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Léon Walras, Irving Fisher and the Cowles Approach to General Equilibrium Analysis

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Abstract: This paper explores the relationship of Walras's work to a particularly influential tradition of general equilibrium, that associated with the Cowles Commission for Research in Economics in Colorado in the 1930s and at the University of Chicago from 1939 to 1955, and its successor, the Cowles Foundation, at Yale University from 1955. Irving Fisher introduced general equilibrium analysis into North America with his 1891 Yale dissertation *Mathematical Investigations in the Theory of Value and Prices* (published 1892) and was responsible in 1892 for the first English translation of a monograph by Walras. Fisher was only able to obtain copies of books by Walras and Edgeworth when his thesis was almost ready for submission, discovering that he had independently reinvented a general equilibrium approach already developed by others, but went beyond Walras in constructing a hydraulic mechanism to simulate computation of general equilibrium and, before Pareto, in using indifference curves. Fisher was closely involved with Alfred Cowles in the Cowles Commission, the Econometric Society and *Econometrica* in the 1930s, promoting formal mathematical and statistical methods in economics, including drawing attention to the contributions of Walras, Edgeworth and Pareto. The first substantial, systematic work on general equilibrium at the Cowles Commission was in international trade, by Theodore Yntema, research director of the Cowles Commission from 1939 to 1942 and author of *A Mathematical Reformulation of the General Theory of International Trade* (1932) and by Yntema's student, Jacob Mosak, author of *General Equilibrium Theory in International Trade* (1944). A subsequent, much better-known body of work on existence of general equilibrium at Cowles was by Kenneth Arrow and Gerard Debreu (initially independently but leading to a major joint publication) and by Lionel McKenzie, all three associated with the Cowles Commission in Chicago in the early 1950s. After Cowles moved to Yale, the focus of general equilibrium research at the Cowles Foundation was Herbert Scarf's pioneering work on computable general equilibrium (which he linked to Fisher's earlier attempt, first presenting his approach in his contribution to *Ten Economic Studies in the Tradition of Irving Fisher*, 1967). Fisher and then the Cowles Commission were the channel through which Walrasian general equilibrium analysis entered North American economics. This paper is part of a larger history of the Cowles Commission and Foundation, commissioned by the Cowles Foundation.

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Introduction

General equilibrium analysis reached the Western Hemisphere in Irving Fisher's 1891 Yale doctoral dissertation (published as Fisher 1892) and developed in Colorado and Chicago at the Cowles Commission for Research in Economics (an institution in whose founding Fisher was closely involved) and then at its successor from 1955, the Cowles Foundation at Yale[1]. Fisher's shock at discovering that his invention of general equilibrium analysis had been preceded by Walras led to the translation of Walras (1892) and the 1892 election of Walras as an honorary foreign member of the American Economic Association. The Cowles Commission and Foundation was the setting for the famous work of Kenneth Arrow and Gerard Debreu (1954) and Lionel McKenzie (1954a) on proving the existence of general equilibrium and of Herbert Scarf (1967, 1973) on computation of equilibria (and other important work in general equilibrium, e.g. Truman Bewley 2007), but also for Martin Shubik's game-theoretic critique of the classical general equilibrium as "incomplete and not adequate" for analysis of a monetary economy (Shubik 1975, 1999-2010) and for now less-remembered general equilibrium reformulations of international trade theory (Jacob Mosak 1944, cf. Theodore Yntema 1932). Walras's writings, like those of Cournot, first reached anglophone readers in translations by Fisher's brother-in-law Nathaniel Bacon, supervised and introduced by Fisher (Cournot [1838] 1897, Walras 1892), and interest in Cournot, Walras, Pareto and other pioneers in mathematical economics was stimulated by public lectures by Fisher and others at the Cowles summer research conferences in Colorado Springs in the 1930s, notably by Harold Davis and René Roy marking the Cournot centenary in 1938 (which Fisher 1938 celebrated in *Econometrica*), and, in the earliest years of *Econometrica*, by articles on early mathematical economists (Jevons, Marshall, Hicks 1934 on Walras, Schneider 1934 on von Thünen).

[1] The Cowles Commission became a foundation when Alfred Cowles 3rd provided an endowment in place of an annual gift, at the time of the move from the University of Chicago to Yale in 1955. Yale had first negotiated for the Cowles Commission to move from Colorado in 1937.

Irving Fisher: Equilibrium, Indifference Curves and a Hydraulic Model

Irving Fisher earned Yale's first PhD in political economy (jointly with mathematics)[2] with a thesis that Paul Samuelson modestly acclaimed as "perhaps the best of all doctoral dissertations in economics," although Robert Dorfman held that "If Fisher's examiners had been better versed in European economic literature than they were, a promising career might have been blighted at its inception" (both quoted by Dimand 2019, p. 18). As a Yale undergraduate and graduate student Fisher studied engineering and mathematics with the mathematical physicist Josiah Willard Gibbs. But he also took political economy courses with William Graham Sumner, the professor of political and social science, because the charismatic Sumner, a Social Darwinist and pioneering sociologist, was a campus celebrity. When Fisher had completed his coursework, he was stumped for a thesis topic that would span both his fields of interest and could be supervised jointly by Gibbs and Sumner. Sumner told Fisher that he had heard of something called mathematical economics and, while he did not claim to know much about it, could direct Fisher to books by W. Stanley Jevons (1871) and by Rudolf Auspitz and Richard Lieben (1889). Visitors to the Centre Walras-Pareto at the University of Lausanne are shown Walras's copy of the 1879 second edition of Jevons's *Theory of Political Economy* ([1871] 1970). Next to Jevons's statement that in principle all prices and quantities should be determined simultaneously but that this would be too difficult mathematically is Walras's marginal comment, "C'est fait, mon garçon." Challenged by Jevons's remark but unaware of Walras, Fisher proceeded to invent general equilibrium analysis for himself.

A few weeks before the submission of his completed thesis in 1891, Fisher finally managed to obtain copies of books by Walras (1874-77) and Francisco Ysidro Edgeworth (1881), having found the titles of

[2] After receiving his PhD in 1891, Fisher was an assistant professor of mathematics at Yale for three years before transferring to economics. He taught courses in and wrote introductory textbooks in calculus and geometry as well as giving a course on "The Mathematical Theory of Price" based on his thesis.

their books in Harriet Jevons's expansion of Jevons's bibliography of mathematical economics in an 1888 posthumous edition of Jevons's *Theory*. Fisher was shocked to discover that general equilibrium analysis, the system of equations describing the simultaneous determination of all prices and quantities, had already been invented by Walras and Edgeworth. Fisher had gone beyond Walras's and Jevons's cardinally-measurable utility by introducing ordinal utility, indifference curves and marginal rates of substitution (before Pareto), but so had Edgeworth. Fisher had gone beyond both Walras and Edgeworth in considering the conditions for the utility function be integrable: if the gradient of utility "is not distributed in the above manner integration is impossible and *there is no such quantity as total utility or gain*" (Fisher 1892, pp. 88-89, Fisher's emphasis). But this advance, while noteworthy, was not comparable to the invention of general equilibrium analysis and had been anticipated by Giovanni Batista Antonelli (1886), which was not translated into English until 1971. Following Harriet Jevons's bibliography of mathematical economics, Antonelli (1886) appeared in Fisher's bibliography, but Fisher, who could read French and German, stated that "In the case of Italian and Danish writings, with which I am wholly unacquainted and in the case of a large number of others which I have not been able to see or examine, I have been guided by book notices or the wording of the title" (Fisher 1892, p. 120).

Such surprises were more common when books and journal articles crossed the ocean by ship[3] and did not mean that Yale was then a scientific backwater (at least, not Gibbs's laboratory). According to Bill Bryson (2003, pp. 116-17, 121), "In 1891 [Max Planck] produced his results and learned to his dismay that the important work on entropy *had* in fact been done already, in this instance by a retiring scholar at Yale University named J. Willard Gibbs." Gibbs's *On the Equilibrium of Heterogenous Substances* "has been called 'the *Principia* of thermodynamics,' but for reasons that defy speculation Gibbs chose to

[3] Distance and publication in obscure local journals were not the only barriers to communication: Albert Einstein formalized Brownian motion for particles in 1905 without knowing that Henri Poincaré's student Louis Bachelier had done so for asset prices in a 1900 Paris doctoral dissertation, published in the annals of the French Academy of Sciences.

publish these landmark observations [as a series of articles from 1875 to 1878] in the *Transactions of the Connecticut Academy of Arts and Sciences*, a journal that managed to be obscure even in Connecticut, why is why Planck did not hear of him until too late … From 1902 to 1904 [Albert Einstein] produced a series of papers on statistical mechanics only to discover that the quietly productive J. Willard Gibbs in Connecticut had done the work as well, in his *Elementary Principles of Statistical Mechanics* of 1901." Notice that Gibbs wrote about equilibrium, that he was a scientist capable of preempting Planck and Einstein, and that his mentor's example explains why Fisher (1892) chose to publish his dissertation in the *Transactions of the Connecticut Academy of Arts and Sciences*.

Even after allowing Walras's priority for general equilibrium and that of Edgeworth for ordinal utility and indifference curves, and apart from his discussion of integrability, Fisher (1892) achieved a major advance. He not only considered how to find the equilibrium prices and quantities but, as befitted a Gibbs student, he constructed a hydraulic mechanism to simulate the determination of equilibrium prices and quantities, represented by the equilibrium depths and volumes of water in the various compartments of Fisher's model (see Brainard and Scarf 2005, Dimand and Ben-El-Mechaiekh 2012, and the photograph of Fisher's 1893 mechanism in Fisher 1997, Vol. 1, p. 41). Fisher's hydraulic mechanism was unfortunately destroyed in transit to be exhibited at the Columbian Exhibition, held in Chicago in 1893 (a year late for the 400th anniversary of Columbus's landing), but Fisher eventually built another in 1925[4] (see photograph in Fisher 1997, Vol. 1, p. 42). Appropriately, Herbert Scarf (1967) announced his algorithm for computation of general economic equilibria in a contribution to *Ten Economic Studies in the Tradition of Irving Fisher*. Not only was Fisher (1892) a striking contribution to general equilibrium analysis, with the first attempt at computable general equilibrium, but, beyond economics, it was a landmark in the history of computing before electronic computers. One can only wonder what a slightly

[4] The machine is no longer extant and perhaps was lost when Fisher's financial reverses forced me to give up his house for an apartment. Brainard and Scarf (2005) reported on their attempt to reconstruct the mechanism.

earlier Yale PhD (in philosophy), Thorstein Veblen, also co-supervised by William Graham Sumner, must have thought of the idea of building a physical analogue of the economy to see what the equilibrium prices and quantities would be. But from the standpoint of mathematical economics and general equilibrium analysis, Vilfredo Pareto wrote to Fisher from Lausanne in 1897, "Ce sont précisément les jeunes économistes, comme vous, qui feront faire les progrès nécéssaires à l'économie politique et qui la feront devenir une vraie science. J'espère avoir fait un 1er pas, mais vous et autres vous ferez *d'autres pas* bien plus grands!" (Pareto 1958, Pareto's emphasis).

The shock of encountering Walras (1874-77) and Edgeworth (1881) made Fisher determined to pay, and draw, attention to his predecessors in mathematical economics, and to avoid intellectual isolation from advances in economics in Europe. Part I of Walras's "Geometrical Theory of the Determination of Prices" (1892) was translated under Fisher's supervision and published with an introductory note by Fisher in the *Annals of the American Academy of Political and Social Science*, while Parts II and III were still forthcoming in French. The unnamed translator was presumably Nathaniel T. Bacon, Fisher's wife's brother-in-law, who went on to translate Cournot ([1838] 1897) with a substantial introduction and notes by Fisher, and with a bibliography of mathematical economics (pp. 173-210) greatly extending Appendix IV of Fisher (1892, "Bibliography of Mathematico-Economic Writings," pp. 120-24), which in turn extended the bibliography of W. Stanley Jevons and Harriet Jevons. Appendix III of Fisher (1892, pp. 106-119) examined "The Utility and History of Mathematical Method in Economics," with extensive quotations from the works that Fisher should have read before writing his thesis. Fisher's introduction to the translation of Cournot was the basis for "Cournot and Mathematical Economics" (Fisher 1898). Walras was elected an honorary foreign member of the American Economic Association in 1892, presumably at Fisher's initiative. Fisher spent 1893-94 meeting almost every prominent European economist outside Scandinavia, visiting Walras and Pareto in Lausanne, Edgeworth in Oxford, Marshall in Cambridge, Barone in Florence, Pantaleoni in Rome, and Menger, Böhm-Bawerk, Wieser and Lieben

(but not Auspitz) in Vienna, and attending mathematics and physics lectures by Frobenius and Helmholtz in Berlin (where the lectures of historical economists such as Schmoller disappointed Fisher).

Fisher's dissertation was translated into French by the general equilibrium theorist Jacques Moret[5] as Fisher (1917), as part of a brief outpouring of books on mathematical economics published in French in Paris and Lausanne, outside the French university system, by French, Swiss, Italian and Polish admirers of Walras and Pareto: Leseine and Suret (1911), Boven (1912), Osorio (1913), Étienne Antonelli (1914), Zawadzki (1914), Moret (1915)[6] and Suret's 1914 translation of Auspitz and Lieben (see Zylberberg 1990). But although Maurice Allais dedicated *Intérêt et Économie* (1947) to Irving Fisher and cited Moret (1915) and wrote the 1947 *Revue d'Économie Politique* memorial article in Fisher, he seems not to have then known Fisher's dissertation on general equilibrium and so could not introduce Fisher (1892, 1917) to his student Gerard Debreu. Although Allais knew Fisher (1892) by 1968 when he wrote the entry on Fisher in the *International Encyclopedia of the Social Sciences*, all the Fisher citations in Allais (1947) were to French translations of Fisher's books on capital and interest and on the purchasing power of money. Thus, although Charles F. Roos and Harold T. Davis, the two research directors of the Cowles Commission in Colorado Springs in the 1930s, were both strong admirers of Fisher (1892), Fisher had only an indirect influence on general equilibrium analysis at the Cowles Commission in Chicago in the 1950s through his role in fostering the Commission as a space for mathematical economic theory, rather than any direct influence through knowledge at that time of his dissertation by Arrow, Debreu or McKenzie.

The Early Years of the Cowles Commission

[5] In 1931, Moret coauthored a *Revue d'Économie Politique* article with Ragnar Frisch about "Methodes nouvelles pour mesurer l'utilité marginale," building on a 1927 essay by Fisher. Fisher reviewed Suret's 1914 translation of Auspitz and Lieben in the *American Economic Review* in 1915.

[6] Except for Zawadzki (1914) and the French translations of Fisher and of Auspitz and Lieben, these books were unknown to Batson's *Select Bibliography of Modern Economic Theory 1870-1929* (1930).

The Wall Street crash of October 1929, in addition to marking the onset of the Great Depression, ruined the reputation and personal finances of Irving Fisher, who notoriously had declared that stock prices appeared to have reached a permanently high plateau. Starting from nothing, Fisher had accumulated in the stock market boom of the 1920s a fortune of ten million dollars, of which he then lost eleven million, a substantial sum of money even for an economics professor as John Kenneth Galbraith remarked (Fisher 1997, Vol. 13, Dimand 2019, chapters 7 and 8). But the stock crash also created an opportunity for Fisher's mathematical economics, econometrics and general equilibrium analysis to shape the development of the economics profession. The crash disillusioned an avid producer and consumer of stock forecasts, Alfred Cowles 3rd, a grandson of one of the founders of the *Chicago Tribune*. Cowles turned to mathematical statistics to demonstrate that stock forecasts were worthless and that stock forecasters had done no better than chance (see Dimand and Veloce 2010, Dimand 2019, chapter 9). Cowles, who lived in Colorado Springs, turned for advice to Harold Davis, an Indiana University mathematics profess and Colorado College graduate who summered in Colorado Springs. Davis told Cowles about the Econometric Society, founded at the very end of 1930. To create a forum for his results, and because his research led him to appreciate the work of other econometricians, Cowles wrote in August 1931 to Irving Fisher, founding president of the new (and impoverished) Econometric Society, offering to pay for a research institute and a scholarly journal, starting with $12,000 a year. Others in the Econometric Society understandably viewed such an unsolicited offer of major funding in 1931 as a crank letter, but Cowles was a Yale graduate (class of 1913) and had been in the Yale senior society Skull and Bones, and his father and uncle had been in Skull and Bones with Fisher when the three were Yale undergraduates in the 1880s (so the Cambridge Apostles are not the only student society with a place in the history of economics). Cowles thus became president of the Cowles Commission for Research in Economics (with Fisher on an advisory council), treasurer of the Econometric Society (and from 1937 also secretary), and business manager of *Econometrica*.

In addition to its permanent research staff and monographs, the Cowles Commission held month-long summer research conferences in econometrics and mathematical economics in Colorado Springs annually from 1936 through 1940, with an entire morning or afternoon for each paper. Thus, on Friday, July 12, 1940, Wassily Leontief reported in the morning on "Some Results of an Empirical Study of the General Equilibrium" (based on his forthcoming book on input-output analysis, Leontief 1941), Paul Samuelson in the afternoon on "The Stability of Equilibrium" (Samuelson 1941-42), at the close of a week that also featured Abraham Wald on a new foundation for maximum likelihood and W. Edwards Deming on sampling. The following Wednesday, July 17, was devoted to Irving Fisher on the velocity of circulation of money and Trygve Haavelmo on "The Problem of Testing Economic Theories by Means of Passive Observations." Paul Samuelson, then a graduate student, later recalled ruefully that the usual excellent wines were missing from dinners at Cowles's home when Fisher, a fervent teetotaler and prohibitionist, attended (Dimand 2019, chapter 9). In contrast to the Cowles Commission summer conferences, the meetings of the Econometric Society, the other venue for advanced research in econometrics and mathematical economics, lasted only three days, with more than one three-paper session at a time and half an hour for presentation and discussion of each paper.

In addition to serving on the advisory council, Irving Fisher was a regular participant in the Cowles conferences, for example opening the July 1936 conference with four days of lectures on "Income in Theory and Income Taxation in Practice" (Fisher 1937) but did not speak at Cowles about general equilibrium or revisit the themes of his dissertation. Abraham Wald, a full-time research fellow at the Cowles Commission from July 1938, had pioneered fixed-point proofs of the existence of general equilibrium in two papers at Karl Menger's mathematics colloquium in Vienna (translated in Baumol and Goldfeld 1968, expounded less formally for a wider audience in Wald [1936] 1951, and analyzed skeptically by Düppe and Weintraub 2016), but his talks at the 1938, 1939 and 1940 Cowles conferences were on mathematical statistics rather than general equilibrium. Wald's contributions to postwar

Cowles conferences and monographs on econometric method, after his move to Columbia's Department of Statistics, also concerned statistical theory, not economic theory. The mathematician Karl Menger (son of the economist Carl Menger) spoke at the 1937 Cowles conference after emigrating from Vienna to the United States, but his talk on "An Exact Theory of Social Relations and Groups" had more affinity to Pareto's sociological writings than to his economics and did not touch on general equilibrium analysis or the Vienna colloquium[7]. Apart from Samuelson on stability of equilibrium and Leontief's input-output empirical application at the 1940 conference, Walrasian general equilibrium analysis appeared in the 1939 Cowles conference when Samuelson's fellow Harvard graduate student Robert Triffin spoke about "Monopoly in Particular-Equilibrium and in General-Equilibrium Economics," citing his forthcoming dissertation on *Monopolistic Competition and General Equilibrium Theory* (1940)[8].

Where general equilibrium analysis did figure prominently in the early years of the Cowles Commission was in international trade theory. International trade was a context that often turned the thoughts of economists to general equilibrium considerations. As Kenneth Arrow remarked, "[John Stuart] Mill's own theory of reciprocal demand is a true general equilibrium theory of international trade" unlike Mill's Ricardian theory of domestic values (Arrow in Kehoe, Srinivasan and Whalley, eds., 2005, p. 15). Alfred Cowles moved to Chicago to take over his family's investments when his father died n 1939, and the Cowles Commission left Colorado to become affiliated with the University of Chicago, filling the gap left by the death of Henry Schultz, who had spoken at the 1938 Cowles conference about "Mathematics in Economics" and "Statistics in Economics"(see Schultz 1938, Lange, McIntyre and

[7] Arrow, in Arrow *et al.* (1991, p. 12), mentioned that Menger, then teaching mathematics at the Illinois Institute of Technology, occasionally attended Cowles Commission seminars after Cowles moved to Chicago in 1939. For example, Menger presented a paper on "Probabilistic Theory of Relations" in the Cowles seminar in November 1951.

[8] Although Triffin taught at Yale for two decades after the Cowles Foundation moved there in 1955, his post-dissertation research was in international monetary economics and he did not return to general equilibrium analysis or mathematical economics. Francis Dresch, a postdoctoral researcher in mathematics at the University of California at Berkeley, where he had studied with Griffith Evans, spoke at the 1939 and 1940 about multi-equation models of business fluctuations, but this work was related to macroeconomic modeling rather than Walrasian general equilibrium.

Yntema, eds., 1942). The international trade theorist Jacob Viner joined the Cowles Commission advisory council to represent the university's Economics Department, and his younger colleague Theodore Otte Yntema, a statistics professor in what was then the university's School of Commerce and Administration (now the Graduate School of Business), became research director of the Commission (from 1939 to the end of 1942). Yntema's 1929 Chicago PhD dissertation, supervised by Viner and Schultz, was published as *A Mathematical Reformulation of the General Theory of International Trade* (1932). Viner, Yntema and Oskar Lange supervised Jacob Mosak's 1941 Chicago doctoral dissertation, published as *General-Equilibrium Theory in International Trade* (Cowles Commission Monograph No. 7, 1944). Although Mosak began his dissertation after Schultz's death, his preface emphasized that Schultz had first introduced him to "the mathematical school of economists" (see also Mosak 1942, in the volume in Schultz's memory). As Mosak (1944, pp. 33-35) pointed out, his formulation of general equilibrium in international trade dropped some simplifications made by Yntema: Yntema assumed the existence of supply and demand function while Mosak (under Yntema's supervision) derived them from utility and transformation or production functions, Mosak wrote demands as functions of all prices where Yntema made demand for each commodity a function of that commodity's price deflated by the general price level.

Taken together, Yntema (1932) and Mosak (1944) marked an advance in the formal, mathematical, general-equilibrium formulation of international trade theory, and Arrow and Hahn (1971, p. 12) credited Mosak (1944) along with a slightly later paper by Lloyd Metzler with initiating "the current trend in comparative statics and stability," yet they have been almost entirely overshadowed in the literature of trade theory (see e.g. the well-known surveys of trade theory by Gottfried Haberler, Jagdish Bhagwati and John Chipman in the 1950s and 1960s reprinted in Dimand, ed., 2004, Vol. 10, or the essays on the history of Cowles in Arrow *et al.* 1991, where Mosak 1944 gets one sentence from Debreu on p. 28, reporting only that it originated as a dissertation). Like Roos and Davis, Yntema and Mosak

barely appeared in the history of the Cowles Commission as presented by, for example, Arrow, Debreu, Malinvaud and Solow (1991). Partly this is because journal articles now have more impact in economics than books (especially if each article makes a single, memorable point), and partly because the authors did not remain academics: Mosak spent his career with the United Nations, and Yntema, after moving from Cowles to the policy-oriented Committee for Economic Development in 1942, joined the Ford Motor Company in 1949 as a vice-president for finance, rising to be chairman of the finance committee (what would now be termed chief financial officer, CFO). But there was also a fortuitous element of timing, also relevant to Mosak's decision not to pursue an academic career. Mosak was clearly one of the two outstanding economics undergraduates of his year at the University of Chicago – but he was inevitably in the shadow of his classmate Paul Samuelson. Rather than follow up his dissertation with further contributions to trade theory and equilibrium analysis, which would have helped retain the profession's attention for his thesis, Mosak chose not to compete and instead had a long, distinguished non-academic, non-publishing career.

Arrow-Debreu-McKenzie: Existence and Stability of General Equilibrium

Lionel McKenzie (1954a) and, in a joint paper after initially working independently (see Arrow 1951b, Debreu 1951, 1952a, 1952b, 1954, Düppe 2012), Kenneth Arrow and Gerard Debreu (1954) used Shizuo Kakutani's fixed-point theorem (Kakutani 1941) to prove existence of general equilibrium with considerably greater simplicity and generality than Abraham Wald had done nearly twenty years before. As Arrow and Hahn (1971, p. 10) remarked, "Wald's papers [translated in Baumol and Goldfeld 1968] were of forbidding mathematical depth, not only in the use of sophisticated tools, but also in the complexity of the argument. As they gradually came to be known among mathematical economists, they probably served as much to inhibit further research by their difficulty as to stimulate it." Like Wald before them, Arrow, Debreu and McKenzie all had Cowles Commission connections. Arrow spent two

years at the Cowles Commission at the University of Chicago, writing his Cowles Monograph on *Social Choice and Individual Values* (1951a), famed for the "Arrow impossibility theorem" showing that no social welfare function can satisfy a specific set of plausible-sounding axioms. He continued to be listed as a research consultant to the Cowles Commission long after moving to Stanford University. Debreu spent eleven years with the Cowles Commission and Foundation in both Chicago and New Haven. McKenzie spent a year beginning with the fall semester of 1950 at the Cowles Commission as a special graduate student visiting the University of Chicago before returning to teaching at Duke University, his undergraduate alma mater, for the fall semester of 1951 and was listed as a Research Consultant in the Cowles Foundation's reports for 1956-58, 1958-61 and 1961-64. He reprinted McKenzie (1954b) in a Cowles Commission Paper, presented a seminar at the Cowles Foundation at Yale in 1956, authored two Cowles Foundation Discussion Papers in 1956 (unrelated to his seminar paper), and reprinted two of his articles as Cowles Foundation Papers in 1957 (one of them a revision of one of the previous year's discussion papers)[9]. Both McKenzie (1954a) and Arrow and Debreu (1954) were presented to the Econometric Society in Chicago, December 27-29, 1952, and McKenzie's paper, which was submitted first by at least two months, was published in the issue of *Econometrica* preceding the one with Arrow and Debreu's article (despite refereeing delays – John Nash never submitted his report), yet a Google Scholar search by Roy Weintraub (2011, p. 211) found some 2,300 citations of Arrow and Debreu (1954a) and only 197 citations of McKenzie (1954a). Even Weintraub (1979, p. 27n, italics in original), when urging that the term Arrow-Debreu model of general equilibrium be replaced by Arrow-Debreu-

[9] The Cowles Commission became the leading center of general equilibrium research just as the University of Lausanne was moving away from any such role. Firmin Oulès, holder of the chair formerly held by Walras and Pareto and editor of selections from their writings (Oulès 1950a), strongly preferred the social and applied economics of the first Lausanne school to their pure economics, and sharply criticized their general equilibrium analysis as insufficient and inappropriate (Oulès 1950b, 1950c). Mathematical economics in Continental Europe outside Switzerland, such as in Karl Menger's Vienna colloquium, was disrupted by the upheavals and emigrations of the 1930s. The *Archiv für Mathematischen Wirtschafts- und Sozialforschung*, founded in 1935 (two years after *Econometrica*) and initially a venue for articles by such notable economists as Erich Schneider, Heinrich von Stackelberg and Jan Tinbergen, declined in quality and finally ceased publication in 1943.

McKenzie, did so on the grounds that "the *proof* of existence on which current work is based came out of McKenzie (1959)," without any mention of McKenzie (1954a) or any of McKenzie's pre-1959 articles. Although, as Weintraub and Gayer (2001) discovered, a referee (a mathematics professor, not an economist) had recommended rejection of Arrow and Debreu (1954), that article figured prominently (together, in Arrow's case, with Arrow 1951a) in the citations for their Nobel Memorial Prizes.

McKenzie's work on general equilibrium emerged from his involvement in international trade theory and from his time at the Cowles Commission[10], although McKenzie (1954a, 1954b, 1955a, 1955b) were all written after his return from Cowles to Duke in the fall of 1951 (and were published well before he finally received his PhD from Princeton in 1957). McKenzie (1954b), showing that bilateral comparison of comparative advantage was insufficient to find the efficient allocation of world production, originated in work done in Tjalling Koopmans's Chicago course on activity analysis (Koopmans, ed., 1951, cited in all four of McKenzie's 1954 and 1955 papers, see also Koopmans 1957, Essay I) and was based Frank Graham's trade theory, which McKenzie had encountered as a pre-World War II doctoral student at Princeton (McKenzie quoted by Weintraub 2011, pp. 200-201, see also Graham 1948, Whitin 1953, McKenzie 1955a, McKenzie 2002, Düppe and Weintraub 2014). McKenzie (1954a) was dedicated "to the memory of my friend and teacher, Frank D. Graham." A suggestion by Koopmans (research director of Cowles 1948-55 and 1961-67) to examine Samuelson's factor-price equalization theorem in the context of activity analysis led to McKenzie (1955b). In the second footnote of McKenzie (1954b) expressed his "wish to thank Professor Tjalling C. Koopmans for his encouragement and support. The debt to my old

[10] In 2009, McKenzie wrote to Roy Weintraub (2011, p. 209) that "Actually I did visit the Cowles Commission in Chicago when Debreu was there and working on existence, but he kept this fact secret from me. I asked him what he was working on and he refused say. The first thing I knew about his work was when we both we both presented papers on existence to the Chicago meeting of the Econometric Society in [December] 1952." Debreu (1951), written before Debreu knew of Arrow's preliminary work on proving existence of general equilibrium, and Debreu (1952a), written after he read Arrow (1951b), were circulated as Cowles Commission Discussion Papers, and Arrow (1951b) and Debreu (1952b) were reprinted as Cowles Commission Papers but, according to McKenzie's recollection, were not noticed by him.

teacher, Frank D. Graham, will be apparent." McKenzie's articles in 1954-55 stemmed both from his time studying activity analysis with Koopmans at the Cowles Commission in 1950-51 and from his earlier study of international trade theory with Graham. Although McKenzie's existence proof applied much more generally than just to Graham's trade model, indeed as generally as Arrow and Debreu (1954), its appearance in a series of articles in international trade theory starting from Graham's model may have contributed to McKenzie (1954a) receiving less attention than Arrow and Debreu (1954) because its title and McKenzie's other 1954 and 1955 articles led economists to think of McKenzie (1954a) as a contribution just to the international trade literature.

Both Arrow and Debreu (1954) and McKenzie (1954a) provided existence proofs for general equilibrium simpler and more general than that of Wald nearly two decades before, as did Nikaido (1956)[11]. There were some differences between the two papers, but these were of secondary importance. Arrow and Debreu started from Nash's proof of existence of a solution for n-person games (Nash 1950), assumed consumer utility functions and, as in Debreu (1952b), used the Eilenburg-Montgomery generalization of Kakutani's fixed point theorem. McKenzie started from Koopmans and the Menger Colloquium papers of Wald and from von Neumann ([1937] 1945), assumed demand functions, and used the Kakutani fixed point theorem (Kakuktani 1941), which he had encountered in a Cowles Commission Discussion Paper by mathematician Morton Slater (McKenzie, quoted in Weintraub

[11] Weintraub (2011, p. 213n13) reports that "Nikaido had independently developed a proof of existence of a general competitive equilibrium using the Kakutani theorem at that time [when his mentor Takuma Yasui heard the Debreu and McKenzie papers in Chicago in December 1952], but it was not until he read McKenzie's paper in 1954 that he believed his own paper might find a place in an English-language journal. His difficulty getting his paper published led to his belief that he might have been ill-treated." However, Kazuo Nishimura and Richard Day, in their biographical foreword to Nikaido (1996, p. ix) state that Nikaido's existence proof "which was published in 1956, was first presented at the annual meeting of the Japan Association of Economics and Econometrics in the autumn of 1954, the same year that the existence proofs of McKenzie and Arrow and Debreu appeared" (but after they were published and nearly two years after Debreu and McKenzie spoke at the Econometric Society, and even longer after the preliminary, less general existence proofs of Arrow 1951b and Debreu 1952b were reprinted as Cowles Commission Papers) and "led to a visiting appointment at Stanford, 1955-56, at the invitation of Kenneth Arrow" so the ill-treatment is not evident. Nikaido (1954) had published an extension of the von Neumann growth model in the *Econometrica* issue preceding that with McKenzie (1954a), so he was not entirely a distant outsider.

2011, p. 201). Later Debreu (1959) and McKenzie (1959) developed still more general existence proofs. McKenzie (1960) and Arrow, working with Leonid Hurwicz of Cowles (Arrow and Hurwicz 1958, 1960, 1962, Arrow, Black and Hurwicz 1959), examined weak gross substitutability and the weak axiom of revealed preference as conditions for stability of general equilibrium (also the subject of Nikaido's articles from 1959 to 1964), so McKenzie's research continued to move in parallel to the work of Arrow and Debreu on general equilibrium, but the general equilibrium papers of all three scholars had their roots in the Cowles Commission in Chicago at the start of the 1950s.

GE at Cowles after Debreu's Departure: Scarf and Computable General Equilibrium, Shubik's Critique

Herbert Scarf's Cowles Monograph on *Computation of Economic Equilibria* (1973) created the field known variously as applied, computable or numerical equilibria. In the spirit of Fisher's dissertation, as Scarf (1967) emphasized, Scarf went beyond proving by contradiction that at least one equilibrium vector of prices and quantities must exist, to devising an algorithm that would compute the actual values of such an equilibrium, a constructive proof of existence. The research program initiated by Scarf and his doctoral students led to a series of conferences that eventually became an annual event at the Cowles Foundation, producing such conferences volumes as Scarf and Shoven, eds. (1984) and Kehoe, Srinivasan and Whalley, eds. (2005). Before turning to computable general equilibrium, Scarf had already advanced general equilibrium theory by finding "reasonable" examples of general equilibria that were globally unstable (Scarf 1960), contrary to conjectures of Debreu (1959), and by collaborating with Debreu on deriving a limit theorem for the core of an economy (Debreu and Scarf 1963), showing that the core converges to the set of competitive allocations, without lump sum income redistribution, as the economy is replicated. Scarf's research program, and the doctoral students that he trained, meant that

the Cowles Foundation at Yale remained central in general equilibrium research[12] even after Debreu departed for the University of California at Berkeley and more than a century after Fisher's dissertation and hydrostatic simulation model. The editors of Kehoe, Srinivasan and Whalley (2005, p. 5), all Yale PhDs, emphasized that "A feature that characterizes the research of the Yale school of AGE [applied general equilibrium] modeling – and distinguishes it from some other AGE modelers – is its heavy interaction with the general equilibrium theory of Arrow, Debreu, McKenzie, and Scarf. Members of the Yale school rely on rigorous theory to guide the development of their models, and they carefully modify and develop new theory when the existing theory is not adequate for their particular applications." Scarf's colleague at the Cowles Foundation, Truman Bewley, who in his dissertation and first articles in the early 1970s had extended proofs of existence of general equilibrium and of equivalence of the core and the set of equilibria to infinitely-dimensional commodity space and a continuum of agents, worked on *General Equilibrium, Overlapping Generations Models, and Optimal Growth Theory* (2007).

But not all at the Cowles Foundation embraced Walrasian general equilibrium as a research program, either the theory expounded in Debreu's *Theory of Value* (1959) or the computable version pioneered by Scarf. Already when reviewing Debreu (1959) in the *Canadian Journal of Economics and Political Science*, Princeton-trained game theorist Martin Shubik (1961, p. 133) was moved to "suggest that the future development of economic theory will rest heavily upon the utilization and exploration of assumptions radically different from those employed in this work. At the level of microeconomics there is a necessity for a better model of the economic actor than *homo economicus*. Economic stability may well depend upon the enforceability of an equilibrium by sets of players larger than one." In an article

[12] Another contributing factor was proximity to Kakutani and his students in fixed-point theory in Yale's Mathematics Department at 12 Hillhouse Avenue, a few doors from the Economics Department at 28 Hillhouse and the Cowles Foundation at 30 Hillhouse. The presence of Bayesian statisticians Leonard Jimmie Savage and I. Richard Savage at the Statistics Department at 24 Hillhouse in the 1960s mattered for econometrics at Yale and for attention to the (L.J.) Savage axioms for rational choice. As the home of the fractalist Benoit Mandelbrot, Yale's Mathematics Department also generated ideas, such as emphasis on fat-tailed probability distributions for asset price movements, less in accord with the mainstream economics of the Cowles Foundation.

whose title paralleled that of Oulès (1950b), Shubik (1975) later dismissed the classical general equilibrium model as "incomplete and not adequate" for the reconciliation of microeconomics with macroeconomic analysis of a monetary economy. Starting in 1970, his monumental *Theory of Money and Financial Institutions* began appearing as Cowles Foundation Discussion Papers and as journal articles reprinted as Cowles Foundation Papers, eventually revised and assembled as two volumes in 1999 plus a third volume in 2010 (Shubik 1999-2010), offering game-theoretic foundations for a theory of monetary economy in place of Arrow-Debreu-McKenzie general equilibrium.

Conclusion

General equilibrium analysis reached North America through independent re-invention by Irving Fisher in his Yale dissertation co-supervised by J. Willard Gibbs, followed promptly by Fisher's energetic efforts to draw attention to the European mathematical economists whose work he had overlooked (Fisher 1892, Walras 1892, and, for partial equilibrium, Cournot [1838] 1897, Fisher 1898, 1938). He also acted to establish communication with the European theorists through personal contact starting in 1893-94 and through publications (Fisher's first four journal articles appeared in Edgeworth's *Economic Journal*). Such European followers of Walras and Pareto as Jacques Moret (1915) and Wladyslaw Zawadzki (1914) took an interest in Fisher (1892), which Moret translated into French. Beyond re-inventing indifference curves and the system of equations for general equilibrium and considering the conditions for integrability of the utility function, Fisher (1892) pioneered computable general equilibrium (and computation more generally) in the era before electronic computers, with his hydrostatic mechanism for simulating determination of equilibrium prices and quantities.

Fisher made a further contribution by sharing with Ragnar Frisch and Charles Roos in the creation of the Econometric Society and in supporting Alfred Cowles's creation of the Cowles Commission and Cowles's funding of *Econometrica*, edited by Frisch. Despite the presence of Fisher, Karl Menger and

Abraham Wald, general equilibrium was not prominent at the Cowles summer conferences in Colorado. But general equilibrium analysis was undertaken in international trade theory by Theodore Yntema (1932), who became the research director of the Cowles Commission when it moved to Chicago in 1939, and in a Cowles monograph by Yntema's student Jacob Mosak (1944). Pathbreaking work on the existence and stability of general equilibrium was carried out in the 1950s by three scholars closely associated with the Cowles Commission (at Chicago to 1955) and Cowles Foundation (at Yale from 1955): Kenneth Arrow, Gerard Debreu and Lionel McKenzie. Herbert Scarf contributed to this theoretical literature (with examples of globally unstable equilibria and, with Debreu, a limit theorem on convergence of the core to set to competitive allocations), but then reestablished the link to Fisher's hydrostatic mechanism (see Scarf 1967), creating the field of computable or applied general equilibrium, with a recognizable Yale or Cowles approach to the field. At the same time, Scarf's Cowles colleague Martin Shubik criticized Arrow-Debreu-McKenzie general equilibrium from a game theoretic perspective whose roots went back to Edgeworth (1881).

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[13] In 1947 the Cowles Commission initiated three series of discussion papers: in economics, starting with no. 201, statistics starting with no. 301, and mathematics starting with no. 401. When the economics series reached number 299 in 1950 (as surely could have been predicted), the economics series jumped to no. 2001 in 1951.

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