is great and growing. But underneath the apparent diversity, all the gap models have a roughly equivalent "two-gap" core, which can be developed in the following indirect but illuminating way.

Assume that an economy has available to it four basic activities: domestic production of investment goods, imports of investment goods, domestic production of consumer goods, and imports of consumer goods. A unit level of an activity can be chosen as "a dollar's worth." All activities require imports. The two production activities also require domestic inputs. The activity matrix is as follows:

\[
\begin{array}{c|cccc}
\text{Activities} & \text{IP} & \text{IM} & \text{CP} & \text{CM} \\
\hline
\text{Output} & I & 1 & 1 & 0 & 0 \\
\text{C} & 0 & 0 & 1 & 1 \\
\hline
\text{Input} & M & a_1 & 1 & a_2 & 1 \\
\text{V} & b_1 & 0 & b_2 & 0 \\
\end{array}
\]

\footnote{Among the most important articles are:}


and, the Vanek book cited above.
The column headings are the four activities, the subscripts, P and M, stand for production and imports respectively. The first two rows are output of investment goods and consumer goods, I and C. The last two rows are use of imports, M, and domestic capacity V (for value added). It is assumed, and this is important, that production of investment goods is more import and less domestic input intensive than production of consumer goods. Thus

\[ \alpha_1 > \alpha_2 > 0 \]

\[ \beta_2 > \beta_1 > 0 \]

Assume a given capacity to import, and a given domestic production capacity. The constraints on the activities then are

\[ M \geq \alpha_1 I_P + \alpha_2 C_P + \beta_1 I_M + \beta_2 C_M \]

\[ V \geq \beta_1 I_P + \beta_2 C_P \]

For our purposes we thus are ignoring any specialization of domestic production capacity between investment and consumer goods.

These constraints limit the choice set for consumption and investment available to the economy to the frontier \( a - b - d \) in Fig. 1.A. If we ignored the two direct import activities the frontier would be \( a - b - d \). But because import capacity not used up for intermediate goods can be directly converted, one to one, for consumer and capital goods, to the right of \( b \) the frontier is \( b - d \), not \( b - c \).

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1. If \( V \) and \( M \) are measured in dollars, then \( 1 = 1 - \alpha_1 \) and \( b_2 = 1 - \alpha_2 \).
2. This certainly is so for Colombia and appears so for most less developed countries. For a discussion of some possible reasons see R. Nelson, A Study of Industrialization in Colombia: Part I, Analysis, RM 5412-AID, The RAND Corporation, December 1967. It probably is not so for more developed countries. See R. Cooper, "The Balance of Payments," in R. Caves et al., Britain's Economic Prospects, Brookings, 1960.
3. If \( V \) is measured in dollars, then the slope of the \( b-d \) facet can be
If the economy is operating in the b - d range, both domestic capacity and import capacity will be fully employed, imports will be employed in production of consumer goods and investment goods, and, in addition, some consumer and investment goods will be directly imported. Along a - b import capacity will be fully utilized, but there will be unutilized domestic capacity. The asymmetry between imports and domestic production capacity is fundamental to the two-gap model. Domestic inputs need to be complemented by imports to produce final product. Imports, in contrast, can directly provide final product (as well as complement domestic inputs).

The standard two-gap model assumes a constant capital to value added (or capital-output) ratio. Although labor may be required in production as well, it is assumed labor is in excess supply. With value added proportional to the capital stock, the increase in value added then will be proportional to the amount of investment. Or (with some changes in units) the earlier figure really depicts the growth, consumption, opportunities open to the economy at any time.\(^1\) In Fig. 1.B, the investment axis has been relabeled and calibrated, and a "savings" or "minimal consumption" constraint introduced. Thus the economy is limited to areas to the right of the savings constraint, as well as below the two input constraints—that is, to the frontier e - b - d.

To bring this formulation in line with the more conventional formulation it is necessary to "get rid" of the V constraint.\(^2\) This we can do by having the horizontal axis refer to \(\frac{C}{V}\) and the vertical axis to the growth rate can be shown to be \(-1\). The slope of the a - b facet is \(-\frac{a_2}{a_1}\).

\(^1\)Specifically \(\Delta V = B I\). Obviously depreciation is being ignored.

\(^2\)By "get rid" I mean, of course, to eliminate it as a variable in the formulation of the problem. The domestic resource constraint on production always will exist (although it may not always be binding).
rate, \( \frac{\Delta V}{V} \), which is proportional to \( \frac{I}{V} \). The former \( V \) constraint now becomes a constant in the problem and over time (so long as \( b_1 \) and \( b_2 \) do not change). The two constraints in Fig. 2 that can be manipulated are the "savings" constraint and the foreign exchange constraint. We now have a "canonical" two gap formulation.

A central concern of the two gap analysis is estimating the effects of shifting one or the other of these constraints. Regarding the savings constraint, notice that the effect on the growth rate of shifting the savings constraint to the left (reducing minimal \( \frac{C}{V} \)) is significantly greater when the savings constraint is to the right of \( b \) than when (as depicted above) it is to the left of \( b \). In Colombia the import coefficient for investment goods appears about two and a half times that for consumer goods (this may actually understate the difference); thus the consumer good cost of a unit increase in investment (or a unit increase in the growth rate) is two and a half times as great to the left of \( b \) than to the right of \( b \).¹ Vanek's pessimistic conclusions regarding Colombia's ability to increase her own growth rate imply that the current position is to the left of \( b \); thus increased savings (decreased consumption) will have limited effect on growth.

And if this model does reflect reality, not only would it take a lot of reduced consumption to gain only a little added growth, there would also be rising unemployment. Although all of the imports released by cutting back consumption would be used for investment activity, the greater import intensity (lesser domestic input intensity) of investment means that the domestic resources so released would not be fully reemployed in investment.

¹See footnote 3, p. 5. We are assuming here that \( a_1 = .5, a_2 = .2 \). These are rough estimates of the direct and indirect dollar (or peso) import content per dollar (or peso) of investor and consumer goods respectively in Colombia.
Thus employment and growth objectives are in conflict.

The relative returns to "more foreign assistance" (holding consumption constant) also vary depending on whether the economy is operating to the left or right of b. In Fig. 2 notice how much more the frontier is shifted upward to the left, as compared with to the right of b, as a result of an increase in import capacity from $M_1$ to $M_2$.

In the model, as posed above, an increase in savings, like an increase in import capacity, will always permit an increase in the growth rate. Thus it is misleading to say that one or the other of these constraints is not binding at any time. Nevertheless there clearly are sharp differences in the returns to more savings and returns to more imports, in terms of an enhanced growth rate, depending on whether one is to the left or right of b.

In a loose sense one can say that to the right of b more savings is needed; to the left of b more import capacity is needed. This is the spirit of the two-gap distinction.

The foreign exchange "gap," which Vanek (and others) attempt to estimate, can be defined as the increase in import capacity needed to achieve a given growth target, given the savings constraint. If, in Fig. 2, the growth target is $g$, the foreign exchange gap as a function of $V$ is proportional to the distance between $M_1$ and $M_2$. That is, if in the absence of assistance the foreign exchange constraint is $M_1$, an increase in foreign exchange proportional to $G$ on the vertical axis will be just sufficient to enable the growth objective to be met, given the existing savings.

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1In some of the earlier formulations imports used in the production of consumer goods were ignored. Thus $a_0$ was assumed to be zero—hence the curve of the import constraint to the left of b is flat. In this case reduced consumption does not permit any increase in investment, once consumer goods imports have been eliminated (that is, to the left of b).
(As a bonus the unemployment rate would be reduced as well). Of course the "gap" would be smaller if the savings constraint could be pushed left. But given that the constraint already is to the left of b, it takes a lot of increased savings to gain only a slight reduction in the "gap."

The two gap model, together with evidence (or assertions) that the economy is to the left of b, is ingeniously well designed to provide a case for more foreign aid. But it is diabolically well designed to engender a sense of helplessness on the part of domestic policy makers regarding what they can do without additional aid. When more aid can be wrung, the two gap allegory is a useful one. When more aid is not likely, it is imperative to develop another way of looking at the problem that illuminates additional options.

The two-gap model assumes fixed proportions. For consideration of short run options this assumption probably is quite realistic, although even for the short run overly strong. When the focus is on the long run, the assumption of no substitution possibilities clearly is absurd. Indeed, for at least the last 20 years the basic thrust of Colombia's industrial development has been import substitution, and policy has deliberately and effectively stimulated this process. Several of the two-gap models do augment the basic core with a time trend in the coefficients.

1 Specifically, the foreign exchange gap in dollars as a fraction of V is \( \frac{a_1}{B} G \) where B is the value added to capital ratio, \( a_1 \) is the import intensity of investment, and \( M_2 - M_1 \) is the vertical distance between the \( M_2 \) and \( M_1 \) constraints. This can be shown as follows. The required increase in the growth rate, given the savings constraint, is \( G \). This will require an increase in \( \frac{1}{V} \) of \( \frac{G}{B} \). The import requirements are \( \frac{a_1}{B} G \).
Yet when focusing on the long run, and factor substitution is admitted—indeed is a major objective of policy—the question of incentives, costs, and factor prices, repressed in the conventional formulation, begs for attention. Colombia's problems are not new to her; indeed Colombia has been suffering from a shortage of imports, and a growing unemployment problem, since the coffee crisis of the mid-1950's, and with extreme severity for at least the last six years. In such a chronic situation the cost of imports relative to domestic factor prices ought to matter. Is it possible that Colombia has compounded her problems, and failed to consider an important path toward alleviating them, by having too high an effective exchange rate?
III. A NEOCLASSICAL INPUT, OUTPUT, AND PRICE MODEL

It seems apparent that any model designed to explore the possibility of substituting domestic inputs for imports (and for exploring the possibility of increasing import capacity through exports) must incorporate explicitly the following characteristic of the Colombian situation. Imports are largely intermediate goods in consumer or producer goods industries, and direct purchases of capital goods, mostly machinery. Thus if one thinks of the economy as having two sectors—one producing consumer goods, the other concerned with building new plants and equipment—the key substitution possibilities are domestic resources—both capital and labor—for imports in both, not substitution of domestically produced consumer goods for imported final consumer goods. And the analysis of these substitution possibilities must recognize that one of the domestic inputs—capital—itself has an import content. Similarly in exploration of export opportunities one must recognize that imports go into exports, both through intermediate inputs and capital goods.¹

This makes for a rather complicated model. But it seems essential to build in this key characteristic explicitly and make simplifications elsewhere. One of the major simplifications that has been required is the use of Cobb-Douglas relationships throughout. The principal reason is that the structure of import demand described above leads to a web of multiplicative relationships that are tractable under Cobb-Douglas assumptions, and next to impossible under other forms. There certainly are difficulties with using Cobb-Douglas throughout; in particular one would be tempted to assume a less than unitary elasticity of substitution between domestic inputs and imports

¹For a more detailed discussion and documentation see Nelson (ibid).
in investment.  However this assumption does bring one partially offsetting bonus. The parameters of a Cobb-Douglas model (like the parameters of a fixed coefficient) are easily estimated.

A second simplifying assumption is that the capital-labor ratio does not differ for investment and consumption. More explicitly, it will be assumed that there are three basic inputs—capital, labor, and imports, which are employed in both consumption and investment activity. However, the domestic input of capital and labor can be viewed as producing a domestic value added input as follows:

\[ V = A L^a K^{1-a} \]  

where \( L \) and \( K \) stand for labor and capital respectively, and \( V \) is value added in real terms (the meaning of which will be apparent shortly). \( A \) is a productivity index, which may change over time. Consumption goods are produced from domestic inputs of capital and labor, and imports, as follows:

\[ C = Z M^\delta \frac{V^{1-\delta}}{c} \]  

where \( C \) and \( M \) are consumer goods and imports and \( V \) was defined above. \( Z \) is a constant that will not be important in the following analysis; all such unimportant constants will be denoted by \( Z \) without a subscript. New capital goods, \( I \), are produced according to the following production function:

\[ I = Z M^b \frac{V^{1-b}}{I} \]  

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1It can be argued that a Cobb-Douglas production function provides a good first approximation to the substitution possibilities in a more general neoclassical model, provided that the (arc) elasticity of substitution is not very low or very high, and the changes in factor prices are not great. See R.R. Nelson, "The CES Production Function and Economic Growth Projections," Review of Economics and Statistics, August 1965.
The Z in Eq. (3) will in general not be the same as in Eq. (2). Notice that both (2) and (3) can also be written in terms of the basic factors of production. The simplifying assumption that permits (2) and (3) to be written in terms of M and V (as well as in M, L, and K) is not necessary, but very convenient.

Note that Eqs. (2) and (3), if taken literally, imply that all imports are intermediate goods. However, they can be interpreted as admitting the possibility of final, as well as intermediate, good imports, but constraining the mix of imported and domestic investment and consumption goods than can be sold (prices will be introduced later).

Also note that the model, if taken literally, assumes that domestic resources are perfectly fungible between investment and consumption. (This also is a characteristic of the basic two-gap model discussed in the preceding section). This is not as bad as it seems. As will be discussed at some length later, the characteristics of the model imply that domestic resources, transferred from consumption to investment, or the other way, yield decreasing returns (yield increases in output at the expense of rising marginal cutbacks in the other). Thus the model can be interpreted as implying a degree of specificity of domestic resources. Put another way, the specification of output as a Cobb-Douglas function of domestic inputs and imports (with different coefficients for consumption and investment) captures the specificity of domestic resources.

\[ C = Z M^a L^{1-a} K^{1-a}(1-\delta) \]
\[ I = Z M^b L^{a(1-b)} K^{(1-a)(1-b)} \]
In order to lend analytic bite to the model it is necessary to make tractable assumptions about pricing and factor choice. Unfortunately I have not yet been able to develop an alternative to the competitive assumptions that permits reasonably easy crank turning. Therefore, it will be assumed that producers act as if labor is supplied elastically, that product markets are elastic, and given the existing capital stock hire labor up to the point where the value of the marginal product equals price. It also will be assumed that investors demand a certain rate of return on capital and that the capital stock is adjusted to achieve that rate of return; thus this is a long run equilibrium model.¹

Let \( W \) be the going wage rate, \( i \) be the equilibrium rate of return on capital, \( P_I \) be the price of capital goods, and \( r = P_I \) the "rental" rate of a unit of capital. Then we have the following equilibrium relationships between inputs and outputs, and their prices.

\[
\frac{L}{V} = \frac{aP}{W} \quad (4)
\]

\[
\frac{K}{V} = \frac{(1-a)P}{r} \quad (5)
\]

where \( P \) is the "price" of domestic value added. Note that (4) simply reflects the labor employment assumption, and (5) is the condition under which there will be a rate of return, \( i \), on capital.²

The relationships can be viewed in three ways. First, they are the familiar "constant share" implications of a Cobb-Douglas competitive model.

¹This "equilibrium" rate of return will later be treated as a variable, as will the wage rate.

²These equations of course are derived from solving for the conditions under which the value of the marginal product of labor equals the wage rate, and the value of the marginal product of capital equals capital's rental rate.
Second, at the same time they are estimating equations for $\alpha$ and $(1-\alpha)$ which are, simply, the observed shares at any time. As discussed above, this simplicity of estimation is one of the motivations for the Cobb-Douglas formulation. Third, they are equations for the input coefficients (as a function of prices).

From the consumer goods production function:

$$\frac{M_C}{C} = \frac{\delta P}{E} \quad (6)$$

$$\frac{V_C}{C} = (1-\delta) \frac{P_C}{P} \quad (7)$$

where $P_C$ is the price of consumer goods, and $E$ the exchange rate. And, from the investment goods production function:

$$\frac{M_I}{I} = \frac{b P_I}{E} \quad (8)$$

$$\frac{V_I}{I} = (1-b) \frac{P_I}{P} \quad (9)$$

$P_I$, as above, is the price of a new capital good. In these formulations the value of inputs is proportional to the value of outputs;
not as in the fixed coefficient model quantity proportional to quantity.

And $\delta$ and $b$ can be estimated from one observation.

Finally we can derive equations for the prices of various outputs as a function of the prices of various inputs. First we can write:

$$p = \frac{Z}{A} W^{\alpha} r^{1-\alpha}$$  \hspace{1cm} (10)

$$p_c = Z E^\delta p^{1-\delta}$$  \hspace{1cm} (11)

$$p_I = Z E^b p^{1-b}$$  \hspace{1cm} (12)

It remains to get (10)-(12) in terms of the prices we will treat as basic—the rate of return on capital, the wage rate, and the exchange rate.

1The derivations are straightforward if tedious. For the first:

(1) \[ P = \frac{L}{V} W + \frac{K}{V} r \]

(2) \[ \frac{V}{K} = A \left( \frac{L}{K} \right)^{\alpha} \quad \text{so} \quad \frac{K}{V} = \frac{\left( \frac{L}{K} \right)^{1-\alpha}}{A} \quad \text{from Eq. (1) in the text.} \]

(3) \[ \frac{L}{K} = \left( \frac{\alpha}{1-\alpha} \right) \frac{r}{W} \quad \text{from Eqs. (4) and (5) in the text.} \]

thus:

\[ P = \frac{\left( \frac{\alpha}{1-\alpha} \right)^{1-\alpha} \left( \frac{r}{W} \right)^{1-\alpha} W + \left( \frac{\alpha}{1-\alpha} \right)^{-\alpha} \left( \frac{r}{W} \right)^{-\alpha} r}{A} \]

(4) \[ P = \frac{\alpha \left( \frac{W^\alpha}{\alpha} \left( \frac{r}{W} \right)^{1-\alpha} \right) + (1-\alpha) \left( \frac{W^\alpha}{\alpha} \frac{r}{W} \right)^{1-\alpha}}{A} \]

(5) \[ P = \frac{Z}{A} W^{\alpha} r^{1-\alpha} \]

The other two equations are derived in the same way.
Recall that $r = P_1i$. Assuming foreign prices are fixed, using (10)-(12) one can derive:

\[
P = \frac{(1-\gamma) \chi b(1-\gamma)}{Z i \lambda W \gamma \epsilon \lambda} \frac{1}{A^\lambda}
\]

(13)

\[
P_c = \frac{(1-\gamma)(1-\epsilon) \chi(1-\epsilon) (1-\omega)b+\omega b}{Z i \lambda W \gamma \epsilon \lambda} \frac{1}{A^\lambda}
\]

(14)

\[
P_\lambda = \frac{(1-\gamma)(1-b) \omega(1-b) b}{Z i \lambda W \gamma \epsilon \lambda} \frac{1}{A^\lambda}
\]

(15)

where $\lambda = \alpha + b - \alpha b$. With $\alpha$ and $b$ both positive but less than one, so must be $\lambda$. Two other price equations will be useful. First the "rental" on capital:

\[
r = \frac{\omega(1-b) b}{Z i \lambda W \gamma \epsilon \lambda} \frac{1}{A^\lambda}
\]

(16)

\[1\]The first equation is derived as follows:

1. $P = \frac{Z_i W^{\alpha} [i Z E^b P^{1-b}]^{1-\gamma}}{A}$

where the term in brackets is the substitution for $r = P_1i$ from Eq. (12).

2. $P(\alpha b - b\omega) = \frac{Z_i W^{\alpha} [i Z E^b P^{1-b}]^{1-\gamma}}{A}$

3. $P = \frac{Z i \lambda W \gamma \epsilon \lambda}{A^\lambda}$
Second, the real wage rate—that is the money wage rate divided by the price of consumer goods:

\[
\frac{W}{P} = \frac{W^*}{E} = 2 \frac{1}{\lambda} \frac{\frac{W_0}{c} (1-\gamma)}{(1-\tau)} \frac{1}{\lambda} \frac{(1-\alpha)\beta + h}{\lambda} \tag{17}
\]

Notice how the structure of imports, described earlier and explicitly built into the model, is reflected in the price equations. Even the price of "domestic inputs" is not independent of the exchange rate; the reason, of course, is the effect of the exchange rate on the price (and rental) of capital as shown in Eqs. (15) and (16). The exchange rate influences the price of consumer goods through both the price of machinery and the price of intermediate goods imports. Thus the real wage (Eq. (17)) involves the exchange rate, as the diatribe relating to the effect on the cost of living of devaluation has stressed. And if, as we shall develop later, it is assumed that demand for Colombian exports are sensitive to the exchange rate, one cannot ignore that the cost of exports may be as well.

The model obviously is strictly neoclassical. It assumes competitive factor and product markets in the short run, and the equations relate to long run equilibrium conditions (in the sense that the capital stock is just such that the target rate of return, \( \lambda \), is achieved). This is a bothersome set of assumptions. I do not know the extent to which the implications derived in the following sections are sensitive to either the Cobb-Douglas specification, or the short run competitive equilibrium assumption, or the long run equilibrium
assumption. The specific quantitative results certainly are. I suspect, however, that most of the qualitative results would hold under much less restrictive, and much more realistic, assumptions. Certainly the implications above regarding prices are qualitatively what one would expect. And these would appear to be sufficient to generate the qualitative results on inputs and outputs derived in the following sections.

An important caveat is on the time frame of the model. The dual assumptions of a considerable (unitary) elasticity of substitution among the factors, and of short and long run equilibriums mean that the effects of changes in factor prices examined in the following sections cannot be assumed to occur overnight. The model is concerned with the long run implications of these changes. I admit I do not know how long the long run is. My conjecture is that something like a five year adjustment period is involved.
IV. THE CONSUMPTION-INVESTMENT CHOICE SET RECONSIDERED

It now is possible to reexamine the consumption-investment choice set within a model that admits input substitution. Recall the following specification of input coefficients in the production of investment and consumer goods in the presentation of the fixed coefficient two-gap model,

\[
\frac{M_I}{I} = a_1 \quad \frac{M_C}{C} = a_2 \quad \frac{V_I}{I} = b_1 \quad \frac{V_C}{C} = b_2 .
\]

Assuming the same capital labor ratios in both consumer and producer goods production:¹

\[
\frac{K}{V} = c_1 \quad \frac{L}{V} = c_2 .
\]

Ignoring the direct import activities the constraints of the two-gap model can be rewritten:²

\[
M \geq a_1 I + a_2 C
\]

\[
K \geq c_1 b_1 I + c_1 b_2 C
\]

\[
L \geq c_2 b_1 I + c_2 b_2 C .
\]

In terms of capital actually employed, \( K^* \);

\[
L \geq \frac{c_2}{c_1} K^*
\]

where, of course, \( K \geq K^* \).

¹ Examination of capital-output ratios in consumer and producer goods industries suggests that this is a reasonable first approximation.

² The analysis will be focused on the region to the right of "b" in Fig. 3; hence there will not be any direct imports.
In our new formulation the constant input coefficients are turned into variables, by Eqs. (4)-(9). Equations (10)-(16), in turn, permit these input coefficients to be written in terms of the basic prices $i, W,$ and $E$. The constraints then take the form:

\[ M \geq \frac{(1-\alpha)(1-b)}{\lambda} \left( \frac{\nu}{E} \right) + \frac{(1-\gamma)(1-\delta)}{\lambda} \left( \frac{\nu}{E} \right) - \frac{(\alpha+\delta)}{\lambda} \left( \frac{\nu}{E} \right) \]

\[ K \geq (1-\alpha)(1-b)i^{-1} I + \frac{Z(1-\alpha)(1-\delta)}{\lambda} \left( \frac{\nu}{E} \right) \]

\[ L \geq \frac{Z i}{\lambda} \left( \frac{\nu}{E} \right) \]

The relationships have the derivatives one would expect. Import requirements of a given $C$, $I$ bill are negatively related to the

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1. The first coefficient is derived as follows:

\[ a_1 = \frac{M}{I} = \frac{b \nu}{E} \]

But

\[ P = \frac{(1-\gamma)(1-b)}{\lambda} \left( \frac{\nu}{E} \right) \frac{b}{1-b} \]

Noting that $\lambda = \alpha + b - \alpha b$ and simplifying

\[ a_1 = \frac{b Z i}{\lambda} \left( \frac{\nu}{E} \right) \]
exchange rate, positively related to domestic factor prices \( W \) and \( i \), reflecting the possibilities to substitute domestic inputs for imports. Capital requirements in the production of consumer goods are positively related to the wage rate, reflecting that a rise in labor costs induces substitution of capital for labor. They are negatively related to the exchange rate which influences the cost of a new machine, and to the expected rate of return on capital. Capital requirements per capital good produced are related only to the expected rate of return. At first glance this is a strange result, but it reflects simply the requirement that capital yield its "own" rate of return in the production of new capital (and the Cobb-Douglas assumptions). Labor employed per unit of capital depends negatively on \( W \), and positively on the exchange rate (for the reason above) and on \( i \).

For any given \( \frac{W}{E} \) and \( i \) there is a set of linear constraints on \( C \) and \( I \), just as in Section II. A number of arbitrary constraints, \( Z \), in effect permit us to define units as we wish. Thus we can define both consumption and investment units as one peso or dollar's worth at some set of basic input prices, just as in Section II. Then, if we assume that capital goods are more import intensive and less domestic input intensive than consumer goods \( (b > 6) \), the \( M \) constraint, and the \( K \) and \( L \) constraints, will have the relative slopes shown in Fig. 3. The special form of Eqs. (1)-(3), which together with cost minimization imply the same capital-labor ratio in both consumer and capital goods production, means that the labor and capital constraints have the same slope. The \( V \) constraint of Section II can be interpreted as the more binding of the \( K \) and \( L \) constraints in Fig. 3.

With two products, three inputs, and a given set of factor prices, in general full employment of all inputs will be impossible. Thus in Fig. 3
there always will be unemployed labor. Further, unless the mix of output is right, one of the other inputs will be underemployed as well. At a high investment rate the import constraint will be binding and capital will be slack; at a high consumption rate the other way around. Of course to the right of b we cannot ignore direct imports of consumer and investor goods, but this range will not concern us in the following analysis. Thus we have, in effect, replicated the two-gap model.

But Fig. 3 assumes a given set of factor prices. We now are able to examine the effects of factor prices on the conclusions of the two-gap model. As factor prices change, the constraints will shift. A decrease in $\frac{W}{E}$ or $i$ will reduce the imports utilized for a given output and shift out the foreign exchange constraint.\(^1\) Capital constraint can be shifted out by a decline in $\frac{W}{E}$, or an increase in $i$. The labor constraint can be shifted out by a rise in $\frac{W}{E}$ or a decline in $i$. There exists an $i, \frac{W}{E}$ combination such that, for any consumption-investment ratio, full employment of all three factors is possible and will be achieved. Further, if these factor prices obtain, the output of the economy, given its $M,K,L$ endowments, is maximal along that $C, I$ ray.

To see this, assume a given target ratio of $\frac{I}{C}$. Then, ignoring the labor constraint, one can plot alternative $i, \frac{W}{E}$ combinations that will assure full utilization of the capital stock and full use of given foreign exchange. The $K, M$ curve of Fig. 4 slopes down from left to right reflect-

\(^1\) A change in $\frac{W}{E}$ will not change capital requirements if all of output is investment.
ing that an increase in either \( i \) or \( \frac{W}{E} \) will induce a rise in \( \frac{M}{K} \).

Therefore to maintain the same ratio, an increase in one must be compensated by a decline in the other. Similarly, ignoring the import constraint, a curve showing alternative \( i, \frac{W}{E} \) pairs that will achieve full utilization of capital and full employment can be drawn. The \( K, L \) curve slopes up from left to right reflecting that a higher \( \frac{W}{E} \) which will itself reduce \( \frac{L}{K} \), requires a higher \( i \), which itself would increase \( \frac{L}{K} \), for the same capital-labor ratio to be cost minimizing. The two curves must cross.\(^2\) Thus in the flexible coefficients model, full utilization of \( M, K \) and \( L \) is possible, for the right factor prices. In terms of Fig. 3, for some set of factor prices, the three constraints can all be made to go through the same point.

Output, for a given \( C, I \) ratio, will be maximal only at the intersection set of factor prices. If factor prices are in region 1 of Fig. 4, there will be excess demand for foreign exchange, together with unemployment of both labor and capital. In region 3, there will be excess demand for domestic labor and capital, and excess supply of imports. Regions 2 and 4 also will be marked by unemployment of at least one factor. Since our basic Cobb-Douglas model implies that the marginal productivity of all factors always is positive, it is clear that output cannot be maximal except at the

\(^1\)From Eqs. (6), (7), (4), (5), and (16), one can solve for the import capital ratio in consumption. From eqs. (6), (9), (4), (5), and (16), one can solve for the ratio in investment. Both equations have the form

\[
\frac{M}{K} = Z i^\lambda \left( \frac{W}{E} \right)^\frac{\alpha(1-b)}{\lambda}
\]

\(^2\)The third curve in the figure, that of constant real wages, will be discussed later.
equilibrium set of factor prices.

One important implication is that, with neoclassical substitution possibilities available, the activity analysis frontier cannot be efficient; there is available a neoclassical frontier that dominates it. In Fig. 5 the inner activity analysis frontier represents the case of unemployed labor, even at point b. It replicates Fig. 4. The analysis above demonstrates that for another set of factor prices output can be greater along the ray. The outer activity analysis frontier, through point c, is drawn for the set of factor prices that will generate full employment of labor as well as capital and imports for the consumption-investment ratio indicated by points b and c. But for other C, I ratios, these factor prices will not be efficient, for there will be unemployment of either domestic resources or imports. The neoclassical frontier (the dashed curve) is a full employment of all resources frontier; along the activity analysis frontier full employment of all resources occurs at most at one point.

In the following section we shall map out how factor prices must vary to achieve different C, I ratios along the frontier. Suffice it to flag here that the K, M curve of Fig. 4 has been drawn assuming a given C, I ratio. As that ratio changes, so does the output maximizing and full employment generating set of factor prices.

Thus far we have ignored the possibility that export earnings, and hence import capacity, may also be sensitive to factor prices. Assume

$$M = B + Z \left( \frac{P_c}{B} \right) - \epsilon$$

(21)
B can be interpreted as net foreign borrowing. The form of the second term on the right hand side of (21) assumes that only consumer goods are exported, and that foreign exchange proceeds of exports are positively related to the exchange rate, negatively related to their price in home currency.¹

There is no reason, of course, why the exchange rate for exports has to be the same as the exchange rate for imports. For the purposes here, however, it is useful to view the two rates as being proportional (perhaps equal) to each other. If this is assumed, then it easily is seen that the introduction of an exchange rate sensitive export terms to the import supply constraint simply makes the K, M curve steeper around any observed point. For, solving for \( \frac{P_c}{E} \) and substituting

\[
M = B + g \left[ \frac{(1-\alpha)(1-\delta) \alpha(1-\delta)}{E} \right]^{-\epsilon}
\]

(22)

Imports thus will be greater at a low \( \frac{W}{E} \), and less at a high \( \frac{W}{E} \), than were import capacity insensitive to the exchange rate.

With exports admitted, the C, I frontier of Fig. 5 is not, of course, the production possibility frontier. Rather, if it is to depict the consumption-investment opportunities available to the country as a whole for a given K, L endowment, and net foreign borrowing of B, the C of the frontier represents production of consumer goods minus exports. The corresponding I represents investment that is possible given domestic resources and imports (including those made possible by export proceeds, minus those resources and imports employed in the production of consumer goods for home use, C, and

¹A necessary condition for a positive ε of course is that world demand is of greater than unitary elasticity.
Exports.

Pulling together the results of this section, we have derived two important implications of admitting substitution possibilities to the basic two-gap model. First, the phenomenon of underutilization of domestic resources at high investment rates no longer is necessary. As we shall see in the following section (but which already should be intuitively obvious), a shift toward investment will require a rise in $\frac{E}{W}$ if employment of domestic inputs is to be maintained. But factor price adjustment can replace the higher unemployment adjustment of the "no substitution" two-gap models. Second, the consumption possibility frontier is both pushed outward and smoothed. The frontier is still concave; that is, the consumption costs of increasing investment rise as one moves from right to left along the frontier. But the sharp kink at $b$ is eliminated.

Of course these conclusions require that factor prices vary as one moves along the frontier. But this would be so for any neoclassical model. Regardless of technological possibilities, no price changes, no substitution. And the qualitative results seem independent of the specific formulation. The admission of continuous substitution policies, or even the admission of a finite number of different activities for the production of $C$ and $I$, can be expected to expand the possibility set, and smooth out its shape or at least eliminate the single kink characteristic of the two-gap models. And relatedly, one would expect to find the "redundant input" phenomenon of the simple two-gap model converted into a smoother diminishing returns

Thus Eqs. (18)-(20) need to be rewritten with the supply of an input minus its use in exports on the left hand side.
phenomenon where relatively full employment of all factors can be assured by suitable factor price adjustments. The qualitative implications regarding C, I possibilities thus far, then, do not seem to depend too much on the specific formulation of the model.
V. THE POLICY OPTIONS RECONSIDERED

The introduction of factor substitution possibilities suggests that domestic policy may be far less helpless, much more potent, in influencing employment and growth possibilities than the simple two-gap model of Section II suggests.

With factor substitution possibilities it is possible to increase employment without reducing investment; indeed it is possible to increase employment and achieve an increase in investment and consumption at the same time. With a given capital stock an increase in employment (a higher labor-capital ratio) shifts the $K,L$ curve of Fig. 4 to the left, for, as Eq. (20) shows, to achieve a higher $L$ requires a lower $W$ for any $i$. The simplest case to analyze geometrically is one of no change in the $C,I$ ratio, hence the $K,M$ curve does not shift. The new equilibrium, with greater employment, more consumption and investment, the same capital stock, and the same level of imports (or imports constrained by the export Eq. (22)) requires, then, a lower $\frac{W}{E}$ and a higher $i$.

The same conclusion can be seen another way. The initial conditions of unemployed labor, but fully employed capital and imports, are depicted by point $b$ in Fig. 6a. A lower $\frac{W}{E}$ will shift the $M$ constraint outward (by reducing the import intensity of both consumption and investment), shift in the labor constraint (by inducing a rise in the labor intensity of both activities), and pivot outwards the $K$ constraint (by reducing capital).

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1In either case the $K,M$ curve of Fig. 4 does not shift.
intensity. As shown in Fig. 6A, output along the given C, I ray will unambiguously be increased. However, a fall in $\frac{W}{E}$ alone will lead to a fall in the import-capital ratio (as shown in footnote 1, p. 29). Thus in the new situation the K and M constraints will intersect to the left of C and there will be slack imports. A rise in $i$ will then be required to restore the old capital-import ratio (which is necessary if full employment of both is to be maintained). It is clear that some decrease in $\frac{W}{E}$, and some increase in $i$ is capable of achieving the situation of Fig. 6B. The K and L constraints have been made to coincide. They intersect the M constraint at d.

The discussion above smacks of the possibility of getting something for nothing. This is both true and untrue. It is true in the sense that the option is open to get both more consumption and more investment. The Keynesian situation obtains. It is untrue in that, as the real wage rate curve on Fig. 4 shows an increase in employment (with balanced increase in C and I) will require a reduced real wage rate for people who are employed. The shifted left K, L curve means that the real wage rate at the new higher employment equilibrium is unambiguously lower. Thus there is a tension between higher employment and lower real wages for those employed. This is a classical tension, and a politically real one in such countries as Colombia.

Given a level of employment, the costs of shifting the mix of output toward more investment, less consumption, are clearly real. But in this model this shift is at least possible to accomplish without the added costs

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The K and C constraints still must have the same slope—thus the L constraint must pivot as well.
of increasing unemployment. In this model, as the fixed coefficient one, investment is more import intensive, less domestic resource intensive, than consumption. Thus an increase in investment relative to consumption will require a reduction in the import intensity of both. This will require a decline in the price of domestic inputs relative to imports. In terms of Fig. 4, an increase in the investment-consumption ratio shifts downward the \( \frac{K}{M} \) curve.\(^1\) The \( \frac{K}{L} \) curve does not change reflecting the assumed equality of the capital labor ratio in consumption and investment. (Note the symmetry to the analysis in the preceding paragraph). Full employment of all factors in the post shift situation thus requires a decline \( \frac{W}{E} \) and a decline in \( i \).

As above, it also is possible to show this conclusion in terms of shifts in the constraints. Starting from the conditions of full employment of all inputs at point \( d \) in Figs. 6. B and 6. C., a decrease in \( \frac{W}{E} \) will shift out the import constraint, shift in the labor constraint, and pivot the capital constraint, just as before. Point \( e \), with a higher ratio of investment to consumption, now is achievable with full employment of imports and labor. But with a fall in \( \frac{W}{E} \) and no decline in \( i \), the labor capital ratio will rise. With the labor constraint binding both before and after the shift; capital now must be redundant. Thus a decline in \( i \), as well as of \( \frac{W}{E} \), is required for full employment of all resources to be achieved at the higher ratio of investment to consumption.

Putting both maneuvers together indicates the wide range of policy

\(^1\)This is because for a given \( \frac{W}{E} \), a reduction in domestic input price relative to \( E \) means a lower \( i \).
options available in a situation where initially there is unemployed labor. It is clear that a given increase in employment can be used totally to increase investment, or totally to increase consumption, or any of a wide variety of combinations in between—the proportional increase in C and I case is no magic. As shown in Fig. 7, the frontier of possible increases in C and I from a given increase in employment must be concave (just as the C, I frontier in general must be concave). A greater than proportional increase in investment will require a greater decline in $\frac{W}{E}$, and a smaller rise in $i$, than the proportional increase case; a greater than proportional increase in C will require less of a decline in $\frac{W}{E}$, more of an increase in $i$. This makes good economic sense. The increase in employment relative to the capital stock calls for a decline in $\frac{W}{E}$ relative to $i$, but any point along this new K,L schedule will sustain the new higher labor-capital ratio. Since investment is more import intensive than consumption, a high investment-consumption ratio requires a higher exchange rate relative to $W$ and $i$ than does a high consumption-investment ratio.

Just as with the two-gap formulation, it is possible to translate the C,I choice set of the new classical model into a growth rate, consumption rate diagram. (See Fig. 8.) In contrast to the fixed coefficient model, where there is a fixed capital-value added ratio and hence the growth rate is uniquely related to $\frac{I}{V}$, in the neoclassical model the growth rate also

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1 This is the well known implication of linear homogeneous production functions, with one of the goods (investment) employing a different factor mix (a higher $\frac{M}{V}$) than the other. The formal proof will not be given here.
is related to the capital-labor ratio, the rate of growth of the labor force, and productivity growth. But taking these variables as given, one can derive Fig. 8 from Fig. 6.

In Fig. 8, if the target growth rate exceeds \( g \), there will be a foreign exchange "gap." But if we are to the right of the savings constraint, say at point \( b \), increases in savings can do more to close the gap than is implied by the fixed coefficient model. Further, if we are on the savings constraint, but there is unemployed labor, by moving to the neoclassical frontier the gap can be reduced.

Notice that with the smoothing out of the frontier—the disappearance of the kink—there also disappears the split of the diagram into two regimes—one with high growth returns from a shift left of the savings constraint and low returns to increased import capacity, the other with high returns to greater import capacity and low returns to a higher savings rate. Instead there are smooth diminishing returns to both increased savings and increased aid alone, and complementarity between them. The diminishing returns to increased savings (reduced consumption) simply reflects the diminishing returns to domestic inputs, holding imports constraint. Diminishing returns to aid reflects the same phenomenon the other way.

That aid yields greater returns in terms of growth at a high savings rate than at a low one is retained in this model, reflecting the greater import intensity of investment. But an increase in aid increases the marginal productivity, in terms of growth, of increased domestic savings.

Thus substitution possibilities mean that domestic policy can do more to increase employment, can have greater impact alone on growth, and is
more important as a complement to foreign aid in influencing the growth rate, than the fixed coefficient two-gap model implies. However, achievement of these potentialities depends on the instruments of policy that a government can employ. In particular, their achievement requires, first, that the government have some ability to influence the C, I level and mix; and second, that it be able to influence the equilibrium rate of return on capital and the effective exchange rate.

I shall not discuss here the assumption, implicit in the basic two-gap model as well as in the neoclassical variant, that government spending, taxing, and credit policy can move consumption and investment to any point within the constraint set. Similarly I shall simply assume (naively) that the government can control the long run rate of interest and the equilibrium rate of return on capital.

For what I want to focus on, and stress, is the role of the real effective exchange rate (in my model E/W) as an instrument of employment and growth policy in a foreign exchange constrained economy. The effective exchange rate, indeed all prices, are repressed in the conventional two-gap formulation. The preceding analysis suggests that if there are substitution possibilities, increasing the effective exchange rate may provide the way out of the impasse of being able to increase neither employment nor investment (without reducing the other) because of the foreign exchange constraint.

An increase in E/W permits and stimulates import substitution, enabling an increase in the employment of domestic factors (principally labor) or a shift toward greater production of the more import intensive
good (investment) without causing an increase in pressure on balance of payments. What the exchange rate (or exchange rate structure) ought to be clearly is a function of objectives; the level of the effective exchange rate may be extremely important in determining the extent to which objectives can be met. If a higher growth rate and more employment is wanted than currently is being achieved, an increase in the effective exchange rate may be the key to enabling better performance. Of course, if higher wages for the employed rather than greater employment is the objective, or higher consumption rather than more investment, then the "optimal" exchange rate would be lower.

To most people the idea that the optimal exchange rate is a function of policy objectives seems reasonable enough. Yet there seems some resistance to an implication that one cannot define, much less measure, the equilibrium exchange rate independently of policy objectives. I will not engage, here, in any exegesis of the conditions under which the optimal exchange rate will also be the rate that equilibrates supply and demand for foreign exchange in a competitive exchange market. Yet whether or not these conditions hold, it obviously is the case that the equilibrium exchange rate is not independent of a nation's objectives, or, more accurately, the policies employed to pursue them. If a country is willing to tolerate a high unemployment rate and a low growth rate, the equilibrium, as well as the "optimal" exchange rate, obviously is lower than if policies are pursued to reduce unemployment and increase investment. The demand for foreign exchange obviously will be lower in the first case than in the second and so will, and ought, its price.
Thus the equilibrium exchange rate certainly does reflect objectives and policies pursuing them, and it should. Let us consider the equations of demand and supply of foreign exchange.
Equation (18) (reproduced below) defines the demand for foreign exchange.

\[ M = \frac{(1-\alpha)(1-\delta)}{\lambda} \frac{\alpha(1-\delta)}{(\frac{W}{E})^\lambda} I + Z \frac{(1-\alpha)(1-\delta)}{\lambda} \frac{\alpha(1-\delta)}{(\frac{W}{E})^\lambda} C. \] (18)

For a given production of I and C, demand will be negatively related to the exchange rate divided by the wage rate, positively related to the equilibrium rate of return on capital, for the reasons discussed earlier. If we assume that \( i \) and \( \frac{W}{E} \) are on the \( K, L \) curve of Fig. 4, import demand is a function of \( C \) and \( I \), given \( K, L \). This is shown in Fig. 9.

Equation (22) (reproduced for convenience) defines the supply of foreign exchange.

\[ M = B + g Z [i \frac{(1-\alpha)(1-\delta)}{\lambda} \frac{\alpha(1-\delta)}{(\frac{W}{E})^\lambda} ] -E \] (22)

The supply will vary positively with the exchange rate divided by the money wage rate, negatively with the equilibrium rate of return on capital (which enters the cost of consumer goods for export). If we assume a given employment of capital and labor, \( i \) is an increasing function of \( \frac{W}{E} \), and we can then draw a supply curve of foreign exchange as a function of \( \frac{E}{W} \) (with \( i \) varying so as to maintain the capital-labor ratio).1 (If we wish we can assume supply is fixed, of course.)

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1From Eq. (20).
We now can follow the policy maneuvers examined earlier in this section in terms of the demand for and supply of foreign exchange. An increase in employment, with an associated rise in output of consumer or investor goods, obviously shifts the demand curve of foreign exchange to the right. The increase in \( \frac{i}{W} \) required to achieve the higher labor capital ratio shifts the supply curve to the left. Thus the conclusion, described earlier in terms of Fig. 5, that an increase in employment requires an increase in the equilibrium effective exchange rate is transparently obvious in terms of analysis of demand for and supply of foreign exchange.

The increase in \( I \) relative to \( C \), holding \( K \) and \( L \) constant, also shifts the demand curve to the right. The supply curve as defined above does not shift. Here, again, the earlier result that an increase in \( I \) relative to \( C \) requires an increase in \( \frac{E}{W} \), described in terms of Fig. 4, is transparent when one considers what happens to the demand for and supply of foreign exchange.

The supply of and demand for foreign exchange diagram also permits us to interpret Colombia's recent economic malaise within the framework of the model. The falloff of coffee prices in the mid 1950's can be interpreted as a shift to the left in the supply of foreign exchange schedule facing Colombia (Fig. 9), or a shift down in the \( K, M \) curve (Fig. 4). For full employment equilibrium to be maintained this required either an allocation of resources away from investment and toward consumption (which would conserve on foreign exchange) or an increase in \( \frac{E}{W} \) which would both increase

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1This is not immediately obvious from the equations because of the constants \( Z \). However, other equations imply that the import intensity of investment exceeds that of consumption.
foreign exchange earnings and tend to push down the import intensity of both investment and consumption.

The 1960 development plan calls, if anything, for an increase in the investment rate. The implication that this required an effective exchange rate above the level of the period prior to the coffee price decline appears not to have been recognized. The result is that the system has operated in region 4 of Fig. 4—excess demand for foreign exchange, excess supply of domestic inputs. The investment rate has in fact fallen, but not sufficiently to avoid the development of significant domestic slack.¹ From time to time devaluation has been undertaken. But almost always the target has been (implicitly) to restore the effective exchange rate of the mid 1950's, not to achieve a higher rate. As a result, even after effective devaluations there still have been the "two-gap" symptoms, and a belief has developed that they are a necessary fact of life. The possibility that the effective exchange rate target has been set too low for the achievement of Colombia's growth and employment objectives—that the 1950 rate is a false parity—does not appear to have received as much consideration as it should.

¹The decline of course has not been deliberate.