Optimal Information Design in Two-Sided Trade

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Cover Page Footnote
I am grateful to my advisor, Professor Dirk Bergemann for his valuable guidance and constant feedback that made this paper possible. His patience and willingness to teach me allowed me to explore a field that was new to me. I would also like to thank Ian Muir, the Head of the Department of Economics at Lyft, for a wonderful summer and helpful suggestions that led me to this research topic.
1. INTRODUCTION

We study optimal information design in a two-sided trading problem where buyers and sellers trade over a single homogeneous product and a broker mediates the trade. We consider individually rational, incentive-compatible mechanisms and optimize for either welfare or the broker’s revenue.

Our study is motivated by and set in the context of rideshare markets. Rideshare platforms present a two-sided marketplace where agents have multi-dimensional preferences. For example, a rider seeking a ride on Uber typically values a ride based on its price, wait time, car type, driver rating, and so on. Similarly, a driver offering a ride on Uber typically values it based on the wage he receives, the opportunity cost of providing the ride, time or distance, passenger rating, and so on. The two most important factors for both agent types here are price and time. Buchholz et al. (2020) conduct an empirical analysis of the Czech-based taxi and ride-hailing application Liftago in order to directly measure a customer’s value of time, or their willingness-to-pay for time savings, and study the impact of heterogeneity in value of time on a market where waiting time is used to ration the good. More recently, Goldszmidt et al. (2020) present an experimental analysis and estimate a consumer’s value of time in the United States using field experiments with the ridesharing company Lyft. Given the significant impact that time has in the rideshare context, we wish to study how a platform can use information about waiting times strategically. Both riders and drivers face a fundamental yet implicit trade-off between time and price. Although waiting times are non-negotiable and agents trade predominantly on price, can information about waiting times influence agent incentives and market outcomes?

The rideshare industry has recently prompted an extensive body of literature on optimal platform design with strategic agents. Ghili and Kumar (2020) study pricing mechanisms to respond to skewed spatial distributions of supply. Afeche et al. (2018) consider the problem of matching riders with strategic drivers under varying levels of platform control. Besbes et al. (2018) propose location-specific pricing to control the spatial distribution of supply while Rheingans-Yoo et al. (2019) analyze revenue optimal pricing and driver compensation with drivers having heterogeneous preferences over location.

Our work differs from these papers in two aspects. First, instead of considering demand and supply as a whole, we fix a rider and driver and view the problem as a stationary game with two agents and a broker. This allows us to model a static Bayesian game of incomplete information, where the prior distributions of rider and driver valuations are public knowledge but they are unaware of each other’s realized valuations. Most importantly, we build on Myerson and Satterthwaite’s seminal 1981 paper, which presents Bayesian incentive compatible and individually rational trade allocation mechanisms that maximize expected total gains from trade or a broker’s revenue.

Secondly, while the above papers study optimal matching and pricing mechanisms, we add information design as an additional lever the platform can use to strategically influence trade. Bergemann and Morris (2019) describe the information design problem as a study of how the mechanism designer can influence the individually optimal behavior of agents only through the choice of the information provided. Even if the broker cannot control outcomes or force agents to choose certain actions, she can influence them simply by choosing whether to and how to reveal information that agents depend upon when choosing an action. Bergemann and Morris (2019) present a broad overview of the literature on this topic and illustrate key insights, the foremost of which is that the mechanism designer may often find it optimal to selectively obfuscate information. We vary information levels along two dimensions: i) the platform revealing information about waiting times to riders and drivers, and ii) riders and drivers revealing their private valuations to the platform.

ABSTRACT

In a two-sided market with a broker, the broker can influence the buyer’s and seller’s optimal trading behaviour through strategic information design. We study the impact of information about waiting times on riders and drivers in a rideshare market. We consider three information regimes: the first in which no information about time is revealed, the second in which true waiting times are communicated, and finally an intermediate regime in which agents are only told whether their waiting time falls within a high or low category. We evaluate the optimality of each information regime by maximizing welfare and revenue for each setting and that by concealing information, the broker can incentivize agents to accept less favorable trades. On the other hand, more information restricts trade to sufficiently favourable bargains, yielding higher expected welfare and expected revenue.
The structure of the paper is as follows. In the next section, we describe our model. In section 3, we allow agent valuations to remain inaccessible to the broker and compute posted price mechanisms that maximize welfare and revenue for zero or complete information about waiting times. In section 4, we use a direct bargaining mechanism to elicit valuations from riders and drivers and again compute welfare-maximizing and revenue-maximizing trading mechanisms. In section 5, we ask if an intermediate revelation about waiting times can achieve a better result and determine the corresponding optimal mechanism. In section 6, we allow for heterogeneous rider preferences over waiting times and study its impact on the market. Finally, in section 7, we present a discussion of our results and conclusion.

2. THE MODEL

Our model contains two agents: riders, denoted by \( r \), and drivers, denoted by \( d \). The rideshare platform acts as a broker. Core parameters of the model include:

- \( w \): the rider’s willingness to pay \( \sim U(0, 1) \)
- \( z \): the driver’s opportunity cost \( \sim U(0, 1) \)
- \( x \): waiting time, or transportation cost \( \sim U(0, X) \)

We assume that all prior distributions are public knowledge. The rider’s valuation is defined as her willingness to pay minus the squared transportation cost.

\[
v_r = w - x^2
\]

Similarly, the driver’s valuation is the sum of her opportunity cost and squared transportation cost.

\[
v_d = z + x^2
\]

In this context of this paper, we assume that the rider’s willingness to pay and the driver’s opportunity cost is independent of transportation cost. While this may be an unrealistic assumption, doing so allows us to separate heterogeneity in rider and driver preferences from the effect of transportation cost on rider values and driver opportunity costs. Then, we can evaluate valuations pointwise and don’t solve for the joint distributions. Both agent’s utilities are defined as the difference between their valuation and the price of the ride, conditional on trade taking place.

\[
\begin{align*}
    u_r &= w - x^2 - p \\
    u_d &= p - z - x^2
\end{align*}
\]

If no trade takes place, utility is 0. We compute trading mechanisms that define an allocation rule and a price. The two outputs of any mechanism are as follows:

- \( p(w, z, x) \): price of the ride
- \( q(w, z, x) \): the probability of trade (1 for rides offered and 0 for rides not offered)

3. POSTED PRICES INDEPENDENT OF \( w, z \)

The values \( w \) and \( z \) are private information of riders and drivers, respectively. We begin with a baseline model in which the broker does not have access to these realized values and only relies on the prior distributions. We consider two levels of information about \( x \).

**Proposition 1.** Providing information about waiting times results in higher volumes of trade and expected welfare.

We show this in the remainder of this section.

3.1. No Information About Waiting Times

First, we assume that agents are given no information about realized waiting times. Instead, they operate on the basis of the expected value \( \mathbb{E}[X] = \frac{X}{3} \). We impose ex-interim individual rationality constraints. That is, agents must have nonnegative expected utility from trade knowing their own willingness to pay or opportunity cost but before any trading outcomes are determined.

\[
w - \frac{X^2}{3} - p \geq 0 \quad \text{and} \quad p - z - \frac{X^2}{3} \geq 0
\]

Thus, agents are willing to trade at the following set of prices

\[
z + \frac{X^2}{3} \leq p \leq w - \frac{X^2}{3}
\]

In order to have nonnegative expected trade, we must have \( x \leq \frac{\sqrt{3}}{2} \). Without any information about the realized values of \( w \) and \( z \), the broker can set a welfare-maximizing fixed posted price that is independent of \( w, z \) and agents will accept the ride whenever (3) is satisfied. Expected welfare is defined as the sum of the two agents’ utilities multiplied by the allocation mechanism.

\[
\begin{align*}
    \text{Expected Welfare} &= \int_0^1 \int_0^1 (w - z - \frac{2X^2}{3}) q(w, z, x) \cdot f_w f_z \, dz \, dw \\
    &= \int_0^1 \int_0^{\frac{X^2}{3}} w - z - \frac{2X^2}{3} \, dz \, dw \\
    \text{Expected Volume of Trade} &= \int_0^1 \int_{\frac{z + X^2}{3}}^{W - \frac{X^2}{3}} 1 \, dz \, dw = \left(1 - \frac{X^2}{3}\right)^2
\end{align*}
\]

3.2. Public Waiting Times

Next, we consider the case where the broker communicates the re-
alized waiting time $x$ to agents and repeat the same computations as above. Agents are willing to trade at the following set of prices:

$$z + x^2 \leq p \leq w - x^2$$

(4)

which imposes the restriction $x \leq \frac{1}{\sqrt{2}}$. With (4) as the allocation mechanism and the broker setting a posted price,

$$\text{Expected Welfare} = \int \int \int_{[0, \infty)^2} p - x^2 \ dz \ dw$$

which is again maximized at $\bar{p} = \frac{1}{2}$. Waiting time has a symmetric effect on riders and drivers in this model as $w$ and $z$ are identically distributed. Thus, information about $x$ doesn’t influence prices in the absence of knowledge about the realized values $w$ and $z$. The welfare-maximizing mechanism is given by

$$q(w, z, x) = \begin{cases} 1 & \text{if } z + x^2 \leq \frac{1}{2} \leq w - x^2 \\ 0 & \text{else} \end{cases}$$

We compute the expected welfare and expected volume of trade in this market for all realizations of $x$, keeping in mind that trade only occurs for $x \leq \frac{1}{\sqrt{2}}$.

$$\text{Expected Welfare} = \frac{1}{X} \int_0^{\infty} \int_0^{\min(w, \frac{x^3}{2})} \int_0^{\frac{1}{2} - x^2} w - z - 2x^2 \ dz \ dw \ dx$$

$$\text{Expected Volume of Trade} = \frac{1}{X} \int_0^{\infty} \int_0^{\min(w, \frac{x^3}{2})} \int_0^{\frac{1}{2} - x^2} 1 \ dz \ dw \ dx$$

Figure 1 displays expected welfare and Figure 2 displays expected volume of trade under the two mechanisms described in this section.

Without knowledge of the agents’ realized valuations, the broker charges a price independent of waiting time. The first mechanism underestimates volume of trade and thereby welfare for realized waiting costs lower than the expected value. Conversely, it overestimates welfare for high waiting times. Revealing waiting times in the second mechanism results in higher volumes of trade and higher expected welfare, but restricts unfavourable trade where $x > \frac{1}{\sqrt{2}}$. This effect becomes more pronounced as $X$ increases and a larger proportion of realized $x^2$ values fall below the expectation.

4. BILATERAL TRADE

In this section, we augment the model with information about the realized values of $w$ and $z$. Since these aren’t publicly observable, the broker must elicit them from the rider and driver. Our model uses the direct bargaining set-up described by Myerson and Satterthwaite (1983), in which each agent simultaneously reports his valuation to the broker and the broker then determines whether trade takes place and the price of the object. Each agent’s only action is to report $w$ and $z$, respectively. In addition to individual rationality, we now impose incentive compatibility constraints. Incentive compatibility is satisfied if honest reporting forms a Bayesian Nash equilibrium. That is, each agent can maximize her expected utility by reporting her true value, given that the other agent is expected to be honest. Myerson and Satterthwaite (1983) compute optimal trading mechanisms that are both individually rational and incentive-compatible. For a proof of their theorems, we encourage the reader to consult their paper. Here we simply follow their methodology as given, with the addition of waiting times in both agents’ valuations. Theorem 1 of their paper characterizes necessary and sufficient conditions for incentive compatibility with individually rational agents:

$$0 \leq \int_0^{V_r} \int_0^{V_d} \left( v_r - \frac{1 - F_r(v_r)}{F_r(v_r)} \right) \left( v_d + \frac{E_d(v_d)}{F_d(v_d)} \right) q(w, z, x)f_r(v_r)f_d(v_d) \ dv_r \ dv_d$$

(5)

where $v_r$, $v_d$ is each agent’s valuation as defined in (1)(2), $V_r$ or $V_d$ is the respective maximum value, and $F(x)$ is the cumulative distribution function. The same equation holds true with the inclusion of $x$. 

![Figure 1](Expected welfare with posted prices)

![Figure 2](Expected volume of trade with posted prices)
to their model. We introduce some new functions as defined in their paper and modify our earlier definition of the allocation mechanism $q(w, z, x)$. For any number $\alpha \geq 0$, let

$$c_d(z, x, \alpha) = (1 + \alpha)(z + x^2)$$
$$c_r(w, x, \alpha) = (1 + \alpha)(w - x^2) - \alpha$$
$$q^*(w, z, x) = \begin{cases} 1 & \text{if } c_d(z, x, \alpha) \leq c_r(w, x, \alpha) \\ 0 & \text{if } c_d(z, x, \alpha) > c_r(w, x, \alpha) \end{cases}$$

As in the previous section, we consider a setting with no information about waiting times and one with complete information. Now that the broker has information about the difference between the rider’s willingness to pay and the driver’s opportunity cost, she can capture some of this difference by offering different prices. We solve for the revenue-maximizing mechanism in addition to the welfare-maximizing calculations and prove two propositions.

**Proposition 2.** Providing information about waiting times restricts trade but increases expected revenue.

**Proposition 3.** The revenue-maximizing mechanism is more restrictive than the welfare-maximizing mechanism.

### 4.1. No Information About Waiting Times

Plugging in our model parameters and $\mathbb{E}[x^2]$ into (5), we must have

$$0 \leq 2 \int_0^1 \int_0^1 \left( w - z - \frac{2X^2}{3} - \frac{1}{2} \right) q(w, z, x) dz dw$$

Thus, conditional on the rider and driver agreeing to trade, the expected difference between their valuations must be at least $\frac{1}{2}$. In other words,

$$w - z \geq \frac{2X^2}{3}$$

This results in a stricter restriction than the one we had with only individual rationality in section 2. Now we must have $X \leq \frac{\sqrt{3}}{2}$.

Theorem 2 by Myerson and Satterthwaite (1983) states that an incentive-compatible mechanism such that $u_d(1) = u_r(0) = 0$ and $q = q^*$ for some $\alpha \in [0, 1]$ maximizes expected total gains from trade among all incentive-compatible individually rational mechanisms. Furthermore, in our context $c_d(z, x, 1)$ and $c_r(w, x, 1)$ are increasing functions and the interiors of the two valuation intervals have a non-empty intersection, thus such a mechanism must exist.

In order for each agent’s minimum utility to be 0, that is that $u_d(1) = u_r(0) = 0$ and $q = q^*$, we must satisfy (6) with equality. Plugging in the definition of $q^*$

$$0 = 2 \int_0^1 \int_0^1 \left( w - z - \frac{2X^2}{3} - \frac{1}{2} \right) dz dw$$

So we must have $\alpha = \frac{3 + 4X^2}{9 - 4X^2}$. The first value results in a mechanism that only allows trade if $w - z \geq 1$, which is not feasible given that $w, z \in [0, 1]$. We pick the second value of $\alpha$ which gives us

$$c_d(z, x, \alpha) = \frac{12z + 4X^2}{9 - 4X^2}$$
$$c_r(w, x, \alpha) = \frac{12w - 8X^2 - 3}{9 - 4X^2}$$
$$q^*(w, z, x) = \begin{cases} 1 & \text{if } w - z \geq X^2 + \frac{1}{4} \\ 0 & \text{if } w - z < X^2 + \frac{1}{4} \end{cases}$$

Thus, welfare is maximized in a mechanism where trade takes place whenever the difference in the rider’s willingness to pay and the driver’s opportunity cost is at least $X^2 + \frac{1}{4}$. The resulting expected welfare is given by

$$\text{Expected Welfare} = \frac{9}{64} - \frac{3X^2}{8} + \frac{X^4}{4}$$

Since price has an opposite and symmetric effect on each agent’s utility, it does not appear in the welfare equation. We do not compute an optimal price here but note that any price that satisfies individual rationality and incentive compatibility can be used. Under this mechanism, the expected volume of trade is

$$\text{Expected Volume of Trade} = \frac{1}{32}(3 - 4X^2)^2$$

For the revenue-maximizing mechanism, let $U_0$ denote the expected revenue of the broker, where

$$U_0 = \int_0^1 \int_0^1 (p_r(w, z, x) - p_d(w, z, x)) \cdot q(w, z, x) \, dz \, dw$$

By Theorem 4 in Myerson and Satterthwaite’s paper, the broker’s expected revenue is maximized by a mechanism in which trade is allowed if and only if $c_r(w, x, 1) \geq c_d(z, x, 1)$. The revenue-maximizing allocation mechanism is

$$q^*(w, z, x) = \begin{cases} 1 & \text{if } w - z \geq \frac{1}{2} + \frac{2X^2}{3} \\ 0 & \text{if } w - z < \frac{1}{2} + \frac{2X^2}{3} \end{cases}$$

All that remains is to construct prices that satisfy individual rationality and incentive compatibility. We define them in the same manner as Myerson and Satterthwaite (1983). If trade occurs, the rider is charged the lowest valuation she could have quoted and still gotten the ride and the driver is paid the highest valuation she could have quoted and still given the ride. It is easy to check that this is incentive compatible. For example, the driver reporting a lower valuation has no effect on prices and reporting a higher valuation decreases the probability of trade. Formally stated,

$$p_r(w, z, x) = q(w, z, x) \cdot \min \left\{ w - \frac{X^2}{3} \mid w \geq \frac{2X^2}{3} \text{ and } c_r(w, x, 1) \geq c_d(z, x, 1) \right\}$$
$$p_d(w, z, x) = q(w, z, x) \cdot \max \left\{ z + \frac{X^2}{3} \mid z \leq \frac{2X^2}{3} \text{ and } c_d(z, x, 1) \geq c_r(w, x, 1) \right\}$$

Under the revenue-maximizing mechanism, the broker should charge the rider $z + \frac{1}{2} + \frac{2X^2}{3}$ and should offer to pay the driver $w - \frac{1}{2} - \frac{2X^2}{3}$, and trade occurs if and only if both agents are willing to trade at these prices. The expected revenue is then
4.2. Public Waiting Times

We now compute welfare-maximizing and revenue-maximizing mechanisms with the broker revealing realized waiting times to agents. To satisfy individual rationality and incentive compatibility, we must have

\[ 0 \leq \int_{0}^{1} \int_{0}^{1} \left( w - z - 2x^{2} - \frac{1}{2} \right) q(w, z, x) \, dz \, dw \tag{9} \]

Conditional on the rider and driver agreeing to trade, the same condition holds as with no information about waiting times:

\[ (w - x^{2}) - (z + x^{2}) \geq \frac{1}{2} \]
\[ w - z \geq \frac{1}{2} + 2x^{2} \]

Note that even before we optimize for welfare or revenue, incentive compatibility imposes the restriction \( x \leq \frac{1}{2} \). If agents know how long they will have to wait, waiting times must be sufficiently small for market participation.

We determine a welfare-maximizing mechanism using the same methodology as the previous subsection. To have \( u_{d}(1) = u_{r}(0) = 0 \) resulting in for \( q = q^{0} \), we must satisfy (9) with equality.

\[ 0 = 2 \int_{0}^{1} \int_{0}^{1} w - 2x^{2} - \frac{1}{2} w - z - 2x^{2} - \frac{1}{2} \, dz \, dw \]

Thus, \( \frac{1 - 2x^{2}}{2x^{2}} \) or \( \frac{1 + 4x^{2}}{3 - 4x^{2}} \) and we pick the second value, as before. The welfare-maximizing mechanism is defined by

\[ c_{d}(z, x, a) = \frac{4z + 4x^{2}}{3 - 4x^{2}} \]
\[ c_{r}(w, x, a) = \frac{4w - 8x^{2} - 1}{3 - 4x^{2}} \]

\[ q^{0}(w, z, x) = \begin{cases} 
1 & \text{if } w - z \geq 3x^{2} + \frac{1}{4} \\
0 & \text{if } w - z < 3x^{2} + \frac{1}{4}
\end{cases} \]

The corresponding maximum expected welfare is given by

\[ \text{Expected Welfare} = \frac{1}{X} \int_{0}^{\min(X, \frac{1}{2})} \int_{0}^{w - 3x^{2} - \frac{1}{2}} \left( w - z - 2x^{2} \right) \, dz \, dw \, dx \]
\[ = \begin{cases} 
\frac{9}{64} - \frac{3x^{2}}{8} + \frac{9x^{4}}{32} & \text{if } X < \frac{1}{2} \\
\frac{3}{40X} & \text{if } X \geq \frac{1}{2}
\end{cases} \]

and the expected volume of trade is

\[ \text{Expected Volume of Trade} = \begin{cases} 
\frac{9}{64} - \frac{3x^{2}}{8} + \frac{9x^{4}}{32} & \text{if } X < \frac{1}{2} \\
\frac{3}{40X} & \text{if } X \geq \frac{1}{2}
\end{cases} \]

The revenue-maximizing mechanism is given by

\[ q^{1}(w, z, x) = \begin{cases} 
1 & \text{if } w - z \geq \frac{1}{2} + 2x^{2} \\
0 & \text{if } w - z \geq \frac{1}{2} + 2x^{2}
\end{cases} \]

The broker should offer the following prices:

\[ p_{r} = z + \frac{1}{2} + x^{2} \]
\[ p_{u} = w - \frac{1}{2} - x^{2} \]

Figure 3 plots expected welfare and Figure 4 plots expected volume of trade for welfare-maximizing mechanisms under no information and complete information about waiting times.

Communicating waiting times incentivizes agents to restrict trade to favourable waiting times, thereby increasing welfare and expected revenue. The revenue-maximizing mechanism under both information regimes is more restrictive than the welfare-maximizing mechanism, as seen in Figure 6, which plots the minimum differ-
ence in $w$ and $z$ needed. In order for the broker to profitably exploit control over trading, she must demand a larger minimum difference in the rider’s willingness to pay and the driver’s opportunity cost.

5. INTERMEDIATE INFORMATION REVELATION

Providing complete information about waiting times allows the broker to optimally restrict trade, but forces her to only serve a limited range of waiting times or distances. The question arises: instead of the two extreme information settings we considered, can we do better through an intermediate revelation?

Suppose the broker informs agents only whether their waiting time fell into a low or high category. That is, for some $\bar{x} \in [0, X]$, the broker reveals

$$x = \begin{cases} x_L & \text{if } x \leq \bar{x} \\ x_H & \text{if } x > \bar{x} \end{cases}$$

$\bar{x}$ could be exactly the halfway point $0.5X$, but this is not necessarily the optimal value as we show later. We restrict our attention to the scenario where the broker elicits $w$ and $z$ from the rider and driver. Without that, the posted price is independent of $x$, as we showed earlier.

**Proposition 4.** An intermediate information revelation regime allows for a larger range of waiting times than complete information with only a small decrease in expected welfare and expected revenue.

We assume that the threshold $\bar{x}$ is public knowledge and agents choose actions based on their expected waiting cost $E[w^2]$.

For low waits, the expected time is given by

$$E[w_L^2] = \frac{1}{\bar{x}} \int_0^{\bar{x}} x_L^2 \, dx_L = \frac{\bar{x}^2}{3}$$

And for high waits, agents expect

$$E[w_H^2] = \frac{1}{X - \bar{x}} \int_{\bar{x}}^{X} x_H^2 \, dx_H = \frac{\bar{x}^2 + X\bar{x} + X^2}{3}$$

We compute welfare-maximizing mechanisms for the two categories separately and pick $\bar{x}$ to maximize the expected total welfare. To satisfy individual rationality and incentive compatibility for each of the time categories, we modify (6) with our new expected waiting times:

**Low Category:** $0 \leq 2 \int_0^1 \int_0^{\bar{x}} \left( w - z - \frac{\bar{x}^2}{3} - \frac{1}{2} \right) q(w, z, x) \, dz \, dw$

**High Category:** $0 \leq 2 \int_0^1 \int_{\bar{x}}^{1} \left( w - z - \frac{\bar{x}^2 + X\bar{x} + X^2}{3} - \frac{1}{2} \right) q(w, z, x) \, dz \, dw$

The second expression is positive only when

$$\bar{x}^2 + X\bar{x} + X^2 \leq \frac{3}{4}$$

We then redo the calculations we did for bilateral trade under no information about $x$ and define $\alpha_L$ and $\alpha_H$ for $x_L$ and $x_H$, respectively.

These model parameters are stated in the appendix while here we...
simply describe the optimal allocation mechanisms.

\[
q^w(x) = \begin{cases} 
1 & \text{if } w - z \geq x^2 + \frac{1}{2} \\
0 & \text{if } w - z < x^2 + \frac{1}{2}
\end{cases}
\]

\[
q^m(x) = \begin{cases} 
1 & \text{if } w - z \geq x^2 + Xx + X^2 + \frac{1}{2} \\
0 & \text{if } w - z < x^2 + Xx + X^2 + \frac{1}{2}
\end{cases}
\]

Total expected welfare is given by

\[
\text{Expected Welfare} = \frac{\bar{e}}{X} \int_0^1 \int_0^1 (w - z - 2x^2)q^w(w, z, x) \, dz \, dw + \frac{X - \bar{e}}{X} \int_0^1 \int_0^1 (w - z - 2x^2)q^m(w, z, x) \, dz \, dw
\]

\[
= \frac{1}{64}((3 - 4X^2)^2 + 16X^3x + 16X^3x^2 - 16X^3 - 16x)
\]

This expression is maximized at

\[
\bar{e} \approx 0.64X
\]

And (10) simplifies to \( X \leq 0.605 \). For low values of \( X \) where \( X \leq 0.605 \), trade will occur in both low and high time segments. This bound on \( X \) is stricter compared to the no information regime but allows trade for a large range of waiting times than those allowed by complete information.

For values of \( X \) beyond 0.605, we can no longer have trade in the higher segment since the required difference between \( w \) and \( z \) becomes greater than 1. As \( X \) keeps increasing, there will come a point when trade will not be possible in the lower segment either since \( \bar{e} \) is a linear function of \( X \). This occurs exactly at the point where \( X > 1.171 \). However, the broker can always pick a different threshold value to still allow trade for low waiting times. When the broker provided no information about waiting times, no trade was possible once \( X \) crossed a threshold since the expected waiting time necessarily grew as \( X \) increased. By having two different time segments instead, the broker can maintain positive trade and still gain welfare and revenue even as \( X \) becomes very large. Specifically, for \( X \) larger than 0.605, the broker can choose a new threshold value to optimize for trade only in the lower segment. Now the objective function becomes

\[
\text{Expected Welfare (for high } X) = \frac{\bar{e}}{X} \int_0^1 \int_0^1 (w - z - 2x^2)q^w(w, z, x) \, dz \, dw + \frac{X - \bar{e}}{X} \int_0^1 \int_0^1 (w - z - 2x^2)q^m(w, z, x) \, dz \, dw
\]

\[
= \frac{1}{64}((3 - 4X^2)^2 + 16X^3x + 16X^3x^2 - 16X^3 - 16x)
\]

which is maximized at

\[
\bar{e} \approx 0.39
\]

This value no longer depends on the maximum waiting time \( X \), but instead maintains a constant volume of trade for a fixed subset of low waiting times. In summary, the welfare-maximizing mechanism for intermediate information is

\[
\bar{e} = \begin{cases} 
0.64X & \text{if } X \leq 0.605 \\
0.30 & \text{if } X > 0.605
\end{cases}
\]

resulting in

\[
\text{Expected Welfare} = \begin{cases} 
0.141 + 0.375X^2 + 0.405X^4 & \text{if } X \leq 0.605 \\
0.004X & \text{if } X > 0.605
\end{cases}
\]

We now construct a mechanism for revenue-maximization. For low waiting times, the revenue-maximizing mechanism is defined by

\[
q^l(w, z, x_L) = \begin{cases} 
1 & \text{if } w - z \geq \frac{1}{2} + \frac{2z^2}{3} \\
0 & \text{if } w - z \geq \frac{1}{2} + \frac{2z^2}{3}
\end{cases}
\]

\[
p_r = z + \frac{1}{2} + \frac{z^2}{3}
\]

\[
p_d = w - \frac{1}{2} - \frac{z^2}{3}
\]

For high waiting times, the broker should implement

\[
q^h(w, z, x_H) = \begin{cases} 
1 & \text{if } w - z \geq \frac{1}{2} + \frac{2(z^2 + Xx + X^2)}{3} \\
0 & \text{if } w - z \geq \frac{1}{2} + \frac{2(z^2 + Xx + X^2)}{3}
\end{cases}
\]

\[
p_r = z + \frac{1}{2} + \frac{z^2}{3}
\]

\[
p_d = w - \frac{1}{2} - \frac{z^2}{3}
\]

Here, too, the high wait category is restricted by (10). With low waiting times, riders are charged less and drivers are paid more compared to agents with high waiting times. Even though the probability of trade with higher waiting times is low, the broker reaps a higher revenue from those trades with higher waiting times that are realized. These results mirror what we often see in practice with ridesharing platforms. For example, rides in the city centre have lower expected waiting times due to a higher supply of drivers and lower expected distances between riders and drivers. Such rides are often cheaper and drivers earn a higher wage by driving in the city centre. By contrast, rides in suburbs outside the city or in less busy areas tend to have higher expected distances, waiting times, and rider prices.

The total expected revenue is calculated as the weighted sum of the expected revenue from each category.

\[
\text{Expected Revenue} = \frac{\bar{e}}{X} \int_0^1 \int_0^1 \left( \int_0^1 \left( w - z - 2x^2 \right)q^l(w, z, x) \, dz \right) \, dw + \frac{X - \bar{e}}{X} \int_0^1 \int_0^1 \left( \int_0^1 \left( w - z - 2x^2 \right)q^h(w, z, x) \, dz \right) \, dw
\]

\[
= \frac{1}{64}((3 - 4X^2)^2 + 16X^3x + 16X^3x^2 - 16X^3 - 16x)
\]

which is maximized at

\[
\bar{e} \approx 0.72X
\]

Plugging this into (10), revenue-maximizing trade is realized in both high and low segments for \( X \leq 0.579 \). As with welfare-maximization, this bound is more restrictive than trade with no informa-
Figure 7. Expected welfare across three levels of information about time.

Figure 8. Expected revenue across three levels of information about time.
tion but less restrictive than trade with complete information. For higher values of $X$, we can restrict our attention to the low waiting time segment and choose a different threshold value.

$$\text{Expected Revenue (for high } X) = \frac{X}{2} \int_0^1 \int_0^1 \left( z + \frac{1}{2} x^2 \right) - \left( w - \frac{1}{2} - \frac{x^2}{3} \right) q'(w, z, x) \, dz \, dw$$

This expression is maximized at

$$x \approx 0.42$$

Thus, the revenue-maximizing mechanism for intermediate information is

$$x = \begin{cases} 0.72X & \text{if } X \leq 0.579 \\ 0.42 & \text{if } X > 0.579 \end{cases}$$

And the expected revenue is

$$\text{Expected Revenue} = \begin{cases} 0.0417 - 0.0833X^2 + 0.160X^6 & \text{if } X \leq 0.579 \\ 0.0415X & \text{if } X > 0.605 \end{cases}$$

Figures 7 and 8 display a comparison of expected welfare and expected revenue for the three information levels of $x$: no information, complete information, and intermediate binary categories. The vertical line markers represent the point at which trade stops taking place in the high waiting time segment and is restricted to a small, fixed subset of waiting times. Intermediate revelation gets us remarkably close to the maximum expected welfare and expected revenue under full revelation and is less restrictive.

### 6. RIDER SENSITIVITY TO WAITING TIME

We now consider a preliminary extension of our model. So far we have assumed that riders have homogeneous preferences over waiting times and both riders and drivers are affected identically by a change in waiting time. In reality, agents often have heterogeneous preferences. We relax this assumption and allow for variation in the rider’s sensitivity to waiting time.

Let $s$ denote the rider’s sensitivity, where $s \sim U(0, S)$. We redefine the rider’s valuation from (1) as

$$v_r = w - sx^2$$

Intuitively, sensitivity represents a rider’s willingness to wait. For more sensitive riders, a unit increase in waiting cost leads to a greater reduction in their valuation of the ride. We restrict our attention to the setting where the broker communicates realized waiting times to agents. We also assume that rider sensitivities are public knowledge. The remaining model parameters are the same, with the small change that they now depend on $s$ as an additional variable. We prove the following propositions.

**Proposition 5.** Increasing rider sensitivity reduces flexibility in the marketplace and has a negative impact on expected revenue.

**Proposition 6.** The cost of a rider’s higher sensitivity must be borne by the driver.

For individually rational, incentive-compatible trade, we plug in the rider’s new valuation into (5) to get

$$0 \leq 2 \int_0^1 \int_0^1 \left( w - z - (1 + s)x^2 - \frac{1}{2} \right) q'(w, z, x) \, dz \, dw$$

Thus, conditional on the rider and driver agreeing to trade, we must have

$$w - z \geq \frac{1}{2} + (1 + s)x^2$$

and we must restrict trade to the subset of waiting times where $s \sim U(0, S)$. The more sensitive riders are to waiting times, the lower is the range of waiting times acceptable to them. The welfare-maximizing mechanism is

$$q'(w, z, x, s) = \begin{cases} 1 & \text{if } w - z \geq \frac{3}{2}(1 + s)x^2 + \frac{1}{8} \\ 0 & \text{if } w - z < \frac{3}{2}(1 + s)x^2 + \frac{1}{8} \end{cases}$$

Intermediate calculations are included in the appendix. Higher values of $s$ restrict trade by demanding that the rider have a higher willingness to pay or the driver have a lower opportunity cost. The negative impact on welfare is illustrated by the expression for expected welfare:

$$\text{Expected Welfare} = \frac{1}{X S} \int_S^s \int_0^{\min(X, (1 + s)x^2 + \frac{1}{8})} \int_{(1 + s)x^2 + \frac{1}{8}}^{w - \frac{3}{2}(1 + s)x^2 + \frac{1}{8}} w - z - (1 + s)x^2 \, dz \, dx \, ds$$

Holding all else constant, a higher sensitivity reduces welfare. Overall, welfare is decreasing in both $X$ and $S$. However, the two variables offset each other. If riders are less sensitive to waiting times, the platform can provide service in larger areas than it would have been able to with homogeneous sensitivity. Conversely, high sensitivities allow trade only for sufficiently low waiting times.

We obtain similar restrictions for revenue maximization. Revenue-maximizing trade occurs for

$$q'(w, z, x, s) = \begin{cases} 1 & \text{if } w - z \geq (1 + s)x^2 + \frac{1}{2} \\ 0 & \text{if } w - z < (1 + s)x^2 + \frac{1}{2} \end{cases}$$

Higher sensitivity to waiting times reduces the probability of trade and revenue-maximizing trade is more restrictive than the welfare-maximizing mechanism. We construct prices as per (7) and (8).

$$p_r(z, x, s) = z + \frac{1}{2} + x^2$$

$$p_d(w, x, s) = w - \frac{1}{2} - sx^2$$
This results in expected revenue

\[
\frac{1}{X S} \int_0^s \int_0^{\min\{X - \frac{w}{z - s - 1}, 1\}} \int_0^{1 - \frac{w}{s + 1}} \left( \frac{1}{2} \int_y^{1 - \frac{w}{s + 1}} dz \right) dx dy dz ds
\]

For higher sensitivities, the broker earns more revenue on realized trades. However, the probability of trade reduces faster than the corresponding increase in the difference in prices. In this particular pricing mechanism, each agent’s price depends on the other’s agent valuation rather than their own. As a result, notice that the cost of the rider’s sensitivity to waiting times is not borne by the rider herself, but is instead transferred to the driver. This must be the case under our model’s assumptions. Sensitivity serves as a proxy for the rider’s waiting time elasticity and we might expect that riders with different sensitivities would be willing to pay different prices.

However, such a mechanism would no longer be incentive compatible as riders would be incentivized to lie about their sensitivities. Instead, it is the driver’s willingness to compensate the rider for her sensitivity that makes trade possible. Thus, the platform should seek to match riders with high sensitivities to drivers with low opportunity costs.

For the broker to charge the rider a price dependent on her sensitivity, it would require that \( s \) is observable by the broker, either publicly or indirectly, through a method other than direct reporting. In practice, rideshare platforms estimate a rider’s willingness to wait or her price elasticity of time based on location, time of the day, and other factors and charge higher prices during work commutes and rush hours. Knowledge about rider sensitivity could allow the broker to strategically vary price and waiting times as complements to each other. Buchholz et al. (2020) show that most people are much more price elastic than waiting time elastic, but a small set of riders have persistent and high willingness to pay a higher price for a reduction in waiting time. The platform could offer higher prices for quicker rides and acquire the flexibility to offer lower prices with higher waiting times to customers who are more willing to wait.

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REFERENCES


APPENDIX

Model Parameters for Welfare-Maximization in Section 5

Under welfare maximization for low waits, \( \alpha_l = \frac{3 + 4x^2}{9 - 4x^2} \), resulting in

\[ c_o(z, x, \alpha_l) = \frac{12z + 4x^2}{9 - 4x^2} \]
\[ c_o(w, x, \alpha_l) = \frac{12w - 8x^2 - 3}{9 - 4x^2} \]

For high waits, \( \alpha_u = \frac{3 + 4(x^2 + Xz + X'^2)}{9 - 4(x^2 + Xz + X'^2)} \), and

\[ c_o(z, x, \alpha_u) = \frac{12z + 4(x^2 + Xz + X'^2)}{9 - 4(x^2 + Xz + X'^2)} \]
\[ c_o(w, x, \alpha_u) = \frac{12w - 8(x^2 + Xz + X'^2) - 3}{9 - 4(x^2 + Xz + X'^2)} \]

Model Parameters for Welfare-Maximization in Section 6

The functions needed for welfare maximization are given by

\[ c_o(z, x, \alpha) = (1 + \alpha)(z + x^2) \]
\[ c_o(w, x, \alpha) = (1 + \alpha)(w - \alpha x^2) - \alpha \]

and expected welfare is maximized when

\[ 0 = 2 \int_0^1 \int_0^{w-z-\frac{x^2}{2}} w-(1+s)x^2-\frac{1}{2} dz \, dw \]

The only value of \( \alpha \) for which trade is feasible is

\[ \alpha = \frac{1 + 2(1+s)x^2}{3 - 2(1+s)x^2} \]

resulting in

\[ c_o = \frac{4z + 4x^2}{3 - 2(1+s)x^2} \]
\[ c_o = \frac{4w - (2 + 6\alpha)x^2 - 1}{3 - 2(1+s)x^2} \]