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Regimes of low-frequency variability in a three-layer quasi-geostrophic ocean model

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ABSTRACT

The temporal variability of the midlatitude double-gyre wind-driven ocean circulation is studied in a three-layer quasi-geostrophic model over a broad range in parameter space. Four different types of flow regimes are found, each characterized by a specific time-mean state and spatio-temporal variability. As the lateral friction is decreased, these regimes are encountered in the following order: the viscous antisymmetric regime, the asymmetric regime, the quasi-homoclinic regime and the inertial antisymmetric regime. The variability in the viscous and the inertial antisymmetric regimes (at high and low lateral friction, respectively) is mainly caused by Rossby basin modes. Low-frequency variability, i.e. on interannual to decadal time-scales, is present in the asymmetric and quasi-homoclinic regime and can be related to relaxation oscillations originating from low-frequency gyre modes. The focus of this paper is on the mechanisms of the transitions between the different regimes. The transition from the viscous antisymmetric regime to the asymmetric regime occurs through a symmetry-breaking pitchfork bifurcation. There are strong indications that the quasi-homoclinic regime is introduced through the existence of a homoclinic orbit. The transition to the inertial antisymmetric regime is due to the symmetrization of the time-mean state zonal velocity field through rectification effects.

1. Introduction

The North Atlantic upper-ocean circulation is an incredibly complex time-dependent flow. A time-mean view reveals a strong western boundary current, i.e. the Gulf Stream, and two gyres of which the subtropical gyre is more pronounced than the subpolar one. Variability occurs on many time scales, but with high energy in the monthly, near-annual and interannual frequency bands (Kelly et al., 1996; Wang et al., 1998).

The ocean flow is forced by a highly variable atmosphere through momentum, heat and freshwater fluxes. Although there appears to be distinct patterns of variability in wintertime atmospheric anomalies, i.e. those associated with the North Atlantic oscillation, no distinct peaks in the spectrum can be distinguished (Hurrell, 1995). Hence, from an oceanographic
point of view, the ocean flow is forced by time-mean fluxes in addition to a variable component with a preferred spatial pattern but with temporally white noise properties (Saravanan and McWilliams, 1995). Through its large thermal inertia, this white noise forcing will be integrated by the ocean to provide energy to the low-frequency part of the spectrum of ocean variability (Hasselmann, 1976; Frankignoul and Hasselmann, 1979; Deser and Blackmon, 1993). In addition, the noise can amplify large-scale ocean flow patterns leading to enhanced decadal variability through spatial resonance (Saravanan and McWilliams, 1997).

On the other hand, detailed studies with eddy-resolving ocean general circulation models (OGCMs) have demonstrated that the ocean is capable of generating a high degree of internal variability (Hurlburt and Hogan, 2000; Smith et al., 2000; Chassignet and Garraffo, 2001). This variability is, for example, caused by mesoscale eddies, which are primarily generated by mixed barotropic/baroclinic instabilities of the wind-driven currents (Holland and Schmitz, 1985; Semtner and Chervin, 1992). The nonlinear interaction of the eddies may introduce large-scale, low-frequency variability of the flow (McCalpin and Haidvogel, 1996). However, eddy-resolving (or eddy-permitting) OGCMs are not the right tool to understand the origin of low-frequency variability in the ocean. The integration times are in most cases too short to extract properties of interannual to decadal variability. Moreover, the results from these models are hard to interpret because of their multi-process and multi-scale character.

An understanding of the physics of low-frequency variability can be obtained by approaching the ‘real’ situation from particular limiting flows of which the variability can be studied in detail. One path proceeds from simple to complex situations through a hierarchy of models. The lowest member of the model hierarchy describing the wind-driven circulation is the barotropic quasi-geostrophic model in a square basin with a flat bottom. The hierarchy of models proceeds upward by inclusion of stratification (multi-layer models), primitive equations (PE) (Lions et al., 1992a,b), continental coastlines, bottom topography and buoyancy-driven flow toward full-blown OGCMs (Ghil et al., 2002). A second path can be taken within one particular member of the model hierarchy where one proceeds from steady, highly-dissipative or weakly forced flows to irregular, weakly dissipative or strongly forced flows (Ghil and Childress, 1987; Dijkstra, 2000).

To understand the variability in the upper ocean, the double-gyre flow in a rectangular basin has become a standard case (Jiang et al., 1995; Cessi and Ierley, 1995). For the (equivalent) barotropic and two-layer quasi-geostrophic models, many results on the bifurcation behavior and internal modes of variability have been obtained. To set the context of the present paper, an overview of these results is presented in the next section.

In this paper, we consider the double-gyre flows in a three-layer quasi-geostrophic model. For this model, the associated bifurcation behavior has not explicitly been considered and no a priori knowledge of the internal modes of variability is available. Flows are computed over quite a range of values of the lateral friction, for different values of the reduced gravity parameters and for two different basin sizes. We demonstrate the
existence of four different flow regimes; each regime is characterized by a distinct
time-mean state and associated variability. The main contribution of this paper is an
explanation of the transitions between the different regimes with help of the knowledge of
the bifurcations which are known to occur in single and two-layer quasi-geostrophic
models.

2. Background

The dynamical behavior of double-gyre flow in the single-layer quasi-geostrophic model
has been analyzed in quite detail over the last decade. Cessi and Ierley (1995) demonstrated
the existence of multiple equilibria in these flows occurring through symmetry-breaking
pitchfork bifurcations. These bifurcations arise when the lateral friction is decreased,
because the antisymmetric double-gyre flow, which is the unique solution at large friction,
becomes unstable to asymmetric perturbations through a barotropic mechanism (Dijkstra
and Katsman, 1997). Beyond this instability boundary, asymmetric steady solutions exist
with either the midlatitude jet displaced north or southward with respect to the mid-axis of
the basin (Dijkstra and Katsman, 1997). These asymmetric flows subsequently become
susceptible to several oscillatory instabilities as the friction is further decreased. These are
either high-frequency Rossby basin modes (indicated with ‘RB’ in the rest of this paper)
with typical oscillation time scales of several months or so-called gyre models (‘G’) that
have interannual to decadal oscillation periods (Nauw and Dijkstra, 2001). The former are
associated with westward propagating basin-sized structures related to unbounded free
Rossby waves. They are deformed by the interaction with the strong currents near the
western boundary (Pedlosky, 1987; Dijkstra and Katsman, 1997). Gyre modes originate
through a merging of two stationary modes and their underlying mechanism of propagation
is basically that of a shear instability and is free of Rossby-wave dynamics (Simonnet and
Dijkstra, 2002). In two-layer models also, ‘classical’ baroclinic modes (‘CB’) are present
that have intermonthly periods. Their spatial patterns resemble those that destabilize an
eastward zonal jet (Eady, 1949; Phillips, 1954; Pedlosky, 1987), but are slightly modified,
because of the boundary conditions on the eastern and western walls of the basin (Dijkstra
and Katsman, 1997). Wall-trapped modes (‘WT’) have also appeared in the literature and
are related to viscous instabilities of the Munk layer (Ierley and Young, 1995).

The brief summary above describes the lowest branches of the bifurcation tree, where
steady-state behavior with highly symmetric spatial patterns changes into richer spatio-
temporal behavior of the flow, once the forcing is increased or the friction decreased. The
periods associated with these first few internal oscillatory models—from months to
toears—are typically comparable to, or longer than, those associated with mesoscale
variability.

When friction is further decreased, more complicated behavior will occur. Meacham
(2000) computed the time-dependent flow with a barotropic and a two-layer baroclinic QG
model by forward integration for several values of the frictional boundary layer thickness
$\delta_M$. At small values of $\delta_M$, aperiodic solutions were found with large excursions in jet
position occurring on long time scales. Meacham (2000) hypothesized that this variability arises through a global bifurcation, in which a periodic orbit expands to the point at which it touches one of the unstable steady states. In doing so, the period becomes infinite and the resulting ‘periodic’ orbit is called a homoclinic orbit. Fairly convincing numerical evidence for the existence of a homoclinic orbit was found in single-layer QG models (Chang et al., 2001; Nadiga and Luce, 2001; Simonnet et al., 2004).

However, it remains difficult to relate the time-dependent behavior of flows computed even within the same models to the robust dynamical building blocks of double-gyre flows (consisting of the stationary states and oscillatory modes) discussed above. Berloff and McWilliams (1999) computed numerical solutions of the wind-driven circulation in a two-layer QG model for five values of the lateral friction coefficient $A_H$ in a basin of realistic size. For $A_H = 1200 \text{ m}^2 \text{s}^{-1}$ an asymmetric steady state was found. When the lateral friction is decreased, the flow first displays variability and two modes of variability can be identified on the basis of an EOF-analysis. The so-called ‘primary’ mode corresponds to intermonthly variability and is characterized by the presence of Rossby waves in the interior. The ‘secondary’ mode introduces variability on an interannual time-scale and is associated with a fluctuating envelope surrounding a standing Rossby wave. At $A_H = 1000 \text{ m}^2 \text{s}^{-1}$, quasi-periodic variability is found containing two dominant frequencies in the sub- and inter-annual range, which could still be related to the two modes. At still smaller friction, a broadband spectrum appears, with the spectral power of the total energy increasing toward lower frequencies. At $A_H = 800 \text{ m}^2 \text{s}^{-1}$, the behavior of the solutions is called ‘chaotic,’ while at $A_H = 600 \text{ m}^2 \text{s}^{-1}$ the flow patterns hover near three states with distinct total energy. These states are characterized by a different penetration length of the eastward jet and the presence or absence of dipole-pattern oscillations in the recirculation region. Moreover, no connection could be made anymore to the two modes identified in flows at higher friction.

McCalpin and Haidvogel (1996) investigated the time-dependent solutions of an equivalent-barotropic QG model also for a basin of realistic size (3600 × 2800 km), as well as the sensitivity of solutions to the magnitude of the wind stress and its meridional profile. They classified solutions according to their basin-averaged kinetic energy and found three persistent states in their simulations. High-energy states are characterized by near-symmetry of the flow with respect to the mid-axis, weak meandering and large jet penetration into the basin interior. Low-energy states have a strongly meandering jet that extends but a short way into the basin, while intermediate-energy states resemble the time-mean flow and have a spatial pattern somewhere between high- and low-energy states. The persistence of the solutions near either state is irregular but can last for more than a decade of simulated time.

Primeau (2002) reproduced the time-dependent behavior of the multiple regimes found by McCalpin and Haidvogel (1996). To test whether unstable steady states act to steer the model trajectories, Primeau (2002) determined the steady states for this particular barotropic QG model using continuation methods. Multiple equilibria, both symmetric and asymmetric, do occur for the parameter values used for the time-dependent integrations;
they arise from several pitchfork bifurcations, which can be accommodated by the basin’s relatively large meridional extent. By projecting the instantaneous flow fields onto four of the steady-state solutions, Primeau (2002) found that a significant amount of the low-frequency variability can be explained by regime switches between the phase-plane neighborhoods of these four steady states. Increasing the asymmetry of the wind-stress forcing reduced the model’s low-frequency variability, which appears to be related to the disappearance of some of the steady states.

To summarize, many of the idealized models of the wind-driven double-gyre circulation exhibit the same type of bifurcation behavior. Robust bifurcations in these models are: the symmetry breaking bifurcation, giving rise to multiple equilibria, several internal oscillatory modes with monthly to interannual periods and the homoclinic bifurcation which introduces large-scale low-frequency oscillations. It appears that the different bifurcations somehow mark the transition boundaries of different time-dependent flow regimes in these models, but this relation is far from clear at the moment. In the present paper, we precisely address this problem by investigating the relation between the bifurcations and flow regimes in detail using a three-layer quasi-geostrophic model.

3. Model configuration and methods

The model used in this study is a quasi-geostrophic model described by Pedlosky (1987) and previously used in Dewar (2001), while the numerical implementation used here is originally described in Holland (1978). The model domain is a square basin with dimensions of $2L \times 2L$. The stratification within this model is idealized by three stacked layers of water with constant densities $\rho_i$, $\rho_1 < \rho_2 < \rho_3$, and mean layer thicknesses $H_i$, with $H = H_1 + H_2 + H_3$. Lateral friction with a constant lateral friction coefficient $A_H$ is considered as the only dissipation mechanism. The governing equations of the model are

$$\frac{\partial q_1}{\partial t} + J(\psi_1, q_1) = A_H \nabla^4 \psi_1 + \frac{f_0}{H_1} w'$$
$$q_1 = \nabla^2 \psi_1 - \frac{f_0}{H_1} h_1 + \beta_0 y$$

$$\frac{\partial q_2}{\partial t} + J(\psi_2, q_2) = A_H \nabla^4 \psi_2$$
$$q_2 = \nabla^2 \psi_2 - \frac{f_0}{H_2} (h_2 - h_1) + \beta_0 y$$

$$\frac{\partial q_3}{\partial t} + J(\psi_3, q_3) = A_H \nabla^4 \psi_3$$
$$q_3 = \nabla^2 \psi_3 + \frac{f_0}{H_3} h_2 + \beta_0 y,$$
where \( q_i \) represents the potential vorticity and \( \psi_i \) the streamfunction in layer \( i \). The Coriolis parameter varies linearly with latitude as \( f = f_0 + \beta_0 y \), where \( \beta_0 \) represents its meridional gradient. The Coriolis parameter, \( f_0 \), is taken at \( \theta_0 \), which is the location of the mid-axis of the basin. The quantities \( h_i \) represent interface perturbations related to the streamfunctions, \( \psi_i \), through \( h_i = f_0 (\psi_i - \psi_{i+1})/g_i^' \) for \( i = 1, 2 \), in which the \( g_i^' = (\rho_{i+1} - \rho_i)/\rho_0 \) represent the reduced gravity parameters. The Jacobian operator is defined in the usual way, \( J(a, b) = a_x b_y - a_y b_x \). Furthermore, a no-slip condition \( \partial \psi_i/\partial n = 0 \), is applied to the lateral boundaries together with a condition of no-normal flow, \( \psi_i = \text{constant} \), where the value of the streamfunction is determined by mass conservation.

The surface wind stress, \( \tau = (\tau^x, \tau^y) \) is given by the analytic function

\[
\tau^x = \tau_0 \left( 1 - 2\alpha \frac{y}{L} \right) \cos \left( \frac{\pi y}{L} \right) + \frac{2\alpha}{\pi} \sin \left( \frac{\pi y}{L} \right); \quad \tau^y = 0, \tag{2}
\]

where \( \tau_0 \) is the amplitude. This wind stress induces an Ekman pumping \( w_e \) of the following form

\[
w_e = \frac{\nabla \times \tau}{\rho_0 f_0} = \mu_0 \left( 1 - 2\alpha \frac{y}{L} \right) \sin \left( \frac{\pi y}{L} \right) \quad y \in [-L, L], \tag{3}
\]

where \( \mu_0 = \tau_0 \pi / (\rho_0 f_0 L) \). The parameter \( \alpha \) is introduced to control the amount of asymmetry in the Ekman pumping with respect to the mid-axis of the basin (\( y = 0 \)).

Standard values of the parameters are provided in Table 1. A relatively large part of parameter space is explored in this study, i.e. two different basin sizes are used, reduced gravity parameters are varied and many values of the lateral friction coefficient \( A_H \) are considered. A standard spatial resolution of 20 km is used, which is sufficient to resolve flows for values of \( A_H \) larger than \( A_H = 300 \text{ m}^2 \text{ s}^{-1} \) for which the Munk western boundary layer, \( \delta_M = (A_H/\beta_0)^{1/3} \approx 26.6 \text{ km} \). Moreover, phenomena with the size of the internal radii of deformation, typically larger than 25 km, can be represented.

After a spin-up for 25 years from a state of rest, the time-dependent development of each flow is computed for 75 years (with a time-step of one hour). Every month, the streamfunctions in every layer, \( \psi_i \), are subsampled at 60 km resolution (at every three grid points).
From these data, time series are calculated for the total kinetic energy, $E_k$, and potential energy, $E_p$, of the flow. These are defined as

\[
E_k = \frac{\rho_0}{2} \sum_{i=1}^{3} \int_{A} \int \left( \frac{\partial \psi_i}{\partial x} \right)^2 + \left( \frac{\partial \psi_i}{\partial y} \right)^2 \, dx \, dy
\]

(4a)

\[
E_p = \frac{\rho_0 g_0^2}{2} \int_{A} \int \left( \frac{(\psi_1 - \psi_2)^2}{g_1^2} + \frac{(\psi_2 - \psi_3)^2}{g_2^2} \right) \, dx \, dy,
\]

(4b)

where $A$ is the area of the basin.

In addition, a normalized transport difference, $\Delta \Phi$, is defined to monitor the asymmetry of the depth-integrated flow with respect to the mid-axis of the basin (Chang et al., 2001; Simonnet et al., 2003a), i.e.

\[
\Delta \Phi = \frac{\Psi_{\text{max}} + \Psi_{\text{min}}}{\max(\Psi_{\text{max}}, -\Psi_{\text{min}})},
\]

(5)

where $\Psi = (H_1\psi_1 + H_2\psi_2 + H_3\psi_3)$ is the barotropic transport function. If the flow is antisymmetric, the transports in the counter-rotating subtropical and subpolar gyre are nearly equal, but of opposite sign ($\Psi_{\text{max}} \approx -\Psi_{\text{min}}$) and hence $\Delta \Phi \approx 0$. On the other hand, if the flow is strongly asymmetric, with clearly different transports in the gyres, then $\Psi_{\text{max}} \neq -\Psi_{\text{min}}$ and $\Delta \Phi$ will be nonzero. For $\Delta \Phi > 0$, the subtropical gyre is stronger and the jet points southeastward (a jet-down solution), whereas $\Delta \Phi < 0$ corresponds to a jet-up solution.

A first inspection of the time series (of $E_k$, $E_p$, $\Delta \Phi$, $\Psi_{\text{max}}$) and their associated power spectra gives a good indication of the type of variability present within the flow. The streamfunction fields are subsequently analyzed with a standard Empirical Orthogonal Functions analysis (Preisendorfer, 1988). To identify propagating patterns of variability, the 10 most dominant principal components are used as input channels for a Multichannel Singular Spectrum Analysis (Plaut et al., 1995). In the latter analysis, a standard window length of 10 yr is used to capture the low-frequency signal.

In the next section, we discuss the results for the three-layer model in a relatively small basin ($L = 5 \cdot 10^5$ m) and compare those with results from two-layer studies in basins with similar size with either quasi-geostrophic models (Dijkstra and Katsman, 1997; Cessi and Ierley, 1995) or shallow-water models (Jiang et al., 1995; Speich et al., 1995; Nauw and Dijkstra, 2001; Simonnet et al., 2003a,b). The results for a larger size basin ($L = 10^6$ m) will be presented in Section 4 and related to the work of Berloff and McWilliams (1999).

4. Small basin ($L = 5 \cdot 10^5$ m)

We consider flows in a basin of $1000 \times 1000$ km forced with the wind-stress shape (2) and for standard values of the parameters as shown in Table 1.
a. Flow regimes

For four different values of $A_H$, the time evolution of $\Delta \Phi$ is shown in Figure 1. At large values of the friction ($A_H = 1600 \text{ m}^2 \text{s}^{-1}$), the time series of $\Delta \Phi$ indicates that the flow settles into a small-amplitude high-frequency periodic orbit (Fig. 1a). A typical flow pattern of the time-mean barotropic transport function, $\Psi$, for this case shows a well-developed double-gyre structure with nearly equal strength of the subtropical and subpolar gyre (Fig. 2a). The flow regime containing this solution is called the viscous antisymmetric regime.

For slightly smaller values of the lateral friction ($A_H = 700 \text{ m}^2 \text{s}^{-1}$), a case is encountered (Fig. 1b) where $\Delta \Phi$ remains a definite sign (after an initial spin-up). The associated time-mean state (Fig. 2b) of the barotropic transport function shows a strongly asymmetric double-gyre circulation with the midlatitude jet directed north-eastward (jet-up solution). The regime containing this solution will be referred to as the asymmetric regime.

The time series associated with the weakly asymmetric circulation at $A_H = 500 \text{ m}^2 \text{s}^{-1}$ (Fig. 1c) shows an alternation between positive and negative values of $\Delta \Phi$. Between $35 < t < 50 \text{ yr}$, the trajectory appears to be attracted to a jet-up solution ($\Delta \Phi < 0$), and between

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Figure 1. The transport difference, $\Delta \Phi$ versus time (in years). (a) $A_H = 1600 \text{ m}^2 \text{s}^{-1}$, viscous antisymmetric regime; (b) $A_H = 700 \text{ m}^2 \text{s}^{-1}$, asymmetric regime; (c) $A_H = 500 \text{ m}^2 \text{s}^{-1}$, quasi-homoclinic regime. (d) $A_H = 300 \text{ m}^2 \text{s}^{-1}$, inertial antisymmetric regime.
$55 < t < 90$ yr a jet-down solution appears to be favored ($\Delta \Phi > 0$). In fact, the slight asymmetry in the time-mean circulation (Fig. 2c) is probably caused by the relatively short integration time. Thus, averaging over only 75 years apparently results in a slightly positive. This is an example of a flow in the quasi-homoclinic regime; the reason for this name will become clear in Section 5.

For the standard value of $A_H = 300$ m$^2$ s$^{-1}$, the temporal behavior of the transport difference is fairly irregular (Fig. 1d). The time-mean flow pattern shows a broad, nearly antisymmetric gyre structure, and even displays additional subgyres near the southern and northern boundary (Fig. 2d). The latter pattern is typical for highly inertial flows and has
been found in similar high-resolution simulations with quasi-geostrophic models (Greatbatch and Nadiga, 2000). This is an example of the flow in a regime which we will refer to as the inertial antisymmetric regime.

For these four flows and others at intermediate values of $A_H$, the time-mean (averaged over the final 75 years of integration) values of the maximum northward barotropic transport, $\Psi_{max}$, and the transport difference, $\Delta \Phi$, are plotted as ‘closed triangles’ in Figure 3. For clarity, the horizontal scale in this figure is logarithmic because changes in the mean circulation and its time-dependent behavior apparently increase from one solution to the other with decreasing $A_H$. From Figure 3, the different regimes can now be identified more clearly.

In the viscous antisymmetric regime ($A_H \geq 1100$ m$^2$ s$^{-1}$) the mean flow is nearly antisymmetric, $\Delta \Phi \approx 0$, (Fig. 3b) and $\Psi_{max}$ increases monotonically with decreasing $A_H$. It was expected that the transport difference would be exactly zero at these high values of the lateral friction because of the mirror-symmetry present in quasi-geostrophic models (Dijkstra and Molemaker, 1997). However, the slightly nonzero value found is probably due to the spatial sampling of the streamfunction, which is every 3 grid-points. Let $d_{min}$ ($d_{max}$) indicate the distance between the location of $\Psi_{min}$ ($\Psi_{max}$) and the mid-axis of the basin, then $|d_{max} - d_{min}|$ will be at least the size of one grid cell $\Delta y$ due to the spatial sampling. Hence, the values of $\Psi_{max}$ and $\Psi_{min}$ in a purely antisymmetric solution are slightly different.

In the inertial antisymmetric regime ($A_H \leq 430$ m$^2$ s$^{-1}$), the mean states are also antisymmetric (Fig. 3b). However, the values of $\Psi_{max}$ are much higher than those in the viscous antisymmetric regime and increase only slightly with decreasing $A_H$ (Fig. 3a). In

![Figure 3. Properties of the mean states in the small-basin case as determined over a 75-year simulation with the three-layer quasi-geostrophic model for different values of $A_H$. (a) The maximum northward barotropic transport, $\Psi_{max}$. (b) Normalized transport difference, $\Delta \Phi$. The open squares mark the solutions with $g' = 0.01$ m s$^{-2}$ and the closed triangles the ones with $g' = 0.03$ m s$^{-2}$ (standard case) and the open circles the ones with $g' = 0.05$ m s$^{-2}$. Note that a logarithmic scale is used on the horizontal axis.](image)
the interval $430 < A_H \leq 1000 \, m^2 \, s^{-1}$, the relation between $A_H$ and $\Psi_{\text{max}}$ is non-monotonic and there is an enormous scatter in the value of $\Delta \Phi$. However, a distinction can be made between strongly asymmetric circulations in the asymmetric regime ($A_H = 650, 700$ and $1000 \, m^2 \, s^{-1}$) and weakly asymmetric circulations in the quasi-homoclinic regime (at $A_H = 465, 500$ and $600 \, m^2 \, s^{-1}$). Although it may seem that the solution at $A_H = 1000 \, m^2 \, s^{-1}$ is a distinct outlier, the solutions at $A_H = 950$ and $1050 \, m^2 \, s^{-1}$ display similar characteristics (not shown). The outlier may be due to a small window, where the behavior is different, such as those that occur in the Lorenz equations (Sparrow, 1982). Moreover, it will become clear in the next section that the solutions at $A_H = 800$ and $900 \, m^2 \, s^{-1}$ do belong to the asymmetric regime, although their amount of asymmetry is rather small.

The four regime behavior also appears to be present in the cases with other values of $g_2'$. The properties of the flows for $g_2' = 0.05$ are very similar to those of the standard case, as can be seen by the open circles in Figure 3. In the case with $g_2' = 0.01$ (open squares in Fig. 3), the asymmetric and quasi-homoclinic regimes apparently extend over a much larger interval of $A_H$ than in the other two cases, thereby reducing the sizes of the viscous and inertial antisymmetric regimes.

### b. Patterns of variability

As mentioned at the end of Section 2, the spatio-temporal variability of the flows is analyzed with Multichannel Singular Spectrum Analysis (MSSA). From the MSSA analysis, statistical pairs are identified on the basis of their dominant period, and their reconstructed components (RCs) provide the spatial patterns associated with these statistical pairs. In the rest of this paper these pairs will be called statistical modes. The statistical modes can be divided into groups having qualitatively similar period, pattern and propagation characteristics. Based on these characteristics, they can (in nearly all cases) be related to known internal modes of variability, which have been identified with linear stability analysis of the steady states of double gyre flows and summarized in Section 2.

Because it might seem a subjective procedure to relate the statistical modes to internal modes of variability, we will provide one example. There are quite a few statistical modes that have a period in the interannual to decadal range and a spatial pattern similar to that of the gyre mode. The variance of these statistical modes will be summed, which gives an indication of the relative importance of the low-frequency gyre mode in the time-dependent flow. However, in some cases the characteristic propagation, which can be determined by superposing the pattern of a statistical mode on the time-mean state, doesn’t resemble that of the gyre mode. This particular statistical model will then be discarded from any of the groups and its variance will be ‘lost.’

For the three different values of $g_2'$, the results of the MSSA are summarized in Figure 4. For each case, a histogram shows the summed variances of the statistical modes within a group. The histograms clearly indicate that the type of variability is different in each of the four regimes identified in the previous paragraph. In the viscous antisymmetric regime at
Figure 4. Histograms of explained variances (%) by each different statistical mode with (a) $g_2' = 0.01 \text{ m s}^{-2}$, (b) $g_2' = 0.03 \text{ m s}^{-2}$ and (c) $g_2' = 0.05 \text{ m s}^{-2}$ in the small-basin case for different values of the lateral friction coefficient, $A_H$. CB1 = classical baroclinic mode, causing meandering of the midlatitude jet, CB2 = classical baroclinic mode causing strengthening and weakening of the midlatitude jet, D = dipole oscillation, G = relaxation gyre oscillation and RB = Rossby basin mode.
large values of $A_H$, statistical modes resembling classical baroclinic modes explain most of the variance. For example, the variability in the viscous antisymmetric regimes for $g'_2 = 0.03$ and $0.05$ m s$^{-1}$ occurs on subannual time-scales. The period and the pattern of variability, identified by MSSA, are very similar to those of the classical baroclinic mode (CB2), which destabilized the steady state at the second Hopf bifurcation in Dijkstra and Katsman (1997) (their Fig. 10). It is slightly different in the viscous antisymmetric regime for $g'_2 = 0.01$ m s$^{-2}$, where the period of the oscillation, $\mathcal{P} = 4.5$ mo, and its spatial pattern strongly resemble the classical baroclinic mode (CB1), which destabilized the steady state at the first Hopf bifurcation in Dijkstra and Katsman (1997) (their Fig. 9).

In the inertial antisymmetric regime at low values of $A_H$, the variability is entirely dominated by Rossby basin modes as can be seen in Figure 4b,c. Note that the inertial antisymmetric regime is not present in the case with $g'_2 = 0.01$ m s$^{-1}$ (Fig. 4a) for the range of values chosen for the lateral friction coefficient (see Fig. 3). The periods of the statistical Rossby basin modes are in the order of several months and their spatial pattern is basin-wide with westward propagating anomalies of opposite sign on either side of the mid-axis of the basin. The anomalies in the upper layer are slightly distorted near the western boundary, because of the strong advection by the midlatitude jet and the recirculation gyres. In general, these modes compare very well with the Rossby basin mode (RB), which destabilizes the steady equivalent barotropic flow in Dijkstra and Katsman (1997) (their Fig. 5).

At intermediate $A_H$, relaxation oscillations are found in the flow, which can be associated with a low-frequency gyre mode. Note that this is also the case for the solution at $A_H = 900$ m$^2$ s$^{-1}$, which supports the view that it resides in the asymmetric regime. These oscillations become visible in the time series of the basin-integrated kinetic, $E_k$, and potential, $E_p$, energy. Figure 5a provides an example of these energy time series at $A_H = 465$ m$^2$ s$^{-1}$ of the solution for $g'_2 = 0.01$ m s$^{-1}$, while Figure 5c shows the SSA-MEM spectra (Simonnet et al., 2004) of these time series (see figure caption for more details). The energy time series are in phase with each other and form an irregular large amplitude sawtooth oscillation. Both SSA-MEM spectra indicate increased variability at frequencies in the interannual range, i.e. at $f = 0.21$, $0.33$ and $0.40$ yr$^{-1}$. The time series of the normalized transport difference, $\Delta \Phi$, are negative on average (Fig. 5b), indicative of a preference for a jet-up circulation and have some short (and one longer) periods with generally positive values, which indicates that the solution is primarily jet-down. The SSA-MEM spectrum of these time series (Fig. 5d) shows variability on a period of $\mathcal{P} = 2.2$ mo, which is caused by the presence of Rossby basin modes. Moreover, increased power is displayed at $f = 0.2$ yr$^{-1}$, which coincides with one of the three interannual peaks in the spectra of the energies. The alternation between jet-up and jet-down solutions and the presence of relaxation oscillations has shown to be characteristic for the quasi-homoclinic regime (Simonnet et al., 2004) as explained in Section 5.

The relaxation oscillation which is related to the gyre mode, also shows up in the MSSA as pairs of eigenvalues associated with low-frequency statistical modes. It seems that only
a small amount of the variance is explained by these modes, however, the selection of these modes was very strict. For example, several of the low-frequency modes were rejected from the count because their propagation was slightly different from that of the gyre mode, although their period and spatial pattern was in agreement with that of the gyre mode. Figure 6 shows the time-mean upper layer streamfunction, $\psi$, at $A_H = 465$ m$^2$ s$^{-1}$ of the solution for $g' = 0.01$ m s$^{-2}$. A prefiltering was applied to the data, retaining only the leading six reconstructed components from the SSA. The MEM-spectrum of the leading seven reconstructed components of the SSA of the transport difference series in (b). For the SSA a window length of 120 (≈ 10 years) was used and the order of the MEM was 40.

Figure 5. Time series of the basin-integrated kinetic, $E_k$, (drawn) and potential, $E_p$, (dashed) energy in panel (a) and of the transport difference, $\Delta \Phi$, in panel (b) at $A_H = 465$ m$^2$ s$^{-1}$ of the solution for $g' = 0.01$ m s$^{-2}$. (c) Maximum-entropy spectra of the kinetic- (drawn) and potential- (dashed) energy series in panel (a). A prefiltering was applied to the data, retaining only the leading six reconstructed components from the SSA. (d) MEM-spectrum of the leading seven reconstructed components of the SSA of the transport difference series in (b). For the SSA a window length of 120 (≈ 10 years) was used and the order of the MEM was 40.
quarter of a period later a tripole pattern is visible (Fig. 6c), which causes a southward shifting of the midlatitude jet (or northward, in the opposite phase). This pattern is similar to that of the low-frequency gyre mode which has been identified in other studies (Dijkstra and Katsman, 1997; Simonnet and Dijkstra, 2002; Jiang et al., 1995; Speich et al., 1995; Nauw and Dijkstra, 2001).

Apart from the recognition of known modes, there is one pattern which cannot immediately be related to an internal mode of variability, as deduced from stability studies. In three simulations with \( g_2' = 0.01 \) m s\(^{-2}\), a low-frequency oscillation is found at \( A_H = 1400, 1600 \) and 1800 m\(^2\) s\(^{-1}\). The period of the oscillation increases from \( \mathcal{P} = 4.1 \) to 8.3 yr as the viscosity is increased. Its upper layer anomalies (for the case with \( A_H = 1600 \) m\(^2\) s\(^{-1}\)) are shown in Figure 7 at four stages during half of a period of an oscillation. It shows a standing pair of anomalies of opposite sign in the area of the recirculation gyres. The anomalies in the middle and lower layer are approximately at the same position, but the sign of the anomalies in the middle layer is opposite to those in the upper and lower layer (not shown). This oscillation induces a fluctuation in the intensity of the subtropical and subpolar recirculation gyres (simultaneously).

In summary, apart from this dipole oscillation, each of the statistical modes of variability can be linked to internal modes of variability found in previous studies of the wind-driven ocean circulation. In addition, in each regime different types of modes contribute to the total variability.

5. Large basin \((L = 10^6 \text{ m})\)

Next, we consider a basin of 2000 \( \times \) 2000 km \((L = 10^6 \text{ m})\) and again the standard case with a reduced gravity of \( g_2' = 0.03 \) m s\(^{-2}\) and with the wind forcing given by (2). The results in this section will bridge the gap between variability found in small basins (as in
the previous section) and the variability in time-dependent flows as considered in larger basins (Berloff and McWilliams, 1999).

a. Flow regimes

For four different values of $A_H$, the value of $\Delta \Phi$ is plotted versus time in Figure 8 and time-mean plots of the barotropic transport streamfunction are shown in Figure 9. The time series of $\Delta \Phi$ in the viscous antisymmetric regime (Fig. 8a) displays a low-frequency modulation of the high-frequency signal, while the time-mean state in Figure 9a is antisymmetric. Also for the large-basin flows, a transition to an asymmetric regime occurs. A typical time series of $\Delta \Phi$ in that regime is shown in Figure 8b. The value of $\Delta \Phi$ remains positive after a spin-up of slightly more than 25 years and the amplitude of the high-frequency oscillation changes quite regularly on a decadal time scale. The time-mean barotropic transport streamfunction is asymmetric and displays a jet-down solution (Fig. 9b) in correspondence with the positive values of $\Delta \Phi$.

For the flow in the quasi-homoclinic regime, several intervals can be distinguished in which there is a preference for either positive or negative values (Fig. 8c). The time-mean flow in this regime is slightly asymmetric (Fig. 9c). The time-series of the case in the inertial antisymmetric regime (Fig. 8d) consists of a mainly high-frequency signal as in the small basin case. The time-mean flow in this regime (Fig. 9b) is also antisymmetric, but the midlatitude jet is much stronger than in the viscous regime. Moreover, the large-scale gyres are accompanied by small-scale subgyres near the northern and southern boundary (Fig. 9d).

In Figure 10, the maximum northward barotropic transport, $\Psi_{\text{max}}$, and the transport difference, $\Delta \Phi$, for the time-mean states are plotted as closed triangles for several intermediate values of the lateral friction coefficient $A_H$. These markers show more scatter than those in Figure 3 for the small-basin case, but the different regimes can again be distinguished. As for the small basin case, the four regimes are robust to changes in the
stratification, since they also appear in the case with $g^2_2 = 0.01 \text{ m s}^{-2}$ (open squares in Fig. 10).

The viscous antisymmetric regime can be found for high values of the lateral friction, i.e. for $A_H \geq 2100 \text{ m}^2\text{s}^{-1}$ for which $\Psi_{\text{max}}$ increases monotonically with decreasing $A_H$. The presence of the inertial antisymmetric regime for $A_H < 700 \text{ m}^2\text{s}^{-1}$ is indicated by the relatively small time-mean transport difference, $\Delta \Phi = 0$ and the much higher maximum northward transport, $\Psi_{\text{max}}$. The three solutions at $A_H = 1500, 1600$ and $1700 \text{ m}^2\text{s}^{-1}$ indicate the presence of an asymmetric regime on the basis of their large time-mean transport difference, $\Delta \Phi$. The quasi-homoclinic regime is now located between $700 < A_H \leq 1400 \text{ m}^2\text{s}^{-1}$.

Actually, the time-mean states in each regime (Fig. 9) are very similar to those for the small-basin case. In fact, if we zoom in on the part between $x \in [0.0, 1.0]$ and $y \in [0.5, 1.5]$, it is clear that these solutions have an almost equal extent of the midlatitude jet and the recirculation gyres as in the small basin. However, the flow in the large basin is much stronger than that in the small basin (compare Figs. 3a and 10a).

For the case with $g^2_2 = 0.03 \text{ m s}^{-2}$, also the effect of an asymmetric wind-stress forcing ($\alpha = 0.15$) was considered. With this wind stress, only one regime is found (open circles in

Figure 8. Time series of the transport difference, $\Delta \Phi$, (including spin-up) for different values of $A_H$ in the large-basin case. (a) $A_H = 2400 \text{ m}^2\text{s}^{-1}$, viscous antisymmetric regime; (b) $A_H = 1600 \text{ m}^2\text{s}^{-1}$, asymmetric regime; (c) $A_H = 900 \text{ m}^2\text{s}^{-1}$, quasi-homoclinic regime; (d) $A_H = 600 \text{ m}^2\text{s}^{-1}$, inertial antisymmetric regime.

Fig. 10). All of the time-mean states in this case belong to the group of asymmetric jet-down solutions (similar as in Fig. 9b) as $\Delta\Phi$ is always larger than zero. This is caused by the wind-stress curl, which has an increased (decreased) vorticity input in the southern (northern) half of the basin. The mirror-symmetry is broken by this asymmetric wind-stress profile and thereby induces a preference for jet-down solutions.

b. Variability

The spatio-temporal variability of the flows in the large basin was also analyzed with MSSA. For each case, a histogram is shown in Figure 11 of the variance explained by each
of the statistical modes, classified into groups that can be related to an internal mode. Most of the variance in the viscous antisymmetric regime can be explained by two types of modes. These are a classical baroclinic mode (CB1) with a period of about $3.0 \text{ mo}$, the latter being somewhat smaller than in the small-basin case, together with the dipole oscillation (D). The patterns of the dipole oscillation are very similar to those of the small-basin case, but slightly more localized near the midlatitude jet.

In the inertial antisymmetric regime the variability is again controlled by basin-wide westward travelling Rossby basin modes (RB) with intermonthly periods. Part of the variance in the asymmetric and quasi-homoclinic regimes can be explained by the relaxation gyre oscillations (G). It causes low-frequency variability with a period of around $3.0 \text{ yr}$. In the large-basin case, this mode is accompanied by high-frequency wall-trapped modes (WT) (Cessi and Ierley, 1993; Sheremet et al., 1997) with a period of $3.0 \text{ mo}$, instead of Rossby basin modes as in the small-basin case. Anomalies of such a statistical wall-trapped mode seem to originate along the northern and southern boundaries and are subsequently advected by the western boundary currents and seem to disappear once they have entered the midlatitude jet.

The case with asymmetric wind-stress forcing is performed to demonstrate that the presence of the gyre mode is linked to the asymmetry of the time-mean state. Hence, if the symmetrization of the mean state is opposed and the regimes transition toward the inertial antisymmetric regime is moved in parameter space, this will have an effect on the low-frequency variability in the system. Moreover, this case is primarily studied to motivate the study after the symmetrization of the time-mean state in the inertial antisymmetric regime, and its discussion is therefore postponed to Section 6b.
Figure 11. Histograms of explained variances (%) for each of the different statistical oscillations with (a) $g_2' = 0.01 \text{ m s}^{-2}$, (b) $g_2' = 0.03 \text{ m s}^{-2}$ and (c) $g_2' = 0.03 \text{ m s}^{-2}$ and asymmetric wind-stress forcing, $\alpha = 0.15$. All calculations have been performed in a large basin. CB1 = classical baroclinic mode, causing meandering of the midlatitude jet; CB2 = classical baroclinic mode causing strengthening and weakening of the midlatitude jet; D = dipole oscillation; G = relaxation gyre oscillations; RB = Rossby basin mode; and WT = wall-trapped mode.
The results of this section show that the presence of the four regimes is only quantitatively affected by the dimension of the basin. Moreover, the associated variability within these regimes can still, for a large part, be understood by the internal oscillatory modes found in previous studies.

6. Transitions between the different regimes

In this section, we analyze the dynamical mechanisms of the transitions between the different regimes. Of these, the transition between the viscous antisymmetric regime and the asymmetric regime is well known and due to the existence of a symmetry-breaking pitchfork bifurcation (Dijkstra and Katsman, 1997). The transition between the asymmetric and quasi-homoclinic regime and that between the quasi-homoclinic and inertial antisymmetric regime are, however, far from trivial.

a. Indications of the nearby presence of a homoclinic orbit

Recently, complex temporal transitions in flows described by the barotropic quasigeostrophic model were analyzed (Simonnet et al., 2004) and a strong case has been made that a homoclinic bifurcation is the cause for the transition from the asymmetric regime to a quasi-homoclinic regime. This homoclinic bifurcation arises through the merging of the relaxation gyre oscillations around the time-mean jet-up and jet-down states.

Indications of the nearby presence of a homoclinic orbit are also found in the solution for the three-layer model with $g'/H_1 = 0.03$ m s$^{-2}$ and $A_H = 1400$ m$^2$ s$^{-1}$. Here, the transport difference (Fig. 12a) undergoes a regular alternation between positive and negative values on a time scale of about 20 years. However, the dominant time scale in the energy time series is approximately 10 years (Fig. 12b; note the different scaling, $10^p$, for the kinetic ($p = 16$) and potential ($p = 17$) energy on the vertical axis). This indicates that two oscillations in the energy time series occur during one oscillation period in the time series of the transport difference. The time series of the transport difference show that the circulation oscillates between a jet-up (with $\Delta \Phi < 0$) and jet-down (with $\Delta \Phi > 0$) phase on an interdecadal time-scale. Furthermore, the energy series indicate that a decadal oscillation occurs during either phase.

The nearby presence of a homoclinic orbit in this flow can be visualized by plotting the maximum barotropic annual mean transport, $\Psi_{max}$, against (the absolute value of) the minimum barotropic annual mean transport, $|\Psi_{min}|$ (thick drawn line in Fig. 12c). The markers, representing the monthly sampled data, show a lot of scatter because of the high-frequency oscillations. The orbit through the unfiltered data is not shown because it fills the entire region between the maximum and minimum value obtained on either side of the diagonal $\Psi_{max} = |\Psi_{min}|$. The time series of the annual mean data demonstrate that two loops with either a stronger subpolar gyre ($\Psi_{max} < |\Psi_{min}|$, jet-up) or a stronger subtropical gyre ($\Psi_{max} > |\Psi_{min}|$, jet-down) are connected through the antisymmetric solution, which is located on the diagonal.
Figure 12. Time series of the transport difference, $\Delta \Phi$, in panel (a) and of the basin-integrated kinetic, $E_k$, (drawn) and potential, $E_p$, (dashed) energy in panel (b) at $A_H = 1400$ m$^2$ s$^{-1}$ of the solution for $g^2 = 0.03$ m s$^{-2}$. The value of the exponent is $p = 16$ for the kinetic and $p = 17$ for the potential energy. (c) Thick drawn line corresponds to the maximum versus the absolute value of the minimum of the annual mean transport function, $\Psi_{max}$. The markers represent the monthly sampled data and the diagonal is plotted as a thin drawn line.
The time-mean upper-layer streamfunction, $\vec{\psi}_1$, displays an antisymmetric circulation (Fig. 13a). The flow changes that occur near the homoclinic orbit can be deduced from the two leading EOFs of the upper-layer streamfunction, $\psi_1$, and (the last 40 years of) their associated PCs (Fig. 13b, c and d). The leading PC (drawn line in Fig. 13d) has a dominant period of \( T = 17 \) yrs, which is twice as long as the dominant period of the second PC (dashed line in Fig. 13d). The primary EOF and its associated PC introduce an alternation between a jet-up and a jet-down circulation on a multidecadal time scale, if it is superposed
on the time-mean circulation. The secondary EOF provides strengthening and weakening of the midlatitude jet. Together, they form a time-dependent circulation in the nearby presence of a homoclinic orbit, which is described in more detail over one loop in Table 2. In summary, it alternates between a relaxation oscillation (related to the gyre mode) around the jet-up and and around the jet-down solution, while switches between both occur through a phase with a strong antisymmetric circulation. This is also characteristic for the flow near the homoclinic bifurcation in the barotropic quasi-geostrophic model (Simonnet et al., 2004).

### b. Symmetrization of the zonal velocity field

The results in both the small- and the large-basin case suggest that (before the occurrence of a homoclinic orbit) an asymmetric time-mean state is essential for the destabilization of the flow by a gyre mode, thereby generating low-frequency relaxation oscillations. The results for the asymmetric wind stress (with $\alpha = 0.15$ and $g_2 = 0.03 \text{ m s}^{-2}$) support this view. Note that the time-mean states in this case are all asymmetric jet-down solutions (Fig. 10). The histogram of the explained variance of the statistical modes (Fig. 11c) indicates that the relaxation oscillations explain a large part of the variance for all values of $A_H$ considered. Although the period of the gyre mode is diminished to $\mathcal{P} = 1.3$ yr, its spatial pattern remains unaffected (not shown). High-frequency variability in these flows is either caused by wall-trapped (with $\mathcal{P} \approx 3.0$ mo) or Rossby basin modes (with $\mathcal{P} \approx 6.0$ mo), which explain only a small fraction of the variance. Hence, the size of the asymmetric and quasi-homoclinic regime can be manipulated by the asymmetry in the wind-stress forcing at the cost of the size of the inertial antisymmetric regime. This enhances the interval of $A_H$ values for which the low-frequency relaxation oscillations, associated with the gyre mode, occur.

The transition of the quasi-homoclinic regime to the inertial antisymmetric regime is associated with a strong symmetrization of the time-mean zonal velocity field. In the latter regime, there is no trace of a relaxation oscillation related to a gyre mode. On the other hand, this mode explains a significant part of the variance in the asymmetric and

### Table 2. Description of the time-dependent circulation near the homoclinic orbit.

<table>
<thead>
<tr>
<th>Time (yr)</th>
<th>PC$_{1}$</th>
<th>PC$_{2}$</th>
<th>Circulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>69</td>
<td>0</td>
<td>max</td>
<td>Strong antisymmetric</td>
</tr>
<tr>
<td>71</td>
<td>min</td>
<td>0</td>
<td>Jet-up</td>
</tr>
<tr>
<td>72</td>
<td>–</td>
<td>min</td>
<td>Weak jet-up</td>
</tr>
<tr>
<td>75</td>
<td>–</td>
<td>0</td>
<td>Jet-up</td>
</tr>
<tr>
<td>78</td>
<td>0</td>
<td>max</td>
<td>Strong antisymmetric</td>
</tr>
<tr>
<td>80</td>
<td>max</td>
<td>0</td>
<td>Jet-down</td>
</tr>
<tr>
<td>81</td>
<td>+</td>
<td>min</td>
<td>Weak jet-down</td>
</tr>
<tr>
<td>83</td>
<td>+</td>
<td>0</td>
<td>Jet-down</td>
</tr>
<tr>
<td>86</td>
<td>0</td>
<td>max</td>
<td>Strong antisymmetric</td>
</tr>
</tbody>
</table>
quasi-homoclinic regime at intermediate $A_H$. The physics of the low-frequency variability hence also changes drastically at the regime boundary. There are no indications that the relaxation oscillations will be significantly influenced directly by high-frequency oscillations in time-dependent flows. However, their destabilization will be essentially dependent on the presence of the asymmetry of the time-mean state. The latter can change through rectification effects induced by the nonlinear interaction of the high-frequency variability. From the results in the previous sections, it appears that the net effect of the high-frequency variability and baroclinicity causes a symmetrization of the time-mean zonal velocity field at low values of the friction. In this section, we investigate the mechanisms that are responsible for this symmetrization.

i. Analysis. The idea is to consider the equation governing the barotropic time-mean flow and then analyze the effects of the forcing terms due to nonlinear interaction of high-frequency instabilities. The equation for the barotropic (or depth averaged) streamfunction, $\psi_d = (H_1 \psi_1 + H_2 \psi_2 + H_3 \psi_3)/H$, can be constructed if we vertically average (1). The resulting equation is very similar to the evolution equation of a simple barotropic flow, apart from an extra term, $F(\psi_1, \psi_2, \psi_3)$ that originates from the advection of the baroclinic relative vorticity due to the baroclinic velocity field (see the Appendix A for a derivation of this term). Its expression is

$$F(\psi_1, \psi_2, \psi_3) = -\sum_{i=1}^{3} \sum_{j=1}^{3} \frac{H_i H_j}{2H^2} J[(\psi_i - \psi_j), \nabla^2 (\psi_i - \psi_j)].$$

The evolution for the time-mean barotropic potential vorticity, $\bar{q}$, can be constructed by separating the time-dependent variables of the resulting equation into a time-mean part (denoted by an overbar) and a fluctuating part (denoted by a prime). By averaging over time, the effect of the interaction of high-frequency modes is incorporated. The final equation becomes

$$J(\bar{\psi}_d, \bar{q}_d) = A_H \nabla^4 \bar{\psi}_d + \frac{f_0}{H} w_r - \frac{J(\psi_d', q_d)}{H} + F(\bar{\psi}_1, \bar{\psi}_2, \bar{\psi}_3) + F(\psi'_1, \psi'_2, \psi'_3)$$

$$\bar{q}_d = \nabla^2 \bar{\psi}_d + \beta_0 y$$

$$q'_d = \nabla^2 \psi'_d.$$  

The first three terms in (7a) represent a barotropic quasi-geostrophic vorticity equation. For the time-mean barotropic streamfunction in our three-layer model, $\bar{\psi}_d$, extra forcing is generated by the time-mean baroclinic flow, $F(\bar{\psi}_1, \bar{\psi}_2, \bar{\psi}_3)$, and by baroclinic and barotropic eddy-interaction, $F(\psi'_1, \psi'_2, \psi'_3)$ and $-J(\psi_d', q_d')$, respectively. These terms and their sum are calculated for a particular case from the asymmetric regime with $(g'_2, A_H) = (0.03 \text{ m s}^{-2}, 700 \text{ m}^2 \text{ s}^{-1})$ and are plotted in Figure 14.

The main contribution to the sum in Figure 14d is from the time-mean baroclinic
advection; its amplitude is 14 times larger than that of the input by the Ekman pumping. The contributions from the eddy interaction terms are a lot smaller, but are not negligible since their amplitudes are still 1.5 (barotropic part) and 4 (baroclinic part) times larger compared to that of the Ekman pumping. Apparently, not all extra forcing terms act in the same direction, since the sum of the terms is (only) 13 times larger than the input by the wind stress.

The antisymmetric part of the forcing term is dominant in the region of the western boundary current, while both symmetric and antisymmetric parts are equally strong in the area of the midlatitude jet. In the interior, the input of the vorticity by the antisymmetric
part of the forcing has the same sign as the Ekman pumping which is positive (negative) north (south) of the mid-axis of the basin. However, it is strongly counteracting the wind-stress forcing in the western boundary region. Hence, the degree of symmetrization and its effect on the low-frequency gyre mode is not immediately clear from this approach because of the complexity of the pattern. To determine this effect more clearly, we use a low-order model in the next subsection.

ii. Effect on gyre modes in a low-order model. Simonnet and Dijkstra (2002) have demonstrated that the merging of stationary modes, which gives rise to gyre modes, is also present in the low-order model of the wind-driven ocean circulation studied by Jiang et al. (1995). Apart from this merging process, the simple model also captures the appearance of multiple asymmetric steady states by a (perturbed) pitchfork bifurcation.

The low-order model can be derived from a nondimensional version of the quasi-geostrophic barotropic vorticity equation, which is forced by a double gyre Ekman pumping and damped by a linear friction. It is therefore comparable to (7) without the extra forcing terms and with the lateral friction substituted by a linear friction, i.e. $A_H \nabla^4 \psi \rightarrow -r \nabla^2 \psi$. A double Fourier expansion is made for the streamfunction, in which the expansion function contains the structure of the western boundary layer. The truncated equations are subsequently projected onto an orthogonal basis and a system of two differential equations is obtained (see Appendix B and Simonnet and Dijkstra (2002) for the derivation):

$$\frac{dA}{dt} - \lambda AB + \nu A = \eta_1$$

$$\frac{dB}{dt} + \lambda A^2 + \nu B = \eta_2.$$  

(8a)

(8b)

This set of equations provides the evolution of $A$ and $B$, which represent the symmetric and antisymmetric part of the streamfunction, respectively. In this way, the flow is antisymmetric if $A = 0$ and asymmetric if both $A$ and $B$ are nonzero. The parameter $\lambda$ measures the influence of the nonlinear advection, $\nu \sim r$ represents the strength of the linear friction and $\eta_1$ and $\eta_2$ are a measure of the strength of the symmetric and antisymmetric part of the wind forcing.

The bifurcation structure of this set of equations is relatively simple, if the double-gyre Ekman pumping is antisymmetric, i.e. $\eta_1 = 0$ and $\eta_2 > 0$. A symmetry breaking pitchfork bifurcation occurs at $\gamma = \lambda \eta_2/\nu^2 = 1$, giving rise to multiple stable asymmetric steady states for $\gamma > 1$. The merging of two stationary modes, from which gyre modes originate, occurs at $\gamma = 9/8$ on the branches with asymmetric steady states. For more details on the derivation of this model and for the results with asymmetric forcing ($\eta_1 \neq 0$), we refer to the paper by Simonnet and Dijkstra (2002).

The time-mean barotropic flow in (7) is not only forced by the Ekman pumping, but also
by the extra forcing generated by baroclinicity and eddy-interaction. The pattern of this forcing (as plotted in Fig. 14d) can also be projected on the orthogonal basis and will thereby enter (8) by changing the values for $\eta_1$ and $\eta_2$. In this way, the projection coefficients indicate whether the extra forcing is counteracting or cooperating the wind-stress forcing. When $\eta_2$ is increased, $\gamma$ will increase and the system will shift toward the regime with multiple asymmetric steady states and the presence of a gyre mode. On the other hand, when $\gamma$ is decreased, the system moves toward the regime where only one antisymmetric solution is present and no gyre modes.

The projection of only the Ekman pumping term, $f_0 w_e/H$, with $w_e$ as in (3) on the basis functions of the truncated model gives standard values for the forcing coefficients ($\eta_1^E$, $\eta_2^E$) = (1.03$\alpha$, 0.79) $f_0\mu_0/H$, which is of course purely antisymmetric if $\alpha = 0.0$ (see also the first column in Table 3). When the sum of the extra forcing terms (Fig. 14d) is projected on this basis, the resulting coefficients are ($\eta_1^F$, $\eta_2^F$) = (−0.38, −0.12) $f_0\mu_0/H$ (second column in Table 3). The contribution of the extra forcing to the antisymmetric part, $\eta_2^F$, is negative and thereby counteracting the effect of the Ekman pumping. This indicates that there is an effective reduction on $\gamma$, moving the system toward the antisymmetric regime. However, the flow cannot become fully antisymmetric, because of the strong asymmetry introduced by this extra forcing, expressed by the negative value of $\eta_1^F$.

For a case in the inertial antisymmetric regime with ($g'_2$, $A_H$) = (0.03 m s$^{-2}$, 400 m$^2$ s$^{-1}$) the patterns of the extra forcing terms are plotted in Figure 15. The coefficients of the projection become ($\eta_1^F$, $\eta_2^F$) = (0.0, −0.37) $f_0\mu_0/H$, which indicates that the opposing effect has become stronger, ($\eta_2^F$ has become stronger negative) effectively decreasing $\gamma$. Moreover, the extra forcing has become entirely antisymmetric because $\eta_1^F = 0.0$. However, the effect of the decreased lateral friction coefficient, $A_H$, cannot be quantified, since it is not incorporated in this low-order model. Hence, it cannot be indicated what path is followed from the solution at ($g'_2$, $A_H$) = (0.03 m s$^{-2}$, 700 m$^2$ s$^{-1}$) to that at ($g'_2$, $A_H$) = (0.03 m s$^{-2}$, 400 m$^2$ s$^{-1}$) in terms of parameters of the low-order model.

This leads to the view that the effect of baroclinicity and eddy-interaction create an extra forcing on the time-mean barotropic flow on top of the Ekman pumping at the surface. The time-mean baroclinicity in this flow is the largest contributor to the extra forcing. This can be seen in Figures 14d and 15d, which are, for the most part, determined by the term $F(\bar{\psi}_1$, $\bar{\psi}_2$, $\bar{\psi}_3$) in the panels (a). In both cases, this term contains an antisymmetric part which is

<table>
<thead>
<tr>
<th>$f_0 w_e/H$</th>
<th>$A_H = 700$ m$^2$ s$^{-1}$</th>
<th>$A_H = 400$ m$^2$ s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_1$</td>
<td>1.03$\alpha$</td>
<td>−0.38</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>0.79</td>
<td>−0.12</td>
</tr>
</tbody>
</table>

Table 3. Values obtained for $\eta_1$ and $\eta_2$ for the projections of the Ekman pumping in Eq. 3 (first column), the extra forcing in Figure 14 (second column) and in Figure 15 (third column) on the orthogonal basis (scaled with the amplitude of the Ekman pumping, $f_0\mu_0/H$).
opposing the Ekman pumping in the area of the western boundary, while it strengthens its effect in the interior of the basin. From the low-order model it is clearly shown that in both cases the extra term is introducing a preference in the direction of the regime with a unique antisymmetric solution.

7. Summary and discussion

In this study, the time-mean states and the variability are explored in simulated ocean flows within a three-layer quasi-geostrophic model. The use of two different basin sizes
enables us to make a connection between earlier bifurcation studies in basin sizes with horizontal dimensions of \( \sim 1000 \) km (Jiang et al., 1995; Speich et al., 1995; Dijkstra and Katsman, 1997; Nauw and Dijkstra, 2001) and studies of variability in much larger basins as in Berloff and McWilliams (1999). Four regimes are identified by the analysis of a combination of the maximum northward transport of the time-mean flow and (the time series of) the normalized transport difference between the subtropical and subpolar gyre. The four regimes are characterized by different types of variability. They are fairly robust to changes in the stratification and, more important, also to the domain size.

At large values of \( A_H \) the time-mean circulation is antisymmetric with respect to the mid-axis of the basin and the transport is relatively weak. The variability in this viscous antisymmetric regime is dominated by high-frequency oscillations, which can be related to classical baroclinic modes (Dijkstra and Katsman, 1997). The regime next to the viscous antisymmetric regime is characterized by a strong asymmetric time-mean circulation with the midlatitude jet either directed north- or southeastward. The results are consistent with the current knowledge of the bifurcation structure of the double-gyre circulation. In quasi-geostrophic double-gyre flows, a symmetry-breaking pitchfork bifurcation destabilizes the antisymmetric steady state and marks the transition to the asymmetric regime as two additional asymmetric steady states are created (Dijkstra and Katsman, 1997). These asymmetric steady states are subsequently destabilized by a low-frequency gyre mode, which generates large-amplitude, low-frequency relaxation oscillations in the time-dependent flow (Simonnet et al., 2004). Although the viscous antisymmetric regime is not considered ‘realistic’ because it only exists at large values of \( A_H \), knowledge of the regime is useful to understand the occurrence of the asymmetric regime. The latter regime exists only for unrealistic values of \( A_H \) under symmetric forcing conditions, but it does broaden under asymmetric wind forcing and hence may be relevant.

A homoclinic bifurcation, caused by the merging of two mirror-symmetric low-frequency relaxation oscillations and the unstable antisymmetric steady state, marks the transition from the asymmetric regime to the quasi-homoclinic regime in a barotropic quasi-geostrophic model (Simonnet et al., 2004). Indications of the nearby presence of a homoclinic bifurcation are found in the large-basin case, where the circulation alternates between a jet-up and a jet-down solution on interdecadal time scales. Furthermore, an interannual relaxation oscillation occurs in each jet-up or jet-down state. The time-mean circulation in the quasi-homoclinic regime is nearly antisymmetric and a large part of the variance is explained by the low-frequency relaxation oscillation related to the gyre mode.

Signatures of the asymmetric and quasi-homoclinic regimes are also found in results from other model studies. For example, in Berloff and McWilliams (1999) the spatial pattern of the ‘secondary’ mode in the symmetrically forced equivalent barotropic case at \( A_H = 1000 \) m\(^2\) s\(^{-1}\) in Berloff and McWilliams (1999) (their Fig. 15) is similar to that of the gyre mode. The meridional position of the separation point of their time-dependent solution at \( A_H = 600 \) m\(^2\) s\(^{-1}\) alternates between locations to the north and to the south of
the mid-axis of the basin on a decadal time scale (their Fig. 18). This indicates an alternation between a jet-up and a jet-down solution and provides support for the nearby presence of a homoclinic orbit. Hence, this solution is likely to reside in a quasi-homoclinic regime.

The streamfunction of the long-term mean flow investigated in McCalpin and Haidvogel (1996) in an equivalent barotropic quasi-geostrophic model (their Fig. 4a) seems to be nearly antisymmetric. The associated energy time series show low-frequency behavior with some characteristics of relaxation oscillations. These oscillations introduce fluctuations of the midlatitude jet on a decadal time scale. Both the nearly antisymmetric time-mean flow and the presence of low-frequency relaxation oscillations have shown to be characteristics of the quasi-homoclinic regime. However, more information on the statistical modes would be needed to make a more precise statement.

The time-mean circulation at low values of $A_H$ is also nearly antisymmetric, but the transports are much higher than in the viscous antisymmetric regime. A large part of the variance in this inertial antisymmetric regime can be explained by high-frequency Rossby basin modes. The transition from the quasi-homoclinic regime to the inertial antisymmetric regime occurs through symmetrization of the zonal velocity field of the time-mean state according to the mechanisms suggested in Section 5. There it is shown that the baroclinicity and barotropic eddy interaction introduce forcing terms that oppose the wind-stress forcing, thereby moving the system toward the regime where multiple equilibria and also the gyre mode cease to exist. This regime transition may also be related to the occurrence of a bifurcation. At least in calculations with a barotropic quasi-geostrophic model in a large basin, it seems to be due to the occurrence of two successive bifurcations, i.e. a reverse pitchfork bifurcation and a Hopf bifurcation (Simonnet et al., 2004). However, the identification of these bifurcations in a multi-layer quasi-geostrophic model is outside of the scope of this paper.

The inertial antisymmetric regime is considered the most ‘realistic’ regime because it occurs at the lowest values of the lateral friction (Berloff and McWilliams, 1999; Dewar, 2001). The time-mean states are nearly antisymmetric and the time series do not show a preference for a particular frequency, but generate red-noise spectra. However, the existence of the latter regime may also be a signature of the limitations of quasi-geostrophic theory, since such a regime has, as far as we know, never been observed in studies using shallow-water models. This motivates a similar study with a shallow-water model in which the role of asymmetry and ageostrophic effects on the quasi-homoclinic dynamics and the presence of the inertial regime will be investigated. The concepts of the theory of dynamical systems may also appear to be particularly useful as a framework of interpretation for these results.

The role of dynamical building blocks, such as the gyre mode and the homoclinic orbit, in transient behavior and regime transitions of the double-gyre wind-driven circulation has clearly been demonstrated. Although similar transient behavior was found in many models of the double-gyre flows, there is still a long way to go to explain the behavior of flows
simulated with high-resolution eddy-resolving ocean models and in observations of the real ocean circulation. These flows have a much smaller lateral eddy viscosity and are characterized by high eddy activity. They display a complex interaction between mean flow and its instabilities. There is no guarantee that the different regimes and the transitions between them actually apply to these flows. However, when the systematic approach as used here is extended to realistic models having increasingly higher resolution and containing more and more physical processes we eventually might understand the physics of such complex flows.

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APPENDIX A

The derivation of the equation for the barotropic (or depth averaged) streamfunction, \( \psi_d = \sum_{i=1}^{3} H_i \psi_i / H \), is rather standard, except for the term including the Jacobian operator, representing the advection of potential vorticity. After averaging (1) over depth, i.e. \( [H_1(1a) + H_2(1b) + H_3(1c)] / H \), where \( H = \sum_{i=1}^{3} H_i \) is the total depth, the total advection of potential vorticity becomes:

\[
\sum_{i=1}^{3} \frac{H_i}{H} J(\psi_i, q_i) = \sum_{i=1}^{3} \frac{H_i}{H} J(\psi_i, \nabla^2 \psi_i) + J(\psi_d, \beta_0 y) + \frac{H_1}{H} J\left(\psi_1, -\frac{f_0}{H_1} h_1\right) + \frac{H_2}{H} J\left(\psi_2, -\frac{f_0}{H_2} (h_2 - h_1)\right) + \frac{H_3}{H} J\left(\psi_3, \frac{f_0}{H_3} h_2\right) \quad (9a)
\]

\[
= \sum_{i=1}^{3} \frac{H_i}{H} J(\psi_i, \nabla^2 \psi_i) + J(\psi_d, \beta_0 y) - \frac{f_0^2}{Hg_1^2} J[\psi_1, (\psi_1 - \psi_2)]
\]

\[
- \frac{f_0^2}{Hg_2^2} J[\psi_2, (\psi_2 - \psi_3)] + \frac{f_0^2}{Hg_1^2} J[\psi_2, (\psi_1 - \psi_2)] + \frac{f_0^2}{Hg_2^2} J[\psi_3, (\psi_2 - \psi_3)]. \quad (9b)
\]

By entering the definition for the layer thickness anomalies, \( h_i = f_0(\psi_i - \psi_{i+1}) / g' \) into (9a), it becomes clear that the terms describing the stretching cancel each other out (last four terms in (9b)).

The extra forcing term introduced by the baroclinicity of the flow, \( F(\psi_1, \psi_2, \psi_3) \) in (6), can be determined by subtracting the total advection of vorticity in (9) from the advection of barotropic potential vorticity by the barotropic flow, \( J(\psi_d, q_d) \).

\[
F(\psi_1, \psi_2, \psi_3) = J(\psi_d, q_d) - \sum_{i=1}^{3} \frac{H_i}{H} J(\psi_i, \nabla^2 \psi_i) - J(\psi_d, \beta_0 y). \quad (10)
\]
Because \( q_d = \nabla^2 \psi_d + \beta_0 y \), part of the first term on the right-hand side cancels with the last term. Hence,

\[
F(\psi_1, \psi_2, \psi_3) = \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{H_i H_j}{H^2} J(\psi_i, \nabla^2 \psi_j) - \sum_{i=1}^{3} \frac{H_i}{H} J(\psi_i, \nabla^2 \psi_i) \tag{11a}
\]

\[
= - \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{H_i H_j}{2H^2} J[(\psi_i - \psi_j), \nabla^2(\psi_i - \psi_j)], \tag{11b}
\]

which can be interpreted as the advection of the baroclinic relative vorticity by the baroclinic part of the flow.

**APPENDIX B**

The low-order model used in Section 6b(ii) was first derived by Jiang et al. (1995) to elucidate the nature of the perturbed pitchfork bifurcation and it was used in Simonnet and Dijkstra (2002) to identify the physics of the oscillatory gyre mode. Here, we repeat the derivation of the low-order model as described in Simonnet and Dijkstra (2002).

The low-order model is derived from the dimensionless barotropic quasi-geostrophic model (Pedlosky, 1987)

\[
\frac{\partial}{\partial t} (\nabla^2 - F)\psi + \epsilon J[\psi, (\nabla^2 - F)\psi] + \beta \frac{\partial \psi}{\partial x} = -r \nabla^2 \psi + w_f, \tag{12}
\]

where the Rossby number, \( \epsilon \), represents the strength of the nonlinear advection and \( r \) the strength of the bottom friction. A square domain is considered \((x, y) \in [0, \pi] \times [0, \pi]\) and conditions of no normal flow are assumed on the boundaries, \( \psi = 0 \). The Ekman pumping, \( w_f \), contains a symmetric and an antisymmetric part, controlled by \( w_1 \) and \( w_2 \), respectively:

\[
w_f = -w_1 \sin y - w_2 \sin 2y. \tag{13}
\]

Solutions to (12) are considered that have a similar symmetric and antisymmetric part (Veronis, 1963). Also, the existence of a western boundary current is accounted for by a decaying exponential in the \( x \) direction, that is

\[
\psi(x, y, t) = A(t)G(x) \sin y + B(t)G(x) \sin 2y \tag{14a}
\]

\[
G(x) = e^{-sx} \sin x, \tag{14b}
\]

where \( G(x) \) represents the zonal asymmetric structure with \( s = 1.3 \).

The truncated equations are obtained by projecting 12 onto the orthogonal basis \( \{ G(x) \sin y, G(x) \sin 2y \} \). Then, the equations are integrated over the domain in such a way that the energy of the truncated system is conserved. A system of two differential equations for the two scalar variables, \( A \) and \( B \), is obtained,
\[
\frac{dA}{dt} - \lambda A b + v_1 A = \eta_1 \\
\frac{dB}{dt} - \lambda A^2 + v_2 B = \eta_2,
\]

where the parameters \( \lambda \sim \epsilon; (v_1, v_2) \sim r; \eta_1 \sim w_1 \) and \( \eta_2 \sim w_2 \) and they are further dependent on \( F \) and \( s \). The precise expressions can be found in Simonnet and Dijkstra (2002) who investigate the case \( F = 0 \) for which \( v_1 = v_2 = \nu \). None of the parameters of the truncated model is dependent on the parameter \( \beta \), because the term \( \partial \psi/\partial x \) is orthogonal to the basis. Hence, Rossby wave dynamics has been filtered out from these equations.

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