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UNIFORM PRICING VERSUS THIRD-DEGREE PRICE DISCRIMINATION

By

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Uniform Pricing Versus Third-Degree Price Discrimination

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We compare the revenue of the optimal third-degree price discrimination policy against a uniform pricing policy. A uniform pricing policy offers the same price to all segments of the market. Our main result establishes that for a broad class of third-degree price discrimination problems with concave revenue functions and common support, a uniform price is guaranteed to achieve one half of the optimal monopoly profits. This revenue bound holds for any arbitrary number of segments and prices that the seller would use in case he would engage in third-degree price discrimination. We further establish that these conditions are tight and that a weakening of common support or concavity leads to arbitrarily poor revenue comparisons.

Additional Key Words and Phrases: First Degree Price Discrimination, Third Degree Price Discrimination, Uniform Price, Approximation, Concave Demand Function, Market Segmentation.

1 INTRODUCTION

1.1 Motivation and Results

An important use of information about demand is to engage in price discrimination. A large body literature, starting with the classic work of Pigou [1920], examines what happens to prices, quantities and various measures of welfare as the market is segmented. A seller is engaging in third-degree price discrimination if he uses information about consumer characteristics to offer different prices to different segments of the market. As every segment is offered a different price, there is scope for the producer to extract more surplus from the consumer. With the increase in available information about consumer demand comes increasing flexibility in the ensuing market segmentation.[1]

Our main contribution is to compare the revenue performance of third-degree price discrimination against a uniform pricing policy. A uniform pricing policy offers the same price to all segments of the market. Theorem 3.1 establishes that for a broad class of third-degree price discrimination problems with concave revenue functions and common support, a uniform price is guaranteed to achieve one half of the optimal monopoly profits. Theorem 3.1 establishes the revenue bound for any arbitrary number of segments and prices that the seller would use in case he would engage in third-degree price discrimination.

We investigate the limits of this result by weakening the assumptions of concavity and common support. First, Proposition 4.2 shows the necessity of the common support assumption by studying a setting with concave profit functions that have finite but different supports. We can then display a sequence of segments under which the ratio of uniform price to third-degree price discrimination goes to zero.[2] Second, Proposition 4.3 notes that our result is not true in general for the commonly studied case in mechanism design and approximate optimal mechanisms of regular distributions. More specifically, we can weaken the concavity of the revenue function to merely assume regular environments. In other words, we assume that the revenue function is only concave in the space of quantities rather than of prices. Proposition 4.3 establishes that for some regular distributions uniform pricing can perform arbitrarily poorly compared to optimal third-degree price discrimination.

[1] Pigou [1920] suggested a classification of different forms of price discrimination. First degree (or perfect) discrimination is given when the monopolist charges each unit with a price that is equal to the consumer's maximum willingness to pay for that unit. Second degree price discrimination arises when the price depends on the quantity (or quality) purchased. Third degree price discrimination occurs when different market segments are offered different prices, e.g. due to temporal or geographical differentiation.

[2] In the related literature section we discuss the relation between this result and Malueg and Snyder [2006].

1.2 Related Literature

Our work builds on the standard literature on third-degree price discrimination. Schmalensee [1981] compares the output and welfare implications of a single price monopoly, in which the same price is applied to all customer segments, and third-degree price discrimination, in which different prices are applied to different customer segments. He argues that, in general, moving from single price monopoly to third-degree price discrimination leads to a drop in welfare, unless output increases. The work of Schmalensee [1981] continues classic studies on welfare, see e.g., Pigou [1920] and Robinson [1933]. In more recent work, Aguirre et al. [2010] identify conditions on the shape of demand function for price discrimination to increase welfare and output compared to the non-discriminating price case.

Bergemann et al. [2015] analyze the limits of price discrimination. They show that the segmentation and pricing induced by the additional information can achieve every combination of consumer and producer surplus such that: (*i*) consumer surplus is nonnegative, (*ii*) producer surplus is at least as high as profits under the uniform monopoly price, and (*iii*) total surplus does not exceed the surplus generated by the efficient trade. Building on this work, Cummings et al. [2020] provide approximate guarantees to segment the market when an intermediary has only partial information of the buyer's values.

In contrast, in this paper we analyze the *profit* implications of uniform pricing versus third-degree price discrimination. We are particularly interested in understanding what are the approximation guarantees that a uniform price can deliver. Closest to our work is a paper by Malueg and Snyder [2006] which examines the profit effects of third-price discrimination compared to uniform pricing. They consider a setting similar to ours in which the monopolist experiences a total cost function for serving different segments. They show that when the demand is continuous and the total cost is supperadditive, the ratio of third-degree price discrimination profit to uniform price profit is bounded above by the number of segments with distinct prices that are served under price discrimination. They provide an example under which this bound is tight. In the worst case, the bound for the ratio equals the total number of segments. In contrast, in the present paper we identify a key and general condition which leads to a bound that is not contingent on the number of segments in the market. Their Proposition 2 is similar to our Proposition 4 in that their result also provides an example that attains the worst case performance for distributions with different support (and linear demand).

Whereas we study the problem of third-degree price discrimination, Armstrong [1999] examines the problem of second-degree price discrimination. This author analyzes how well a simple two-part tariff approximates the optimal tariff's profit in a multi-product monopolist setting. He shows that the performance ratio of the simpler tariff to the optimal one depends on the coefficient of variation of the total surplus function and that it can be arbitrarily close to one as the number of products sold by the monopolist increases.

Since the seminal work of Myerson [1981], there has been a great deal of interest in the research community on simple or approximate mechanisms design. In general, characterizing optimal selling mechanisms is a difficult task, see e.g., Daskalakis et al. [2014] and Papadimitriou et al. [2016]. Hence, deriving simple-practical mechanisms is of utmost importance.

Along these lines, Chawla et al. [2007] study the unit-demand pricing problem in which a seller with n units sells to a consumer who has a unit demand. In their main result, they find a pricing policy that delivers a 3-approximation with respect to the optimal Myersonian revenue (this is true for non-regular distributions). In the regular i.i.d. case they obtain a 2.17-approximation that prices every item the same. They also show that the Vickrey auction with optimal reservation values is a 2-approximation to the optimal single-item auction.

Hartline and Roughgarden [2009] consider general single-parameter environments with n bidders in which each agent has a valuation for receiving service, and there is a system of specifying feasible sets (e.g., in k-unit auctions the sets are those of size k). In these settings, they investigate how VCG with specific reserve prices approximates the seller's optimal expected revenue. They show that for independent valuations, downward-closed(matroid) environments and under MHR (regularity), VCG with monopoly reserves yields a 2-approximation to optimal mechanism (this is tight). For single-item with valuations drawn independently from regular distributions, they establish that *VCG* with an anonymous reserve price yields a 4-approximation to the expected revenue of the optimal auction.

In the single item setting with n buyers, Alaei et al. [2018] provide several approximation guarantees by analyzing the performance of anonymous pricing (posted price) against an ex-ante relaxation. They show that the ratio of ex-ante/posted price is upper bounded by e (and this is tight) for independent (non-identically) distributed values from regular distributions. As a corollary they improve the 4-approximation in Hartline and Roughgarden [2009] to an e-approximation. Their key insight is to show that triangular instances—instances for which the revenue functions in the *quantile space* are triangle-shaped—are the worst case in terms of guarantee performance. In the single item setting with independent and regular distributions, Jin et al. [2018] establishes a tight 2.62-approximation of anonymous pricing compared to the optimal Myerson auction. In the n heterogeneous items, unit-demand buyer setting they provide the same approximation guarantee for the ratio of optimal item pricing to uniform pricing. The main technique is to find a worst-case scenario instance. Because the ratio is more complex than in Alaei et al. [2018], the worst-case scenario instance is no longer triangular. The present paper differs from the aforementioned works in that we consider the problem faced by a monopolist selling to a single buyer whose valuation can come from one of many segments. Nevertheless, in line with these works, we aim to obtain performance guarantees when comparing the best possible pricing for the monopolist—third-degree price discrimination—to the simple pricing scheme—uniform pricing. In terms of techniques we resort to related triangular instances as worst case performance settings as we discuss in the next paragraph.

Our work also shares some similarities with the approach taken by Dhangwatnotai et al. [2015], see also Hartline [2013], to study the prior-independent single sample mechanism. In this mechanism bidders are allocated according to VCG mechanism with reserves randomly computed from other bidders' bids. A key insight of Dhangwatnotai et al. [2015] is a novel interpretation of the classic work by Bulow and Klemperer [1996], which establishes that the optimal reserve auction for n bidders is revenue dominated by a second price auction without reserve with $n+1$ bidders. Suppose that bidders draw i.i.d. values from the same distribution. Then, if $n = 1$ the optimal auction with reserve is a simple posted price mechanism. When $n = 2$ each bidder's contribution to the profit is the same and the winner pays a random price. In turn, two times the profit from a random price is larger than the profit from the optimal posted price. That is, in this case, random pricing achieves half of the optimal profit. Dhangwatnotai et al. [2015] expand this idea to more complex settings and they formalize it by using an intuitive geometric approach similar to the one we present in Section 3. In particular, under the assumption of regular distributions, the profit function in the quantile space turns out to be concave. Consequentially, the profit function is bounded below by a triangle with height equal to the maximum profit. This implies that the expected profit from uniformly selecting a quantile is bounded below by the area of the triangle or, equivalently, by half the maximum profit. The proof of our result in Theorem 3.1 relies on a similar insight. However, we must assume that the profit functions are *concave in the price space*, otherwise our half approximation result might not hold. Indeed, in Proposition 4.3 we show that for regular distribution simple pricing can obtain arbitrarily bad guarantees.

2 MODEL

We consider a monopolist selling to K different customer segments. Each segment k is in proportion α_k in the market where $\alpha_k \geq 0$ for all $k \in \{1, \ldots, K\}$ and $\sum_{k=1}^{K} \alpha_k = 1$. If the monopolist offers price p_k to segment k, then the monopolist receives an associated profit of $R_k(p_k) \triangleq p_k \cdot (1 - \overline{F}_k(p_k))$, where $F_k(\cdot)$ is the cumulative density function of a distribution with support in $\Theta_k \subset \mathbb{R}_+$. The total profit the monopolist receives by pricing according to $\boldsymbol{p} = (p_1, \ldots, p_K)$ the different types is

$$\Pi(\boldsymbol{p}) = \sum_{k=1}^{K} \alpha_k R_k(p_k).$$

The monopolist wishes to choose $\boldsymbol{p}$ to maximize $\Pi(\boldsymbol{p})$.

The monopolist can choose prices in different manners. First, for each type k, the monopolist can set the price $p_k^\star$ where

$$p_k^\star \in \arg\max_{p \in \Theta_k} R_k(p).$$

Let $\boldsymbol{p}^\star$ be the vector of prices $\{p_k^\star\}_{k=1}^K$, we refer to these prices as the *per-segment optimal prices*. Note that $\boldsymbol{p}^\star$ correspond to the case of third-degree price discrimination. We use Π^M to denote $\Pi(\boldsymbol{p}^\star)$. We sometimes denote $\Pi^M(\alpha, R)$ to make explicit the dependence of the monopolist profit on the model parameters.

Another way of setting prices that is relevant to practice is to simply set the uniform price for all segments. In this case the monopolist must solve the problem

$$\Pi^U \triangleq \max_{p \in \cup_{k=1}^K \Theta_k} \sum_{k=1}^{K} \alpha_k R_k(p).$$

We use p^U to denote the optimal price in the above problem, which we refer to as the *optimal uniform price*. With some abuse of notation we sometimes use $\Pi^U(p)$ to denote $\Pi(\boldsymbol{p})$ when all the components of $\boldsymbol{p}$ are equal to p. Our main objective in this paper is to study how the best third-degree price discrimination scheme compares to the best uniform price scheme. That is, we are interested in analyzing the ratio

$$\min_{\alpha, R} \frac{\Pi^U(\alpha, R)}{\Pi^M(\alpha, R)}, \tag{$\mathcal{P}$}$$

under different parameters environments.

3 CONCAVE PROFIT FUNCTIONS

In this section we assume that the profit functions, $R_k(\cdot)$, are concave. We will further assume that the supports, Θ_k, are well behaved in the sense that $\Theta_k = \Theta$ for all k where Θ is a closed and bounded interval,$[0, \overline{p}]$, of $\mathbb{R}_+$. In later sections, we analyze ($\mathcal{P}$) under relaxed assumptions. In what follows, we provide a simple geometric argument to show that the current environment the optimal value of problem ($\mathcal{P}$) is bounded below by 1/2. The argument is similar to the one presented in Dhangwatnotai et al. [2015] and Hartline [2013], but they assume regular distributions, and therefore concavity of the profit function, in quantile space. Later we show that our 1/2 approximation result does not hold under regularity.

Let

$$r_k \triangleq \alpha_k R_k(p_k^\star),$$

that is, r_k corresponds to the maximum profit the seller can obtain from the fraction α_k of segment k customer. Note that Π^M equals $\sum_{k=1}^{K} r_k$. Since for each segment k the profit function is concave

in Θ we can lower bound it by a triangular-shaped function that we denote $R_k^L(p)$ as depicted in Figure 1 (a).

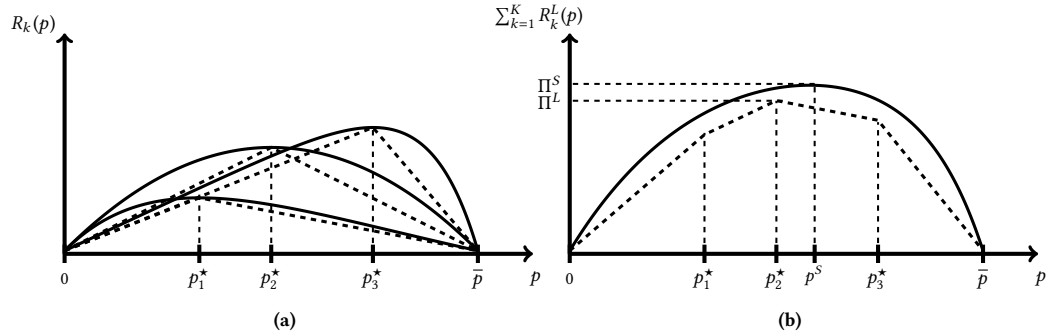


Fig. 1. **(a)** The solid lines depict the concave profit function of each type, $\alpha_k R_k(p)$. The dashed lines depict the lower bounds $R_k^L(p)$ for each type. **(b)** The solid line shows the sum of the profit functions over types, $\sum_{k=1}^K \alpha_k R_k(p)$. The dashed (thick) line shows the sum of the lower bound over types, $\sum_{k=1}^K R_k^L(p)$.

More precisely, we define the lower bound functions

$$R_k^L(p) \triangleq \begin{cases} \frac{r_k}{p_k^\star} \cdot p & \text{if } p \in [0, p_k^\star] \\ \frac{r_k}{\overline{p}-p_k^\star} \cdot (\overline{p} - p) & \text{if } p \in [p_k^\star, \overline{p}]. \end{cases}$$

Observe that $\sum_{k=1}^K R_k^L(p)$ is a concave-piecewise linear function that achieves its maximum at some $\{p_k^\star\}_{k=1}^K$. We use Π^L to denote its maximum value. Then, it is easy to see that $\Pi^U \geq \Pi^L$ because $\sum_{k=1}^K R_k^L(p)$ lower bounds $\sum_{k=1}^K \alpha_k R_k(p)$, see Figure 1 (b). Next we argue that

$$\Pi^L = \max_{p \in \{p_1^\star, \ldots, p_K^\star\}} \left\{ \sum_{k=1}^K R_k^L(p) \right\} \geq \frac{1}{2} \sum_{k=1}^K r_k = \frac{1}{2} \Pi^M. \tag{1}$$

See Theorem 3.1 below for a formal statement. Consider Figure 2 and note that $\Pi^L \cdot \overline{p}$ is equal to the area of the smallest rectangle that contains the graph of $\sum_{k=1}^K R_k^L(p)$. As a consequence, $\Pi^L \cdot \overline{p}$ is an upper bound for the area below the curve $\sum_{k=1}^K R_k^L(p)$, that is,

$$\Pi^L \cdot \overline{p} \geq \int_0^{\overline{p}} \sum_{k=1}^K R_k^L(p) dp = \sum_{k=1}^K \int_0^{\overline{p}} R_k^L(p) dp = \sum_{k=1}^K \frac{r_k \cdot \overline{p}}{2},$$

where in the last equality we have used that $R_k^L(p)$ is triangle-shaped and, therefore, the area below its curve equals $r_k \cdot \overline{p}/2$. Dividing both sides in the expression above by $\overline{p}$ yields Eq. (1), completing the proof. We next state this result as a formal theorem and then provide some comments about it.

Theorem 3.1 (**Uniform price is a half approximation**).
Suppose that the profit functions $R_k(p)$ are concave and defined in the same bounded interval $\Theta \subset \mathbb{R}_+$ for all $k \in \{1, \ldots, K\}$. Then Π^U is at least half as large as Π^M. In particular,

$$\Pi^U = \max_{p \in \Theta} \left\{ \sum_{k=1}^K \alpha_k R_k(p) \right\} \geq \max_{p \in \{p_1^\star, \ldots, p_K^\star\}} \left\{ \sum_{k=1}^K \alpha_k R_k(p) \right\} \geq \frac{1}{2} \Pi^M.$$

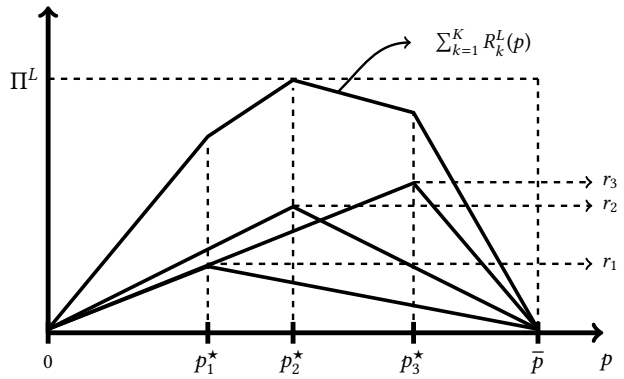


Fig. 2. Proof of Theorem 3.1: the area below $\sum_{k=1}^{K} R_{k,L}(p)$ equals the sum of the areas below $R_{k,L}(p)$ for all k.

Theorem 3.1 provides a fundamental guarantee of uniform pricing compared to optimal third-degree price discrimination. In particular, the monopolist can simple use a judiciously chosen price across all customer segments to ensure half of the best possible profit from perfectly discriminating across the different segments in the market. The theorem also suggests two simple and appealing ways of selecting the price. First, the monopolist can optimize against the mixture of customer segments to derive the optimal uniform price. This is advantageous for situations in which the monopolist possesses aggregate market information and perfectly discriminating segments is not an available option. When the monopolist has more granular market information, for example the monopolist knows the prices $\{p_k^\star\}_{k=1}^K$, then it is not necessary for the monopolist to optimize over the full range of prices but he can simply choose one of the K prices at hand.

As shown by the next example, the profit guarantee in Theorem 3.1 is tight.

Example 3.2.
Consider for example a case with two segments in the same proportion and triangle-shaped profit functions. Furthermore, assume that that $p_1^\star$ is close to zero, say $\varepsilon > 0$, and $p_2^\star$ is close to $\overline{p}$; and that for both profit functions, the maximum profit is the same and equal to one. Then Π^M equals to $\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$ whereas the optimal uniform price is any price between $p_1^\star$ and $p_2^\star$, therefore, Π^U equals $\frac{1}{2}R_1(\varepsilon) + \frac{1}{2}R_2(\varepsilon) \approx \frac{1}{2} + 0$.

The result in Theorem 3.1 is intimately related to the problem of ex-post individually rational screening in which the seller must optimally design a menu of contracts that incentivize buyers of different type to self select, see e.g. Krähmer and Strausz [2015] and Bergemann et al. [2018]. The optimal static pricing in the screening setting is the same as our optimal uniform pricing. However, the optimal screening pricing is different to our optimal third-degree discrimination pricing because screening imposes incentive constraints that translate into information rents the monopolist must give up; in the screening problem buyers have private information about their types. In turn the optimal screening profit is upper bounded by Π^M but is an upper bound to Π^U. This leads us to the following corollary of Theorem 3.1.

Corollary 3.3 (**Half approximation in sequential screening**).
Suppose that the assumptions of Theorem 3.1 hold. Then in the ex-post individually rational screening setting of Krähmer and Strausz [2015] and Bergemann et al. [2018], the optimal static contract delivers a 1/2-approximation for seller's profits.

As discussed in Section 1.2, our result and approach shares some similarities to that in Dhangwatnotai et al. [2015]. For the case of one buyer—and regular distributions—they show that the expected profit of randomly selecting a price achieves half of the optimal profit. Their approach uses the fact that the profit function in the quantile space for regular distributions is concave and then a uniform randomization over quantities. We can proceed in a similar fashion to show that the expected profit of uniformly choosing prices achieves half the profit of third-degree price discrimination. Indeed, suppose we set a price p at random such that $p \sim U[0, \overline{p}]$. Then for the expected profit we have,

$$\mathsf{E}[\Pi^U(p)] = \int_0^{\overline{p}} \sum_{k=1}^{K} \alpha_k R_k(p) \cdot \frac{1}{\overline{p}}\, dp \geq \frac{1}{\overline{p}} \cdot \int_0^{\overline{p}} \sum_{k=1}^{K} R_k^L(p)\, dp = \frac{1}{\overline{p}} \cdot \sum_{k=1}^{K} \frac{r_k \cdot \overline{p}}{2} = \frac{1}{2}\Pi^M. \qquad (2)$$

We summarize this discussion in the following proposition.

Proposition 3.4 (**Uniformly at random pricing**).
Suppose that the profit functions $R_k(p)$ are concave and defined in the same bounded interval $\Theta \subset \mathbb{R}_+$ for all $k \in \{1, \ldots, K\}$. Then for $p \sim U[0, \overline{p}]$ we have that $\mathsf{E}_p[\Pi^U(p)]$ is at least half as large as Π^M.

The difference of Theorem 3.1 with Eq. (2) stems from the fact that in the former, we choose a unique price, p^U, whereas in the latter, we randomize over Θ. In Section 4, we further elaborate on the differences with Dhangwatnotai et al. [2015].

A natural question that emerges from Theorem 3.1 is about what uniform price to use. According to the theorem, in order to achieve the approximation guarantee, it is enough to choose the price $p^\star_{k^\star}$ that solves $\max_k \Pi^U(p^\star_k)$. However, if we consider the next best price, $p^{\mathsf{2nd}} \in \arg\max_{k \neq k^\star} \Pi^U(p^\star_k)$, is it possible to achieve a *good* approximation guarantee? The next proposition answers this question.

Proposition 3.5 (**Second best price**).
The second best price among the per-segment-optimal prices, p^{2nd}, can deliver an arbitrarily low profit guarantee. More precisely, for any $\varepsilon > 0$ there exists (α, R) such that $\Pi^U(p^{\mathsf{2nd}})/\Pi^M = O(\varepsilon)$.

Proof. Consider $K \geq 2$, $\overline{p} = 1$, $\alpha_1 = \cdots = \alpha_K = 1/K$ and let $n \geq 1$. Define the profit functions by

$$R_1(p) \triangleq \begin{cases} \frac{n}{1/n} \cdot p & \text{if } p \in [0, 1/n] \\ \frac{n}{1-1/n} \cdot (1-p) & \text{if } p \in [1/n, 1], \end{cases} \quad \text{and} \quad R_k(p) \triangleq \begin{cases} \frac{\varepsilon_k}{(1-\varepsilon_k)} \cdot p & \text{if } p \in [0, 1-\varepsilon_k] \\ 1-p & \text{if } p \in [1-\varepsilon_k, 1], \end{cases}$$

where $\varepsilon_k \in (0, \varepsilon)$ for all $k \in \{2, \ldots, K\}$ are fixed. Note that the price of type 1 is the optimal uniform price when n is large, that is, $p^U = 1/n$. Therefore, the second best price, p^{2nd}, must equal $1 - \varepsilon_{\hat{k}}$ for some $\hat{k} \geq 2$. Then,

$$\Pi^U(p^{\mathsf{2nd}}) = \frac{1}{K}\left(R_1(1-\varepsilon_{\hat{k}}) + \sum_{k \geq 2} R_k(1-\varepsilon_{\hat{k}})\right) \leq \frac{1}{K}\left(\frac{n}{1-1/n}\varepsilon_{\hat{k}} + \sum_{k \geq 2} \max\{\varepsilon_{\hat{k}}, \frac{\varepsilon_k}{1-\varepsilon_k} \cdot (1-\varepsilon_{\hat{k}})\}\right)$$

Whereas the profit of the perfect discrimination is

$$\Pi^M = \frac{1}{K}R_1(1/n) + \frac{1}{K}\sum_{k \geq 2} R_2(1-\varepsilon_k) = \frac{1}{K}n + \frac{1}{K}\varepsilon = \frac{1}{K}\left(n + \sum_{k \geq 2}\varepsilon_k\right) \geq \frac{n}{K}.$$

Then,

$$\frac{\Pi^U(p^{\mathsf{2nd}})}{\Pi^M} \leq \frac{\frac{n}{1-1/n}\varepsilon + \sum_{k \geq 2} \max\{\varepsilon_{\hat{k}}, \frac{\varepsilon_k}{1-\varepsilon_k} \cdot (1-\varepsilon_{\hat{k}})\}}{n} \to \varepsilon \text{ as } n \to \infty.$$

□

Proposition 3.5 establishes that by using the second best price among the per-segment-optimal prices, the monopolist profit can be arbitrarily small. In the proof, we construct concave profit functions such that the profit from the first type is arbitrarily large at $p_1^\star$ but the profit from any other type $k \geq 2$ at $p_k^\star$ is arbitrarily low. In turn, p^U coincides with $p_1^\star$ and $p^{2\text{nd}}$ is among segments $k \geq 2$. As a consequence the monopolist profit from using $p^{2\text{nd}}$ across segments is arbitrarily small compared to the profit from perfect price discrimination.

4 PROFIT PERFORMANCE IN GENERAL ENVIRONMENTS

In this section we examine deviations from the environment studied is Section 3. In particular, we aim to understand how ($\mathcal{P}$) behaves when we relax the assumptions in Theorem 3.1. We first look into the assumption of finite support and consider profit functions with different supports. Then, we study non-concave environments. In the latter, we are specially interested in common well-behaved environments such as regular and MHR (monotone hazard rate) value distributions.

4.1 All Concave with Unbounded Support

In Theorem 3.1 we considered concave profit functions supported on some common finite interval Θ. In the next proposition, we relax the finite support assumption while keeping a common support and concave profit functions across customer segments.

Proposition 4.1 (**No gap with unbounded support**).
Suppose that the profit functions for all segments are concave with common and unbounded support $\Theta = [0, \infty]$ *then* $\Pi^U = \Pi^M$.

Proof. Without loss of generality assume that $p_1^\star \leq \cdots \leq p_K^\star$. Note that the concavity of the profit functions together with the unbounded support assumption obliges each $R_k(p)$ to be increasing up to $p_k^\star$ and then constant and equal to $R_k(p_k^\star)$ for any price p larger than $p_k^\star$ for all $k \geq 1$. In turn, by setting p^U equal to $p_K^\star$ the profit Π^U becomes $\sum_{k=1}^K \alpha_k R_k(p_k^\star) = \Pi^M$. □

The proposition establishes that in the concave case with unbounded and common support there is no gap between no price discrimination and full price discrimination. The intuition behind Proposition 4.1 is simple. Concavity, together with the unbounded support assumption, implies that the marginal profit for each segment must equal zero for sufficiently large prices. As a consequence, setting an equal and sufficiently large price for every segment achieves the optimal third-degree price discrimination outcomes.

4.2 Necessity of Common Support

Here we consider concave profit functions supported on some finite interval Θ_k for each segment $k \geq 1$. In contrast to the previous sections, we will not assume that $\Theta_k = \Theta$ for all segments.

Proposition 4.2 (**Necessity of common support**).
If we allow for different supports among segments then the optimal uniform price can deliver an arbitrarily small profit guarantee as the number of segments increases.

Proof. We construct concave profit functions with finite support but such that $\Theta_k \neq \Theta_j$ for all $k \neq j$. Let the profit functions be defined by

$$R_k(p) = \begin{cases} p & \text{if } p \in [0, v_k] \\ \frac{v_k}{\varepsilon_k}(v_k + \varepsilon_k - p) & \text{if } p \in [v_k, v_k + \varepsilon_k] \end{cases}$$

with

$$v_k = \frac{1}{(K-k+1)}, \quad \alpha_k = \frac{1}{K}, \quad \varepsilon_k \in (0, v_{k+1} - v_k), \quad \forall k \in \{1, \ldots, K\}.$$

The perfect price discrimination profit is

$$\Pi^M = \sum_{k=1}^{K} \alpha_k v_k = \frac{1}{K} \sum_{k=1}^{K} \frac{1}{K-k+1} = \frac{1}{K} \sum_{k=1}^{K} \frac{1}{k}.$$

The optimal uniform price must be achieved at one of the v_k, hence

$$\Pi^U = \frac{1}{K} \max_{k=1,\ldots,K} \left\{ \sum_{j=k}^{K} v_k \right\} = \frac{1}{K} \max_{k=1,\ldots,K} \left\{ \frac{1}{K-k+1} \cdot (K-k+1) \right\} = \frac{1}{K}.$$

Hence,

$$\frac{\Pi^U}{\Pi^M} = \frac{\frac{1}{K}}{\frac{1}{K}\sum_{k=1}^{K}\frac{1}{k}} = \frac{1}{\sum_{k=1}^{K}\frac{1}{k}} \approx \frac{1}{\log(K)} \to 0 \quad \text{as } K \uparrow \infty.$$

□

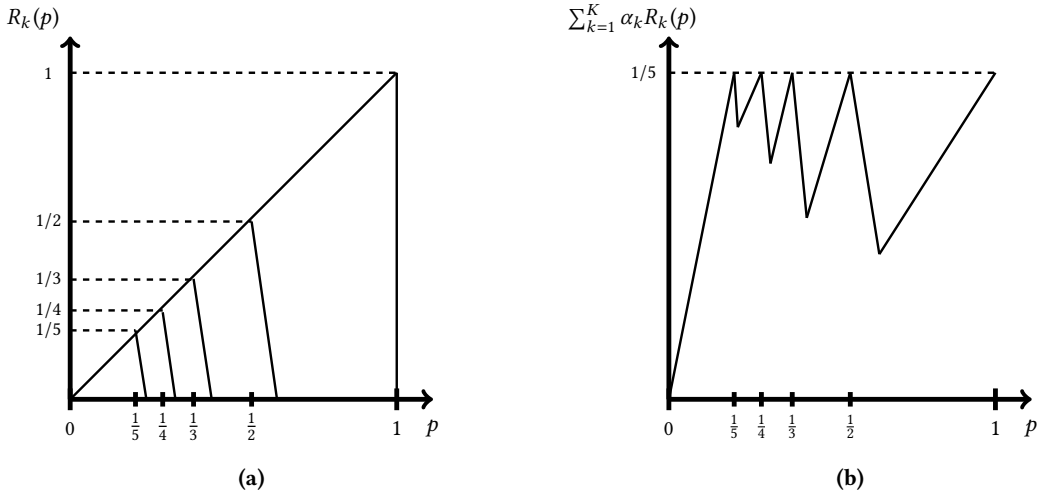


Fig. 3. Example for construction of concave profit functions with different support as in the proof of Proposition 4.2 for $K = 5$. In **(a)** we illustrate the profit functions for each segment where the value and optimal per-segment price decay as $1/k$. In **(b)** we show $\sum_{k=1}^{K} \alpha_k R_k(p)$, it is maximized at any of the per-segment optimal prices and is bounded above by $1/K$.

Proposition 4.2 shows the necessity of the common support assumption. In the proof, we construct concave profit functions that have finite support, but the end points of the supports are increasing, see Figure 4 **(a)**. In the construction, all segments are given the same weight and the profit functions are triangle shaped. All of them start at zero, go up along the 45 degree line, peak at $1/k$ and then go down sharply such that the upper end of the support of type k is strictly between $\frac{1}{k}$ and $\frac{1}{k-1}$ (solid lines in Figure 4 **(a)**). In turn, Π^M (normalized by the per-segment proportions) grows logarithmically with K. Since the end of the supports are strictly increasing and non-overlapping, the uniform price profit at the per-segment optimal prices, $\Pi^U(1/k)$, is constant and equal to 1 (normalized by the per-segment proportions), see Figure 4 **(b)**. For example consider $p_3^\star = 1/3$, at this price $R_1(1/3) = R_2(1/3) = 0$ and $R_3(1/3) = R_4(1/3) = R_5(1/3) = 1/3$ hence $5 \cdot \Pi^U = 0 + 0 + 3 \cdot \frac{1}{3}$. As a result, the ratio in ($\mathcal{P}$) goes to zero as the number of segments increases. Note that this only works because the profit functions do not have common support. The non-common support allows

for the possibility of having profit functions such that at the per-segment optimal prices some of them equal zero. In turn, in terms of Π^U, at each $p_k^\star$ there is some profit loss compared to the same-support case and that profit loss is large enough to be outperformed by Π^M. We note that a similar version of this result was stated in Malueg and Snyder [2006]. Their proof is inductive whereas our proof in constructive and provides an intuitive characterization of the necessity of common support.

4.3 Non-Concave Environments

One of the most important families of distributions in the mechanism design literature correspond to regular distributions. These are distributions such that the virtual value function is non-decreasing. Formally, for any distribution with cdf F, pdf f and virtual value function defined by

$$\phi(p) \triangleq p - \frac{(1 - F(p))}{f(p)},$$

we say that F is regular if and only if $\phi(p)$ is non-decreasing. As pointed out in Section 1.2, for these distributions several approximation guarantees have been obtained in diverse settings. One of the main insights used in the literature is that the profit function associated to this family of distribution is concave in the quantile space. Indeed, let $R(p) = p \cdot (1 - F(p))$ and consider the change of variables $q = 1 - F(p)$. Define the profit function in the quantile space as $\hat{R}(q) = q \cdot F^{-1}(1 - q)$ then

$$\frac{d}{dq}\hat{R}(q) = F^{-1}(1 - q) - \frac{q}{f(F^{-1}(1 - q))} = \phi(F^{-1}(1 - q)),$$

since $\phi(\cdot)$ is non-decreasing we can conclude that $\hat{R}(q)$ is concave. The concavity of $\hat{R}(q)$ allows arguments similar to the ones leveraged in Theorem 3.1. For example, Dhangwatnotai et al. [2015] leverage this property to show that with one bidder the expected profit from random pricing(uniformly selecting a quantile) is half the profit of the optimal monopoly price. In turn, it becomes appealing if a similar approach would work in our framework. In particular, in the regular distribution setting is it possible to work in the quantile space, leverage the concavity of the profit functions and then show that a good approximation guarantee obtains?

Recall that for each segment $k \geq 1$ the profit function comes from a cdf F_k for which we assume its pdf f_k is well defined. In order to switch to the quantile space, for any $p \in \Theta$, we would need to define

$$q_k = 1 - F_k(p) \quad \text{and} \quad \hat{R}_k(q) = q \cdot F_k^{-1}(q).$$

The optimal uniform price profit, Π^U, is given by

$$\begin{aligned} \Pi^U = \max_{0 \leq \boldsymbol{q} \leq 1} & \sum_{k=1}^{K} \alpha_k \cdot \hat{R}_k(q_k) \\ \text{s.t} \quad & F_k^{-1}(1 - q_k) = F_j^{-1}(1 - q_j), \quad \forall k, j. \end{aligned}$$

Note that in this formulation we have gained that the objective function is the summation of concave functions. However, we have introduced additional constraints compared to the original formulation of Π^U in Section 2. These constraints stems from the fact that under uniform pricing each segment receives the uniform price p, and since $q_k = 1 - F_k(p)$ we must have that $F^{-1}(1 - q_k) = F^{-1}(1 - q_j)$ for all segments $k, j \geq 1$. At this point, the natural approach would be to lower bound each $\hat{R}_k$ by a triangle shaped function—similar to Figure 1 **(a)** but in the quantile space. Then solve the resulting optimization problem and, hopefully, obtain a good approximation guarantee. Unfortunately, for common regular distributions the former approach fails as established by the following proposition.

Proposition 4.3 (**Failure of regular distributions**).
There exist regular distributions for which the optimal uniform price delivers an arbitrarily small profit guarantee as the number of segments increases.

Proof. We construct regular distributions $\{F_k\}_{k=1}^K$ such that $\Pi^U/\Pi^M \to 0$ as $K \uparrow \infty$. Define

$$F_k(p) = 1 - e^{-(K-k+1)p}, \quad \forall p \geq 0, \quad \text{and} \quad \alpha_k = 1/K \quad \forall k \in \{1, \dots, K\}.$$

Thus we consider exponential distributions with increasing means, given by $1/(K-k+1)$, as k increases. Note that this distributions are regular

$$p - \frac{(1-F_k(p))}{f_k(p)} = p - \frac{e^{-(K-k+1)p}}{(K-k+1)e^{-(K-k+1)p}} = p - \frac{1}{K-k+1}.$$

For all $k \geq 1$ the profit functions are

$$R_k(p) = p \cdot (1 - F_k(p)) = p \cdot e^{-(K-k+1)p},$$

whereas the per-segment optimal prices are

$$p_k^\star = \frac{1}{K-k+1}, \quad \text{and} \quad R_k(p_k^\star) = \frac{e^{-1}}{K-k+1}.$$

Then the full price discrimination profit is

$$\Pi^M = \sum_{k=1}^K \alpha_k R_k(p_k^\star) = \sum_{k=1}^K \frac{1}{K} \frac{e^{-1}}{K-k+1} = \frac{e^{-1}}{K} \sum_{k=1}^K \frac{1}{K-k+1} \approx \frac{e^{-1}}{K} \cdot \log(K).$$

The uniform price profit for some price p is

$$\sum_{k=1}^K \alpha_k R_k(p) = \frac{1}{K} \sum_{k=1}^K p \cdot e^{-(K-k+1)p} = \frac{p}{K} \sum_{k=1}^K e^{-kp} = \frac{p}{K} \cdot \frac{1-e^{-Kp}}{e^p - 1} \leq \frac{1-e^{-Kp}}{K} \leq \frac{1}{K},$$

where the second to last inequality holds because we always have that $p + 1 \leq e^p$. With this we can conclude that

$$\frac{\Pi^U}{\Pi^M} = \frac{\max_{p \geq 0} \left\{ \sum_{k=1}^K \alpha_k R_k(p) \right\}}{\Pi^M} \leq \frac{\frac{1}{K}}{\frac{e^{-1}}{K} \cdot \log(K)} = \frac{1}{e^{-1} \cdot \log(K)} \to 0, \quad K \uparrow \infty.$$

□

Proposition 4.3 establishes that for some regular distributions, uniform pricing can perform arbitrarily poorly compared to optimal third-degree price discrimination. Moreover, in the proof, we use exponential distributions and, therefore, the result is also true for MHR (monotone hazard rate) distribution. The intuition behind this result is similar to that of Proposition 4.2. We consider exponential distributions such that at the optimal uniform price, most of the per-segment profits will be low and, therefore, they will not contribute much to Π^U, see Figure 4. Since for exponentials the associated profit functions decay quickly after they peak, they behave very similar to the case of non-common support in which the upper end of the support are increasing. Indeed, in the proof of Proposition 4.3 we obtain a similar profit guarantee as in the proof of Proposition 4.2, namely, $O(1/\log(K))$.

To conclude this section, we consider the case of triangular instances on quantile space. These are instances for which the profit functions in the quantile space are triangle-shaped. They are widely used in the literature of approximate mechanism design as a bridge to provide good profit guarantees. However, in our setting they can perform arbitrarily poorly.

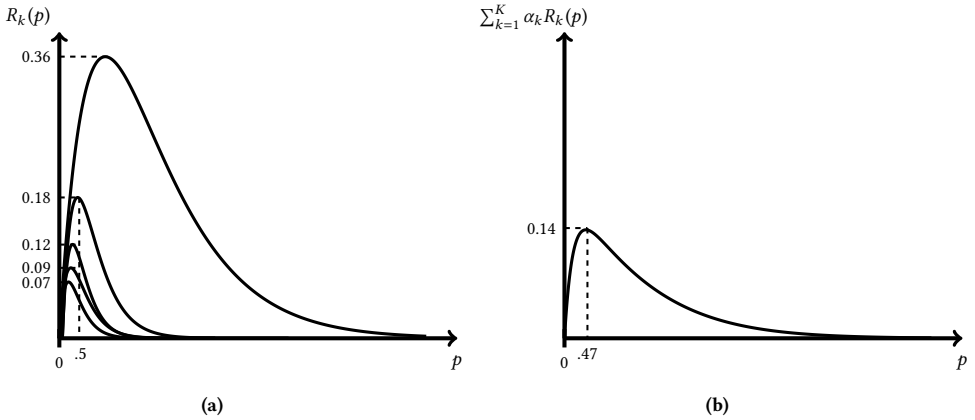


Fig. 4. Example for construction of profit functions from regular distributions in Proposition 4.2 with $K = 5$. In **(a)** we illustrate the profit functions for each segment where the per-segment optimal profit is e^{-1}/k and the per-segment optimal price $1/k$. In **(b)** we show $\sum_{k=1}^{K} \alpha_k R_k(p)$ with maximum value 0.14 (bounded above by $1/K$).

Proposition 4.4 (**Triangular instances**).
For triangular instances defined by

$$F_k(p) = \begin{cases} 1 & \text{if } v \geq v_k; \\ \frac{p\cdot(1-q_k)}{p\cdot(1-q_k)+v_k\cdot q_k} & \text{if } v < v_k, \end{cases}$$

there exists a choice of $\{\alpha_k\}_{k=1}^K \in (0,1)$, $\{v_k\}_{k=1}^K \in \mathbb{R}_+$ *and* $\{q_k\}_{k=1}^K \in (0,1)$ *such that* $\Pi^U/\Pi^M \to 0$ *as the number of segments increases.*

Proof. Let us start by considering triangular instances, the profit functions are

$$R_k(p) = p \cdot \bar{F}_k(p) = \begin{cases} 0 & \text{if } v \geq v_k \\ \frac{p\cdot v_k\cdot q_k}{p\cdot(1-q_k)+v_k\cdot q_k} & \text{if } v < v_k. \end{cases}$$

Note that $R_k(p)$ is increasing and concave up to v_k and then is constant and equal to zero for $p \geq v_k$. For each curve the optimal price is v_k (minus small $\varepsilon > 0$), thus

$$\Pi^M = \sum_{k=1}^{K} \alpha_k \cdot v_k \cdot q_k \tag{3}$$

For the static contract the optimal price must be achieved at one of the $v_1, \ldots, v_K$. Therefore,

$$\Pi^U = \max_{i\in\{1,\ldots,K\}} \left\{ \sum_{k=i}^{K} \alpha_k \cdot \frac{v_i \cdot v_k \cdot q_k}{v_i \cdot (1-q_k) + v_k \cdot q_k} \right\}$$

Next we will establish that $\Pi^U/\Pi^M \to 0$ as $K \to \infty$. Consider the instance

$$v_k = \frac{1}{(K-k+1)}, \quad q_k = 0.5, \quad \text{and} \quad \alpha_k = \frac{1}{K}, \quad \forall k \in \{1,\ldots,K\}.$$

Hence,

$$\sum_{k=i}^{K} \alpha_k \cdot \frac{v_i \cdot v_k \cdot q_k}{v_i \cdot (1-q_k) + v_k \cdot q_k} = \frac{1}{K} \sum_{k=i}^{K} \frac{\frac{1}{(K-i+1)} \cdot \frac{1}{(K-k+1)}}{\frac{1}{(K-i+1)} + \frac{1}{(K-k+1)}} = \frac{1}{K} \sum_{k=i}^{K} \frac{1}{2(K+1)-(k+i)}.$$

It is possible to show that the last term above is decreasing in i and, therefore,

$$\Pi^U = \frac{1}{K}\sum_{k=1}^{K}\frac{1}{2K+1-k} = \frac{1}{K}\sum_{k=1}^{K}\frac{1}{K+k} \approx \frac{1}{K}\int_1^K \frac{1}{K+x}dx = \frac{1}{K}\log\left(\frac{2K}{K+1}\right).$$

We also have that

$$\Pi^M = \sum_{k=1}^{K}\alpha_k \cdot v_k \cdot q_k = \frac{1}{2K}\sum_{k=1}^{K}\frac{1}{(K-k+1)} = \frac{1}{2K}\sum_{k=1}^{K}\frac{1}{k} \approx \frac{1}{2K}\log(K).$$

Thus,

$$\frac{\Pi^U}{\Pi^M} \approx \frac{\frac{1}{K}\log\left(\frac{2K}{K+1}\right)}{\frac{1}{2K}\log(K)} = 2\frac{\log\left(\frac{2K}{K+1}\right)}{\log(K)} \to 0 \quad (\approx 2\cdot\log(2)/\log(K)).$$

□

5 WORST CASE PERFORMANCE

In this brief section, our objective is to investigate the worst case performance of uniform pricing. For this purpose suppose there are K segments and without loss of generality let us assume that

$$0 \le \alpha_1 R_1(p_1^\star) \le \alpha_2 R_2(p_2^\star) \le \cdots \le \alpha_K R_K(p_K^\star). \tag{4}$$

Using this condition we can verify that the ratio Π^U/Π^M is always bounded below by $1/K$. Indeed, note that $\Pi^U \ge \Pi^U(p_K^\star)$ and

$$\Pi^U(p_K^\star) = \sum_{k=1}^{K}\alpha_k R_k(p_K^\star) \ge \alpha_K R_K(p_K^\star) = \frac{\overbrace{\alpha_K R_K(p_K^\star) + \cdots + \alpha_K R_K(p_K^\star)}^{K \text{ times}}}{K} \ge \frac{1}{K}\sum_{k=1}^{K}\alpha_k R_k(p_k^\star),$$

where in the last inequality above we use Eq. (4). This proves that the worst case performance of uniform pricing with respect to third-degree price discrimination is potentially $1/K$. In what follows, we argue that this lower bound performance can indeed be achieved. [3]

Let us consider atomic valuations, that is, every segment k has a unique possible value denoted by p_k. Under perfect price discrimination the monopolist can charge the price p_k to the buyer and extract full surplus: $\sum_{k=1}^{K}\alpha_k p_k$. Under uniform pricing, the monopolist charges a fixed price p across all segments and collects profit only from those segments whose price, p_k, is larger that p: $\sum_{k=1}^{K}\alpha_k p \mathbf{1}_{p_k \ge p}$. In turn, the optimization problem we would like to solve to assess the performance of uniform pricing becomes

$$\min_{\alpha_k, p_k}\left\{\frac{\max_{p\ge 0}\sum_{k=1}^{K}\alpha_k p \mathbf{1}_{p_k\ge p}}{\sum_{k=1}^{K}\alpha_k p_k}, \quad \text{s.t} \quad \sum_{k=1}^{K}\alpha_k = 1,\ \ \alpha_k \ge 0\ \ \forall k\right\}.$$

Observe that in the above problem, without loss of generality, we can assume that the prices p_k are ordered. This allows us to simplify the numerator in the objective above. Note that the maximum in the numerator must be achieved at some price p_j for $j \in \{1,\ldots,K\}$ and for any p_j we have

$$\sum_{k=1}^{K}\alpha_k p_j \mathbf{1}_{p_k \ge p_j} = p_j \sum_{k=j}^{K}\alpha_k.$$

[3] Proposition 2 in Malueg and Snyder [2006] provides an inductive argument with linear demand functions. Here we provide a constructive argument with atomic distributions.

In turn, this enables us to reformulate the problem as

$$\min_{\alpha_k} \left\{ \frac{\max_{j\in\{1,\dots,K\}} p_j \sum_{k=j}^{K} \alpha_k}{\sum_{k=1}^{K} \alpha_k p_k}, \quad \text{s.t} \quad p_1 \le \cdots \le p_K, \quad \sum_{k=1}^{K} \alpha_k = 1, \ \ \alpha_k \ge 0 \ \ \forall k \right\}. \tag{5}$$

Next, we exhibit values of p_k and α_k such that the ratio in the problem above is arbitrarily close to the lower bound $1/K$. Let $\varepsilon > 0$ be small, define the prices

$$p_k = \frac{\varepsilon}{K}\left(\frac{1+\varepsilon}{\varepsilon}\right)^k, \quad k \in \{1,\dots,K-1\}, \quad \text{and} \quad p_K = \frac{1}{K}\left(\frac{1+\varepsilon}{\varepsilon}\right)^{K-1},$$

and let the per-segment proportions be

$$\alpha_k = \frac{1}{K}\frac{1}{p_k}, \quad k \in \{1,\dots,K\}.$$

Next, we verify that the above prices and proportions are feasible. Indeed, it easy to verify that p_k is increasing in k while for the per-segment proportions we have that $\alpha_k > 0$ and

$$\sum_{k=1}^{K} \alpha_k = \sum_{k=1}^{K-1} \frac{1}{\varepsilon}\left(\frac{\varepsilon}{1+\varepsilon}\right)^k + \left(\frac{\varepsilon}{1+\varepsilon}\right)^{K-1} = 1.$$

Now let us look at the objective in problem (5). Given our choice of prices and proportions, we have that $\alpha_k p_k = 1/K$ and, therefore, the denominator in Eq. (5) equals one. For the numerator consider $j \in \{1,\dots,K-1\}$ then

$$\begin{aligned} p_j \sum_{k=j}^{K} \alpha_k &= \frac{\varepsilon}{K}\left(\frac{1+\varepsilon}{\varepsilon}\right)^j \left(\sum_{k=j}^{K-1} \frac{1}{\varepsilon}\left(\frac{\varepsilon}{1+\varepsilon}\right)^k + \left(\frac{\varepsilon}{1+\varepsilon}\right)^{K-1} \right) \\ &= \frac{\varepsilon}{K}\left(\frac{1+\varepsilon}{\varepsilon}\right)^j \left(\frac{\varepsilon}{1+\varepsilon}\right)^{j-1} \\ &= \frac{1+\varepsilon}{K}, \end{aligned}$$

and $p_K \alpha_K = 1/K$. Therefore the objective in Eq. (5) evaluated in our current choice of prices and proportions equals

$$\max_{j\in\{1,\dots,K\}} \left\{ p_j \sum_{k=j}^{K} \alpha_k \right\} = \max\left\{ \frac{1}{K}, \frac{1+\varepsilon}{K} \right\} = \frac{1+\varepsilon}{K}. \tag{6}$$

In conclusion, we have exhibited an instance for which the performance of the optimal uniform price is as close as one may desire to the worst performance guarantee, $1/K$. We summarize this discussion in the following proposition.

Proposition 5.1 (**Worst performance achieved**).
There exists an atomic instance and $\varepsilon_0 > 0$ such that

$$\min_{\alpha_k, \theta_k} \frac{\Pi^U}{\Pi^M} \le \frac{1+\varepsilon}{K}, \quad \forall \varepsilon \in (0, \varepsilon_0).$$

6 CONCLUSION

In this paper we consider the revenue performance of uniform pricing in settings where a monopolist may engage in third-degree price discrimination. We establish that, for concave revenue functions with common support, using a single price can achieve half of the optimal revenue the monopolist could potentially garner by engaging in third-degree price discrimination. Our revenue guarantee is robust. It does not depend on the number of market segments or prices the seller might use.

We then investigate the scope of our revenue guarantee. We establish that by relaxing either the concavity or the common support assumptions, uniform pricing can deliver arbitrarily bad revenue guarantees as the number of market segments increases. Interestingly, for regular distributions and triangular instancesâĂŤleading cases in the literature of approximate mechanism designâĂŤwe show that uniform pricing can again obtain poor revenues guarantees that scales with the number of market segments.

In current work, we are numerically investigating the performance of uniform pricing in empirically relevant settings. We conjecture that the actual performance in practice may be better than our theoretical bound of one half.

Further, depending on the nature of a market, different types of price discrimination are possible. A plausible direction of future work is to consider an environment in which the seller can exercise second-degree price discrimination by creating a menu of prices and quantities. For example, consider a setting with K different markets each of which is characterized by a different distribution of valuations. Within each market the seller may offer an optimal menu of prices and quantities. However, due to legal or business constraints, it is often the case in practice that such strong price discrimination is not implementable. Instead, the seller might only be able to offer the same menu across all markets. In turn, it becomes a natural question to explore the performance of this limited menu versus the full discriminating one. Another related question arises in the same domain of non-linear pricing in which, within a market, the seller offers different quantities and prices to buyers of different valuations. In this setting, it would be interesting to investigate the revenue performance of a simple menu consisting of a single price in which buyers self-select their quantities. These are all directions of current and future research.

REFERENCES

I. Aguirre, S. Cowan, and J. Vickers. 2010. Monopoly Price Discrimination and Demand Curvature. *American Economic Review* 100 (2010), 1601–1615.

S. Alaei, J. Hartline, R. Niazadeh, E. Pountourakis, and Y. Yuan. 2018. Optimal Auctions vs Anonymous Pricing. *Games and Economic Behavior* (2018).

M. Armstrong. 1999. Price Discrimination by a Many-Product Firm. *The Review of Economic Studies* 66, 1 (1999), 151–168.

D. Bergemann, B. Brooks, and S. Morris. 2015. The Limits of Price Discrimination. *American Economic Review* 105 (2015), 921–957.

D. Bergemann, F. Castro, and G. Weintraub. 2018. The Scope of Sequential Screening With Ex Post Participation Constraints. In *Working paper, available at SSRN: https://ssrn.com/abstract=3206783.*

J. Bulow and P. Klemperer. 1996. Auctions vs Negotiations. *American Economic Review* 86 (1996), 180–194.

S. Chawla, J. Hartline, and R. Kleinberg. 2007. Algorithmic Pricing Via Virtual Valuations. In *Proceedings of the 8th ACM Conference on Electronic Commerce*. ACM, 243–251.

R. Cummings, N. Devanur, Z. Huang, and X. Wang. 2020. Algorithmic Price Discrimination. In *Proceedings of the Fourteenth Annual ACM-SIAM Symposium on Discrete Algorithms*. SIAM, 2432–2451.

C. Daskalakis, A. Deckelbaum, and C. Tzamos. 2014. The Complexity of Optimal Mechanism Design. In *Proceedings of the twenty-fifth annual ACM-SIAM symposium on Discrete algorithms*. SIAM, 1302–1318.

P. Dhangwatnotai, T. Roughgarden, and Q. Yan. 2015. Revenue Maximization With a Single Sample. *Games and Economic Behavior* 91 (2015), 318–333.

J. Hartline and T. Roughgarden. 2009. Simple Versus Optimal Mechanisms. In *Proceedings of the 10th ACM conference on Electronic commerce*. 225–234.

Jason D Hartline. 2013. Mechanism Design and Approximation. *Book draft. October* 122 (2013).

Y. Jin, P. Lu, Q. Qi, Z. G. Tang, and T. Xiao. 2018. Tight Approximation Ratio of Anonymous Pricing. *arXiv preprint arXiv:1811.00763* (2018).

D. Krähmer and R. Strausz. 2015. Optimal Sales Contracts with Withdrawal Rights. *Review of Economic Studies* 82 (2015), 762–790.

D. Malueg and C. Snyder. 2006. Bounding The Relative Profitability of Price Discrimination. *International Journal of Industrial Organization* 24, 5 (2006), 995–1011.

R. Myerson. 1981. Optimal Auction Design. *Mathematics of Operations Research* 6 (1981), 58–73.

C. Papadimitriou, G. Pierrakos, C. Psomas, and A. Rubinstein. 2016. On The Complexity of Dynamic Mechanism Design. In *Proceedings of the twenty-seventh annual ACM-SIAM symposium on Discrete algorithms*. 1458–1475.

A. Pigou. 1920. *The Economics of Welfare.* Macmillan, London.

J. Robinson. 1933. *The Economics of Imperfect Competition.* Macmillan, London.

R. Schmalensee. 1981. Output and Welfare Implications of Monopolistic Third-Degree Price Discrimination. *American Economic Review* 71 (1981), 242–247.